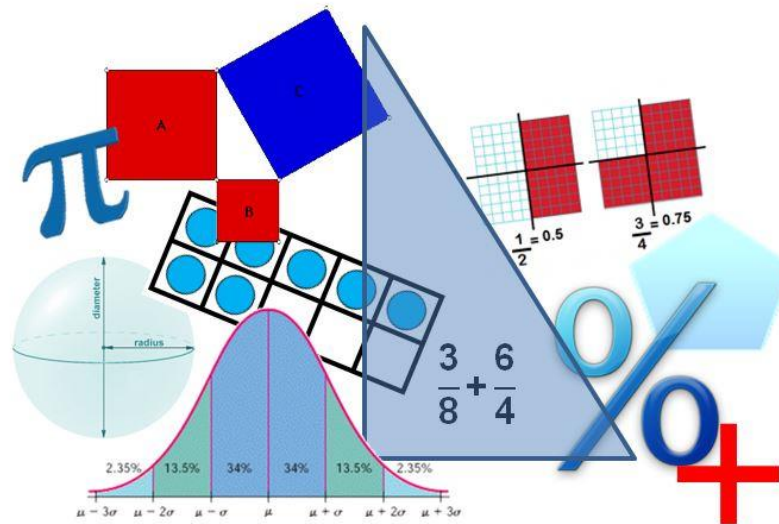


# Mathematics

## 2016 Standards of Learning

### Probability and Statistics Curriculum Framework



Board of Education  
Commonwealth of Virginia

Copyright © 2016  
by the  
Virginia Department of Education  
P.O. Box 2120  
Richmond, Virginia 23218-2120  
<http://www.doe.virginia.gov>

All rights reserved. Reproduction of these materials for instructional purposes in public school classrooms in Virginia is permitted.

**Superintendent of Public Instruction**  
Steven R. Staples

**Chief Academic Officer/Assistant Superintendent for Instruction**  
Steven M. Constantino

**Office of Mathematics and Governor's Schools**  
Debra Delozier, Mathematics Specialist  
Tina Mazzacane, Mathematics and Science Specialist  
Christa Southall, Mathematics Specialist

**Acknowledgements**

The Virginia Department of Education wishes to express sincere thanks to Michael Bolling, who assisted in the development of the 2016 *Mathematics Standards of Learning* and 2016 *Mathematics Standards of Learning Curriculum Framework*.

**NOTICE**

The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

# Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*

## Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

### *Understanding the Standard*

This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

### *Essential Knowledge and Skills*

This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

## **Mathematical Process Goals for Students**

---

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

### **Mathematical Problem Solving**

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

### **Mathematical Communication**

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

### **Mathematical Reasoning**

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

### **Mathematical Connections**

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

### **Mathematical Representations**

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

## **Instructional Technology**

---

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “... the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (\*).

## **Computational Fluency**

---

Mathematics instruction must develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

## **Algebra Readiness**

---

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

## Equity

---

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

**PS.1†** The student will analyze graphical displays of univariate data, including dotplots, stemplots, boxplots, cumulative frequency graphs, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>• Data are collected for a purpose and have meaning in a context.</li> <li>• Measures of central tendency describe how the data cluster or group.</li> <li>• Measures of dispersion describe how the data spread (disperse) around the center of the data.</li> <li>• Graphical displays of data may be analyzed informally.</li> <li>• Data analysis must take place within the context of the problem.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Create and interpret graphical displays of data, including dotplots, stemplots, boxplots, cumulative frequency graphs, and histograms, using appropriate technology.</li> <li>• Examine graphs of data for clusters and gaps, and relate those phenomena to the data in context.</li> <li>• Examine graphs of data for outliers, and explain the outlier(s) within the context of the data.</li> <li>• Examine graphs of data and identify the central tendency of the data as well as the spread.</li> <li>• Explain the central tendency and the spread of the data within the context of the data.</li> </ul>

† Standard should be included in a one-semester course in Probability and Statistics.

**PS.2<sup>†</sup>** The student will analyze numerical characteristics of univariate data sets to describe patterns and departures from patterns, using mean, median, mode, variance, standard deviation, interquartile range, range, and outliers.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>• Data are collected for a purpose and have meaning within a context.</li> <li>• Analysis of the descriptive statistical information generated by a univariate data set should include the interplay between central tendency and dispersion as well as among specific measures.</li> <li>• Data points identified algorithmically as outliers should not be excluded from the data unless sufficient evidence exists to show them to be in error.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Interpret mean, median, mode, range, interquartile range, variance, and standard deviation of a univariate data set in terms of the problem’s context.</li> <li>• Identify possible outliers, using an algorithm.</li> <li>• Explain the influence of outliers on a univariate data set.</li> <li>• Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.</li> </ul>

<sup>†</sup> Standard should be included in a one-semester course in Probability and Statistics.



**PS.3<sup>†</sup>** The student will compare distributions of two or more univariate data sets, numerically and graphically, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>• Data are collected for a purpose and have meaning in a context.</li> <li>• Statistical tendency refers to typical cases but not necessarily to individual cases.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Compare and contrast two or more univariate data sets, numerically and graphically, by analyzing measures of center and spread within a contextual framework.</li> <li>• Describe any unusual features of the data, such as clusters, gaps, or outliers, within the context of the data.</li> <li>• Analyze skewness in conjunction with measures of center and spread in a contextual framework.</li> </ul>

<sup>†</sup> Standard should be included in a one-semester course in Probability and Statistics.

**PS.4<sup>†</sup>** The student will analyze scatterplots to identify and describe the relationship between two variables, using shape; strength of relationship; clusters; positive, negative, or no association; outliers; and influential points.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>• Data are collected for a purpose and have meaning in a context.</li> <li>• A scatterplot serves two purposes:                             <ul style="list-style-type: none"> <li>– to determine whether there is a useful relationship between two variables, and</li> <li>– to determine the family of equations that describes the relationship.</li> </ul> </li> <li>• Association between two variables considers both the direction and strength of the association.</li> <li>• The strength of an association between two variables reflects how accurately the value of one variable can be predicted based on the value of the other variable.</li> <li>• Outliers are observations with large residuals and do not follow the pattern apparent in the other data points.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Examine scatterplots of data, and describe skewness, and correlation within the context of the data.</li> <li>• Describe and explain any unusual features of the data, such as clusters, gaps, or outliers, within the context of the data.</li> <li>• Identify influential data points (observations that have a great effect on a line of best fit because of extreme x-values) and describe the effect of the influential points.</li> </ul>

<sup>†</sup> Standard should be included in a one-semester course in Probability and Statistics.

**PS.5 The student will determine and interpret linear correlation, use the method of least squares regression to model the linear relationship between two variables, and use the residual plot to assess linearity.**

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>• Least squares regression generates the equation of the line that minimizes the sum of the squared distances from the data points to the line.</li> <li>• Each data point may be considered to be comprised of two parts: fit (the part explained by the model) and residual (the result of chance variation or of variables not measured).</li> <li>• Residual = Actual – Fitted</li> <li>• A correlation coefficient measures the degree of association between two variables that are related linearly.</li> <li>• Two variables may be strongly associated without a cause-and-effect relationship existing between them.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Calculate a correlation coefficient, <math>r</math>.</li> <li>• Explain how the correlation coefficient, <math>r</math>, measures association by looking at its formula.</li> <li>• Interpret the coefficient of determination, <math>r^2</math>, in a contextual framework.</li> <li>• Use regression lines to make predictions, and identify the limitations of the predictions.</li> <li>• Use residual plots to determine whether a linear model is satisfactory for describing the relationship between two variables.</li> <li>• Describe the errors inherent in extrapolation beyond the range of the data.</li> <li>• Use least squares regression to determine the equation of the line of best fit for a set of data.</li> <li>• Interpret the slope and <math>y</math>-intercept of the least squares regression line in a contextual framework.</li> <li>• Explain how least squares regression generates the equation of the line of best fit by examining the formulas used in computation.</li> </ul>

**PS.6** The student will make logarithmic and power transformations to achieve linearity.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"><li>• A logarithmic transformation reduces positive skewness because it compresses the upper tail of the distribution while stretching the lower tail.</li><li>• Nonlinear transformations do not preserve relative spacing between data points.</li></ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"><li>• Apply a logarithmic transformation to data.</li><li>• Explain how a logarithmic transformation works to achieve a linear relationship between variables.</li><li>• Apply a power transformation to data.</li><li>• Explain how a power transformation works to achieve a linear relationship between variables.</li></ul>

**PS.7<sup>†</sup>** The student, using two-way tables and other graphical displays, will analyze categorical data to describe patterns and departures from patterns and to determine marginal frequency and relative frequencies, including conditional frequencies.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>• Simpson’s paradox refers to the fact that aggregate proportions can reverse the direction of the relationship seen in the individual parts.</li> <li>• Two categorical variables are independent if the conditional frequencies of one variable are the same for every category of the other variable.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Produce a two-way table as a summary of the information obtained from two categorical variables.</li> <li>• Create and interpret graphical displays of categorical data including bar charts.</li> <li>• Calculate marginal, relative, and conditional frequencies in a two-way table.</li> <li>• Use marginal, relative, and conditional frequencies to analyze data in two-way tables within the context of the data.</li> </ul>

<sup>†</sup>Standard should be included in a one-semester course in Probability and Statistics.

**PS.8<sup>†</sup>** The student will describe the methods of data collection in a census, sample survey, experiment, and observational study and identify an appropriate method of solution for a given problem setting.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>• Data are collected for a purpose and have meaning in a context.</li> <li>• The value of a sample statistic varies from sample to sample if the simple random samples are taken repeatedly from the population of interest.</li> <li>• Poor data collection can lead to misleading and meaningless conclusions.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Compare and contrast controlled experiments and observational studies and the conclusions one can draw from each.</li> <li>• Compare and contrast population and sample, and parameter and statistic.</li> <li>• Identify biased sampling methods.</li> <li>• Describe simple random sampling.</li> <li>• Select a data collection method appropriate for a given context.</li> </ul>

<sup>†</sup> Standard should be included in a one-semester course in Probability and Statistics.

**PS.9<sup>†</sup>** The student will plan and conduct a survey. The plan will address sampling techniques and methods to reduce bias.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>• The purpose of sampling is to provide sufficient information so that population characteristics may be inferred.</li> <li>• Inherent bias diminishes as sample size increases.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Distinguish between a population and a sample.</li> <li>• Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling.</li> <li>• Determine which sampling technique is best, given a particular context.</li> <li>• Plan a survey to answer a question or address an issue.</li> <li>• Given a plan for a survey, identify possible sources of bias, and describe ways to reduce bias.</li> <li>• Design a survey instrument.</li> <li>• Conduct a survey.</li> </ul>

<sup>†</sup> Standard should be included in a one-semester course in Probability and Statistics.

**PS.10<sup>†</sup>** The student will plan and conduct a well-designed experiment. The plan will address control, randomization, replication, blinding, and measurement of experimental error.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>Experiments must be well-designed in order to detect a cause-and-effect relationship between variables.</li> <li>Principles of experimental design include comparison with a control group, randomization, and blindness.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>Plan and conduct a well-designed experiment. The experimental design should address control, randomization, replication, blinding and minimization of experimental error.</li> <li>Identify treatments, levels, factors, control groups, and experimental units in an experimental design.</li> <li>Identify sources of bias and confounding, including the placebo effect.</li> <li>Identify a situation when a block design, including matched pairs, would reduce the effects of confounding variables.</li> </ul>

<sup>†</sup> Standard should be included in a one-semester course in Probability and Statistics.



**PS.11<sup>†</sup>** The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>• The complement of event <math>A</math> consists of all outcomes in which event <math>A</math> does not occur.</li> <li>• Two events, <math>A</math> and <math>B</math>, are independent if the occurrence of one does not affect the probability of the occurrence of the other. If <math>A</math> and <math>B</math> are not independent, then they are said to be dependent.</li> <li>• Events <math>A</math> and <math>B</math> are mutually exclusive if they cannot occur simultaneously.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events.</li> <li>• Given two or more events in a problem setting, determine whether the events are complementary, dependent, independent, and/or mutually exclusive.</li> </ul>

<sup>†</sup> Standard should be included in a one-semester course in Probability and Statistics.

**PS.12<sup>†</sup>** The student will determine probabilities (relative frequency and theoretical), including conditional probabilities for events that are either dependent or independent, by applying the Law of Large Numbers concept, the addition rule, and the multiplication rule.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>• Data are collected for a purpose and have meaning in a context.</li> <li>• Venn diagrams may be used to determine conditional probabilities.</li> <li>• The Law of Large Numbers states that as a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Calculate relative frequency and expected frequency.</li> <li>• Determine conditional probabilities for dependent, independent, and mutually exclusive events.</li> </ul>

<sup>†</sup> Standard should be included in a one-semester course in Probability and Statistics.

**PS.13** The student will develop, interpret, and apply the binomial and geometric probability distributions for discrete random variables, including computing the mean and standard deviation for the binomial and geometric variables.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>• A probability distribution is a complete listing of all possible outcomes of an experiment together with their probabilities.</li> <li>• In a binomial distribution, the procedure has a fixed number of independent trials.</li> <li>• In a geometric distribution, the procedure is repeated until the first success.</li> <li>• A random variable assumes different values depending on the event outcome.</li> <li>• A probability distribution combines descriptive statistical techniques and probabilities to form a theoretical model of behavior.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Develop the binomial and geometric probability distributions within a practical context.</li> <li>• Calculate the mean and standard deviation for the binomial and geometric variables.</li> <li>• Use the binomial and geometric distributions to calculate probabilities associated with experiments for which there are only two possible outcomes.</li> </ul>

**PS.14** The student will simulate probability distributions, including binomial and geometric.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"><li>• A probability distribution combines descriptive methods and probabilities to form a theoretical model of behavior.</li><li>• A probability distribution gives the probability for each value of the random variable.</li></ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"><li>• Design and conduct a simulation of a binomial distribution.</li><li>• Design and conduct a simulation of a geometric distribution.</li><li>• Calculate probabilities resulting from simulations of binomial and geometric distributions.</li></ul>

**PS.15** The student will identify random variables as independent or dependent and determine the mean and standard deviations for random variables and sums and differences of independent random variables.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>A random variable is a variable that has a single numerical value, determined by chance, for each outcome of a procedure.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>Compare and contrast independent and dependent random variables.</li> <li>Determine the mean (expected value) and standard deviation for a random variable and linear transformation of a random variable.</li> <li>Determine the mean (expected value) for sums and differences of random variables.</li> <li>Determine the standard deviation for sums and differences of independent random variables.</li> </ul>

**PS.16<sup>†</sup>** The student will identify properties of a normal distribution and apply the normal distribution to determine probabilities.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>• The normal distribution curve is a family of symmetrical curves defined by the mean and the standard deviation.</li> <li>• Areas under the curve represent probabilities associated with continuous distributions.</li> <li>• The normal curve is a probability distribution and the total area under the curve is 1.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Identify the properties of a normal distribution.</li> <li>• Describe how the standard deviation and the mean affect the graph of the normal distribution.</li> <li>• Calculate and interpret the z-score of a given data value from a normal distribution.</li> <li>• Determine the probability of a given event, using the normal distribution.</li> <li>• Use a graphing utility and a table of Standard Normal Probabilities to determine probabilities.</li> </ul>

<sup>†</sup>Standard should be included in a one-semester course in Probability and Statistics.

**PS.17** The student, given data from a large sample, will determine and interpret appropriate point estimates and confidence intervals for parameters. The parameters will include proportion and mean, difference between two proportions, difference between two means (independent and paired), and slope of a least-squares regression line.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>• A primary goal of sampling is to estimate the value of a parameter based on a statistic.</li> <li>• Confidence intervals use the sample statistic to construct an interval of values that one can be reasonably certain contains the true (unknown) parameter.</li> <li>• Confidence intervals and tests of significance are complementary procedures.</li> <li>• Paired comparisons experimental design allows control for possible effects of extraneous variables.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Construct confidence intervals to estimate a population parameter, such as a proportion or the difference between two proportions; a mean or the difference between two means; or slope of a least-squares regression line.</li> <li>• Select a value for the confidence level of a confidence interval.</li> <li>• Interpret confidence intervals and confidence levels in the context of the data.</li> <li>• Explain the importance of random sampling for confidence intervals.</li> <li>• Explain how changes in confidence level and sample size effect width of the confidence interval and margin of error.</li> <li>• Calculate point estimates for parameters and discuss the limitations of point estimates.</li> </ul>

**PS.18** The student will apply and interpret the logic of an appropriate hypothesis-testing procedure. Tests will include large sample test for proportion, mean, difference between two proportions, difference between two means (independent and paired); chi-squared tests for goodness of fit, homogeneity of proportions, and independence; and slope of a least-squares regression line.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>• Confidence intervals and tests of significance are complementary procedures.</li> <li>• Paired comparisons experimental design allows control for possible effects of extraneous variables.</li> <li>• Tests of significance assess the extent to which sample data support a hypothesis about a population parameter.</li> <li>• The purpose of a goodness of fit test is to decide whether the sample results are consistent with results that would have been obtained if a random sample had been selected from a population with a known distribution.</li> <li>• Practical significance and statistical significance are not necessarily congruent.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Use the chi-squared test for goodness of fit to decide whether the population being analyzed fits a particular distribution pattern.</li> <li>• Use hypothesis-testing procedures to determine whether or not to reject the null hypothesis. The null hypothesis may address proportion, mean, difference between two proportions or two means, goodness of fit, homogeneity of proportions, independence, and the slope of a least-squares regression line.</li> <li>• Compare and contrast Type I and Type II errors.</li> <li>• Explain how and why the hypothesis-testing procedure allows one to reach a statistical decision.</li> </ul>



**PS.19** The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> <li>• The Central Limit Theorem states:                             <ul style="list-style-type: none"> <li>– The mean of the sampling distribution of means is equal to the population mean.</li> <li>– If the sample size is sufficiently large, the sampling distribution approximates the normal probability distribution.</li> <li>– If the population is normally distributed, the sampling distribution is normal regardless of sample size.</li> </ul> </li> <li>• Sampling distributions have less variability with larger sample sizes.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Describe the use of the Central Limit Theorem for drawing inferences about a population parameter based on a sample statistic.</li> <li>• Describe the effect of sample size on the sampling distribution and on related probabilities.</li> <li>• Use the normal approximation to calculate probabilities of sample statistics falling within a given interval.</li> <li>• Identify and describe the characteristics of a sampling distribution of a sample proportion, mean, difference between two sample proportions, or difference between two sample means.</li> </ul>

**PS.20** The student will identify properties of a  $t$ -distribution and apply  $t$ -distributions to single-sample and two-sample (independent and matched pairs)  $t$ -procedures.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"><li>• Paired comparisons experimental design allows control for possible effects of extraneous variables.</li><li>• The sampling distribution of means with a small sample size follows a <math>t</math>-distribution.</li></ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"><li>• Identify the properties of a <math>t</math>-distribution.</li><li>• Compare and contrast a <math>t</math>-distribution and a normal distribution.</li><li>• Use a <math>t</math>-test for single-sample and two-sample data.</li></ul>