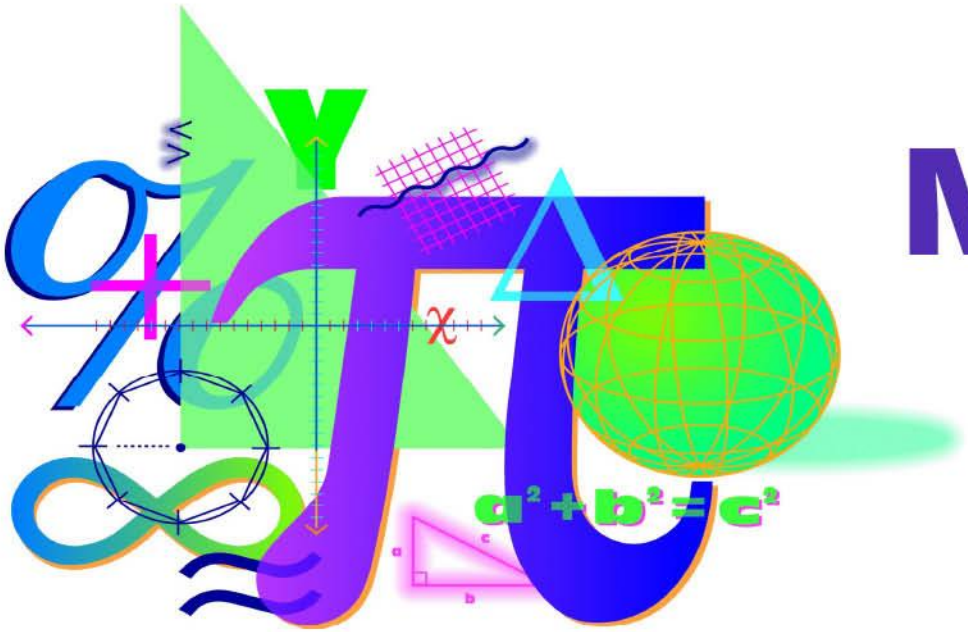
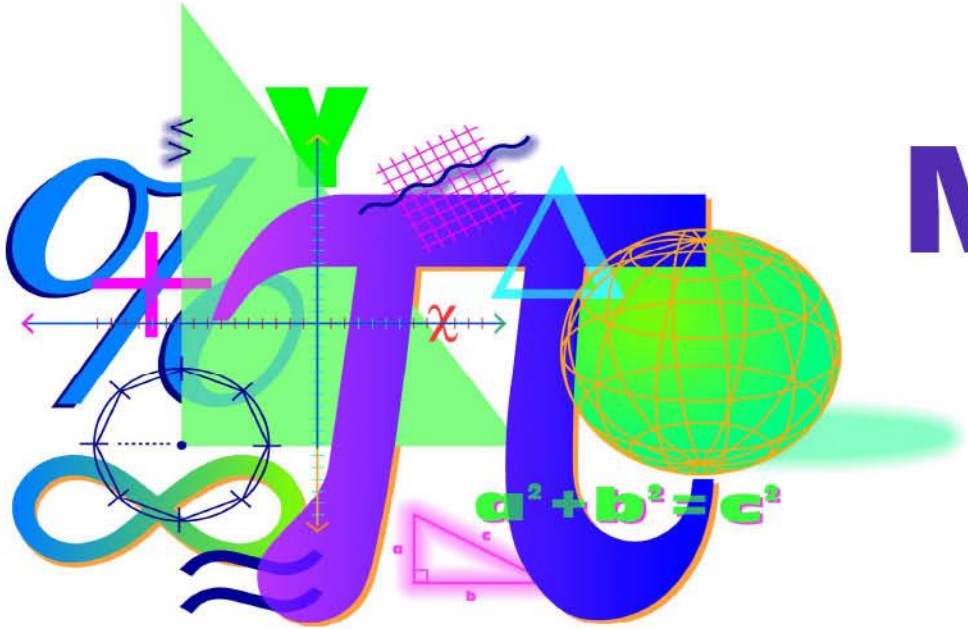


Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Proposed Draft for First Review

Board of Education
Commonwealth of Virginia



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Kindergarten

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> "Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

Students in grades K–~~3~~ 2 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, fractions, number relationships ("more than," "less than," and "equal to"), and the effects of single-step and multistep computations. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

~~K.1 The student, given two sets, each containing 10 or fewer concrete objects, will identify and describe one set as having more, fewer, or the same number of members as the other set, using the concept of one-to-one correspondence.~~[Moved to K.2]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~A set is a collection of distinct elements or items.~~ • ~~A one-to-one correspondence exists when two sets have an equal number of items.~~ • ~~Strategies for developing the concept of one-to-one matching involve set comparisons without counting. Hands-on experiences in matching items between two sets by moving, touching, and aligning objects, using one-to-one correspondence, enable visual as well as kinesthetic comparisons of the number of items in the two sets.~~ • ~~Students can also count to make comparisons between two sets without matching the sets, using one-to-one correspondence.~~ • ~~Students are generally familiar with the concept of *more*, but have had little experience with the term *less*. It is important to use the terms together to build an understanding of their relationship. For example, when asking which group has more, follow with which group has less and vice versa.~~	~~**All students should**~~ • ~~Understand how quantities relate to each other, which leads to an understanding of how numbers are related to each other.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Match each member of one set with each member of another set, using the concept of one-to-one correspondence to compare the number of members between sets, where each set contains 10 or fewer objects.~~ • ~~Compare and describe two sets of 10 or fewer objects, using the terms *more*, *fewer*, and *the same*.~~ • ~~Given a set of objects, construct a second set which has more, fewer or the same number of objects.~~

K.~~2~~1 The student~~, given a set containing 15 or fewer concrete objects,~~ will

a) tell how many are in ~~the~~ a given set of 20 or fewer objects by counting ~~the number of objects~~ orally; and

b) read, write, and represent numbers from 0 through 20. ~~the numeral to tell how many are in the set; and~~

~~c) select the corresponding numeral from a given set of numerals.~~[Moved to EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Counting involves two separate skills: verbalizing the list of standard number words in order ("one, two, three, …") and connecting this sequence with the objects in the set being counted, using one-to-one correspondence. Association of number words with collections of objects is achieved by moving, touching, or pointing to objects as the number words are spoken. Objects may be presented in random order or arranged for easy counting. • When counting objects, students should: – Say the number names in standard order; – Count one item for each number word (one-to-one correspondence); – Understand that the number of objects is the same regardless of their arrangement or the order in which they were counted (conservation of number); – Understand that the last number names the total amount of objects counted (cardinality). By asking the question, "How many are there?" after the student has counted a collection of objects, teachers can determine if students have cardinality of number; and – Understand that each successive number name refers to a quantity that is one larger. • Kinesthetic involvement (e.g., tracing the ~~numbers~~numerals, using tactile materials, such as sand, sandpaper, carpeting, or finger paint) facilitates the writing of numerals. ~~• Articulating the characteristics of each numeral when writing numbers has been found to reduce the amount of time it takes to learn to write numerals.~~	~~All students should~~ ~~• Read and write numerals from 0 through 15.~~[Moved to EKS] ~~• Understand that the total number of objects can be found by counting.~~ ~~• Understand that the last counted number describes the total amount in the set.~~ ~~• Understand that if the set is empty, it has 0 elements.~~[Moved to US] ~~• Understand that changing the spatial arrangement of a set of objects does not change the total amount of the set.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count orally to tell how many are in a given ~~the number of objects in a~~ set containing 20~~15~~ or fewer concrete objects, using one-to-one correspondence, and identify the corresponding numeral. (a) • Read, write, and represent numbers from 0-20 to include: – Construct a set of objects that corresponds to a given numeral, including an empty set. (b)[Reordered] – Read and write the numerals from 0 through 20.[Reordered] – Identify written numerals from 0 through 20~~15~~ represented in random order. (b) – Identify the numeral that corresponds to the total number of objects in a given ~~Select the numeral from a given set of numerals that corresponds to a~~ set of 20~~15~~ or fewer concrete objects. (b) ~~• Write the numerals from 0 through 15.~~[Reordered] – Write a numeral that corresponds to a set of 20~~15~~ or fewer concrete objects. (b) ~~• Construct a set of objects that corresponds to a given numeral, including an empty set.~~[Reordered]

K.~~2~~1 The student~~, given a set containing 15 or fewer concrete objects,~~ will

a) tell how many are in ~~the~~ a given set of 20 or fewer objects by counting ~~the number of objects~~ orally; and

b) read, write, and represent numbers from 0 through 20. ~~the numeral to tell how many are in the set; and~~

~~c) select the corresponding numeral from a given set of numerals.~~[Moved to EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• If a set is empty, it has zero (0) objects or elements. Zero (0) is both a number and a digit. ~~As a number, it plays a central role in mathematics as the additive identity of the integers, real numbers, and many other algebraic structures. As a digit, zero~~ It is used as a placeholder in our number system~~s~~. • Symbolic reversals in numeral writing are common for this age and should not be mistaken for lack of understanding. ~~•~~ Describing a teen number as a ten and some more, will help students name how many are in a set of 13-19 objects. This also lays a foundation for place value.~~*Conservation of number* and *cardinality principle* are two important milestones in development to attaching meaning to counting.~~ ~~• The cardinality principle refers to the concept that the last counted number describes the total amount of the counted set. It is an extension of one-to-one correspondence.~~ ~~• Conservation of number is the understanding that the number of objects remains the same when they are rearranged spatially.~~		

K.2~~1~~ The student, given no more than three ~~two~~ sets, each set containing 10 or fewer concrete objects, will

a) ~~identify~~compare and describe one set as having more, fewer, or the same number of ~~members~~ objects as the other set(s)~~,~~; and ~~using the concept of one-to-one correspondence.~~

b) compare and order sets from least to greatest and greatest to least.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A set is a collection of objects~~distinct elements or items~~. • ~~A one-to-one correspondence exists when two sets have an equal number of items.~~[Included in K.1] • Sets can be compared by matching, lining up the objects, visually estimating magnitude, recognizing quantities without counting (subitizing), or counting the number of objects in each set. • ~~Strategies for developing the concept of one-to-one matching involve set comparisons without counting. Hands-on experiences in matching items between two sets by moving, touching, and aligning objects, using one-to-one correspondence, enable visual as well as kinesthetic comparisons of the number of items in the two sets.~~ • ~~Students can also count to make comparisons between two sets without matching the sets, using one-to-one correspondence.~~ • Comparing sets is an extension of conservation of number (e.g., 5 is 5 whether it is 5 marbles or 5 basketballs even though 5 basketballs take up more space). When comparing objects, the set can be arranged differently while still containing the same number (e.g., 5 marbles in a cup is the same as 5 marbles on the floor). • Comparing objects is an extension of cardinality. Cardinality is knowing how many are in a set by recognizing that the last counting word tells the total number in a set.	~~**All students should**~~ • ~~Understand how quantities relate to each other, which leads to an understanding of how numbers are related to each other.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Match each member of one set with each member of another set, using the concept of one-to-one correspondence to compare the number of members between sets, where each set contains 10 or fewer objects.~~[Included in US] • Compare and describe ~~two~~ no more than three sets of 10 or fewer objects, using the terms *more*, *fewer*, and *the same*. (a) • Given a set of objects, construct a second set which has more, fewer or the same number of objects. (a) • Compare and order three of fewer sets, each set containing 10 or fewer concrete objects, from least to greatest and greatest to least. (b)

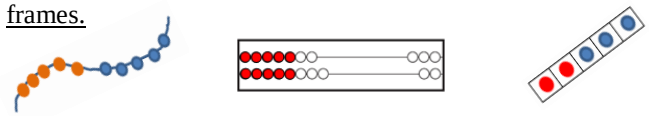
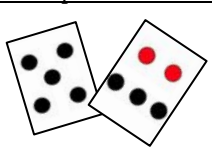
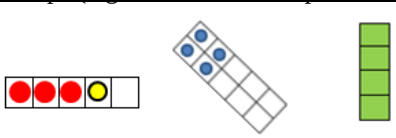
K.2~~1~~ The student, given no more than three ~~two~~ sets, each set containing 10 or fewer concrete objects, will

a) ~~identify~~compare and describe one set as having more, fewer, or the same number of ~~members~~ objects as the other set(s)~~,~~; and ~~using the concept of one-to-one correspondence.~~

b) compare and order sets from least to greatest and greatest to least.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• After having a student count a collection of objects, the teacher may be able to assess whether the student has cardinality of number by asking the question, "How many are there?" Students who do not yet have cardinality of number are often unable to tell you how many objects there were without recounting them. • Students are generally familiar with the concept of *more*, but may have had little experience with the term *less*. It is important to use the terms together to build an understanding of their relationship. For example, when asking which group has more, follow with which group has less ~~and vice versa~~.		

K.3 **The student will~~, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth, and the ordered position of each object.~~[Ordinal positions moved to 1.3]**
a) recognize and describe with fluency part-whole relationships for numbers up to 5; and
b) investigate and describe part-whole relationships for numbers up to 10.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently. • Flexibility requires knowledge of more than one approach to solving a particular kind of problem. Being flexible allows students to choose an appropriate strategy for the numbers involved. • Composing and decomposing numbers flexibly forms a basis for understanding properties of the operations and later formal algebraic concepts and procedures. • Parts of 5 and 10 should be represented in a variety of ways, such as five frames, ten frames, strings of beads, arrangements of tiles or tooth picks, dot cards, beaded strings or number frames.  • Dot patterns should be presented in both regular and irregular arrangements.  • Numbers can be composed and decomposed using part-part-whole relationships (e.g., 4 can be decomposed as 3 and 1, 2 and 2, 4 and 0). 	~~**All students should**~~ ~~Use ordinal numbers to describe the position of objects in a sequence.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize and describe with fluency part-whole relationships for numbers up to 5 in a variety of configurations. (a) • Investigate and describe part-whole relationships for numbers up to 10 using a variety of configurations. (b) ~~• Identify the ordinal positions first through tenth using ordered sets of ten concrete objects and/or pictures of such sets presented from~~ ~~- left-to-right;~~ ~~- right-to-left;~~ ~~- top-to-bottom; and/or~~ ~~bottom-to-to~~[Ordinal positions moved to 1.3]

K.3 **The student will~~, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth, and the ordered position of each object.~~[Ordinal positions moved to 1.3]**
a) recognize and describe with fluency part-whole relationships for numbers up to 5; and
b) investigate and describe part-whole relationships for numbers up to 10.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Quickly recognizing and naming the number of objects in a small group without counting is called subitizing. The size of the group a student can subitize is dependent upon the arrangement of the dots or objects. At this age, students should subitize regular arrangements up to 5. • When students are able to combine or separate groups to create a number, they are building a foundation for addition and subtraction. • Decomposition of 5 and 10 develops benchmarks of 5 and 10 that are essential in building place value understanding. • Flexibility requires knowledge of more than one approach to solving a particular kind of problem. Being flexible allows students to choose an appropriate strategy for the numbers involved. • Composing and decomposing numbers flexibly forms a basis for understanding properties of the operations and later formal algebraic concepts and procedures. • Accuracy is the ability to determine a correct answer using knowledge of number facts and other important number relationships. • Efficiency is the ability to carry out a strategy easily when solving a problem without getting bogged down in too many steps or lose trace of the logic of the strategy being used. • Mathematically fluent students are not only able to provide correct answers quickly but also to use facts and computation strategies they know to efficiently determine answers that they do not know. ~~• Understanding the cardinal and ordinal meanings of numbers are necessary to quantify, measure, and identify the order of objects.~~		

K.3 **The student will~~, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth, and the ordered position of each object.~~[Ordinal positions moved to 1.3]**
a) recognize and describe with fluency part-whole relationships for numbers up to 5; and
b) investigate and describe part-whole relationships for numbers up to 10.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~An ordinal number is a number that names the place or position of an object in a sequence or set (e.g., first, third). *Ordered position*, *ordinal position*, and *ordinality* are terms that refer to the place or position of an object in a sequence or set.~~ • ~~The ordinal position is determined by where one starts in an ordered set of objects or sequence of objects.~~ • ~~The ordinal meaning of numbers is developed by identifying and verbalizing the place or position of objects in a set or sequence (e.g., the student's position in line when students are lined up alphabetically by first name).~~ [Moved to Grade 1]		

K.4 The student will

a) count forward to 100 starting at any number between 0 and 100;

b) count ~~and~~ backward by ones when given any number between 1 and 10~~from 10~~; and

c) identify ~~one more than a number and one less than a number; and~~the number after and the number before when given any number between 0 and100~~count by fives and tens to 100~~.[Included in 1.2]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Counting skills are essential components of the development of number ideas; however, they are only one of the indicators of the understanding of numbers. • Counting forward by rote, supported by visuals such as the hundred chart or number path, advances the child's development of sequencing. • The natural numbers are 1, 2, 3, 4…. The whole numbers are 0, 1, 2, 3, 4…. Students should count the whole numbers 0, 1, 2, 3, 4…. • A number path is a counting model where each number is represented within a square and the squares can be clearly counted. Example of a Number Path 1 2 3 4 5 6 7 8 9 10 • A number line is a length model where each number represents its length from zero. When young children use a number line as a counting tool in solving problems, they often confuse what should be counted-- the numbers or the spaces between the numbers. A number path is more appropriate for students at this age. • Counting backward by rote lays the foundation for subtraction. Students should count backward beginning with 10, 9, 8,… through …3, 2, 1, 0. • Counting forward and backward leads to the development of counting on and counting back. • Connecting rote counting to the counting of collections is necessary for students to understand the meaning of a number.	~~All students should~~ • ~~Use the correct oral counting sequence in both forward and backward counting situations.~~ • ~~Understand that skip counting can be used to count a collection of objects.~~ • ~~Describe patterns in skip counting and use those patterns to predict the next number or numbers in the skip counting sequence.~~ • ~~Understand that numeric relationships include one more than, one less than, two more than, two less than, etc.~~ • ~~Understand benchmarks of five and ten.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count forward from 0 to 100, starting at any number between 0 and 100. (a) • Count backward ~~from 10 to 0~~when given any number between 0 and 10. (b) • Identify the number after and the number before when given any number between 0 and 100. (c) • ~~Recognize the relationship of one more than and one less than a number using objects (i.e., five and one more is six; and one less than ten is nine).~~ • ~~Group 100 or fewer objects together into sets of fives or tens and then count them by fives or by tens.~~ • ~~Investigate and recognize the pattern of counting by fives to 100, using a variety of tools.~~ [Included in 1.2] • ~~Investigate and recognize the pattern of counting by tens to 100, using a variety of tools.~~

K.4 The student will

a) count forward to 100 starting at any number between 0 and 100;

b) count ~~and~~ backward by ones when given any number between 1 and 10~~from 10~~; and

c) identify ~~one more than a number and one less than a number; and~~the number after and the number before when given any number between 0 and100~~count by fives and tens to 100~~.[Included in 1.2]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Identifying the number after and/or the number before any given numbers demonstrates an understanding of number relationships as opposed to a memorized sequence of numbers. • Providing experiences in counting beyond 100 will help students as children often struggle with going over the century mark. • ~~The patterns developed as a result of skip counting are precursors for recognizing numeric patterns, functional relationships, and concepts underlying money, time telling, and multiplication. Powerful models for developing these concepts include, but are not limited to, counters, hundred chart, and calculators.~~ • ~~Skip counting by fives lays the foundation for reading a clock effectively and telling time to the nearest five minutes, counting money, and developing the multiplication facts for five.~~ • ~~Skip counting by tens is a precursor for use of place value, addition, counting money, and multiplying by multiples of 10.~~ • ~~Calculators can be used to display the numeric patterns that result from skip counting.~~		

K.5 **The student will ~~identify the parts of a set and/or region that represent fractions for halves and fourths.~~ investigate fractions by solving practical problems involving equal sharing with two or four sharers.**

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Practical situations with fractions should involve real life problems in which students themselves are determining how to subdivide a whole into equal parts, test those parts to be sure they are equal, and use those parts to recreate the whole. • The focus of this grade level is to develop the idea of equal sharing and part-whole relationships. Fraction notation will be introduced in second grade. • An equal sharing problem is an idea that young children understand intuitively because of their experiences sharing objects with siblings, friends, etc. Consider the following examples: – Two children sharing six sandwiches – Two children sharing one sandwich – Four children sharing one piece of paper – Four children sharing two sticks of clay – Two children sharing three brownies – Four children sharing five brownies • Teachers should use vocabulary such as halves and fourths but students should not be expected to use fraction vocabulary or notation at this age. Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions furthers this development. • Fraction models at this level should be able to be continuously divided (e.g., cookies, brownies). It is important to use models that can be continuously divided when there are remainders so those remainders can be cut into as many equal parts as needed. ~~• A fraction is a way of representing part of a whole (as in a region/area model) or part of a group (as in a set model).~~ • In each fraction model, the fractional parts must ~~have the same area~~be equal shares of a whole.~~-~~[Moved from middle column.]	~~All students should~~ ~~• Understand that fractional parts are equal shares of a whole region or a whole set.~~ [Moved to US] ~~• Understand that the fraction name (*half, fourth*) tells the number of equal parts in the whole.~~ [Moved to US] ~~• Understand that the fraction name (*half, fourth*) of the set model is a subset of the whole set with equal numbers.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given a practical situation, share a whole equally with two or four sharers. • Given a practical situation, represent fair shares concretely or pictorially. • Given a practical situation, describe shares as equal pieces or parts of the whole. ~~• Recognize fractions as representing parts of equal size of a whole.~~ ~~• Given a region, identify half and/or a fourth of the region.~~ ~~• Given a set, identify half and/or a fourth of the set.~~

K.5 **The student will ~~identify the parts of a set and/or region that represent fractions for halves and fourths.~~ investigate fractions by solving practical problems involving equal sharing with two or four sharers.**

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Equal parts may be different shapes but maintain the same value (e.g., a sandwich could be cut in two equal pieces vertically, horizontally, or diagonally to represent halves). • The fraction name (*half* or *fourth*) tells the number of equal parts in the whole.[Moved from US] ~~The fractional parts of a set model are subsets of an equal number. For example, in a set of ten cubes, each half would be a subset of five cubes.~~ • ~~Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions furthers this development (e.g., fourths means "four equal parts of a whole" or $\frac{1}{4}$ represents one of four parts of equal size when a pizza is shared among four students).~~		

A variety of contexts and problem types are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from real-life experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as "How many books do we have altogether?" or "How many cookies are left if I eat two?" or "I have three more candies than you do." Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling, discussing, and recording a variety of problem situations. This approach helps students transition from the concrete to the representation to the symbolic in order to develop meaning for the operations and how they relate to each other.

In grades K–~~3~~2, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition and~~,~~ subtraction~~, multiplication, division and related facts~~;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as skip counting and solving problems that involve equal groupings of objects ~~and~~as well as problems that involve sharing equally, and initial work ~~; and performing initial operations~~ with fractions.

K.6 The student will model and solve single-step story and picture problems with sums to 10 and differences within 10~~adding and subtracting whole numbers~~, using ~~up to 10~~ concrete objects.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• Whole numbers are 0, 1, 2, 3, 4, 5, 6, and so on.~~ • Students should experience a variety of problem types related to addition and subtraction. • The problem types most appropriate for students at this level include: Join (result unknown) - Example: Sue had 4 pennies. Josh gave her 2 more. How many pennies does Sue have altogether? Separate (result unknown) – Example: Sue had 8 pennies. She gave 5 pennies to Josh. How many pennies does Sue have now? Part-part-whole (whole unknown) – Example: Josh has 4 red balloons and 3 blue balloons. How many balloons does he have? Part-part-whole (both parts unknown) – Example: Josh has 5 balloons. Some of them are red and some of them are blue. How many balloons can be blue and how many can be red? ~~• Addition is~~Join problems involve the process of combining or joining sets or quantities. Separate problems • ~~Subtraction~~ can be viewed as a taking away or separating process ~~or as comparing to find the difference between two sets~~. Part-part-whole problems involve two quantities that are combined into one whole but no physical action is required. Comparison problems that ask *how many more* or *how many fewer* should be reserved for first and second grade. • Operation symbols (+, -) are introduced in first grade. • Single-step refers to the least number of steps necessary to solve a problem.	~~**All students should**~~ ~~• Understand that addition means putting things together and that subtraction is the inverse of addition and means to separate things out.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model and solve various types of story and picture problems using 10 or fewer concrete objects. (Types of problems should include joining, separating, and part-part-whole scenarios.) ~~• Combine two sets with known quantities in each set, and count the combined set using up to 10 concrete objects, to determine the sum, where the sum is not greater than 10.~~ ~~• Given a set of 10 or fewer concrete objects, remove, take away, or separate part of the set and determine the result.~~

K.6 The student will model and solve single-step story and picture problems with sums to 10 and differences within 10~~adding and subtracting whole numbers~~, using ~~up to 10~~ concrete objects.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Number relationships help students develop strategies for addition and subtraction. These strategies include: – Instant recognition of the amount in a set of objects (subitize) that are arranged in a familiar pattern such as the dots on number cubes; and – One more than, one less than, two more than, two less than.[Reordered] • Counting on from the larger set to determine the sum of the combined sets is a strategy for ~~finding~~ determining a sum. • Counting backward from the larger set to determine the difference between two sets is a strategy for subtraction. ~~• Number relationships, including the following, help students develop strategies for adding and subtracting:~~ ~~– Instant recognition of the amount in a set of objects that are arranged in a familiar pattern such as the dots on number cubes~~ ~~– One more than, one less than, two more than, two less than.~~[Reordered]		

The exploration of measurement and geometry in the primary grades allows students to learn more about the world around them. Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K~~–3~~ 2~~should~~ encounter measurement in ~~many normal situations, from~~ their daily lives, from their use of the calendar and ~~from~~ science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, ~~area, perimeter, volume,~~ and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both techniques and nonstandard and standard tools to ~~find~~ determine measurements and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

[Moved from Geometry strand introduction]

Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on

- observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.

K.7 The student will recognize the attributes of a penny, nickel, dime, and quarter and identify the value of each coin. ~~and will determine the value of a collection of pennies and/or nickels whose total value is 10 cents or less.~~[Value of a collection included in 1.7]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Involvement in varied activities such as physically manipulating coins and making comparisons about their sizes, colors, and values are~~is~~ prerequisites to the skills of coin recognition and valuation. • ~~Counting money helps students gain an awareness of consumer skills and the use of money in everyday life.~~ • ~~A variety of classroom experiences in which students manipulate physical models of money and count forward to determine the value of a collection of coins are important activities to ensure competence with using money.~~ • Students need experiences counting collections of pennies. This can promote one-to-one correspondence, as a penny is worth one cent. • Students need experiences to develop the concept that a nickel has a value of five cents even though it is one object.	~~**All students should**~~ • ~~Develop common referents for identifying pennies, nickels, dimes, and quarters.~~ • ~~Understand the value of a collection of coins whose value is 10 cents or less.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the ~~properties/characteristics~~attributes (e.g., color, relative size) of a penny, nickel, dime, and quarter. • Identify a penny, nickel, dime, and quarter. • Identify ~~that~~ the value of each coin (a penny is worth one cent, a nickel is worth five cents, a dime is worth ten cents, and a quarter is worth twenty-five cents).~~a nickel is the same value as five pennies.~~ • ~~Count a randomly placed collection of pennies and/or nickels (or models of pennies and/or nickels) whose value is 10 cents or less, and determine the value of the collection.~~

~~K.8~~ ~~The student will identify the instruments used to measure length (ruler), weight (scale), time (clock: digital and analog; calendar: day, month, and season), and temperature (thermometer).~~**[Moved each instrument to the standard where the content is first taught (ruler – SOL 2.8; scale – SOL 2.8; clock – SOL 1.9; thermometer – SOL 2.11]**

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Many experiences in measuring physical objects, using nonstandard and standard units of measure, help to develop an intuitive understanding of measurement and will help students connect a tool with its purpose in measuring.~~ • ~~Selecting from among various measuring instruments and determining which can be used to solve various real-life problems are introduced at this level.~~ • ~~A precursor to connecting tools to a type of measurement is an introduction to the concepts of length, weight, time, and temperature.~~	~~All students should~~ • ~~Identify an appropriate measuring tool for a given unit of measure.~~	~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~ • ~~Identify a ruler as an instrument to measure length.~~[Moved to 2.8] • ~~Identify different types of scales as instruments to measure weight.~~[Moved to 2.8] • ~~Identify different types of clocks (analog and digital) as instruments to measure time.~~[Moved to 1.9] • ~~Identify the components of a calendar, including days, months, and seasons.~~[Moved days and months to K.8] • ~~Identify different types of thermometers as instruments used to measure temperature.~~ [Moved to 2.11]

K.8~~9~~ The student will investigate the passage of time by reading and interpreting a calendar.[Moved from 1.11] ~~tell time to the hour, using analog and digital clocks.~~[Time to hour moved to 1.9a]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Practical situations are appropriate to develop a sense of the interval of time between events (e.g., club or team meetings occur every week on Monday: there is a week between meetings). • The calendar is a way to represent units of time (e.g., days, weeks, months, and a year). • Using a calendar develops the concept of a day as a 24-hour period rather than a period of time from sunrise to sunset. [Moved from 1.11] • ~~Many experiences in relating time on the hour to daily routines and school schedules (e.g., catching the bus, lunch time, recess time, and resource time) help students develop personal referents for time.~~ • ~~Making sense of telling time to the nearest hour is reinforced when students recognize the positions of the hands on an analog clock and identify the corresponding time to the hour.~~	~~**All students should**~~ • ~~Apply an appropriate technique, depending on the type of clock, to determine time to the nearest hour.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the months of the year.[Moved from 1.11] • Identify the seven days in a week.[Moved from 1.11] • Determine the days/dates before and after a given day/date (e.g., yesterday, today, tomorrow).[Moved from 1.11] • ~~Tell time on an analog clock to the hour.~~ • ~~Tell time on a digital clock to the hour.~~

K.9~~10~~ The student will compare two objects or events, using direct comparisons ~~or nonstandard units of measure~~, according to one or more of the following attributes: length (shorter, longer), height (taller, shorter), weight (heavier, lighter), temperature (hotter, colder), volume (more, less), and time (longer, shorter). ~~Examples of nonstandard units include foot length, hand span, new pencil, paper clip, and block.~~[Nonstandard units included in 1.10]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Students need to identify the attribute that they are measuring (e.g., length, height, weight, temperature, volume) before they begin to measure.[Reordered] • Multiple hands-on experiences are needed to gain the ability to compare the attributes of objects.[Reordered] • Students develop conservation of measurement when they understand that the attributes do not change when the object is manipulated (e.g., a piece of string that is coiled maintains its length as it is straightened; the volume of water does not change when poured from a pitcher into a fish tank.) • Length is the distance between two points. • Height is the ~~vertical length of a perpendicular to its base~~distance from the bottom or base of something to the top. • Weight is a measure of the heaviness of an object. • Temperature is the degree of hotness or coldness of an object ~~(e.g., a body)~~ or environment. • Volume is the measure of the capacity of a container. • Time is the measure of an event from its beginning to end. Students could compare the difference between the time spent sliding down the slide versus the time spent walking around the school building. • ~~Students need to identify the attribute that they are measuring (e.g., length, height, weight, temperature) before they begin to measure.~~[Reordered] • ~~Multiple hands-on experiences are needed to gain the ability to compare the attributes of objects.~~[Reordered]	~~All students should~~ • ~~Compare and order objects according to their attributes.~~ • ~~Develop an understanding of measuring with nonstandard and standard units of measure.~~ • ~~Recognize attributes (length, height, weight, temperature) that can be measured.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and describe lengths of two objects (as shorter or longer), using direct comparison ~~or nonstandard units of measure (e.g., foot length, hand span, new pencil, paper clip, block)~~. • Compare and describe heights of two objects (as taller or shorter), using direct comparison ~~or nonstandard units of measure (e.g., book, hand span, new pencil, paper clip, block)~~. • Compare and describe weights of two objects (as heavier or lighter), using direct comparison ~~or nonstandard units of measure (e.g., book, cubes, new pencil, paper clip, block)~~. • Compare and describe temperatures of two objects or environment (as hotter or colder), using direct comparison. • Compare and describe volumes of two containers (as more or less), using direct comparison. • Compare and describe the amount of time spent on two events (as longer or shorter), using direct comparison.

[Moved to Measurement and Geometry strand introduction]

~~Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.~~

~~The focus of instruction at this level is on~~

- ~~observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;~~
- ~~sorting objects and ordering them directly by comparing them one to the other;~~
- ~~describing, comparing, contrasting, sorting, and classifying figures; and~~
- ~~exploring symmetry, congruence, and transformation.~~

~~In the primary grades, children begin to develop basic vocabulary related to figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).~~

~~The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.~~

- ~~**Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.~~
- ~~**Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)~~
- ~~**Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.~~

K.10~~11~~ The student will

a) identify~~,~~ and describe~~, and trace~~ plane ~~geometric~~ figures (circle, triangle, square, and rectangle); ~~and~~

b) compare the size (smaller, larger~~, smaller~~) and shape of plane ~~geometric~~ figures (circle, triangle, square, and rectangle)~~.~~; and

c) describe the location of one object relative to another (above, below, next to) and identify representations of plane figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.[Moved from K.12]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• An important part of the geometry strand in grades K through 2 is the naming and describing of figures. Children move from their own vocabulary and begin to incorporate conventional terminology as the teacher uses geometric terms. • Early experiences with comparing, sorting, and subdividing figures assist students in analyzing the characteristics of plane figures. • Attribute blocks, relational attribute blocks, and tangrams, are among the manipulatives that are particularly appropriate for sorting and comparing size. • Students should be given opportunities to construct geometric figures using multiple tools (e.g., clay, straws, and paper and scissors). • Presentation of triangles, rectangles, and squares should be made in a variety of spatial orientations so that students are less likely to develop common misconception that triangles, rectangles, and squares must have one side parallel to the bottom of the page on which they are printed.[Moved from K.12 EKS] • A plane ~~geometric~~ figure is any two-dimensional shape that can be drawn in a plane.~~plane, closed figure. Circles and polygons are examples of plane geometric figures.~~ • A vertex is the point at which two or more lines, line segments, or rays meet to form an angle. The term "vertices" is the plural form of "vertex". ~~• Presentation of triangles, rectangles, and squares should be made in a variety of spatial orientations so that students do not develop the common misconception that triangles, rectangles, and squares must have one side parallel to the bottom of the~~	~~All students should~~ ~~• Use their knowledge of plane figures to help them systematically represent and describe their world.~~ ~~• Identify the characteristics of plane geometric figures (circle, triangle, square, and rectangle).~~ ~~• Compare the size and shape of plane geometric figures by using strategies to sort and/or group and begin to refine the vocabulary used to explain their strategies.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify a circle, triangle, square, and rectangle. (a) • Describe the characteristics of triangles, squares, and rectangles, including number of sides and number of vertices~~angles~~. (a) • Describe a circle using terms such as round and curved. (a) ~~• Trace a circle, triangle, square, and rectangle.~~ [Included in 1.11] • Compare and group plane ~~geometric figures~~figures (circle, triangle, square, and rectangle) according to their relative sizes (larger, smaller). (b) • Compare and group plane ~~geometric~~ figures (circle, triangle, square, and rectangle) according to their shapes. (b) • Distinguish between examples and nonexamples of identified ~~geometric~~ plane figures (circle, triangle, square, and rectangle). (b) • Identify pictorial representations of a circle, triangle, square, and rectangle, regardless of their position and orientation in space. (c)[Moved from K.12] • Describe the location of one object relative to another, using the terms *above*, *below*, and *next to*. (c)[Moved from K.12]

K.10~~11~~ The student will

a) identify~~,~~ and describe~~, and trace~~ plane ~~geometric~~ figures (circle, triangle, square, and rectangle); ~~and~~

b) compare the size (smaller, larger~~, smaller~~) and shape of plane ~~geometric~~ figures (circle, triangle, square, and rectangle)~~.~~; and

c) describe the location of one object relative to another (above, below, next to) and identify representations of plane figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.[Moved from K.12]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~page on which they are printed.~~ ~~• The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.~~ ~~• **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.~~ ~~• **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., "I know it's a rectangle because it looks like a door, and I know that a door is a rectangle.")~~ ~~• **Level 2: Analysis.** Properties are perceived, but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., "I know it's a rectangle because it is closed; it has four sides and four right angles.").~~ • A polygon is a closed plane figure composed of at least three line segments that do not cross.~~geometric figure that~~ ~~– has sides that are line segments;~~ ~~– is simple (its sides do not cross);~~ ~~– is closed; and~~ ~~– is two-dimensional (lies in a plane).~~ • A triangle is a polygon with three ~~angles and three~~ sides.		

K.10~~11~~ The student will

a) identify~~,~~ and describe~~, and trace~~ plane ~~geometric~~ figures (circle, triangle, square, and rectangle); ~~and~~

b) compare the size (smaller, larger~~, smaller~~) and shape of plane ~~geometric~~ figures (circle, triangle, square, and rectangle)~~.~~; and

c) describe the location of one object relative to another (above, below, next to) and identify representations of plane figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.[Moved from K.12]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Children should ~~be shown~~have experiences with different types of triangles (e.g., ~~such as~~ equilateral, isosceles, scalene, right, acute, and obtuse); however, at this level, they are not expected to name the various types. • A quadrilateral is a polygon with four sides. • A rectangle is a quadrilateral with four right angles. • A square is a quadrilateral~~rectangle~~ with ~~all~~ four congruent (equal length) sides and four right angles.~~of equal length.~~ • A circle is the set of~~a closed curve with all~~ points in a~~one~~ plane that are~~and~~ the same distance from a ~~fixed~~ point called (the center~~)~~. • Representations of circles, squares, rectangles, and triangles can be found in the students' environment at school and at home. Students should have opportunities to identify/classify things in their environment by the type of figures those things represent. ~~• Early experiences with comparing and sorting figures assist students in analyzing the characteristics of plane geometric figures.~~ ~~• Attribute blocks, relational attribute blocks, and tangrams are among the manipulatives that are particularly appropriate for sorting and comparing size.~~ • ~~Clay, straws, and paper and scissors are several manipulatives that are appropriate for constructing geometric figures.~~		

~~**K.12 The student will describe the location of one object relative to another (above, below, next to) and identify representations of plane geometric figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.**~~**[Moved to K.10c]**

~~**UNDERSTANDING THE STANDARD**~~ ~~**(Background Information for Instructor Use Only)**~~	~~**ESSENTIAL UNDERSTANDINGS**~~	~~**ESSENTIAL KNOWLEDGE AND SKILLS**~~
• ~~Representations of circles, squares, rectangles, and triangles can be found in the students' environment at school and at home. Students should have opportunities to identify/classify things in their environment by the type of figures those things represent.~~ • ~~Children are often confused when a figure such as a square is rotated: they frequently refer to the rotated square as a diamond. Clarification needs to be ongoing — i.e., a square is a square regardless of its location in space; there is no such geometric figure as a diamond.~~	~~**All students should**~~ • ~~Understand that objects can have different orientations in space.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Identify pictorial representations of a circle, triangle, square, and rectangle, regardless of their position and orientation in space.~~[Moved to K.10c] • ~~Describe the location of one object relative to another, using the terms *above*, *below*, and *next to*.~~[Moved to K.10c]

Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible, unlikely, equally likely, more likely,* and *certain.*

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

K.11~~13~~ The student will ~~gather data by counting and tallying.~~[Tallying included in 1.14a]
a) collect, organize, and represent data; and
b) read and interpret data in object graphs, pictures graphs, and tables.[Moved from K.14]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are pieces of information collected about people or things. The primary purpose of collecting data is to answer questions. The primary purpose of interpreting data is to inform decisions (e.g., which type of clothing to pack for a vacation based on a weather graph or which type of lunch to serve based upon class favorites). • Methods for organizing data could include five or ten frames, surveys, checklists, or various methods of grouping concrete materials. • At this level, data gathered and displayed by students should be limited to 16 or fewer data points for no more than 4 categories. • Students should have opportunities to interpret graphs, created with the assistance of the teacher, that contain data points where their entire class is represented (e.g., tables that show who brought their lunch and who will buy their lunch for any given day, picture graph showing how students traveled to school – bus, car, walk). • A key should be included where appropriate. ~~• Tallying is a method for gathering information. Tally marks are used to show how often something happens or occurs. Each tally mark represents one occurrence. Tally marks are clustered into groups of five, with four vertical marks representing the first four occurrences and the fifth mark crossing the first four on a diagonal to represent the fifth occurrence.~~ • When data are presented in an organized manner, students can interpret and discuss~~describe~~ the results and implications of their investigation (e.g., identifying parts of the data that have special characteristics, including categories with the greatest, the least, or the same number of responses).	~~**All students should**~~ ~~• Pose questions and gather data.~~ ~~• Understand how data are collected and presented in an organized manner by counting and tallying.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Use counting and tallying to gather~~Collect data on categories identified by the teacher and/or student (e.g., ~~favorites,~~ number of siblings, ~~days of various types of weather during a given month,~~ types/numbers of pets, types of ~~shoes~~flowers in the garden). (a) • Represent data by arranging concrete objects into organized groups to form a simple object graph. (a) • Represent gathered data, using pictures to form a simple picture graph (e.g., a picture graph of the weather for a month). (a) • Represent gathered data in tables (vertically or horizontally). (a) • Answer questions related to the gathered data displayed in object graphs, picture graphs, and tables: – Read the graph to determine the categories of data and the data as a whole (e.g., the total number of responses) and its parts (e.g., five people are wearing sneakers); and (b) – Interpret the data that represents numerical relationships, including categories with the greatest, the least, or the same. (b) [Moved from K.14]

K.11~~13~~ The student will ~~gather data by counting and tallying.~~[Tallying included in 1.14a]
a) collect, organize, and represent data; and
b) read and interpret data in object graphs, pictures graphs, and tables.[Moved from K.14]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• In the process of collecting ~~gathering~~ data, students make decisions about what is relevant to their investigation (e.g., when collecting data on their classmates' favorite pets, deciding to limit the categories to common pets). • When students begin to collect data, they recognize the need to categorize, which helps develop the understanding of "things that go together." Categorical data are used when constructing picture graphs and bar graphs. • Different types of representations emphasize different things about the same data.[Moved from K.14 US] • Object graphs are graphs that use concrete materials to represent the categorical data that are collected (e.g., cubes stacked by the month, with one cube representing the birthday month of each student).[Moved from K.14 US] • Picture graphs are graphs that use pictures to represent and compare information.[Moved from K.14 US] At this level, each picture should represent 1 data point. • Tables are an orderly arrangement of data in which the data are arranged in columns and rows in an essentially rectangular format. Tables may be used to display numerical relationships or to organize lists.[Moved from K.14 US] • Students represent data to convey results of their investigations at a glance, using concrete objects, pictures, and numbers to give a "picture" of the organized data.[Moved from K.14 Us] • Graphs can be used to make connections between mathematics and science or social studies (e.g., types of plants found in the school yard, how students get to school).[Moved from K.14 US]		

K.11~~13~~ The student will ~~gather data by counting and tallying.~~[Tallying included in 1.14a]
a) collect, organize, and represent data; and
b) read and interpret data in object graphs, pictures graphs, and tables.[Moved from K.14]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Students should have experiences answering questions related to the analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses).[Moved from K.14 US]		

~~K.14 The student will display gathered data in object graphs, picture graphs, and tables, and will answer questions related to the data.~~[Moved to K.11b]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Object graphs are graphs that use concrete materials to represent the categorical data that are collected (e.g., cubes stacked by the month, with one cube representing the birthday month of each student).~~ • ~~Picture graphs are graphs that use pictures to show and compare information.~~ • ~~Tables are an orderly arrangement of data in which the data are arranged in columns and rows in an essentially rectangular format. Tables may be used to display some type of numerical relationship or organized lists (e.g., input/output functions, tables showing one candy costs five cents and two candies cost 10 cents).~~ • ~~Students represent data to convey results of their investigations at a glance, using concrete objects, pictures, and numbers to give a "picture" of the organized data.~~ • ~~When data are displayed in an organized manner, children can describe the results of their investigations.~~ • ~~Graphs can be used to make connections between mathematics and social studies and/or science (e.g., job areas and the different people that work in these areas: health — doctors and nurses; education — teachers and principals).~~ • ~~Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be asked.~~[Moved to K.11b]	~~**All students should**~~ • ~~Understand that data can be represented using concrete objects, pictures, and graphs.~~ • ~~Understand that different types of representations emphasize different things about the same data.~~ • ~~Understand that picture graphs use pictures to show and compare information; object graphs use concrete materials to represent categorical data; and tables can be used to show an orderly arrangement of data in columns and rows.~~ • ~~Answer questions related to the gathered data from object graphs, picture graphs, and tables.~~ • ~~Relate their ideas about the data to concepts such as part-part-whole and number relationships.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Display data by arranging concrete objects into organized groups to form a simple object graph.~~ • ~~Display gathered data, using pictures to form a simple picture graph (e.g., a picture graph of the types of shoes worn by students on a given day).~~ • ~~Display gathered data in tables, either in rows or columns.~~ • ~~Answer questions related to the gathered data displayed in object graphs, picture graphs, and tables by:~~ ~~– Describing the categories of data and the data as a whole (e.g., the total number of responses) and its parts.~~ ~~– Identifying parts of the data that represent numerical relationships, including categories with the greatest, the least, or the same.~~

Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children's mathematical development.

Students in ~~kindergarten through third~~ grades K-2 develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns. ~~These s~~Students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to algebra.

K.12~~15~~ The student will sort and classify objects according to attributes.

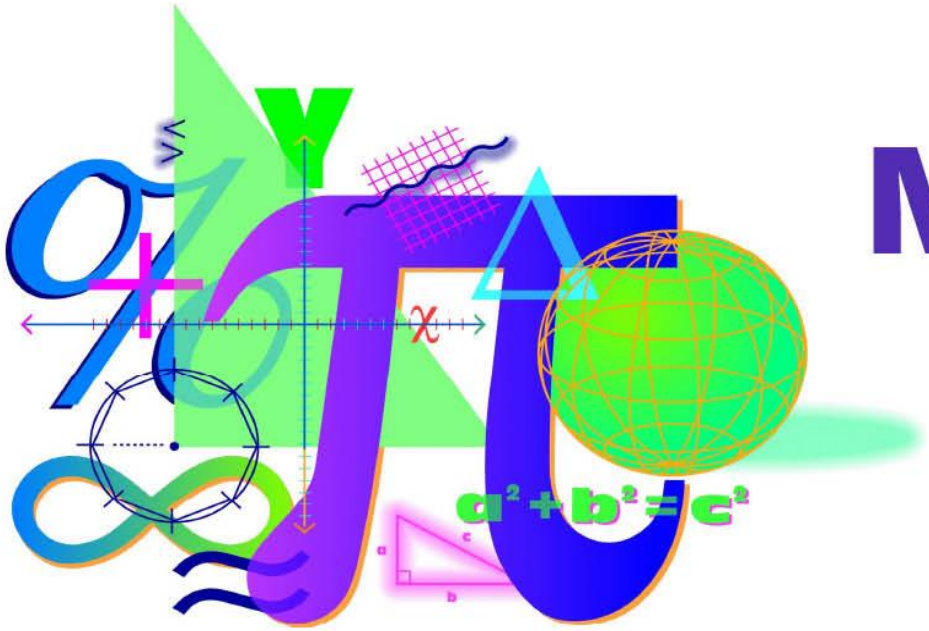
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Objects can be sorted and classified in different ways.[Moved from middle column]. • To classify is to arrange or organize a set of materials according to a category or attribute (a quality or characteristic). ~~An object has many attributes such as color, size, shape, thickness, etc.~~ [Moved to EKS] • General similarities and differences among objects are easily observed by children entering kindergarten, who are able to focus on any one attribute. The teacher's task is to move students toward a more sophisticated understanding of classification in which two or more attributes connect or differentiate sets, such as those found in nature (e.g., leaves having both~~with~~ different colors and different figures).	~~**All students should**~~ • ~~Understand that the same set of objects can be sorted and classified in different ways.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the attributes of an object such as color, size, shape, thickness, etc.[Moved from US] • Sort objects into appropriate groups (categories) based on one attribute. • Classify sets of objects into groups (categories) of one attribute. • Label attributes of a set of objects that has been sorted. • Name multiple ways to sort a set of objects.

K.13~~16~~ The student will identify, describe, ~~and~~ extend, create, and transfer repeating patterns.[Create added to align with EKS bullets]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Patterning ~~recognition~~ is a fundamental cornerstone of mathematics, particularly algebra. The process of generalization leads to the foundation of algebraic reasoning. • Opportunities to create, identify, describe, extend, and transfer repeating patterns are essential to the primary school experience and lay the foundation for thinking algebraically. • Patterning should include – creating a given pattern, using objects, sounds, movements, and pictures; – recording a pattern with pictures or symbols; – transferring a pattern into a different representation (e.g., snap, snap, clap to a blue, blue, red to an AAB repeating pattern); and – analyzing patterns in practical situations (e.g., calendar, seasons, days of the week). • The part of the pattern that repeats is called the core. • At this level students should have experiences extending patterns when given a complete repetition of a core (e.g., ABCABCABC) as well as when the final repetition of the core is incomplete (e.g., ABCABCA... or Red, Blue, Green, Red, Blue, Green, Red, Blue…). ~~• In a repeating pattern, the core of the pattern is the string of elements that repeats. By identifying the core, students demonstrate their understanding of the pattern.~~ ~~• Students should recognize that the sound pattern 'snap, clap, snap, clap' is the same in form as the color pattern 'red, blue, red, blue'.~~ ~~• Pattern recognition and the extension of the pattern allow students to make predictions.~~ ~~• The simplest types of patterns are repeating patterns. The patterns can be oral, such as the refrain in "Old MacDonald's Farm" ("e i e i o"), or physical with clapping and snapping~~	~~**All students should**~~ ~~• Understand that patterns are a way to recognize order and organize their world and to predict what comes next in an arrangement.~~ ~~• Understand that the sound pattern 'snap, clap, snap, clap' is the same in form as the color pattern 'red, blue, red, blue'.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Observe and i~~Identify and describe the ~~basic repeating pattern (~~core~~)~~ found in repeating patterns of common objects, sounds, ~~and~~ movements, and pictures ~~that occur in practical situations~~. ~~• Identify the core in a repeating pattern.~~ • Extend a repeating pattern by adding at least two repetitions to the pattern. • Create a repeating pattern. • Compare similarities and differences between patterns. • Transfer a repeating pattern from one representation to another.

K.13~~16~~ The student will identify, describe, ~~and~~ extend, create, and transfer repeating patterns.[Create added to align with EKS bullets]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~patterns, or combinations of both, such as is found in songs like the "Hokey Pokey." In each case, students need to identify the basic unit of the pattern and repeat it. Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience.~~ • Ex~~S~~amples of repeating patterns: ~~(repeating the core) are~~ – ABABABAB; – ABCABC; – ABBAABBA; – AABBAABBAABB; and – AABAAB.~~;~~ ~~AABCAABC; and~~ ~~ABACABAC.~~		



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Grade 1

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

Students in grades K–~~3~~ 2 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, fractions, number relationships ("more than," "less than," and "equal to"), and the effects of single-step and multistep computations. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

1.1 The student will

a) count forward to 110~~from 0 to 100~~, starting at any number between 0 and 110, and write the corresponding numerals; ~~and~~

b) ~~group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.~~[Moved to 1.2]represent a number from 0-110 using concrete objects;

c) count backward by ones when given any number between 1 and 30;[Moved from 1.2]and

d) count forward by ones, twos, fives, and tens to determine the total number of objects to 110.[Moved from 1.2]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• There are three developmental levels of counting: – rote sequence; – one-to-one correspondence; and – the cardinality of numbers. • Counting involves two separate skills: verbalizing the list of standard number words in order ("one, two, three, …") and connecting this sequence with the items in the set being counted, using one-to-one correspondence. Association of number words with collections of objects is achieved by moving, touching, or pointing to objects as the number words are spoken. • The last number stated represents the number of objects in the set. ~~This is~~ The total number of objects in the set is known as the cardinality of the set. After having a student count a collection of objects, the teacher may be able to assess whether the student has cardinality of number by asking the question, "How many are there?" Students who do not yet have cardinality of number are often unable to tell you how many objects there were without recounting them. • Rote counting is a prerequisite skill for the understanding of addition (1 more), subtraction (1 less), and the ten-to-one concept of place value. ~~• Articulating the characteristics of each numeral when writing numbers has been found to reduce the amount of time it takes to learn to write numerals.~~	~~**All students should**~~ ~~• Associate oral number names with the correct numeral and set of objects.~~ ~~• Understand that 1 and 10 are special units of numbers (e.g., 10 is 10 ones, but it is also 1 ten).~~ ~~• Understand the ten-to-one relationship of ones and tens (10 ones equals 1 ten).~~ ~~• Understand that numbers are written to show how many tens and how many ones are in the number.~~ ~~• Understand that groups of tens and ones can be used to tell how many.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count by rote from 0 to 110.~~100, using the correct name for each numeral.~~ (a) • Use the ~~correct~~ oral counting sequence to tell how many objects are in a set. (a, b, c, d) • Write numerals 0-110 ~~correctly~~. (a, b) ~~• Write each numeral from 0 to 100.~~ ~~• Read two-digit numbers when shown a numeral, a Base-10 model of the number, or a pictorial representation of the number.~~ • Represent a given two-digit number with a model or picture. (b). • Given a model or a pictorial representation, write the corresponding numeral. (a, b, d) • Count backward by ones when given any number between 1 and 30. (c)[Moved from 1.2] • Count forward by ones, twos, fives, and tens to determine the total number of objects to 110. (d)[Moved from 1.2] ~~• Identify the place value (ones, tens) of each digit in a two-digit numeral (e.g., The place value of the 2 in the number 23 is tens. The value of the 2 in the number 23 is 20).~~ ~~• Group a collection of objects into sets of tens and ones. Write the numeral that corresponds to the total number of~~

1.1 The student will

a) count forward to 110~~from 0 to 100~~, starting at any number between 0 and 110, and write the corresponding numerals; ~~and~~

b) ~~group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.~~[Moved to 1.2]represent a number from 0-110 using concrete objects;

c) count backward by ones when given any number between 1 and 30;[Moved from 1.2]and

d) count forward by ones, twos, fives, and tens to determine the total number of objects to 110.[Moved from 1.2]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Using objects and asking the questions, "How many are in each group?, "How many groups are there?" and "How many total do you have?" as students learn to skip count supports students as they solidify their understanding of cardinality and develop multiplicative reasoning. • The patterns developed as a result of skip counting are precursors for recognizing numeric patterns, functional relationships, and concepts underlying money, time telling, and multiplication. Powerful models for developing these concepts include counters, number paths, and hundred charts. Example of a Number Path 1 2 3 4 5 6 7 8 9 10 • A number path is a counting model where each number is represented within a square and the squares can be clearly counted. • A number line is a length model with each number being represented by its length from zero. When young children use a number line as a counting tool in solving problems, they often confuse what should be counted, the numbers or the spaces between the numbers. A number path is more appropriate for students at this age. • Counting backward by rote lays the foundation for subtraction. • Skip counting by twos supports the development of the concept of even numbers and the development of multiplication facts for two.		~~objects in a given collection of objects that have been grouped into sets of tens and ones.~~

1.1 The student will

a) count forward to 110~~from 0 to 100~~, starting at any number between 0 and 110, and write the corresponding numerals; ~~and~~

b) ~~group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.~~[Moved to 1.2]represent a number from 0-110 using concrete objects;

c) count backward by ones when given any number between 1 and 30;[Moved from 1.2]and

d) count forward by ones, twos, fives, and tens to determine the total number of objects to 110.[Moved from 1.2]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Skip counting by fives lays the foundation for telling time to the nearest five minutes, counting money, and developing the multiplication facts for five. • Skip counting by tens is a precursor for place value, addition (10 more), subtraction (10 less), counting money, and multiplying by multiples of 10. • Ten-to-one trading activities with manipulatives on place value mats and base ten blocks are more appropriate in grade 2. ~~• The number system is based on a pattern of tens where each place has ten times the value of the place to its right. This is known as the ten-to-one concept of place value.~~ ~~• Opportunities to experience the relationships among tens and ones through hands-on experiences with manipulatives are essential to developing the ten-to-one place value concept of our number system and to understanding the value of each digit in a two-digit number. Ten-to-one trading activities with manipulatives on place value mats provide excellent experiences for developing the understanding of the places in the Base-10 system.~~ ~~• Models that clearly illustrate the relationships among tens and ones are physically proportional (e.g., the tens piece is ten times larger than the ones piece).~~ ~~• Providing students with opportunities to model two-digit numbers expressed with groups of ones and tens will help students understand the ideas of trading, regrouping, and equality.~~ • ~~Recording the numeral when using physical and pictorial models leads to an understanding that the position of each digit in a~~		

1.1 The student will

a) count forward to 110~~from 0 to 100~~, starting at any number between 0 and 110, and write the corresponding numerals; ~~and~~

b) ~~group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.~~[Moved to 1.2]represent a number from 0-110 using concrete objects;

c) count backward by ones when given any number between 1 and 30;[Moved from 1.2]and

d) count forward by ones, twos, fives, and tens to determine the total number of objects to 110.[Moved from 1.2]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~numeral determines the size of the group it represents.~~[Moved to 1.2]		

1.2 The student, given up to 110 objects, ~~will count forward by ones, twos, fives, and tens to 100 and backward by ones from 30.~~[Moved to 1.1]

a) group a collection into tens and ones and write the corresponding numeral;[Moved from 1.1]

b) compare two numbers between 0 and 110 represented pictorially or with concrete objects, using the words *greater than*, *less than* or *equal to*; and

c) order three or fewer sets from least to greatest and greatest to least.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• The patterns developed as a result of skip counting are precursors for recognizing numeric patterns, functional relationships, and concepts underlying money, time telling, and multiplication. Powerful models for developing these concepts include counters, number line, hundred chart, and calculators.~~ ~~• Skip counting by twos supports the development of the concept of even numbers.~~ ~~• Skip counting by fives lays the foundation for reading a clock effectively and telling time to the nearest five minutes, counting money, and developing the multiplication facts for five.~~ ~~• Skip counting by tens is a precursor for use of place value, addition, counting money, and multiplying by multiples of 10.~~ ~~• Counting backward by rote lays the foundation for subtraction. Students should count backward beginning with 30, 29, 28, … through …3, 2, 1, 0.~~[Moved to 1.1] • Conservation of number is applied when students understand that a group of ten objects is still ten objects regardless of whether they are arranged in a cup, group, stack, etc. • Unitizing is the concept that a group of objects can be counted as one unit (e.g., 10 ones can be counted as 1 ten.) • The number system is based on a pattern of tens where each place has ten times the value of the place to its right. This is known as the ten-to-one concept of place value.	~~**All students should**~~ ~~• Understand that collections of objects can be grouped and skip counting can be used to count the collection.~~ ~~• Describe patterns in counting by ones (both forward and backward) and skip counting and use those patterns to predict the next number in the counting sequence.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** ~~• Count by ones, twos, fives, and tens to 100, using concrete objects, such as counters, connecting cubes, pennies, nickels, and dimes.~~ ~~• Demonstrate a one-to-one correspondence when counting by ones with concrete objects or representations.~~ ~~• Skip count orally by twos, fives and tens to 100 starting at various multiples of 2, 5, or 10.~~ ~~• Count backward by ones from 30.~~ • Group a collection of objects into sets of tens and ones. (a) • Write the numeral that corresponds to the total number of objects in a given collection of objects that have been grouped into sets of tens and ones. (a) • Identify the place value of each digit in a two-digit numeral (e.g., in the number 23, the 2 is in the tens place and the value of the 2 is 20). (a) • Knows the number of tens and ones that can be made from any number (e.g., 47 is 47 ones or can also be grouped into 4 tens with 7 ones leftover). (a) • Compare two numbers between 0 and 110 represented pictorially or with concrete objects, using the words *greater than*, *less than* or *equal to*. (b) • Order three or fewer sets, each set containing up to 110 objects, from least to greatest and greatest to least. (c)

1.2 The student, given up to 110 objects, ~~will count forward by ones, twos, fives, and tens to 100 and backward by ones from 30.~~[Moved to 1.1]

a) group a collection into tens and ones and write the corresponding numeral;[Moved from 1.1]

b) compare two numbers between 0 and 110 represented pictorially or with concrete objects, using the words *greater than*, *less than* or *equal to*; and

c) order three or fewer sets from least to greatest and greatest to least.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Opportunities to experience the relationships among tens and ones through hands-on experiences with manipulatives are essential to developing the ten-to-one place value concept of our number system and to understanding the value of each digit in a two-digit number. • Manipulatives that can be physically connected and separated into groups of tens and leftover ones, such as connecting or snap cubes, beans on craft sticks, pennies in cups, bundle of sticks, or beads on pipe cleaners should be used. • Models that clearly illustrate the relationships among tens and ones as physically proportional are most appropriate for this grade (e.g., the tens piece is ten times larger than the ones piece). • Knowledge of place value is essential when comparing numbers. • Recording the numeral when using physical and pictorial models leads to an understanding that the position of each digit in a numeral determines the size of the group it represents. • Ten-to-one trading activities with manipulatives on place value mats and base ten blocks are more appropriate in grade 2. ~~• Calculators can be used to reinforce skip counting.~~		

1.3 The student, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth, and the ordered position of each object.[Moved from K.3]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~		ESSENTIAL KNOWLEDGE AND SKILLS
• An understanding of the cardinal and ordinal meanings of numbers is necessary to quantify, measure, and identify the order of objects.[Moved from K.3] • An ordinal number is a number that names the place or position of an object in a sequence or set (e.g., first, third). *Ordered position*, *ordinal position*, and *ordinality* are terms that refer to the place or position of an object in a sequence or set.[Moved from K.3] • The ordinal position is determined by where one starts in an ordered set of objects or sequence of objects.[Moved from K.3] • The ordinal meaning of numbers is developed by identifying and verbalizing the place or position of objects in a set or sequence (e.g., the student's position in line when students are lined up alphabetically by first name).[Moved from K.3] • Practical applications of ordinal numbers can be experienced through calendar and patterning activities.		**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the ordinal positions first through tenth using ordered sets of ten concrete objects and/or pictures of such sets presented from – left-to-right; – right-to-left; – top-to-bottom; and/or – bottom-to-top.[Moved from K.3]

1.4~~3~~ The student will ~~identify the parts of a set and/or region that represent fractions for halves, thirds, and fourths and write the fractions.~~

a) investigate fractions by solving equal sharing problems with two, four, or eight sharers; and

b) verbally identify the parts of a region that represent fractions for halves, fourths, and eighths.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Practical situations with fractions should involve real life problems in which students themselves are determining how to subdivide a whole into equal parts, test those parts to be sure they are equal, and use those parts to recreate the whole. • When working with fractions, the whole must be defined. • An equal sharing problem is an idea that young children understand intuitively because of their experiences sharing objects with siblings, friends, etc. Consider the following examples: – Two children sharing six sandwiches – Two children sharing one sandwich. – Four children sharing one piece of paper. – Four children sharing two sticks of clay. – Two children sharing three brownies. – Four children sharing five brownies. – Eight children sharing one chocolate bar. – Eight children sharing five sandwiches. • Fraction models that can be continuously divided should be used at this grade (e.g., cookies, brownies). It is important to use models that can be continuously divided when there are remainders so those remainders can be cut into as many equal parts as needed. • Equal parts may be different shapes but maintain the same value (e.g., a sandwich could be cut in two equal pieces vertically, horizontally, or diagonally to represent halves). • The words *denominator* and *numerator* are not required at this grade, but the concepts of part and whole are required for understanding of a fraction.[Reordered]	~~**All students should**~~ • ~~Understand that a fraction represents a part of a whole.~~ • ~~Understand that fractional parts are equal shares of a whole.~~ • ~~Understand that the fraction name (*half, third, fourth*) tells the number of equal parts in the whole.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given a practical situation, share a whole equally with two, four, or eight sharers. (a) • Given a practical situation, represent fair shares pictorially. (a) • Given a practical situation, describe shares as equal pieces or parts of the whole. (a) • ~~Represent a whole to show it having two equal parts and identify one-half ($\frac{1}{2}$), and two halves ($\frac{2}{2}$).~~ • ~~Represent a whole to show it having three equal parts and identify one-third ($\frac{1}{3}$), two-thirds ($\frac{2}{3}$) and three-thirds ($\frac{3}{3}$).~~ • ~~Represent a whole to show it having four equal parts and identify one-fourth ($\frac{1}{4}$), two-fourths ($\frac{2}{4}$), three-fourths ($\frac{3}{4}$) and four-fourths ($\frac{4}{4}$).~~ • ~~Identify and model halves, thirds, and fourths~~ Represent halves, fourths, and eighths of a whole, using ~~the set model (e.g., connecting cubes and counters), and~~ a region/area model~~s~~ (e.g., pie pieces, pattern blocks, ~~geoboards,~~ paper folding, and drawings). (a)

1.4~~3~~ The student will ~~identify the parts of a set and/or region that represent fractions for halves, thirds, and fourths and write the fractions.~~
a) investigate fractions by solving equal sharing problems with two, four, or eight sharers; and
b) verbally identify the parts of a region that represent fractions for halves, fourths, and eighths.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The focus of this grade level is to develop the idea of equal sharing and part-whole relationships. Fraction notation will be introduced in grade 2. • Students should use the vocabulary for halves, fourths, and eighths, but should not be expected to use fraction notation at this age. Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions furthers this development.[Reordered] ~~• A fraction is a way of representing part of a whole set or a whole region.~~ ~~• In a set fraction model, students need opportunities to make fair shares. For example, when sharing a set of 12 markers with three friends, each person would have one-third of the whole set. Also, each element of the set, no matter its size, is considered to be an equal share of the whole.~~ ~~• In a region/area fraction model, a whole is broken into equal-size parts and reassembled into one whole.~~ • ~~The words *denominator* and *numerator* are not required at this grade, but the concepts of part and whole are required for understanding of a fraction.~~[Reordered] ~~• At this level, students are expected to first understand the part-whole relationship (e.g., three out of four equal parts) before being expected to recognize or use symbolic representations for fractions (e.g., $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$).~~		• Name ~~and write~~ fractions represented by drawings or concrete materials for halves, ~~thirds, and~~ fourths, and eighths. (b) ~~• Represent a given fraction using concrete materials, pictures, and symbols for halves, thirds, and fourths. For example, write the symbol for one-fourth, and represent it with concrete materials and pictures.~~

1.4~~3~~ The student will ~~identify the parts of a set and/or region that represent fractions for halves, thirds, and fourths and write the fractions.~~

a) investigate fractions by solving equal sharing problems with two, four, or eight sharers; and

b) verbally identify the parts of a region that represent fractions for halves, fourths, and eighths.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Students should have opportunities to make connections and comparisons among fraction representations by connecting concrete or pictorial representations with spoken representations (e.g., "one-half," "one out of two equal parts, "or ~~"two thirds," "two out of three equal parts," and~~ "one half is more than one fourth of the same whole"). • ~~Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions (e.g., *thirds* means "three equal parts of a whole") furthers this development.~~		

A variety of contexts and problem types are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from practical experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as "How many books do we have altogether?" or "How many cookies are left if I eat two?" or "I have three more candies than you do." Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling, discussing, and recording a variety of problem situations. This approach helps students transition from the concrete to the representation to the symbolic in order to develop meaning for the operations and how they relate to each other.

In grades K–~~3~~2, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition, and subtraction~~, multiplication, division and related facts~~;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as skip counting and solving problems that involve equal groupings of objects ~~and~~as well as problems that involve sharing equally, the initial work ~~; and performing initial operations~~ with fractions.

1.5~~4~~ The student, given a familiar problem situation involving magnitude, will

a) select a reasonable order of magnitude from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, 500); and

b) explain the reasonableness of the choice.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Magnitude refers to the size of a set. • Exploring ways to estimate the number of objects in a set, based on appearance (e.g., clustering, grouping, comparing), enhances the development of number sense. • To estimate means to ~~find~~ determine a number that is close to the exact amount. When asking for an estimate, teachers might ask, "*about* how much?" or "*about* how many?" or "Is this about 10 or about 50?" • Students should be provided opportunities to estimate a quantity, given a benchmark of 10 and/or 100 objects.	~~**All students should**~~ • ~~Develop an understanding of the order of magnitude (size) of whole numbers and use this knowledge to estimate quantities.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Select a reasonable order of magnitude for a given set from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50~~, and~~or 500 jelly beans in jars) in a familiar problem situation. (a) • Given a familiar problem situation involving magnitude, explain why a particular estimate was chosen as the most reasonable from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral. (b)

~~1.5 The student will recall basic addition facts with sums to 18 or less and the corresponding subtraction facts.~~[Moved to 1.7]

~~UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Associate the terms *addition*, *adding*, and *sum* with the concept of joining or combining.~~ • ~~Associate the terms *subtraction*, *subtracting*, *minus*, and *difference* with the process of taking away or separating (i.e., removing a set of objects from the given set of objects, finding the difference between two numbers, or comparing two numbers).~~ • ~~Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the basic addition and subtraction facts include~~ ~~counting back;~~ ~~"one-more-than," "two-more-than" facts;~~ ~~"one-less-than," "two-less-than" facts;~~ ~~"doubles" to recall addition facts (e.g., 2 + 2 = __; 3 + 3 = __);~~ ~~"near doubles" [e.g., 3 + 4 = (3 + 3) + 1 = __];~~ ~~"make-ten" facts (e.g., at least one addend of 8 or 9);~~ ~~"think addition for subtraction" (e.g., for 9 − 5 = __, think "5 and what number makes 9?");~~ ~~use of the commutative property, without naming the property (e.g., 4 +3 is the same as 3 + 4);~~ ~~use of related facts (e.g., 4 + 3 = 7 , 3 + 4 = 7, 7 − 4 = 3, and 7 − 3 = 4);~~ ~~use of the additive identity property (e.g., 4 + 0 = 4), without naming the property but saying, "When you add zero to a number, you always get the original number."; and use patterns to make sums (e.g., 0 + 5 = 5, 1 + 4 = 5, 2 + 3 = 5, etc.).~~ • ~~Manipulatives should be used to develop an understanding of addition and subtraction facts. Automaticity of facts can be achieved through constant practice which may include games, hands-on activities, flash cards, and paper and pencil.~~ • ~~Students should first master facts to 10 and then master facts to 18.~~	~~**All students should**~~ • ~~Develop an understanding of the addition and subtraction relationship.~~ • ~~Develop addition and subtraction strategies for fact recall.~~ • ~~Develop fluency with basic number combinations for addition and subtraction.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Identify + as a symbol for addition, − as a symbol for subtraction, and = as a symbol for equality.~~ • ~~Recall and state orally the basic addition facts for sums with two addends to 18 or less and the corresponding subtraction facts.~~ • ~~Recall and write the basic addition facts for sums to 18 or less and the corresponding subtraction facts, when addition or subtraction problems are presented in either horizontal or vertical written format.~~

1.6 The student will create and solve ~~one~~single-step story and picture problems using ~~basic~~ addition facts with sums to ~~18~~ 20 or less and the corresponding subtraction facts.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship. • The problem solving process is enhanced when students: – create their own story problems; ~~and~~ – visualize the action in the story problem and draw a picture to show their thinking; and – model the~~word~~ problem~~s,~~ using manipulatives, representations, or number sentences/equations. • The least number of steps necessary to solve a single-step problem is one. • In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as *in all*, *altogether*, *difference*, *etc.* encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions. • Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the basic addition and subtraction facts include – counting on; – counting back; – "one more than," "two more than"; – "one less than," "two less than"; – "doubles" (e.g., 6 + 6 = __); – "near doubles" [e.g., 7 + 8 = (7 + 7) + 1 = or (8 + 8) - 1]; – "make ten" (e.g., 7 + 4 can be thought of as 7 + 3 + 1 in order to make a 10); – "think addition for subtraction" (e.g., for 9 – 5 = , think "5 and what number makes 9?"); – use of the commutative property (e.g., 14 +3 is the same as 3 + 14); – use of related facts (e.g.,14 + 3 = 17 , 3 + 14 = 17, 17 – 4 = 13, and 17 – 13 = 4);	~~All students should~~ ~~Understand various meanings of addition and subtraction in a variety of situations.~~ ~~Understand that creating and solving problems involves the use of addition and/or subtraction.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Interpret~~Create and solve single-step oral or written story and picture problems ~~involving one-step solutions~~, using ~~basic~~ addition and subtraction facts (sums to~~18~~20 or less and the corresponding subtraction facts). • Identify a correct number sentence to solve an oral or written story and picture problem, selecting from among basic addition and/or subtraction facts. • Combine parts contained in larger numbers up to 20 by using related combinations (e.g., 9 + 7 can be thought of as 9 broken up into 2 and 7; using doubles 7 + 7 = 14; 14 + 2 = 16 or 7 broken up into 1 and 6; making a ten 1 + 9 = 10; 10+ 6 = 16). • Explain strategies used to solve addition and subtraction problems up to 20 using spoken words, objects, pictorial models, and number sentences.

~~**1.5 The student will recall basic addition facts with sums to 18 or less and the corresponding subtraction facts.**~~**[Moved to 1.7]**

~~UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
– use of the additive identity property (e.g., 14 + 0 = 14); and – use patterns to make sums (e.g., 0 + 15 = 15, 1 + 14 = 15, 2 + 13 = 15, etc.). • Students at this level are not expected to use the parentheses or to name the properties. • Students should develop fluency with facts to 10 and then utilize strategies and known facts to 10 to determine facts to 20. • Flexibility with facts to 10 should be applied to facts to 20 (e.g., when adding 4 + 7, it is appropriate to think of 4 as 3 + 1 in order to combine 3 and 7 to make a 10 whereas adding 4 + 8, it is appropriate to think of 4 as 2 + 2 in order to combine 8 and 2 to make a 10).~~Students should experience a variety of problem types related to addition and subtraction, including~~ ~~– join and separate problems (action involved); join (for example: Sam had 8 pennies. Tom gave him 3 more. How many pennies does Sam have now? separate (for example: Sam had 11 pennies. He gave 3 to Tom. How many pennies does Sam have now?)~~ ~~– part-part-whole problems (no action involved); missing part (for example: There are 8 marbles. Five are shown. How many are missing?)~~ ~~– classification problems (for example: Jane had 12 hats. Only 3 of the hats are blue. How many are not blue?)~~ ~~– comparison problems (for example: Bill is 7 years old. Alice is 4 years old. How much younger is Alice than Bill?).~~ • Students should have exposure to a variety of problem types related to addition and subtraction. Examples are represented in the chart below. It is important to note that Join Problems (with start unknown), Separate Problems (with start unknown, Compare Problems (with larger unknown – using 'fewer') and Compare problems (with smaller unknown – using 'more') are the most difficult and should be mastered in grade 2.		

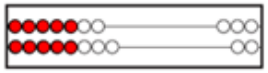
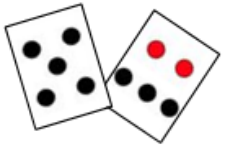
~~1.5 The student will recall basic addition facts with sums to 18 or less and the corresponding subtraction facts.~~**[Moved to 1.7]**

~~UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~

GRADE 1 - ADDITION AND SUBTRACTION PROBLEM TYPES

Join (Result Unknown)	Join (Change Unknown)	Join (Start Unknown)
Sue had 8 pennies. Josh gave her 4 more. How many pennies does Sue have altogether?	Sue had 8 pennies. Josh gave her some more. Now Sue has 12 pennies. How many did Josh give her?	Sue had some pennies. Josh gave her 4 more. Now Sue has 12 pennies. How many pennies did Sue have to begin with?
Separate (Result Unknown)	**Separate (Change Unknown)**	**Separate (Start Unknown)**
Sue had 12 pennies. She gave 4 pennies to Josh. How many pennies does Sue have now?	Sue had 12 pennies. She gave some to Josh. Now she has 8 pennies. How many did she give to Josh?	Sue had some pennies. She gave 8 to Josh. Now she has 4 pennies left. How many pennies did Sue have to begin with?
Part-Part-Whole (Whole Unknown)	**Part-Part-Whole (Part Unknown)**	**Part-Part-Whole (Both Parts Unknown)**
Josh has 4 pennies and 8 nickels. How many coins does he have?	Josh has 12 coins. Eight of the coins are pennies, and the rest are nickels. How many nickels does Josh have?	Josh has 12 coins. Some are pennies and some are nickels. How many pennies and nickels could he have?
Compare (Difference Unknown)	**Compare (Larger Unknown)**	**Compare (Smaller Unknown)**
Josh has 12 pennies and Sue has 8 pennies. How many more pennies does Josh have than Sue?	Josh has 4 more pennies than Sue. Sue has 8 pennies. How many pennies does Josh have?	Josh has 8 more pennies than Sue. Josh has 12 pennies. How many pennies does Sue have?
Josh has 12 pennies. Sue has 8 pennies. How many fewer pennies does Sue have than Josh?	Sue has 4 fewer pennies than Josh. Sue has 8 pennies. How many pennies does Josh have?	Sue has 4 fewer pennies than Josh. Josh has 12 pennies. How many pennies does Sue have?

1.7~~5~~ The student will ~~recall basic~~
a) recognize and describe with fluency part-whole relationships for numbers up to 10; and
b) demonstrate fluency with addition facts ~~with~~ for sums to 10~~18~~ or less and the corresponding subtraction facts.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently. • Flexibility requires knowledge of more than one approach to solving a particular kind of problem. Being flexible allows students to choose an appropriate strategy for the numbers involved. • Mathematically fluent students are not only able to provide correct answers quickly but also to use facts and computation strategies they know to efficiently determine answers that they do not know. • Composing and decomposing numbers flexibly forms a basis for understanding properties of the operations and later formal algebraic concepts and procedures. • Parts of numbers to 10 should be represented in a variety of different ways, such as five frames, ten frames, strings of beads, arrangements of tiles or tooth picks, dot cards, rekenreks or beaded number frames.  • Dot patterns should be presented in both regular and irregular arrangements.  • Accuracy is the ability to determine a correct answer using knowledge of number facts and other important number relationships.	~~All students should~~ • ~~Develop an understanding of the addition and subtraction relationship.~~ • ~~Develop addition and subtraction strategies for fact recall.~~ • ~~Develop fluency with basic number combinations for addition and subtraction.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize and describe with fluency part-whole relationships for numbers up to 10 in a variety of configurations. (a) • Identify + as a symbol for addition, – as a symbol for subtraction, and = as a symbol for equality. (b) • Demonstrate fluency with~~Recall and state orally the basic~~ addition facts for sums ~~with two addends to 18~~to 10 or less and the corresponding subtraction facts. (b) • ~~Recall and write the basic addition facts for sums to 18 or less and the corresponding subtraction facts, when addition or subtraction problems are presented in either horizontal or vertical written format.~~

1.7~~5~~ The student will ~~recall basic~~
a) recognize and describe with fluency part-whole relationships for numbers up to 10; and
b) demonstrate fluency with addition facts ~~with~~ for sums to 10~~18~~ or less and the corresponding subtraction facts.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Efficiency is the ability to carry out a strategy easily when solving a problem without getting bogged down in too many steps or lose trace of the logic of the strategy being used. • Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship. • Manipulatives should be used to develop an understanding of addition and subtraction facts.[Reordered] • Automaticity of facts can be achieved through meaningful~~constant~~ practice which may include games, hands-on activities, ~~flash~~ dot cards, and ~~paper and pencil~~ten frames.[Reordered] • Subtraction is the inverse of addition. Subtraction can be viewed as a process of taking away or separating, or as a process of comparing two sets to determine the difference between them. • Associate the terms addition, adding, and sum with the concept of joining or combining. • Associate the terms subtraction, subtracting, minus, and difference with the process of taking away or separating (i.e., removing a set of objects from the given set of objects, ~~finding~~ determining the difference between two numbers, or comparing two numbers to determine the difference). • Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the ~~basic~~ addition and subtraction facts include – counting on; – counting back; – "one more than," "two more than" ~~facts~~; – "one less than," "two less than" ~~facts~~; – "doubles" ~~to recall addition facts~~ (e.g., 2 + 2 =__; 3 + 3 =__);		

1.7~~5~~ The student will ~~recall basic~~
a) recognize and describe with fluency part-whole relationships for numbers up to 10; and
b) demonstrate fluency with addition facts ~~with~~ for sums to 10~~18~~ or less and the corresponding subtraction facts.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– "near doubles" [e.g., 3 + 4 = (3 + 3) + 1 = __]; – "make ten"~~ facts~~ (7 + 4 can be thought of as 7 + 3 + 1 in order to make a ten)~~e.g., at least one addend of 8 or 9~~); – "think addition for subtraction" (e.g., for 9 – 5 = __, think "5 and what number makes 9?"); – use of the commutative property~~, without naming the property~~ (e.g., 4 +3 is the same as 3 + 4); – use of related facts (e.g., 4 + 3 = 7 , 3 + 4 = 7, 7 – 4 = 3, and 7 – 3 = 4); – use of the additive identity property (e.g., 4 + 0 = 4); ~~without naming the property but saying, "When you add zero to a number, you always get the original number.";~~ and – use patterns to make sums (e.g., 0 + 5 = 5, 1 + 4 = 5, 2 + 3 = 5, etc.). • Students at this level are not expected to name the properties. • ~~Manipulatives should be used to develop an understanding of addition and subtraction facts.~~[Reordered] • ~~Automaticity of facts can be achieved through constant practice which may include games, hands-on activities, flash cards, and paper and pencil.~~[Reordered] • ~~Students should first master facts to 10 and then master facts to 18.~~		

The exploration of measurement and geometry in the primary grades allows students to learn more about the world around them. Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K–~~3~~ 2~~should~~ encounter measurement in ~~many normal situations, from~~ their daily lives, from their use of the calendar and ~~from~~ science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature~~, area, perimeter, volume~~, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both techniques and nonstandard and standard tools to ~~find~~ determine measurements and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

[Moved from Geometry strand introduction]
Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.
The focus of instruction at this level is on

- observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).
The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.

1.8~~7~~ The student will

a) identify the number of pennies equivalent to a nickel, a dime, and a quarter; and

b) determine the value of a collection of like coins (pennies, nickels, or ~~and~~ dimes) whose total value is 100 cents or less.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Many experiences with coins help students develop an understanding of money, such as – drawing pennies to show the value of a given coin (e.g., a nickel, a dime, or a quarter); – playing store and purchasing classroom objects, using play money (pennies); – using skip counting to count a collection of like coins; – representing the value of coins using a variety of organizers, such as five/ten frames or hundreds charts, pictures; and – ~~representing the value of a nickel, a dime, and a quarter, using pennies; and~~ – trading the equivalent value of pennies for a nickel, a dime, and a quarter, using play money. • Counting coins is an application of unitizing. • Unitizing is the concept that a group of objects can be counted as one unit (e.g., 10 pennies can be counted as 1 dime.) • Counting money helps students gain an awareness of consumer skills and the use of money in everyday life. • A variety of classroom experiences in which students manipulate physical models of money and count forward to determine the value of a collection of coins are important activities to ~~ensure~~ develop competence with counting money. • ~~Establishing a one-to-one correspondence between the number names and the items in a set of coins (pennies, nickels, or dimes) is essential for an accurate count.~~ • The last number stated represents the value of a collection of coins being counted. ~~This is known as the cardinality of the set.~~	~~**All students should**~~ • ~~Develop an understanding of exchanging the appropriate number of pennies for a nickel, a dime, or a quarter.~~ • ~~Develop an understanding of place value by skip counting a collection of coins by ones, fives, and tens.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the value of a nickel, a dime, and a quarter in terms of pennies. (a) • ~~Recognize the characteristics of pennies, nickels, and dimes (e.g., color, size).~~[Included in K.7] • Count by ones to determine the value of a collection of pennies whose total value is 100 cents or less. (b) • Group a collection of pennies by fives and tens as a way to determine the value. The total value of the collection is 100 cents or less. (b) • Count by fives to determine the value of a collection of nickels whose total value is 100 cents or less. (b) • Count by tens to determine the value of a collection of dimes whose total value is 100 cents or less. (b) • ~~Count by ones, fives, and tens to determine the value of a collection of pennies and nickels, pennies and dimes, and nickels and dimes whose total value is 100 cents or less.~~ • ~~Count by ones, fives, and tens to determine the value of a collection of pennies, nickels, and dimes whose total value is 100 cents or less.~~[Moved to 2.7]

1.9~~8~~ The student will investigate the passage of time by
a) telling time to the hour and half-hour, using analog and digital clocks;~~.~~ and[Time to hour moved from K.9]
b) reading and interpreting a calendar.[Moved from 1.11]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Many experiences using clocks help students develop an understanding of the telling of time to the hour and half-hour, including – identifying the parts of an analog clock (minute and hour hands); – demonstrating a given time to the hour and half-hour, using a model clock; – writing correct digital time to the hour and half-hour;~~ and~~ – relating time on the hour and half-hour to daily routines and school schedules (e.g., ~~the times of TV programs,~~ bedtime, ~~resource time,~~ lunch time, recess time); and~~.~~ – connecting the hour and half hour to fraction concepts. • Practical situations are appropriate to develop a sense of the interval of time between events (e.g., Boy Scout meetings occur every week on Monday: there is a week between meetings). • The calendar is a way to represent units of time (e.g., days, weeks, and months). • Using a calendar develops the concept of day as a 24-hour period rather than a period of time from sunrise to sunset. [Moved from 1.11]	~~**All students should**~~ • ~~Understand how to tell time to the half-hour, using an analog and digital clock.~~ • ~~Understand the concepts of a.m., p.m., minutes, and hours.~~ • ~~Understand that there are sixty minutes in an hour.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify different types of clocks (analog and digital) as instruments to measure time.[Moved from K.8] (a) • Tell time shown on an analog clock to the hour and half-hour. (a) • Tell time shown on a digital clock to the hour and half-hour. (a) • Match a written time (e.g., 1:00, 3:30, 11:00) to the time shown on a digital and analog clock to the half-hour. (a) • Read a calendar to locate a given day or date. (b) • Determine the days/dates before and after a given day/date (e.g., today is the 30th, so yesterday must have been the ?). (b) • Determine the date that is a specific number of days or weeks in the past or in the future from a given date, using a calendar. (b)[Moved from 1.11] • Identify specific dates (e.g., the third Monday in a given month). (b)[Moved from 1.11]

1.10~~9~~ The student will use nonstandard units to measure and compare length, weight/mass, and volume.[Compare moved from 1.10]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The process of measurement involves selecting a unit of measure, comparing the unit to the object to be measured, counting the number of times the unit is used to measure the object, and arriving at an approximate total number of units. • Measurement involves comparing an attribute of an object to the same attribute of the unit of measurement (e.g., the length of a cube measures the length of a book; the weight/mass of the cube measures the weight/mass of the book; the volume of the cube measures the volume of a book).[Moved from middle column] • Premature use of instruments or formulas leaves children without the understanding necessary for solving measurement problems. • When children's initial explorations of length, weight/mass, and volume involve the use of nonstandard units, they develop some understanding about the need for standard measurement units for length, weight/mass, and volume, especially when they communicate about these measures. • The level of difficulty in measuring volume can be increased by varying and mixing the sizes and/or shapes of the containers (e.g., using short, wide containers as well as tall, narrow containers). • Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?").	~~**All students should**~~ • ~~Understand that measurement involves comparing an attribute of an object to the same attribute of the unit of measurement (e.g., the length of a cube measures the length of a book. The weight/mass of the cube measures the weight/mass of the book. The volume of the cube measures the volume of a book).~~[Moved to US] • ~~Understand how to measure length, weight/mass, and volume using various nonstandard units of measurement.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Measure the length of objects, using various nonstandard units (e.g., connecting cubes, paper clips, erasers). • Compare the length of two objects, using the terms *longer/shorter*, *taller/shorter*, *same as*. • Measure the weight/mass of objects, using a balance or pan scale with various nonstandard units (e.g., paper clips, bean bags, cubes). • Identify a balance scale or a pan scale as a tool for measuring weight. • Compare the weight/mass of two objects, using the terms *lighter*, *heavier*, or *the same*, using a balance scale.[Moved from 1.10] • Measure the volume of objects, using various nonstandard units (e.g., connecting cubes, blocks, rice, water). • Compare the volumes of two containers to determine if the volume of one is more, less, or equivalent to the other, using nonstandard units of measure (e.g., a spoonful or scoopful of rice, sand, jellybeans).[Moved from 1.10] • Compare the volumes of two containers to determine if the volume of one is more, less, or equivalent to the other by pouring the contents of one container into the other. [Moved from 1.10]

1.10~~9~~ The student will use nonstandard units to measure and compare length, weight/mass, and volume.[Compare moved from 1.10]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Balance scales are instruments used for comparing weight/mass. A balance scale usually has a beam that is supported in the center. On each side of the beam are two identical trays. When the trays hold equal weights, the beam is level, and the scale is "balanced."–The tray containing less weight/mass will rise and the tray containing more weight/mass will fall. • Physically measuring the weights of objects, using a balance scale, helps students develop an intuitive idea of what it means to say something is "lighter," "heavier," or "the same." • Experience estimating the weights of two objects (one in each hand) using the terms "lighter", "heavier", or "the same" promotes an understanding of the concept of balance.[Moved from 1.10]		

~~1.10 The student will compare, using the concepts of more, less, and equivalent,~~ [Compare moved to 1.10]

~~a) the volumes of two given containers; and~~[Moved to 1.10 EKS]

~~b) the weight/mass of two objects, using a balance scale.~~[Moved to 1.10 EKS]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Estimation is a commonly used strategy to compare the volumes of two containers.~~ • ~~Determining the volume of a container by counting the number of nonstandard units (e.g., a spoonful, or scoopful of concrete material, such as jelly beans, sand, water, or rice) that can be held by the container is a precursor to comparing volumes.~~ • ~~A variety of activities that focus on directly comparing the volume of objects leads to an understanding of volume.~~ • ~~The level of difficulty in measuring volume can be increased by varying and mixing the sizes of the containers (e.g., using short, wide containers as well as tall, narrow containers).~~ • ~~Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?").~~ • ~~Balance scales are instruments used for comparing weight/mass. A balance scale usually has a beam that is supported in the center. On each side of the beam are two identical trays. When the trays hold equal weights, the beam is level, and the scale is "balanced."~~ • ~~Physically measuring the weights of objects, using a balance scale, helps students develop an intuitive idea of what it means to say something is "lighter," "heavier," or "the same."~~ • ~~Experience estimating the weights of two objects (one in each hand) using the terms lighter, heavier, or the same promotes an understanding of the concept of balance.~~	~~**All students should**~~ • ~~Understand how to fill containers with objects to determine their volume and compare the volumes of two containers.~~ • ~~Understand that a balance scale can be used to compare the weights of two objects using the terms more, less, or equivalent.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Compare the volumes of two containers to determine if the volume of one is more, less, or equivalent to the other, using nonstandard units of measure (e.g., a spoonful or scoopful).~~[Moved to 1.10] • ~~Compare the volumes of two containers to determine if the volume of one is more, less, or equivalent to the other by pouring the contents of one container into the other.~~ [Moved to 1.10] • ~~Compare the weight/mass of two objects, using the terms *lighter*, *heavier*, or *the same*, using a balance scale. The pan containing less weight/mass will rise and the pan containing more weight/mass will fall. If the objects are of equivalent weight/mass, the two pans will balance.~~ [Moved to 1.10]

~~1.11~~ ~~The student will use calendar language appropriately (e.g., names of the months, *today*, *yesterday*, *next week*, *last week*).~~[Moved to K.9 and 1.9]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Practical situations are appropriate to develop a sense of the interval of time between events (e.g., Boy Scout meetings occur every week on Monday: there is a week between meetings).~~ • ~~The calendar is a way to represent units of time (e.g., days, weeks, and months).~~ • ~~Using a calendar develops the concept of day as a 24-hour period rather than a period of time from sunrise to sunset.~~	~~**All students should**~~ • ~~Understand how to use a calendar as a way to measure time.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Read a calendar to locate a given day or date.~~ [Moved to 1.9] • ~~Identify the months of the year.~~[Moved to K.9] • ~~Identify the seven days in a week.~~[Moved to K.9] • ~~Determine the days/dates before and after a given day/date (e.g., yesterday, today, tomorrow).~~[Moved to 1.9 and K.9] • ~~Determine the date that is a specific number of days or weeks in the past or in the future from a given date, using a calendar.~~[Moved to 1.9] • ~~Identify specific dates (e.g., the third Monday in a given month).~~[Moved to 1.9]

[Moved to Measurement and Geometry strand introduction]

~~Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.~~

~~The focus of instruction at this level is on~~

- ~~observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;~~
- ~~sorting objects and ordering them directly by comparing them one to the other;~~
- ~~describing, comparing, contrasting, sorting, and classifying figures; and~~
- ~~exploring symmetry, congruence, and transformation.~~

~~In the primary grades, children begin to develop basic vocabulary related to these figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).~~

~~The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.~~

- ~~**Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.~~
- ~~**Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)~~
- ~~**Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)~~

1.11~~12~~ The student will

a) identify and trace, describe, and sort plane ~~geometric~~ figures (triangle, square, rectangle, and circle) according to number of sides, vertices, and ~~right~~ angles; and~~.~~

b) identify and describe representations of circles, squares, rectangles, and triangles in different environments and explain reasoning.[Moved from 1.13]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Early experiences with comparing, sorting, and subdividing figures assist students in analyzing the characteristics of plane geometric figures. • A plane ~~geometric~~ figure is any two-dimensional shape that can be drawn in a plane~~closed figure. Circles and polygons are examples of plane geometric figures~~. ~~• The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.~~ ~~• **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.~~ ~~• **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., "I know it's a rectangle because it looks like a door, and I know that a door is a rectangle").~~ ~~• **Level 2: Analysis.** Properties are perceived, but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., "I know it's a rectangle because it is closed; it has four sides and four right angles, and opposite sides are parallel.").~~ ~~• Students should have experiences with various plane geometric polygons.~~ ~~Triangles could be equilateral, right, obtuse, acute, etc.~~ ~~Quadrilaterals could be rectangles, squares, trapezoids,~~	~~**All students should**~~ ~~• Develop strategies to sort and/or group plane geometric figures and refine the vocabulary used to explain their strategies.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the name of the plane figure when given information about the number of sides, vertices, and angles. (a)[Reordered] • Trace triangles, squares, rectangles, and circles. (a) [Reordered] • Describe a circle. (a)[Reordered] • Describe triangles, squares, and rectangles by the number of sides, vertices, and ~~right~~ angles. Recognize that rectangles and squares have special types of angles called right angles. (a) ~~• Identify the name of the geometric figure when given information about the number of sides, vertices, and right angles.~~ [Reordered] • Sort plane ~~geometric~~ figures ~~into appropriate subsets (categories)~~ based on their characteristics (number of sides, vertices, angles, curved, etc.). (a)[Reordered] • Identify and describe representations of circles, squares, rectangles, and triangles in different environments and explain reasoning. (b)[Moved from 1.13]

1.11~~12~~ The student will

a) identify and trace, describe, and sort plane ~~geometric~~ figures (triangle, square, rectangle, and circle) according to number of sides, vertices, and ~~right~~ angles; and~~.~~

b) identify and describe representations of circles, squares, rectangles, and triangles in different environments and explain reasoning.[Moved from 1.13]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~rhombi, etc.~~ • A vertex is the point at which two or more lines, line segments, ~~lines,~~ or rays meet to form an angle. The term "vertices" is the plural form of vertex. • An angle is formed by two rays that share a common endpoint called the vertex. Angles are found wherever lines or line segments intersect. • A polygon is a closed plane figure composed of at least three line segments that do not cross. ~~• A polygon is a plane geometric figure which~~ ~~– has sides that are line segments;~~ ~~– is simple (its sides do not cross);~~ ~~– is closed; and~~ ~~– is two-dimensional (lies in a plane).~~ • A triangle is a polygon with three ~~angles and~~ three sides. • A quadrilateral is a polygon with four sides. • A rectangle is a quadrilateral with four right angles. • A square is a ~~rectangle~~ quadrilateral with four congruent (equal length) sides ~~of equal length.~~ and four right angles. • A right angle measures exactly 90°. • A circle is the set of ~~a closed curve with all~~ points in a~~one~~ plane ~~and~~ that are the same distance from a ~~fixed~~ point ~~(~~called the center~~)~~. A circle is not a polygon • ~~Transformations (translations, reflections, and rotation) can be used to change the location of objects.~~		

1.11~~12~~ The student will

a) identify and trace, describe, and sort plane ~~geometric~~ figures (triangle, square, rectangle, and circle) according to number of sides, vertices, and ~~right~~ angles; and~~.~~

b) identify and describe representations of circles, squares, rectangles, and triangles in different environments and explain reasoning.[Moved from 1.13]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Presentation of triangles, rectangles, and squares should be made in a variety of spatial orientations so that students do not develop the common misconception that triangles, rectangles, and squares must have one side parallel to the bottom of the page on which they are printed. • Representations of circles, squares, rectangles, and triangles can be found in the students' environment at school and at home. Students should have opportunities to identify/classify things in the environment by the type of figure those things represent. [Moved from 1.13] • A common misconception students have when a figure such as a square is rotated is they will frequently refer to the rotated square as a diamond. Clarification needs to be ongoing — e.g., a square is a square regardless of its location in space; there is no geometric figure called a diamond.[Moved from 1.13] • Building geometric and spatial capabilities fosters enthusiasm for mathematics while providing a context to develop spatial sense.[Moved from 1.13] • Polygons can be constructed using other polygons (e.g., six equilateral triangles can be used to construct a hexagon; a triangle can be added to a rectangle to create a pentagon, etc.). [Moved from 1.13] • Early experiences with comparing, sorting, and subdividing figures assist students in analyzing the characteristics of plane geometric figures.		

~~1.13 The student will construct, model, and describe objects in the environment as geometric shapes (triangle, rectangle, square, and circle) and explain the reasonableness of each choice.~~[Moved to 1.11b]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Representations of circles, squares, rectangles, and triangles can be found in the students' environment at school and at home. Students should have opportunities to identify/classify things in their environment by the type of figure those things represent.~~ • ~~A common misconception students have when a figure such as a square is rotated is they will frequently refer to the rotated square as a diamond. Clarification needs to be ongoing — i.e., a square is a square regardless of its location in space; there is no such geometric figure as a diamond.~~ • ~~Building geometric and spatial capabilities fosters enthusiasm for mathematics while providing a context to develop spatial sense.~~ • ~~Polygons can be constructed using other polygons (e.g., six equilateral triangles can be used to construct a hexagon, a triangle can be added to a rectangle to create a pentagon, etc.).~~	~~**All students should**~~ • ~~Understand that geometric figures are integral parts of the environment.~~ • ~~Use familiarity with the figure, structure, and location to develop spatial reasoning.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Construct plane geometric figures.~~ • ~~Identify models of representations of circles, squares, rectangles, and triangles in the environment at school and home and tell why they represent those figures.~~ • ~~Describe representations of circles, squares, rectangles, and triangles in the environment and explain the reasonableness of the choice.~~ [Moved to 1.11 EKS]

Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible, unlikely, equally likely, more likely,* and *certain.*

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

1.12~~14~~ The student will

a) ~~investigate, identify, and describe~~collect, organize, and represent various forms of data collection ~~(e.g., recording daily temperature, lunch count, attendance, favorite ice cream),~~ using tables, picture graphs, and object graphs; and[Examples moved to US]

b) read and interpret data displayed in tables, picture graphs, and object graphs, using the vocabulary *more*, *less*, *fewer*, *greater than*, *less than*, and *equal to*.[Moved from 1.15]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are pieces of information collected about people or things. The primary purpose of collecting data is to answer questions. The primary purpose of interpreting data is inform decisions (e.g., which type of clothing to pack for a vacation based on a weather graph or which type of lunch to serve based upon class favorites). • ~~Students' questions about the physical world can often be answered by collecting data and observing the results.~~ • ~~Data are information collected about people or things. The primary purpose of collecting data is to answer questions.~~ • After generating questions, students decide what information is needed and how it can be collected. • The collection of the data often leads to new questions to be investigated. • ~~The entire process broadens children's views of mathematics and its usefulness.~~ • Data collection could involve voting, informal surveys, tallying, and charts (e.g., recording daily temperature, lunch count, attendance, favorite ice cream). • Surveys, which are data-collecting tools that list choices, should have a limited number of questions at the primary grades. • Tallying is a method for gathering information. Tally marks are used to show how often something happens or occurs. Each tally mark represents one occurrence. Tally marks are clustered into groups of five, with four vertical marks representing the first four occurrences and the fifth mark crossing the first four on a diagonal to represent the fifth occurrence.	~~**All students should**~~ ~~Understand how data can be collected and presented in an organized manner.~~ ~~Understand that data gathered and analyzed from observations and surveys can have an impact on our everyday lives.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Investigate~~ Collect and organize data using various forms of data collection~~, including~~ (e.g., counting and tallying, informal surveys, observations, ~~and~~ voting). (a) • Represent data in tables, picture graphs, and object graphs. (a) • ~~Identify and describe various forms of data collection in practical situations (e.g., recording daily temperature, lunch count, attendance, and favorite ice cream.)~~ • Analyze information displayed in tables, picture graphs, and object graphs (horizontally or vertically represented): (b) – Read the graph to determine the categories of data and the data as a whole (e.g., the total number of responses) and its parts (e.g., 15 people are wearing sneakers); and – Interpret the data that represents numerical relationships, to include using the words *more*, *less*, *fewer*, *greater than*, *less than*, *and equal to*. (b)[Moved from 1.15 EKS]

1.12~~14~~ The student will

a) ~~investigate, identify, and describe~~collect, organize, and represent various forms of data collection ~~(e.g., recording daily temperature, lunch count, attendance, favorite ice cream),~~ using tables, picture graphs, and object graphs; and[Examples moved to US]

b) read and interpret data displayed in tables, picture graphs, and object graphs, using the vocabulary *more*, *less*, *fewer*, *greater than*, *less than*, and *equal to*.[Moved from 1.15]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Picture graphs are graphs that use pictures to represent and compare information.[Moved from 1.15 US] At this level, each picture should represent 1 data point. • Object graphs are graphs that use concrete materials to represent and compare the categorical data that are collected (e.g., cubes stacked by the month, with one cube representing the birthday month of each student).[Moved from 1.15 US] • Tables are an orderly arrangement of data in which the data are arranged in columns and rows in an essentially rectangular format. Tables may be used to display some type of numerical relationship or organized lists. • At this level, data gathered and displayed by students should be limited to 16 or fewer data points for no more than 4 categories. • Students should have opportunities to interpret graphs, created with the assistance of the teacher, that contain data points where their entire class is represented (e.g., tables that show who brought their lunch and who will buy their lunch for any given day, picture graph showing how students traveled to school – bus, car, walk).		

~~1.15 The student will interpret information displayed in a picture or object graph, using the vocabulary *more, less, fewer, greater than, less than,* and *equal to*.~~[Moved to 1.12b]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Statistics is used to describe and interpret with numbers the world around us; it is a tool for problem solving.~~ • ~~Students' questions about everyday life can often be answered by collecting and interpreting data.~~ • ~~Organized data provides a clearer picture for interpretation. Data may be described in object graphs and picture graphs.~~ • ~~Picture graphs are graphs that use pictures to show and compare information.~~ • ~~Object graphs are graphs that use concrete materials to represent the categorical data that are collected (e.g., cubes stacked by the month, with one cube representing the birthday month of each student).~~ • ~~Interpretation of the data could lead to additional questions to be investigated.~~	~~**All students should**~~ • ~~Understand that picture graphs use pictures to represent and compare data while object graphs use concrete objects to represent and compare data.~~ • ~~Understand that data can be analyzed and interpreted, using the terms *more*, *less*, *fewer*, *greater than*, *less than*, and *equal to*.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Compare one category to another in a graph, indicating which has more or which has less, or which is equal to.~~ • ~~Interpret information displayed in object graphs and picture graphs, using the words *more*, *less*, *fewer*, *greater than*, *less than*, and *equal to*.~~ • ~~Find answers to questions, using graphs (e.g., "Which category has more?", "How many more?", and "How many in all?").~~ [Moved to 1.12b EKS]

Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children's mathematical development.

Students in kindergarten through ~~third~~ second grade develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns. ~~These s~~Students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to algebra.

1.13~~16~~ The student will sort and classify concrete objects according to one or more attributes.~~, including color, size, shape, and thickness.~~[Included in EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Sorting, classifying, and ordering objects facilitate work with patterns, geometric shapes, and data. • The same set of objects can be sorted and classified in different ways.[Moved from middle column] • To classify is to arrange or organize a set of materials according to a category or attribute (a quality or characteristic). • One way to explore attributes is to investigate non-examples (e.g., a triangle could be a non-example in a sort of rectangles and circles). • General similarities and differences among items are easily observed by primary students, who can begin to focus on more than one attribute at a time. During the primary grades, the teacher's task is to move students toward a more sophisticated understanding of classification in which two or more attributes connect or differentiate sets, such as those found in nature (e.g., leaves with different colors and different shapes).	~~**All students should**~~ • ~~Understand the same set of objects can be sorted and classified in different ways.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Sort and classify objects into appropriate subsets (categories) based on one or two attributes, such as size, shape, color, and/or thickness (e.g., sort a set of objects that are both red and thick). • Label attributes of a set of objects that has been sorted. • Name multiple ways to sort a set of objects.

1.14~~17~~ The student will identify~~recognize~~, describe, extend, ~~and~~ create, and transfer ~~a wide variety of~~ growing and repeating patterns. [Transfer included to match EKS bullet.]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Opportunities to create, identify, describe, extend, and transfer patterns are essential to the primary school experience and lay the foundation for thinking algebraically. • Opportunities to create, identify, describe, extend, and transfer patterns are essential to the primary school experience and lay the foundation for thinking algebraically. • ~~Patterns allow children to recognize order, generalize, and predict.~~ • Patterning should include: – ~~reproducing~~creating a given pattern, using objects, sounds, movements and pictures~~manipulatives~~; – recording a pattern with pictures or symbols; – transferring a pattern into a different form or different representation (e.g., blue–blue–red–green to an AABC repeating pattern); and – analyzing patterns in practical situations (e.g., calendar, seasons, days of the week). • ~~The simplest types of patterns are~~In a repeating pattern ~~s. T~~the part of the pattern that repeats is the core. ~~The patterns can be oral, such as the refrain in "Old MacDonald's Farm" ("e-i-e-i-o"), or physical with clapping and snapping patterns, or combinations of both, such as is found in songs like the "Hokey Pokey." In each case, students need to identify the basic unit of the pattern and repeat it.~~ • ~~Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience.~~ [Reordered] • At this level students should have experiences extending patterns when given a complete repetition of a core (e.g., ABACABACABAC) as well as when the final repetition of the core is incomplete (e.g., AABBAABBA …; Red, Blue, Green, Red, Blue, Green, Red, Blue....).	~~All students should~~ • ~~Understand that patterns are a way to recognize order, to organize their world, and to predict what comes next in an arrangement.~~ • ~~Recognize and state the core of a pattern.~~ • ~~Analyze how both repeating and growing patterns are generated.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Recognize~~ Identify the pattern in a given rhythmic, color, geometric figure, or numerical sequence. • Describe the pattern in a given rhythmic, color, geometric figure, or numerical sequence in terms of the core (the part of the sequence that repeats). • Extend a pattern, using manipulatives, geometric figures, numbers, or calculators. • Create a repeating or growing pattern, using manipulatives, geometric figures, numbers, or calculators (e.g., the growing patterns 2, 3, 2, 4, 2, 5, 2, 6, 2, …).[Reordered] • Transfer a pattern from one form to another. • ~~Create a repeating or growing pattern, using manipulatives, geometric figures, numbers, or calculators (e.g., the growing patterns 2, 3, 2, 4, 2, 5, 2, 6, 2, …).~~[Reordered]

1.14~~17~~ The student will identify~~recognize~~, describe, extend, ~~and~~ create, and transfer ~~a wide variety of~~ growing and repeating patterns. [Transfer included to match EKS bullet.]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Examples of repeating patterns include: – AABCAABC; – ABACABAC; – ABBCABBC; – AABCAABC; and – ABACDABACD. • Growing patterns involve a progression from step to step which make them more difficult for students than repeating patterns. Students must determine what comes next and also begin the process of generalization, which leads to the foundation of algebraic reasoning. Students need experiences identifying what changes and what stays the same in a growing pattern. Growing patterns may be represented in various ways, including dot patterns, staircases, pictures, etc. • Examples of growing patterns include: – 1 2 3 4 – **5, 10, 15, 20…** – ☆ △ ☆ ☆ △ ☆ ☆ ☆ △ • Transferring a pattern is creating the pattern in a different form or representation. • Examples of pattern transfers include: – 1, 2, 3, 4.... has the same structure as 10, 11, 12, 13… – ABABAB… has the same structure as red, blue, red, blue, red blue – Snap, clap, jump, clap, snap, clap, jump, clap has the same structure as ABCAABCA		

1.14~~17~~ The student will identify~~recognize~~, describe, extend, ~~and~~ create, and transfer ~~a wide variety of~~ growing and repeating patterns. [Transfer included to match EKS bullet.]

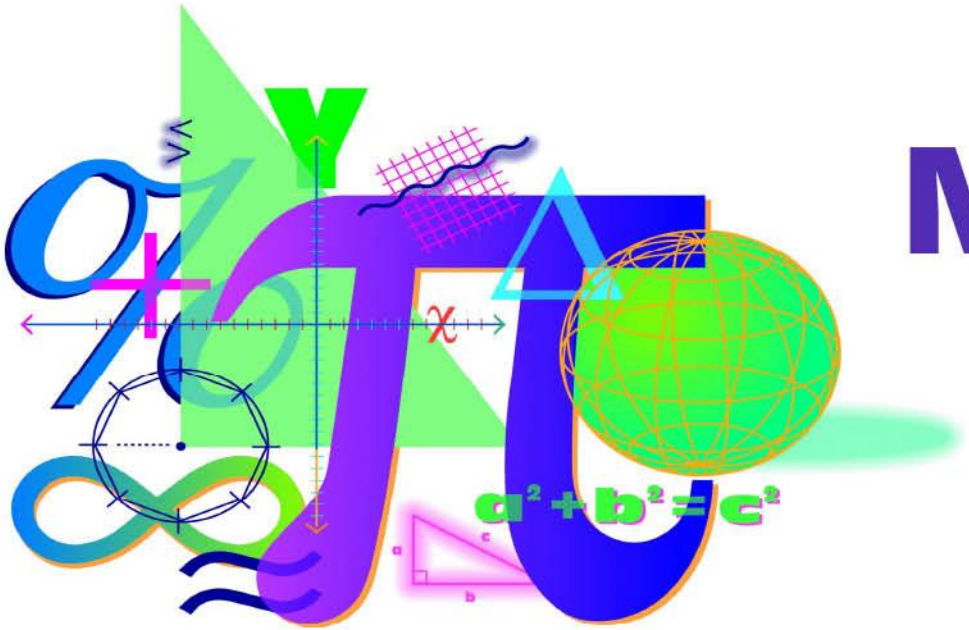
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns in both arithmetic and geometric formats.~~. • ~~Create an arithmetic number pattern. Sample numeric patterns include~~ ~~6, 9, 12, 15, 18, …(growing pattern);~~ ~~20, 18, 16, 14, … (growing pattern);~~ ~~1, 2, 4, 7, 11, 16, … (growing pattern); and~~ ~~1, 3, 5, 1, 3, 5, 1, 3, 5… (repeating pattern).~~ • ~~In geometric patterns, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation is the result of turning a figure around a point or a vertex, and reflection is the result of flipping a figure over a line.~~		

1.15~~18~~ The student will demonstrate an understanding of equality through the use of the equal ~~sign~~symbol.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Equality can be shown by measuring with a balance scale or a number balance.~~ [Reordered] • An equation (number sentence) is a mathematical statement representing two expressions that are equivalent. It consists of two expressions, one on each side of an 'equal' symbol (e.g., 5 + 3 = 8, 8 = 5 + 3 and 4 + 3 = 9 - 2). An equation can be represented using a balance scale, with equal amounts on each side (e.g., 3 + 5 = 6 + 2). Students at this level are not expected to use the term expression. • Equations can be written with an equal sign. The equal sign means "is the same as" or "another name for" or "equal in value". [Moved from middle column] • A common misunderstanding is that the equal sign always means "the answer comes next." The equal sign can represent a balance between equations.[Reordered] • Equations have expressions of equal value on both sides (e.g., 5 + 3 = 8, 8 = 5 + 3 and 4 + 3 = 5 + 2). [Reordered] • At this level, students should represent equality symbolically through the use of the equal sign while inequality should be communicated primarily through words such as "not equal," "not equivalent," etc. • Equality can be shown by measuring with a balance scale or a number balance. [reordered] • Manipulatives such as ~~rods,~~ connecting cubes, and counters can be used to model equations. • An expression is a representation of a quantity. It contains~~is made up of~~ numbers, variables, and/or computational operation symbols. It does not have an equal sign (e.g., ~~x~~5, 4 + 3, 8-2). • ~~An equation is a mathematical statement that two expressions are the same. Equations are written with an equal sign.~~ • ~~Equations have expressions of equal value on both sides (e.g., 5 +~~	~~**All students should**~~ • ~~Understand that the equal sign means "is the same as" or "another name for" or "equal in value".~~ • ~~Understand that equality represents a balance concept. Both sides of the equation balance because they are equal (they have the same value).~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the concept~~s~~ of equality. • Represent equalities through the use of the equal (=) symbol. • Identify expressions that are not equal (e.g., 4 + 3 is not equal to 3 + 5). • Recognize that ~~the~~ equations ~~4 + 2 = 2 + 4 and 6 + 1= 4 + 3~~ can be used to represent the relationship between two expressions of equal value (e.g., 4 + 2 = 2 + 4 and 6 + 1= 4 + 3). • Model an equation that represents the relationship of two expressions of equal value. • Identify equivalent values (e.g., 3 = 3, 4 + 3 = ~~8 – 1~~ 3+ 4, 7 = 2 + 5, etc.).

1.15~~18~~ The student will demonstrate an understanding of equality through the use of the equal ~~sign~~symbol.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~3 = 8, 8 = 5 + 3 and 4 + 3 = 9 – 2).~~ ~~• An equation can be represented using balance scales. There must be the same amount on each side of an equal sign (e.g., 5 + 3 = 3 + 5).~~ ~~• A common misunderstanding is that the equal sign always means the answer. The equal sign can represent an equality.~~ ~~• Equations should be shown in many different forms (e.g., 6 = 6, 4 + 2 = 6, 5 + 1 = 4 + 2, 6 = 4 + 2, 4 = 6 – 2).~~ • Inequalities such as 5 < 4 + 3 are not equations. Equations must have the equal sign. ~~• It is important for children to understand that the expression 3 + 5 is another representation of eight.~~ ~~• The equal sign is used when two representations name the same number, 5 + 3 = 10 – 2. These two expressions in the equation represent the same number, eight.~~ • Equations should be routinely modeled in conjunction with story problems. • Solving missing addend problems and stories helps with the understanding of equality and the equal sign (e.g., There are 4 red birds in the tree. Some black birds fly to the tree. Now there are six birds in the tree. How many black birds flew to the tree? 4+__ =6)		



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Grade 2

Board of Education
Commonwealth of Virginia

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The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

Students in grades K–2~~3~~ have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, fractions, number relationships ("more than," "less than," and "equal to"), and the effects of single-step and multistep computations. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

2.1 The student will

a) read, write, and identify the place value of each digit in a three-digit numeral, ~~using numeration~~ with and without models;

b) identify the number that is 10 more, 10 less, 100 more, and 100 less than a given number up to 999;

c) round two-digit numbers to the nearest ten; and[Reordered]

d) compare and order ~~two~~ whole numbers between 0 and 999~~, using symbols (>, <, or =) and words (*greater than, less than*, or *equal to*)~~.[Symbols and words included in EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The number system is based on a simple pattern of tens where each place has ten times the value of the place to its right. • Numbers are written to show how many hundreds, tens, and ones are in the number.[Moved from middle column] • Opportunities to experience the relationships among hundreds, tens, and ones through hands-on experiences with manipulatives are essential to developing the ten-to-one place value concept of our number system and to understanding the value of each digit in a three-digit number. This structure is helpful when comparing and ordering numbers. • Manipulatives that can be physically connected and separated into groups of tens and leftover ones (e.g., snap cubes, beans on craft sticks, pennies in cups, bundle of sticks, beads on pipe cleaners, etc.) should be used. • Ten-to-one trading activities with manipulatives on place value mats provide ~~excellent~~ experiences for developing the understanding of the places in the Base-10 system.[Reordered] • Models that clearly illustrate the relationships among ~~hundreds, tens, and~~ ones, tens, and hundreds, are physically proportional (e.g., the tens piece is ten times larger than the ones piece). ~~• Students need to understand that 10 and 100 are special units of numbers (e.g., 10 is 10 ones, but it is also 1 ten).~~	~~All students should~~ ~~• Understand the ten-to-one relationship of ones, tens, and hundreds (10 ones equals 1 ten; 10 tens equals 1 hundred).~~ ~~• Understand that numbers are written to show how many hundreds, tens, and ones are in the number.~~[Moved to US] ~~• Understand that rounding gives a close, easy-to-use number to use when an exact number is not needed for the situation at hand.~~ ~~• Understand that a knowledge of place value is essential when comparing numbers.~~ ~~• Understand the relative magnitude of numbers by comparing numbers.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Demonstrate the understanding of the ten-to-one relationships among ones, tens, and hundreds, using manipulatives (e.g., beans and cups, Base-10 blocks, bundles of 10 sticks). (a) ~~• Determine the place value of each digit in a three-digit numeral presented as a pictorial representation (e.g., a picture of Base-10 blocks) or as a physical representation (e.g., actual Base-10 blocks).~~ • Write numerals, using a Base-10 model or ~~picture.~~ pictorial representation (a picture of Base-10 Blocks). (a) • Read three-digit numbers when shown a numeral, a Base-10 model of the number, or a pictorial representation of the number. (a) • Identify and write the place ~~value~~ (ones, tens, hundreds) of each digit in a three-digit numeral. (a) • Determine the value of each digit in a three-digit numeral (e.g., in 352, the 5 represents 5 tens and its value is 50). • Use models to represent numbers in multiple ways, according to place value (e.g., 256 can be 1 hundred, 14 tens, and 16 ones, but also 25 tens and 6 ones). (a) • Use place value understanding to identify the number that is 10 more, 10 less, 100 more, or 100 less than a given number. (b)

2.1 The student will

a) read, write, and identify the place value of each digit in a three-digit numeral, ~~using numeration~~ with and without models;

b) identify the number that is 10 more, 10 less, 100 more, and 100 less than a given number up to 999;

c) round two-digit numbers to the nearest ten; and[Reordered]

d) compare and order ~~two~~ whole numbers between 0 and 999~~, using symbols (>, <, or =) and words (*greater than, less than, or equal to*)~~.[Symbols and words included in EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Flexibility in thinking about numbers is critical~~. For example, 123 is 123 ones; or 1 hundred, 2 tens, and 3 ones; or 12 tens and 3 ones.~~ (e.g., 84 is 8 tens and 4 ones, or 7 tens and 14 ones, or 5 tens and 34 ones, etc.). This flexibility builds background understanding for the ideas used when regrouping. When subtracting 18 from 174, a student may choose to regroup and think of 174 as 1 hundred, 6 tens, and 14 ones. • Rounding a number to the nearest ten means determining which 10 the number is closest to (e.g., 48 rounded to the nearest 10 is 50). • Rounding is an estimation strategy that is often used to assess the reasonableness of a solution or to give an estimate of an amount. ~~• Rounding is finding the nearest easy-to-use number (e.g., the nearest 10) for the situation at hand.~~ • Number lines are useful tools for developing the concept of rounding~~ the nearest ten~~. Rounding to the nearest ten using a number line is done as follows: – Locate the number on the number line. Identify the two closest tens the number comes between. – Determine the closest ten. If the number in the ones place is 5 (halfway between the two tens), round the number to the higher ten. ~~• Once the concept for rounding numbers using a number line is developed, the procedure for rounding numbers to the nearest ten is as follows:~~ ~~– Look one place to the right of the digit in the place you wish to round to.~~		• Round two-digit numbers to the nearest ten. (c) • Compare two numbers between 0 and 999 represented with concrete objects, pictorially or ~~with concrete objects~~ symbolically ~~(e.g., Base-10 blocks),~~ using the symbols (>, <, or =) and the words *greater than*, *less than* or *equal to*. (d) • Order three whole numbers between 0 and 999 represented with concrete objects, pictorially, or symbolically from least to greatest and greatest to least. (d)

2.1 The student will

a) read, write, and identify the place value of each digit in a three-digit numeral, ~~using numeration~~ with and without models;

b) identify the number that is 10 more, 10 less, 100 more, and 100 less than a given number up to 999;

c) round two-digit numbers to the nearest ten; and[Reordered]

d) compare and order ~~two~~ whole numbers between 0 and 999~~, using symbols (>, <, or =) and words (*greater than*, *less than*, or *equal to*)~~.[Symbols and words included in EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~- If the digit is less than 5, leave the digit in the rounding place as it is, and change the digit to the right of the rounding place to zero.~~ ~~- If the digit is 5 or greater, add 1 to the digit in the rounding place, and change the digit to the right of the rounding place to zero.~~ ~~• A procedure for comparing two numbers by examining place value may include the following:~~ ~~- Line up the numbers by place value lining up the ones.~~ ~~- Beginning at the left, find the first place value where the digits are different.~~ ~~- Compare the digits in this place value to determine which number is greater (or which is less).~~ ~~- Use the appropriate symbol > or < or words *greater than* or *less than* to compare the numbers in the order in which they are presented.~~ ~~- If both numbers are the same, use the symbol = or the words *equal to*.~~ • Mathematical symbols (>, <) used to compare two unequal numbers are called *inequality symbols*.		

2.24 The student will

a) count forward by twos, fives, and tens to 120~~100~~, starting at various multiples of 2, 5, or 10;
b) count backward by tens from 120~~100~~; and[Increased to 120 to correlate to Grade 1 change]
c) ~~recognize~~ use objects to determine whether a number is even or~~and~~ odd ~~numbers~~.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Collections of objects can be grouped and skip counting can be used to count the collection.[Moved from middle column.] • The patterns developed as a result of grouping and/or skip counting are precursors for recognizing numeric patterns, functional relationships, ~~and~~ concepts underlying money, and telling time ~~telling, multiplication, and division~~. Powerful models for developing these concepts include counters, a hundred chart, and calculators. ~~• Skip counting by twos supports the development of the concept of even numbers.~~ • Skip counting by fives lays the foundation for reading a clock ~~effectively and telling time~~ to the nearest five minutes and~~,~~ counting nickels~~money, and developing the multiplication facts for five~~. • Skip counting by tens ~~is a precursor~~lays the foundation for use of place value and ~~, addition,~~ counting dimes.~~money, and multiplying by multiples of 10.~~ • Calculators can be used to display the numeric patterns resulting from skip counting. Use the constant feature of the four-function calculator to display the numbers in the sequence when skip counting by that constant. • ~~Odd and even numbers can be explored in different ways (e.g., dividing collections of objects into two equal groups or pairing objects).~~[Moved to EKS]	~~All students should~~ ~~• Understand that collections of objects can be grouped and skip counting can be used to count the collection.~~[Moved to US] ~~• Describe patterns in skip counting and use those patterns to predict the next number in the counting sequence.~~ ~~• Understand that the starting point for skip counting by 2 does not always begin at 2.~~ ~~• Understand that the starting point for skip counting by 5 does not always begin at 5.~~ ~~• Understand that the starting point for skip counting by 10 does not always begin at 10.~~ • ~~Understand that every counting number is either even or odd.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine patterns created by counting by twos, fives, and tens to 120 on a hundred chart. (a) • Describe patterns in skip counting and use those patterns to predict the next number in the counting sequence. (a)[Moved from middle column] • Skip count by twos, fives, and tens to 120~~100~~ from various multiples of 2, 5 or 10, using manipulatives, a hundred chart, mental mathematics, a calculator, and/or paper and pencil. (a) • Skip count by twos to 120 starting from any multiple of 2~~, fives, and tens to 100~~. (a) • Skip count by five to 120 starting at any multiple of 5. (a) • Skip count by ten to 120 starting at any multiple of 10. (a) • Count backward by tens from 120~~100~~. (b) • Use objects to determine whether a number is odd or even (e.g., dividing collections of objects into two equal groups or pairing objects).[Moved example from US] (c)

2.3~~2~~ The student will

a) count and identify the ordinal positions first through twentieth, using an ordered set of objects; and

b) write the ordinal numbers~~.~~ 1st through 20th.[Edited to match EKS]

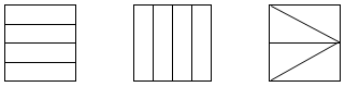
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Understanding t~~The cardinal and ordinal ~~meanings~~ understanding of numbers ~~are~~ is necessary to quantify, measure, and identify the order of objects. • The ordinal meaning of numbers is developed by identifying and verbalizing the place or position of objects in a set or sequence (e.g., a student's position in line when students are lined up alphabetically by first name). • ~~An ordinal number is a number that names the place or position of an object in a sequence or set (e.g., first, third). *Ordered position*, *ordinal position*, and *ordinality* are terms that refer to the place or position of an object in a sequence or set.~~ • The ordinal position is determined by where one starts in an ordered set of objects or sequence of objects (e.g., left, right, top, bottom). • ~~The ordinal meaning of numbers is developed by identifying and verbalizing the place or position of objects in a set or sequence (e.g., a student's position in line when students are lined up alphabetically by first name).~~ • Ordinal position can also be emphasized through sequencing events (e.g., ~~months in a year~~days in a month or sequencing in a story). • ~~Cardinality can be compared with ordinality when comparing the results of counting. There is obvious similarity between the ordinal number words *third* through *twentieth* and the cardinal number words *three* through *twenty*.~~	~~All students should~~ • ~~Use ordinal numbers to describe the position of an object in a sequence or set.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count an ordered set of objects, using the ordinal number words *first* through *twentieth*. (a) • ~~Identify the ordinal positions first through twentieth, using an ordered set of objects.~~ • Identify the ordinal positions first through twentieth, using an ordered set of objects presented in lines or rows from – left to right; – right to left; – top to bottom; and – bottom to top. (a) • Write 1st, 2nd, 3rd, through 20th in numerals. (b)

2.4~~3~~ The student will

a) name and write fractions represented by a set, region, or length model ~~identify the parts of a set and/or region that represent fractions~~ for halves, fourths, eighths, thirds, and~~fourths,~~ sixths~~, eighths, and tenths~~;

b) ~~write the~~ represent fractional~~s~~ parts with models and with symbols; and

c) compare the unit fractions for halves, fourths, eighths, thirds, and~~fourths,~~ sixths~~, eighths, and tenths~~.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Students need opportunities to solve practical problems involving fractions in which students themselves are determining how to subdivide a whole into equal parts, test those parts to be sure they are equal, and use those parts to recreate the whole. • When working with fractions, ~~T~~the whole ~~should~~ must be defined. • A fraction is a way of representing part of a whole (as in a region/area model), ~~or~~ part of a group (as in a set model), or part of a length model. • In ~~each fractiona~~ region/area model, the parts must be equal (i.e.., each ~~pie~~ piece must have the same area)~~; the size of each chip in a set must be equal)~~. ~~In problems with fractions, a whole is broken into equal-size parts and reassembled into one whole.~~ The pieces could have a different shape as shown in the examples below:  • In a set model, each item represents an equal part of the set. For example, in a set of six counters, three counters represent one-half and in a set of three counters, one counter represents one-third.	~~All students should~~ • ~~Understand that fractional parts are equal shares of a whole or a whole set.~~ • ~~Understand that the fraction name (*half, fourth*) tells the number of equal parts in the whole.~~ • ~~Understand that when working with unit fractions, the larger the denominator, the smaller the part and therefore the smaller the fraction.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize fractions as representing equal-size parts of a whole. (a) • Name and write fractions represented by a set model showing halves, fourths, eighths, thirds, and sixths. (a, b) • Name and write fractions represented by a region/area model showing halves, fourths, eighths, thirds, and sixths. (a, b) • Name and write fractions represented by a length model showing halves, fourths, eighths, thirds, and sixths. (a, b) • Using same-size fraction pieces, from region/area models or length/measurement models, count the pieces (e.g., *one-fourth*, *two-fourths*, *three-fourths*, etc.) and compare those pieces to one whole (e.g., *four-fourths will make one whole; five-fourths is more than one whole* or *is equal to one whole and one fourth*). (a) • ~~Identify the fractional parts of a whole or a set for~~ $\cancel{\frac{2}{2}, \frac{2}{3}, \frac{3}{4}, \frac{2}{6}, \frac{7}{8}, \frac{7}{10}}$~~, etc.~~ • ~~Identify the fraction names (halves, thirds, fourths, sixths, eighths, tenths) for the fraction notations~~ $\cancel{\frac{2}{2}, \frac{2}{3}, \frac{3}{4}, \frac{2}{6}, \frac{7}{8}, \frac{7}{10}}$~~, etc.~~

2.4~~3~~ The student will

a) name and write fractions represented by a set, region, or length model ~~identify the parts of a set and/or region that represent fractions~~ for halves, fourths, eighths, thirds, and~~fourths,~~ sixths~~, eighths, and tenths~~;

b) ~~write the~~ represent fractional~~s~~ parts with models and with symbols; and

c) compare the unit fractions for halves, fourths, eighths, thirds, and~~fourths,~~ sixths~~, eighths, and tenths~~.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• In a length model, each length represents an equal part of the whole. For example, in a cube train with a length of 8 cubes, each cube represents one-eighth. Students would be able to state that there are eight one-eighths and that is the same as one whole. If they have 9 cubes, they could make one whole train with one-eighth left over. Students will learn to write names for fractions greater than one and for mixed numbers in grade 3. • Students need opportunities to use models (region/area or length/measurement) to count fractional parts that go beyond one whole. For instance, if students are counting five pie pieces, where each piece is equivalent to one-fourth of a pie, they might say "one-fourth, two-fourth, three-fourths, four-fourths, five-fourths." As a result, they begin to develop flexibility in naming this amount in different ways (e.g., five-fourths or one and one-fourth). They will begin to generalize that when the numerator and the denominator are the same, they have one whole. They also see a fraction as the sum of unit fractions (e.g., $\frac{5}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$). • Creating models that have a fractional value greater than one whole and describing those models as having a whole and leftover equal-sized pieces are the foundation for understanding mixed numbers in grade 3.		• Represent with models and with symbols fractional parts of a whole for halves, fourths, eighths, thirds, and sixths. ~~halves, thirds, fourths, sixths, eighths, tenths using~~ – region/area models (e.g., pie pieces, pattern blocks, geoboards); – sets (e.g., chips, counters, cubes); and – length/measurement models (e.g., fraction strips or bars, rods, connecting cube trains). (b) • Compare unit fractions with models or pictures ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$ ~~, and~~) using the words greater than, less than or equal to and the symbols (>, <, =). (c)

2.4~~3~~ The student will

a) name and write fractions represented by a set, region, or length model ~~identify the parts of a set and/or region that represent fractions~~ for halves, fourths, eighths, thirds, and~~fourths,~~ sixths~~, eighths, and tenths~~;

b) ~~write the~~ represent fractional~~s~~ parts with models and with symbols; and

c) compare the unit fractions for halves, fourths, eighths, thirds, and~~fourths,~~ sixths~~, eighths, and tenths~~.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• When given a fractional part of a whole and its value (e.g., one-third), students should explore how many one-thirds it will take to build one whole to build two wholes, etc. If this $\square$ is $\frac{1}{3}$, then this is the whole $\square\square\square$. If this is the whole $\square\square\square$,then this $\square$ is $\frac{1}{3}$. • Students should have experiences dividing a whole into additional parts. As the whole is divided into more parts, students understand that each part becomes smaller (e.g., folding a paper in half one time, creates 2 halves; folding it in half again, creates 4 fourths, which are smaller; folding it in half again, creates 8 eighths, which are even smaller). The same concept can be applied to thirds and sixths. • The value of a fraction is dependent on both the number of equal parts in a whole (denominator) and the number of those parts being considered (numerator). • ~~The denominator tells how many equal parts are in the whole or set. The numerator tells how many of those parts are being described.~~ • Students should have opportunities to make connections among fraction representations by connecting concrete or pictorial representations with spoken or symbolic representations.		

2.4~~3~~ The student will

a) name and write fractions represented by a set, region, or length model ~~identify the parts of a set and/or region that represent fractions~~ for halves, fourths, eighths, thirds, and~~fourths,~~ sixths~~, eighths, and tenths~~;

b) ~~write the~~ represent fractional~~s~~ parts with models and with symbols; and

c) compare the unit fractions for halves, fourths, eighths, thirds, and~~fourths,~~ sixths~~, eighths, and tenths~~.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions will further this development (e.g., *thirds* means "three equal parts of a whole" or $\frac{1}{3}$ represents one of three equal-size parts when a pizza is shared among three students)~~ will further this development~~. • A unit fraction is when there is a one as ~~in which~~ the numerator ~~is one~~. • Using models when comparing unit fractions builds a mental image of fractions and the understanding that as the number of pieces of a whole increases, the size of one single piece decreases ~~will assist in developing the concept that~~ (i.e., the larger the denominator the smaller the piece; therefore, $\frac{1}{3} > \frac{1}{4}$).		

A variety of contexts and problem types are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from real-life experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as "How many books do we have altogether?" or "How many cookies are left if I eat two?" or "I have three more candies than you do." Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling, discussing, and recording a variety of problem situations. This approach helps students transition from the concrete to the representation to the symbolic in order to develop meaning for the operations and how they relate to each other.

In grades K–2~~3~~, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition and~~,~~ subtraction~~, multiplication, division and related facts~~;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as skip counting and solving problems that involve equal groupings of objects ~~and~~as well as problems that involve sharing equally, the initial work ~~; and performing initial operations~~ with fractions.

2.5 The student will ~~recall~~

a) demonstrate fluency with addition facts ~~with~~ for sums to 20 or less and the corresponding subtraction facts; and

b) recognize and use the relationships between addition and subtraction to solve single-step practical problems.[Moved from 2.9]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently. • Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship. • Associate the terms *addition, adding, plus,* and *sum* with the concept of joining or combining. • Associate the terms *subtraction, subtracting, minus*, and *difference* with the concept of ~~process of "taking away" or separating (i.e.,~~ removing a set of objects from the given set of objects, ~~finding~~ determining the difference between two numbers, or comparing two numbers~~)~~. • Concrete models should be used initially to develop an understanding of addition and subtraction facts. • Recognizing and using patterns and learning to represent situations mathematically are important aspects of primary mathematics.[Moved from EKS 2.21] • An equation (number sentence) is a mathematical statement that two expressions are equivalent. It consists of two expressions, one on each side of an 'equal' symbol (e.g., 5 + 3 = 8, 8 = 5 + 3 and 4 + 3 = 9 - 2). • An equation can be represented using balance scales, with equal amounts on each side (e.g., 3 + 5 = 6 + 2). • An expression is a representation of a quantity. It contains numbers, variables, and/or computational operation symbols. It does not have an equal sign (e.g., 5, 4 + 3, 8-2). It is not necessary for students at this level to use the term 'expression.'	~~All students should~~ • ~~Understand that addition involves combining and subtraction involves separating.~~ • ~~Develop fluency in recalling facts for addition and subtraction.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Recall and write the basic~~Demonstrate fluency with addition facts for sums to 20 or less and the corresponding subtraction facts~~, when addition or subtraction problems are presented in either horizontal or vertical written format~~. (a)[Moved to US] • Recognize and use the relationship between addition and subtraction to solve single-step practical problems. (b) • Determine the missing number in an equation (number sentence) (e.g., 3 + __ = 5 or __ + 2 = 5; 5 – __ = 3 or 5 – 2 = __). (b)[Moved from 2.9 EKS] • Write the related facts for a given addition or subtraction fact (e.g., given 3 + 4 = 7, write 7 – 4 = 3 and 7 – 3 = 4). (b)[Moved from 2.9 EKS]

2.5 The student will ~~recall~~
a) demonstrate fluency with addition facts ~~with~~ for sums to 20 or less and the corresponding subtraction facts; and
b) recognize and use the relationships between addition and subtraction to solve single-step practical problems.[Moved from 2.9]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The patterns formed by related facts facilitate the solution of problems involving a missing addend in an addition sentence or a missing part in a subtraction sentence.[Moved from EKS 2.21] • Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the ~~basic~~ addition and subtraction facts include – counting on; – counting back; – "one more than," "two more than" ~~facts~~; – "one less than," "two less than" ~~facts~~; – "doubles" ~~to recall addition facts~~ (e.g., 2 + 2 =__; 3 + 3 =__); – "near doubles" [e.g., 3 + 4 = (3 + 3) + 1 = __]; – "make ten" facts (7 + 4 can be thought of as 7 + 3 + 1 in order to make a ten)~~(e.g., at least one addend of 8 or 9)~~; – "think addition for subtraction," (e.g., for 9 – 5 = __, think "5 and what number makes 9?"); – use of the commutative property, without naming the property (e.g., 4 + 3 is the same as 3 + 4); – use of related facts (e.g., 4 + 3 = 7 , 3 + 4 = 7, 7 – 4 = 3, and 7 – 3 = 4); ~~and~~ – use of the additive identity property (e.g., 4 + 0 = 4), without naming the property but saying, "When you add zero to a number, you always get the original number~~.~~"; and – use patterns to make sums (e.g., 0 + 5 = 5, 1 + 4 = 5, 2 + 3 = 5, etc.) • Addition and subtraction problems should be presented in both horizontal and vertical written format. Equations may be written with sums and differences at the beginning of the equation (e.g., 8 = 5 + 3). [Moved from EKS]		

2.5 The student will ~~recall~~

a) demonstrate fluency with addition facts ~~with~~ for sums to 20 or less and the corresponding subtraction facts; and

b) recognize and use the relationships between addition and subtraction to solve single-step practical problems.[Moved from 2.9]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Models such as 10 or 20 frames and part-part-whole diagrams help develop an understanding of relationships between equations and operations. • ~~Manipulatives should be used initially to develop an understanding of addition and subtraction facts and to engage students in meaningful memorization. Rote recall of the facts is often achieved through constant practice and may come from a variety of formats, including presentation through counting on, related facts, flash cards, practice sheets, and/or games.~~		

2.6 The student~~, given two whole numbers whose sum is 99 or less,~~ will

a) estimate ~~the~~ sums and differences; ~~and~~[Differences moved from 2.7a]

b) ~~find the~~ determine sums and differences, using various methods ~~of calculation.~~; and[Differences moved from 2.7b]

c) create and solve single-step and two-step practical problems involving addition and subtraction.[Moved from 2.8 and 2.21]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• An equation (number sentence) is a mathematical statement representing two expressions that are equivalent. It consists of two expressions, one on each side of an 'equal' symbol (e.g., 5 + 3 = 8, 8 = 5 + 3 and 4 + 3 = 9 - 2). An equation can be represented using a balance scale, with equal amounts on each side (e.g., 3 + 5 = 6 + 2). • An expression represents a quantity. It contains numbers, variables, and/or computational operation symbols. It does not have an equal sign (e.g., 5, 4 + 3, 8-2). At this level, students are not expected to understand use the terms expression or variable. • Estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed. • ~~Estimation is a number sense skill used instead of finding an exact answer. When an actual computation is not necessary, an estimate will suffice.~~ • Rounding is one strategy used to estimate. • Estimation ~~is also~~can be used ~~before solving a problem~~ to check the reasonableness of the sum or difference when an exact answer is required. • Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship. • Students should experience a variety of problem types related to addition and subtraction. Problem type examples are included in the following chart:	~~All students should~~ • ~~Understand that estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed.~~ • ~~Understand that estimation skills are also valuable in determining the reasonableness of the sum when solving for the exact answer is needed.~~ • ~~Understand that addition is used to join groups in practical situations when exact answers are needed.~~ • ~~Develop flexible methods of adding whole numbers by combining numbers in a variety of ways to find the sum, most depending on place values.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Regroup 10 ones for 1 ten, using Base-10 models, when finding the sum of two whole numbers whose sum is 99 or less.~~ • Estimate the sum of two whole numbers whose sum is 99 or less and recognize whether the estimation is reasonable. (a) • Estimate the difference between two whole numbers each 99 or less and recognize whether the estimate is reasonable. (a) • ~~Find~~ Determine the sum of two whole numbers whose sum is 99 or less~~, using Base-10 models, such as Base-10 blocks and bundles of tens~~. (b) • Determine the difference of two whole numbers each 99 or less. (b)[Moved to 2.6b] • Solve single-step and two-step practical problems ~~presented vertically or horizontally~~ that require ~~finding~~ determining the sum of two whole numbers whose sum is 99 or less~~, using paper and pencil~~. (c) • ~~Solve problems, using mental computation strategies, involving addition of two whole numbers whose sum is 99 or less.~~ • Solve single-step and two-step practical problems that require determining the difference between two whole numbers each 99 or less. (c)

2.6 The student~~, given two whole numbers whose sum is 99 or less,~~ will

a) estimate ~~the~~ sums and differences; ~~and~~[Differences moved from 2.7a]

b) ~~find the~~ determine sums and differences, using various methods ~~of calculation.~~; and[Differences moved from 2.7b]

c) create and solve single-step and two-step practical problems involving addition and subtraction.[Moved from 2.8 and 2.21]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS

GRADE 2 - ADDITION AND SUBTRACTION PROBLEM TYPES

Join (Result Unknown)	Join (Change Unknown)	Join (Start Unknown)
Sue had 28 pennies. Josh gave her 14 more. How many pennies does Sue have altogether?	Sue had 28 pennies. Josh gave her some more. Now Sue has 56 pennies. How many did Josh give her?	Sue had some pennies. Josh gave her 14 more. Now Sue has 53 pennies. How many pennies did Sue have to begin with?
Separate (Result Unknown)	**Separate (Change Unknown)**	**Separate (Start Unknown)**
Sue had 45 pennies. She gave 26 pennies to Josh. How many pennies does Sue have now?	Sue had 61 pennies. She gave some to Josh. Now she has 30 pennies. How many did she give to Josh?	Sue had some pennies. She gave 14 to Josh. Now she has 34 pennies left. How many pennies did Sue have to begin with?
Part-Part-Whole (Whole Unknown)	**Part-Part-Whole (Part Unknown)**	**Part-Part-Whole (Both Parts Unknown)**
Josh has 14 pennies and 18 nickels. How many coins does he have?	Josh has 72 coins. Fifty of the coins are pennies, and the rest are nickels. How many nickels does Josh have?	Josh has 23 coins. Some are pennies and some are nickels. How many pennies and nickels could he have?
Compare (Difference Unknown)	**Compare (Larger Unknown)**	**Compare (Smaller Unknown)**
Josh has 22 pennies and Sue has 8 pennies. How many more pennies does Josh have than Sue?	Josh has 12 more pennies than Sue. Sue has 38 pennies. How many pennies does Josh have?	Josh has 8 more pennies than Sue. Josh has 12 pennies. How many pennies does Sue have?
Josh has 22 pennies. Sue has 8 pennies. How many fewer pennies does Sue have than Josh?	Sue has 12 fewer pennies than Josh. Sue has 18 pennies. How many pennies does Josh have?	Sue has 4 fewer pennies than Josh. Josh has 12 pennies. How many pennies does Sue have?

2.6 The student~~, given two whole numbers whose sum is 99 or less,~~ will

a) estimate ~~the~~ sums and differences; ~~and~~[Differences moved from 2.7a]

b) ~~find the~~ determine sums and differences, using various methods ~~of calculation.~~; and[Differences moved from 2.7b]

c) create and solve single-step and two-step practical problems involving addition and subtraction.[Moved from 2.8 and 2.21]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• By estimating the result of an addition problem, a place value orientation for the answer is established.~~ • In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as *in all, altogether, difference, etc.* encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions. • Strategies for adding and subtracting two-digit numbers can include, but are not limited to, using a hundred chart, number line, and invented strategies. • Mental computation helps build number sense in students. Strategies for mentally adding two-digit numbers should be ~~include~~ student-invented strategies. Some of these strategies may include:~~, making ten, partial sums, and counting on, among others.~~ ~~• partial sums: 56 + 41 = __~~ ~~50 + 40 = 90~~ ~~6 + 1 = 7~~ ~~90 + 7 = 97~~ ~~• counting on: 36 + 62 = __~~ ~~36 + 60 = 96~~ ~~96 + 2 = 98~~ 56 + 41 = __ 36 + 62 = __ 50 + 40 = 90 36 + 60 = 96 6 + 1 = 7 96 + 2 = 98 90 + 7 = 97 ~~• Addition means to combine or join quantities.~~		

2.6 The student~~, given two whole numbers whose sum is 99 or less,~~ will

a) estimate ~~the~~ sums and differences; ~~and~~[Differences moved from 2.7a]

b) ~~find the~~ determine sums and differences, using various methods ~~of calculation.~~; and[Differences moved from 2.7b]

c) create and solve single-step and two-step practical problems involving addition and subtraction.[Moved from 2.8 and 2.21]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• The terms used in addition are~~ ~~23 › *addend*~~ ~~+ 46 › *addend*~~ ~~69 › *sum*~~ ~~• Strategies for adding two-digit numbers can include, but are not limited to, using a hundreds chart, number line, and invented strategies.~~ ~~• Building an understanding of the algorithm by first using concrete materials and then a do-and-write approach connects it to the written form of the algorithm.~~ ~~• The traditional algorithm for two-digit numbers is contrary to the natural inclination to begin with the left-hand number.~~ ~~• Regrouping is used in addition when a sum in a particular place value is 10 or greater.~~ • Mental computational strategies for subtracting two-digit numbers should be student-invented strategies. Some of these strategies may include: 87 – 25 = __ 20 + 60 = 80 5 + 2 = 7 60 + 2 = 62 87 – 25 = __ 25 + 60 = 85 85 + 2 = 87 60 + 2 = 62 87 – 25 = __ 25 + 2 = 27 27 + 60 = 87 2 + 60 = 62 98 – 41 = __ 98 – 40 = 58 58 – 1 = 57 56 – 21 = __ 50 – 20 = 30 6 – 1 = 5 30 + 5 = 35		

2.6 The student~~, given two whole numbers whose sum is 99 or less,~~ will

a) estimate ~~the~~ sums and differences; ~~and~~[Differences moved from 2.7a]

b) ~~find the~~ determine sums and differences, using various methods ~~of calculation.~~; and[Differences moved from 2.7b]

c) create and solve single-step and two-step practical problems involving addition and subtraction.[Moved from 2.8 and 2.21]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The terms used in addition are 23 → *addend* + 46 → *addend* 69 → *sum* • The terms often used in subtraction are *minuend* → 98 *subtrahend* → – 41 *difference* → 57 • At this level, students do not need to use the terms shown above. • Problem solving means engaging in a task for which a solution or a method of solution is not known in advance. Solving problems using data and graphs offers one way to connect mathematics to practical situations. • The problem solving process is enhanced when students – create their own story problems; and – model word problems, using manipulatives or drawings. • The least number of steps necessary to solve a single-step problem is one.		

~~**2.7 The student, given two whole numbers, each of which is 99 or less, will**~~

~~a) estimate the difference; and~~[Moved to 2.6a]

~~b) find the difference, using various methods of calculation.~~[Moved to 2.6b]

~~UNDERSTANDING THE STANDARD~~ ~~**(Background Information for Instructor Use Only)**~~	~~**ESSENTIAL UNDERSTANDINGS**~~	~~**ESSENTIAL KNOWLEDGE AND SKILLS**~~
• ~~Estimation is a number sense skill used instead of finding an exact answer. When an estimate is needed, the actual computation is not necessary.~~ • ~~Rounding is one strategy used to estimate.~~ • ~~Estimation is also used before solving a problem to check the reasonableness of the sum when an exact answer is required.~~ • ~~By estimating the result of a subtraction problem, a place value orientation for the answer is established.~~ • ~~Subtraction is the inverse operation of addition and is used for different reasons:~~ ~~- to remove one amount from another;~~ ~~- to compare one amount to another; and~~ ~~- to find the missing quantity when the whole quantity and part of the quantity are known.~~ • ~~Three terms often used in subtraction are~~ ~~*minuend* › 98~~ ~~*subtrahend* › 41~~ ~~*difference* › 57~~ • ~~Regrouping is a process of renaming a number to make subtraction easier.~~ • ~~An understanding of the subtraction algorithm should be built by first using concrete materials and then employing a do-and-write approach (i.e., use the manipulatives, then record what you have done). This connects the activity to the written form of the algorithm.~~ • ~~Mental computational strategies for subtracting two-digit numbers might include~~ ~~- lead-digit or front-end strategy:~~	~~All students should~~ • ~~Understand that estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed.~~ • ~~Understand that estimation skills are also valuable in determining the reasonableness of the difference when solving for the exact answer is needed.~~ • ~~Understand that subtraction is used in practical situations when exact answers are needed.~~ • ~~Develop flexible methods of subtracting whole numbers to find the difference, by combining numbers in a variety of ways, most depending on place values.~~	~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~ • ~~Regroup 1 ten for 10 ones, using Base-10 models, such as Base-10 blocks and bundles of tens.~~ • ~~Estimate the difference of two whole numbers each 99 or less and recognize whether the estimation is reasonable.~~[Moved to 2.6a] • ~~Find the difference of two whole numbers each 99 or less, using Base-10 models, such as Base-10 blocks and bundles of tens.~~[Moved to 2.6b] • ~~Solve problems presented vertically or horizontally that require finding the difference between two whole numbers each 99 or less, using paper and pencil.~~[Moved to 2.6b] • ~~Solve problems, using mental computation strategies, involving subtraction of two whole numbers each 99 or less.~~[Moved to 2.6 US]

~~2.7 The student, given two whole numbers, each of which is 99 or less, will~~

~~a) estimate the difference; and~~[Moved to 2.6a]

~~b) find the difference, using various methods of calculation.~~[Moved to 2.6b]

~~UNDERSTANDING THE STANDARD~~ ~~**(Background Information for Instructor Use Only)**~~	~~**ESSENTIAL UNDERSTANDINGS**~~	~~**ESSENTIAL KNOWLEDGE AND SKILLS**~~
~~56 − 21 = __~~ ~~50 − 20 = 30~~ ~~6 − 1 = 5~~ ~~30 + 5 = 35~~ ~~- counting up:~~ ~~87 − 25 = __~~ ~~20 + 60 = 80~~ ~~5 + 2 = 7~~ ~~60 + 2 = 62~~ ~~or~~ ~~87 − 25 = __~~ ~~25 + 60 = 85~~ ~~85 + 2 = 87~~ ~~60 + 2 = 62~~ ~~or~~ ~~87 − 25 = __~~ ~~25 + 2 = 27~~ ~~27 + 60 = 87~~ ~~2 + 60 = 62~~ ~~- partial differences:~~ ~~98 − 41 = __~~ ~~90 − 40 = 50~~ ~~8 − 1 = 7~~ ~~50 + 7 = 57.~~ ~~• Strategies for subtracting two-digit numbers may include using a hundreds chart, number line, and invented strategies.~~		

~~2.8~~ ~~The student will create and solve one- and two-step addition and subtraction problems, using data from simple tables, picture graphs, and bar graphs.~~[Combined with 2.6c]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Problem solving means engaging in a task for which a solution or a method of solution is not known in advance. Solving problems using data and graphs offers a natural way to connect mathematics to practical situations.~~ • ~~The ability to retrieve information from simple charts and picture graphs is a necessary prerequisite to solving problems.~~ • ~~An example of an approach to solving problems is Polya's four-step plan:~~ ~~- Understand: Retell the problem.~~ ~~- Plan: Decide what the operation is.~~ ~~- Solve: Write a number sentence.~~ ~~- Look back: Does the answer make sense?~~ • ~~The problem solving process is enhanced when students~~ ~~- create their own story problems; and~~ ~~- model word problems, using manipulatives or drawings.~~	~~All students should~~ • ~~Develop strategies for solving practical problems.~~ • ~~Enhance problem solving skills by creating their own problems.~~	~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~ • ~~Identify the appropriate data and the operation needed to solve an addition or subtraction problem where the data are presented in a simple table, picture graph, or bar graph.~~[Combined with 2.6] • ~~Solve addition and subtraction problems requiring a one- or two-step solution, using data from simple tables, picture graphs, bar graphs, and everyday life situations.~~[Combined with 2.6] • ~~Create a one- or two-step addition or subtraction problem using data from simple tables, picture graphs, and bar graphs whose sum is 99 or less.~~[Combined with 2.6]

~~2.9 The student will recognize and describe the related facts that represent and describe the inverse relationship between addition and subtraction.~~[Moved to 2.5b]

~~UNDERSTANDING THE STANDARD~~ ~~**(Background Information for Instructor Use Only)**~~	~~**ESSENTIAL UNDERSTANDINGS**~~	~~**ESSENTIAL KNOWLEDGE AND SKILLS**~~
• ~~Addition and subtraction are inverse operations, that is, one undoes the other:~~ ~~3 + 4 = 7 7 − 3 = 4~~ ~~7 − 4 = 3 4 + 3 = 7~~ • ~~For each addition fact, there is a related subtraction fact.~~ • ~~Developing strategies for solving missing addends problems and the missing part of subtraction facts builds an understanding of the link between addition and subtraction. To solve~~ ~~9 − 5 = __, think 5 + __ = 9.~~ • ~~Demonstrate joining and separating sets to investigate the relationship between addition and subtraction.~~	~~All students should~~ • ~~Understand how addition and subtraction relate to one another.~~	~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~ • ~~Determine the missing number in a number sentence (e.g., 3 + __ = 5 or __ + 2 = 5; 5 − __ = 3 or~~ ~~5 − 2 = __).~~ [Moved to 2.5 EKS] • ~~Write the related facts for a given addition or subtraction fact (e.g., given 3 + 4 = 7, write 7 − 4 = 3 and 7 − 3 = 4).~~ [Moved to 2.5 EKS]

The exploration of measurement and geometry in the primary grades allows students to learn more about the world around them. Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K~~–3~~ 2~~should~~ encounter measurement in ~~many normal situations, from~~ their daily lives, from their use of the calendar and ~~from~~ science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, ~~area, perimeter, volume,~~ and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both techniques and nonstandard and standard tools to ~~find~~ determine measurements and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

[Moved from Geometry strand introduction]
Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.
The focus of instruction at this level is on

- observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).
The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.

2.7~~10~~ The student will

a) count and compare a collection of pennies, nickels, dimes, and quarters whose total value is $2.00 or less; and

b) ~~correctly~~ use the cent symbol ~~(¢)~~, dollar symbol ~~($)~~, and decimal point ~~(.)~~.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~•~~ The money system used in the United States consists of coins and bills based on ones, fives, and tens.~~, making it easy to count money.~~ • The dollar is the basic unit. ~~• Emphasis is placed on the verbal expression of the symbols for cents and dollars (e.g., $0.35 and 35¢ are both read as "thirty-five cents"; $3.00 is read as "three dollars").~~[Reordered] ~~• Money can be counted by grouping coins and bills to determine the value of each group and then adding to determine the total value.~~ • The value of a collection of coins and bills can be determined by counting on, beginning with the highest value, and/or by grouping the coins and bills into groups that are easier to count. • Simulate everyday opportunities to count and compare a collection of coins and one-dollar bills whose total value is $2.00 or less. • Emphasis is placed on the verbal expression of the symbols for cents and dollars (e.g., $0.35 and 35¢ are both read as "thirty-five cents"; $2.00 is read as "two dollars"). • ~~The most common way to add amounts of money is to "count on" the amount to be added.~~	~~All students should~~ ~~• Understand how to count and compare a collection of coins and one-dollar bills whose total value is $2.00 or less.~~ ~~• Understand the proper use of the cent symbol (¢), dollar sign ($), and decimal point (.).~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the value of a collection of coins and one-dollar bills whose total value is $2.00 or less. (a) • Count by ones, fives, tens, and twenty-fives to determine the value of a collection of coins whose total value is $2.00 or less. (a)[Moved from 1.7 EKS] • Compare the values of two sets of coins and one-dollar bills (each set having a total value of $2.00 or less), using the terms *greater than*, *less than*, or *equal to*. ~~• Simulate everyday opportunities to count and compare a collection of coins and one-dollar bills whose total value is $2.00 or less.~~[Moved to US] • Use the cent (¢) and dollar ($) symbols and decimal point (.) to write a value of money which is $2.00 or less. (b)

2.8~~11~~ The student will estimate and measure

a) length to the nearest ~~centimeter and~~ inch;[Centimeters included in 3.8]

b) weight/mass of objects ~~in~~ to the nearest pound~~s/ounces and kilograms/grams, using a scale; and~~. [Ounces, kilograms/grams included in 4.8]

~~c) liquid volume in cups, pints, quarts, gallons, and liters.~~[Included in 3.8]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The process of measurement involves selecting a unit of measure, comparing the unit to the object to be measured, counting the number of times the unit is used to measure the object, and arriving at an approximate total number of units. [Copied from 1.10] • Measurement involves comparing an attribute of an object to the same attribute of the unit of measurement (e.g., the length of a cube measures the length of a book; the weight/mass of the cube measures the weight/mass of the book.). • A clear concept of the size of one unit is necessary before one can measure to the nearest unit. ~~• Knowledge of the exact relationships within the metric or U.S. Customary system of measurement for measuring liquid volume, such as 4 cups to a quart, is not required at this grade level.~~ [Included in 4.6 and 5.8] ~~• Practical experiences measuring liquid volume, using a variety of actual measuring devices (e.g., containers for a cup, pint, quart, gallon, and liter), will help students build a foundation for estimating liquid volume with these measures.~~ • The experience of making a ruler out of individual units of length can lead to greater understanding of using one. A ruler takes those units of length and numbers them. Measurement of length is counting the number of units. A "broken ruler" is a useful tool for students to use in order to develop an understanding of counting the number of units. • Students benefit from experiences that allow them to explore the relationship between the size of the unit of measurement and the number of units needed to measure the length of an object.	~~All students should~~ ~~• Understand that centimeters/inches are units used to measure length.~~ ~~• Understand how to estimate and measure to determine a linear measure to the nearest centimeter and inch.~~ ~~• Understand that pounds/ounces and kilograms/grams are units used to measure weight/mass.~~ ~~• Understand how to use a scale to determine the weight/mass of an object and use the appropriate unit for measuring weight/mass.~~ ~~• Understand that cups, pints, quarts, gallons, and liters are units used to measure liquid volume.~~ ~~• Understand how to use measuring devices to determine liquid volume in both metric and customary units.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify a ruler as an instrument to measure length. (a)[Moved from K.8 EKS] • Estimate and then measure the length of various line segments and objects to the nearest inch ~~and centimeter.~~using a ruler. (a) • Identify different types of scales as instruments to measure weight/mass.[Moved from K.8 EKS] (b) • Estimate and then measure the weight/mass of objects to the nearest pound.~~s/ounces and kilograms/grams,~~ using a scale. (b) ~~• Estimate and measure liquid volume in cups, pints, quarts, gallons, and liters.~~[Included in 3.8 EKS]

2.8~~11~~ The student will estimate and measure

a) length to the nearest ~~centimeter and~~ inch;[Centimeters included in 3.8]

b) weight/mass of objects ~~in~~ to the nearest pound~~s/ounces and kilograms/grams, using a scale; and~~. [Ounces, kilograms/grams included in 4.8]

~~c) liquid volume in cups, pints, quarts, gallons, and liters.~~[Included in 3.8]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• Proper placement of a ruler when measuring length (i.e., placing the end of the ruler at one end of the item to be measured) should be demonstrated.~~ • Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depend~~ent~~ing on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • A balance is a scale for measuring mass. To determine the mass of an object by using a two-pan balance, first level both sides of the balance by putting standard units of mass on one side to counterbalance the object on the other; then ~~find~~ determine the sum of the standard units of mass required to level the balance. • Benchmarks of common objects need to be established for one pound ~~and one kilogram~~. Practical experience measuring the mass of familiar objects helps to establish benchmarks. ~~• Pounds and kilograms are not compared at this level.~~ ~~• The terms *cups*, *pints*, *quarts*, *gallons*, and *liters* are introduced as terms used to describe the liquid volume of everyday containers.~~ • ~~The exact relationship between a quart and a liter is not expected at this level.~~		

2.9~~12~~ The student will tell and write time to the nearest five minutes, using analog and digital clocks.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Telling time requires reading a clock. The position of the two hands on an analog clock is read to tell the time. A digital clock shows the time by displaying the time in numbers which are read as the hour and minutes. • Counting by fives is beneficial when telling time to the nearest five minutes.[Moved from middle column.] • Students should develop an understanding that there are sixty minutes in an hour. • The use of a demonstration clock with gears ensures that the positions of the hour hand and the minute hand are precise at all times. • ~~The face of an analog clock can be divided into 4 equal parts, called *quarter hours*, of 15 minutes each.~~	~~All students should~~ • ~~Apply an appropriate technique to determine time to the nearest five minutes, using analog and digital clocks.~~ • ~~Demonstrate an understanding of counting by fives to predict five minute intervals when telling time to the nearest five minutes.~~[Moved to US and edited]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Show, tell, and write time to the nearest five minutes, using an analog and digital clock. • Match a written time (e.g., 4:20, 10:05, 1:50) to a time shown on a clock face to the nearest five minutes.

2.10~~13~~ The student will

a) determine past and future days of the week; and

b) identify specific days and dates on a given calendar.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The calendar is a way to represent units of time (e.g., days, weeks, ~~and~~ months, and years). • Using a calendar develops the concept of day as a 24-hour period rather than a period of time from sunrise to sunset. • Practical situations are appropriate to develop a sense of the interval of time between events (e.g., ~~Boy Scout~~club meetings occur every week on Monday: there is a week between meetings).	~~All students should~~ ~~Understand how to use a calendar as a way to measure time.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the days/dates before and after a given day/date. (a) • Determine the day that is a specific number of days or weeks in the past or in the future from a given date, using a calendar. (a) • Identify specific days and dates (e.g., What is the third Monday in a given month? On ~~or~~ what day of the week does May 11th fall? ~~on~~). (b)

2.11~~14~~ The student will read the temperature ~~on a Celsius and/or Fahrenheit thermometer~~ to the nearest 10 degrees.[Celsius/Fahrenheit included in EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The symbols for degrees in Celsius (°C) and degrees in Fahrenheit (°F) should be used to write temperatures. • Celsius and Fahrenheit temperatures should be related to everyday occurrences by measuring the temperature of ~~the classroom, the outside, liquids, body temperature, and other things found in the environment~~things found in the student's environment (e.g., temperature of the classroom; temperature on the playground; temperature of warm and cold liquids; body temperature). • Estimating and measuring temperatures in the environment in Fahrenheit and Celsius require the use of real thermometers. • A variety of physical models (e.g., circular, linear) should be used to represent the temperature determined by a real thermometer. • A physical or pictorial model can be used to represent the temperature ~~determined by~~measured using a real thermometer.	~~All students should~~ • ~~Understand how to measure temperature in Celsius and Fahrenheit with a thermometer.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify different types of thermometers as instruments used to measure temperature.[Moved from K.8 EKS] • Read temperature in Celsius and Fahrenheit to the nearest ten degrees on thermometers (real world, physical model, and pictorial representation). ~~Read temperature to the nearest 10 degrees from real Celsius and Fahrenheit thermometers and from physical models (including pictorial representations) of such thermometers.~~

[Moved to Measurement and Geometry strand introduction]

~~Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.~~

~~The focus of instruction at this level is on~~

- ~~observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;~~
- ~~sorting objects and ordering them directly by comparing them one to the other;~~
- ~~describing, comparing, contrasting, sorting, and classifying figures; and~~
- ~~exploring symmetry, congruence, and transformation.~~

~~In the primary grades, children begin to develop basic vocabulary related to these figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).~~

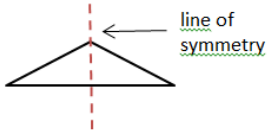
~~The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.~~

- ~~**Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.~~
- ~~**Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)~~
- ~~**Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)~~

2.12~~15~~ The student will

a) draw a line of symmetry in a figure; and

b) identify and create figures with at least one line of symmetry.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A line of symmetry divides a ~~symmetrical~~ figure~~, object, or arrangement of objects~~ into two congruent parts~~, that are congruent if one part is reflected over the line of symmetry.~~each of which is the mirror image of the other. An example is shown below:  Lines of symmetry are not limited to horizontal and vertical lines. • Children learn about symmetry through hands-on experiences with geometric figures and the creation of geometric pictures and patterns. • Guided explorations of the study of symmetry ~~by~~ using mirrors, ~~miras,~~ paper folding, and pattern blocks will enhance students' understanding of the attributes of symmetrical figures. • Congruent figures have exactly the same size and shape. Noncongruent figures do not have exactly the same size and shape. Congruent figures remain congruent even if they are in different spatial orientations. • While investigating symmetry, children move figures, such as pattern blocks, intuitively, thereby exploring transformations of those figures. A transformation is the movement of a figure — either a translation, rotation, or reflection. A translation is the result of sliding a figure in any direction; rotation is the result of turning a figure around a point or a vertex; and reflection is the result of flipping a figure over a line. Children at this level do not need to know the terms related to transformations of figures.	~~All students should~~ • ~~Develop strategies to determine whether or not a figure has at least one line of symmetry.~~ • ~~Develop strategies to create figures with at least one line of symmetry.~~ • ~~Understand that some figures may have more than one line of symmetry.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Draw a line of symmetry in a figure. (a)[Reordered] • Identify figures with at least one line of symmetry, using various concrete materials (e.g., mirrors, paper folding, pattern blocks). (b) • ~~Draw a line of symmetry~~[Reordered]~~ — horizontal, vertical, and diagonal — in a figure.~~[Moved to US] • Determine a line of symmetry that results in two congruent parts and explain reasoning. (a, b) • Create figures with at least one line of symmetry using various concrete materials. (b)

2.13~~16~~ The student will identify, describe, compare, and contrast plane and solid ~~geometric~~ figures (circle/sphere, square/cube, and rectangle/rectangular prism).

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A plane figure is any two-dimensional shape that can be drawn in a plane. • ~~The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.~~ ~~**Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.~~ ~~**Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., "I know it's a rectangle because it looks like a door, and I know that a door is a rectangle.").~~ ~~**Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., "I know it's a rectangle because it is closed; it has four sides and four right angles, and opposite sides are parallel.").~~ • ~~An important part of geometry is naming and describing figures in two-dimensions (plane figures) and three-dimensions (solid figures).~~ • A vertex is a point at which~~where~~ two or more lines, line segments, ~~lines,~~ or rays meet to form an angle. • An angle is formed by two rays that share a~~n~~ common endpoint called the vertex. Angles are found wherever lines or line segments intersect. • A ~~P~~plane figures is any~~are~~ two-dimensional shape that can be drawn in a plane~~ figures formed by lines that are curved, straight, or a combination of both. They have angles and sides~~.	~~All students should~~ • ~~Understand the differences between plane and solid figures while recognizing the inter-relatedness of the two.~~ • ~~Understand that a solid figure is made up of a set of plane figures.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine similarities and differences between related plane and solid figures (e.g., circle/sphere, square/cube, rectangle/rectangular prism), using models and cutouts. • Trace faces of solid figures (e.g., cube and rectangular solid) to create the set of plane figures related to the solid figure. • Identify and describe plane ~~and solid~~ figures (e.g., circle~~/sphere~~, square~~/cube~~, and rectangle~~/rectangular prism~~), according to their characteristics (~~the~~ number of sides, vertices, and angles).~~and shape of their faces, edges, and vertices using models.~~ Squares and rectangles have four right angles. • Compare and contrast plane and solid ~~geometric~~ figures (e.g., circle/sphere, square/cube, and rectangle/rectangular prism) according to their characteristics (number and shape of their faces ~~(sides, bases)~~, edges, vertices, and angles).

2.13~~16~~ The student will identify, describe, compare, and contrast plane and solid ~~geometric~~ figures (circle/sphere, square/cube, and rectangle/rectangular prism).

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~The identification of plane and solid figures is accomplished by working with and handling objects.~~ • A solid figure is a three-dimensional figure. • ~~Tracing faces of solid figures is valuable to understanding the set of plane figures related to the solid figure (e.g., cube and rectangular prism).~~[Reordered] • A circle is ~~a closed curve in a plane with all its~~the set of points that are the same distance from a point called the center. • A sphere is a solid figure with all of its points the same distance from its center. • A square is a ~~rectangle~~ quadrilateral with four congruent (equal length) sides ~~of equal length~~and four right angles. • A right angle measures exactly 90°. • A rectangular prism is a solid figure in which all six faces are rectangles. A rectangular prism has 8 vertices and 12 edges. • A cube is a solid figure with six congruent, square faces. All edges are the same length. A cube has 8 vertices and 12 edges. It is a type of rectangular prism. • A rectangle is a quadrilateral ~~plane figure~~ with four right angles. A square is a special type of rectangle. • The edge is the line segment where two faces of a solid figure intersect. • A face is any flat side of a ~~a polygon that serves as one side of a~~ solid figure (e.g., a square is a face of a cube). • ~~A base is a special face of a solid figure.~~		

2.13~~16~~ The student will identify, describe, compare, and contrast plane and solid ~~geometric~~ figures (circle/sphere, square/cube, and rectangle/rectangular prism).

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Tracing faces of ~~solid figures is valuable to~~cubes and rectangular prisms helps students understand~~ing~~ the set of plane figures related to the solid figure~~ (e.g., cube and rectangular prism)~~.[Reordered] • The relationship between plane and solid ~~geometric~~ figures, such as the square and the cube or the rectangle and the rectangular prism helps build the foundation for future geometric study of faces, edges, angles, and vertices. The following chart defines the characteristics of solid geometric figures:		

Solid Geometric Figure	# of Faces	Shape of Faces	# of Edges	# of Vertices
Cube	6	Squares	12	8
Rectangular Prism	6	Rectangles	12	8
Square Pyramid	5	Square/Triangles	8	5
Sphere	0	N/A	0	0
Cone	1	Circle*	0	1
Cylinder	2	Circle*	0	0

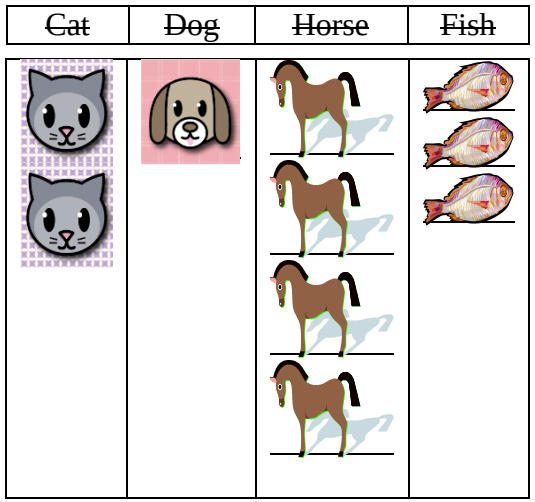
* The most commonly referenced cones and cylinders have circular bases.

Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

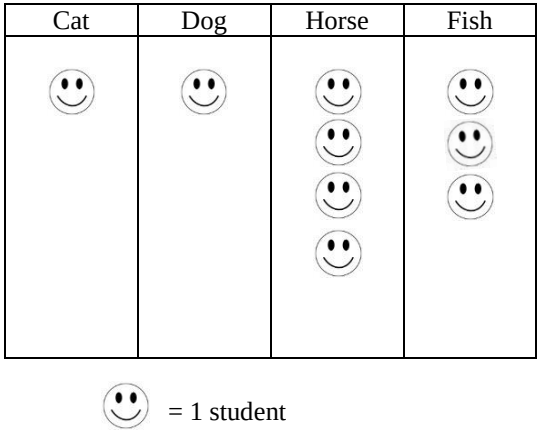
The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible, unlikely, equally likely, more likely,* and *certain.*

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

2.14~~17~~ The student will ~~use data from experiments to construct picture graphs, pictographs, and bar graphs.~~
a) collect, organize, and represent data in picture graphs and bar graphs; and
b) read and interpret the data represented in picture graphs and bar graphs.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Data can be collected and organized in picture graphs and bar graphs.[Moved from center column.] • The purpose of a graph is to represent data gathered to answer a question. ~~Our Favorite Pets~~  • Picture graphs (also called pictographs) are graphs that use pictures to show and compare information. ~~An example of a picture graph is:~~ • A picture graph uses pictures or symbols to represent one or more objects (data points). • A key is provided in a graph to assist in the analysis of the displayed data.[Moved from 2.19 US]	~~All students should~~ • ~~Understand that data may be generated from experiments.~~ • ~~Understand how data can be collected and organized in picture graphs, pictographs, and bar graphs.~~[Moved to US] • ~~Understand that picture graphs use pictures to show and compare data.~~ • ~~Understand that pictographs use a symbol of an object, person, etc.~~ • ~~Understand that bar graphs can be used to compare categorical data.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect and ~~O~~organize data ~~from experiments,~~ using various forms of data collection (e.g., lists, tables, objects, pictures, symbols, tally marks, ~~and~~ charts~~,~~) ~~in order to construct a graph~~. Data points collected should be limited to 16 or few for no more than four categories. (a) • ~~Read the information presented horizontally and vertically on picture graphs, pictographs, and bar graphs.~~ • ~~Collect no more than 16 pieces of data to answer a given question.~~ • Represent data ~~from experiments by constructing~~in picture graphs~~, pictographs,~~ and bar graphs (limited to 16 or few data points for no more than four categories). (a) • ~~Label the axes on a bar graph, limiting the number of categories (categorical data) to four and the increments to multiples of whole numbers (e.g., multiples of 1, 2, or 5).~~[Moved to EKS] • ~~On a pictograph, limit the number of categories to four and include a key where appropriate.~~[Moved to EKS] • Analyze information displayed in picture graphs and bar graphs with up to 25 data points for no more than 6 categories (represented horizontally or vertically). State orally and in writing (at least one statement) that includes one or more of the following:

2.14~~17~~ The student will ~~use data from experiments to construct picture graphs, pictographs, and bar graphs.~~
a) collect, organize, and represent data in picture graphs and bar graphs; and
b) read and interpret the data represented in picture graphs and bar graphs.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The key should be provided for the symbol in a picture graph when the symbol represents more than one piece of data (e.g., represents five people in a graph). At this level, each symbol should represent nor more than 1, 2, 5, or 10 pieces of data. • ~~Pictographs are graphs that use symbols to show and compare information. A student can be represented as a stick figure in a pictograph. A key should be used to indicate what the symbol represents (e.g., one picture of a sneaker represents five sneakers in a graph of shoe types).~~ An example of a ~~pictograph~~ picture graph is: The Types of Pets We Have  		– Describes the categories of data and the data as a whole (e.g., the total number of responses); – Identifies parts of the data that have special characteristics; including categories with the greatest, the least, or the same; – Uses the data to make comparisons; and – Makes predictions and generalizations. [Moved from 2.19 EKS] (b)

2.14~~17~~ The student will ~~use data from experiments to construct picture graphs, pictographs, and bar graphs.~~
a) collect, organize, and represent data in picture graphs and bar graphs; and
b) read and interpret the data represented in picture graphs and bar graphs.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Bar graphs are used to compare counts of different categories (categorical data). Using grid paper may ensure more accurate graphs. – A bar graph uses ~~parallel,~~ horizontal or vertical parallel bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category. – There is space before, between, and after each of the bars. – The axis displaying the scale that represents the count for the categories should begin at zero and extend one increment above the greatest recorded piece of data. Second grade students should ~~be~~ collect~~ing~~ data that are recorded in increments of whole numbers limited to ~~, usually~~ multiples of 1, 2, or 5. – At this level, the number of categories on a picture graph should be limited to four. A key should be included where appropriate.[Moved from EKS] – Each axis should be labeled, and the graph should be given a title. • Statements that represent an analysis and interpretation of the data in the graph should be discussed with students and written (e.g., similarities and differences, least and greatest, the categories, total number of responses, etc.).[Moved from 2.19 US] • Data gathered and displayed by students should be limited to 16 or fewer data points for no more than 4 categories. However, students at this level should be able to interpret graphs that contain data points that represent their entire class (e.g., approximately 25 data points).		

2.15~~18~~ The student will use data from experiments to predict outcomes when the experiment is repeated.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in investigations and have opportunities to use manipulatives. • Investigation of experimental probability is continued through informal activities, such as dropping a two-colored counter (usually a chip that has a different color on each side), using a multicolored spinner (a circular spinner that is divided equally into two, three, four, six or eight ~~or more~~ equal ~~"pie"~~ parts where each part is filled with a different color), using spinners with numbers, or rolling random number cubes ~~generators (dice)~~. • Probability is the chance of an event occurring. ~~(e.g., the probability of landing on a particular color when flipping a two-colored chip is $\frac{1}{2}$, representing one of two possible outcomes).~~ • An event is a possible outcome in probability. Simple events include the possible outcomes when tossing a coin (heads or tails), when rolling a random number cube ~~generator (a number cube or a die where there are six equally likely outcomes and the probability of one outcome is $\frac{1}{6}$),~~ or when spinning a spinner. ~~• If all the outcomes of an event are equally likely to occur, the probability of an event is equal to the number of favorable outcomes divided by the total number of possible outcomes: the probability of the event = $\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$.~~ ~~• At this level, students need to understand only this fractional representation of probability (e.g., the probability of getting heads when flipping a coin is $\frac{1}{2}$).~~ ~~• Students should have opportunities to describe in informal terms~~	~~All students should~~ ~~• **Understand that data may be generated from experiments.**~~ • ~~Understand that the likelihood of an event occurring is to predict the probability of it happening.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Conduct probability experiments~~,~~ using multicolored spinners, colored tiles, or number cubes and use the data from the experiments to predict outcomes if the experiment is repeated. • Record the results of probability experiments, using tables, charts, and tally marks. • Interpret the results of probability experiments ~~(e.g., the two-colored spinner landed on red 5 out of 10 times)~~. • Predict which of two events is more or less likely to occur if an experiment is repeated.

2.15~~18~~ The student will use data from experiments to predict outcomes when the experiment is repeated.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~(i.e., *impossible*, *unlikely*, *as likely as*, *equally likely*, *likely*, and *certain*) the degree of likelihood of an event occurring. Activities should include practical examples.~~		

~~2.19 The student will analyze data displayed in picture graphs, pictographs, and bar graphs.~~[Moved to 2.14b]

~~UNDERSTANDING THE STANDARD~~ ~~**(Background Information for Instructor Use Only)**~~	~~**ESSENTIAL UNDERSTANDINGS**~~	~~**ESSENTIAL KNOWLEDGE AND SKILLS**~~
• ~~Statements that represent an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be discussed with students and written.~~ • ~~When data are displayed in an organized manner, the results of investigations can be described, and the questions posed can be answered.~~	~~All students should~~ • ~~Understand how to read the key used in a graph to assist in the analysis of the displayed data.~~ • ~~Understand how to interpret data in order to analyze it.~~ • ~~Understand how to analyze data in order to answer the questions posed, make predictions, and generalizations.~~	~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~ • ~~Analyze information from simple picture graphs, pictographs, and bar graphs by writing at least one statement that covers one or both of the following:~~ ~~- Describe the categories of data and the data as a whole (e.g., the total number of responses).~~ ~~- Identify parts of the data that have special characteristics, including categories with the greatest, the least, or the same.~~[Moved to 2.14 EKS] • ~~Select the best analysis of a graph from a set of possible analyses of the graph.~~[Moved to 2.14 US]

Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children's mathematical development.

Students in kindergarten through ~~third~~ second grade develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

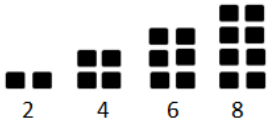
- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns. ~~These s~~Students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to algebra.

2.16~~20~~ The student will identify, describe, create, ~~and~~ extend, and transfer ~~a wide variety of~~ patterns found in objects, pictures, and numbers.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Patterning is a fundamental cornerstone of mathematics, particularly algebra. The process of generalization leads to the foundation of algebraic reasoning. • Opportunities to create, identify, describe, extend, and transfer patterns are essential to the primary school experience and lay the foundation for thinking algebraically.[Reordered and edited] • The part of the pattern that repeats is called the core. ~~• Identifying and extending patterns is an important process in mathematical thinking.~~ ~~• Analysis of patterns in the real world (e.g., patterns on a butterfly's wings, patterns on a ladybug's shell) leads to the analysis of mathematical patterns such as number patterns and geometric patterns.~~ ~~• Reproduction of a given pattern in a different manifestation, using symbols and objects, lays the foundation for writing numbers symbolically or algebraically.~~ • ~~The simplest types of patterns are repeating patterns. Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience.~~ [Reordered] • Growing patterns ~~are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns in both arithmetic and geometric formats.~~ involve a progression from step to step which make them more difficult for students than repeating patterns. Students must determine what comes next and also begin the process of generalization, which leads to the foundation of algebraic reasoning. Students need experiences identifying what changes and what stays the same in a growing pattern. Growing patterns may be represented in various ways,	~~All students should~~ ~~• Understand patterns are a way to recognize order and to predict what comes next in an arrangement.~~ ~~• Analyze how both repeating and growing patterns are generated.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify a growing ~~and/~~ or repeating pattern. ~~from a given geometric or numeric sequence.~~ ~~• Predict the next number, geometric figure, symbol, picture, or object in a given pattern.~~ • Describe a given growing or repeating pattern. • Create a growing or repeating pattern, using numbers, pictures, or objects.[Reordered] • Extend a given pattern, using numbers, geometric figures, symbols, pictures, or objects. • ~~Create a new pattern, using numbers, geometric figures, pictures, symbols, or objects.~~[Reordered] • ~~Recognize the same pattern in different manifestations.~~ Transfer a given growing or repeating pattern using numbers, pictures, or objects.

2.16~~20~~ The student will identify, describe, create, ~~and~~ extend, and transfer ~~a wide variety of~~ patterns found in objects, pictures, and numbers.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
including dot patterns, staircases, pictures, etc. • In numeric patterns, students must determine the difference, called the *common difference*, between each succeeding number in order to determine what is added to each previous number to obtain the next number. Students do not need to use the term common difference at this level. ~~Create an arithmetic number pattern.~~ • Sample numeric patterns include: – 6, 9, 12, 15, 18, …(growing pattern); – ~~20, 18, 16, 14, … (growing pattern);~~ – ~~1, 2, 4, 7, 11, 16~~1, 22, 333, 444, … (growing pattern); and – 1, 3, 5, 1, 3, 5, 1, 3, 5… (repeating pattern). • In ~~geometric~~ patterns using objects or figures, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation is the result of turning a figure ~~around a point or a vertex~~, and reflection is the result of flipping a figure over a line. • Examples of patterns using objects or figures include: –  – – • Transferring a pattern is creating the pattern in a different form or representation.		

2.16~~20~~ The student will identify, describe, create, ~~and~~ extend, and transfer ~~a wide variety of~~ patterns found in objects, pictures, and numbers.

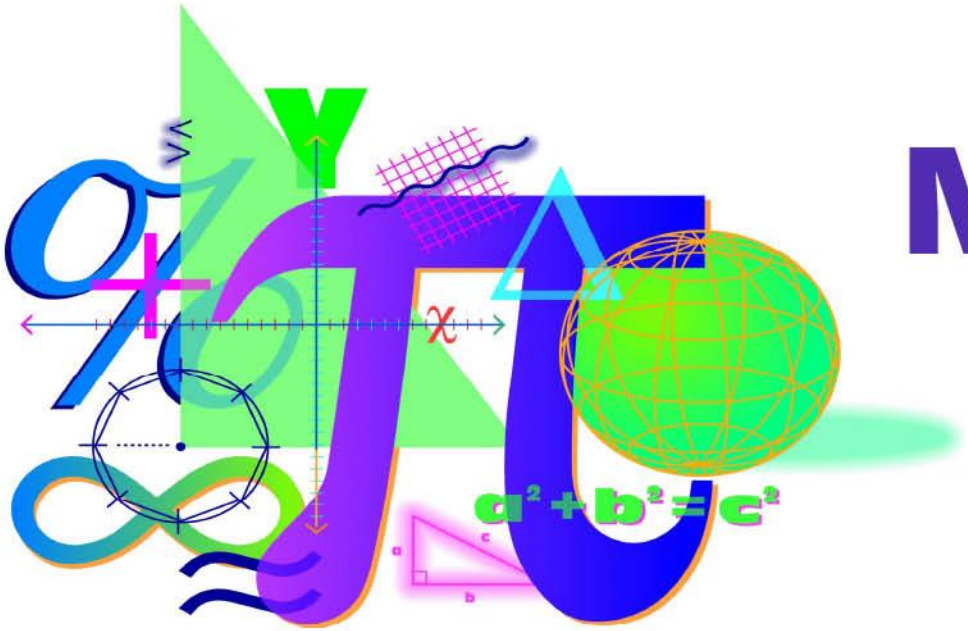
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Examples of pattern transfers include: – 10, 20, 30, 40 has the same structure as 14, 24, 34, 44 – has the same structure as – 1, 3, 5, 1, 3, 5, 1, 3, 5 has the same structure as ABCABC		

~~2.21 The student will solve problems by completing numerical sentences involving the basic facts for addition and subtraction. The student will create story problems, using the numerical sentences.~~[Moved to 2.5 and 2.6]

~~UNDERSTANDING THE STANDARD~~ ~~**(Background Information for Instructor Use Only)**~~	~~**ESSENTIAL UNDERSTANDINGS**~~	~~**ESSENTIAL KNOWLEDGE AND SKILLS**~~
• ~~Recognizing and using patterns and learning to represent situations mathematically are important aspects of primary mathematics.~~ • ~~Discussing what a word problem is saying and writing a number sentence are precursors to solving word problems.~~ • ~~The patterns formed by related basic facts facilitate the solution of problems involving a missing addend in an addition sentence or a missing part (subtrahend) in a subtraction sentence.~~ • ~~Making mathematical models to represent simple addition and subtraction problems facilitates their solution.~~ • ~~By using story problems and numerical sentences, students begin to explore forming equations and representing quantities using variables.~~ • ~~Students can begin to understand the use of a symbol (e.g., __, ?, or ☐) to represent an unknown quantity.~~	~~All students should~~ • ~~Use mathematical models to represent and understand quantitative relationships.~~ • ~~Understand various meanings of addition and subtraction and the relationship between the two operations.~~ • ~~Understand how to write missing addend and missing subtrahend sentences.~~	~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~ • ~~Solve problems by completing a numerical sentence involving the basic facts for addition and subtraction (e.g., 3 + __ = 7, or 9 __ = 2).~~[Moved to 2.5] • ~~Create a story problem for a given numerical sentence.~~[Moved to 2.6]

2.17~~22~~ The student will demonstrate an understanding of equality ~~by recognizing that~~ through the use of the equal symbol = ~~in an equation indicates equivalent quantities~~ and the use of the not equal symbol ≠ ~~indicates that quantities are not equivalent~~.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The equal = symbol (=) means that the values on either side are ~~the same~~equivalent (balanced). • The not equal (≠) symbol means that the values on either side are not ~~the same~~equivalent (not balanced). • In order for students to develop the concept of equality, students need to see the = symbol used in various locations (e.g., 3 + 4 = 7 and 5 = 2 + 3). • An equation (number sentence) is a mathematical statement representing two expressions that are equivalent. It consists of two expressions, one on each side of an 'equal' symbol (e.g., 5 + 3 = 8, 8 = 5 + 3 and 4 + 3 = 9 - 2). An equation can be represented using a balance scale, with equal amounts on each side (e.g., 3 + 5 = 6 + 2). • An expression represents a quantity. It contains numbers, variables, and/or computational operation symbols. It does not have an equal sign (e.g., 5, 4 + 3, 8-2). Students at this level are not expected to use the terms expression or variable. • ~~A number sentence is an equation with numbers (e.g., 6 + 3 = 9; or 6 + 3 = 4 + 5).~~	~~All students should~~ • ~~Understand that the equal symbol means equivalent (same as) quantities.~~ • ~~The inequality symbol (≠) means not equivalent.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the equal~~ity~~ symbol (=) as the symbol used to indicate that the values on either side are equivalent. • Identify the not equal symbol ~~and inequality~~(≠) as the symbol used to indicate that two values on either side are not equivalent.~~symbols.~~ • Identify ~~equivalen~~t values and ~~equations~~expressions that are equal~~.~~ (e.g., 8 = 8 and 8 = 4 + 4). • Identify ~~nonequivalent~~ values and ~~equations~~expressions that are not equal~~.~~ (e.g., 8 ≠ 9 and 4 + 3 ≠ 8). • Identify and use the appropriate symbol to distinguish between equal and not equal quantities~~.~~ (e.g., 8 + 2 = 7 + 3 and 1 + 4 ≠ 6 + 2).



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Grade 3

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

~~Students in grades K–3 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.~~

~~Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, fractions, number relationships ("more than," "less than," and "equal to"), and the effects of single-step and multistep computations. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.~~

[Moved from grade 4 strand introduction]
Mathematics instruction in grades 3 through 5 should continue to foster the development of number sense, with greater emphasis on decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction in grades allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names), and making connections among mathematics concepts as well as to other content areas. Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g., $\frac{1}{2}$ or 0.5). Students should apply their knowledge of number and number sense to investigate and solve a variety of problem types.

3.1 The student will

a) read and write six-digit numerals and identify the place ~~value~~ and value of each digit;

b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and

c) compare ~~two~~ and order whole numbers ~~between 0 and~~each 9,999 or less~~, using symbols (>, <, or =) and words (*greater than, less than*, or *equal to*)~~.[Symbols and words included in EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The structure of the ~~B~~base-10 number system is based upon a simple pattern of tens, where each place is ten times the value of the place to its right. This structure,~~is~~ known as a ten-to-one place value relationship, is helpful in comparing and ordering numbers. • ~~The structure of the Base-10 blocks is based on the ten-to-one place value relationship (e.g., 10 units make a long, 10 longs make a flat, 10 flats make a cube).~~ • Models that clearly illustrate the relationships among hundreds, tens, and ones are physically proportional (e.g., the tens piece is ten times larger than the ones piece). • Place value refers to the value of each digit and depends upon the position of the digit in the number. In the number 7,864, the eight is in the hundreds place, and the value of the 8 is eight hundred. • Flexibility in thinking about numbers — or "decomposition" of numbers (e.g., ~~1~~2,345 is ~~1~~23 hundreds, 4 tens, and 5 ones, or 2 thousands, 34 tens, and 5 ones, or 22 hundreds, 13 tens, and 15 ones, etc.) — is critical and supports understandings essential to addition/subtraction and multiplication/~~and~~ division. This flexibility also builds background understanding for the ideas that students use when regrouping (e.g., when subtracting 18 from 174, a student may choose to regroup and think of 174 as 1 hundred, 6 tens, and 14 ones). • Whole numbers may be written in a variety of formats: – Standard: 123,456; – Written: one hundred twenty-three thousand, four hundred fifty-six; and – Expanded: 100,000 + 20,000 + 3,000 + 400 + 50 + 6 ~~(1 × 100,000) + (2 × 10,000) + (3 × 1,000) + (4 × 100) + (5 × 10) + (6 × 1)or 100,00 + 20,000 + 3,000 + 400 + 50 + 6(1 ×~~	~~All students should~~ • ~~Understand that knowledge of place value is essential when comparing numbers.~~ • ~~Understand the relationships in the place value system, where each place is ten times the value of the place to its right.~~ • ~~Understand that rounding gives an estimate to use when exact numbers are not needed for the situation.~~ • ~~Understand the relative magnitude of numbers by comparing numbers.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent numbers up to 9,999 in multiple ways, according to place value (e.g., 256 can be 1 hundred, 14 tens, and 16 ones, but also 25 tens and 6 ones). (a) • ~~Investigate and identify the place and value for each digit in a six-digit numeral, using Base-10 manipulatives (e.g., Base-10 blocks).~~ • ~~Use the patterns in the place value system to read and write numbers.~~ • Identify the place (ones, tens, hundreds, thousands, etc.) of each digit in a 6-digit numeral. (a) • Determine the value of each digit in a 6-digit numeral (e.g., in 165,724, the 7 represents 7 hundreds and its value is 700). (a) • Read six-digit numerals orally. (a) • Write six-digit numerals in standard form that are stated verbally or written in words. (a) • Round a given whole number, 9,999 or less, to the nearest ten, hundred, and thousand. (b) • Solve problems, using rounding of numbers, 9,999 or less, to the nearest ten, hundred, and thousand. (b) • Determine which of two whole numbers between 0 and 9,999 is greater. (c) • Determine which of two whole numbers between 0 and 9,999 is less. (c)

3.1 The student will

a) read and write six-digit numerals and identify the place ~~value~~ and value of each digit;

b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and

c) compare ~~two~~ and order whole numbers ~~between 0 and~~each 9,999 or less~~, using symbols (>, <, or =) and words (*greater than, less than*, or *equal to*)~~.[Symbols and words included in EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~100,000) + (2 × 10,000) + (3 × 1,000) + (4 × 100) + (5 × 10) + (6 × 1).~~ • Numbers are arranged into groups of three places called *periods* (ones, thousands, millions, and so on). Places within the periods repeat (hundreds, tens, ones). Commas are used to separate the periods. Knowing the place value and period of a number helps students ~~find~~ determine the value of a digit in any number as well as read and write numbers. ~~• To read a whole number through the hundred thousands place,~~ ~~- read the digits to the first comma;~~ ~~- say the name of the period (e.g., "thousands"); then~~ ~~- read the last three digits, but do not say the name of the ones period.~~ ~~•~~ Reading and writing large numbers should be related to numbers that have meanings (e.g., numbers found in the students' environment). ~~Concrete materials, such as Base-10 blocks may be used to represent whole numbers through thousands. Larger numbers may be represented on place value charts.~~ • Rounding is ~~one of the~~an estimation strategy~~ies~~ that is often used to assess the reasonableness of a solution or to ~~give an~~ estimate ~~of~~ an amount. • Students should explore reasons for estimation, using practical experiences, and use rounding to solve practical ~~situations~~ problems. • The concept of rounding may be introduced through the use of a number line. When given a number to round, locate it on the number line. Next, determine the closest multiples of ten, hundred, or thousand it is between. Then identify to which it is closer.		• Compare two whole numbers ~~between 0 and~~each 9,999 or less, using the symbols >, <, ~~or~~ =, or ≠. (c) • Use the terms *greater than*, *less than*, ~~and~~ *equal to*, and *not equal to* when comparing two whole numbers. (c) • Order up to three numbers, each 9,999 or less, represented with concrete objects, pictorially, or symbolically from least to greatest and greatest to least. (c)

3.1 The student will

a) read and write six-digit numerals and identify the place ~~value~~ and value of each digit;

b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and

c) compare ~~two~~ and order whole numbers ~~between 0 and~~each 9,999 or less~~, using symbols (>, <, or =) and words (*greater than, less than*, or *equal to*)~~.[Symbols and words included in EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• A procedure for rounding numbers to the nearest ten, hundred, or thousand is as follows:~~ ~~– Look one place to the right of the digit to which you wish to round.~~ ~~– If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero.~~ ~~– If the digit is 5 or greater, add 1 to the digit in the rounding place, and change the digits to the right of the rounding place to zero.~~ ~~• A procedure for comparing two numbers by examining may include the following:~~ ~~– Line up the numbers by place value by lining up the ones.~~ ~~– Beginning at the left, find the first place value where the digits are different.~~ ~~– Compare the digits in this place value to determine which number is greater (or which is less).~~ ~~– Use the appropriate symbol > or < or the words *greater than* or *less than* to compare the numbers in the order in which they are presented.~~ ~~– If both numbers are the same, use the symbol = or the words *equal to*.~~		

~~3.2~~ ~~The student will recognize and use the inverse relationships between addition/subtraction and multiplication/division to complete basic fact sentences. The student will use these relationships to solve problems.~~[Addition/subtraction included in 2.5; multiplication/division moved to 3.4 EKS]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Addition and subtraction are inverse operations, as are multiplication and division.~~ • ~~In building thinking strategies for subtraction, an emphasis is placed on connecting the subtraction fact to the related addition fact. The same is true for division, where the division fact is tied to the related multiplication fact. Building fact sentences helps strengthen this relationship.~~ • ~~Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship.~~ • ~~Multiplication and division should be taught concurrently in order to develop understanding of the inverse relationship.~~	~~**All students should**~~ • ~~Understand how addition and subtraction are related.~~ • ~~Understand how multiplication and division are related.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Use the inverse relationships between addition/subtraction and multiplication/division to solve related basic fact sentences. For example, 5 + 3 = 8 and 8 − 3 = __;~~ ~~4 × 3 = 12 and 12 ÷ 4 = __.~~ • ~~Write three related basic fact sentences when given one basic fact sentence for addition/subtraction and for multiplication/division. For example, given~~ ~~3 × 2 = 6, solve the related facts __× 3 = 6,~~ ~~6 ÷ 3 = __, and 6 ÷ __ = 3.~~

3.2~~3~~ The student will

a) name and write fractions (including improper fractions and mixed numbers) represented by a model;

b) ~~model~~ represent fractions (including improper fractions and mixed numbers) ~~and write the fractions' names~~with models and symbols; and

c) use models to compare proper fractions having like and unlike denominators, using words and symbols (>, <, ~~or~~ =, or ≠).

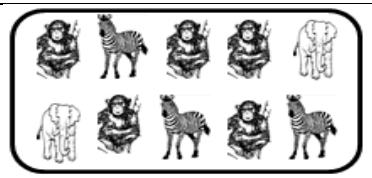
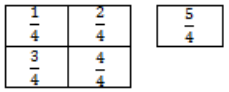
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The whole must be defined. • A fraction is a way of representing part of a whole (as in a region/area model or a length/measurement model) or part of a group (as in a set model). ~~Fractions are used to name a part of one thing or a part of a collection of things. Models can include pattern blocks, fraction bars, rulers, number line, etc.~~ • The value of a fraction is dependent on both the number of equal parts in a whole (denominator) and the number of those parts being considered (numerator). • Model a fractional part of a whole, using – region/area models (e.g., pie pieces, pattern blocks, geoboards, drawings); – set models (e.g., chips, counters, cubes, drawings); and – length/measurement models (e.g., rods, connecting cubes, number lines, rulers, and drawings). (b)[Moved from 3.7) • In each area/region and length/measurement model, the whole is ~~parts must be equal-sized (congruent). Wholes are~~ divided or partitioned into equal-sized (congruent) parts. • In the set model, each member of the set is an equal part of the set. ~~The members of the set do not have to be equal in size.~~ In set models, the whole needs to be defined, but members of the set may have different sizes and shapes. For instance, if a whole is defined as a set of 10 animals, the animals within the set may be different. Further, students should be able to write a fraction that represents the number of monkeys in the following set of animals.	~~All students should~~ • ~~Understand that the whole must be defined.~~ • ~~Understand that the denominator tells the number of equal parts that represent a whole.~~ • ~~Understand that the numerator is a counting number that tells how many equal size parts are being considered.~~ • ~~Understand that the value of a fraction is dependent on both the number of parts in a whole (denominator) and the number of those parts being considered (numerator).~~[Moved to US] • ~~Understand that a proper fraction is a fraction whose numerator is smaller than its denominator.~~[Moved to US] • ~~that an improper fraction is a fraction whose numerator is greater than or equal to the denominator and is one or greater than one.~~[Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Name and write fractions (including improper fractions and mixed numbers) with denominators of 12 or less in symbols represented by ~~a~~both concrete and pictorial models (~~drawings~~set, region/area, and length/measurement models or concrete materials).~~to include halves, thirds, fourths, eighths, tenths, and twelfths.~~ (a) • ~~Use concrete materials and pictures to model at least halves, thirds, fourths, eighths, tenths, and twelfths.~~ • Represent a given fraction (including improper or mixed numbers), using concrete materials, pictures, models (set, area/region, length/measurement), and symbols. (b)[Moved from 3.7] • Identify a fraction represented by a model as the sum of unit fractions (e.g., $\frac{5}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$). • Using a model of a fraction greater than one, count the fractional parts to name and write it as an improper fraction and as a mixed number (e.g., $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4} = 1\frac{1}{4}$, or $2\frac{1}{3} = \frac{7}{3}$). (b) • Compare proper fractions using the terms *greater than, less than,* ~~or~~ *equal to, or not equal to* and the symbols (<, >, ~~and~~ =, and ≠). Comparisons are made between fractions with both like and unlike denominators, using models (length/measurement, region/area, and set), concrete materials and pictures. (c)

3.2~~3~~ The student will

a) name and write fractions (including improper fractions and mixed numbers) represented by a model;

b) ~~model~~ represent fractions (including improper fractions and mixed numbers) ~~and write the fractions' names~~with models and symbols; and

c) use models to compare proper fractions having like and unlike denominators, using words and symbols (>, <, ~~or~~ =, or ≠).

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
 • In the primary grades, students may benefit from experiences with sets that are comprised of congruent figures (e.g., 12 eggs in a carton) before working with sets that have noncongruent parts. • ~~The denominator tells how many equal parts are in the whole or set. The numerator tells how many of those parts are being considered.~~ • Students need opportunities to use models to count fractional parts that go beyond a whole. For instance, if students are counting five slices of cake, where each slice is equivalent to one-fourth, they might say "one-fourth, two-fourth, three-fourths, four-fourths, five-fourths." As a result, they begin to develop flexibility in naming this amount in different ways (e.g., five-fourths or one and one-fourth). They will begin to generalize that when the numerator and the denominator are the same, there is one whole and when the numerator is larger than the denominator, there is more than one whole.  $\frac{5}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ and $\frac{5}{4} = 1\frac{1}{4}$ • A proper fraction is a fraction whose numerator is less than the denominator. A proper fraction is a fraction that is always less than one.	• ~~Understand that an improper fraction can be expressed as a whole number or a mixed number.~~ • ~~Understand that a mixed number is written as a whole number and a proper fraction.~~	• Use models to compare fractions to the benchmarks of 0, $\frac{1}{2}$, and 1. (c)

3.2~~3~~ The student will

a) name and write fractions (including improper fractions and mixed numbers) represented by a model;

b) ~~model~~ represent fractions (including improper fractions and mixed numbers) ~~and write the fractions' names~~with models and symbols; and

c) use models to compare proper fractions having like and unlike denominators, using words and symbols (>, <, ~~or~~ =, or ≠).

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• An improper fraction is a fraction whose numerator is greater than or equal to the denominator. An improper fraction is a fraction that is greater than or equal to one. • An improper fraction greater than one can also be expressed as a mixed number. A mixed number is written as a whole number and a proper fraction. • Models, benchmarks, and equivalent forms are helpful in judging the size of fractions.[Copied from 4.2] • Comparing fractions to a benchmark on a number line (e.g., close to 0, less than $\frac{1}{2}$, exactly $\frac{1}{2}$, greater than $\frac{1}{2}$, or close to 1) facilitates the comparison of fractions when using concrete materials or pictorial models.[Reordered] • Comparing unit fractions (a fraction in which the numerator is one) builds a mental image of fractions and the understanding that as the number of pieces of a whole increases, the size of one single piece decreases (e.g., $\frac{1}{5}$ of a bar is smaller than $\frac{1}{4}$ of the same bar).[Reordered] • Provide opportunities to make connections among fraction representations by connecting concrete or pictorial representations with oral language and symbolic representations. • Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions (e.g., *thirds* means "three equal parts of a whole," $\frac{1}{3}$ represents one of three equal-size parts when a pizza is shared among three students, or *three-fourths* means "three of four equal parts of a whole") furthers this development.		

3.2~~3~~ The student will

a) name and write fractions (including improper fractions and mixed numbers) represented by a model;

b) ~~model~~ represent fractions (including improper fractions and mixed numbers) ~~and write the fractions' names~~with models and symbols; and

c) use models to compare proper fractions having like and unlike denominators, using words and symbols (>, <, ~~or~~ =, or ≠).

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Comparing unit fractions (a fraction in which the numerator is one) builds a mental image of fractions and the understanding that as the number of pieces of a whole increases, the size of one single piece decreases (e.g., $\frac{1}{5}$ of a bar is smaller than $\frac{1}{4}$ of a bar).~~ [Reordered] • ~~Comparing fractions to a benchmark on a number line (e.g., close to 0, less than $\frac{1}{2}$, exactly $\frac{1}{2}$, greater than $\frac{1}{2}$, or close to 1) facilitates the comparison of fractions when using concrete materials or pictorial models.~~[Reordered]		

~~A variety of contexts are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from real-life experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as "How many books do we have altogether?" or "How many cookies are left if I eat two?" or "I have three more candies than you do." Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling, discussing, and recording a variety of problem situations. This approach helps students transition from the concrete to the representation to the symbolic in order to develop meaning for the operations and how they relate to each other.~~

~~In grades K–3, computation and estimation instruction focuses on~~

- ~~relating the mathematical language and symbolism of operations to problem situations;~~
- ~~understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;~~
- ~~developing proficiency with basic addition, subtraction, multiplication, division and related facts;~~
- ~~gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;~~
- ~~developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;~~
- ~~choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;~~
- ~~recognizing whether numerical solutions are reasonable;~~
- ~~experiencing situations that lead to multiplication and division, such as equal groupings of objects and sharing equally; and performing initial operations with fractions.~~

[Moved from grade 4 Computation and Estimation strand introduction]

Computation and estimation in grades 3-5 should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students' understanding of these operations to work with decimals. Instruction should focus on computation activities that enable students to model, explain, and develop proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability not only to identify but to use relationships between operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use properties of operations to solve problems [e.g., 7×28 is equivalent to $(7 \times 20) + (7 \times 8)$].
Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals, using benchmarks (e.g., $\frac{2}{5} + \frac{1}{3}$ must be less than 1 because both fractions are less than $\frac{1}{2}$).
Using estimation, students should develop strategies to recognize the reasonableness of their solutions.

Additionally, students should enhance their ability to select an appropriate problem solving method from among estimation, mental mathematics, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.

3.34 The student will

a) estimate ~~solutions to~~ and determine the sum or difference of two whole numbers; and

b) create and solve single-step and multistep practical problems involving ~~the~~ sums or differences of two whole numbers, each 9,999 or less~~, with or without regrouping~~.

UNDERSTANDING THE STANDARD
~~(Background Information for Instructor Use Only)~~

- Students should experience a variety of problem types related to addition and subtraction. Examples are included in the following chart:

GRADE 3 - ADDITION AND SUBTRACTION PROBLEM TYPES		
Join (Result Unknown)	Join (Change Unknown)	Join (Start Unknown)
Sue had 328 pennies. Josh gave her 214 more. How many pennies does Sue have altogether?	Sue had 214 pennies. Josh gave her some more. Now Sue has 328 pennies. How many did Josh give her?	Sue had some pennies. Josh gave her 214 more. Now Sue has 328 pennies. How many pennies did Sue have to begin with?
Separate (Result Unknown)	Separate (Change Unknown)	Separate (Start Unknown)
Sue had 145 pennies. She gave 26 pennies to Josh. How many pennies does Sue have now?	Sue had 161 pennies. She gave some to Josh. Now she has 30 pennies. How many did she give to Josh?	Sue had some pennies. She gave 14 to Josh. Now she has 134 pennies left. How many pennies did Sue have to begin with?
Part-Part-Whole (Whole Unknown)	Part-Part-Whole (Part Unknown)	Part-Part-Whole (Both Parts Unknown)
Josh has 314 pennies and 298 nickels. How many coins does he have?	Josh has 172 coins. Twenty-five of the coins are pennies, and the rest are nickels. How many nickels does Josh have?	Josh has 123 coins. Some are pennies and some are nickels. How many pennies and nickels could he have?
Compare (Difference Unknown)	Compare (Larger Unknown)	Compare (Smaller Unknown)
Josh has 422 pennies and Sue has 509 pennies. How many more pennies does Josh have than Sue?	Josh has 99 more pennies than Sue. Sue has 38 pennies. How many pennies does Josh have?	Josh has 18 more pennies than Sue. Josh has 112 pennies. How many pennies does Sue have?
Josh has 322 pennies. Sue has 18 pennies. How many fewer pennies does Sue have than Josh?	Sue has 100 fewer pennies than Josh. Sue has 18 pennies. How many pennies does Josh have?	Sue has 15 fewer pennies than Josh. Josh has 285 pennies. How many pennies does Sue have?

~~ESSENTIAL UNDERSTANDINGS~~

~~**All students should**~~

- ~~Understand that estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed.~~[Moved to US]
- ~~Understand that estimation skills are also valuable in determining the reasonableness of the sum or difference when solving for the exact answer is needed.~~[Moved to US]
- ~~Develop and use strategies to estimate whole number sums and differences to determine the reasonableness of an exact answer.~~
- ~~Develop flexible methods of adding whole numbers by combining numbers in a variety of ways, most depending on place values.~~[Moved to US]

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Determine whether an estimate or an exact answer is an appropriate solution for practical addition and subtraction problem~~s~~ ~~situations~~ involving single-step and multistep problems. (a, b)
- ~~Determine whether to add or subtract in practical problem situations.~~
- Estimate the sum ~~or difference~~ of two whole numbers with sums to~~, each~~ 9,999 ~~or less when an exact answer is not required~~. (a)
- Estimate the difference of two whole numbers, each 9,999 or less. (a)
- Add ~~or subtract~~ two whole numbers with sums to~~, each~~ 9,999 ~~or less~~. (b)
- Subtract two whole numbers, each 9,999 or less. (b)
- ~~Recognize and use related~~Use inverse relationships between addition and ~~/~~subtraction facts to solve practical problems. (b)[Moved from 3.2]
- ~~Solve practical problems involving the sum of two whole numbers, each 9,999 or less, with or without regrouping, using calculators, paper and pencil, or mental computation in practical problem situations.~~
- ~~Solve practical problems involving the difference of two whole numbers, each 9,999 or less, with or without regrouping, using calculators, paper and pencil, or mental computation in practical problem situations.~~

3.34 The student will

a) estimate ~~solutions to~~ and determine the sum or difference of two whole numbers; and

b) create and solve single-step and multistep practical problems involving ~~the~~ sums or differences of two whole numbers, each 9,999 or less~~, with or without regrouping~~.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Rate and combination problem types are introduced in grade 5. • In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as *in all*, *altogether*, *difference*, *etc.* encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions. • Addition is the combining of quantities; it uses the following terms: *addend* → 423 *addend* → + 246 *sum* → 669 • Subtraction is the inverse of addition; it yields the difference between two numbers and uses the following terms: *minuend* → 7,698 *subtrahend* → – 5,341 *difference* → 2,357 • An algorithm is a step-by-step method for computing. • Flexible methods of adding whole numbers by combining numbers in a variety of ways, most depending on place values, are useful.[Moved from middle column] • The least number of steps necessary to solve a single-step problem is one. ~~• An example of an approach to solving problems is Polya's four-step plan:~~ ~~• Understand: Retell the problem; read it twice; take notes; study the charts or diagrams; look up words and symbols that are new.~~ ~~• Plan: Decide what operation(s) and sequence of steps to use to~~		• Use strategies when adding and subtracting, including application of the properties of addition. (a, b)[Moved from 3.20] • Create and ~~S~~solve single-step and multistep practical problems involving the sum or difference of two whole numbers, each 9,999 or less~~, with or without regrouping~~. (b)

3.34 The student will

a) estimate ~~solutions to~~ and determine the sum or difference of two whole numbers; and

b) create and solve single-step and multistep practical problems involving ~~the~~ sums or differences of two whole numbers, each 9,999 or less~~, with or without regrouping~~.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~solve the problem.~~ • ~~Solve: Follow the plan and work accurately. If the first attempt does not work, try another plan.~~ • ~~Look back: Does the answer make sense?~~ • Estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed.[Moved from middle column] • Estimation skills are also valuable in determining the reasonableness of the sum or difference when solving for the exact answer is needed.[Moved from middle column] • ~~Knowing whether to find an exact answer or to make an estimate is learned through practical experiences in recognizing which is appropriate.~~ • When an exact answer is required, opportunities to explore whether the answer can be determined mentally or must involve paper and pencil or calculators help students select the ~~correct~~most efficient approach. • Determining whether an estimate is appropriate and using a variety of strategies to estimate requires experiences with problem situations involving estimation. • There are a variety of mental mathematics strategies for each basic operation, and opportunities to practice these strategies give students the tools to use them at appropriate times. For example, with addition, mental mathematics strategies include: – adding doubles; – using addition by counting up for solving subtraction problems; – adding 9: add 10 and subtract 1; and – ~~M~~making 10: for column addition, look for numbers that group together to make 10.		

3.34 The student will

a) estimate ~~solutions to~~ and determine the sum or difference of two whole numbers; and

b) create and solve single-step and multistep practical problems involving ~~the~~ sums or differences of two whole numbers, each 9,999 or less~~, with or without regrouping~~.

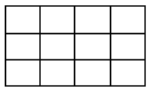
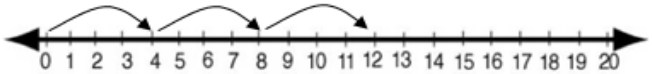
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4). [Moved from 3.20] • The identity property of addition states that if zero is added to a given number, the sum is the same as the given number.[Moved from 3.20] • At this level, it is not necessary for students to use the formal terms for these properties. • Using concrete materials (e.g., ~~B~~base-10 ~~materials to~~ blocks, connecting cubes, beans and cups, etc.) to explore, model and stimulate discussion about a variety of problem situations helps students understand regrouping and enables them to move from the concrete to the abstract. Regrouping is used in addition and subtraction algorithms. • Conceptual understanding begins with concrete experiences. Next, the children must make connections that serve as a bridge to the symbolic. ~~One strategy used to make connections is~~ Student created representations, such as drawings, diagrams, tally marks, graphs, or written comments are strategies that help students make these connections.		

3.4~~5~~ The student will ~~recall multiplication facts through the twelves table, and the corresponding division facts.~~[Recall moved to 4.4a]

a) represent multiplication and division, using a variety of approaches and models;[Moved from 3.6]

b) create and solve single-step practical problems that involve multiplication and division through 10 × 10; and[Create and solve multiplication moved from 3.6; create and solve with division is new expectation]

c) create and solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less.[Moved from 3.6]

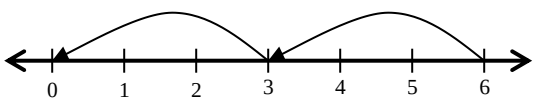
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and practical problems involving equal-sized groups, arrays, and length models. • To extend the understanding of multiplication, three models may be used: • The equal-sets or equal-groups model lends itself to sorting a variety of concrete objects into equal groups and reinforces repeated addition or skip counting. • The array model, consisting of rows and columns (e.g., 3 rows of 4 columns for a 3-by-4 array), helps build an understanding of the commutative property.  • The length model (e.g., a number line) also reinforces repeated addition or skip counting.[Reordered]  • There is an inverse relationship between multiplication and division.	~~All students should~~ • ~~Develop fluency with number combinations for multiplication and division.~~ • ~~Understand that multiplication is repeated addition.~~ • ~~Understand that division is the inverse of multiplication.~~[Moved to US] • ~~Understand that patterns and relationships exist in the facts.~~[Moved to US] • ~~Understand that number relationships can be used to learn and retain the facts.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Recall and state the multiplication and division facts through the twelves table.~~[Moved to 4.4 EKS] • ~~Recall and write the multiplication and division facts through the twelves table.~~ • Represent multiplication using a variety of approaches and models (e.g., repeated addition, equal-sized groups, arrays, equal jumps on a number line, skip counting). (a) • Represent division using a variety of approaches (e.g., repeated subtraction, equal sharing, forming equal groups). (a) • Create practical problems to represent a multiplication or division fact. (b) • Use multiplication and division basic facts to represent a given situation, using a number sentence. (b) • Recognize and use the inverse relationship between multiplication and division to solve practical problems. (b)[Moved from 3.2] • Write three related equations ~~basic~~ (fact sentences) when given one ~~basic~~ equation (fact sentence) for ~~addition/subtraction and for~~ multiplication and ~~/~~division. (b)[Moved from 3.2 EKS]

3.4~~5~~ The student will ~~recall multiplication facts through the twelves table, and the corresponding division facts.~~[Recall moved to 4.4a]

a) represent multiplication and division, using a variety of approaches and models;[Moved from 3.6]

b) create and solve single-step practical problems that involve multiplication and division through 10 × 10; and[Create and solve multiplication moved from 3.6; create and solve with division is new expectation]

c) create and solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less.[Moved from 3.6]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The terms associated with multiplication are listed below: *factor* → 54 *factor* → × 3 *product* → 162[Moved from 3.6] • The number line model can be used to solve a multiplication problem such as 3 × 4. This is represented on the number line by three jumps of four or four jumps of three, depending on the context of the problem.[Moved from 3.6] • The number line model can be used to solve a division problem such as 6 ÷ 3 and is represented on the number line by noting how many jumps of three go from 6 to 0.[Moved from 3.6]  The number line model above shows two jumps of three between 6 and 0, answering the question of how many jumps of three go from 6 to 0; therefore, 6 ÷ 3 = 2. • An algorithm is a step-by-step method for computing.[Moved from 3.6] • Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently. • The development of computational fluency relies on quick access to number facts. There are patterns and relationships that exist in the facts. These relationships can be used to learn and retain the facts. By studying patterns and relationships, students build a foundation for fluency with multiplication and division facts.		• Solve single-step practical problems that involve multiplication and division of whole numbers through 10 × 10. (b) • Create and solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less. (c)[Moved from 3.5] • Apply the properties of multiplication when multiplying and dividing whole numbers. (a, b, c)[Moved from 3.20]

3.4~~5~~ The student will ~~recall multiplication facts through the twelves table, and the corresponding division facts.~~[Recall moved to 4.4a]

a) represent multiplication and division, using a variety of approaches and models;[Moved from 3.6]

b) create and solve single-step practical problems that involve multiplication and division through 10 × 10; and[Create and solve multiplication moved from 3.6; create and solve with division is new expectation]

c) create and solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less.[Moved from 3.6]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~A certain amount of practice is necessary to develop fluency with computational strategies; however, the practice must be motivating and systematic if students are to develop fluency in computation, whether mental, with manipulative materials, or with paper and pencil.~~[Moved to 4.4a US] • Strategies that allow students to derive unknown facts from facts they do know include: doubles (2s facts), doubling twice (4s facts), five facts (half of ten), decomposing into known facts (e.g., 7 × 8 can be thought of as 5 × 8 + 2 × 8). • ~~Strategies to learn the multiplication facts through the twelves table include an understanding of multiples/skip counting, properties of zero and one as factors, pattern of nines, commutative property, and related facts.~~[Moved to 4.4a US] • Students should experience a variety of problem types related to multiplication and division. Some examples are included in the following chart:		

3.4~~5~~ The student will ~~recall multiplication facts through the twelves table, and the corresponding division facts.~~[Recall moved to 4.4a]

a) represent multiplication and division, using a variety of approaches and models;[Moved from 3.6]

b) create and solve single-step practical problems that involve multiplication and division through 10 × 10; and[Create and solve multiplication moved from 3.6; create and solve with division is new expectation]

c) create and solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less.[Moved from 3.6]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS

MULTIPLICATION AND DIVISION PROBLEM TYPES

EQUAL GROUP PROBLEMS		
Whole Unknown	Size of Groups Unknown	Number of Groups Unknown
There are 4 bags of apples. There are 6 apples in each bag. How many apples in all?	If 24 apples are shared equally among 4 friends, how many apples will each friend get?	If 24 apples are packed into bags containing 6 apples each. How many bags are needed?

MULTIPLICATIVE COMPARISON PROBLEMS		
Result Unknown	Start Unknown	Comparison Factor Unknown
Bob picked 6 apples. Sue picked 4 times as many apples as Bob. How many did Sue pick?	Sue picked 24 apples. She picked 4 times as many apples as Bob. How many apples did Bob pick?	Sue picked 24 apples. Bob only picked 6 apples. How many times more apples did Sue pick than Bob?

ARRAY OR AREA PROBLEMS	
Whole Unknown	One Dimension Unknown
There are 9 rows of children at the assembly with 12 children in each row. How many children are there together? Joe's rectangular garden measures 5 feet by 12 feet. How many square feet are in the garden?	There are 108 children at the assembly. The children are divided equally in 9 rows. How many children are in each row? There are 108 children at the assembly. There are 12 children in each row. How many rows of children are there? Joe's rectangular garden covers 60 square feet. The length of the garden is 5 feet. What is the width of the garden?

RATE PROBLEMS	
Whole Unknown	Size of Groups Unknown
If you need 15 cents to buy one sticker how much money do you need to buy 4 stickers?	Peter bought 15 folders for $75.00. If each folder cost the same price how much did one folder cost? How much did 7 folders cost?

COMBINATION PROBLEMS	
Outcomes Unknown	Factor Unknown
An experiment involves tossing a coin and rolling a die. How many different outcomes are possible? Sue has 2 pairs of pants and 3 shirts that can all be worn together. How many different outfits consisting of a pair of pants and a shirt does she have?	Sue bought some new pants and shirts that can all be worn together. He has a total of 12 different outfits. If he bought 3 pairs of pants, how many shirts did he buy?

3.4~~5~~ The student will ~~recall multiplication facts through the twelves table, and the corresponding division facts.~~[Recall moved to 4.4a]

a) represent multiplication and division, using a variety of approaches and models;[Moved from 3.6]

b) create and solve single-step practical problems that involve multiplication and division through 10 × 10; and[Create and solve multiplication moved from 3.6; create and solve with division is new expectation]

c) create and solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less.[Moved from 3.6]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The least number of steps necessary to solve a single-step problem is one. • The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 2 × 3 = 3 × 2).[Moved from 3.20] • The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number.[Moved from 3.20] • The associative property of addition states that the sum stays the same when the grouping of addends is changed [e.g., 15 + (35 + 16) = (15 + 35) + 16]. The associative property of multiplication states that the product stays the same when the grouping of factors is changed [e.g., 6 × (3 × 5) = (6 × 3) × 5].[Moved from 4.16] • The distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products:[Copied from 5.19] 8 × 7 = (8 × 5) + (8 ×2) 40 + 16 56 5 × 23 = 5(20+3) 100+ 15 115 • At this level, it is not necessary for students to use the formal terms for the properties.		

3.4~~5~~ The student will ~~recall multiplication facts through the twelves table, and the corresponding division facts.~~[Recall moved to 4.4a]

a) represent multiplication and division, using a variety of approaches and models;[Moved from 3.6]

b) create and solve single-step practical problems that involve multiplication and division through 10 × 10; and[Create and solve multiplication moved from 3.6; create and solve with division is new expectation]

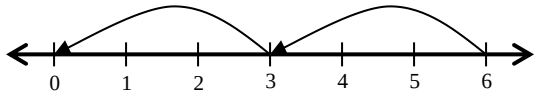
c) create and solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less.[Moved from 3.6]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Strategies for solving problems that involve multiplication or division may include mental strategies, partial products, the standard algorithm, and the commutative, associative, and distributive properties.[Moved from 3.20] • ~~In order to develop and use strategies to learn the multiplication facts through the twelves table, students should use concrete materials, hundred chart, and mental mathematics.~~ • ~~To extend the understanding of multiplication, three models may be used:~~ • ~~The equal-sets or equal-groups model lends itself to sorting a variety of concrete objects into equal groups and reinforces repeated addition or skip counting.~~ • ~~The array model, consisting of rows and columns (e.g., 3 rows of 4 columns for a 3-by-4 array) helps build the commutative property.~~ • ~~The length model (e.g., a number line) also reinforces repeated addition or skip counting.~~[Reordered]		

~~3.6 The student will represent multiplication and division, using area, set, and number line models, and create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 5 or less.~~[Moved to 3.4a and 3.4c]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~The multiplication and division facts through the twelves tables should be modeled.~~ • ~~Multiplication is a shortcut for repeated addition. The terms associated with multiplication are listed below:~~ ~~*factor* → 54~~ ~~*factor* → × 3~~ ~~*product* →162~~ [Moved to 3.4 US] • ~~Creating real-life problems and solving them facilitates the connection between mathematics and everyday experiences (e.g., area problems).~~ • ~~The use of Base-10 blocks and repeated addition can serve as a model. For example, 4 × 12 is read as four sets consisting of one rod and two units. The sum is renamed as four rods and eight units or 48. This can be thought of as~~ ~~12 + 12 + 12 + 12 = (SET)~~ • ~~The use of Base-10 blocks and the array model can be used to solve the same problem. A rectangle array that is one rod and two units long by four units wide is formed. The area of this array is represented by 4 rods and 8 units.~~ • ~~The number line model can be used to solve a multiplication problem such as 3 × 4. This is represented on the number line by three jumps of four.~~ • ~~The number line model can be used to solve a division problem such as 6 ÷ 3 and is represented on the number line by noting how many jumps of three go from 6 to 0.~~	~~**All students should**~~ • ~~Understand the meanings of multiplication and division.~~ • ~~Understand the models used to represent multiplying and dividing whole numbers.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Model multiplication, using area, set, and number line models.~~ • ~~Model division, using area, set, and number line models.~~ • ~~Solve multiplication problems, using the multiplication algorithm, where one factor is 99 or less and the second factor is 5 or less.~~ • ~~Create and solve word problems involving multiplication, where one factor is 99 or less and the second factor is 5 or less.~~[Moved to 3.4 EKS]

~~3.6 The student will represent multiplication and division, using area, set, and number line models, and create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 5 or less.~~[Moved to 3.4a and 3.4c]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
 ~~The number of jumps (two) of a given length (three) is the answer to the question.~~ • ~~An algorithm is a step-by-step method for computing.~~		

3.5~~7~~ The student will solve practical problems that involve addition and subtraction with proper fractions having like denominators of 12 or less.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A proper fraction is a fraction whose numerator is less than the denominator. A proper fraction is a fraction that is always less than one. • An improper fraction is a fraction whose numerator is greater than or equal to the denominator. An improper fraction is a fraction that is equal to or greater than one. • An improper fraction can be expressed as a mixed number. A mixed number is written as a whole number and a proper fraction. When adding or subtracting fractions, an answer greater than one can be expressed as an improper fraction or the equivalent mixed number. For example: $\frac{3}{5}+\frac{4}{5}=\frac{7}{5}=1\frac{2}{5}$. ~~• The strategies of addition and subtraction applied to fractions are the same as the strategies applied to whole numbers.~~ • Reasonable answers to problems involving addition and subtraction of fractions can be established by using benchmarks such as 0, $\frac{1}{2}$, and 1. For example, $\frac{3}{5}$ and $\frac{4}{5}$ are each greater than $\frac{1}{2}$, so their sum is greater than 1. • Concrete materials and pictorial models representing area/regions (e.g., circles, squares, and rectangles), length/measurements (fraction bars and strips), and sets (counters) can be used to add and subtract fractions having like denominators of 12 or less.	~~All students should~~ ~~• Understand that a proper fraction is a fraction whose numerator is smaller than its denominator.~~ ~~• Understand that an improper fraction is a fraction whose numerator is greater than or equal to the denominator and is one or greater than one.~~ ~~• Understand that an improper fraction can be expressed as a whole number or a mixed number.~~ ~~• Understand that a mixed number is written as a whole number and a proper fraction. A mixed number is the sum of a whole number and the proper fraction.~~ ~~• Understand that computation with fractions uses the same strategies as whole number computation.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** ~~• Demonstrate a fractional part of a whole, using~~ ~~- region/area models (e.g., pie pieces, pattern blocks, geoboards, drawings);~~ ~~- set models (e.g., chips, counters, cubes, drawings); and~~ ~~- length/measurement models (e.g., nonstandard units such as rods, connecting cubes, and drawings).~~[Moved to 3.2 EKS] ~~• Name and write fractions and mixed numbers represented by drawings or concrete materials.~~[Moved to 3.2 EKS] • ~~Represent a given fraction or mixed number, using concrete materials, pictures, and symbols. For example, write the symbol for one-fourth and represent it with concrete materials and/or pictures.~~[Moved to 3.2 EKS] • Solve practical problems that involve ~~A~~addition and subtraction with proper fractions having like denominators of 12 or less, using concrete materials and pictorial models representing area/regions (e.g., circles, squares, and rectangles), length/measurements (e.g., fraction bars and strips), and sets (e.g., counters).

~~Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K–3 should encounter measurement in many normal situations, from their daily use of the calendar and from science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.~~

~~Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, area, perimeter, volume, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.~~

~~As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both techniques and nonstandard and standard tools to find measurements and to develop an understanding of the use of simple U.S. Customary and metric units.~~

~~Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.~~

[Moved from grade 4 Measurement strand introduction]

Students in grades 3-5 should be actively involved in measurement activities that require a dynamic interaction among students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects.

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, perimeter, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary), to measure length, weight/mass, liquid volume, area, perimeter, temperature, and time. Student understanding of measurement continues to be enhanced by using appropriate tools such as rulers, balances, clocks, and thermometers.

[Moved from grade 4 Geometry stand introduction]
The study of geometry helps students represent and make sense of the world. In grades 3 through 5 reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.
Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles.

[Moved from geometry strand introduction]
The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)

3.6~~8~~ The student will

a) determine~~, by counting,~~ the value of a collection of bills and coins whose total value is $5.00 or less~~,~~;
b) compare the value of ~~the~~ two sets of coins or two sets of coins and bills ~~and coins,~~; and
c) make change from $5.00 or less.[Edited to match EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Simulate everyday opportunities to count and compare a collection of coins and one-dollar bills whose total value is $5.00 or less. • The value of a collection of coins and bills can be determined by counting on, beginning with the highest value, and/or by grouping the coins and bills. • A variety of skills can be used to determine the change after a purchase, including: – counting on, using coins and bills, ~~i.e.~~e.g., starting with the amount to be paid (purchase price),~~-~~; – counting forward to the next dollar, and then counting forward by dollar bills to reach the amount from which to make change; and – mentally calculating the difference.	~~All students should~~ • ~~Understand that a collection of coins and bills has a value that can be counted.~~ • ~~Understand how to make change from $5.00 or less.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Count~~ Determine the value of collections of coins and bills up to $5.00. (a) • Compare the values of two sets of coins or two sets of coins and bills, up to $5.00, using the terms *greater than*, *less than*, and *equal to*. (b) • Make change from $5.00 or less. (c)

3.7~~9~~ The student will estimate and use U.S. Customary and metric units to measure

a) length to the nearest $\frac{1}{2}$ inch, inch, foot, yard, centimeter, and meter; and

b) liquid volume in cups, pints, quarts, gallons, and liters~~;~~.

~~c) weight/mass in ounces, pounds, grams, and kilograms; and~~[Included in 4.8]

~~d) area and perimeter.~~[Moved to 3.8a,b]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?").~~ • The concept of a standard measurement unit is one of the major ideas in understanding measurement. Familiarity with standard units is developed through hands-on experiences of comparing, estimating, measuring, and constructing. • Benchmarks of common objects need to be established for each of the specified units of measure (e.g., the liquid volume of a small glass of orange juice is about one cup~~ mass of a mathematics book is about one kilogram~~). Practical experiences measuring the ~~mass~~volume of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate measures. • One unit of measure may be more appropriate than another to use when measure~~ing~~ an object, depending on the size of the object and the degree of accuracy desired. • Correct use of measurement tools is essential to understanding the concepts of measurement. • ~~Perimeter is the distance around any two-dimensional figure and is found by adding the measures of the sides.~~ • ~~Area is a two-dimensional measure and is therefore measured in square units.~~ • ~~Area is the number of square units needed to cover a figure, or~~	~~**All students should**~~ • ~~Understand how to estimate measures of length, liquid volume, weight/mass, area and perimeter.~~ • ~~Understand how to determine the actual measure of length, liquid volume, weight/mass, area and perimeter.~~ • ~~Understand that perimeter is a measure of the distance around a polygon.~~ • ~~Understand that area is a measure of square units needed to cover a surface.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate and use U.S. Customary and metric units to measure lengths of objects to the nearest $\frac{1}{2}$ ~~of an~~ inch, inch, foot, yard, centimeter, and meter. (a) • Determine the actual measure of length using U.S. Customary and metric units to measure objects to the nearest $\frac{1}{2}$ ~~of an~~ inch, foot, yard, centimeter, and meter. (a) • Estimate and use U.S. Customary and metric units to measure liquid volume to the nearest cup, pint, quart, gallon, and liter. (b) • Determine the actual measure of liquid volume using U.S. Customary and metric units to measure to the nearest cup, pint, quart, gallon, and liter. (b) • ~~Estimate and use U.S. Customary and metric units to measure the weight/mass of objects to the nearest ounce, pound, gram, and kilogram.~~ • ~~Determine the actual measure of weight/mass using U.S. Customary and metric units to measure the weight/mass of objects to the nearest ounce, pound, gram, and kilogram.~~[Included in 4.6] • ~~Estimate and use U.S. Customary and metric units to measure area and perimeter.~~[Moved to 3.8 EKS] • ~~Determine the actual measure of area or perimeter using U.S. Customary and metric units.~~[Moved to 3.8 EKS]

3.7~~9~~ The student will estimate and use U.S. Customary and metric units to measure

a) length to the nearest $\frac{1}{2}$ inch, inch, foot, yard, centimeter, and meter; and

b) liquid volume in cups, pints, quarts, gallons, and liters~~;~~.

~~c) weight/mass in ounces, pounds, grams, and kilograms; and~~[Included in 4.8]

~~d) area and perimeter.~~[Moved to 3.8a,b]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~more precisely, it is the measure in square units of the interior region of a two-dimensional figure.~~		

3.8~~10~~ The student will estimate and[Moved from 3.9d EKS]

a) measure the distance around a polygon in order to determine its perimeter using U.S. Customary and metric units[Moved from 3.9d EKS]; and

b) count the number of square units needed to cover a given surface in order to determine its area.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A polygon is a closed plane figure composed of at least three line segments that do not cross. ~~None of the sides are curved.~~ • Perimeter is ~~a measure of~~ the path or distance around ~~a polygon and is found by adding the measures of the sides~~any plane figure. • Area is the number of iterations of a two-dimensional unit needed to cover a surface. The two-dimensional unit is usually a square, but it could also be another shape such as a rectangle or an equilateral triangle. • Opportunities to explore the concepts of perimeter and area should involve hands-on experiences (e.g., placing ~~tiles~~toothpicks (units) around a polygon and counting the number of ~~tiles~~toothpicks to determine its perimeter and filling or covering a polygon with ~~cubes~~tiles (square units) and counting ~~the cubes~~the tiles to determine its area).	~~**All students should**~~ • ~~Understand the meaning of a polygon as a closed figure with at least three sides. None of the sides are curved and there are no intersecting lines.~~ • ~~Understand that perimeter is a measure of the distance around a polygon.~~ • ~~Understand how to determine the perimeter by counting the number of units around a polygon.~~ • ~~Understand that area is a measure of square units needed to cover a surface.~~ • ~~Understand how to determine the area by counting the number of square units.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate and use U.S. Customary and metric units to ~~M~~measure the distance around a polygon with no more than 6 sides ~~each side of a variety of polygons and add the measures of the sides~~ to determine the perimeter ~~of each polygon~~. (a)[Estimate and use moved from 3.9d] • Determine the area of a given surface by estimating and then counting the number of square units needed to cover the surface. (b)

3.9~~11~~ The student will

a) tell time to the nearest minute, using analog and digital clocks; and

b) ~~determine~~ solve practical problems related to elapsed time in one-hour increments ~~over~~within a 12-hour period; and

c) identify equivalent periods of time and solve practical problems related to equivalent periods of time.[Moved from 3.12]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• While digital clocks make reading time easy, it is necessary to ensure that students understand that there are sixty minutes in an hour.~~ • Use of a demonstration clock with gears ensures that the positions of the hour hand and the minute hand are precise when time is read. • Students need to understand that time has passed or will pass in equal increments (i.e., seconds, minutes, or hours). • Elapsed time is the amount of time that has passed between two given times. • Elapsed time should be modeled and demonstrated using geared analog clocks and timelines. • ~~It is necessary to ensure that s~~Students need to understand that there are sixty minutes in an hour when using analog and digital clocks. • Elapsed time can be found by counting on from the beginning time or counting back from the ending time. ~~to the finishing time.~~ ~~• Count the number of whole hours between the beginning time and the finishing time.~~ ~~• For example, to find the elapsed time between 7 a.m. and 10 a.m., students can count on to find the difference between the times (7 and 10), so the total elapsed time is 3 hours.~~ • The knowledge that a year has 365 and $\frac{1}{4}$ days will help students understand the necessity of adding a full day every fourth year, called a *leap year*.[Moved from 3.12]	~~**All students should**~~ ~~• Apply appropriate techniques to determine time to the nearest minute, using analog and digital clocks.~~ ~~• Understand how to determine elapsed time in one-hour increments over a 12-hour period.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Tell time to the nearest minute, using analog and digital clocks. (a) • Match a written time (e.g., 4:38, 7:09, 12:51) to the time~~s~~ shown on analog and digital clocks to ~~written times and to each other~~the nearest minute. (a) ~~• When given the beginning time and ending time, determine the elapsed time in one-hour increments within a 12-hour period (times do not cross between a.m. and p.m.).~~ • Solve practical problems ~~in relation to time that has~~ related to elapsed time in one-hour increments, within a 12-hour period (within a.m. or within p.m.): – when given the beginning time and the ending time, determine the time that has elapsed; (b) – when given the beginning time and amount of elapsed time in one-hour increments, determine the ending time; or (b) – when given the ending time and the elapsed time in one-hour increments, determine the beginning time. (b) • Identify the number of minutes in an hour and the number of hours in a day.[Moved from 3.12] • Identify equivalent relationships observed in a calendar, including the approximate number of days in a given month (about 30), the number of days in a week, the number of days in a year (365 ¼), and the number of months in a year.[Moved from 3.12]

3.9~~11~~ The student will

a) tell time to the nearest minute, using analog and digital clocks; and

b) ~~determine~~ solve practical problems related to elapsed time in one-hour increments ~~over~~within a 12-hour period; and

c) identify equivalent periods of time and solve practical problems related to equivalent periods of time.[Moved from 3.12]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The use of a calendar facilitates the understanding of time relationships between days and months, days and weeks, days and years, and months and years. Students need to know the relationship between periods of time (e.g., if there are 24 hours in one day, how many hours are in three days?).[Moved from 3.12] • The use of an analog clock facilitates the understanding of time relationships between minutes and hours and hours and days.[Moved from 3.12]		• Solve practical problems in relation to equivalent periods of time to include: – approximate days in 5 or fewer months; – days in 5 or fewer weeks; – months in 5 or fewer years; – minutes in 5 or fewer hours; and – hours in 5 or fewer days.

~~3.12 The student will identify equivalent periods of time, including relationships among days, months, and years, as well as minutes and hours.~~[Moved to 3.9c]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~The knowledge that a year has 365 and $\frac{1}{4}$ days will help students understand the necessity of adding a full day every fourth year, called a *leap year*.~~[Moved to 3.9 US] • ~~The use of a calendar facilitates the understanding of time relationships between days and months, days and weeks, days and years, and months and years.~~[Moved to 3.9 US] • ~~Recognize that students need to know the relationships, such as if there are 24 hours in one day, how many hours are in three days?~~[Moved to 3.9]~~If the date is January 6, what date would it be in two weeks? How many weeks are in March, April, and May?~~ • ~~The use of an analog clock facilitates the understanding of time relationships between minutes and hours and hours and days.~~[Moved to 3.9 US]	~~**All students should**~~ • ~~Understand the relationship that exists among periods of time, using calendars, and clocks.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Identify equivalent relationships observed in a calendar, including the number of days in a given month, the number of days in a week, the number of days in a year, and the number of months in a year.~~[Moved to 3.9 EKS] • ~~Identify the number of minutes in an hour and the number of hours in a day~~.[Moved to 3.9 EKS]

3.10~~13~~ The student will read temperature to the nearest degree ~~from a Celsius thermometer and a Fahrenheit thermometer. Real thermometers and physical models of thermometers will be used.~~[Thermometer information moved to EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Estimating and measuring temperatures in the environment in Fahrenheit and Celsius require the use of real thermometers. • A variety of physical models (e.g., circular, linear) should ~~can~~ be used to represent the temperature determined by a real thermometer. • The symbols for degrees in Celsius (°C) and degrees in Fahrenheit (°F) should be used to write temperatures. • Celsius and Fahrenheit temperatures should be related to everyday occurrences by measuring ~~the temperature of the classroom, the outside, liquids, body temperature, and other things found in the environment.~~ things found in the student's environment (e.g., the temperature of the classroom; temperature on the playground; temperature of warm and cold liquids; body temperature). • At this level, scale increments should be limited to 1 or 2.	~~**All students should**~~ ~~Understand how to measure temperature in Celsius and Fahrenheit with a thermometer.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Read Celsius and Fahrenheit temperatures to the nearest degree ~~from~~ using real ~~Celsius and Fahrenheit~~ thermometers, physical models, or ~~(including~~ using pictorial representations~~)~~ of ~~such~~ thermometers.

~~Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.~~

~~The focus of instruction at this level is on~~

- ~~observing, identifying, describing, comparing, contrasting and investigating solid objects and their faces;~~
- ~~sorting objects and ordering them directly by comparing them one to the other;~~
- ~~describing, comparing, contrasting, sorting, and classifying figures; and~~
- ~~exploring symmetry, congruence, and transformation.~~

~~In the primary grades, children begin to develop basic vocabulary related to these figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).~~

~~The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.~~

- ~~**Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.~~
- ~~**Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)~~
- ~~**Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)~~

3.11 The student will

a) define polygon;[Moved from 4.12a]

b) identify and name polygons with 10 or fewer sides; and[Moved from 4.12b]

c) combine and subdivide polygons with 3 or 4 sides and name the resulting polygons.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~		ESSENTIAL KNOWLEDGE AND SKILLS
• A polygon is a closed figure composed of at least 3 line segments that do not cross. None of the sides are curved. • Polygons may be described using line segments and angles. • A rectangle, square, trapezoid, parallelogram, and rhombus are all quadrilaterals.		**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Define a polygon. (a)[Moved from 4.12 EKS] • Classify figures as polygons or not polygons. (a)[Moved from 4.12 EKS] • Identify and name polygons with 10 or fewer sides in various orientations: – triangle is a 3-sided polygon. – quadrilateral is a 4-sided polygon. – pentagon is a 5-sided polygon. – hexagon is a 6-sided polygon. – heptagon is a 7-sided polygon. – octagon is a 8-sided polygon. – nonagon is a 9-sided polygon. – decagon is a 10-sided polygon. (b)[Moved from 4.12 EKS] • Combine no more than three polygons, where each has 3 or 4 sides, and name the resulting polygon. (c) • Subdivide a 3- or 4-sided polygon into no more than three parts and name the resulting polygons. (c)

~~3.14 The student will identify, describe, compare, and contrast characteristics of plane and solid geometric figures (circle, square, rectangle, triangle, cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models.~~[Moved to 4.11]

~~UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.~~ ~~**Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.~~ ~~**Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., "I know it's a rectangle because it looks like a door, and I know that the door is a rectangle.").~~ ~~**Level 2: Analysis**. Properties are perceived, but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., "I know it's a rectangle because it's closed, it has four sides and four right angles, and opposite sides are parallel.").~~ • ~~A plane geometric figure is any two-dimensional closed figure. Circles and polygons are examples of plane geometric figures.~~ • ~~Three-dimensional figures are called solid figures or simply solids. Solids enclose a region of space. The interior of both plane and solid figures are not part of the figure. Solids are classified by the types of surfaces they have. These surfaces may be flat, curved, or both.~~ • ~~The study of geometric figures must be active, using visual images and concrete materials.~~ • ~~Access to a variety of concrete tools such as graph paper, pattern blocks, geoboards, and geometric solids is greatly enhanced by computer software tools that support exploration.~~	~~**All students should**~~ • ~~Understand how to identify and describe plane and solid geometric figures by using relevant characteristics.~~ • ~~Understand the similarities and differences between plane and solid figures.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Identify models and pictures of plane geometric figures (circle, square, rectangle, and triangle) and solid geometric figures (cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by name.~~ • ~~Identify and describe plane geometric figures by counting the number of sides and angles.~~ • ~~Identify and describe solid geometric figures by counting the number of angles, vertices, edges, and by the number and shape of faces.~~ • ~~Compare and contrast characteristics of plane and solid geometric figures (e.g., circle/sphere, square/cube, triangle/square pyramid, and rectangle/rectangular prism), by counting the number of sides, angles, vertices, edges, and the number and shape of faces.~~ • ~~Compare and contrast characteristics of solid geometric figures (i.e., cube, rectangular prism, square pyramid, sphere, cylinder, and cone) to similar objects in everyday life (e.g., a party hat is like a cone).~~ • ~~Identify characteristics of solid geometric figures (cylinder, cone, cube, square pyramid, and rectangular prism).~~ [Moved to 4.11 EKS]

~~3.14~~ ~~The student will identify, describe, compare, and contrast characteristics of plane and solid geometric figures (circle, square, rectangle, triangle, cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models.~~[Moved to 4.11]

~~UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Opportunity must be provided for building and using geometric vocabulary to describe plane and solid figures.~~ • ~~A cube is a solid figure with six congruent square faces and with every edge the same length. A cube has 8 vertices and 12 edges.~~ • ~~A cylinder is a solid figure formed by two congruent parallel circles joined by a curved surface.~~ • ~~A cone is a solid, pointed figure that has a flat, round face (usually a circle) that is joined to a vertex by a curved surface.~~ • ~~A rectangular prism is a solid figure in which all six faces are rectangles with three pair of parallel congruent opposite faces.~~ • ~~A sphere is a solid figure with all of its points the same distance from its center.~~ • ~~A square pyramid is a solid figure with one square face and four triangular faces that share a common vertex.~~ • ~~A face is a polygon that serves as one side of a solid figure (e.g., a square is a face of a cube).~~ • ~~An angle is formed by two rays with a common endpoint. This endpoint is called the *vertex*. Angles are found wherever lines intersect. An angle can be named in three different ways by using~~ ~~- three letters to name, in this order, a point on one ray, the vertex, and a point on the other ray;~~ ~~- one letter at the vertex; or~~ ~~- a number written inside the rays of the angle.~~ • ~~An edge is the line segment where two faces of a solid figure intersect.~~ • ~~A vertex is the point at which two lines, line segments, or rays meet to form an angle. It is also the point on a three dimensional figure where three or more faces intersect.~~		

~~**3.14 The student will identify, describe, compare, and contrast characteristics of plane and solid geometric figures (circle, square, rectangle, triangle, cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models.**~~**[Moved to 4.11]**

~~**UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)**~~	~~**ESSENTIAL UNDERSTANDINGS**~~	~~**ESSENTIAL KNOWLEDGE AND SKILLS**~~
• ~~Students should be reminded that a solid geometric object is hollow rather than solid. The "solid" indicates a three-dimensional figure.~~		

3.12~~15~~ The student will identify and draw representations of points, lines, line segments, rays, and angles~~, and lines~~.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A point is an exact location in space. It has no length, ~~or~~ width, or height. ~~Usually, a point is named with a capital letter.~~[Naming is included in grade 4] • A line is a collection of points extending indefinitely ~~going on and on~~ in both directions. It has no endpoints. ~~When a line is drawn, at least two points on it can be marked and given capital letter names. The line can also be named with a single, lower-case letter. Arrows must be drawn to show that the line goes on in both directions infinitely.~~[Naming is included in grade 4] • A line segment is part of a line. It has two endpoints and includes all the points between and including those endpoints. ~~The endpoints are used to name a line segment.~~[Naming is included in grade 4] • A ray is part of a line. It has one endpoint and ~~continues on and on~~extends indefinitely in one direction. • An angle is formed by two rays that share~~having~~ a common endpoint ~~. This endpoint is~~ called the vertex. Angles are found wherever lines or line segments intersect. ~~• Angles are found wherever lines and line segments intersect. An angle can be named in three different ways by using~~ ~~- three letters to name, in this order, a point on one ray, the vertex, and a point on the other ray;~~ ~~- one letter at the vertex; or~~ ~~- a number written inside the rays of the angle.~~ ~~Angle rulers may be particularly useful in developing the concept of an angle.~~	~~All students should~~ ~~• Understand that line segments and angles are fundamental components of plane polygons.~~ ~~• Understand that a line segment is a part of a line, has two end points, and contains all the points between those two endpoints.~~ ~~• Understand that points make up a line.~~ ~~• Understand that a line continues indefinitely in two opposite directions.~~ ~~• Understand that a ray is part of a line, has one endpoint, and continues indefinitely in only one direction.~~ ~~• Understand that an angle is formed by two rays having a common endpoint.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify examples of points, lines, line segments, rays, and angles~~, and lines~~. • Describe endpoints and vertices as they relate to lines, line segments, rays, and angles. • Draw representations of points, line segments, rays, angles, and lines, using a ruler or straightedge.

3.13~~16~~ The student will identify and describe congruent and noncongruent ~~plane~~ figures.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Congruent ~~plane~~ figures ~~are figures having exactly~~have the same size and shape. Noncongruent ~~plane~~ figures ~~are figures that are~~do not have exactly the same size and shape. Opportunities for exploring figures that are congruent and/or noncongruent can best be accomplished by using physical models. • Congruent plane figures remain congruent even if they are in different spatial orientations.[Moved from middle column] • ~~Have students identify f~~Figures that are congruent or noncongruent may be identified by using direct comparisons and/or tracing procedures.	~~**All students should**~~ • ~~Understand that congruent plane figures match exactly.~~ • ~~Understand that noncongruent plane figures do not match exactly.~~ • ~~Understand that congruent plane figures remain congruent even if they are in different spatial orientations.~~ [Moved to US] • ~~Understand that noncongruent plane figures remain noncongruent even if they are in different spatial orientations.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify examples of congruent and noncongruent figures. ~~Verify their congruence by laying one on top of the other using drawings or models.~~ • Determine and explain why plane figures are congruent or noncongruent~~, using tracing procedures~~.

~~Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.~~

~~The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible*, *unlikely*, *equally likely*, *more likely*, and *certain*.~~

~~The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.~~

[Moved from grade 4 strand introduction]
Students entering grades 3-5 have begun to explore~~d~~ the concepts of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students have had initial experiences in predicting whether a game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction at grades 3-5 is to deepen their understanding of the concepts of probability by:

- offering opportunities to set up models simulating practical events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities that instill a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data collection strategies. Students should continue to focus on:

- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to practical experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions (“How many?” “How much?”) from the data displays;
- identifying and justifying comparisons (“Which is the most? Which is the least?” “Which is the same? Which is different?”) about the information;
- comparing their initial predictions to the actual results; and
- communicating to others their interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data analysis methods as powerful means for decision making.

3.14~~17~~ The student will

a) collect, ~~and~~ organize, and represent data~~, using observations, measurements, surveys, or experiments~~ in picture graphs or bar graphs; and~~;~~[Process included in EKS]

b) ~~construct a line plot, a picture graph, or a bar graph to represent the data; and~~[Line plot moved to 5.16; construct picture and bar graphs moved to 3.14a]

~~c)~~ read and interpret the data represented in bar graphs~~,~~ and picture graphs~~ and write a sentence analyzing the data~~.

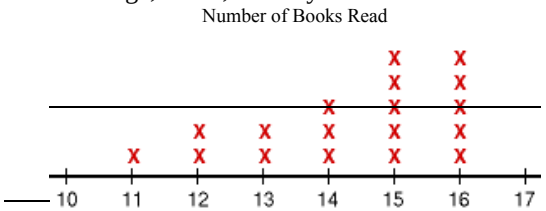
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Investigations involving data should occur frequently and relate to students' experiences, interests, and environment. • Formulating questions for investigations is student-generated at this level. For example: What is the cafeteria lunch preferred by students in the class when four lunch menus are offered? • The purpose of a graph is to represent data gathered to answer a question. • A picture graph uses symbols to represent one or more objects. • A key should be provided for the symbol in a picture graph when the symbol represents more than one piece of data (e.g., represents five people in a graph). The key is used in a graph to assist in the analysis of the displayed data. • Bar graphs are used to compare counts of different categories (categorical data). Using grid paper ensures more accurate graphs. • A bar graph uses ~~parallel,~~ horizontal or vertical parallel bars to represent counts for categories. One bar is used for each category, with the length of the bar representing the count for that category. • There is space before, between, and after each of the bars. • The axis displaying the scale representing the count for the categories should begin at zero and extend one increment above the greatest recorded piece of data. Third grade students should collect data that are recorded in increments of whole numbers, ~~usually~~ limited to multiples of 1, 2, 5, or 10. • Each axis should be labeled, and the graph should be given a title.	~~All students should~~ • ~~Understand how data can be collected and organized.~~ • ~~Understand that data can be displayed in different types of graphs depending on the data.~~[Moved to US] • ~~Understand how to construct a line plot, picture graph, or bar graph.~~ • ~~Understand that data sets can be interpreted and analyzed to draw conclusions.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Formulate questions to investigate. (a) • Design data investigations to answer formulated questions, limiting the number of categories for data collection to four. (a) • Collect and organize data, using various forms of data collections (e.g., surveys, polls, questionnaires, scientific experiments, ~~and~~ observations). (a) • ~~Organize data and construct~~Represent data in a bar graph ~~on grid paper representing~~ (limited to16 or fewer data points for no more than four categories). (a) • ~~Construct a line plot with no more than 30 data points.~~[Moved to 5.16 EKS] • ~~Read, interpret and analyze information from line plots by writing at least one statement.~~[Moved to 5.16 EKS] – Label each axis on a bar graph and give the bar graph a title. Limit increments on the numerical axis to whole numbers representing multiples of 1, 2, 5, or 10. (a) • Represent data in a picture graph (limited to16 or fewer data points for no more than four categories). (a) • Analyze the data represented in picture graphs and bar graphs, orally and in writing. (b) • – Read the information presented on a ~~simple~~ bar or picture graph (e.g., the title, the categories, the description of the two axes). (b)

3.14~~17~~ The student will

a) collect, ~~and~~ organize, and represent data~~, using observations, measurements, surveys, or experiments~~ in picture graphs or bar graphs; and~~;~~[Process included in EKS]

b) ~~construct a line plot, a picture graph, or a bar graph to represent the data; and~~[Line plot moved to 5.16; construct picture and bar graphs moved to 3.14a]

~~c)~~ read and interpret the data represented in bar graphs~~,~~ and picture graphs~~ and write a sentence analyzing the data~~.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Statements representing an analysis and interpretation of the characteristics of the data in the graph ~~(e.g., similarities and differences, least and greatest, the categories, and total number of responses)~~ should be written (e.g., similarities and differences, least and greatest, the categories, and total number of responses). • ~~A line plot shows the frequency of data on a number line. Line plots are used to show the spread of the data and quickly identify the range, mode, and any outliers.~~  ~~Each x represents one student~~ • ~~When data are displayed in an organized manner, the results of the investigations can be described and the posed question answered.~~ • ~~Recognition of appropriate and inappropriate statements begins at this level with graph interpretations.~~		• ~~Analyze and i~~Interpret information from picture and bar graphs, with up to 30 data points and up to 8 categories, describe interpretation orally and by writing at least one sentence. (b) • Describe the categories of data and the data as a whole (e.g., data were collected on four ways to cook or prepare eggs — scrambled, fried, hard boiled, and egg salad — eaten by students). (b) • Identify parts of the data that have special characteristics, including categories with the greatest, the least, or the same (e.g., most students prefer scrambled eggs). (b) • Select a correct interpretation of a graph from a set of interpretations~~ of the graph~~, where one is correct and the remaining are incorrect.~~ For example, a bar graph containing data on four ways to cook or prepare eggs — eaten by students show that more students prefer scrambled eggs. A correct answer response, if given, would be that more students prefer scrambled eggs than any other way to cook or prepare eggs~~. (b)

3.15~~18~~ The student will investigate and describe the concept of probability as a measurement of chance and list possible ~~results~~ outcomes ~~of~~for a given situation.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives. • Investigation of experimental probability is continued at this level through informal activities using materials such as two-colored counters, spinners, and random number ~~generators (number~~ cubes~~)~~. • Probability is the measurement of chance of an event occurring. ~~• The probability of an event occurring is the ratio of desired outcomes to the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event = $\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$.~~ ~~• The probability of an event occurring is represented by a ratio between 0 and 1. An event is "impossible" if it has a probability of 0 (e.g., the probability that the month of April will have 31 days). An event is "certain" if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow morning).~~ • When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time). • Students should have opportunities to describe in informal terms (e.g., *impossible*, *unlikely*, *as likely as*, ~~*equally likely,*~~ *likely*, and *certain*) the degree of likelihood of an event occurring. Activities should include real-life examples. • For any event, such as flipping a coin, spinning a spinner, or rolling a number cube, the ~~equally likely~~ things that can happen are called *outcomes*. For example, there are two ~~equally likely~~ possible outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up; when flipping a coin, each of the outcomes is equally likely.	~~**All students should**~~ ~~• Investigate, understand, and apply basic concepts of probability.~~ ~~• Understand that probability is the chance of an event happe ning.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Define probability as the measurement of chance that an event will happen. • List all possible outcomes for a given situation (e.g., heads and tails are the two possible outcomes of flipping a coin). The number of outcomes may be limited to 12 or fewer. • ~~Identify~~ Describe the degree of likelihood of an outcome occurring using terms such as *impossible*, *unlikely*, *as likely as*, ~~*equally likely,*~~ *likely*, and *certain*.

3.15~~18~~ The student will investigate and describe the concept of probability as a measurement of chance and list possible ~~results~~ outcomes ~~of~~for a given situation.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~A sample space represents a~~All possible outcomes of an experiment ~~. The sample space~~ may be organized in a list, table, or chart.		

~~Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children's mathematical development.~~

~~Students in kindergarten through third grade develop the foundation for understanding various types of patterns and functional relationships through the following experiences:~~

- ~~sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;~~
- ~~identifying, analyzing, and extending patterns;~~
- ~~creating repetitive patterns and communicating about these patterns in their own language;~~
- ~~analyzing simple patterns and making predictions about them;~~
- ~~recognizing the same pattern in different representations;~~
- ~~describing how both repeating and growing patterns are generated; and~~
- ~~repeating predictable sequences in rhymes and extending simple rhythmic patterns.~~

~~The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns. These students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to algebra.~~

[Moved from grade 4 Patterns, Functions, and Algebra strand introduction]

Students entering grades 3-5 have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships within a pattern.

The focus of instruction is to help students develop a solid use of patterning as a problem solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They interpret the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By interpreting mathematical situations and models, students begin to represent these, using symbols and variables to write "rules" for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.

3.16~~19~~ The student will ~~recognize and~~ identify, describe, create, extend and transfer patterns found in objects, pictures, numbers and tables. ~~a variety of patterns formed using numbers, tables, and pictures, and extend the patterns, using the same or different forms.~~

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Developing fluency and flexibility in identifying, describing, transferring, and extending patterns is fundamental to mathematics, particularly algebraic reasoning. • ~~Exploring patterns requires active physical and mental involvement.~~ • The use of materials to extend patterns permits experimentation or trial-and-error approaches that are almost impossible without them. • ~~Reproduction of a given pattern in a different representation, using symbols and objects, lays the foundation for writing numbers symbolically or algebraically.~~ • The simplest types of patterns are repeating patterns. In each case, students need to identify the ~~basic unit~~core of the pattern and repeat it. ~~Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience.~~ • Growing patterns are more difficult for students to understand than repeating patterns because not only must they identify the core, they must also look for a generalization or relationship that will tell them how the pattern is changing. ~~determine what comes next, they must also begin the process of generalization~~. Growing patterns can increase or decrease by a constant value. Students need experiences with growing patterns ~~in both arithmetic and geometric formats~~using objects, pictures, numbers, and tables. • Sample numeric patterns include: – 6, 9, 12, 15, 18, …(growing pattern); – 1, 2, 4, 7, 11, 16, …(growing pattern); – 20, 18, 16, 14,…(growing pattern); and – 1, 3, 5, 1, 3, 5, 1, 3, 5 …(repeating pattern). • Students should have experiences with numeric patterns that are additive in nature.	~~All students should~~ • ~~Understand that numeric and geometric patterns can be expressed in words or symbols.~~[Moved to US] • ~~Understand the structure of a pattern and how it grows or changes.~~ • ~~Understand that mathematical relationships exist in patterns.~~ • ~~Understand that patterns can be translated from one representation to another.~~[Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Recognize~~ Identify repeating and growing ~~numeric and geometric~~ patterns using objects, pictures, numbers, and tables~~(e.g., skip counting, addition tables, and multiplication tables)~~. • Identify a missing term in a pattern (e.g., 4, 6, ?, 10, 12, 14). • Describe repeating and growing ~~numeric and geometric~~ patterns formed using objects, pictures, numbers, and tables.~~numbers, tables, and/or pictures, using the same or different forms.~~ • Extend repeating and growing patterns ~~of numbers or figures~~ using ~~concrete~~ objects, pictures, numbers, and tables~~, and/or pictures~~. • Transfer repeating and growing patterns using numbers, tables, objects, and pictures.[Moved from US] • Apply input and output rules involving addition and subtraction to solve problems in various contexts.

3.16~~19~~ The student will ~~recognize and~~ identify, describe, create, extend and transfer patterns found in objects, pictures, numbers and tables. ~~a variety of patterns formed using numbers, tables, and pictures, and extend the patterns, using the same or different forms.~~

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• In geometric figure patterns, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation is the result of turning a figure around a point or a vertex, and reflection is the result of flipping a figure over a line. • Sample geometric figure patterns include: – ◇□◇ ◇□□ ◇◇◇□□□ – ▭ ⇨ ⇩ ▭ ⇧ ▭ ⇨ ⇩ ▭ ⇧ – O Δ O O Δ Δ O O O Δ Δ Δ …; and – □□★★□★□□★★□★…. • A table of values can be analyzed to determine the pattern that has been used, and that pattern can then be used to ~~find~~ determine the next value. • Sample pattern transfers include: – 2, 5, 8, 11, 14, 17 is the same pattern as 4, 7, 10, 13, 16, 19 – ▽○⇩⇧▽○⇩⇧ is the same pattern as ⬡▭▽△⬡▭▽△ • Input/output tables with a given rule can be used to determine an unknown value. Sample input/output tables: Rule: Add 4 Input: 4, 5, 8 — Output: 8, ?, ? Rule: Subtract 2 Input: 6, 8, 9 — Output: ?, 6, ?		

3.17~~20~~ The student will create equations to represent equivalent mathematical relationships.[Moved from 3.20 EKS]

~~a) investigate the identity and the commutative properties for addition and multiplication; and~~[Use of properties moved to 3.3 EKS and 3.4 EKS]

~~b) identify examples of the identity and commutative properties for addition and multiplication.~~

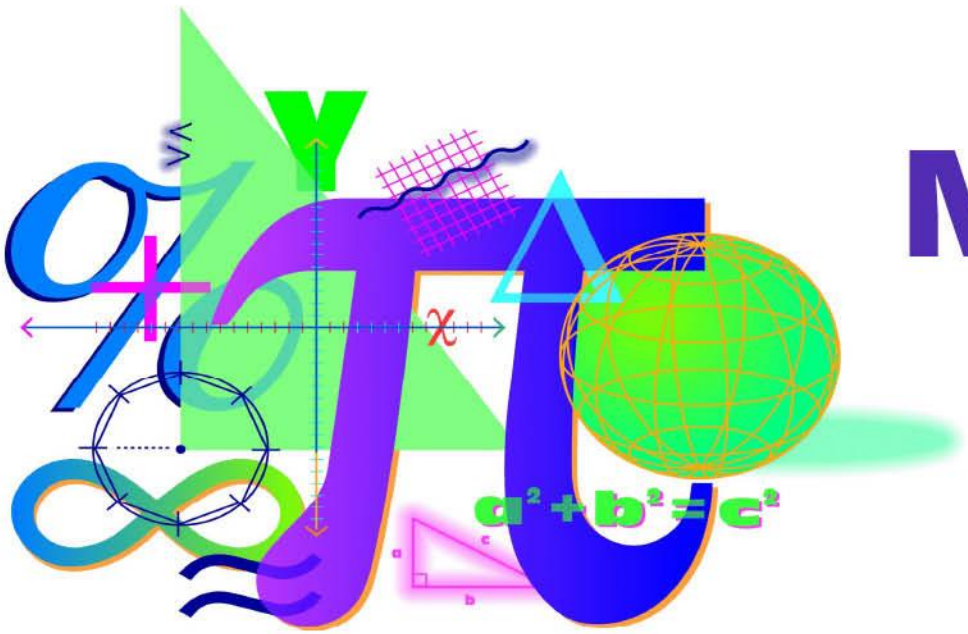
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Mathematical relationships can be expressed using equations (number sentences).[Moved from middle column] • A number sentence is an equation with numbers (e.g., 6 + 3 = 9; or 6 + 3 = 4 + 5).[Reordered] • The equal-sign (=) means that the values on either side are equivalent (balanced). • The not equal (≠) symbol means that the values on either side are not equivalent (not balanced). • Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers. • An expression is a representation of a quantity. It contains numbers, variables, and/or computational operation symbols. It does not have an equal sign (e.g., 5, 4 + 3, 8 - 2, 2 × 7). • An equation is a mathematical sentence that two expressions are equivalent. It consists of two expressions, one on each side of an 'equal' symbol (e.g., 5 + 3 = 8, 8 = 5 + 3 and 4 + 3 = 9 - 2). • An equation can be represented using balance scales, with equal amounts on each side (e.g., 3 + 5 = 6 + 2). • ~~The commutative property for addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4). Similarly, the commutative property for multiplication states that changing the order of the factors does not affect the product (e.g., 2 × 3 = 3 × 2).~~ • ~~The identity property for addition states that if zero is added to a given number, the sum is the same as the given number. The identity property of multiplication states that if a given number is~~	~~All students should~~ • ~~Understand that mathematical relationships can be expressed using number sentences.~~[Moved to US] • ~~Understand the identity property for addition.~~ • ~~Understand the identity property for multiplication.~~ • ~~Understand the commutative property of addition.~~ • ~~Understand the commutative property of multiplication.~~ • ~~Understand that quantities on both sides of an equals sign must be equal.~~ [Moved to US] • ~~Understand that quantities on both sides of the not equal sign are not equal.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Investigate the identity property for addition and determine that when the number zero is added to another number or another number is added to the number zero, that number remains unchanged. Examples of the identity property for addition are 0 + 2 = 2; 5 + 0 = 5.~~[Moved to 3.3 US] • ~~Investigate the identity property for multiplication and determine that when the number one is multiplied by another number or another number is multiplied by the number one, that number remains unchanged. Examples of the identity property for multiplication are 1 x 3 = 3; 6 x 1 = 6.~~[Moved to 3.4 US] • ~~Recognize that the commutative property for addition is an order property. Changing the order of the addends does not change the sum (5 + 4 = 9 and 4 + 5 = 9).~~[Moved to 3.3 US] • ~~Recognize that the commutative property for multiplication is an order property. Changing the order of the factors does not change the product (2 × 3 = 3 × 2).~~[Moved to 3.4 US] • Identify equivalent values and expressions. (e.g., 5 + 3 = 4 × 2 and 8 = 4 + 4) • Create equations ~~Write number sentences~~ to represent equivalent mathematical relationships (e.g., 4 × 3 = 14 - 2). • ~~Identify examples of the identity and commutative properties for addition and multiplication.~~

3.17~~20~~ The student will create equations to represent equivalent mathematical relationships.[Moved from 3.20 EKS]

~~a) investigate the identity and the commutative properties for addition and multiplication; and~~[Use of properties moved to 3.3 EKS and 3.4 EKS]

~~b) identify examples of the identity and commutative properties for addition and multiplication.~~

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~multiplied by one, the product is the same as the given number.~~ • ~~A number sentence is an equation with numbers (e.g., 6 + 3 = 9; or 6 + 3 = 4 + 5).~~[Reordered]		



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Grade 4

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

Mathematics instruction in grades ~~4 and~~ 3-5 should continue to foster the development of number sense, ~~especially~~ with greater emphasis on decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction ~~at~~ in grades ~~4 and~~3- 5 allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names), and making connections among mathematics concepts as well as to other content areas. Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g., $\frac{1}{2}$ or 0.5). Students should apply their knowledge of number and number sense to investigate and solve a variety of problem types.

4.1 The student will

a) identify orally and in writing the place and value for each digit in a whole number expressed through millions;

b) compare and order ~~two~~ whole numbers expressed through millions~~, using symbols (>, <, or =)~~; and[Symbols included in EKS]

c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The structure of the base-10 number system is based upon a simple pattern of tens, in which the value of each place is ten times the value of the place to its right. • Place value refers to the value of each digit and depends upon the position of the digit in the number. For example, in the number 7,864,352, the eight is in the hundred thousand place, and the value of the 8 is eight hundred thousand or 800,000. • Whole numbers may be written in a variety of form~~at~~s: – Standard: 1,234,567 – Written: one million, two hundred thirty-four thousand, five hundred sixty-seven – Expanded: (1 × 1,000,000) + (2 × 100,000) + (3 × 10,000) + (4 × 1,000) + (5 × 100) + (6 × 10) + (7 × 1) • Numbers are arranged into groups of three places called *periods* (ones, thousands, millions, …). Places within the periods repeat (hundreds, tens, ones). Commas are used to separate the periods. Knowing the place value and period of a number helps students ~~find~~ determine values of digits in any number as well as read and write numbers. • Reading and writing large numbers should be meaningful for students. Experiences can be provided that relate practical situations (e.g., numbers found in the students' environment including population, number of school lunches sold statewide in a day, etc.). • Concrete materials such as base-10 blocks or, ~~and~~ bundles of sticks may be used to represent whole numbers through thousands. Larger numbers may be represented by digit cards and place value charts or on number lines.	~~**All students should**~~ ~~• Understand the relationships in the place value system in which the value of each place is ten times the value of the place to its right.~~ ~~• Use the patterns in the place value system to read and write numbers.~~ ~~• Understand that reading place value correctly is essential when comparing numbers.~~ ~~• Understand that rounding gives a close number to use when exact numbers are not needed for the situation at hand.~~ ~~• Develop strategies for rounding.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and communicate, ~~both~~ orally and in written form, the place and value for each digit in whole numbers expressed through ~~the one~~ millions ~~place~~. (a) • Read whole numbers, presented in standard form, expressed through ~~the one~~ millions ~~place that are presented in standard format~~, and ~~select the matching~~ represent the same number in written form~~at~~. (a) • Write whole numbers through ~~the one~~ millions ~~place~~ in standard form~~at~~ when the numbers are presented orally or in written form~~at~~. (a) ~~• Identify and use the symbols for *greater than*, *less than*, and *equal to*.~~[Added to next bullet] • Compare two whole numbers expressed through ~~the one~~ millions, using the words *greater than*, *less than*, *equal to*, *and not equal to* or using the symbols >, <, ~~or~~ =, or ≠. (b) • Order up to four whole numbers expressed through millions. (b) • Round whole numbers expressed through ~~the one~~ millions ~~place~~ to the nearest thousand, ten thousand, and hundred-thousand place. (c) • Identify the greatest whole number and least whole number that round to a given thousand, ten thousand, and hundred thousand. (c)

4.1 The student will

a) identify orally and in writing the place and value for each digit in a whole number expressed through millions;

b) compare and order ~~two~~ whole numbers expressed through millions~~, using symbols (>, <, or =)~~; and[Symbols included in EKS]

c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Number lines are useful tools when developing a conceptual understanding of rounding with whole numbers. • Mathematical symbols (>, <) used to compare two unequal numbers are called *inequality symbols*. ~~• A procedure for comparing two numbers by examining place value may include the following:~~ ~~Compare the digits in the numbers to determine which number is greater (or which is less).~~ ~~Use a number line to identify the appropriate placement of the numbers based on the place value of the digits.~~ ~~Use the appropriate symbol > or < or words *greater than* or *less than* to compare the numbers in the order in which they are presented.~~ ~~If both numbers have the same value, use the symbol = or words *equal to*.~~ ~~• A strategy for rounding numbers to the nearest thousand, ten thousand, and hundred thousand is as follows:~~ ~~Use a number line to determine the rounded number (e.g., when rounding 4,367,925 to the nearest thousand, identify the 'thousands' the number would fall between on the number line, then determine the thousand that the number is closest to):~~ 4,36**7,000** ? 4,36**8,000** ~~Look one place to the right of the digit to which you wish to round.~~ ~~If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the~~		

4.1 The student will

a) identify orally and in writing the place and value for each digit in a whole number expressed through millions;

b) compare and order ~~two~~ whole numbers expressed through millions~~, using symbols (>, <, or =)~~; and[Symbols included in EKS]

c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~rounding place to zero.~~ – ~~If the digit is 5 or greater, add 1 to the digit in the rounding place and change the digits to the right of the rounding place to zero.~~		

4.2 The student will

a) compare and order fractions and mixed numbers, with and without models;*

b) represent equivalent fractions;* and

c) identify the division statement that represents a fraction with models and in context.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A fraction is a way of representing part of a whole (as in a region/area model~~ or a measurement model~~) or part of a group (as in a set model) or as a location on a number line (measurement model). A fraction is used to name a part of one thing or a part of a collection of things. • In the area/region and length/measurement fraction models, the parts must be equal. In the set model, the elements of the set do not have to be equal (e.g., "What fraction of the class is wearing the color red?"). • The value of a fraction is dependent on both the number of equal parts in a whole (denominator) and the number of those parts being considered (numerator). • ~~The denominator tells how many equal parts are in the whole or set. The numerator tells how many of those parts are being counted or described.~~ • The more parts the whole is divided into, the smaller the parts (e.g., $\frac{1}{5} < \frac{1}{3}$).[Moved from middle column] • When fractions have the same denominator, they are said to have "common denominators" or "like denominators." Comparing fractions with like denominators involves comparing only the numerators.	~~All students should~~ • ~~Develop an understanding of fractions as parts of unit wholes, as parts of a collection, and as locations on a number line.~~ • ~~Understand that a mixed number is a fraction that has two parts: a whole number and a proper fraction. The mixed number is the sum of these two parts.~~[Moved to US] • ~~Use models, benchmarks, and equivalent forms to judge the size of fractions.~~ • ~~Recognize that a whole divided into nine equal parts has smaller parts than if the whole had been divided into five equal parts.~~ • ~~Recognize and generate equivalent forms of commonly used fractions and decimals.~~ • ~~Understand the division statement that represents a fraction.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and order no more than four fractions having denominators of 12 or less, using manipulative models and drawings~~, such as region/area models~~. (a) • Use benchmarks (e.g., 0, $\frac{1}{2}$ or 1) to compare and order no more than four fractions having unlike denominators of 12 or less. (a)[Reordered] • Compare and order no more than four fractions with like denominators of 12 or less by comparing number of parts (numerators) (e.g., $\frac{1}{5} < \frac{3}{5}$). (a) • Compare and order no more than four fractions with like numerators and unlike denominators of 12 or less by comparing the size of the parts (e.g., $\frac{3}{9} < \frac{3}{5}$). (a)~~Compare and order fractions having unlike denominators of 12 or less by comparing the fractions to benchmarks (e.g., 0, $\frac{1}{2}$ or 1) to determine their relationships to the benchmarks or by finding a common denominator.~~ • Compare and order no more than four proper fractions, mixed numbers, and improper fractions having denominators of 12 or less. (a) • Use the symbols >, <, ~~and~~ =, and $\neq$ to compare the numerical value of proper fractions, improper fractions, and mixed numbers having denominators of 12 or less. (a)

4.2 The student will

a) compare and order fractions and mixed numbers, with and without models;*

b) represent equivalent fractions;* and

c) identify the division statement that represents a fraction with models and in context.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Strategies for comparing fractions having unlike denominators may include – comparing fractions to familiar benchmarks (e.g., 0, $\frac{1}{2}$, 1); – ~~finding~~ determining equivalent fractions, using ~~manipulative~~ models such as fraction strips, number lines, fraction circles, rods, pattern blocks, cubes, base-10 blocks, tangrams, graph paper, or ~~a~~patterns in a multiplication chart ~~and patterns~~; and – ~~finding~~ determining a common denominator by ~~finding~~ determining the least common multiple (LCM) of both denominators and then rewriting each fraction as an equivalent fraction, using the LCM as the denominator. • A variety of fraction models should be used to expand students' understanding of fractions and mixed numbers: – Region/area models: a surface or area is subdivided into smaller equal parts, and each part is compared with the whole (e.g., fraction circles, pattern blocks, geoboards, grid paper, color tiles). – Set models: the whole is understood to be a set of objects, and subsets of the whole make up fractional parts (e.g., counters, chips). – Measurement models: similar to area models but lengths instead of areas are compared (e.g., fraction strips, rods, cubes, number lines, rulers). • A proper fraction is a fraction whose numerator is less than the denominator. A proper fraction is a fraction that is always less than one.	~~• Understand that the more parts the whole is divided into, the smaller the parts (e.g., $\frac{1}{5} < \frac{1}{3}$).~~	• Represent equivalent fractions through twelfths, using region/area models, set models, and measurement models. (b) • Identify the division statement that represents a fraction with models and in context (e.g., $\frac{3}{5}$ means the same as 3 divided by 5 or $\frac{3}{5}$ represents the amount of muffin each of five children will receive when sharing 3 muffins equally). (c)

4.2 The student will

a) compare and order fractions and mixed numbers, with and without models;*

b) represent equivalent fractions;* and

c) identify the division statement that represents a fraction with models and in context.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• An improper fraction is a fraction whose numerator is greater than or equal to the denominator. An improper fraction is a fraction that is equal to or greater than one. • An improper fraction can be expressed as a mixed number. A mixed number is written as a whole number and a proper fraction. • ~~A mixed number has two parts: a whole number and a fraction.~~ • Equivalent fractions name the same amount. Students should use a variety of models to identify different names for equivalent fractions. • ~~Students should focus on finding equivalent fractions of familiar fractions such as halves, thirds, fourths, sixths, eighths, tenths, and twelfths.~~ • ~~Decimals and fractions represent the same relationships; however, they are presented in two different formats. The decimal 0.25 is written as~~ $\cancel{\frac{1}{4}}$~~.~~ When presented with the fraction $\frac{3}{5}$, the division expression representing a fraction is written as 3 divided by 5. • The fraction $\frac{3}{4}$ may be interpreted as the amount of cake each person will receive when 3 cakes are divided equally among 4 people.		

4.3 The student will

a) read, write, represent, and identify decimals expressed through thousandths;
b) round decimals to the nearest whole number~~, tenth, and hundredth~~;[Round to tenth and hundredth included in 5.1]
c) compare and order decimals; and
d) given a model, write the decimal and fraction equivalents.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Decimal numbers are another way of writing fractions. • The structure of the base-10 number system is based upon a simple pattern of tens, where each place is ten times the value of the place to its right. This is known as a ten-to-one place value relationship. • Base-10 models concretely relate fractions to decimals (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, decimal circles, money).Reordered] • ~~Understanding the system of tens means that ten tenths represents one whole, ten hundredths represents one tenth, ten thousandths represents one hundredth.~~ • A decimal point separates the whole number places from the places that are less than one. Place values extend infinitely in two directions from a decimal point. A number containing a decimal point is called a *decimal number* or simply a *decimal*. • To read decimals, – read the whole number to the left of the decimal point~~, if there is one~~; – read the decimal point as "and"; – read the digits to the right of the decimal point just as you would read a whole number; and – say the name of the place value of the digit in the smallest place. • Any decimal less than 1 will include a leading zero (e.g., 0.125). • Decimals may be written in a variety of forms: – Standard: 26.537 – Written: twenty-six and five hundred thirty-seven thousandths – Expanded: $(2 \times 10) + (6 \times 1) + (5 \times 0.1) + (3 \times 0.01) + (7 \times 0.001)$.	~~**All students should**~~ • ~~Understand the place value structure of decimals and use this structure to read, write, and compare decimals.~~ • ~~Understand that decimal numbers can be rounded to an estimate when exact numbers are not needed for the situation at hand.~~ • ~~Understand that decimals are rounded in a way that is similar to the way whole numbers are rounded.~~ • ~~Understand that decimals and fractions represent the same relationship; however, they are presented in two different formats.~~ • ~~Understand that models are used to show decimal and fraction equivalents.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate the ten-to-one place value relationship for decimals through thousandths, using base-10 manipulatives (e.g., place value mats/charts, decimal squares, and base-10 blocks~~, money~~). (a) • Represent and identify decimals expressed through thousandths, using base-10 manipulatives, pictorial representations, and numerical symbols (e.g., relate the appropriate drawing to 0.05). (a) • Identify and communicate, both orally and in written form, the position and value of a decimal through thousandths. For example, in 0.385, the 8 is in the hundredths place and has a value of 0.08. (a) • Read and write decimals expressed through thousandths, using base-10 manipulatives, drawings, and numerical symbols. (a) • Round decimals expressed through thousandths to the nearest whole number~~, tenth, and hundredth~~. (b) • Compare two decimals, using the symbols >, <, =, and ≠ and the words *greater than*, *less than*, *equal to*, *and not equal to*. (c) • Order a set of up to four decimals from least to greatest or greatest to least. (c)

4.3 The student will

a) read, write, represent, and identify decimals expressed through thousandths;
b) round decimals to the nearest whole number~~, tenth, and hundredth~~;[Round to tenth and hundredth included in 5.1]
c) compare and order decimals; and
d) given a model, write the decimal and fraction equivalents.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Decimals and fractions represent the same relationships; however, they are presented in two different form~~at~~s. The decimal 0.25 is written as $\frac{1}{4}$. Decimal numbers are another way of writing fractions. ~~When presented with the fraction $\frac{3}{5}$, the division expression representing a fraction is written as 3 divided by 5. The Base-10 models concretely relate fractions to decimals (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, money).~~ • ~~The procedure for~~ Strategies for rounding whole numbers can be applied to rounding decimals~~ numbers is similar to the procedure for rounding whole numbers~~. • Number lines are useful tools when developing a conceptual understanding of rounding with decimals. ~~• A strategy for rounding decimal numbers to the nearest tenth and hundredth is as follows:~~ ~~– Look one place to the right of the digit you want to round to.~~ ~~– If the digit is 5 or greater, add 1 to the digit in the rounding place, and drop the digits to the right of the rounding place.~~ ~~– If the digit is less than 5, leave the digit in the rounding place as it is, and drop the digits to the right of the rounding place.~~ ~~• Different strategies for rounding decimals include:~~ ~~– Use a number line to locate a decimal between two numbers. For example, 18.83 is closer to 18.8 than to 18.9.~~ ~~– Compare the digits in the numbers to determine which number is greater (or which is less).~~ ~~– Compare the value of decimals, using the symbols >, <, = (e.g., 0.83 > 0.8 or 0.19 < 0.2).~~		• Represent fractions for halves, fourths, fifths, and tenths as decimals through hundredths, using concrete objects ~~(e.g., demonstrate the relationship between the fraction $\frac{1}{4}$ and its decimal equivalent 0.25)~~. (d) • Relate fractions to decimals, using concrete objects (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, decimal circles, money [coins]). (d) • Write the decimal and fraction equivalent for a given model (e.g., $\frac{1}{4}$ = 0.25 or 0.25 = $\frac{1}{4}$; 1.25 = $\frac{5}{4}$ or $1\frac{1}{4}$). (d)

4.3 The student will

a) read, write, represent, and identify decimals expressed through thousandths;
b) round decimals to the nearest whole number~~, tenth, and hundredth~~;[Round to tenth and hundredth included in 5.1]
c) compare and order decimals; and
d) given a model, write the decimal and fraction equivalents.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~Order the value of decimals, from least to greatest and greatest to least (e.g., 0.83, 0.821, 0.8).~~ • ~~Decimal numbers are another way of writing fractions (halves, fourths, fifths, and tenths). The Base-10 models concretely relate fractions to decimals (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, decimal circles money)~~[Reordered] ~~Provide a fraction model (halves, fourths, fifths, and tenths) and ask students for its decimal equivalent.~~ • ~~Provide a decimal model and ask students for its fraction equivalent (halves, fourths, fifths, and tenths).~~		

Computation and estimation in grades ~~4 and~~3- 5 should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students' understanding of these operations to work with decimals. Instruction should focus on computation activities that enable students to model, explain, and develop proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract ~~with common~~ fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability to identify and use relationships between operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use ~~(not identify)~~ properties of operations to solve problems [e.g., 7×28 is equivalent to $(7 \times 20) + (7 \times 8)$].

Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals, using benchmarks (e.g., $\frac{2}{5} + \frac{1}{3}$ must be less than 1 because both fractions are less than $\frac{1}{2}$). Using estimation, students should develop strategies to recognize the reasonableness of their ~~computations~~solutions.
Additionally, students should enhance their ability to select an appropriate problem solving method from among estimation, mental mathematics, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.

4.4 The student will

a) demonstrate fluency with multiplication facts through 12 × 12 and the corresponding division facts;*[Moved from 3.5]~~estimate sums, differences, products, and quotients of whole numbers;~~

b) estimate and determine sums, differences, and products of ~~add, subtract, and multiply~~ whole numbers;*

c) estimate and determine quotients of ~~divide~~ whole numbers, ~~finding quotients~~ with and without remainders;* and

d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division ~~problems~~ with whole numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently. • The development of computational fluency relies on quick access to number facts. There are patterns and relationships that exist in the facts. These relationships can be used to learn and retain the facts. • A certain amount of practice is necessary to develop fluency with computational strategies; however, the practice must be motivating and systematic if students are to develop fluency in computation, whether mental, with manipulative materials, or with paper and pencil.[Reordered] • In order to develop and use strategies to learn the multiplication facts through the twelves table, students should use concrete materials, hundred chart, and mental mathematics. Strategies to learn the multiplication facts include an understanding of multiples/skip counting, properties of zero and one as factors, pattern of nines, commutative property, and related facts. • The identity property of addition states that if zero is added to a given number, the sum is the same as the given number. The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number.	~~All students should~~ • ~~Develop and use strategies to estimate whole number sums and differences and to judge the reasonableness of such results.~~ • ~~Understand that addition and subtraction are inverse operations.~~ • ~~Understand that division is the operation of making equal groups or equal shares. When the original amount and the number of shares are known, divide to find the size of each share. When the original amount and the size of each share are known, divide to find the number of shares.~~ • ~~Understand that multiplication and division are inverse operations.~~ • ~~Understand various representations of division and the terms used in~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Demonstrate fluency with multiplication through 12 × 12, and the corresponding division facts. (a)[Moved from 3.5] • Use strategies when adding and subtracting, including application of the properties of addition. (b, d)[Use of properties moved from 3.20 and 5.19] • Use strategies when multiplying and dividing, including application of the properties of multiplication. (b, c, d)[Use of properties moved from 3.20 and 5.19] • Estimate whole number sums, differences, products, and quotients. (b, c) • ~~Refine estimates by adjusting the final amount, using terms such as *closer to*, *between*, and *a little more than*.~~ [Reordered] • Determine the sum or difference of two whole numbers, each 999,999 or less, in vertical and horizontal form with or without regrouping.~~, using paper and pencil, and using a calculator.~~ (b) • Estimate and ~~find~~ determine the products of two whole numbers when ~~one factor~~both factors have~~s~~ two digits or fewer~~ and the other factor has three digits or fewer, using paper and pencil and calculators~~. (b)

4.4 The student will

a) demonstrate fluency with multiplication facts through 12 × 12 and the corresponding division facts;*[Moved from 3.5]~~estimate sums, differences, products, and quotients of whole numbers;~~

b) estimate and determine sums, differences, and products of ~~add, subtract, and multiply~~ whole numbers;*

c) estimate and determine quotients of ~~divide~~ whole numbers, ~~finding quotients~~ with and without remainders;* and

d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division ~~problems~~ with whole numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 24 + 136 = 136 + 24). Similarly, the commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 12 × 43 = 43 × 12). • The associative property of addition states that the sum stays the same when the grouping of addends is changed [e.g., 15 + (35 + 16) = (15 + 35) + 16]. The associative property of multiplication states that the product stays the same when the grouping of factors is changed [e.g., 16 × (40 × 5) = (16 × 40) × 5]. • Using the distributive property with whole numbers helps with understanding mathematical relationships.[Moved from middle column] The distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products. Several examples are shown below: – 3(9) = 3(5 + 4) 3(5 + 4) = 3 × 5 + 3 × 4 – 5 × (3 + 7) = (5 × 3) + (5 × 7) – (2 × 3) + (2 × 5) = 2 × (3 + 5) – 9 × 23 9(20+3) 180+ 27 207	~~division are *dividend*, *divisor*, and *quotient*.~~ ~~*dividend ÷ divisor = quotient*~~ ~~*quotient*~~ ~~*divisor*)*dividend*~~ ~~• Understand how to solve single-step and multistep problems using whole number operations.~~	• Estimate and ~~find~~ determine the quotient of two whole numbers, given a one-digit divisor and a two- or three-digit dividend. (c) • Refine estimates by adjusting the final amount, using terms such as *closer to*, *between*, and *a little more than*. [Reordered] • Create and ~~S~~solve single-step and multistep practical problems ~~using~~ involving addition, subtraction, and multiplication with whole numbers~~ operations~~. • Create and solve single-step practical problems involving division with whole numbers.(d) • Use the context in which a practical problem is situated to interpret the quotient and remainder. (d) • ~~Verify the reasonableness of sums, differences, products, and quotients of whole numbers using estimation.~~[Moved to US]

4.4 The student will

a) demonstrate fluency with multiplication facts through 12 × 12 and the corresponding division facts;*[Moved from 3.5]~~estimate sums, differences, products, and quotients of whole numbers;~~

b) estimate and determine sums, differences, and products of ~~add, subtract, and multiply~~ whole numbers;*

c) estimate and determine quotients of ~~divide~~ whole numbers, ~~finding quotients~~ with and without remainders;* and

d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division ~~problems~~ with whole numbers.

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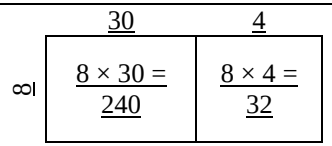
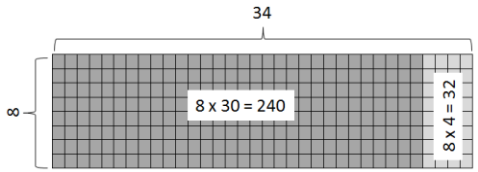
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– 34 × 8   • Students will begin to use the formal terms for the properties in grade 5. • ~~A sum is the result of adding two or more numbers.~~ • ~~A difference is the amount that remains after one quantity is subtracted from another.~~ • The problem solving process is enhanced when students create and solve their own practical problems and model problems using manipulatives and drawings. • The least number of steps necessary to solve a single-step problem is one. • In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as *in all*, *altogether*, *difference*, *etc.* encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions.		

4.4 The student will

a) demonstrate fluency with multiplication facts through 12 × 12 and the corresponding division facts;*[Moved from 3.5]~~estimate sums, differences, products, and quotients of whole numbers;~~

b) estimate and determine sums, differences, and products of ~~add, subtract, and multiply~~ whole numbers;*

c) estimate and determine quotients of ~~divide~~ whole numbers, ~~finding quotients~~ with and without remainders;* and

d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division ~~problems~~ with whole numbers.

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UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Students should experience a variety of problem types related to multiplication and division. Some examples are included in the following chart:		

MULTIPLICATION AND DIVISION PROBLEM TYPES		
EQUAL GROUP PROBLEMS		
Whole Unknown	Size of Groups Unknown	Number of Groups Unknown
There are 20 bags of apples. There are 14 apples in each bag. How many apples in all?	If 126 apples are shared equally among 4 friends, how many apples will each friend get?	If 168 apples are packed into bags containing 12 apples each. How many bags are needed?
MULTIPLICATIVE COMPARISON PROBLEMS		
Result Unknown	Start Unknown	Comparison Factor Unknown
Bob picked 26 apples. Sue picked 10 times as many apples as Bob. How many did Sue pick?	Sue picked 168 apples. She picked 4 times as many apples as Bob. How many apples did Bob pick?	Sue picked 168 apples. Bob only picked 42 apples. How many times more apples did Sue pick than Bob?

ARRAY OR AREA PROBLEMS	
Whole Unknown	One Dimension Unknown
There are 25 rows of children at the assembly with 15 children in each row. How many children are there together? Joe's rectangular garden measures 15 feet by 22 feet. How many square feet are in the garden?	There are 180 children at the assembly. The children are divided equally in 15 rows. How many children are in each row? There are 180 children at the assembly. There are 15 children in each row. How many rows of children are there? Joe's rectangular garden covers 60 square feet. The length of the garden is 15 feet. What is the width of the garden?

4.4 The student will

a) demonstrate fluency with multiplication facts through 12 × 12 and the corresponding division facts;*[Moved from 3.5]~~estimate sums, differences, products, and quotients of whole numbers;~~

b) estimate and determine sums, differences, and products of ~~add, subtract, and multiply~~ whole numbers;*

c) estimate and determine quotients of ~~divide~~ whole numbers, ~~finding quotients~~ with and without remainders;* and

d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division ~~problems~~ with whole numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Estimation can be used to verify the reasonableness of sums, differences, products, and quotients of whole numbers.[Moved from middle column] An estimate is a number close to an exact solution. An estimate tells about how much or about how many. • Strategies such as rounding, front-end, and compatible numbers may be used to estimate sums, differences, products, and quotients of whole numbers. ~~• Different strategies for estimating include using compatible numbers to estimate sums and differences and using front-end estimation for sums and differences.~~ ~~• Compatible numbers are numbers that are easy to work with mentally. Number pairs that are easy to add or subtract are compatible. When estimating a sum, replace actual numbers with compatible numbers (e.g., 52 + 74 can be estimated by using the compatible numbers 50 + 75). When estimating a difference, use numbers that are close to the original numbers. Tens and hundreds are easy to subtract (e.g., 83 – 38 is close to 80 – 40).~~ ~~• The front-end strategy for estimating is computing with the front digits. Front-end estimation for addition can be used even when the addends have a different number of digits. The procedure requires the addition of the values of the digits in the greatest of the smallest number. For example:~~ ~~2367 › 2300~~ ~~243 › 200~~ ~~+ 1186 › + 1100~~		

4.4 The student will

a) demonstrate fluency with multiplication facts through 12 × 12 and the corresponding division facts;*[Moved from 3.5]~~estimate sums, differences, products, and quotients of whole numbers;~~

b) estimate and determine sums, differences, and products of ~~add, subtract, and multiply~~ whole numbers;*

c) estimate and determine quotients of ~~divide~~ whole numbers, ~~finding quotients~~ with and without remainders;* and

d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division ~~problems~~ with whole numbers.

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UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~3600~~ ~~• Front-end or leading-digit estimation always gives a sum less than the actual sum; however, the estimate can be adjusted or refined so that it is closer to the actual sum.~~ • Addition is the combining of quantities; it uses the following terms: *addend* → 45,623 *addend* → + 37,846 *sum* → 83,469 • Subtraction is the inverse of addition; it yields the difference between two numbers and uses the following terms: *minuend* → 45,698 *subtrahend* → – 32,741 *difference* → 12,957 ~~• Before adding or subtracting with paper and pencil, addition and subtraction problems in horizontal form should be rewritten in vertical form by lining up the places vertically.~~ ~~• Using Base-10 materials to model and stimulate discussion about a variety of problem situations helps students understand regrouping and enables them to move from the concrete to the pictorial, to the abstract. Regrouping is used in addition and subtraction algorithms. In addition, when the sum in a place is 10 or more, is used to regroup the sums so that there is only one digit in each place. In subtraction, when the number (minuend) in a place is not enough from which to subtract, regrouping is required.~~		

4.4 The student will

a) demonstrate fluency with multiplication facts through 12 × 12 and the corresponding division facts;*[Moved from 3.5]~~estimate sums, differences, products, and quotients of whole numbers;~~

b) estimate and determine sums, differences, and products of ~~add, subtract, and multiply~~ whole numbers;*

c) estimate and determine quotients of ~~divide~~ whole numbers, ~~finding quotients~~ with and without remainders;* and

d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division ~~problems~~ with whole numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~A certain amount of practice is necessary to develop fluency with computational strategies for multidigit numbers; however, the practice must be meaningful, motivating, and systematic if students are to develop fluency in computation, whether mentally, with manipulative materials, or with paper and pencil.Calculators are an appropriate tool for computing sums and differences of large numbers, particularly when mastery of the algorithm has been demonstrated.~~ • In multiplication, one factor represents the number of equal groups and the other factor represents the size of each group. The product is the total of all the parts. Multiplication can also refer to a multiplicative comparison, such as: "Gwen has six times as many stickers as Phillip". Both situations should be modeled with manipulatives. • The terms associated with multiplication are *factor* → 376 *factor* → × 23 *product* → 8,648 • Models of multiplication may include repeated addition (set model); partial products; and area or array models. ~~• One model of multiplication is repeated addition.~~ ~~• Another model of multiplication is the "Partial Product" model.~~ ~~24~~ ~~× 3~~ ~~12 ← Multiply the ones: 3 × 4 = 12~~		

4.4 The student will

a) demonstrate fluency with multiplication facts through 12 × 12 and the corresponding division facts;*[Moved from 3.5]~~estimate sums, differences, products, and quotients of whole numbers;~~

b) estimate and determine sums, differences, and products of ~~add, subtract, and multiply~~ whole numbers;*

c) estimate and determine quotients of ~~divide~~ whole numbers, ~~finding quotients~~ with and without remainders;* and

d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division ~~problems~~ with whole numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~+ 60 ‹ Multiply the tens: 3 × 20 = 60~~ ~~72~~ ~~• Another model of multiplication is the "Area Model" (which also represents partial products) and should be modeled first with Base 10 blocks. (e.g., 23 x 68)~~ ~~• Students should continue to develop fluency with single-digit multiplication facts and their related division facts.~~ ~~• Calculators should be used to solve problems that require tedious calculations.~~ ~~• Estimation should be used to check the reasonableness of the product. Examples of estimation strategies include the following:~~ ~~• The front-end method: multiply the front digits and then complete the product by recording the number of zeros found in the factors. It is important to develop understanding of this process before using the step-by-step procedure.~~ ~~523 › 500~~ ~~× 31 › × 30~~ ~~15,000~~ ~~• This is 3 × 5 = 15 with 3 zeros.~~ ~~• Compatible numbers: replace factors with compatible numbers, and then multiply. Opportunities for students to discover patterns with 10 and powers of 10 should be provided.~~ ~~64 › 64~~ ~~• × 11 › × 10~~		

4.4 The student will

a) demonstrate fluency with multiplication facts through 12 × 12 and the corresponding division facts;*[Moved from 3.5]~~estimate sums, differences, products, and quotients of whole numbers;~~

b) estimate and determine sums, differences, and products of ~~add, subtract, and multiply~~ whole numbers;*

c) estimate and determine quotients of ~~divide~~ whole numbers, ~~finding quotients~~ with and without remainders;* and

d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division ~~problems~~ with whole numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Division is the operation of making equal groups or equal shares. When the original amount and the number of shares are known, divide to find the size of each share. When the original amount and the size of each share are known, divide to find the number of shares. Both situations may be modeled with Base-10 manipulatives.~~ • ~~Multiplication and division are inverse operations.~~ • Division is the inverse of multiplication. Terms used in division are *dividend, divisor*, and *quotient.* dividend ÷ divisor = quotient quotient divisor)dividend • ~~Opportunities to invent division algorithms help students make sense of the algorithm. Teachers should teach division by~~Students benefit from experiences with various methods of division, such as repeated multiplication, ~~and~~ repeated subtraction, and ~~(~~partial quotients~~) before teaching the traditional long division algorithm.~~ • Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the whole numbers.[Moved from 4.16] • Students need exposure to various types of practical problems in which they must interpret the quotient and remainder based on the context. The chart below includes one example of each type of problem.		

4.4 The student will

a) demonstrate fluency with multiplication facts through 12 × 12 and the corresponding division facts;*[Moved from 3.5]~~estimate sums, differences, products, and quotients of whole numbers;~~

b) estimate and determine sums, differences, and products of ~~add, subtract, and multiply~~ whole numbers;*

c) estimate and determine quotients of ~~divide~~ whole numbers, ~~finding quotients~~ with and without remainders;* and

d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division ~~problems~~ with whole numbers.

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UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS

MAKING SENSE OF THE REMAINDER IN DIVISION

TYPE OF PROBLEM	EXAMPLE
Remainder is not needed and can be left over (or discarded).	Bill has 29 pencils to share fairly with 6 friends. How many pencils will each friend receive? 4 pencils with 5 pencils left over
Remainder is partitioned and represented as a fraction or decimal.	Six friends will share 29 ounces of juice. How many ounces will each person get if all of the juice is shared equally? $4\frac{5}{6}$ ounces
Remainder forces the answer to be increased to the next whole number.	There are 29 people going to the party by car. How many cars will be needed if each car holds 6 people? 5 cars
Remainder forces the answer to be rounded (giving an approximate answer).	Six children will share a bag of candy containing 29 pieces. About how many pieces of candy will each child get? about 5 pieces of candy

4.5 The student will

a) determine common multiples and factors, including least common multiple and greatest common factor;

b) add and subtract fractions and mixed numbers having like and unlike denominators ~~that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors;~~* and

~~c) add and subtract with decimals; and~~[Moved to 4.6a]

c~~d~~) solve single-step and multistep practical problems involving addition and subtraction with fractions ~~and with decimals~~.[Decimals limited to single-step only and moved to 4.6b; solve multistep with decimals included in 5.5b]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A factor of a whole number is a~~n integer~~ whole number that divides evenly into that number with ~~a~~no remainder~~ of zero~~. • A factor of a number is a divisor of the number. • A common factor of two or more numbers is a divisor that all of the numbers share.[Reordered] • The greatest common factor of two or more numbers is the largest of the common factors that all of the numbers share.[Reordered] • A multiple of a number is the product of the number and any natural number. ~~• A common factor of two or more numbers is a divisor that all of the numbers share.~~ • Common multiples and common factors can be useful when simplifying fractions.[Moved from middle column] • The least common multiple of two or more numbers is the ~~smallest common~~lowest number that is a multiple of all of the given numbers. ~~• The greatest common factor of two or more numbers is the largest of the common factors that all of the numbers share.~~ • Estimation keeps the focus on the meaning of the numbers and operations, encourages reflective thinking, and helps build informal number sense with fractions. Students can reason with benchmarks to get an estimate without using an algorithm.	~~**All students should**~~ ~~• Understand and use common multiples and common factors for simplifying fractions.~~ ~~• Develop and use strategies to estimate addition and subtraction involving fractions and decimals.~~ ~~• Use visual models to add and subtract with fractions and decimals.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Find~~ Determine common multiples and common factors of numbers. (a) • Determine the least common multiple and greatest common factor of no more than 3 numbers. (a) • ~~Use least common multiple and/or greatest common factor to find~~ Determine a common denominator for fractions, using common multiples and factors. Common denominators should not exceed 60. (b) • Estimate the sum or difference of two fractions.(b, c) • Add and subtract ~~with~~ fractions and mixed numbers having like and unlike denominators whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction ~~using common multiples and factors~~. (b) ~~• Add and subtract with fractions having unlike denominators whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction using common multiples and factors.~~[Included in bullet above]

4.5 The student will

a) determine common multiples and factors, including least common multiple and greatest common factor;

b) add and subtract fractions and mixed numbers having like and unlike denominators ~~that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors;~~* and

~~c) add and subtract with decimals; and~~[Moved to 4.6a]

c~~d~~) solve single-step and multistep practical problems involving addition and subtraction with fractions ~~and with decimals~~.[Decimals limited to single-step only and moved to 4.6b; solve multistep with decimals included in 5.5b]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Students should investigate addition and subtraction with fractions, using a variety of models (e.g., fraction circles, fraction strips, rulers, linking cubes, pattern blocks). • While this standard requires instruction in solving problems with denominators of 2, 3, 4, 5, 6, 8, 10, and 12, students would benefit from experiences with other denominators. • When students use the least common multiple to determine common denominators to add or subtract fractions with unlike denominators, the least common multiple may be greater than 12, but will not exceed 60. • Instruction on addition and subtraction of fractions should include experiences with improper fractions and mixed numbers as addends, minuends, subtrahends, sums, and differences. • At this level, subtraction with fractions will be limited to problems that do not require regrouping. • A fraction is in simplest form when its numerator and denominator have no common factors other than 1. The numerator can be greater than the denominator.[Copied from SOL5.6] • At this level, denominators of fractions resulting from simplification will be limited to 12 or less. • The least number of steps necessary to solve a single-step problem is one.		• Solve single-step and multistep practical problems that involve adding and subtracting ~~with~~ fractions and mixed numbers having like and unlike denominators whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction using common multiples and factors. (c) • ~~Add and subtract with decimals through thousandths, using concrete materials, pictorial representations, and paper and pencil.~~[Moved to 4.6 EKS] • ~~Solve single-step and multistep problems that involve adding and subtracting with fractions and decimals through thousandths.~~ [Moved to 4.6 EKS]

4.5 The student will

a) determine common multiples and factors, including least common multiple and greatest common factor;

b) add and subtract fractions and mixed numbers having like and unlike denominators ~~that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors;~~* and

~~c) add and subtract with decimals; and~~[Moved to 4.6a]

c~~d~~) solve single-step and multistep practical problems involving addition and subtraction with fractions ~~and with decimals~~.[Decimals limited to single-step only and moved to 4.6b; solve multistep with decimals included in 5.5b]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Addition and subtraction of decimals may be explored, using a variety of models (e.g., 10-by-10 grids, number lines, money). [Moved from 4.5 US] • The problem-solving process is enhanced when students create and solve their own practical problems and model problems using manipulatives and drawings. • In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as *in all*, *altogether*, *difference*, *etc.* encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions. • The least number of steps necessary to solve a single-step problem is one. ~~• When adding or subtracting with fractions having like denominators, add or subtract the numerators and use the same denominator. Write the answer in simplest form using common multiples and factors.~~ ~~• When adding or subtracting with fractions having unlike denominators, rewrite them as fractions with a common denominator. The least common multiple (LCM) of the unlike denominators is a common denominator (LCD). Write the answer in simplest form using common multiples and factors.~~ ~~• Addition and subtraction of decimals may be explored, using a variety of models (e.g., 10-by-10 grids, number lines, money).~~ [Moved to 4.6]		

4.5 The student will

a) determine common multiples and factors, including least common multiple and greatest common factor;

b) add and subtract fractions and mixed numbers having like and unlike denominators ~~that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors;~~* and

~~c) add and subtract with decimals; and~~[Moved to 4.6a]

c~~d~~) solve single-step and multistep practical problems involving addition and subtraction with fractions ~~and with decimals~~.[Decimals limited to single-step only and moved to 4.6b; solve multistep with decimals included in 5.5b]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• For decimal computation, the same ideas developed for whole number computation may be used, and these ideas may be applied to decimals, giving careful attention to the placement of the decimal point in the solution. Lining up tenths to tenths, hundredths to hundredths, etc. helps to establish the correct placement of the decimal.~~ ~~• Fractions may be related to decimals by using models (e.g., 10-by-10 grids, decimal squares, money).~~		

4.6 The student will

a) add and subtract with decimals;* and[Moved from 4.5c]

b) solve single-step practical problems involving addition and subtraction with decimals.[Moved from 4.5d; decimals changed to single-step only; multistep with decimals included in 5.5b]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Addition and subtraction of decimals may be explored, using a variety of models (e.g., 10-by-10 grids, number lines, money). [Moved from 4.5 US] • The problem solving process is enhanced when students create and solve their own practical problems and model problems using manipulatives and drawings. • In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as *in all*, *altogether*, *difference*, *etc.* encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions. • The least number of steps necessary to solve a single-step problem is one.		**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate sums and differences of decimals. (a)[Moved from 4.5 EKS] • Add and subtract with decimals through thousandths, using concrete materials, pictorial representations, and paper and pencil. (a)[Moved from 4.5 EKS] • Solve single-step practical problems that involve adding and subtracting with decimals through thousandths. (b) [Moved from 4.5 EKS; multistep included in 5.5b]

Students in grades ~~4 and~~3- 5 should be actively involved in measurement activities that require a dynamic interaction between students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects.

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, perimeter, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary), to measure ~~perimeter~~length, weight/mass, liquid volume/capacity, area, perimeter, temperature, and time. Students~~ continue to enhance their~~ understanding of measurement ~~by~~continues to be enhanced through experiences using appropriate tools such as rulers, balances, clocks, and thermometers. ~~The process of measuring is identical for any attribute (i.e., length, weight/mass, liquid volume/capacity, area): choose a unit, compare that unit to the object, and report the number of units.~~

[Moved from geometry stand introduction]
The study of geometry helps students represent and make sense of the world. In grades 3 through 5, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)

~~4.6 The student will~~

~~**a) estimate and measure weight/mass and describe the results in U.S. Customary and metric units as appropriate; and**~~**[Moved to 4.8b]**

~~**b) identify equivalent measurements between units within the U.S. Customary system**~~**[Moved to 4.8c]**~~**(ounces, pounds, and tons)**~~**[Moved to 4.8 EKS]**~~ **and between units within the metric system (grams and kilograms).**~~**[Included in 5.9a]**

~~**UNDERSTANDING THE STANDARD**~~ ~~**(Background Information for Instructor Use Only)**~~	~~**ESSENTIAL UNDERSTANDINGS**~~	~~**ESSENTIAL KNOWLEDGE AND SKILLS**~~
• ~~Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?").~~[Moved to 4.8 US] • ~~Balances are appropriate measuring devices to measure weight in U.S. Customary units (ounces, pounds) and mass in metric units (grams, kilograms).~~[Moved to 4.8 US] • ~~Practical experience measuring the mass of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate weight/mass.~~[Moved to 4.8 US] ~~Students should estimate the mass/weight of everyday objects (e.g., foods, pencils, book bags, shoes), using appropriate metric or U.S. Customary units.~~	~~**All students should**~~ • ~~Use benchmarks to estimate and measure weight/mass.~~ • ~~Identify equivalent measures between units within the U.S. Customary and between units within the metric measurements.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Determine an appropriate unit of measure (e.g., ounce, pound, ton, gram, kilogram) to use when measuring everyday objects in both metric and U.S. Customary units.~~[Moved to 4.8 EKS] • ~~Measure objects in both metric and U.S. Customary units (e.g., ounce, pound, ton, gram, or kilogram) to the nearest appropriate measure, using a variety of measuring instruments.~~[Moved to 4.8 EKS] • ~~Record the mass of an object including the appropriate unit of measure (e.g., 24 grams).~~[Moved to 4.8 EKS]

~~4.7 The student will~~

~~a) estimate and measure length, and describe the result in both metric and U.S. Customary units; and~~[Moved to 4.8a]

~~b) identify equivalent measurements between units within the U.S. Customary system (inches and feet; feet and yards; inches and yards; yards and miles)~~[Moved to 4.8c]~~ and between units within the metric system (millimeters and centimeters; centimeters and meters; and millimeters and meters).~~[Included in 5.9a].

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Length is the distance along a line or figure from one point to another.~~[Moved to 4.8 US] • ~~U.S. Customary units for measurement of length include inches, feet, yards, and miles. Appropriate measuring devices include rulers, yardsticks, and tape measures. Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter ruler, meter stick, and tape measure.~~[Moved to 4.8 US] • ~~Practical experience measuring the length of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate length.~~ • ~~Students should estimate the length of everyday objects (e.g., books, windows, tables) in both metric and U.S. Customary units of measure.~~ • ~~When measuring with U.S. Customary units, students should be able to measure to the nearest part of an inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$), inch, foot, or yard.~~[Moved to 4.8 US]	~~**All students should**~~ • ~~Use benchmarks to estimate and measure length.~~ • ~~Understand how to convert units of length between the U.S. Customary and metric systems, using ballpark comparisons.~~ • ~~Understand the relationship between U.S. Customary units and the relationship between metric units.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Determine an appropriate unit of measure (e.g., inch, foot, yard, mile, millimeter, centimeter, and meter) to use when measuring everyday objects in both metric and U.S. Customary units.~~[Moved to 4.8 EKS] • ~~Estimate the length of everyday objects (e.g., books, windows, tables) in both metric and U.S. Customary units of measure.~~[Moved to 4.8 EKS] • ~~Measure the length of objects in both metric and U.S. Customary units, measuring to the nearest inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$), foot, yard, mile, millimeter, centimeter, or meter, and record the length including the appropriate unit of measure (e.g., 24 inches).~~[Moved to 4.8 EKS] • ~~Compare estimates of the length of objects with the actual measurement of the length of objects.~~[Moved to 4.8 EKS] • ~~Identify equivalent measures of length between units within the U.S. Customary measurements .~~[Moved to 4.8 EKS]~~and between units within the metric measurements.~~ [Included in 5.9a]

4.7 The student will solve practical problems that involve determining perimeter and area in standard units of measure.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**		**ESSENTIAL KNOWLEDGE AND SKILLS**
• Perimeter is the path or distance around any plane figure. • Area is the number of square units needed to cover a surface or plane figure. • To determine the perimeter of any polygon, determine the sum of the lengths of the sides. • Students should have opportunities to investigate and discover, using manipulatives, the formulas for the area of a square and the area of a rectangle. – Area of a square = side × side – Area of rectangle = length × width		**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the perimeter of a polygon with no more than 10 sides when the lengths of the sides are given. • Determine the perimeter and area of a rectangle when given the measure of two adjacent sides.[Moved from 5.8 EKS] • Determine the perimeter and area of a square when the measure of one side is given.[Moved from 5.8 EKS] • Label the perimeter and area with the appropriate unit of measure. • Solve practical problems that involve determining perimeter and area in standard units of measure.

4.8 The student will

a) estimate and measure length and describe the result in U.S. Customary and metric units;[Moved from 4.7a]

~~a~~b)~~estimate and measure liquid volume and describe the results in U.S. Customary units~~[Included in 3.7b]estimate and measure weight/mass and describe the results in U.S. Customary and metric units[Moved from 4.6a];~~ and~~

~~b~~c)identify equivalent measure~~ment~~s of length, weight/mass and liquid volume between units within the U.S. Customary system;~~ (cups, pints, quarts, and gallons)~~[Moved to EKS]~~.~~ and

d) solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units.

~~UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~U.S. Customary units for measurement of liquid volume include cups, pints, quarts, and gallons.~~ • The measurement of the object must include the unit of measure along with the number of iterations. • ~~Students should measure the liquid volume of everyday objects in U.S. Customary units, including cups, pints, quarts, gallons, and record the volume including the appropriate unit of measure (e.g., 24 gallons).~~[Reordered] • ~~Practical experience measuring liquid volume of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate liquid volume.~~ • ~~Students should estimate the liquid volume of containers in U.S. Customary units to the nearest cup, pint, quart, and gallon.~~ • Length is the distance between two points along a line.[Moved from 4.7 US] • U.S. Customary units for measurement of length include inches, feet, yards, and miles. Appropriate measuring devices include rulers, yardsticks, and tape measures. • Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter ruler, meter stick, and tape measure. [Moved from 4.7 US] • When measuring with U.S. Customary units, students should be	~~**All students should**~~ • ~~Use benchmarks to estimate and measure volume.~~ • ~~Identify equivalent measurements between units within the U.S. Customary system.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • Determine an appropriate unit of measure (inch, foot, yard, mile, millimeter, centimeter, and meter) to use when measuring length in both metric and U.S. Customary units. (a)[Moved from 4.7 EKS] • Estimate and measure length in both metric and U.S. Customary units, measuring to the nearest part of an inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$), and to the nearest foot, yard, millimeter, centimeter, or meter, and record the length including the unit of measure (e.g., 24 inches). (a)[Moved from 4.7 EKS] • Compare estimates of the length with the actual measurement of the length. (a)[Moved from 4.7 EKS] • Determine an appropriate unit of measure (ounce, pound, gram, kilogram) to use when measuring everyday objects in both metric and U.S. Customary units. (b)[Moved from 4.6 EKS] • Estimate and measure the weight/mass of objects in both U.S. Customary and metric units (ounce, pound, gram, or kilogram) to the nearest appropriate measure, using a variety of measuring instruments. (b)[Moved from 4.6 EKS] • Record the weight/mass of an object with the unit of

4.8 The student will

a) estimate and measure length and describe the result in U.S. Customary and metric units;[Moved from 4.7a]

~~a~~b)~~estimate and measure liquid volume and describe the results in U.S. Customary units~~[Included in 3.7b]estimate and measure weight/mass and describe the results in U.S. Customary and metric units[Moved from 4.6a];~~ and~~

~~b~~c)identify equivalent measure~~ments~~s of length, weight/mass and liquid volume between units within the U.S. Customary system; ~~(cups, pints, quarts, and gallons)~~[Moved to EKS]~~.~~ and

d) solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units.

~~UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
able to measure to the nearest part of an inch ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$), foot, or yard.[Moved from 4.7 US] • Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?").[Moved from 4.6 US] • Balances are appropriate measuring devices to measure weight in U.S. Customary units (ounces, pounds) and mass in metric units (grams, kilograms).[Moved from 4.6 US] • Practical experience measuring the weight/mass of familiar objects (e.g., foods, pencils, book bags, shoes) helps to establish benchmarks and facilitates the student's ability to estimate weight/mass.[Moved from 4.6 US] • Students should measure the liquid volume of everyday objects in U.S. Customary units, including cups, pints, quarts, gallons, and record the volume including the appropriate unit of measure (e.g., 24 gallons).[Reordered]		measure (e.g., 24 grams). (b)[Moved from 4.6 EKS] • ~~Determine an appropriate unit of measure (cups, pints, quarts, gallons) to use when measuring liquid volume in U.S. Customary units.~~ • ~~Estimate the liquid volume of containers in U.S. Customary units of measure to the nearest cup, pint, quart, and gallon.~~[Include in 3.7 EKS] • ~~Measure the liquid volume of everyday objects in U.S. Customary units, including cups, pints, quarts, and gallons, and record the volume including the appropriate unit of measure (e.g., 24 gallons).~~[Included in 3.7 EKS] • Identify equivalent measures ~~of volume~~ between units within the U.S. Customary system for: – length: inches and feet, feet and yards, inches and yards, yards and miles;[Moved from 4.7b], – weight/mass: ounces, pounds;[Moved from 4.6b] and – liquid volume: cups, pints, quarts, and gallons[Moved from 4.8b]. (c) • Solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary.

4.9 The student will ~~determine~~ solve practical problems related to elapsed time in hours and minutes within a 12-hour period.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Elapsed time is the amount of time that has passed between two given times. • Elapsed time should be modeled and demonstrated using analog clocks and timelines. • Elapsed time can be found by counting on from the beginning time or counting back from the ending time. ~~– Count the number of whole hours between the beginning time and the finishing time.~~ ~~– Count the remaining minutes.~~ ~~– Add the hours and minutes.~~ ~~For example, to find the elapsed time between 10:15 a.m. and 1:25 p.m., count 10 minutes; and then, add 3 hours to 10 minutes to find the total elapsed time of 3 hours and 10 minutes.~~	~~**All students should**~~ ~~• Understanding the "counting on" strategy for determining elapsed time in hour and minute increments over a 12-hour period from a.m. to a.m. or p.m. to p.m.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** ~~• Determine the elapsed time in hours and minutes within a 12-hour period (times can cross between a.m. and p.m.).~~ • Solve practical problems ~~in relation~~ related to elapsed time ~~that has elapsed~~in hours and minutes, within a 12-hour period (within a.m., within p.m., and across a.m. and p.m.): – when given the beginning time and the ending time, determine the time that has elapsed; – when given the beginning time and amount of elapsed time in hours and minutes, determine the ending time; or – when given the ending time and the elapsed time in hours and minutes, determine the beginning time.

[Moved to Measurement and Geometry strand introduction]~~The study of geometry helps students represent and make sense of the world. At the fourth- and fifth-grade levels, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.~~

~~Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.~~

~~Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles.~~

~~The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.~~

- ~~**Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.~~
- ~~**Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)~~
- ~~**Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)~~
- ~~**Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)~~

4.10 The student will

a) identify and describe ~~representations of~~ points, lines, line segments, rays, and angles, including endpoints and vertices; and

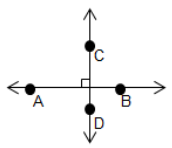
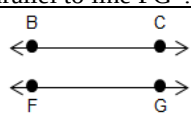
b) identify and describe ~~representations of lines that illustrate~~ intersecting~~on~~, parallel~~ism~~, and perpendicular~~ity~~ lines.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Points, lines, line segments, rays, and angles, including endpoints and vertices are fundamental components of noncircular geometric figures.[Moved from middle column] • A point is a location in space. It has no length, width, or height. A point is usually named with a capital letter.[Reordered] • The shortest distance between two points on a flat surface is a line segment.[Moved from middle column] • ~~A point is a location in space. It has no length, width, or height. A point is usually named with a capital letter.~~[Reordered] • A line is a collection of points extending ~~going on and on~~ infinitely in both directions. It has no endpoints. When a line is drawn, at least two points on it can be marked and given capital letter names. Arrows must be drawn to show that the line goes on in both directions infinitely (e.g., $\overleftrightarrow{AB}$ read as "line AB"). • A line segment is part of a line. It has two endpoints and includes all the points between and including the~~ose~~ endpoints. To name a line segment, name the endpoints (e.g., $\overline{AB}$ read as "line segment AB"). • A ray is part of a line. It has one endpoint and extends~~continues~~ infinitely in one direction. To name a ray, say the name of its endpoint first and then say the name of one other point on the ray (e.g., $\overrightarrow{AB}$ read as "ray AB"). • An angle is formed by ~~T~~two rays that share a common ~~have the same~~endpoint ~~form an angle~~ ~~This endpoint is~~ called the vertex. Angles are found wherever lines or ~~and~~ line segments intersect. • An angle can be named in three different ways by using: – three letters ~~to name,~~ in ~~this~~ order~~,~~: a point on one ray, the vertex, and a point on the other ray; – one letter at the vertex; or – a number written inside the rays of the angle.	~~All students should~~ • ~~Understand that points, lines, line segments, rays, and angles, including endpoints and vertices are fundamental components of noncircular geometric figures.~~ • ~~Understand that the shortest distance between two points on a flat surface is a line segment.~~ • ~~Understand that lines in a plane either intersect or are parallel. Perpendicularity is a special case of intersection.~~ • ~~Identify practical situations that illustrate parallel, intersecting, and perpendicular lines.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and describe ~~representations~~ of points, lines, line segments, rays, and angles, including endpoints and vertices. (a) • ~~Understand that lines in a plane can intersect or are parallel. Perpendicularity is a special case of intersection.~~[Moved to US] • Use symbolic notation to name points, lines, line segments, rays, and angles. (a) • Identify practical situations that illustrate parallel, intersecting, and perpendicular lines. (b) • Identify parallel, perpendicular, and intersecting line segments in plane and solid figures. (b) • Use symbolic notation to describe parallel lines, and perpendicular lines. (b)

4.10 The student will

a) identify and describe ~~representations of~~ points, lines, line segments, rays, and angles, including endpoints and vertices; and

b) identify and describe ~~representations of lines that illustrate~~ intersecting~~on~~, parallel~~ism~~, and perpendicular~~ity~~ lines.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A vertex is the point at which two lines, line segments, or rays meet to form an angle. In three-dimensional figures, a vertex is the point at which three or more edges meet. • Lines in a plane either intersect or are parallel. Perpendicularity is a special case of intersection. • Intersecting lines have one point in common. • Perpendicular lines ~~are special~~ intersect~~ing lines that form~~ at right angles ~~where they intersect~~. The symbol ⊥ is used to indicate that two lines are perpendicular. For example, the notation $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ is read as "line CD is perpendicular to line AB."  $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ • Students need experiences using geometric markings in figures to indicate congruence of sides and angles and to indicate parallel sides. • Parallel lines ~~are lines that~~ lie in the same plane and never ~~do not~~ intersect. Parallel lines are always the same distance apart and do not share any points. The symbol ∥ indicates that two or more lines are parallel. For example, the notation $\overleftrightarrow{BC} \parallel \overleftrightarrow{FG}$ is read as "line BC is parallel to line FG".  $\overleftrightarrow{BC} \parallel \overleftrightarrow{FG}$ • ~~Students should explore intersection, parallelism, and perpendicularity in both two and three dimensions. For example, students should analyze the relationships between the edges of a cube. Which edges are parallel? Which are perpendicular? What~~		

4.10 The student will

a) identify and describe ~~representations of~~ points, lines, line segments, rays, and angles, including endpoints and vertices; and

b) identify and describe ~~representations of lines that illustrate~~ intersecting~~on~~, parallel~~ism~~, and perpendicular~~ity~~ lines.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~plane contains the upper left edge and the lower right edge of the cube? Students can visualize this by using the classroom itself to notice the lines formed by the intersection of the ceiling and walls, of the floor and wall, and of two walls.~~		

~~4.11 The student will~~

~~a) investigate congruence of plane figures after geometric transformations, such as reflection, translation, and rotation, using mirrors, paper folding, and tracing; and~~

~~b) recognize the images of figures resulting from geometric transformations, such as translation, reflection, and rotation.~~

[Moved to 5.14]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.~~ ~~**Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.~~ ~~**Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)~~ ~~**Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)~~ ~~**Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusion are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)~~ • ~~Congruent figures are figures having exactly the same size and shape.~~[Moved to 5.14 US] • ~~Opportunities for exploring figures that are congruent and/or noncongruent can best be accomplished by using physical models.~~	~~**All students should**~~ • ~~Understand the meaning of the term *congruent*.~~ • ~~Understand how to identify congruent figures.~~ • ~~Understand that the orientation of figures does not affect congruency or noncongruency.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Recognize the congruence of plane figures resulting from geometric transformations such as translation, reflection, and rotation, using mirrors, paper folding and tracing.~~ [Moved to 5.14]

~~4.11 The student will~~

~~a) investigate congruence of plane figures after geometric transformations, such as reflection, translation, and rotation, using mirrors, paper folding, and tracing; and~~

~~b) recognize the images of figures resulting from geometric transformations, such as translation, reflection, and rotation.~~

[Moved to 5.14]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~A translation is a transformation in which an image is formed by moving every point on a figure the same distance in the same direction.~~[Moved to 5.14 US] • ~~A reflection is a transformation in which a figure is flipped over a line called the *line of reflection*. All corresponding points in the image and preimage are equidistant from the line of reflection.~~ [Moved to 5.14 US] • ~~A rotation is a transformation in which an image is formed by turning its preimage about a point.~~[Moved to 5.14] • ~~The resulting figure of a translation, reflection, or rotation is congruent to the original figure.~~[Moved to 5.14 US]		

4.11 The student will identify, describe, compare, and contrast characteristics of plane and solid figures by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models.[Moved from 3.14]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The study of geometric figures must be active, using visual images and concrete materials (tools such as graph paper, pattern blocks, geoboards, and geometric solids, and computer software tools). • Opportunity must be provided for building and using geometric vocabulary to describe plane and solid figures. • A plane figure is any two-dimensional shape that can be drawn in a plane. A plane is a flat surface with no thickness. • A solid figure is a three-dimensional figure. • A face is any flat surface of a solid figure. • An angle is formed by two rays with a common endpoint called the *vertex*. Angles are found wherever lines and/or line segments intersect. • An edge is the line segment where two faces of a solid figure intersect. • A vertex is the point at which two or more lines, line segments, or rays meet to form an angle. A vertex in a solid figure is where three or more faces meet. • A cube is a solid figure with six congruent, square faces. All edges are the same length. A cube has 8 vertices and 12 edges. • A cylinder is a solid figure formed by two congruent parallel faces called bases joined by a curved surface. The most commonly referenced cylinders have circular bases. • A base is a special face of a solid figure. • A cone is a solid figure formed by a face called a base that is joined to a vertex by a curved surface. The most commonly referenced cones have a circular base.		**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify models and representations of solid figures (cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by name.[Moved from 3.14 EKS] • Identify and describe characteristics of solid figures (cube, rectangular prism, square pyramid, sphere, cone, cylinder) by counting the number of angles, vertices, edges, and by the number and shape of faces.[Moved from 3.14 EKS] • Compare and contrast characteristics of plane and solid figures (e.g., circle/sphere, square/cube, triangle/square pyramid, rectangle/rectangular prism), by counting the number of sides, angles, vertices, edges, and the number and shape of faces.[Moved from 3.14 EKS]

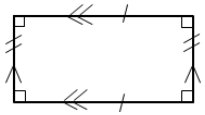
4.11 **The student will identify, describe, compare, and contrast characteristics of plane and solid figures by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models.[Moved from 3.14]**

<table>
<tr><th>UNDERSTANDING THE STANDARD
~~(Background Information for Instructor Use Only)~~</th><th>~~ESSENTIAL UNDERSTANDINGS~~</th><th>ESSENTIAL KNOWLEDGE AND SKILLS</th></tr>
<tr><td>

A rectangular prism is a solid figure in which all six faces are rectangles. A rectangular prism has 8 vertices and 12 edges.
A sphere is a solid figure with all of its points the same distance from its center.
A square pyramid is a solid figure with a square base and four faces that are triangles with a common vertex. A square pyramid had 5 vertices and 8 edges.
Characteristics of solid geometric figures included at this grade level are defined in the chart:

<table>
<tr><th>Solid Geometric Figure</th><th># of Faces</th><th>Shape of Faces</th><th># of Edges</th><th># of Vertices</th></tr>
<tr><td>Cube</td><td>6</td><td>Squares</td><td>12</td><td>8</td></tr>
<tr><td>Rectangular Prism</td><td>6</td><td>Rectangles</td><td>12</td><td>8</td></tr>
<tr><td>Square Pyramid</td><td>5</td><td>Square/Triangles</td><td>8</td><td>5</td></tr>
<tr><td>Sphere</td><td>0</td><td>N/A</td><td>0</td><td>0</td></tr>
<tr><td>Cone</td><td>1</td><td>Circle*</td><td>0</td><td>1</td></tr>
<tr><td>Cylinder</td><td>2</td><td>Circle*</td><td>0</td><td>0</td></tr>
</table>
* The most commonly referenced cones and cylinders have circular faces.
</td><td></td><td></td></tr>
</table>

4.12 The student will classify quadrilaterals as a parallelograms, rectangles, squares, rhombuses, and/or trapezoids.
~~a) define polygon; and~~
~~b) identify polygons with 10 or fewer sides.~~[Polygons other than quadrilaterals moved to 3.11]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~A polygon is a closed plane geometric figure composed of at least three line segments that do not cross. None of the sides are curved.~~ • ~~A triangle is a polygon with three angles and three sides.~~ • A quadrilateral is a polygon with four sides. • A parallelogram is a quadrilateral with both pairs of opposite sides parallel and congruent.[Reordered] • Congruent figures have the same size and shape. Congruent sides are the same length. • A rectangle is a quadrilateral with four right angles and opposite sides that are parallel and congruent. • The geometric markings shown on the rectangle below indicate parallel sides with an equal number of arrows and congruent sides indicated with an equal number of hatch (hash) marks.  • A square is a rectangle with four congruent sides ~~of equal length~~and four right angles. • A trapezoid is a quadrilateral with exactly one pair of parallel sides. • ~~A parallelogram is a quadrilateral with both pairs of opposite sides parallel.~~[Reordered] • A rhombus is a quadrilateral with ~~4~~ four congruent sides. Properties of a rhombus include the following: – opposite sides are congruent – opposite sides are parallel – opposite angles are congruent	~~**All students should**~~ • ~~Identify polygons with 10 or fewer sides in everyday situations.~~ • ~~Identify polygons with 10 or fewer sides in multiple orientations (rotations, reflections, and translations of the polygons).~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • ~~Define and identify properties of polygons with 10 or fewer sides.~~[Polygons other than quadrilaterals moved to 3.13] • ~~Identify polygons by name with 10 or fewer sides in multiple orientations (rotations, reflections, and translations of the polygons).~~[Polygons other than quadrilaterals moved to 3.13] • Develop definitions for parallelograms, rectangles, squares, rhombi, and trapezoids.[Reworded] • Identify properties of quadrilaterals including parallel, perpendicular, and congruent sides.[Reworded] • Classify quadrilaterals as a parallelograms, rectangles, squares, rhombuses, and/or trapezoids.[Reworded] • Compare and contrast the properties of quadrilaterals. • Identify parallel sides, congruent sides, and right angles using geometric markings to denote properties of quadrilaterals.

4.12 The student will classify quadrilaterals as a parallelograms, rectangles, squares, rhombuses, and/or trapezoids.
~~a) define polygon; and~~
~~b) identify polygons with 10 or fewer sides.~~[Polygons other than quadrilaterals moved to 3.11]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~A pentagon is a 5-sided polygon.~~ • ~~A hexagon is a 6-sided polygon.~~ • ~~A heptagon is a 7-sided polygon.~~ • ~~An octagon is an 8-sided polygon.~~ • ~~A nonagon is a 9-sided polygon.~~ • ~~A decagon is a 10-sided polygon.~~[Polygon information moved to 3.13]		

Students entering grades ~~4 and~~3- 5 have begun to explore~~d~~ the concepts of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students ~~are able to predict~~ have had initial experiences in predicting whether ~~the~~ a game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction ~~at~~in grades ~~4 and~~3- 5 is to deepen their understanding of the concepts of probability by

- offering opportunities to set up models simulating practical events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities that instill a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data collection strategies. Students should continue to focus on

- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to practical experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions (“How many?” “How much?”) from the data displays;
- identifying and justifying comparisons (“Which is the most? Which is the least?” “Which is the same?” “Which is different?”) about the information;
- comparing their initial predictions to the actual results; and
- ~~writing a few sentences to~~ communicate~~e~~ing to others their interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data analysis methods as powerful means for decision making.

4.13 The student will

a) ~~predict~~ determine the likelihood of an outcome of a simple event; and

b) represent probability as a number between 0 and 1, inclusive; and

c) create a model or practical problem to represent a given probability.

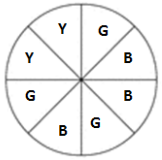
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives. • Probability is the ~~chance~~ measure of likelihood that an event ~~occurring.~~will occur. An event is a collection of outcomes from an investigation or experiment. • The terms *certain*, *likely*, *equally likely*, *unlikely*, and *impossible* can be used to describe the likelihood of an event.[Moved from middle column] • The probability of an event ~~occurring is the ratio of desired~~can be expressed as a fraction, where the numerator represents the number of favorable outcomes ~~to~~and the denominator represents the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event = number of favorable outcomes / total number of possible outcomes. • Probability is quantified as a number ~~The probability of an event occurring is represented by a ratio~~ between 0 and 1. An event is "impossible" if it has a probability of 0 (e.g., the probability that the month of April will have 31 days). An event is "certain" if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow morning). • When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time). • ~~Conduct experiments to determine the probability of an event occurring for a given number of trials (no more than 25 trials), using manipulatives (e.g., the number of times "heads" occurs when flipping a coin 10 times; the chance that when the names of~~	~~**All students should**~~ • ~~Understand and apply basic concepts of probability.~~ • ~~Describe events as likely or unlikely and discuss the degree of likelihood, using the terms *certain*, *likely*, *equally likely*, *unlikely*, and *impossible*.~~ [Moved to US] • ~~Predict the likelihood of an outcome of a simple event and test the prediction.~~ • ~~Understand that the measure of the probability of an event can be represented by a number between 0 and 1, inclusive.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model and determine all possible outcomes of a given simple event where there are no more than 24 possible outcomes, using a variety of manipulatives~~,~~ (e.g., ~~such as~~ coins, number cubes, and spinners). (a) • Determine the outcome of an event that is least likely to occur (less than half) or most likely to occur (greater than half) where there are no more than 24 possible outcomes. (a)[Reordered] • Write the probability of a given simple event as a fraction, where there are no more than 24 possible outcomes~~the total number of possible outcomes is 24 or fewer~~. (b) • ~~Identify~~ Determine the likelihood of an event occurring and relate it to its fractional representation (e.g., impossible/0; equally likely/$\frac{1}{2}$; certain/1). (b) • ~~Determine the outcome of an event that is least likely to occur (less than half) or most likely to occur (greater than half) when the number of possible outcomes is 24 or less.~~ • Create a model or practical problem to represent a given probability. (c) ~~Represent probability as a point between 0 and 1, inclusively, on a numbe~~

4.13 The student will

a) ~~predict~~ determine the likelihood of an outcome of a simple event; and

b) represent probability as a number between 0 and 1, inclusive; and

c) create a model or practical problem to represent a given probability.

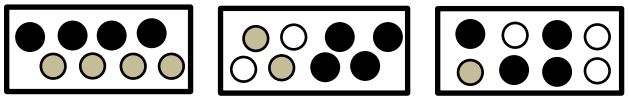
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~12 classmates are put in a shoebox, a name that begins with *D* will be drawn).~~Equally likely events can be represented with fractions of equivalent value. For example, on a spinner with 8 sections of equal size, where three of the sections are labeled G (green) and three are labeled B (blue), the chances of landing on green or on blue are equally likely; the probability of each of these events is the same, or $\frac{3}{8}$.  ~~• Students should have opportunities to describe in informal terms (i.e., *impossible*, *unlikely*, *as likely as unlikely*, *equally likely*, *likely*, and *certain*) the degree of likelihood of an event occurring.~~ ~~• Activities should include practical examples.~~ • For an event such as flipping a coin, the ~~equally likely~~ things that can happen are called *outcomes*. For example, there are two ~~equally likely~~ possible outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up. • For another event such as spinning a spinner that is one-third red and two-thirds blue, the two outcomes, red and blue, are not equally likely.~~ This is an unfair spinner (since it is not divided equally), therefore, the outcomes are not equally likely.~~ 		

4.13 The student will

a) ~~predict~~ determine the likelihood of an outcome of a simple event; and

b) represent probability as a number between 0 and 1, inclusive; and

c) create a model or practical problem to represent a given probability.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Students need opportunities to create a model or practical problem that represents a given probability. For example, if asked to create a box of marbles where the probability of selecting a black marble is $\frac{4}{8}$, sample responses might include: 		

4.14 The student will

a) collect, organize, and represent data in a bar graph and line graph~~display,~~; and

b) make observations and inferences about data represented in bar graphs and line graphs.~~interpret data from a variety of graphs.~~

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Data analysis helps describe data, recognize patterns or trends, and make predictions. • Investigations involving practical data should occur frequently, and data can be collected through brief class surveys or through more extended projects taking many days. • Students formulate questions, predict answers to questions under investigation, collect and represent initial data, and consider whether the data answer the questions. • Data displayed in bar and line graphs can be interpreted so that informed decisions can be made.[Move from middle column] • The title and labels of the graph provide the foundation for interpreting the data.[Moved from middle column] • Bar graphs should be used to compare counts of different categories (categorical or qualitative data). ~~Using g~~Grid paper can assist students in creating~~ensures more accurate~~ graphs with greater accuracy.[Reordered] – A bar graph uses horizontal or vertical bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category. – There is space before, between, and after the bars. – The axis that displays the scale representing the count for the categories should begin at zero and extend one increment above the greatest recorded piece of data. Fourth-grade students should collect data that are recorded in increments of whole numbers, usually multiples of 1, 2, 5, 10, or 100. – Each axis should be labeled, and the graph should be given a title.[Reordered]	~~**All students should**~~ • ~~Understand the difference between representing categorical data and representing numerical data.~~ • ~~Understand that line graphs show change over time (numerical data).~~ • ~~Understand that bar graphs should be used to compare counts of different categories (categorical data).~~ • ~~Understand how data displayed in bar and line graphs can be interpreted so that informed decisions can be made.~~ [Moved to US] • ~~Understand that the title and labels of the graph provide the foundation for interpreting the data.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect data, using, for example, observations, measurement, surveys, scientific experiments, polls, or questionnaires. (a) • Organize data into a chart or table. (a) • ~~Construct and display~~ Represent data in bar graphs, labeling one axis with equal whole number increments of 1 or more (numerical data) (e.g., 2, 5, 10, or 100) and the other axis with categories related to the title of the graph (categorical data) (e.g., swimming, fishing, boating, and water skiing as the categories of "Favorite Summer Sports"). (a) • ~~Construct and display~~ Represent data in line graphs, labeling the vertical axis with equal whole number increments of 1 or more and the horizontal axis with continuous data commonly related to time (e.g., hours, days, months, years, and age). Line graphs will have no more than 10 identified points along a continuum for continuous data.~~ For example, growth charts showing age versus and the height place age on the horizontal axis (e.g., 1 month, 2 months, 3 months, and 4 months).~~[Example moved to US] (a) • Title the graph or identify an appropriate title.~~the title in a given graph and label or identify the axes.~~ Label the axis or identify the appropriate labels. (a)

4.14 The student will

a) collect, organize, and represent data in a bar graph and line graph~~display,~~; and

b) make observations and inferences about data represented in bar graphs and line graphs.~~interpret data from a variety of graphs.~~

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be written.[Reordered] • Line graphs are used to show how two ~~continuous~~ variables are related. Line graphs may be used to show how one variable changes over time (numerical or quantitative data). ~~If this one variable is not continuous, then a broken line is used.~~ By looking at a line graph, it can be determined whether the variable is increasing, decreasing, or staying the same over time. – The values along the horizontal axis represent continuous data on a given variable, usually some measure of time (e.g., time in years, months, or days). The data presented on a line graph is referred to as "continuous data," as it represents data collected over a continuous period of time. – The values along the vertical axis are the scale and represent the frequency with which those values occur in the data set. The values should represent equal increments of multiples of whole numbers, fractions, or decimals, depending upon the data being collected. The scale should extend one increment above the greatest recorded piece of data. – Each axis should be labeled, and the graph should be given a title. – ~~A line graph tells whether something has increased, decreased, or stayed the same with the passage of time.~~ Statements representing an analysis and interpretation of the characteristics of the data in the graph should be included (e.g., trends of increase and/or decrease, and least and greatest). • For example, a line graph documenting data gathered during a planting cycle might show length of time and the height of a plant at any given interval.		• Interpret data by making observations from ~~simple~~ line and bar graphs by describing the characteristics of the data and the data as a whole (e.g., the category with the greatest/least, categories with the same number of responses, similarities and differences, the total number). ~~Data points will be limited to 30 and categories to 8.~~One set of data will be represented on a graph. (b) • Interpret data by making inferences from line and bar graphs. (b) • Interpret the data to answer the question posed, and compare the answer to the prediction (e.g., "The summer sport preferred by most is swimming, which is what I predicted before collecting the data."). (b) • Write at least one sentence to describe the analysis and interpretation of the data, identifying parts of the data that have special characteristics, including categories with the greatest, the least, or the same. (b) • Compare two different representations of the same data (e.g., A set of data displayed on chart and bar graphs; chart and line graphs; picture graph and bar graphs). (b)

4.14 The student will

a) collect, organize, and represent data in a bar graph and line graph~~display,~~; and

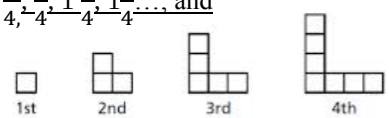
b) make observations and inferences about data represented in bar graphs and line graphs.~~interpret data from a variety of graphs.~~

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• Bar graphs should be used to compare counts of different categories (categorical data). Using grid paper ensures more accurate graphs.~~ ~~– A bar graph uses parallel, horizontal or vertical bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category.~~ ~~– There is space before, between, and after the bars.~~ ~~– The axis that displays the scale representing the count for the categories should extend one increment above the greatest recorded piece of data. Fourth-grade students should collect data that are recorded in increments of whole numbers, usually multiples of 1, 2, 5, 10, or 100.~~ ~~– Each axis should be labeled, and the graph should be given a title.~~ ~~– Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be written.~~[Bar graph information reordered]		

Students entering grades ~~4 and~~3- 5 have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships ~~in open number sentences~~ within a pattern.

The focus of instruction is to help students develop a solid use of patterning as a problem solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They interpret the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By interpreting mathematical situations and models, students begin to represent these, using symbols and variables to write “rules” for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.

4.15 The student will ~~recognize~~identify, describe, create, and extend ~~numerical and geometric~~ patterns found in objects, pictures, numbers, and tables.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Patterns and functions can be represented in many ways and described using words, tables, graphs, and symbols.[Moved from middle column] • ~~Most p~~Patterning activities should involve making connections between~~some form of~~ concrete materials and numerical representations~~to make up a pattern~~. • ~~Students will identify and extend a wide variety of patterns, including rhythmic, geometric, graphic, numerical, and algebraic.~~ The patterns will include both growing (addition and subtraction only) and repeating patterns. • Students need experiences with growing patterns using concrete materials and calculators. Growing patterns can increase or decrease by a constant value. • Reproduction of a given pattern in a different representation, using symbols and objects, lays the foundation for writing the relationship symbolically or algebraically. • Sample numerical (arithmetic) patterns include: – 20, 18, 16, 14, …, – 0.35, 0.55, 0.75… – $\frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, 1\frac{3}{4}$…; and –  • Tables of values should be analyzed for a pattern to determine an unknown value and/or rule~~ the next value~~. Examples include:	~~**All students should**~~ • ~~Understand that patterns and functions can be represented in many ways and described using words, tables, graphs, and symbols.~~[Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and ~~D~~describe ~~geometric and numerical~~ patterns, using objects, pictures, numbers, and tables~~, symbols, or words~~. • Create ~~geometric and numerical~~ patterns~~,~~ using ~~concrete materials~~objects, pictures, numbers~~ lines~~, and tables~~, and words~~. • Extend ~~geometric and numerical~~ patterns, using objects, pictures, numbers, and tables~~concrete materials, number lines, tables, and words~~. • Identify, describe, create, and extend single-operation input and output rules involving addition and subtraction to solve problems in various contexts. • Identify the rule in a single-operation, addition or subtraction, numerical pattern found in a list or table.

4.15 The student will ~~recognize~~identify, describe, create, and extend ~~numerical and geometric~~ patterns found in objects, pictures, numbers, and tables.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS

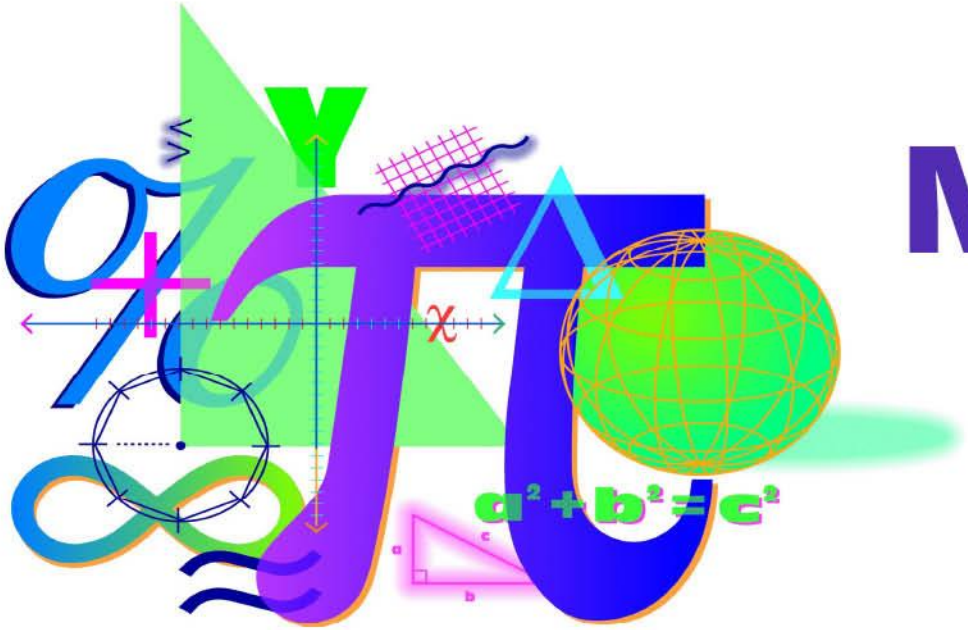
Rule: ?		Rule: ?		Rule: ?	
Input	Output	Input	Output	Input	Output
4	11	145	130	8.9	9.4
5	12	100	85	6.6	7.1
6	13	75	60	?	3.5
10	17	50	?	0.5	1.0

4.16 The student will

~~a)~~ recognize and demonstrate the meaning of equality in an equation.~~; and~~

~~b) investigate and describe the associative property for addition and multiplication.~~[Moved to 5.19]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Mathematical relationships can be expressed using equations.[Moved from middle column] • ~~Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers.~~[Moved to 4.4] • ~~The commutative property for addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4). Similarly, the commutative property for multiplication states that changing the order of the factors does not affect the product (e.g., 2 × 3 = 3 × 2).~~[Moved to 4.4] • ~~The associative property for addition states that the sum stays the same when the grouping of addends is changed [e.g., 15 + (35 + 16) = (15 + 35) + 16]. The associative property for multiplication states that the product stays the same when the grouping of factors is changed [e.g., 6 × (3 × 5) = (6 × 3) × 5].~~[Moved to 5.19] • An expression is a representation of a quantity. It is made up of numbers, variables, and/or computational symbols. It does not have an equal sign (e.g., 8, 15 × 12). • An equation represents the relationship between two expressions of equal value. (e.g., 12 × 3 = 72 ÷2) • The equal-sign (=) means that the values on either side are equivalent (balanced). • The not equal (≠) symbol means that the values on either side are not equivalent (not balanced).	~~All students should~~ • ~~Understand that mathematical relationships can be expressed using equations.~~[Moved to US] • ~~Understand that quantities on both sides of an equation must be equal.~~ • ~~Understand that the associative property for addition means you can change the groupings of three or more addends without changing the sum.~~ • ~~Understand that the associative property for multiplication means you can change the groupings of three or more factors without changing the product.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Recognize and demonstrate that the equals sign (=) relates equivalent quantities in an equation.~~ • Write an equation to represent the relationship between equivalent mathematical expressions~~relationships~~ (e.g., 4 × 3 = 2 × 6). • Recognize and demonstrate appropriate use of the equal~~s~~ sign (=) and the not equal sign (≠)~~in an equation~~. • Identify and use the appropriate symbol to distinguish between expressions that are equal and expressions that are not equal quantities, using addition, subtraction, multiplication, and division (e.g., 4 × 12 = 6 × 8 and 64 ÷ 8 ≠ 8 × 8). • ~~Investigate and describe the associative property for addition as (6 + 2) + 3= 6 + (2 + 3).~~[Moved to 5.19] • ~~Investigate and describe the associative property for multiplication as (3 x 2) x 4 = 3 x (2 x 4).~~[Moved to 5.19]



Mathematics
Standards of Learning

Curriculum Framework ~~2008~~ 2016

Grade 5

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

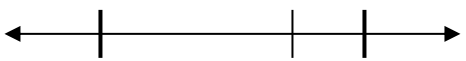
Mathematics instruction in grades ~~4 and~~ 3-5 should continue to foster the development of number sense, ~~especially~~ with greater emphasis on decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction ~~at~~in grades ~~4 and~~3- 5 allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names), and making connections among mathematical concepts as well as to other content areas. Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g., $\frac{1}{2}$ or 0.5). Students should apply their knowledge of number and number sense to investigate and solve a variety of problem types.

5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The structure of the base-10 number system is based upon a simple pattern of tens in which each place is ten times the value of the place to its right. This is known as a ten-to-one place value relationship. Use base-10 manipulatives, such as place value mats/charts, decimal squares, base-10 blocks, and money to investigate this relationship. • A decimal point separates the whole number places from the places less than one. Place values extend infinitely in two directions from a decimal point. A number containing a decimal point is called a *decimal number* or simply a *decimal*. ~~• To read decimals,~~ ~~– read the whole number to the left of the decimal point, if there is one;~~ ~~– read the decimal point as "and";~~ ~~– read the digits to the right of the decimal point just as you would read a whole number; and~~ ~~– say the name of the place value of the digit in the smallest place.~~ ~~• Decimals may be written in a variety of forms:~~ ~~– Standard: 23.456~~ ~~– Written: Twenty-three and four hundred fifty-six thousandths~~ ~~– Expanded: $(2 \times 10) + (3 \times 1) + (4 \times 0.1) + (5 \times 0.01) + (6 \times 0.001)$~~ • ~~To help students identify the ten-to-one place value relationship for decimals through thousandths, use Base-10 manipulatives, such as place value mats/charts, decimal squares, Base-10 blocks, and money.~~Decimals can be rounded ~~to the nearest whole number, tenth or hundredth~~ in situations when exact numbers are not needed. Strategies for rounding whole numbers can be applied to rounding decimals. • Number lines are tools that can be used in developing a conceptual understanding of rounding decimals. One strategy includes creating a number line that shows the decimal that is to be rounded.	~~**All students should**~~ ~~• Understand that decimals are rounded in a way that is similar to the way whole numbers are rounded.~~ ~~• Understand that decimal numbers can be rounded to estimate when exact numbers are not needed for the situation at hand.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Round decimal numbers to the nearest whole number, tenth, or hundredth.

5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• Strategies for rounding decimal numbers to the nearest whole number, tenth and hundredth are as follows:~~ ~~– Look one place to the right of the digit to which you wish to round.~~ ~~– If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero.~~ ~~– If the digit is 5 or greater, add 1 to the digit in the rounding place and change the digits to the right of the rounding place to zero.~~ ~~• Create a number line that shows the decimal that is to be rounded.~~ ~~• The position of the decimal will help children conceptualize the number's placement relative for rounding. An example is to round 5.747 to the nearest hundredth:~~  ~~5.74 5.747 5.75~~		

5.2 The student will

a) ~~recognize and name~~ represent and identify equivalencies among fractions and ~~in their equivalent~~ decimals, with and without models; ~~form and vice versa;~~* and

b) compare and order fractions and decimals in a given set from least to greatest and greatest to least.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Students should ~~recognize, name, and~~ focus on ~~finding~~ determining equivalent decimals of familiar fractions with denominators that are factors of 100 making connections to tenths and hundredths. (e.g., 2/5 = 4/10 or 0.4) and (e.g., 7/20 = 35/100 or 0.35). ~~such as halves, fourths, fifths, eighths, and tenths.~~ • ~~Students should be able to determine equivalent relationships between decimals and fractions with denominators up to 12.~~ • Students should have experience with fractions such as $\frac{1}{8}$, whose decimal representation is a terminating decimal (e. g., $\frac{1}{8}$ = 0.125) and with fractions such as $\frac{2}{3}$ $\frac{2}{9}$, whose decimal representation does not end but continues to repeat (e. g., $\frac{2}{3}$ $\frac{2}{9}$ = 0.~~222~~666…). The repeating decimal can be written with ellipses (three dots) as in 0.~~222~~666… or denoted with a bar above the digits that repeat as in ~~0.$\overline{2}$~~ 0.$\overline{6}$~~.~~ • To help students compare the value of two decimals through thousandths, use manipulatives, such as place value mats/charts, 10-by-10 grids, decimal squares, base-10 blocks, meter sticks, number lines, and money. • A proper fraction is a fraction whose numerator is less than the denominator. A proper fraction is a fraction that is always less than one. • An improper fraction is a fraction whose numerator is greater than or equal to the denominator. An improper fraction is a fraction that is equal to or greater than one.	~~**All students should**~~ • ~~Understand the relationship between fractions and their decimal form and vice versa.~~ • ~~Understand that fractions and decimals can be compared and ordered from least to greatest and greatest to least.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent fractions with denominators that are thirds, eighths, and factors of 100 in their equivalent decimal form with models. (a)~~(halves, fourths, fifths, eighths, tenths, and twelfths) in their equivalent decimal form and vice versa.~~ • Represent decimals in their equivalent fraction form (thirds, eighths, and factors of 100) with models. (a) • ~~Recognize and name~~Identify equivalent relationships between decimals and fractions with denominators ~~up to 12~~that are thirds, eighths, and factors of 100 in their equivalent decimal form without models. (a) • Compare and order from least to greatest and greatest to least a given set of no more than five numbers written as decimals, fractions, and mixed numbers with denominators of 12 or less. (b) • Use the symbols >, <, =, and ≠ to compare decimals through thousandths, proper fractions, improper fractions, and mixed numbers, having denominators of 12 or less. (b)

5.2 The student will

a) ~~recognize and name~~ represent and identify equivalencies among fractions and ~~in their equivalent~~ decimals, with and without models; ~~form and vice versa;~~* and

b) compare and order fractions and decimals in a given set from least to greatest and greatest to least.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• An improper fraction can be expressed as a mixed number. A mixed number is written as a whole number and a proper fraction. • ~~A procedure for comparing two decimals by examining may include the following:~~ ~~– Line up the decimal numbers at their decimal points.~~ ~~– Beginning at the left, find the first place value where the digits are different.~~ ~~– Compare the digits in this place value to determine which number is greater (or which is less).~~ ~~– Use the appropriate symbol > or < or the words *greater than* or *less than* to compare the numbers in the order in which they are presented.~~ ~~– If both numbers are the same, use the symbol = or words *equal to*.~~ ~~Two numbers can be compared by examining place value and/or using a number line.~~ • Decimals and fractions represent the same relationships; however, they are presented in two different formats. Decimal numbers are another way of writing fractions. • Base-10 models (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, money) ~~concretely relate fractions to decimals and vice versa~~demonstrate the relationship between fractions and decimals.		

5.3 The student will

a) identify and describe the characteristics of prime and composite numbers; and

b) identify and describe the characteristics of even and odd numbers.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A prime number is a natural number that has exactly two different factors, one and the number itself. • A composite number is a natural number that has more than two different factors. • The number 1 is neither prime nor composite because it has only one factor, itself. • The prime factorization of a number is a representation of the number as the product of its prime factors. For example, the prime factorization of 18 is $2 \times 3 \times 3$. • Prime factorization concepts can be developed by using factor trees. • Prime or composite numbers can be represented by rectangular models or rectangular arrays on grid paper. A prime number can be represented by only one rectangular array (e.g., 7 can be represented by a 7×1 and a 1×7). A composite number can always be represented by more than two rectangular arrays (e.g., 9 can be represented by a 9×1, a 1×9, or a 3×3). • Divisibility rules are useful tools in identifying prime and composite numbers. • Students should use manipulatives (e.g., base-10 blocks, cubes, tiles, hundreds board, etc.) to explore and categorize numbers into groups of odd or even. • Examples of ways to use manipulatives to show even and odd numbers may include (but are not limited to): – for an even number (e.g., 12) six pairs of counters can be formed with no remainder, or two groups of six counters can be formed with no remainder; and – for an odd number, such as 13: (a) six pairs of counters can be formed with one counter remaining, or (b) two groups of six counters can be formed with one counter remaining.	~~**All students should**~~ ~~• Understand and use the unique characteristics of certain sets of numbers, including prime, composite, even, and odd numbers.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify prime numbers less than or equal to 100. (a) • Identify composite numbers less than or equal to 100. (a) • Demonstrate with manipulatives or pictorial representations and ~~E~~explain orally ~~and~~ or in writing why a number is prime or composite. (a) • Identify which numbers are even or odd. (b) • ~~Explain and d~~Demonstrate with manipulatives or~~,~~ pictorial representations and explain orally or in writing~~, oral language, or written language~~ why a number is even or odd. (b) • Demonstrate with manipulatives or pictorial representations and explain orally or in writing why the sum or difference of two numbers is even or odd. (b)

5.3 The student will

a) identify and describe the characteristics of prime and composite numbers; and

b) identify and describe the characteristics of even and odd numbers.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Students should use rules to categorize numbers into groups of odd or even. Rules can include: – An odd number does not have 2 as a factor or is not divisible by 2. – The sum of two even numbers is even. – The sum of two odd numbers is even. – The sum of an even and an odd is odd. – Even numbers have an even number or zero in the ones place. – Odd numbers have an odd number in the ones place. – An even number has 2 as a factor or is divisible by 2.		

Computation and estimation in grades ~~4 and~~3- 5 should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students' understanding of these operations to work with decimals. Instruction should focus on computation activities that enable students to model, explain, and develop proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract ~~with common~~ fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability not only to identify but to use relationships between operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use ~~(not identify)~~ properties of operations to solve problems [e.g., 7×28 is equivalent to $(7 \times 20) + (7 \times 8)$].

Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals, using benchmarks (e.g., $\frac{2}{5} + \frac{1}{3}$ must be less than 1 because both fractions are less than $\frac{1}{2}$).
Using estimation, students should develop strategies to recognize the reasonableness of their ~~computations~~solutions.
Additionally, students should enhance their ability to select an appropriate problem solving method from among estimation, mental mathematics, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.

5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• An example of an approach to solving problems is Polya's four-step plan:~~ ~~• Understand: Retell the problem; read it twice; take notes; study the charts or diagrams; look up words and symbols that are new.~~ ~~• Plan: Decide what operation(s) to use and what sequence of steps to use to solve the problem.~~ ~~• Solve: Follow the plan and work accurately. If the first attempt doesn't work, try another plan.~~ ~~• Look back: Does the answer make sense?~~ • The problem-solving process is enhanced when students create and solve their own practical problems and model problems using manipulatives and drawings. • In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as *in all, altogether, difference, etc.* encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions. ~~• Estimation gives a rough idea of an amount.~~ Strategies such as front-end, rounding, and mental computation may be used to estimate sums, differences, products, and quotients~~addition, subtraction, multiplication, and division~~ of whole numbers. ~~• Examples of problems to be solved by using estimation strategies are encountered in shopping for groceries, buying school supplies, budgeting allowance, and sharing the cost of a pizza or the prize money from a contest.~~ • Estimation can be used to verify the reasonableness of sums, differences, products, and quotients of whole numbers.~~check the reasonableness of the results.~~	~~**All students should**~~ ~~• Understand the meaning of mathematical operations and how these operations relate to one another when creating and solving single-step and multistep word problems.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** ~~• Select appropriate methods and tools from among paper and pencil, estimation, mental computation, and calculators according to the context and nature of the computation in order to compute with whole numbers.~~ • Create single-step and multistep problems involving the operations of addition, subtraction, multiplication, and division with and without remainders of whole numbers, using practical situations. • Estimate the sum, difference, product, and quotient of whole number computations. • Solve single-step and multistep problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers, ~~using paper and pencil, mental computation, and calculators~~ in which: – sums, differences, and products ~~will~~ do not exceed five digits; – ~~multipliers will~~ factors do not exceed two digits by three digits;[Moved from 4.4] – divisors ~~will~~ do not exceed two digits; or – dividends ~~will~~ do not exceed four digits. • Use the context of a practical problem to interpret the quotient and remainder. ~~• Use two or more operational steps to solve a multistep problem. Operations can be the same or different.~~[Moved to US]

5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The least number of steps necessary to solve a single-step problem is one. • A multistep problem incorporates two or more operational steps (operations can be the same or different).[Moved from EKS] • Students should experience a variety of problem types related to multiplication and division. Some examples are included in the following chart:		

5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.

UNDERSTANDING THE STANDARD

~~(Background Information for Instructor Use Only)~~

MULTIPLICATION AND DIVISION PROBLEM TYPES		
EQUAL GROUP PROBLEMS		
Whole Unknown	Size of Groups Unknown	Number of Groups Unknown
There are 20 bags of apples. There are 14 apples in each bag. How many apples in all?	If 126 apples are shared equally among 4 friends, how many apples will each friend get?	If 168 apples are packed into bags containing 12 apples each. How many bags are needed?
MULTIPLICATIVE COMPARISON PROBLEMS		
Result Unknown	Start Unknown	Comparison Factor Unknown
Bob picked 26 apples. Sue picked 10 times as many apples as Bob. How many did Sue pick?	Sue picked 168 apples. She picked 4 times as many apples as Bob. How many apples did Bob pick?	Sue picked 168 apples. Bob only picked 42 apples. How many times more apples did Sue pick than Bob?

ARRAY OR AREA PROBLEMS	
Whole Unknown	One Dimension Unknown
There are 25 rows of children at the assembly with 15 children in each row. How many children are there together? Joe's rectangular garden measures 15 feet by 22 feet. How many square feet are in the garden?	There are 180 children at the assembly. The children are divided equally in 15 rows. How many children are in each row? There are 180 children at the assembly. There are 15 children in each row. How many rows of children are there? Joe's rectangular garden covers 60 square feet. The length of the garden is 15 feet. What is the width of the garden?
RATE PROBLEMS**	
Whole Unknown	Size of Groups Unknown
If you need 47 cents to buy one stamp how much money do you need to buy 25 stamps?	Peter bought 15 folders for $75.00. If each folder cost the same price how much did one folder cost? How much did 7 folders cost?
COMBINATION PROBLEMS	
Outcomes Unknown	Factor Unknown
An experiment involves tossing a coin and rolling a die. How many different outcomes are possible? Sue has 2 pairs of pants and 3 shirts that can all be worn together. How many different outfits consisting of a pair of pants and a shirt does she have?	Sue bought some new pants and shirts that can all be worn together. He has a total of 12 different outfits. If he bought 3 pairs of pants, how many shirts did he buy?

** Students in grade 5 will not be asked to find the unit rate in practical problems.

~~ESSENTIAL UNDERSTANDINGS~~

ESSENTIAL KNOWLEDGE AND SKILLS

5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Students also need exposure to various types of practical problems in which they must interpret the quotient and remainder based on the context. The chart below includes one example of each type of problem.		

MAKING SENSE OF THE REMAINDER IN DIVISION	
TYPE OF PROBLEM	EXAMPLE
Remainder is not needed and can be left over (or discarded).	Bill has 29 pencils to share fairly with 6 friends. How many pencils will each friend receive? 4 pencils with 5 pencils left over
Remainder is partitioned and represented as a fraction or decimal.	Six friends will share 29 ounces of juice. How many ounces will each person get if all of the juice is shared equally? $4\frac{5}{6}$ ounces
Remainder forces the answer to be increased to the next whole number.	There are 29 people going to the party by car. How many cars will be needed if each car holds 6 people? 5 cars
Remainder forces the answer to be rounded (giving an approximate answer).	Six children will share a bag of candy containing 29 pieces. About how many pieces of candy will each child get? about 5 pieces of candy

5.5 The student will

a) ~~find the sum, difference,~~determine the product~~,~~ and quotient of two numbers ~~expressed as~~involving decimals~~ decimals through thousandths (divisors with only one nonzero digit);~~* and

b) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals, and single-step practical problems involving division of decimals.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Addition and subtraction of decimals may be investigated using a variety of models (e.g., 10-by-10 grids, number lines, money). • Decimal computation uses similar procedures as those developed for whole number computation and applies them to decimal place values, giving careful attention to the placement of the decimal point in the solution. • Multiplication of decimals follows the same procedure as multiplication of whole numbers. The only difference is that a decimal point must be correctly placed in the product giving careful attention to the placement of the decimal point in the solution. ~~• The product of decimals is dependent upon the two factors being multiplied.~~ • In cases where an exact product is not required, the product of decimals can be estimated using strategies for multiplying whole numbers, such as front-end and compatible numbers, or rounding. In each case, the student needs to determine where to place the decimal point to ensure that the product is reasonable. • Estimation keeps the focus on the meaning of the numbers and operations, encourages reflective thinking, and helps build informal number sense with decimals. Students can reason with benchmarks to get an estimate without using an algorithm. • Division is the operation of making equal groups or shares. When the original amount and the number of shares are known, divide to ~~find~~ determine the size of each share. When the original amount and the size of each share are known, divide to ~~find~~ determine the number of shares. Both situations may be modeled with base-10 manipulatives.	~~**All students should**~~ ~~• Use similar procedures as those developed for whole number computation and apply them to decimal place values, giving careful attention to the placement of the decimal point in the solution.~~ ~~• Select appropriate methods and tools from among paper and pencil, estimation, mental computation, and calculators according to the context and nature of the computation in order to compute with decimal numbers.~~ ~~• Understand the various meanings of *division* and its effect on whole numbers.~~ ~~• Understand various representations of division, i.e.,~~ ~~*dividend ÷ divisor = quotient*~~ ~~*quotient*~~ ~~*divisor* ⟌ *dividend*~~ ~~$\frac{dividend}{divisor}$ = *quotient*.~~[Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use estimation to check the reasonableness of a product and quotient. (a)[Reordered] • Determine the product of two decimal numbers, in which – the multipliers do not exceed two digits by two digits (e.g., 2.3 × 4.5, 0.08 × 0.9, 0.85 × 2.3); and – the products do not exceed the thousandths place. (a) Leading zeroes will not be considered when counting digits. • Determine the quotient of numbers, in which – quotients do not exceed 4 digits with or without a decimal point; – quotients may include whole numbers, tenths, hundredths, or thousandths; – divisors are limited to a single digit whole number or a decimal expressed as tenths; and – no more than one additional zero will need to be annexed.(a) • Use multiple representations to model multiplication and division of decimals and whole number.(a) • Create and solve single-step and multistep practical

5.5 The student will

a) ~~find the sum, difference,~~determine the product~~,~~ and quotient of two numbers ~~expressed as~~involving decimals ~~decimals through thousandths (divisors with only one nonzero digit);~~;* and

b) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals, and single-step practical problems involving division of decimals.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The fair-share concept of decimal division can be modeled, using manipulatives (e.g., base-10 blocks). Multiplication and division of decimals can be represented with arrays, paper folding, repeated addition, repeated subtraction, base-10 models, and area models. • Division with decimals is performed the same way as division of whole numbers. The only difference is the placement of the decimal point in the quotient. • Annexing zeros is sometimes necessary when dividing decimal numbers. This occurs when a zero must be added in the dividend as a place holder. • The quotient can be estimated, given a dividend expressed as a decimal through thousandths (and no adding of zeros to the dividend during the division process) and a single-digit divisor. • Estimation can be used to check the reasonableness of a quotient. • Division is the inverse of multiplication; therefore, multiplication and division are inverse operations. • Terms used in division are *dividend, divisor,* and *quotient.* *dividend* ÷ *divisor* = *quotient* *quotient* *divisor*)*dividend* $\frac{dividend}{divisor}$ = *quotient* [Moved from US]		problems involving addition, subtraction, and multiplication of decimals, and single-step practical problems involving division of decimals.(b) • ~~Determine an appropriate method of calculation to find the sum, difference, product, and quotient of two numbers expressed as decimals through thousandths, selecting from among paper and pencil, estimation, mental computation, and calculators.~~ • ~~Estimate to find the number that is closest to the sum, difference, and product of two numbers expressed as decimals through thousandths.~~ • ~~Find the sum, difference, and product of two numbers expressed as decimals through thousandths, using paper and pencil, estimation, mental computation, and calculators.~~ • ~~Determine the quotient, given a dividend expressed as a decimal through thousandths and a single-digit divisor. For example, 5.4 divided by 2 and 2.4 divided by 5.~~ • ~~Use estimation to check the reasonableness of a sum, difference, product, and quotient.~~[Reordered] • ~~Create and solve single-step and multistep problems.~~ • ~~A multistep problem needs to incorporate two or more operational steps (operations can be the same~~

5.5 The student will

a) ~~find the sum, difference,~~determine the product~~,~~ and quotient of two numbers ~~expressed as~~involving decimals~~ decimals through thousandths (divisors with only one nonzero digit)~~;* and

b) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals, and single-step practical problems involving division of decimals.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• There are a variety of algorithms for division such as repeated multiplication and subtraction. Experience with these algorithms may enhance understanding of the traditional long division algorithm. ~~• A multistep problem needs to incorporate no more than two operational steps (operations can be the same or different).~~ • The least number of steps necessary to solve a single-step problem is one.		~~or different).~~

5.6 The student will

a) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers; and ~~express answers in simplest form.~~[Included in EKS]

b) solve single-step practical problems that involve modeling and determining the product of a whole number and a unit fraction.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

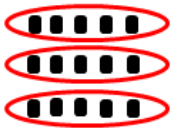
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A fraction can be expressed in simplest form (simplest equivalent fraction) by dividing the numerator and denominator by their greatest common factor. • When the numerator and denominator have no common factors other than 1, then the fraction is in simplest form. • Fractions having like denominators means the same as fractions having common denominators. ~~• Equivalent fractions name the same amount. To find equivalent fractions, multiply or divide the numerator and denominator by the same nonzero number.~~ • Addition and subtraction with fractions and mixed numbers can be modeled using a variety of concrete materials and pictorial representations~~ as well as paper and pencil~~. • Estimation keeps the focus on the meaning of the numbers and operations, encourages reflective thinking, and helps build informal number sense with fractions. Students can reason with benchmarks to get an estimate without using an algorithm. ~~• To add, subtract, and compare fractions and mixed numbers, it often helps to find the least common denominator. The least common denominator (LCD) of two or more fractions is the least common multiple (LCM) of the denominators.~~ ~~• To add or subtract with fractions having the same or like denominators, add or subtract the numerators and write in simplest form.~~ ~~• To add or subtract with fractions that do not have the same denominator, first find equivalent fractions with the least~~	~~**All students should**~~ ~~• Develop and use strategies to estimate and compute addition and subtraction of fractions.~~ ~~• Understand the concept of least common multiple and least common denominator as they are important when adding and subtracting fractions.~~ ~~• Understand that a fraction is in simplest form when its numerator and denominator have no common factors other than 1. The numerator can be greater than the denominator.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve single-step and multistep practical problems involving addition and subtraction with fractions having like and unlike denominators. Denominators in the problems should be limited to 12 or less (e.g., $\frac{1}{5}+\frac{1}{4}$) and answers should be expressed in simplest form. (a) • Solve single-step and multistep practical problems involving addition and subtraction with mixed numbers having like and unlike denominators, with and without regrouping. Denominators in the problems should be limited to 12 or less, and answers should be expressed in simplest form. (a) • Use estimation to check the reasonableness of a sum or difference. (a) • Solve single-step practical problems that include modeling and determining the product of a whole number and a unit fraction (e.g.,$9\times\frac{1}{3}$, $12\times\frac{1}{4}$). The denominator will be a factor of the whole number, the whole number will not exceed 60, and answers should be expressed in simplest form. (b) • Apply the inverse property of multiplication in models. (For example, use a visual fraction model to represent $\frac{4}{4}$ (or 1) as the product of $4\times\frac{1}{4}$). (b)

5.6 The student will

a) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers; and ~~express answers in simplest form.~~[Included in EKS]

b) solve single-step practical problems that involve modeling and determining the product of a whole number and a unit fraction.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~common denominator. Then add or subtract and write the answer in simplest form.~~ • A mixed number has two parts: a whole number and a fraction. The value of a mixed number is the sum of its two parts. ~~To add or subtract with mixed numbers, students may use a number line, draw a picture, rewrite fractions with like denominators, or rewrite mixed numbers as fractions.~~ • A unit fraction is a fraction in which the numerator is one. • The inverse property of multiplication states that the product of a number and its multiplicative inverse always equals one. • Multiplying a whole number by a unit fraction, can be related to dividing the whole number by the denominator of the fraction.  $\frac{15}{3}$ or 5 • At this level, students will multiply a whole number and a unit fraction where the denominator is a factor of the whole number. Examples of problems students in grade 5 should be able to solve include, but are not limited to, the following: – $12 \times \frac{1}{4}$ – If nine children each bring $\frac{1}{3}$ of a cup of candy for the party, how many thirds will there be? What will be the total number of cups of candy? • Answers should be expressed in simplest form.		

5.6 The student will

a) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers; and ~~express answers in simplest form.~~[Included in EKS]

b) solve single-step practical problems that involve modeling and determining the product of a whole number and a unit fraction.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Problems such as $\frac{1}{8} \times 6$ or $6 \times \frac{1}{8}$, will be a focus in grade 6. • The fraction $\frac{3}{4}$ may be interpreted as the amount of cake each person gets when 3 cakes are divided equally among 4 people. • The least number of steps necessary to solve a single-step problem is one.		

5.7 The student will evaluate whole number numerical expressions, using the order of operations limited to parentheses, addition, subtraction, multiplication, and division.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• An expression is a representation of a quantity. It is made up of numbers, variables, and/or computational symbols. It does not have an equal sign (e.g.,15 × 12). • ~~An expression, like a phrase, has no equal sign.~~ • Expressions containing more than one operation are simplified by using the order of operations. • The order of operations is a convention that defines the computation order to follow in simplifying an expression. It ensures that there is only one correct value. • The order of operations is as follows: – First, complete all operations within grouping symbols. If there are grouping symbols within other grouping symbols, do the innermost operation first. (Students in grade 5 are not expected to simplify expressions having parentheses within parentheses.) – If there are multiple operations within the parentheses, apply the order of operations. – Second, evaluate all exponential expressions. (Students in grade 5 are not expected to simplify expressions with exponents.) – Third, multiply and/or divide in order from left to right. – Fourth, add and/or subtract in order from left to right.	~~**All students should**~~ • ~~Understand that the order of operations describes the order to use to simplify expressions containing more than one operation.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Simplify expressions by using the order of operations in a demonstrated step-by-step approach. • Find the value of numerical expressions, using the order of operations. • Given an expression involving more than one operation, describe which operation is completed first, which is second, etc.

Students in grades ~~4 and~~3- 5 should be actively involved in measurement activities that require a dynamic interaction among students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects.

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, perimeter, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary), to measure ~~perimeter~~length, weight/mass, liquid volume~~/capacity~~, area, perimeter, temperature, and time. Student~~s continue to enhance their~~ understanding of measurement ~~by~~continues to be enhanced through experiences using appropriate tools such as rulers, balances, clocks, and thermometers. ~~The process of measuring is identical for any attribute (i.e., length, weight/mass, liquid volume/capacity, area): choose a unit, compare that unit to the object, and report the number of units.~~

[Moved from Geometry Strand introduction]
The study of geometry helps students represent and make sense of the world. In grades 3 through 5, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.
Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)

5.8 The student will

a) ~~find~~determine perimeter, area, and volume in standard units of measure; and

b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation~~;~~.

~~c) identify equivalent measurements within the metric system;~~[Moved to 5.9a]

~~d) estimate and then measure to solve problems, using U.S. Customary and metric units; and~~[U.S. Customary included in 4.8d; Metric moved to 5.9b]

~~e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units.~~[Moved to 5.9EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Perimeter is the path or distance around any plane figure~~object~~. It is a measure of length. • Area is the number of square units needed to cover a surface or plane figure. • Volume of a three-dimensional figure is a measure of capacity and is measured in cubic units. • To ~~find~~ determine the perimeter of any polygon, add the lengths of the sides. • Students should label the perimeter, area, and volume with the appropriate unit of linear, square, or cubic measure. ~~• Area is the number of square units needed to cover a surface or figure.~~ • Students should explore area ~~investigate,~~ using manipulatives~~,~~ to discover the formulas for the area of a ~~square, rectangle, and~~ right triangle~~;~~ and volume of a rectangular solid. ~~Area of a rectangle = Length × Width~~ ~~Area of a square = Side × Side~~ – Area of a right triangle = $\frac{1}{2}$ Base × Height – Volume of a rectangular solid = Length × Width × Height • Students would benefit from opportunities that include the use of benchmark fractions (e.g., $\frac{1}{2}$, $\frac{1}{4}$) in determining perimeter.	~~All students should~~ ~~• Understand the concepts of perimeter, area, and volume.~~ ~~• Understand and use appropriate units of measure for perimeter, area, and volume.~~ ~~• Understand the difference between using perimeter, area, and volume in a given situation.~~ ~~• Understand how to select a measuring device and unit of measure to solve problems involving measurement.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the perimeter of a polygon, with or without diagrams, when – the lengths of all sides of a polygon that is not a rectangle or a square are given; – the length and width of a rectangle are given; or – the length of a side of a square is given. (a) • Estimate and determine the ~~perimeter of a polygon, and~~ area of a square and rectangle using only whole number measurements given in metric or U.S. Customary units, and record the solution with the appropriate unit of measure (e.g., 24 square inches). • Develop a procedure for determining the area of a~~, and~~ right triangle ~~following the parameters listed above,~~ using only whole number measurements given in metric or U.S. Customary units, and record the solution with the appropriate unit of measure (e.g., ~~24~~ 12 square inches). (a) ~~• Estimate and determine the area of a square, with or without diagrams, when the length of a side is given.~~[Moved to 4.7 EKS] ~~• Estimate and determine the area of a rectangle, with or without diagrams, when the length and width are given.~~[Moved to 4.7 EKS]

5.8 The student will

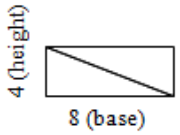
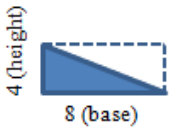
a) ~~find~~determine perimeter, area, and volume in standard units of measure; and

b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation~~;~~.

~~c) identify equivalent measurements within the metric system;~~[Moved to 5.9a]

~~d) estimate and then measure to solve problems, using U.S. Customary and metric units; and~~[U.S. Customary included in 4.8d; Metric moved to 5.9b]

~~e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units.~~[Moved to 5.9EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The area of a rectangle can be determined by multiplying the length of the base by the length of the height. • The rectangle below has been divided in half diagonally creating two right triangles. The legs of the right triangles are congruent to the side lengths of the rectangle. The area of each right triangle is half the area of the rectangle. Exploring decomposition of shapes helps students develop algorithms for determining area of various shapes (e.g., area of a triangle is ½ × b × h).  • The distance from the top of the right triangle to its base is called the height of the triangle.  • Two congruent right triangles can always be arranged to form a square or a rectangle. • ~~Length is the distance along a line or figure from one point to another.~~[Moved to 5.9 EKS] • ~~U.S. Customary units for measurement of length include inches, feet, yards, and miles. Appropriate measuring devices include~~		• Estimate and determine the area of a right triangle, with ~~or without~~ diagrams, when the base and the height are given. (a) • ~~Differentiate among the concepts of area, perimeter, and volume.~~ • Develop a procedure for ~~finding~~ determining volume using manipulatives (e.g., cubes). (a) • Estimate and ~~D~~determine volume of a rectangular prism with diagrams, when the length, width, and height are given using whole number measurements. ~~in standard units.~~(a) • Describe practical situations where ~~area,~~ perimeter, area, and volume are appropriate measures to use, and justify ~~their choices~~ orally or in writing. (b) • Identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation. (b) • ~~Identify equivalent measurements within the metric system for the following:~~ ~~- length: millimeters, centimeters, meters, and kilometers;~~[Moved to 5.9 EKS] ~~- mass: grams and kilograms;~~[Moved to 5.9 EKS] ~~- liquid volume: milliliters, and liters.~~[Moved to 5.9 EKS]

5.8 The student will

a) ~~find~~determine perimeter, area, and volume in standard units of measure; and

b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation~~;~~.

~~c) identify equivalent measurements within the metric system;~~[Moved to 5.9a]

~~d) estimate and then measure to solve problems, using U.S. Customary and metric units; and~~[U.S. Customary included in 4.8d; Metric moved to 5.9b]

~~e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units.~~[Moved to 5.9EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~rulers, yardsticks, and tape measures. Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter ruler, meter stick, and tape measure.~~[Moved to 5.9 EKS] • ~~When measuring with U.S. Customary units, students should be able to measure to the nearest part of an inch ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$), foot, or yard.~~ [Moved to 5.9 EKS] • ~~Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight that an object changes is dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?").~~[Moved to 5.9 EKS] • ~~Appropriate measuring devices to measure mass in U.S. Customary units (ounces, pounds) and metric units (grams, kilograms) are balances.~~ [Moved to 5.9 EKS] • ~~U.S. Customary units to measure liquid volume (capacity) include cups, pints, quarts, and gallons. Metric units to measure liquid volume (capacity) include milliliters and liters.~~[Moved to 5.9 EKS] • ~~Temperature is measured using a thermometer. The U.S.~~		• ~~Solve problems involving measurement by selecting an appropriate measuring device and a U.S. Customary or metric unit of measure for the following:~~[Moved to 5.9b] ~~– length: part of an inch ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$), inches, feet, yards, millimeters, centimeters, meters, and kilometers;~~[Moved to 5.9b] ~~– weight: ounces, pounds, and tons;~~[Moved to 5.9b] ~~– mass: grams and kilograms;~~[Moved to 5.9b] ~~– liquid volume: cups, pints, quarts, gallons, milliliters, and liters;~~[Moved to 5.9b] ~~– area: square units; and~~ [Moved to 5.8a] ~~– temperature: Celsius and Fahrenheit units.~~ ~~– Water freezes at 0°C and 32°F.~~ ~~– Water boils at 100°C and 212°F.~~ ~~– Normal body temperature is about 37°C and 98.6°F.~~

5.8 The student will

a) ~~find~~determine perimeter, area, and volume in standard units of measure; and

b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation~~;~~.

~~c) identify equivalent measurements within the metric system;~~[Moved to 5.9a]

~~d) estimate and then measure to solve problems, using U.S. Customary and metric units; and~~[U.S. Customary included in 4.8d; Metric moved to 5.9b]

~~e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units.~~[Moved to 5.9EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~Customary unit of measure is degrees Fahrenheit; the metric unit of measure is degrees Celsius.~~ • ~~Practical experience measuring familiar objects helps students establish benchmarks and facilitates students' ability to use the units of measure to make estimates.~~ [Moved to 5.9 EKS]		

5.9 The student will

a) identify equivalent measurements within the metric system; and[Moved from 5.8c]

b) solve practical problems involving length, mass, and liquid volume using metric units.[Moved from 5.8d]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Length is the distance between two points along a line.[Moved from 4.7 US] • Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter ruler, meter stick, and tape measure.[Moved from 4.7 US] • Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?").[Moved from 5.8 US] • Balances are appropriate measuring devices to measure mass in U.S. Customary units (ounces, pounds) and metric units (grams, kilograms).[Moved from 5.8 US] • Metric units to measure liquid volume (capacity) include milliliters and liters.[Moved from 5.8 US] • Practical experience measuring familiar objects helps students establish benchmarks and facilitates students' ability to use the appropriate units of measure to make estimates.[Moved from 5.8 US] • Appropriate examples of linear metric measurement equivalents at this grade level may include, but are not limited to: – 4.35 meters = 435 centimeters – 148 millimeters = 14.8 centimeters – Seth ran 2.78 kilometers on Saturday. How many meters are equivalent to 2.78 kilometers? 2.78 kilometers = 2,780 meters		**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify equivalent measurements within the metric system for the following: – length: millimeters, centimeters, meters, and kilometers; – mass: grams and kilograms; – liquid volume: milliliters, and liters. (a)[Moved from 5.8 EKS] • Estimate and measure to solve practical problems that involve metric units: – length: millimeters, centimeters, meters, and kilometers; – mass: grams and kilograms; and – liquid volume: milliliters, and liters. (b)[Moved from 5.8 EKS]

5.10~~9~~ The student will identify and describe the diameter, radius, chord, and circumference of a circle.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A circle is a set of points ~~on a flat surface~~ (in a plane) ~~with every point equidistant~~ that are the same distance from a ~~given~~ point called the *center*. • A chord is a line segment connecting any two points on a circle. A chord may or may not go through the center of a circle. The diameter is the longest chord of a circle. ~~• Students will benefit from understanding that a chord goes from one side of the circle to the other, but does not need to pass through the center.~~ • A diameter is a chord that goes through the center of a circle. ~~The diameter is two times the radius.~~ The length of the diameter of a circle is twice the length of the radius.[Moved from EKS] • A radius is a line segment joining~~from~~ the center of a circle to any point on the circle. Two radii end-to-end form a diameter of a circle. • Circumference is the distance around or "perimeter" of a circle. The circumference is about ~~3~~three times ~~larger than~~ the diameter of a circle. The circumference is about six times the radius of a circle.	~~**All students should**~~ ~~• Understand that a chord is a line segment that extends between any two unique points of a circle.~~ ~~• Understand that a diameter is also a special chord that goes through the center of a circle.~~ ~~• Understand the relationship between the measures of diameter and radius and the relationship between the measures of radius and circumference.~~ ~~• Understand that a radius is a line segment that extends between the center and the circumference of the circle.~~ ~~• Understand that the circumference is the distance around the circle. Perimeter is the measure of the circumference.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and describe the diameter, radius, chord, and circumference of a circle. • Investigate and ~~D~~describe the relationship between – diameter and radius; – diameter and chord; – radius and circumference; and – diameter and circumference. • ~~The length of the diameter of a circle is twice the length of the radius.~~[Moved to US]

5.11~~10~~ The student will ~~determine an amount of~~ solve practical problems related to elapsed time in hours and minutes within a 24-hour period.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Elapsed time is the amount of time that has passed between two given times. • Elapsed time can be found by counting on from the beginning time or counting back from ~~to~~ the ~~finish~~ending time. ~~– Count the number of whole hours between the beginning time and the finishing time.~~ ~~– Count the remaining minutes.~~ ~~– Add the hours and minutes.~~ ~~For example, to find the elapsed time between 10:15 a.m. and 1:25 p.m., count on as follows:~~ ~~from 10:15 a.m. to 1:15 p.m., count 3 hours; from 1:15 p.m. to 1:25 p.m., count 10 minutes; and then add 3 hours to 10 minutes to find the total elapsed time of 3 hours and 10 minutes.~~	~~**All students should**~~ ~~• Understand that elapsed time can be found by counting on from the beginning time to the finishing time.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** ~~• Determine elapsed time in hours and minutes within a 24-hour period.~~ • Solve practical problems related to elapsed time in hours and minutes within a 24-hour period: – When given the beginning time and the ending time, determine the time that has elapsed; – When given the beginning time and amount of elapsed time in hours and minutes, determine the ending time; or – When given the ending time and the elapsed time in hours and minutes, determine the beginning time.

5.12~~11~~ The student will classify and measure right, acute, obtuse, and straight angles.[Classify moved from 5.12a]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Angles can be classified as right, acute, obtuse, or straight according to their measures.[Moved from 5.12 US] • Angles are measured in degrees. There are up to 360 degrees in an angle. A degree is $\frac{1}{360}$ of a complete rotation of a full circle. There are 360 degrees in a circle. • To measure the number of degrees in an angle, use a protractor or an angle ruler. • A right angle measures exactly 90°. • An acute angle measures greater than 0° but less than 90°. • An obtuse angle measures greater than 90° but less than 180°. • A straight angle forms an angle measures exactly 180°. • Before measuring an angle, students should first compare it to a right angle to determine whether the measure of the angle is less than or greater than 90°. • Students should recognize angle measure as additive. When an angle is decomposed into nonoverlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.[Moved from EKS] • Students should understand how to work with a protractor or angle ruler as well as available computer software to measure and draw angles and triangles.	~~All students should~~ ~~• Understand how to measure acute, right, obtuse, and straight angles.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Classify angles as right, acute, obtuse, or straight.[Moved from 5.12a] • Identify the appropriate tools (e.g., protractor and straightedge or angle ruler as well as available software) used to measure and draw angles ~~and triangles~~. • Measure right, acute, straight, and obtuse angles, using appropriate tools, and identify their measures in degrees. • ~~Recognize angle measure as additive. When an angle is decomposed into nonoverlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.~~[Moved to US]~~†~~ • Solve addition and subtraction problems to ~~find~~ determine unknown angle measures on a diagram in practical and mathematical problems, (e.g., by using an equation with a symbol for the unknown angle measure).~~‡~~ ~~‡Revised March 2011~~

[Moved to Measurement and Geometry strand introduction]

~~The study of geometry helps students represent and make sense of the world. At the fourth- and fifth-grade levels, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.~~

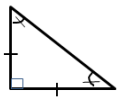
~~Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.~~

~~Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles.~~

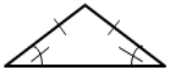
~~The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.~~

- ~~**Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.~~
- ~~**Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)~~
- ~~**Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)~~
- ~~**Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)~~

5.13~~12~~ The student will ~~classify~~
~~a) angles as right, acute, obtuse, or straight; and~~[Classification of angles moved to 5.12]
~~b~~a) classify triangles as right, acute, or obtuse ~~,~~and equilateral, scalene, or isosceles; and
b) investigate the sum of the interior angles in a triangle and determine an unknown angle measure.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Angles can be classified as right, acute, obtuse, or straight according to their measures.[Moved from middle column] • A triangle can be classified as either right, acute, or obtuse according to the measure of its largest angle.[Moved from middle column] • Triangles may also be classified according to the measure of their sides, e.g., scalene (no sides congruent), isosceles (at least two sides congruent) and equilateral (all sides congruent).[Moved from old 5.13 US] • An equilateral triangle (with three congruent sides) is a special case of an isosceles triangle (which has at least two congruent sides). • Triangles can be classified by the measure of their largest angle and by the measure of their sides (i.e., an isosceles right triangle).  Isosceles Right Triangle • A right angle measures exactly 90°. • An acute angle measures greater than 0° but less than 90°. • An obtuse angle measures greater than 90° but less than 180°. • A straight angle forms an angle that measures exactly 180°. • A right triangle has one right angle. • The hypotenuse of a right triangle is the side opposite the right angle. The hypotenuse of a right triangle is always the longest side of the right triangle. The legs of a right triangle form the right angle. • An obtuse triangle has one obtuse angle.	~~All students should~~ • ~~Understand that angles can be classified as right, acute, obtuse, or straight according to their measures.~~ • ~~Understand that a triangle can be classified as either right, acute, or obtuse according to the measure of its largest angle.~~ • ~~Understand that a triangle can be classified as equilateral, scalene, or isosceles according to the number of sides with equal length.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Classify angles as right, acute, straight, or obtuse.~~[Moved to 5.11] • Classify triangles as right, acute, or obtuse. (a) • Classify triangles as ~~equilateral,~~ scalene, ~~or~~ isosceles, or equilateral. (a) • Compare and contrast the properties of triangles. (a) • Identify congruent sides and right angles using geometric markings to denote properties of triangles. (a) • Identify the hypotenuse and legs of a right triangle. (a) • Use models to prove that the sum of the interior angles of a triangle is 180° and use that relationship to determine an unknown angle measure in a triangle. (b)

5.13~~12~~ The student will ~~classify~~
~~a) angles as right, acute, obtuse, or straight; and~~[Classification of angles moved to 5.12]
~~b~~a) classify triangles as right, acute, or obtuse ~~,~~and equilateral, scalene, or isosceles; and
b) investigate the sum of the interior angles in a triangle and determine an unknown angle measure.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• An acute triangle has three acute angles ~~(or no angle measuring 90° or greater)~~. • A scalene triangle has no congruent sides. • An isosceles triangle has at least two congruent sides. The angles opposite of the congruent sides are congruent.  • An equilateral triangle has three congruent sides. All angles of an equilateral triangle are congruent and measure 60°. • ~~To facilitate the exploration of relationships, ask students whether a right triangle can have an obtuse angle. Why or why not? Can an obtuse triangle have more than one obtuse angle? Why or why not? What type of angles are the two angles other than the right angle in a right triangle? What type of angles are the two angles other than the obtuse angle in an obtuse triangle?~~		

5.14~~13~~ The student~~, using plane figures (square, rectangle, triangle, parallelogram, rhombus, and trapezoid),~~ will

a) ~~develop definitions of these plane figures~~[Included in 4.12]recognize and apply transformations, such as translation, reflection, and rotation;[Moved from 4.11b] and

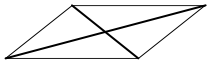
b) investigate and describe the results of combining and subdividing ~~plane figures~~ polygons.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A transformation of a figure (preimage) changes the size, shape, or position of the figure to a new figure (image). Transformations can be explored using mirrors, paper folding and tracing. • Congruent figures have the same size and shape.[Moved from 4.11 US] • A translation is a transformation in which an image is formed by moving every point on the preimage the same distance in the same direction.[Moved from 4.11 US] • A reflection is a transformation in which an image is formed by reflecting the preimage over a line called the *line of reflection*. All corresponding points in the image and preimage are equidistant from the line of reflection.[Moved from 4.11 US] • A rotation is a transformation in which an image is formed by rotating the preimage about a point called the center of rotation. The center of rotation may or may not be on the preimage. [Moved from 4.11 US] • The resulting figure of a translation, reflection, or rotation is congruent to the original figure.[Moved from 4.11 US] • The orientation of figures does not affect congruency or noncongruency. ~~• A triangle is a polygon with three sides. Triangles may be classified according to the measure of their angles, i.e., right, acute, or obtuse. Triangles may also be classified according to the measure of their sides, i.e., scalene (no sides congruent), isosceles (at least two sides congruent) and equilateral (all sides congruent).~~[Moved to 5.13] ~~• A quadrilateral is a polygon with four sides.~~	~~All students should~~ ~~• Understand that simple plane figures can be combined to make more complicated figures and that complicated figures can be subdivided into simple plane figures.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • ~~Develop definitions for squares, rectangles, triangles, parallelograms, rhombi, and trapezoids.~~ [Included in 4.12] • Apply transformations to polygons in order to determine congruence.[Moved from 4.11a and edited] • Recognize that translations, reflections, and rotations preserve congruency.[Moved from 4.11 EKS] • Identify the image of a polygon resulting from a single transformation (translation, reflection, or rotation).[Moved from 4.11b] • Investigate and describe the results of combining and subdividing polygons.~~plane figures.~~ • Compare and contrast the characteristics of a given figure that has been subdivided, with the characteristics of the resulting parts.

5.14~~13~~ The student~~, using plane figures (square, rectangle, triangle, parallelogram, rhombus, and trapezoid),~~ will

a) ~~develop definitions of these plane figures~~[Included in 4.12]recognize and apply transformations, such as translation, reflection, and rotation;[Moved from 4.11b] and

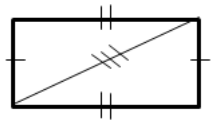
b) investigate and describe the results of combining and subdividing ~~plane figures~~ polygons.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. Properties of a parallelogram include the following:~~ ~~– A diagonal (a segment that connects two vertices of a polygon but is not a side) divides the parallelogram into two congruent triangles.~~ ~~– The opposite sides of a parallelogram are congruent.~~ ~~– The opposite angles of a parallelogram are congruent.~~ ~~– The diagonals of a parallelogram bisect each other. To bisect means to cut a geometric figure into two congruent halves. A bisector is a line segment, line, or plane that divides a geometric figure into two congruent halves. A sample of a bisected parallelogram is below.~~  ~~• A rectangle is a parallelogram with four right angles. Since a rectangle is a parallelogram, a rectangle has the same properties as those of a parallelogram.~~ ~~• A square is a rectangle with four congruent sides. Since a square is a rectangle, a square has all the properties of a rectangle and of a parallelogram.~~ ~~• A rhombus is a parallelogram with four congruent sides. Opposite angles of a rhombus are congruent. Since a rhombus is a parallelogram, the rhombus has all the properties of a parallelogram.~~ ~~• A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called *bases*, and the nonparallel sides are called *legs*. If the legs have the same length, then the trapezoid is an isosceles trapezoid.~~		

5.14~~13~~ The student~~, using plane figures (square, rectangle, triangle, parallelogram, rhombus, and trapezoid),~~ will

a) ~~develop definitions of these plane figures~~[Included in 4.12]recognize and apply transformations, such as translation, reflection, and rotation;[Moved from 4.11b] and

b) investigate and describe the results of combining and subdividing ~~plane figures~~ polygons.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Two or more figures can be combined to form a new figure. Students should be able to identify the figures that have been combined. • ~~The region of a polygon may be subdivided into two or more regions that represent figures.~~ A figure that can be divided into more than one of the basic figures is said to be a composite figure (or shape). Students should understand how to divide ~~the region of~~ a polygon into familiar figures using concrete materials (e.g., pattern blocks, tangrams, geoboards, grid paper, paper (folding), etc.).  • This rectangle has been divided in half diagonally creating two right triangles. The legs of the right triangles are congruent to the side lengths of the rectangle. The area of each right triangle is half the area of the rectangle. Exploring decomposition of shapes helps students develop algorithms for determining area of various shapes (e.g., area of a triangle is ½ × b × h).		

Students entering grades ~~4 and~~3- 5 have begun to explore~~d~~ the concepts of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students ~~are able~~ have had initial experiences in ~~to~~ predicting whether ~~the~~a game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction ~~at~~ in grades ~~4 and~~3- 5 is to deepen their understanding of the concepts of probability by:

- offering opportunities to set up models simulating practical events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities that instill a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data collection strategies. Students should continue to focus on:

- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to practical experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions (“How many?” “How much?”) from the data displays;
- identifying and justifying comparisons (“Which is the most? Which is the least?” “Which is the same? Which is different?”) about the information;
- comparing their initial predictions to the actual results; and
- ~~writing a few sentences to~~ communicate~~e~~ing to others their interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data analysis methods as powerful means for decision making.

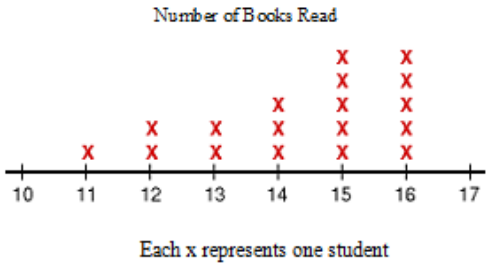
5.15~~14~~ The student will ~~make predictions and~~ determine the probability of an outcome by constructing a sample space.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives, tables, tree diagrams, and lists.[Copied from 4.13] • Probability is the chance of an event occurring. • The probability of an event occurring is the ratio of desired outcomes to the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event = $\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$. • The probability of an event occurring is represented by a ratio between 0 and 1. An event is "impossible" if it has a probability of 0 (e.g., the probability that the month of April will have 31 days). An event is "certain" if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow morning). • When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time). • Students should have opportunities to describe in informal terms (i.e., *impossible*, *unlikely*, ~~*as likely as unlikely*, *as likely as*,~~ *equally likely*, *likely*, and *certain*) the degree of likelihood of an event occurring. Activities should include practical examples. ~~• For any event such as flipping a coin, the equally likely things that can happen are called *outcomes*. For example, there are two equally likely outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up.~~ • A sample space represents all possible outcomes of an experiment. The sample space may be organized in a list, chart, or tree diagram.	~~**All students should**~~ ~~• Understand that the basic concepts of probability can be applied to make predictions of outcomes of simple experiments.~~ ~~• Understand that a sample space represents all possible outcomes of an experiment.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct a sample space, using a tree diagram to identify all possible outcomes~~ of a single event~~. • Construct a sample space, using a list or chart to represent all possible outcomes~~ of a single event~~. • ~~Predict and d~~Determine the probability of an outcome by constructing a sample space. The sample space will have a total of 24 or ~~less~~ fewer equally likely possible outcomes. • Compute the number of possible outcomes by using the Fundamental (Basic) Counting Principle.[Moved from 7.10]

5.15~~14~~ The student will ~~make predictions and~~ determine the probability of an outcome by constructing a sample space.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Tree diagrams can be used to illustrate~~show~~ all possible outcomes in a sample space. ~~The Fundamental Counting Principle describes how to find the number of outcomes when there are multiple choices.~~ For example, how many different outfit combinations can you make from 2 shirts (red and blue) and 3 pants (black, white, khaki)? The sample space displayed in a tree diagram would show that there are $2 \times 3 = 6$ (Fundamental Counting Principle) outfit combinations: red-black; red-white; red-khaki; blue-black; blue-white; blue-khaki. • The Fundamental (Basic) Counting Principle is a computational procedure to determine the number of possible outcomes when there are multiple choices or several events. It is the product of the number of outcomes for each choice or event that can be chosen individually.[Moved from 7.10] ~~A spinner with eight equal-sized sections is equally likely to land on any one of the sections, three of which are red, three green, and two yellow. Have students write a problem statement involving probability, such as, "What is the probability that the spinner will land on green?"~~		

5.16~~15~~ The student, given a practical problem ~~situation,~~ will ~~collect, organize, and interpret data in a variety of forms, using~~
a) represent data in line plots, ~~a~~ stem-and-leaf plots, and line graphs; and[Line plot moved from 3.17]
b) make observations and inferences about data represented in a line plot, stem-and-leaf plot, and line graph.[Line plot moved from 3.17]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The emphasis in all work with statistics should be on the analysis and the communication of the analysis, rather than on a single correct answer. Data analysis should include opportunities to describe the data, recognize patterns or trends, and make predictions. • Statistical investigations should be active, with students formulating questions about something in their environment and ~~finding~~ determining quantitative ways to answer the questions. • Investigations can be brief class surveys or more extended projects taking many days. • Through experiences displaying data in a variety of graphical representations, students learn to select an appropriate representation. • A line plot shows the frequency of data on a number line. Line plots are used to show the spread of the data and quickly identify the range, mode, and any outliers.  • Line graphs are used to show how two continuous variables are related. Line graphs may be used to show how one variable changes over time. If one variable is not continuous, then a broken line is used. By looking at a line graph, it can be determined whether the variable is increasing, decreasing, or staying the same over time.	~~All students should~~ • ~~Understand how to interpret collected and organized data.~~ • ~~Understand that stem-and-leaf plots list data in a meaningful array. It helps in finding median, modes, minimum and maximum values, and ranges.~~ • ~~Understand that line graphs show changes over time.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Formulate the question that will guide the data collection.~~ Collect data, using observations (e.g., weather), measurement (e.g., shoe sizes), surveys (e.g., hours watching television), or experiments (e.g., plant growth). (a) • Organize the data into a chart, table, line plot, stem-and-leaf plot~~s~~, and line graph~~s~~. (a)[Line plot moved from 3.17] • Represent data in a line plot. Line plots will have no more than 30 data points.[Moved from 3.17] (a) • ~~Display data in line graphs and stem-and-leaf plots.~~ • ~~Construct~~ Represent data in a line graph~~s~~, labeling the vertical axis with equal whole number, decimal, or fractional increments and the horizontal axis with continuous data commonly related to time (e.g., hours, days, months, years, and age). Line graphs will have no more than six identified points along a continuum for continuous data (e.g., the decades: 1950~~s~~, 1960~~s~~, 1970~~s~~, 1980~~s~~, 1990~~s~~, and 2000~~s~~). (a) • ~~Construct~~ Represent data in a stem-and-leaf plot ~~to organize and display data,~~ where the stem is listed in ascending order and the leaves are in ascending order, with or without commas between leaves. (a) • Title the given graph or identify an appropriate~~the~~ title.~~Interpret the data in a variety of forms (e.g., orally or in written form).~~ (a)

5.16~~15~~ The student, given a practical problem~~ situation,~~ will ~~collect, organize, and interpret data in a variety of forms, using~~
a) represent data in line plots, ~~a~~ stem-and-leaf plots, and line graphs; and[Line plot moved from 3.17]
b) make observations and inferences about data represented in a line plot, stem-and-leaf plot, and line graph.[Line plot moved from 3.17]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– The values along the horizontal axis represent continuous data on a given variable, usually some measure of time (e.g., time in years, months, or days). The data presented on a line graph is referred to as "continuous data" because it represents data collected over a continuous period of time. – The values along the vertical axis are the scale and represent the frequency with which those values occur in the data set. The values should represent equal increments of multiples of whole numbers, fractions, or decimals depending upon the data being collected. The scale should extend one increment above the greatest recorded piece of data. – Each axis should be labeled and the graph should have a title. – A line graph tells whether something has increased, decreased, or stayed the same with the passage of time. Statements representing an analysis and interpretation of the characteristics of the data in the graph should be included (e.g., trends of increase and/or decrease, and least and greatest). A broken line is used if the data collected is not continuous data (such as test scores); a solid line is used if the data is continuous (such as height of a plant).		• Interpret data by making observations from line plots, stem-and-leaf plots, and line graphs by describing the characteristics of the data and the data as a whole. One set of data will be represented on a graph. (b) • Interpret data by making inferences from line plots, stem-and-leaf plots, and line graphs. (b) • Compare two different representations of the same data (e.g., a set of data displayed on- chart and line graph; line plot and chart; line plot and stem-and-leaf plot). (b)

5.16~~15~~ The student, given a practical problem ~~situation,~~ will ~~collect, organize, and interpret data in a variety of forms, using~~
a) represent data in line plots, ~~a~~ stem-and-leaf plots, and line graphs; and[Line plot moved from 3.17]
b) make observations and inferences about data represented in a line plot, stem-and-leaf plot, and line graph.[Line plot moved from 3.17]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A ~~S~~stem-and-leaf plot~~s~~ is useful when data have a range greater than 25 and allows the exact values of data to be listed in a meaningful array. ~~Data covering a range of 25 numbers are best displayed in a stem-and-leaf plot and are utilized to organize numerical data from least to greatest, using the digits of the greatest to group data.~~		

Stem	Leaf
0	3,6
1	2,5,7,8
2	4,6
3	1,3,7,7,7
4	0,0,4,8
5	
6	1,2,2,3,8

3|5 = 35

- The data is organized from least to greatest.
  - Each value should be separated into a stem and a leaf [e.g., two-digit numbers are separated into stems (tens) and leaves (ones)].
  - The stems are listed vertically from least to greatest with a line to their right. The leaves are listed horizontally, also from least to greatest, and can be separated by spaces or commas. Every value is recorded regardless of the number of repeats. No stem can be skipped. For example, in the stem and leaf plot above, there are no data for the stem 5, 5 should be listed showing no leaves.
  - A key is ~~often~~ included to explain how to read the plot.

5.17~~16~~ The student will solve practical problems that involve

a) describe~~e~~ing mean, median, and mode as measures of center;
b) describe~~e~~ing mean as fair share;
c) describing the range of a set of data as a measure of spread; and[Reordered]
d~~c~~) ~~find~~ determining the mean, median, mode, and range of a set of data. ~~; and~~
~~d) describe the range of a set of data as a measure of variation.~~[Reordered and edited]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Statistics is the science of conducting studies to collect, organize, summarize, analyze, and draw conclusions from data. • A measure of center is a value at the center or middle of a data set. Mean, median, and mode are measures of center. • The mean, median, and mode are three of the various ways that data can be analyzed. • Mean represents a fair share concept of the data. Dividing the data constitutes a fair share. This is done by equally dividing the data points. This should be demonstrated visually and with manipulatives. The arithmetic way is to add all of the data points then divide by the number of data points to determine the average or mean. • The median is the ~~piece of data that lies in the middle of the set of data arranged in order.~~ middle value of a data set in ranked order. If there is an odd number of pieces of data, the median is the middle value in ranked order. If there is an even number of pieces of data, the median is the numerical average of the two middle values. • The mode is the piece of data that occurs most frequently in the data set. There may be one, more than one, or no mode in a data set. Students should order the data from least to greatest so they can better ~~find~~ determine the mode.	~~All students should~~ • ~~Understand that mean, median, and mode are described as measures of center.~~ • ~~Understand that mean, median, and mode are three of the various ways that data can be described or summarized.~~ • ~~Understand that mean as fair share is described as equally dividing the data set or the data set has already been divided equally.~~ • ~~Understand how to find the mean, median, and mode of a set of data as measures of center.~~ • ~~Understand values in the context of other characteristics of the data in order to best describe the results.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe and ~~find~~ determine the mean of a group of numbers representing data from a given context as a measure of center. (a, d) • Describe and ~~find~~ determine the median of a group of numbers representing data from a given context as a measure of center. (a, d) • Describe and ~~find~~ determine the mode of a group of numbers representing data from a given context as a measure of center. (a, d) • Describe mean as fair share. (b) • Describe and ~~find~~ determine the range of a group of numbers representing data from a given context as a measure of spread~~variation~~. (c, d) • ~~Describe the impact on measures of center when a single value of a data set is added, removed, or changed~~[Moved to 6.15].[‡] ~~[‡]Revised March 2011~~

5.17~~16~~ The student will solve practical problems that involve

a) describe~~e~~ing mean, median, and mode as measures of center;

b) describe~~e~~ing mean as fair share;

c) describing the range of a set of data as a measure of spread; and[Reordered]

d~~c~~) ~~find~~ determining the mean, median, mode, and range of a set of data. ~~; and~~

~~d) describe the range of a set of data as a measure of variation.~~[Reordered and edited]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The range is the spread of a set of data. The range of a set of data is the difference between the greatest and least values in the data set. It is determined by subtracting the least number in the data set from the greatest number in the data set. An example is ordering test scores from least to greatest: 73, 77, 84, 87, 89, 91, 94. The greatest score in the data set is 94 and the least score is 73, so the least score is subtracted from the greatest score or 94 - 73 = 21. The range of these test scores is 21. • Students need to learn more than how to identify the mean, median, mode, and range of a set of data. They need to build an understanding of what the number tells them about the data, and they need to see those values in the context of other characteristics of the data in order to best describe the results.		

Students entering grades ~~4 and~~3-5 have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships ~~in open number sentences~~within a pattern.

The focus of instruction is to help students develop a solid use of patterning as a problem solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They interpret the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By interpreting mathematical situations and models, students begin to represent these, using symbols and variables to write “rules” for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.

5.18~~17~~ The student will describe and express the relationship of ~~found in a~~ number patterns ~~and express the relationship~~found in objects, pictures, numbers and tables.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Mathematical relationships exist in patterns.[Moved from middle column] There are an infinite number of patterns. • Patterns and functions can be represented in many ways and described using words, tables, and symbols.[Moved from middle column] • ~~The simplest types of patterns are repeating patterns. In such patterns, students need to identify the basic unit of the pattern and repeat it.~~ • Growing patterns ~~are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization.~~ can increase or decrease by a constant value. • Students need experiences ~~with~~ exploring growing patterns using concrete materials and calculators.[Moved from EKS] Calculators are valuable tools for generating and analyzing patterns. The emphasis is not on computation but on identifying and describing patterns. • Several ~~S~~sample numerical patterns are included below: – ~~6, 9, 12, 15, 18, …;~~ – ~~5, 7, 9, 11, 13, …;~~ – 1, 2, 4, 7, 11, 16, …; – 2, 4, 8, 16, 32, …; – 32, 30, 28, 26, 24…; ~~and~~ – 1, 5, 25, 125, 625,…. – 0.15, 0.35, 0.55, 0.75…; – $\frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, 1\frac{3}{4}$…; and – 128, 64, 32, 16, 8 • An expression is a representation of a quantity. It is made up of numbers, variables, and/or computational symbols. It does not have an equal sign (e.g.,15 × 12). • ~~An expression, like a phrase, has no equal sign.~~	~~All students should~~ • ~~Understand that patterns and functions can be represented in many ways and described using words, tables, and symbols.~~ • ~~Understand the structure of a pattern and how it grows or changes using concrete materials and calculators.~~ • ~~Understand that mathematical relationships exist in patterns.~~ [Moved to US] • ~~Understand that an expression uses symbols to define a relationship and shows how each number in the list, after the first number, is related to the preceding number.~~ • ~~Understand that expressions can be numerical or variable or a combination of numbers and variables.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Describe numerical and geometric patterns formed by using concrete materials and calculators.~~[Moved to US] • Create, describe, and extend patterns using concrete materials, number lines, tables, or pictures. • Describe and express the relationship found in patterns, using words, tables, and symbols ~~to express the relationship~~.[Reordered] • Identify, describe, create, and extend single-operation input and output rules involving addition, subtraction, multiplication, or division to solve problems in various contexts. • Identify the rule in a single-operation (addition, subtraction, multiplication, or division) numerical pattern found in a list or table.

5.18~~17~~ The student will describe and express the relationship of ~~found in a~~ number patterns ~~and express the relationship~~found in objects, pictures, numbers and tables.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A verbal expression involving one operation can be represented by a variable expression that describes the relationship. Numbers are used when they are known; variables are used when the numbers are unknown. The example in the table below defines the relationship as $x + 3$.[Reordered] • ~~When the pattern data are expressed in a T-table, an expression can represent that data. An example is:~~		

X	Y
6	9
7	10
11	14
15	18

• ~~A verbal quantitative expression involving one operation can be represented by a variable expression that describes what is going on. Numbers are used when they are known; variables are used when the numbers are unknown. For example, "a full box of cookies and four extra" can be represented by $b + 4$; "three full boxes of cookies" by $3b$; "a full box of cookies shared among four" by $\frac{b}{4}$.~~[Reordered] • A variable~~mathematical~~ expression ~~contain~~ is a variable or a combination of variables, numbers, and/or operation symbols and represents a mathematical relationship. ~~An expression cannot be solved.~~		

5.19~~18~~ The student will

a) investigate and describe the concept of variable;

b) write an ~~open sentence~~equation to represent a given mathematical relationship, using a variable;

c) use a variable expression to represent a verbal quantitative expression involving one operation; and ~~model one-step linear equations in one variable, using addition and subtraction; and~~[Moved to 6.14]

d) create a problem situation based on a given ~~open sentence~~equation, using a single variable.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A variable is a symbol that can stand for an unknown number or object. • A variable expression is like a phrase: as a phrase does not have a verb, so an expression does not have an equals sign (=). • A verbal expression involving one operation can be represented by a variable expression that describes the relationship~~what is going on~~. Numbers are used when they are known; variables are used when the numbers are unknown. For example, "a full box of cookies and four extra" can be represented by $b + 4$; "three full boxes of cookies" by $3b$; "a full box of cookies shared among four" by $\frac{b}{4}$. • A problem situation can be expressed as an equation.[Moved from middle column] • An equation represents the relationship between two expressions of equal value. (e.g., $12 \times 3 = 72 \div 2$) • An ~~open sentence~~equation may contain~~s~~ a variable and an equals sign (=). For example, the sentence, "A full box of cookies and four extra equal 24 cookies." can be written as $b + 4 = 24$, where b stands for the number of cookies in one full box. "Three full boxes of cookies equal 60 cookies." can be written as $3b = 60$. • Another example of an ~~open sentence~~equation is $b + 3 = 23$ and represents the answer to the word problem, "How many cookies are in a box if the box plus three more equals 23 cookies, where b stands for the number of cookies in the box?	~~All students should~~ • ~~Understand that a variable is a symbol that can stand for an unknown number or object.~~ • ~~Understand that a variable expression is a variable or combination of variables, numbers, and symbols that represents a mathematical relationship.~~ • ~~Understand that verbal expressions can be translated to variable expressions.~~ • ~~Understand that an open sentence has a variable and an equal sign (=).~~ • ~~Understand that problem situations can be expressed as open sentences.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the concept of a variable (presented as boxes, letters, or other symbols) as a representation of an unknown quantity. (a) • Write an ~~open sentence~~equation with addition, subtraction, multiplication, or division, using a variable to represent an unknown quantity. ~~missing number.~~ (b) • ~~Model one-step linear equations using a variety of concrete materials such as colored chips on an equation mat or weights on a balance scale.~~[Moved to 6.18] • Use a variable expression to represent a given verbal expression involving one operation (e.g., "5 more than a number" can be represented by $y + 5$). (c) • Create and write a word problem to match a given ~~open sentence~~equation with a single variable and one operation. (d)

5.19~~18~~ The student will

a) investigate and describe the concept of variable;

b) write an ~~open sentence~~equation to represent a given mathematical relationship, using a variable;

c) use a variable expression to represent a verbal quantitative expression involving one operation; and ~~model one-step linear equations in one variable, using addition and subtraction; and~~[Moved to 6.14]

d) create a problem situation based on a given ~~open sentence~~equation, using a single variable.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• At this level, discuss how the symbol × used to represent multiplication can often be confused with the variable *x*. Students can minimize this confusion by using parentheses [e.g., 4(*x*) = 20 or 4*x* = 20] or a small dot raised off the line to represent multiplication [4 • *x* = 20]. • By using story problems and numerical sentences, students begin to explore forming equations and representing quantities using variables. • An ~~open sentence~~equation containing a variable is neither true nor false until the variable is replaced with a number.		

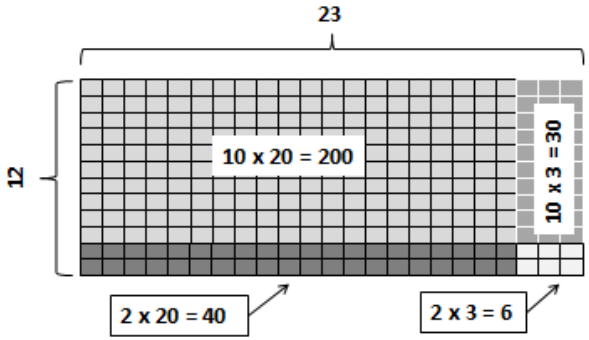
5.2~~019~~ The student will investigate and identify~~recognize the distributive property of multiplication over addition.~~

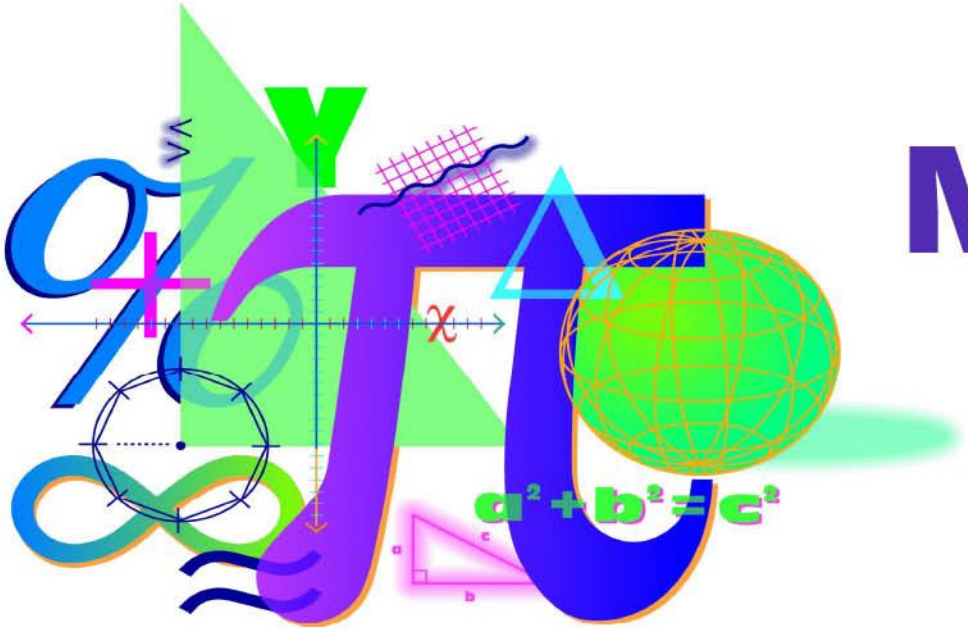
a) the identity properties of addition and multiplication;[Moved from 3.20]
b) the commutative properties of addition and multiplication;[Moved from 3.20] and
c) the associative properties of addition and multiplication; and[Moved from 4.16b]
d) the distributive property of multiplication over addition.[Reordered]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers.[Moved from 3.20] • The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4). Similarly, the commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 2 × 3 = 3 × 2).[Moved from 3.20] • The identity property of addition states that if zero is added to a given number, the sum is the same as the given number. The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number.[Moved from 3.20] • The associative property of addition states that the sum stays the same when the grouping of addends is changed [e.g., 15 + (35 + 16) = (15 + 35) + 16]. The associative property of multiplication states that the product stays the same when the grouping of factors is changed [e.g., 6 × (3 × 5) = (6 × 3) × 5].[Moved from 4.16] • The distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products. Examples of the distributive property include: ~~(e.g.,~~ – 3(9) = 3(5 + 4) – 3(54 + 4) = 3 × 54 + 3 × 4 – 5 × (3 + 7) = (5 × 3) + (5 × 7) – (2 × 3) + (2 × 5) = 2 × (3 + 5)	~~All students should~~ • ~~Understand that the distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products.~~ • ~~Understand that using the distributive property with whole numbers helps with understanding mathematical relationships.~~ [Moved to US] • ~~Understand when and why the distributive property is used.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and identify the identity properties of addition and multiplication of whole numbers, using diagrams and manipulatives. (a)[Moved from 3.20] • Investigate and identify equations that represent the identity properties of addition and multiplication of whole numbers. (a)[Moved from 3.20] • Investigate and identify the commutative properties of addition and multiplication of whole numbers, using diagrams and manipulatives. (b)[Moved from 3.20] • Investigate and identify equations that represent the commutative properties of addition and multiplication of whole numbers. (b)[Moved from 3.20] • Investigate and identify the associative properties of addition and multiplication of whole numbers, using diagrams and manipulatives. (c))[Moved from 4.16] • Investigate and identify equations that represent the associative properties of addition and multiplication of whole numbers. (c)[Moved from 4.16] • Investigate and ~~recognize~~ identify the distributive property of whole numbers, limited to multiplication over addition using diagrams and manipulatives. (d) • Investigate and ~~recognize~~ identify ~~an~~ equations that represent~~s~~ the distributive property~~, when given several whole number equations,~~ limited to multiplication over

5.20~~19~~ The student will investigate and identify~~recognize the distributive property of multiplication over addition.~~

a) the identity properties of addition and multiplication;[Moved from 3.20]
b) the commutative properties of addition and multiplication;[Moved from 3.20] and
c) the associative properties of addition and multiplication; and[Moved from 4.16b]
d) the distributive property of multiplication over addition.[Reordered]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– 9 × 23 9(20+3) 180+ 27 207 – 34 × 8 30 4 8 | 8 × 30 = 240 | 8 × 4 = 32 – 23 × 12 (20 + 3) × (10+2) (20 × 10) + (20 × 2) + (3 × 10) + (3 × 2) 200 + 40 + 30 + 6 276  • ~~The distributive property can be used to simplify expressions (e.g., 9 x 23 = 9(20+3) =180+ 27 = 207; or 5 x 19 = 5(10 + 9) = 50 + 45 = 95).~~[Reorganized and reordered]		addition. (d)



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Grade 6

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> "Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students in the middle grades focus on mastering rational numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to most middle school mathematics topics.~~
- ~~Students develop an understanding of integers and rational numbers by using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.~~
- ~~Students develop an understanding of the properties of operations on real numbers through experiences with rational numbers and by applying the order of operations.~~
- ~~Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.~~

Mathematics instruction in grades 6 – 8 continues to focus on the development of number sense, with emphasis on rational and real numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to many middle school mathematics topics.

Students develop an understanding of integers and rational numbers using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation.
Flexible thinking about rational number representations is encouraged when students solve problems.

Students develop an understanding of real numbers and the properties of operations on real numbers through experiences with rational and irrational numbers and apply the order of operations.

Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

6.1 The student will describe and compare ~~data~~sets, using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A ratio is a comparison of any two ~~quantities~~sets. A ratio is used to represent relationships within and between sets. Ratios are used in practical situations when there is a need to compare quantities. [Moved from middle column] • In the elementary grades, students are taught that fractions represent a part to whole relationship. • A ratio may be written using a colon (*a*:*b*), or the word *to* (*a* to *b*). It may also be written as a fraction $\left(\frac{a}{b}\right)$, if the ratio expresses a part to whole comparison. • The order of the values in a ratio is directly related to the order in which the sets are compared. For example, if asked for the ratio of the number of cats to dogs in a park, the ratio must be expressed as the number of cats to the number of dogs, in that order. [Reordered] • A ratio can represent different comparisons within or between sets. <table><tr><th>Ratio</th><th>Comparison</th></tr><tr><td>part to whole (same set)</td><td>compare part of a set to the entire set</td></tr><tr><td>part to part (same set)</td><td>compare part of a set to another part of the same set</td></tr><tr><td>whole to whole (different sets)</td><td>compare all of one set to all of another set</td></tr><tr><td>part to part (different sets)</td><td>compare part of one set to part of another set</td></tr></table>	• ~~What is a ratio? A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a set and between two sets. A ratio can be written using fraction form ($\frac{2}{3}$), a colon (2:3), or the word *to* (2 to 3).~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Describe~~ Represent a relationship within a set by comparing part of the set to the entire set. • ~~Describe~~ Represent a relationship between two sets by comparing part of one set to ~~a corresponding~~ part of the other set. • ~~Describe~~ Represent a relationship between two sets by comparing all of one set to all of the other set. • ~~Describe~~ Represent a relationship within a set by comparing one part of the set to another part of the same set. • Represent a relationship in words that makes a comparison by using the notations $\frac{a}{b}$ *a*:*b*, and *a to b*. • Create a relationship in words for a given ratio expressed symbolically.

6.1 The student will describe and compare ~~data~~sets, using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
Examples: Given Set A and Set B, the following comparisons could be expressed. Set A: Set B:		

Ratio	Example	Ratio Notation(s)
part to whole (same set)	compare the number of unfilled stars to the total number of stars in Set A	3:8; 3 to 8; $\frac{3}{8}$; 0.375; 37.5%
part to part (same set)	compare the number of unfilled stars to the number of filled stars in Set A	3:5 or 3 to 5
whole to whole (different sets)	compare the number of stars in Set A to the number of stars in Set B	8:5 or 8 to 5
part to part (different sets)	compare the number of unfilled stars in Set A to the number of filled stars in Set B	3:3, 3 to 3, 1:1 or 1 to 1

- ~~A ratio can compare part of a set to the entire set (part-whole comparison).~~ [Included in table above]
- ~~A ratio can compare part of a set to another part of the same set (part-part comparison).~~ [Included in table above]
- ~~A ratio can compare part of a set to a corresponding part of another set (part-part comparison).~~ [Included in table above]

6.1 The student will describe and compare ~~data~~sets, using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A ratio can compare ~~all of a set to all of another set~~two real-world quantities. ~~(whole-whole comparison).~~ For example, miles per gallon, unit rate, and circumference to diameter (pi). • ~~The order of the quantities in a ratio is directly related to the order of the quantities expressed in the relationship. For example, if asked for the ratio of the number of cats to dogs in a park, the ratio must be expressed as the number of cats to the number of dogs, in that order.~~ [Reordered] • ~~A ratio is a multiplicative comparison of two numbers, measures, or quantities.~~ • ~~All fractions are ratios and vice versa~~. • Ratios may or may not be written in simplest form. • ~~Ratios can compare two parts of a whole.~~ • ~~Rates can be expressed as ratios.~~ A rate is a ratio that compares two quantities measured in different units. A unit rate is a rate with a denominator of 1. Examples of rates include miles/hour and revolutions/second. Other examples: – rate: 142 heartbeats per 2 minutes; unit rate: 71 beats/min. – rate: $4.90 for 10 stamps; $0.49/stamp		

6.2 The student will

a) ~~investigate and describe fractions, decimals and percents as ratios~~ [Moved to EKS] represent and determine equivalencies among fractions, decimals, and percents;* and

~~b) identify a given fraction, decimal or percent from a representation;~~ [Included in 6.2a]

~~c) demonstrate equivalent relationships among fractions, decimals, and percents; and~~

b) compare and order ~~fractions, decimals, and percents~~positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Fractions, decimals, and percents are three different ways to express the same number. [Moved from middle column.] • Equivalent relationships among fractions, decimals, and percents may be determined by using concrete materials and pictorial representations (e.g., fraction bars, Base-10 blocks, fraction circles, number lines, colored counters, cubes, decimal squares, shaded figures, shaded grids, or calculators). [Reordered and reworded] • *Percent* means "per 100" or how many "out of 100"; *percent* is another name for *hundredths*. • A number followed by a percent symbol (%) is equivalent to a fraction with that number as the numerator and with ~~a~~ 100 as the denominator ~~of 100~~ (e.g., 30% = $\frac{30}{100} = \frac{3}{10}$ = 0.3). • Percents can be expressed as fractions with a denominator of 100 (e.g., 175% = $\frac{175}{100} = 1\frac{3}{4}$). • Percents can be expressed as decimals (e.g., 38% = $\frac{38}{100}$ = 0.38). • Some fractions can be rewritten as equivalent fractions with denominators of powers of 10, and can be represented as decimals or percents (e.g., $\frac{3}{5} = \frac{6}{10} = \frac{60}{100}$ = 0.60 = 60%).	~~What is the relationship among fractions, decimals and percents? Fractions, decimals, and percents are three different ways to express the same number. A ratio can be written using fraction form ($\frac{2}{3}$), a colon (2:3), or the word *to* (2 to 3). Any number that can be written as a fraction can be expressed as a terminating or repeating decimal or a percent.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and describe fractions, decimals, and percents as ratios. (a) [Moved from 6.2a] • ~~Identify~~ Determine the decimal and percent equivalents for numbers written in fraction form including repeating decimals. (a) • ~~Represent fractions, decimals, and percents on a number line.~~[Combined] • ~~Describe orally and in writing the~~Represent and determine ~~equivalent relationships~~ equivalencies among decimals, percents, and fractions that have denominators that are 12 or less or factors of 100. (a) • ~~Represent, by shading a grid, a fraction, decimal, and percent.~~ [Combined] • ~~Represent in fraction, decimal, and percent form a given shaded region of a grid~~. [Combined] • Compare two decimals through thousandths using manipulatives, pictorial representations, number lines, and symbols (<, ≤, ≥, >, =). (b)

6.2 The student will

a) ~~investigate and describe fractions, decimals and percents as ratios~~ [Moved to EKS] represent and determine equivalencies among fractions, decimals, and percents;* and

~~b) identify a given fraction, decimal or percent from a representation;~~ [Included in 6.2a]

~~c) demonstrate equivalent relationships among fractions, decimals, and percents; and~~

b) compare and order ~~fractions, decimals, and percents~~positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Decimals, fractions, and percents can be represented using concrete materials (e.g., Base 10 blocks, number lines, decimal squares, or grid paper).~~Reordered] • Fractions, decimals, and ~~P~~percents can be represented by ~~drawing shaded regions on grids~~using an area model, a set model, or ~~by finding a location on number lines~~ a measurement model. For example, the fraction $\frac{1}{3}$ is shown using each of the three models. • Percents are used ~~in real life~~to solve practical problems involving for ~~taxes~~, sales, data description, and data comparison. • The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where a and b are integers and b does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of rational numbers are $\sqrt{25}$, $\frac{1}{4}$, 82, 75%, $4.\overline{59}$.		• Compare two fractions (proper, improper, and mixed numbers) with denominators of 12 or less or factors of 100 using manipulatives, pictorial representations, number lines, and symbols (<, ≤, ≥, >, =). (b) • Compare two percents using pictorial representations and symbols (<, ≤, ≥, >, =). (b) • Order no more than ~~3~~ four fractions, decimals, and percents (decimals through thousandths, fractions with denominators of 12 or less), in ascending or descending order. Fractions may be expressed as proper, improper, or mixed numbers. (b)

6.2 The student will

a) ~~investigate and describe fractions, decimals and percents as ratios~~ [Moved to EKS] represent and determine equivalencies among fractions, decimals, and percents;* and

~~b) identify a given fraction, decimal or percent from a representation;~~ [Included in 6.2a]

~~c) demonstrate equivalent relationships among fractions, decimals, and percents; and~~

b) compare and order ~~fractions, decimals, and percents~~positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A proper fraction is a fraction whose numerator is less than the denominator. A proper fraction is a fraction that is always less than one. • An improper fraction is a fraction whose numerator is greater than or equal to the denominator. An improper fraction is a fraction that is equal to or greater than one. • Fractions, decimals and percents are equivalent forms representing a given number. Any number that can be written as a fraction can be expressed as a terminating or repeating decimal or a percent. [Moved from middle column] ~~• The decimal point is a symbol that separates the whole number part from the fractional part of a number.~~ ~~• The decimal point separates the whole number amount from the part of a number that is less than one.~~ • ~~The symbol • can be used in Grade 6 in place of "x" to indicate multiplication.~~[Moved to 6.4] • Strategies using 0, $\frac{1}{2}$ and 1 as benchmarks can be used to compare fractions.		

6.2 The student will

a) ~~investigate and describe fractions, decimals and percents as ratios~~ [Moved to EKS] represent and determine equivalencies among fractions, decimals, and percents;* and

~~b) identify a given fraction, decimal or percent from a representation;~~ [Included in 6.2a]

~~c) demonstrate equivalent relationships among fractions, decimals, and percents; and~~

b) compare and order ~~fractions, decimals, and percents~~positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• When comparing two fractions, ~~use~~ $\frac{1}{2}$ may be used as a benchmark. Example: Which is greater, $\frac{4}{7}$ or $\frac{3}{9}$? $\frac{4}{7}$ is greater than $\frac{1}{2}$ because 4, the numerator, represents more than half of 7, the denominator. The denominator tells the number of parts that make the whole. $\frac{3}{9}$ is less than $\frac{1}{2}$ because 3, the numerator, is less than half of 9, the denominator, which tells the number of parts that make the whole. Therefore, $\frac{4}{7} > \frac{3}{9}$. • When comparing two fractions close to 1, use the distance from 1 as your benchmark. Example: Which is greater, $\frac{6}{7}$ or $\frac{8}{9}$? $\frac{6}{7}$ is $\frac{1}{7}$ away from 1 whole. $\frac{8}{8}$ is $\frac{1}{9}$ away from 1 whole. Since, $\frac{1}{7} > \frac{1}{9}$, then $\frac{6}{7}$ is a greater distance away from 1 whole than $\frac{8}{9}$ so $\frac{8}{9} > \frac{6}{7}$.		

6.2 The student will

a) ~~investigate and describe fractions, decimals and percents as ratios~~ [Moved to EKS] represent and determine equivalencies among fractions, decimals, and percents;* and

~~b) identify a given fraction, decimal or percent from a representation;~~ [Included in 6.2a]

~~c) demonstrate equivalent relationships among fractions, decimals, and percents; and~~

b) compare and order ~~fractions, decimals, and percents~~positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Students should have experience with~~Some fractions such as $\frac{1}{8}$, ~~whose~~ have a decimal representation that is a terminating decimal (e. g., $\frac{1}{8} = 0.125$) and ~~with~~ some fractions such as $\frac{2}{9}$, ~~whose~~ have a decimal representation that does not ~~end~~ terminate but continues to repeat (e. g., $\frac{2}{9} = 0.222\ldots$). The repeating decimal can be written with ellipses (three dots) as in 0.222… or denoted with a bar above the digits that repeat as in $0.\overline{2}$.		

6.3 The student will

a) identify and represent integers;
b) order and compare integers; and
c) identify and describe absolute value of integers.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The set of ~~I~~integers ~~are~~ includes the set of whole numbers~~,~~ and their opposites {…-2, -1, 0, 1, 2, …}~~,~~. ~~and zero.~~Zero has no opposite and is an integer that is neither positive nor negative. [Reordered] • Integers are used in practical situations, such as temperature (above/below zero), deposits/withdrawals in a checking account, golf (above/below par), time lines, football yardage, and altitude (above/below sea level). [Moved from middle column] • Integers should be explored by modeling on a number line and using manipulatives, such as two-color counters, drawings, or algebra tiles. • The opposite of a positive number is negative and the opposite of a negative number is positive. • Positive integers are greater than zero. • Negative integers are less than zero. ~~• Zero is an integer that is neither positive nor negative.~~ [Reordered] • A negative integer is always less than a positive integer. • When comparing two negative integers, the negative integer that is closer to zero is greater. • An integer and its opposite are the same distance from zero on a number line. For example, the opposite of 3 is -3 and the opposite of -10 is 10.	~~• What role do negative integers play in practical situations? Some examples of the use of negative integers are found in temperature (below 0), finance (owing money), below sea level. There are many other examples.~~ [Moved to US] ~~• How does the absolute value of an integer compare to the absolute value of its opposite? They are the same because an integer and its opposite are the same distance from zero on a number line.~~ [Included in US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model integers, including models derived from practical situations. (a) • Identify an integer represented by a point on a number line. (a) • Represent integers on a number line. (a) • Order and compare integers using a number line. (b) • Compare integers, using mathematical symbols (<, >, =). (b) • Identify and describe the absolute value of an integer. (c)

6.3 The student will

a) identify and represent integers;
b) order and compare integers; and
c) identify and describe absolute value of integers.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• On a conventional number line, a smaller number is always located to the left of a larger number (e.g.,–7 lies to the left of –3, thus –7 < –3; 5 lies to the left of 8 thus 5 is less than 8) [Reordered] • The absolute value of a number is the distance of a number from zero on the number line regardless of direction. Absolute value is represented using the symbol | | ~~as~~ {e.g., $\|-6\| = 6$ and $\|6\| = 6$). [Includes middle column]. ~~• On a conventional number line, a smaller number is always located to the left of a larger number (e.g., –7 lies to the left of –3, thus –7 < –3; 5 lies to the left of 8 thus 5 is less than 8).~~Reordered] • The absolute value of zero is zero.		

~~6.4 The student will demonstrate multiple representations of multiplication and division of fractions.~~ [Moved to 6.5 EKS]

~~UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Using manipulatives to build conceptual understanding and using pictures and sketches to link concrete examples to the symbolic enhance students' understanding of operations with fractions and help students connect the meaning of whole number computation to fraction computation.~~ • ~~Multiplication and division of fractions can be represented with arrays, paper folding, repeated addition, repeated subtraction, fraction strips, pattern blocks and area models.~~ • ~~When multiplying a whole by a fraction such as $3 \times \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole.~~ • ~~When multiplying a fraction by a fraction such as $\frac{2}{3} \cdot \frac{3}{4}$, we are asking for part of a part.~~ • ~~When multiplying a fraction by a whole number such as $\frac{1}{2} \times 6$, we are trying to find a part of the whole.~~ • ~~For measurement division, the divisor is the number of groups. You want to know how many are in each of those groups. Division of fractions can be explained as how many of a given divisor are needed to equal the given dividend. In other words, for $\frac{1}{4} \div \frac{2}{3}$, the question is, "How many $\frac{2}{3}$ make $\frac{1}{4}$?"~~	• ~~When multiplying fractions, what is the meaning of the operation?~~ ~~When multiplying a whole by a fraction such as $3 \times \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole.~~ • ~~When multiplying a fraction by a fraction such as $\frac{2}{3} \cdot \frac{3}{4}$, we are asking for part of a part.~~ ~~When multiplying a fraction by a whole number such as $\frac{1}{2} \times 6$, we are trying to find a part of the whole.~~ • ~~What does it mean to divide with fractions?~~ ~~For measurement division, the divisor is the number of groups and the quotient will be the number of groups in the dividend. Division of fractions can be explained as how many of a given divisor are needed to equal the given dividend. In other words, for $\frac{1}{4} \div \frac{2}{3}$ the question is, "How many $\frac{2}{3}$ make $\frac{1}{4}$?"~~ ~~For partition division the divisor is the size of the group, so the quotient answers the question, "How much is the whole?" or "How much for one?"~~	**~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~** • ~~Demonstrate multiplication and division of fractions using multiple representations.~~ • ~~Model algorithms for multiplying and dividing with fractions using appropriate representations.~~

~~6.4 The student will demonstrate multiple representations of multiplication and division of fractions.~~ [Moved to 6.5 EKS]

~~UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~For partition division the divisor is the size of the group, so the quotient answers the question, "How much is the whole?" or "How much for one?"~~		

6.4~~5~~ The student will ~~investigate and describe concepts of positive~~recognize and represent patterns with whole number exponents and perfect squares.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The symbol • can be used in Grade 6 in place of "x" to indicate multiplication. [Moved from 6.2 US] • In exponential notation, the base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In 8^3, 8 is the base and 3 is the exponent. (e.g., $8^3 = 8 \cdot 8 \cdot 8$) [Same as middle column] • ~~A power of a number represents repeated multiplication of the number by itself (e.g., $8^3 = 8 \times 8 \times 8$ and is read "8 to the third power").~~ [Combined with bullet above] • Any real number other than zero raised to the zero power is 1. Zero to the zero power (0^0) is undefined. • A ~~Perfect~~ perfect square~~s are the numbers that result from multiplying any~~ is a whole number ~~by itself~~whose square root is an integer. Zero (a whole number) is a perfect square. (e.g., $36 = 6 \times 6 = 6^2$). • Perfect squares ~~can~~ may be represented geometrically as the areas of squares the length of whose sides are whole numbers (e.g., 1×1, 2×2, or 3×3). This can be modeled with grid paper, tiles, geoboards and virtual manipulatives. [Same as middle column] • The examination of patterns in place value of the powers of ten in grade 6 leads to the development of scientific notation in grade 7.	~~• What does exponential form represent? Exponential form is a short way to write repeated multiplication of a common factor such as $5 \times 5 \times 5 \times 5 = 5^4$.~~ ~~• What is the relationship between perfect squares and a geometric square? A perfect square is the area of a geometric square whose side length is a whole number.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize and ~~describe~~ represent patterns with bases and exponents that are ~~natural~~ whole numbers~~, by using a calculator~~. • Recognize and ~~describe~~ represent patterns of perfect squares not to exceed 20^2, by using grid paper, square tiles, tables, and calculators. • Recognize powers of ten with whole number exponents by examining patterns in ~~a~~ place value~~chart: $10^4 = 10{,}000$, $10^3 = 1000$, $10^2 = 100$, $10^1 = 10$, $10^0 = 1$~~.

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense.~~
- ~~Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate.~~
- ~~Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonableness of answers.~~
- ~~Students reinforce skills with operations with whole numbers, fractions, and decimals through problem solving and application activities.~~

Computation and estimation in grades 6-8 focus on developing conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring an understanding as to why procedures work and make sense.

Students develop and refine estimation strategies based on an understanding of number concepts, properties and relationships. The development of problem solving, using operations with integers and rational numbers, builds upon the strategies developed in the elementary grades. Students will reinforce these skills and build on the development of proportional reasoning and more advanced mathematical skills.

Students learn to make sense of the mathematical tools available by making valid judgments of the reasonableness of answers. Students will balance the ability to make precise calculations through the application of the order of operations with knowing when calculations may require estimation to obtain appropriate solutions to practical problems.

6.5~~6~~ The student will

a) multiply and divide fractions and mixed numbers;* ~~and~~

b) ~~estimate solutions and then~~ solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and

c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals. [Moved from 6.7]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A fraction can be expressed in simplest form (simplest equivalent fraction) by dividing the numerator and denominator by their greatest common factor. • When the numerator and denominator have no common factors other than 1, then the fraction is in simplest form. ~~• Simplifying fractions to simplest form assists with uniformity of answers.~~ • Addition and subtraction are inverse operations as are multiplication and division. • Multiplication and division of fractions can be represented with arrays, paper folding, repeated addition, repeated subtraction, fraction strips, fraction rods, pattern blocks, and area models. [Moved from 6.4] • When a number is multiplied by a number greater than 1, the product is greater than the original number. But when a number is multiplied by a number between 0 and 1, the product is less than the original number. For example, $\frac{1}{2} \cdot 4 = 2$. 2 is greater than $\frac{1}{2}$. However, $\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$. $\frac{1}{8}$ is less than $\frac{1}{2}$. [Moved from 6.4 middle column] • It is helpful to use estimation to develop computational strategies. For example, $2\frac{7}{8} \cdot \frac{3}{4}$ is about $\frac{3}{4}$ of 3, so the answer is between 2 and 3.	~~• How are multiplication and division of fractions and multiplication and division of whole numbers alike? Fraction computation can be approached in the same way as whole number computation, applying those concepts to fractional parts.~~ ~~• What is the role of estimation in solving problems? Estimation helps determine the reasonableness of answers.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Demonstrate/model multiplication and division of fractions and mixed numbers using multiple representations. (a) [Moved from 6.4] • Multiply and divide ~~with~~ fractions and mixed numbers. Answers are expressed in simplest form. (a) • Solve single-step and multistep practical problems that involve addition and subtraction with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less. Answers are expressed in simplest form. (b) • Solve single-step and multistep practical problems that involve multiplication and division with fractions and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form. (b) • Solve ~~single step and~~ multistep practical problems involving addition, subtraction, multiplication and division with decimals. (c) [Moved from 6.7] – Divisors are limited to a three digit number, with decimal divisors limited to tenths and hundredths.

6.5~~6~~ The student will

a) multiply and divide fractions and mixed numbers;* ~~and~~

b) ~~estimate solutions and then~~ solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and

c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals. [Moved from 6.7]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• When multiplying a whole number by a fraction such as $3 \cdot \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. • When multiplying a fraction by a fraction such as $\frac{2}{3} \cdot \frac{3}{4}$, we are asking for part of a part. • When multiplying a fraction by a whole number such as $\frac{1}{2} \cdot \cancel{6}$ 6, we are trying to ~~find~~ determine a part of the whole. • Partition division can be thought of as repeated subtraction or equal groups. For partition division, the divisor is the number of groups. You want to know how many are in each of those groups. For example, Diana is painting statues. She has $\frac{7}{8}$ gallon of paint remaining. Each statue requires $\frac{1}{16}$ of a gallon of paint. How many statues can she paint? Determine how many $\frac{1}{16}$ are in $\frac{7}{8}$.[Moved from SOL 6.4 middle column] • In measurement division, the divisor is the size of the group, so the quotient answers the question, "How much is the whole?" or "How much for one?" For example, Jane bought $3\frac{1}{3}$ pounds of apples for $4.50. How much did she pay per pound? In $3\frac{1}{3}$ there are 10 thirds. If $4.50 is the cost of 10 thirds, then one third is $0.45. There are 3 thirds in one pound so 1 pound is 3($0.45) or $1.35. [Moved from SOL 6.4 middle column]		

6.5~~6~~ The student will

a) multiply and divide fractions and mixed numbers;* ~~and~~

b) ~~estimate solutions and then~~ solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and

c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals. [Moved from 6.7]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Students do not need to use the terms partition division or measurement division. • A multistep problem is a problem that requires two or more steps to solve. • Different strategies can be used to estimate the result of computations and judge the reasonableness of the result. For example: What is an approximate answer for 2.19 ÷ 0.8? The answer is around 2 because 2.19 ÷ 0.8 is about 2 ÷ 1 = 2. [Moved from 6.7] • Understanding the placement of the decimal point is very important when determining quotients of decimals. Examining patterns with successive decimals provides meaning, such as dividing the dividend by 6, by 0.6, and by 0.06. [Moved from 6.7] • Solving multistep problems in the context of practical situations enhances interconnectedness and proficiency with estimation strategies. [Moved from 6.7] • Examples of practical situations solved by using estimation strategies include shopping for groceries, buying school supplies, budgeting an allowance, and sharing the cost of a pizza or the prize money from a contest. [Moved from 6.7]		

~~6.7 The student will solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of decimals.~~ [Moved to 6.5c]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Different strategies can be used to estimate the result of computations and judge the reasonableness of the result. For example: What is an approximate answer for 2.19 ÷ 0.8? The answer is around 2 because 2 ÷ 1 = 2.~~ • ~~Understanding the placement of the decimal point is very important when finding quotients of decimals. Examining patterns with successive decimals provides meaning, such as dividing the dividend by 6, by 0.6, by 0.06, and by 0.006.~~ • ~~Solving multistep problems in the context of real-life situations enhances interconnectedness and proficiency with estimation strategies.~~ • ~~Examples of practical situations solved by using estimation strategies include shopping for groceries, buying school supplies, budgeting an allowance, deciding what time to leave for school or the movies, and sharing a pizza or the prize money from a contest.~~	• ~~What is the role of estimation in solving problems? Estimation gives a reasonable solution to a problem when an exact answer is not required. If an exact answer is required, estimation allows you to know if the calculated answer is reasonable.~~	**~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~** • ~~Solve single-step and multistep practical problems involving addition, subtraction, multiplication and division with decimals expressed to thousandths with no more than two operations.~~

6.6 The student will

a) add, subtract, multiply, and divide integers;*and[Moved from 7.3]

b) solve practical problems involving operations with integers. [Moved from 7.3]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Knowledge for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The set of integers is the set of whole numbers and their opposites (e.g., …-3, -2, -1, 0, 1, 2, 3…). Zero has no opposite and is neither positive nor negative. [Moved from 7.3] • Integers are used in practical situations, such as temperature changes (above/below zero), balance in a checking account (deposits/withdrawals), golf, time lines, football yardage, and changes in altitude (above/below sea level). [Moved from 7.3] • Concrete experiences in formulating rules for adding, subtracting, multiplying, and dividing integers should be explored by examining patterns using calculators, along a number line, and using manipulatives, such as two-color counters, drawings, or by using algebra tiles. [Moved from 7.3] • Sums, differences, products and quotients of integers are either positive, negative, or zero. This can be demonstrated through the use of patterns and models.	• ~~The sums, differences, products and quotients of integers are either positive, zero, or negative. How can this be demonstrated? This can be demonstrated through the use of patterns and models.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model addition, subtraction, multiplication and division of integers using pictorial representations or concrete manipulatives. (a) • Add, subtract, multiply, and divide integers. (a) • Solve practical problems involving addition, subtraction, multiplication, and division with integers. (b)

6.7~~8~~ The student will ~~evaluate whole number~~simplify numerical expressions involving integers, using the order of operations.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The order of operations is a convention that defines the computation order to follow in simplifying an expression. Having an established convention ensures that there is only one correct result when simplifying an expression. [Moved from middle column] • The order of operations is as follows: – First, complete all operations within grouping symbols.[1] If there are grouping symbols within other grouping symbols, do the innermost operation first. – Second, evaluate all exponential expressions. – Third, multiply and/or divide in order from left to right. – Fourth, add and/or subtract in order from left to right. [1]Parentheses ( ), brackets [ ], braces { }, absolute value | | (e.g., $\|3(-5+2)\|$ – 7), and the division bar ~~as in~~ (e.g., $\frac{3+4}{5+6}$) should be treated as grouping symbols. • Expressions are simplified using the order of operations and applying the properties of real numbers. – The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 5.6 + -4 = -4 + 5.6). – The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 5.6 · -4 = -4 · 5.6). – The associative property of addition states that regrouping the addends does not change the sum [e.g., 5.6 + (-4 + 3) = (5.6 + -4) + 3]. – The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5.6 · (-4 · 3) = (5.6 · -4) · 3].	~~• What is the significance of the order of operations? The order of operations prescribes the order to use to simplify expressions containing more than one operation. It ensures that there is only one correct answer.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use the order of operations and apply the properties of real numbers to ~~S~~simplify numerical expressions involving integers~~by using the order of operations in a demonstrated step-by-step approach~~. The expressions should ~~be limited to positive values and~~ not include braces { }, but may use brackets [ ] or absolute value | |. Exponents are limited to 1, 2, and 3 and bases are limited to whole numbers. • ~~Find the value of numerical expressions, using order of operations, mental mathematics, and appropriate tools. Exponents are limited to positive values.~~ [Combined with previous bullet]

6.7~~8~~ The student will ~~evaluate whole number~~simplify numerical expressions involving integers, using the order of operations.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– Subtraction and division are neither commutative nor associative. – The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true. Distributive property examples: • $-5.2 \cdot (3 + 7) = (-5.2 \cdot 3) + (-5.2 \cdot 7)$, or • $-5.2 \cdot (3 - 7) = (-5.2 \cdot 3) - (-5.2 \cdot 7)$ Converse of the distributive property examples: • $(-5.2 \cdot 3) + (-5.2 \cdot 7) = -5.2 \cdot (3 + 7)$, or • $(-5.2 \cdot 3) - (-5.2 \cdot 7) = -5.2 \cdot (3 - 7)$ – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division. – The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real number (e.g., $\frac{5}{3} + 0 = \frac{5}{3}$). – The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number (e.g., $\sqrt{8} \cdot 1 = \sqrt{8}$). – Inverses are numbers that combine with other numbers and result in identity elements [e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$]. – The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., $5.6 + (-5.6) = 0$).		

6.7~~8~~ The student will ~~evaluate whole number~~simplify numerical expressions involving integers, using the order of operations.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., $-4 \cdot -\frac{1}{4} = 1$). – Zero has no multiplicative inverse. – The multiplicative property of zero states that the product of any real number and zero is zero (e.g., $7 \cdot 0 = 0$ and $0 \cdot 7 = 0$). – Division by zero is not a possible mathematical operation. It is undefined. – The substitution property of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality. (e.g., if $x = -5 + 8$ then $x = 3$. Thus, 3 was substituted for -5 + 8 since they are equivalent). • The power of a number represents repeated multiplication of the number (e.g., $8^3 = 8 \cdot 8 \cdot 8$). The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In the example, 8 is the base, and 3 is the exponent. • Any number, except 0, raised to the zero power is 1. Zero to the zero power (0^0) is undefined.		

~~Measurement geometry In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students develop the measurement skills that provide a natural context and connection among many mathematics concepts. Estimation skills are developed in determining length, weight/mass, liquid volume/capacity, and angle measure. Measurement is an essential part of mathematical explorations throughout the school year.~~
- ~~Students continue to focus on experiences in which they measure objects physically and develop a deep understanding of the concepts and processes of measurement. Physical experiences in measuring various objects and quantities promote the long-term retention and understanding of measurement. Actual measurement activities are used to determine length, weight/mass, and liquid volume/capacity.~~
- ~~Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, pyramids, and cones.~~

[Moved from Geometry Strand Introduction and Rewritten]

Measurement and geometry in the middle grades provide a natural context and connection among many mathematical concepts. Students expand informal experiences with geometry and measurement in the elementary grades and develop a solid foundation for further exploration of these concepts in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.

Students develop measurement skills through exploration and estimation. Physical exploration to determine length, weight/mass, liquid volume/capacity, and angle measure are essential to develop a conceptual understanding of measurement. Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, square-based pyramids, and cones.

Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provide experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.

Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, square-based pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.

Students explore and develop an understanding of the Pythagorean Theorem. Understanding how the Pythagorean Theorem can be applied in practical situations has a far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

Level 0: Pre-recognition. Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

Level 2: Analysis. Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)

Level 4: Deduction. Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)

~~6.9 The student will make ballpark comparisons between measurements in the U.S. Customary System of measurement and measurements in the metric system.~~ [Included in 7.3 EKS]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Making sense of various units of measure is an essential life skill, requiring reasonable estimates of what measurements mean, particularly in relation to other units of measure.~~ ~~- 1 inch is about 2.5 centimeters.~~ ~~- 1 foot is about 30 centimeters.~~ ~~- 1 meter is a little longer than a yard, or about 40 inches.~~ ~~- 1 mile is slightly farther than 1.5 kilometers.~~ ~~- 1 kilometer is slightly farther than half a mile.~~ ~~- 1 ounce is about 28 grams.~~ ~~- 1 nickel has the mass of about 5 grams.~~ ~~- 1 kilogram is a little more than 2 pounds.~~ ~~- 1 quart is a little less than 1 liter.~~ ~~- 1 liter is a little more than 1 quart.~~ ~~- Water freezes at 0°C and 32°F.~~ ~~- Water boils at 100°C and 212°F.~~ ~~- Normal body temperature is about 37°C and 98°F.~~ ~~- Room temperature is about 20°C and 70°F.~~ • ~~Mass is the amount of matter in an object. Weight is the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight*, as shown by the questions: "How much does it weigh?" versus "What is its mass?"~~ • ~~The degree of accuracy of measurement required is determined by the situation.~~ • ~~Whether to use an underestimate or an overestimate~~	• ~~What is the difference between weight and mass? Weight and mass are different. Mass is the amount of matter in an object. Weight is the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location.~~ • ~~How do you determine which units to use at different times?~~ ~~Units of measure are determined by the attributes of the object being measured. Measures of length are expressed in linear units, measures of area are expressed in square units, and measures of volume are expressed in cubic units.~~ • ~~Why are there two different measurement systems? Measurement systems are conventions invented by different cultures to meet their needs. The U.S. Customary System is the preferred method in the United States. The metric system is the preferred system worldwide.~~	**~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~** • ~~Estimate the conversion of units of length, weight/mass, volume, and temperature between the U.S. Customary system and the metric system by using ballpark comparisons.~~ ~~Ex: 1 L ≈ 1qt. Ex: 4L ≈ 4 qts.~~ • ~~Estimate measurements by comparing the object to be measured against a benchmark.~~

~~6.9~~ ~~The student will make ballpark comparisons between measurements in the U.S. Customary System of measurement and measurements in the metric system.~~ [Included in 7.3 EKS]

~~UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
~~is determined by the situation.~~ • ~~Physically measuring objects along with using visual and symbolic representations improves student understanding of both the concepts and processes of measurement.~~		

6.8~~10~~ The student will

a) ~~define~~ derive π (pi) ~~as the ratio of the circumference of a circle to its diameter~~;

b) solve problems, including practical problems, involving circumference and area of a circle~~, given the diameter or radius~~; and

c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles~~; and~~.

~~d) describe and determine the volume and surface area of a rectangular prism.~~ [Included in 7.4a]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The value of pi (π) is the ratio of the circumference of a circle to its diameter. Thus, the circumference of a circle is proportional to its diameter. • The calculation of determining area and circumference may vary depending upon the approximation for pi. Common approximations for π include 3.14, $\frac{22}{7}$, or the pi button on a calculator. • Experiences in deriving the formulas for area, perimeter, and volume using manipulatives such as tiles, one-inch cubes, ~~adding machine tape,~~ graph paper, geoboards, or tracing paper, promote an understanding of the formulas and ~~facility in~~ their use.‡ ~~• The perimeter of a polygon is the measure of the distance around the polygon.~~ • Perimeter is the path or distance around any plane figure. The perimeter of a circle is called the circumference.~~Circumference is the distance around or "perimeter" of a circle.~~ • The circumference of a circle is about three times the measure of its diameter. [Moved from middle column] • The circumference of a circle is computed using $C = \pi d$ or $C = 2\pi r$, where d is the diameter and r is the radius of the circle. [Reordered] • The area of a closed curve is the number of nonoverlapping square units required to fill the region enclosed by the curve. • The area of a circle is computed using the formula $A = \pi r^2$, where r is the radius of the circle. [Reordered]	~~• What is the relationship between the circumference and diameter of a circle?~~ ~~The circumference of a circle is about 3 times the measure of the diameter.~~ [Moved to US] ~~• What is the difference between area and perimeter? Perimeter is the distance around the outside of a figure while area is the measure of the amount of space enclosed by the perimeter.~~ [Moved to US] ~~• What is the relationship between area and surface area?~~ ~~Surface area is calculated for a three-dimensional figure. It is the sum of the areas of the two-dimensional surfaces that make up the three-dimensional figure.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Derive an approximation for pi (3.14 or $\frac{22}{7}$) by gathering data and comparing the circumference to the diameter of various circles, using concrete materials or computer models. (a) ~~• Find the circumference of a circle by substituting a value for the diameter or the radius into the formula $C = \pi d$ or $C = 2\pi r$.~~ ~~• Find the area of a circle by using the formula $A = \pi r^2$.~~ • Solve problems, including practical problems involving circumference and area of a circle when given the length of the diameter or radius. (b) • ~~Apply formulas to s~~Solve problems, including practical problems involving area and perimeter of triangles and rectangles.(c) ~~• Solve practical problems involving circumference and area of a circle when given the diameter or radius.~~ ~~• Create and solve problems that involve finding the circumference and area of a circle when given the diameter or radius.~~ [Reordered] • ~~Solve problems that require finding the surface area of a rectangular prism, given a diagram of the prism with~~

6.8~~10~~ The student will

a) ~~define~~ derive π (pi) ~~as the ratio of the circumference of a circle to its diameter~~;

b) solve problems, including practical problems, involving circumference and area of a circle~~, given the diameter or radius~~; and

c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles~~; and~~.

~~d) describe and determine the volume and surface area of a rectangular prism.~~ [Included in 7.4a]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• The area of a closed curve is the number of nonoverlapping square units required to fill the region enclosed by the curve.~~ • The perimeter of a square whose side measures s ~~is~~ can be determined by multiplying 4 ~~times~~by s ($P = 4s$), and its area ~~is~~ can be determined by squaring the length of one side ~~times side~~ ($A = s^2$). • The perimeter of a rectangle ~~is~~ can be determined by computing the sum of twice the length and twice the width [$P = 2l + 2w$, or $P = 2(l + w)$], and its area ~~is~~ can be determined by computing the product of the length and the width ($A = lw$). ~~• The value of pi (π) is the ratio of the circumference of a circle to its diameter.~~ [Reordered] ~~• The ratio of the circumference to the diameter of a circle is a constant value, pi (π), which can be approximated by measuring various sizes of circles.~~ [Reordered and rewritten] ~~• The fractional approximation of pi generally used is $\frac{22}{7}$.~~ [Incorporated in bullet above] ~~• The decimal approximation of pi generally used is 3.14.~~ [Incorporated in bullet above] ~~• The circumference of a circle is computed using $C = \pi d$ or $C = 2\pi r$, where d is the diameter and r is the radius of the circle.~~ [Reordered] ~~• The area of a circle is computed using the formula $A = \pi r^2$, where r is the radius of the circle.~~ [Reordered]		~~the necessary dimensions labeled~~. [Moved to 7.4a] ~~• Solve problems that require finding the volume of a rectangular prism given a diagram of the prism with the necessary dimensions labeled.~~ [Moved to 7.4a]

6.8~~10~~ The student will

a) ~~define~~ derive π (pi) ~~as the ratio of the circumference of a circle to its diameter~~;

b) solve problems, including practical problems, involving circumference and area of a circle~~, given the diameter or radius~~; and

c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles~~; and~~.

~~d) describe and determine the volume and surface area of a rectangular prism.~~ [Included in 7.4a]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• The surface area of a rectangular prism is the sum of the areas of all six faces ($SA = 2lw + 2lh + 2wh$).~~ Moved to 7.4a] ~~• The volume of a rectangular prism is computed by multiplying the area of the base, B, (length x width) by the height of the prism ($V = lwh = Bh$).~~ [Moved to 7.4a] ~~†Revised March 2011~~		

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students expand the informal experiences they have had with geometry in the elementary grades and develop a solid foundation for the exploration of geometry in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.~~
- ~~Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provides experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.~~
- ~~Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.~~
- ~~Students explore and develop an understanding of the Pythagorean Theorem. Mastery of the use of the Pythagorean Theorem has far-reaching impact on subsequent mathematics learning and life experiences.~~

~~The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.~~

- ~~**Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.~~
- ~~**Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)~~
- ~~**Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)~~

- ~~**Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)~~
- ~~**Level 4: Deduction.** Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)~~

[Included in Measurement and Geometry Strand Introduction]

6.9~~11~~ The student will

a) identify the ~~coordinates of a point in a~~components of the coordinate plane; and

b) identify the coordinates of a point and graph ordered pairs in a coordinate plane.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• In a coordinate plane, the coordinates of a point are typically represented by the ordered pair (*x*, *y*), where *x* is the first coordinate and *y* is the second coordinate. ~~However, any letters may be used to label the axes and the corresponding ordered pairs~~. • Any given point is defined by only one ordered pair in the coordinate plane. [Moved from middle column] • The grid lines on a coordinate plane are proportional and perpendicular. • The quadrants of a coordinate plane are the four regions created by the two intersecting perpendicular ~~number~~ lines. Quadrants are named in counterclockwise order. The signs on the ordered pairs for quadrant I are (+,+); for quadrant II, (–,+); for quadrant III, (–, –); and for quadrant IV, (+,–). • In a coordinate plane, the origin is the point at the intersection of the *x*-axis and *y*-axis; the coordinates of this point are (0,0). • The axes of the coordinate plane are the two intersecting perpendicular lines that divide it into its four quadrants. The *x*-axis is the horizontal axis and the *y*-axis is the vertical axis. • For all points on the *x*-axis, the *y*-coordinate is 0. For all points on the *y*-axis, the *x*-coordinate is 0. • The coordinates may be used to name the point. (e.g., the point (2,7)). It is not necessary to say "the point whose coordinates are (2,7)". The first coordinate tells the location or distance of the point to the left or right of the *y*-axis and the second coordinate tells the location or distance of the point above or below the *x*-axis. For example, (2, 7) is 2 units to the right of the *y*-axis and 7 units above the *x*-axis. [Moved from middle column]	• ~~Can any given point be represented by more than one ordered pair? The coordinates of a point define its unique location in a coordinate plane. Any given point is defined by only one ordered pair.~~ [Moved to US] • ~~In naming a point in the plane, does the order of the two coordinates matter? Yes. The first coordinate tells the location of the point to the left or right of the y-axis and the second point tells the location of the point above or below the x-axis. Point (0, 0) is at the origin.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and label the axes, origin, and quadrants of a coordinate plane. (a) • ~~Identify and label the quadrants of a coordinate plane.~~ [Included in bullet above] • Identify the quadrant or the axis on which a point is positioned by examining the coordinates (ordered pair) of the point. (a) • Ordered pairs will be limited to coordinates expressed as integers. (b) • Graph ordered pairs in the four quadrants and on the axes of a coordinate plane. (b) • Identify ordered pairs represented by points in the four quadrants and on the axes of the coordinate plane. (b) • Relate the coordinates of a point to the distance from each axis and relate the coordinates of a single point to another point on the same horizontal or vertical line.[‡] (b) [‡]~~Revised March 2011~~

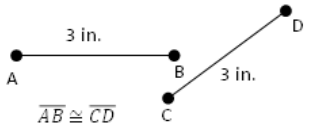
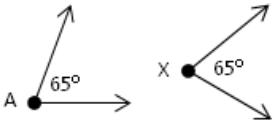
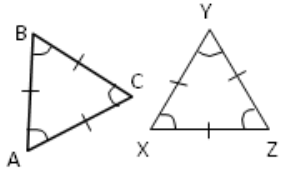
6.9~~11~~ The student will

a) identify the ~~coordinates of a point in a~~components of the coordinate plane; and

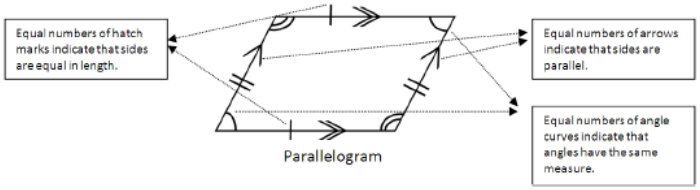
b) identify the coordinates of a point and graph ordered pairs in a coordinate plane.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Coordinates of points having the same *x*-coordinate are located on the same vertical line. For example, (2, 4) and (2, -3) are both 2 units to the right of the *y*-axis and are vertically 7 units from each other. • Coordinates of points having the same *y*-coordinate are located on the same horizontal line. For example, (-4, -2) and (2, -2) are both 2 units below the *x*-axis and are horizontally 6 units from each other.		

6.10~~12~~ The student will determine congruence of segments, angles, and polygons.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The symbol for congruency is ≅ . [Reordered] • Congruent figures have exactly the same size and the same shape. Line segments are congruent if they have the same length. Angles are congruent if they have the same measure. Congruent polygons have an equal number of sides, and all the corresponding sides and angles are congruent. [Same as middle column] For example:  $\overline{AB} \cong \overline{CD}$  ∠A ≅ ∠X  $\Delta ABC \cong \Delta XYZ$ • A regular polygon has congruent sides and congruent interior angles. • The number of lines of symmetry of a regular polygon is equal to the number of sides of the polygon. • A line of symmetry divides a figure into two congruent parts, each of which is the mirror image of the other. Lines of symmetry are not limited to horizontal and vertical lines.	~~• Given two congruent figures, what inferences can be drawn about how the figures are related?~~ ~~The congruent figures will have exactly the same size and shape.~~ [Included in US] ~~• Given two congruent polygons, what inferences can be drawn about how the polygons are related?~~ ~~Corresponding angles of congruent Polygons will have the same measure.~~ ~~Corresponding sides of congruent polygons will have the same measure.~~[Included in US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify regular polygons. • Draw the lines of symmetry for regular polygons. • Determine the congruence of segments, angles, and polygons given their ~~attributes~~properties. [Reordered] • ~~Characterize~~ Determine if polygons ~~as~~ are congruent ~~and~~ or noncongruent according to the measures of their sides and angles. ~~• Determine the congruence of segments, angles, and polygons given their attributes.~~ [Reordered] • Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to ~~find~~ determine the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving practical and mathematical problems.[†] [†]~~Revised March 2011~~

6.10~~12~~ The student will determine congruence of segments, angles, and polygons.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Noncongruent figures may have the same shape but not the same size. • ~~The symbol for congruency is ≅.~~ [Reordered] • ~~The corresponding angles of congruent polygons have the same measure, and the corresponding sides of congruent polygons have the same measure.~~ [Included in bullet above] • Students should be familiar with geometric markings in figures to indicate congruence of sides and angles and to indicate parallel sides. An equal number of hatch (hash) marks indicate that those sides are equal in length. An equal number of arrows indicate that those sides are parallel. An equal number of angle curves indicate that those angles have the same measure. See included diagram.  • The determination of the congruence or noncongruence of two figures can be accomplished by placing one figure on top of the other or by comparing the measurements of all corresponding sides and angles. [Same as middle column] • Construction of congruent line segments, angles, and polygons helps students understand congruency.		

~~6.13~~ **~~The student will describe and identify properties of quadrilaterals.~~** **[Moved to 7.6]**

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~A quadrilateral is a closed planar (two-dimensional) figure with four sides that are line segments.~~ • ~~A parallelogram is a quadrilateral whose opposite sides are parallel and opposite angles are congruent.~~ • ~~A rectangle is a parallelogram with four right angles.~~ • ~~Rectangles have special characteristics (such as diagonals are bisectors) that are true for any rectangle.~~ • ~~To bisect means to divide into two equal parts.~~ • ~~A square is a rectangle with four congruent sides or a rhombus with four right angles.~~ • ~~A rhombus is a parallelogram with four congruent sides.~~ • ~~A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called *bases*, and the nonparallel sides are called *legs*. If the legs have the same length, then the trapezoid is an isosceles trapezoid.~~ • ~~A kite is a quadrilateral with two pairs of adjacent congruent sides. One pair of opposite angles is congruent.~~ • ~~Quadrilaterals can be sorted according to common attributes, using a variety of materials.~~ • ~~Quadrilaterals can be classified by the number of parallel sides: a parallelogram, rectangle, rhombus, and square each have two pairs of parallel sides; a trapezoid has only one pair of parallel sides; other quadrilaterals have no parallel sides.~~ • ~~Quadrilaterals can be classified by the measures of their angles: a rectangle has four 90° angles; a trapezoid may have zero or two 90° angles.~~ • ~~Quadrilaterals can be classified by the number of congruent sides: a rhombus has four congruent sides; a square, which is a rhombus with four right angles, also has four congruent sides; a~~	• ~~Can a figure belong to more than one subset of quadrilaterals?~~ ~~Any figure that has the attributes of more than one subset of quadrilaterals can belong to more than one subset. For example, rectangles have opposite sides of equal length. Squares have all 4 sides of equal length thereby meeting the attributes of both subsets.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Sort and classify polygons as quadrilaterals, parallelograms, rectangles, trapezoids, kites, rhombi, and squares based on their properties. Properties include number of parallel sides, angle measures and number of congruent sides.~~ • ~~Identify the sum of the measures of the angles of a quadrilateral as 360°.~~

~~6.13~~ **~~The student will describe and identify properties of quadrilaterals.~~** **[Moved to 7.6]**

~~UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
~~parallelogram and a rectangle each have two pairs of congruent sides.~~ • ~~A square is a special type of both a rectangle and a rhombus, which are special types of parallelograms, which are special types of quadrilaterals.~~ • ~~The sum of the measures of the angles of a quadrilateral is 360°.~~ • ~~A chart, graphic organizer, or Venn Diagram can be made to organize quadrilaterals according to attributes such as sides and/or angles.~~		

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students develop an awareness of the power of data analysis and probability by building on their natural curiosity about data and making predictions.~~
- ~~Students explore methods of data collection and use technology to represent data with various types of graphs. They learn that different types of graphs represent different types of data effectively. They use measures of center and dispersion to analyze and interpret data.~~
- ~~Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability.~~
- ~~Students explore experimental and theoretical probability through experiments and simulations by using concrete, active learning activities.~~

In the middle grades, students develop an awareness of the power of data analysis and the application of probability through fostering their natural curiosity about data and making predictions.

The exploration of various methods of data collection and representation allows students to become effective at using different types of graphs to represent different types of data. Students use measures of center and dispersion to analyze and interpret data.

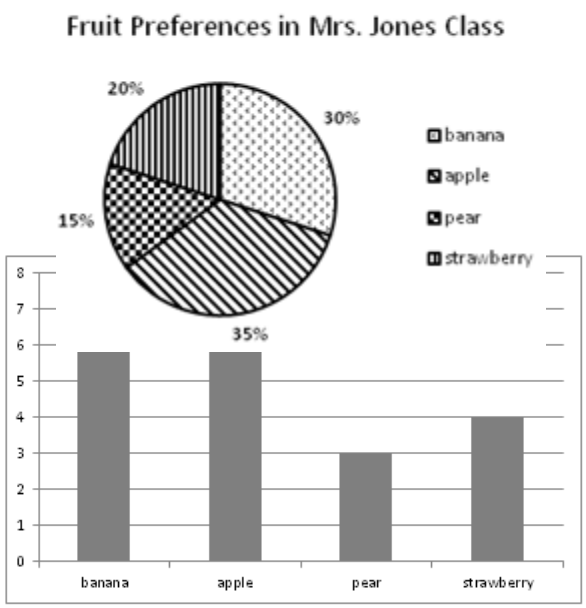
Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability. Through experiments and simulations, students build on their understanding of the Fundamental Counting Principle from elementary mathematics to learn more about probability in the middle grades.

6.11~~14~~ The student, given a ~~problem~~ practical situation, will

a) ~~construct~~ represent data in a circle graphs;

b) ~~draw conclusions and~~ make ~~predictions,~~observations and inferences about data represented in a ~~using~~ circle graphs; and

c) compare ~~and contrast~~circle graphs with the same data represented in bar graphs, picture graphs, and line plots. ~~that present information from the same data set.~~

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Circle graphs are used for data showing a relationship of the parts to the whole. [Moved from middle column] Example: the favorite fruit of 20 students in Mrs. Jones class was recorded in the table. Compare the same data displayed in both a circle graph and a bar graph. banana 6 apple 7 pear 3 strawberry 4  	• ~~What types of data are best presented in a circle graph?~~ ~~Circle graphs are best used for data showing a relationship of the parts to the whole.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect, organize and ~~display~~ represent data in a circle graphs~~ by depicting information as fractional~~. Data should be limited to comparisons that have denominators of 12 or less or those that are factors of 100. (a) • ~~Draw conclusions and m~~Make ~~predictions~~ observations and inferences about data represented in a circle graph. (b) • Compare ~~and contrast~~ data represented in a circle graph with the same data represented in ~~other graphical forms.~~a bar graph, picture graph, and line plot. (c)

6.11~~14~~ The student, given a ~~problem~~ practical situation, will

a) ~~construct~~ represent data in a circle graphs~~;~~

b) ~~draw conclusions and~~ make ~~predictions,~~observations and inferences about data represented in a ~~using~~ circle graphs~~;~~ and

c) compare ~~and contrast~~circle graphs with the same data represented in bar graphs, picture graphs, and line plots. ~~that present information from the same data set.~~

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Circle graphs can represent percent or frequency. • Circle graphs are not useful for data with large numbers of categories. • Students are not expected to construct circle graphs by multiplying the percentage of data in a category by 360° in order to determine the central angle measure. Limiting comparisons to fraction parameters noted will assist students in estimating the central angle measures needed to construct circle graphs. • To collect data for any problem situation, an experiment can be designed, a survey can be conducted, or other data-gathering strategies can be used. The data can be organized, displayed, analyzed, and interpreted to ~~answer~~ solve the problem. • Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. • Different types of graphs ~~are~~ can be used to display ~~different types of~~categorical data. The way data is displayed is often dependent on what someone is trying to communicate. – A line plot is used for categorical and discrete numerical data and is used to show frequency of data on a number line. It is a simple way to organize data. – A ~~B~~bar ~~graphs~~ graph is used for categorical ~~(~~and discrete~~)~~ numerical data (e.g., number of months or number of people with a particular eye color) and is used to show comparisons. – A picture ~~graphs~~ graph is mainly used to show categorical data.~~use continuous data (e.g., temperature and time).~~		

6.11~~14~~ The student, given a ~~problem~~ practical situation, will

a) ~~construct~~ represent data in a circle graphs~~;~~

b) ~~draw conclusions and~~ make ~~predictions,~~observations and inferences about data represented in a ~~using~~ circle graphs~~;~~ and

c) compare ~~and contrast~~circle graphs with the same data represented in bar graphs, picture graphs, and line plots. ~~that present information from the same data set.~~

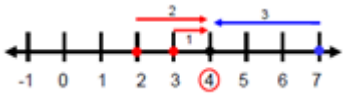
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
Picture graphs are used to show frequency and compare items. However, the use of partial pictures can give misleading information. – A ~~C~~circle graph is used for categorical and discrete numerical data. Circle graphs are used to show a relationship of the parts to a whole. ~~s show a relationship of the parts to a whole.~~ • All graphs must include a title, percent or number labels, ~~and data categories should have labels~~for data categories, and a key. A key is essential to explain how to read the graph. A title is essential to explain what the graph represents. • A scale should be chosen that is appropriate for the data values being represented. ~~• A key is essential to explain how to read the graph.~~ [Included in bullet above] ~~• A title is essential to explain what the graph represents.~~ [Included in bullet above] ~~• Data are analyzed by describing the various features and elements of a graph.~~ • Comparisons, predictions, and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. • The information displayed in different graphs may be examined to determine how data are or are not related, differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inferences).		

6.12~~15~~ The student will

a) ~~describe~~ represent the mean of a data set graphically as the balance point; ~~and~~

b) ~~decide which~~determine an appropriate measure of center ~~is appropriate~~ for a given ~~purpose~~situation; and

c) determine the impact on measures of center when a single value of a data set is added, removed, or changed. [Moved from 5.16]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. • Measures of center are types of averages for a data set. They represent numbers that describe a data set. Mean, median, and mode are measures of center that are useful for describing the average for different situations. – Mean works well for sets of data with no very high or low numbers. – Median is a good choice when data sets have a couple of values much higher or lower than most of the others. – Mode is a good descriptor to use when the set of data has some identical values, ~~or~~ when data is non-numeric (categorical) or when asked to choose the most popular item~~are not conducive to computation of other measures of central tendency, as when working with data in a yes or no survey~~. • Mean can be defined as the point on a number line where the data distribution is balanced. This requires that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances of all the data points below the mean. This is the concept of mean as the balance point. For example: Given the data set: 2, 3, 4, 7 The mean value of 4 can be represented on a number line as the balance point:  	~~• What does the phrase "measure of center" mean? This is a collective term for the 3 types of averages for a set of data – mean, median, and mode.~~ [Included in US] ~~• What is meant by mean as balance point? Mean can be defined as the point on a number line where the data distribution is balanced. This means that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances of all the data points below the mean. This is the concept of mean as the balance point.~~ [Included in US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Find~~ Represent the mean ~~for~~ of a set of data graphically as the balance point represented in a line plot. (a) • ~~Describe~~ Determine the ~~three~~ appropriate measures of center for a given situation ~~in which each would best represent a set of data~~. (b) ~~• Identify and draw a number line that demonstrates the concept of mean as balance point for a set of data.~~ • Determine the impact on measures of center when a single value of a data set is added, removed, or changed. (c) [Moved from 5.16]

6.12~~15~~ The student will

a) ~~describe~~ represent the mean of a data set graphically as the balance point; ~~and~~

b) ~~decide which~~determine an appropriate measure of center ~~is appropriate~~ for a given ~~purpose~~situation; and

c) determine the impact on measures of center when a single value of a data set is added, removed, or changed. [Moved from 5.16]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The mean can also be found by calculating ~~is~~ the numerical average of the data set ~~and is found by adding the numbers in the data set together and dividing the sum by the number of data pieces in the set~~. • In grade 5 mathematics, mean is defined as fair- share. • ~~Mean can be defined as the point on a number line where the data distribution is balanced. This means that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances of all the data points below the mean. This is the concept of mean as the balance point.~~ [Reordered] • Defining mean as the balance point is a prerequisite for understanding standard deviation. • The median is the middle value of a data set in ranked order. If there are an odd number of pieces of data, the median is the middle value in ranked order. If there is an even number of pieces of data, the median is the numerical average of the two middle values. • The mode is the piece of data that occurs most frequently. If no value occurs more often than any other, there is no mode. If there is more than one value that occurs most often, all these most-frequently-occurring values are modes. When there are exactly two modes, the data set is bimodal.		

~~6.16 The student will~~

~~a) compare and contrast dependent and independent events; and~~ [Moved to 8.11a]

~~b) determine probabilities for dependent and independent events.~~ [Included in 8.11b]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~The probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes (sample space).~~ • ~~The probability of an event occurring can be represented as a ratio or the equivalent fraction, decimal, or percent.~~ • ~~The probability of an event occurring is a ratio between 0 and 1.~~ ~~- A probability of 0 means the event will never occur.~~ ~~- A probability of 1 means the event will always occur.~~ • ~~A simple event is one event (e.g., pulling one sock out of a drawer and examining the probability of getting one color).~~ • ~~Events are independent when the outcome of one has no effect on the outcome of the other. For example, rolling a number cube and flipping a coin are independent events.~~ • ~~The probability of two independent events is found by using the following formula:~~ ~~$P(A \text{ and } B) = P(A) \cdot P(B)$~~ ~~Ex: When rolling two number cubes simultaneously, what is the probability of rolling a 3 on one cube and a 4 on the other?~~ ~~$P(3 \text{ and } 4) = P(3) \cdot P(4) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$~~	• ~~How can you determine if a situation involves dependent or independent events?~~ ~~Events are independent when the outcome of one has no effect on the outcome of the other. Events are dependent when the outcome of one event is influenced by the outcome of the other.~~	**~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~** • ~~Determine whether two events are dependent or independent.~~ • ~~Compare and contrast dependent and independent events.~~ • ~~Determine the probability of two dependent events.~~ • ~~Determine the probability of two independent events.~~

~~6.16~~ ~~The student will~~

~~a) compare and contrast dependent and independent events; and~~ [Moved to 8.11a]

~~b) determine probabilities for dependent and independent events.~~ [Included in 8.11b]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
~~• Events are dependent when the outcome of one event is influenced by the outcome of the other. For example, when drawing two marbles from a bag, *not* replacing the first after it is drawn affects the outcome of the second draw.~~ ~~• The probability of two dependent events is found by using the following formula:~~ ~~$P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$~~ ~~Ex: You have a bag holding a blue ball, a red ball, and a yellow ball. What is the probability of picking a blue ball out of the bag on the first pick and then *without* replacing the blue ball in the bag, picking a red ball on the second pick?~~ ~~$P(\text{blue } \textit{and} \text{ red}) = P(\text{blue}) \cdot P(\text{red } \textit{after} \text{ blue}) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$~~		

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students extend their knowledge of patterns developed in the elementary grades and through life experiences by investigating and describing functional relationships.~~
- ~~Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable*, *term*, *coefficient*, *exponent*, *expression*, *equation*, *inequality*, *domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.~~
- ~~Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.~~
- ~~Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.~~

Patterns, functions and algebra become a larger mathematical focus in the middle grades as students extend their knowledge of patterns developed in the elementary grades.

Students make connections between the numeric concepts of ratio and proportion and the algebraic relationships that exist within a set of equivalent ratios. Students use variable expressions to represent proportional relationships between two quantities and begin to connect the concept of a constant of proportionality to rate of change and slope. Representation of relationships between two quantities using tables, graphs, equations, or verbal descriptions allow students to connect their knowledge of patterns to the concept of functional relationships. Graphing linear equations in two variables in the coordinate plane is a focus of the study of functions which continues in high school mathematics.

Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable*, *term*, *coefficient*, *exponent*, *expression*, *equation*, *inequality*, *domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades. Students learn to solve equations by using concrete materials. They expand their skills from one-step to multistep equations and inequalities through their application in practical situations.

~~6.17 The student will identify and extend geometric~~ [Included in AFDA.1 and AII.5] ~~and arithmetic~~ [Included in AI.7] ~~sequences.~~

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Numerical patterns may include linear and exponential growth, perfect squares, triangular and other polygonal numbers, or Fibonacci numbers.~~ • ~~Arithmetic and geometric sequences are types of numerical patterns.~~ • ~~In the numerical pattern of an arithmetic sequence, students must determine the difference, called the *common difference*, between each succeeding number in order to determine what is added to each previous number to obtain the next number. Sample numerical patterns are 6, 9, 12, 15, 18, …; and 5, 7, 9, 11, 13, ….~~ • ~~In geometric number patterns, students must determine what each number is multiplied by to obtain the next number in the geometric sequence. This multiplier is called the *common ratio*. Sample geometric number patterns include 2, 4, 8, 16, 32, …; 1, 5, 25, 125, 625, …; and 80, 20, 5, 1.25, …~~ • ~~Strategies to recognize and describe the differences between terms in numerical patterns include, but are not limited to, examining the change between consecutive terms, and finding common factors. An example is the pattern 1, 2, 4, 7, 11, 16,…~~	• ~~What is the difference between an arithmetic and a geometric sequence?~~ ~~While both are numerical patterns, arithmetic sequences are additive and geometric sequences are multiplicative.~~	**~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~** • ~~Investigate and apply strategies to recognize and describe the change between terms in arithmetic patterns.~~ • ~~Investigate and apply strategies to recognize and describe geometric patterns.~~ • ~~Describe verbally and in writing the relationships between consecutive terms in an arithmetic or geometric sequence.~~ • ~~Extend and apply arithmetic and geometric sequences to similar situations.~~ • ~~Extend arithmetic and geometric sequences in a table by using a given rule or mathematical relationship.~~ • ~~Compare and contrast arithmetic and geometric sequences.~~ • ~~Identify the common difference for a given arithmetic sequence.~~ • ~~Identify the common ratio for a given geometric sequence.~~

6.13 The student will

a) write an equation in the form *y* = *kx*, where *k* is the constant of proportionality, to represent a proportional relationship between two quantities, including practical problems; and

b) make connections among representations of a proportional relationship between two quantities using verbal descriptions, tables, graphs, and equations.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~		ESSENTIAL KNOWLEDGE AND SKILLS
• A ratio is a comparison of any two quantities or sets. • Proportions represent equivalent ratios. Equivalent ratios arise by multiplying each value in a ratio pair by the same positive number. For example, the ratio of 3:2 would be equivalent to the ratio 6:4. • A proportion is an equation that states that two ratios are equal. For example: $\frac{2}{4} = \frac{1}{2}$ represents a proportion. • A proportional relationship can be represented using verbal descriptions, ratio tables, graphs, and equations. Proportional relationships can be described verbally using the phrases "for each," "for every," and "per." • Proportional relationships involve collections of pairs of equivalent ratios that may be graphed in the coordinate plane. The graph of a proportional relationship includes ordered pairs (x, y) that represent pairs of values that may be represented in a table. • Proportional relationships can be represented by the equation $y = kx$, where k represents the constant of proportionality. An example of a proportional relationship: Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges $8 for each medium pizza. This table represents the cost per number of pizza ordered.		**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write an equation in the form $y = kx$ representing a proportional relationship between quantities. The constant of proportionality, k, will be limited to positive integers. (a) • Make connections among multiple representations of the same proportional relationship using verbal descriptions, tables, graphs and equations. (b)

6.13 The student will

a) write an equation in the form *y* = *kx*, where *k* is the constant of proportionality, to represent a proportional relationship between two quantities, including practical problems; and

b) make connections among representations of a proportional relationship between two quantities using verbal descriptions, tables, graphs, and equations.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~		ESSENTIAL KNOWLEDGE AND SKILLS

x number of pizzas	1	2	3	4
y total cost	8	16	24	32

x8 x8

This relationship can be represented by $y = 8x$ where y is the cost and x is the number of medium pizzas ordered. The constant of proportionality is evident with the table as the multiplicative relationship that exists within each x:y ratio. In this example, it is 8.

An example of a non-proportional relationship: Uptown Pizza sells medium pizzas for $7 each but charges a $3 delivery fee per order. This table represents the cost per number of pizzas ordered.

x number of pizzas	1	2	3	4
y total cost	10	17	24	31

The ratios represented in the table are not equivalent. Other non-proportional relationships that are not linear will be studied in later courses.

- Proportional relationships can be expressed using verbal descriptions, tables, graphs, and equations. Example: (verbal description) To make a drink, mix 1 liter of syrup with 3 liters of water. If x represents how many liters of syrup are in the mixture and y represents how many liters of water are in the mixture, this proportional relationship can be represented using a table:

6.13 The student will

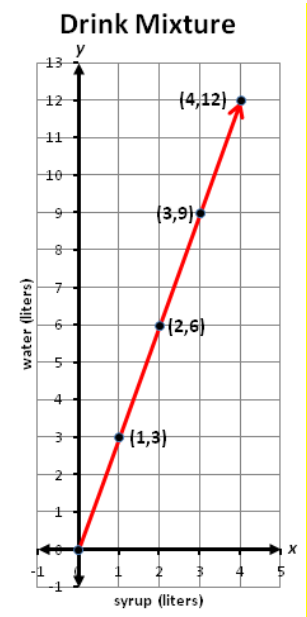
a) write an equation in the form $y = kx$, where k is the constant of proportionality, to represent a proportional relationship between two quantities, including practical problems; and

b) make connections among representations of a proportional relationship between two quantities using verbal descriptions, tables, graphs, and equations.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~		ESSENTIAL KNOWLEDGE AND SKILLS

Syrup (liters) x	1	2	3	4
Water (liters) y	3	6	9	12

This proportional relationship can be represented using the equation $y = 3x$, since for every one liter of syrup in the drink mixture there are 3 liters of water. The constant of proportionality is 3. Additionally, the proportional relationship may be graphed using the ordered pairs in the table.



6.13 The student will

a) write an equation in the form *y* = *kx*, where *k* is the constant of proportionality, to represent a proportional relationship between two quantities, including practical problems; and

b) make connections among representations of a proportional relationship between two quantities using verbal descriptions, tables, graphs, and equations.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~		ESSENTIAL KNOWLEDGE AND SKILLS
• A graph representing a proportional relationship is a straight line that passes through the origin (0, 0). • If the relationship between *x* and *y* is proportional, it means that as *x* changes, *y* also changes by the same percentage.		

6.14~~18~~ The student will solve one-step linear equations in one variable, ~~involving whole number coefficients and positive rational solutions~~including practical problems that require the solution of a one-step linear equation in one variable.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A one-step linear equation ~~is an equation that requires one operation to solve~~may include, but not be limited to equations such as the following:~~.~~ $2x = 5$; $y - 3 = -6$; $\frac{1}{5}x = -3$; $a - (-4) = 11$. • A variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable. • An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an equal sign (e.g., $\frac{3}{4}$, $5x$, $140 - 38.2$, $18 \cdot 21$, $5 + x$.) • An expression that contains a variable is a variable expression. A variable expression is like a phrase: as a phrase does not have a verb, so an expression does not have an equal sign (=). An expression cannot be solved. • A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used when the numbers are unknown. For example, the verbal expression "a number multiplied by 5" could be represented by the variable expression "$n \cdot 5$" or "$5n$". • An algebraic expression is a variable expression that contains at least one variable (e.g., $x - 3$). • A verbal sentence is a complete word statement (e.g., "The sum of a number and two is five." could be represented by "$n + 2 = 5$"). • An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., $2x = 7$). • ~~A mathematical expression contains a variable or a combination of variables, numbers, and/or operation symbols and represents a mathematical relationship. An expression cannot be~~	• ~~When solving an equation, why is it necessary to perform the same operation on both sides of an equal sign?~~ • ~~To maintain equality, an operation performed on one side of an equation must be performed on the other side.~~ [Included in US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Identify examples of the following algebraic vocabulary: equation, variable, expression, term, and coefficient.[Reordered] • Represent and solve ~~a~~ one-step linear equations in one variable, using a variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale. • Apply properties of real numbers and properties of equality to ~~S~~solve a one-step equation ~~by demonstrating the steps algebraically~~. Coefficients are limited to integers and unit fractions. Numeric terms are limited to integers. • ~~Identify and use the following algebraic terms appropriately: *equation*, *variable*, *expression*, *term*, and *coefficient*.~~[Reordered] • Write verbal expressions and sentences as algebraic expressions and equations. • Write algebraic expressions and equations as verbal expressions and sentences. • Represent and solve a practical problem with a one-step linear equation in one variable.

6.14~~18~~ The student will solve one-step linear equations in one variable, ~~involving whole number coefficients and positive rational solutions~~including practical problems that require the solution of a one-step linear equation in one variable.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~solved.~~[Reordered] • A term is a number, variable, product, or quotient in an expression of sums and/or differences. In $7x^2 + 5x - 3$, there are three terms, $7x^2$, $5x$, and 3. • A coefficient is the numerical factor in a term. For example, in the term $3xy^2$, 3 is the coefficient; in the term z, 1 is the coefficient. ~~• Positive rational solutions are limited to whole numbers and positive fractions and decimals.~~ • An equation is a mathematical sentence stating that two expressions are equal. • A variable is a symbol ~~(placeholder)~~ used to represent an ~~unspecified member of a set~~unknown quantity. • The solution to an equation is a value that makes it a true statement. Many equations have one solution and are represented as a point on a number line. • Properties of real numbers and properties of equality can be used to solve equations, justify equation solutions, and express simplification. • The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 5.6 + -4 = -4 + 5.6). • The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 5.6 · -4 = -4 · 5.6). • The associative property of addition states that regrouping the addends does not change the sum [e.g., 5.6 + (-4 + 3) = (5.6 + -4) + 3].		

6.14~~18~~ The student will solve one-step linear equations in one variable, ~~involving whole number coefficients and positive rational solutions~~including practical problems that require the solution of a one-step linear equation in one variable.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5.6 · (-4 · 3) = (5.6 · -4) · 3]. • Subtraction and division are neither commutative nor associative. • The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true. Distributive property examples: • -5.2 · (3 + 7) = (-5.2 · 3) + (-5.2 · 7), or • -5.2 · (3 – 7) = (-5.2 · 3) – (-5.2 · 7) Converse of the distributive property examples: • (-5.2 · 3) + (-5.2 · 7) = -5.2 · (3 + 7), or • (-5.2 · 3) – (-5.2 · 7) = -5.2 · (3 – 7) • The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division. • The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real number (e.g., $\frac{5}{3} + 0 = \frac{5}{3}$). • The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number (e.g., $\sqrt{8} \cdot 1 = \sqrt{8}$). • Inverses are numbers that combine with other numbers and result in identity elements		

6.14~~18~~ The student will solve one-step linear equations in one variable, ~~involving whole number coefficients and positive rational solutions~~including practical problems that require the solution of a one-step linear equation in one variable.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
[e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$]. • The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., 5.6 + (–5.6) = 0). • The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., $-4 \cdot -\frac{1}{4} = 1$). • Zero has no multiplicative inverse. • The multiplicative property of zero states that the product of any real number and zero is zero (e.g., 7 · 0 = 0 and 0 · 7 = 0). • Division by zero is not a possible mathematical operation. It is undefined. • The addition property of equality states that any number must be added to both sides of an equation to maintain equality. • The subtraction property of equality states that any number must be subtracted from both sides of an equation to maintain equality. • The multiplication property of equality states that an equation must be multiplied on both sides by the same number to maintain equality. • The division property of equality states that an equation must be divided on both sides by the same number to maintain equality. • The substitution property of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality. (e.g., if x = -5 + 8 then x = 3. Thus, 3 was substituted for -5 + 8 since they are equivalent).		

6.14~~18~~ The student will solve one-step linear equations in one variable, ~~involving whole number coefficients and positive rational solutions~~including practical problems that require the solution of a one-step linear equation in one variable.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS

~~6.19 The student will investigate and recognize~~

~~a) the identity properties for addition and multiplication;~~

~~b) the multiplicative property of zero; and~~

~~c) the inverse property for multiplication.~~

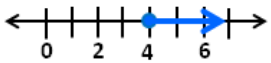
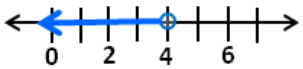
[Included in the EKS for 6.7, 6.14, 6.15]

~~UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~Identity elements are numbers that combine with other numbers without changing the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). There are no identity elements for subtraction and division.~~ • ~~The additive identity property states that the sum of any real number and zero is equal to the given real number (e.g., $5 + 0 = 5$).~~ • ~~The multiplicative identity property states that the product of any real number and one is equal to the given real number (e.g., $8 \cdot 1 = 8$).~~ • ~~Inverses are numbers that combine with other numbers and result in identity elements.~~ • ~~The multiplicative inverse property states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., $4 \cdot \frac{1}{4} = 1$).~~ • ~~Zero has no multiplicative inverse.~~ • ~~The multiplicative property of zero states that the product of any real number and zero is zero.~~ • ~~Division by zero is not a possible arithmetic operation. Division by zero is undefined.~~	~~How are the identity properties for multiplication and addition the same? Different? For each operation the identity elements are numbers that combine with other numbers without changing the value of the other numbers. The additive identity is zero (0). The multiplicative identity is one (1).~~ ~~What is the result of multiplying any real number by zero?~~ ~~The product is always zero.~~ • ~~Do all real numbers have a multiplicative inverse? No. Zero has no multiplicative inverse because there is no real number that can be multiplied by zero resulting in a product of one.~~	**~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~** ~~Identify a real number equation that represents each property of operations with real numbers, when given several real number equations.~~ ~~Test the validity of properties by using examples of the properties of operations on real numbers.~~ ~~Identify the property of operations with real numbers that is illustrated by a real number equation.~~ ~~NOTE: The commutative, associative and distributive properties are taught in previous grades.~~

6.15~~20~~ The student will

a) represent a practical situation with a~~graph inequalities~~ linear inequality in one variable; and

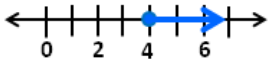
b) solve one-step linear inequalities in one variable and graph the solution on a number line.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The solution set to an inequality is the set of all numbers that make the inequality true. [Reordered] • Inequalities can represent practical situations. For example, Jaxon works at least 4 hours per week mowing lawns. Write an inequality representing this situation and graph the solution. $x \geq 4$ or $4 \leq x$.  Would Jaxon ever work 3 hours in a week? 6 hours? • The variable in an inequality may represent values that are limited by the context of the problem or situation. For example, if the variable represents all children in a classroom who are taller than 36 inches, the variable will be limited to have a minimum and maximum value based on the heights of the children. • Inequalities using the < or > symbols are represented on a number line with an open circle on the number and a shaded line over the solution set. For example: When graphing $x < 4$, use an open circle above the 4 to indicate that the 4 is not included.  ~~• When graphing x ≤ 4 fill in the circle above the 4 to indicate that the 4 is included.~~ [Included in bullet below] • Inequalities using the $\leq$ *or* $\geq$ symbols are represented on a number line with a closed circle on the number and shaded line in the direction of the solution set.	~~• In an inequality, does the order of the elements matter?~~ ~~Yes, the order does matter. For example, x > 5 is not the same relationship as 5 > x. However, x > 5 is the same relationship as 5 < x.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Represent a practical situation with a one-variable linear inequality. (a) • Apply properties of real numbers and properties of inequality to solve ~~Given~~ a ~~simple~~ one-step linear inequality in one variable ~~with integers~~, and graph the ~~relationship~~ solution on a number line. (b) • Given the graph of an ~~simple~~ linear inequality with integers, represent the inequality two different ways using symbols (<, >, ≤, ≥). (b) • Identify a numerical value(s) that satisfies an inequality. (a, b)

6.15~~20~~ The student will

a) represent a practical situation with a~~graph inequalities~~ linear inequality in one variable; and

b) solve one-step linear inequalities in one variable and graph the solution on a number line.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
For example: When graphing $x \geq 4$ fill in the circle above the 4 to indicate that the 4 is included.  • ~~The solution set to an inequality is the set of all numbers that make the inequality true.~~ [Reordered] • It is important for students to see inequalities written with the variable before the inequality symbol and after. For example ~~x > -6 and 7 > y.~~ $x > 5$ is not the same relationship as $5 > x$. However, $x > 5$ is the same relationship as $5 < x$. [Moved from middle column] • A one-step linear inequality may include, but not be limited to inequalities such as the following: $2x > 5$; $y - 3 \leq -6$; $\frac{1}{5}x < -3$; $a - (-4) \geq 11$. • Properties of real numbers and properties of inequality can be used to solve inequalities, justify solutions, and express simplification. – The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 5.6 + -4 = -4 + 5.6). – The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 5.6 · -4 = -4 · 5.6). – The associative property of addition states that regrouping the addends does not change the sum [e.g., 5.6 + (-4 + 3) = (5.6 + -4) + 3]. – The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5.6 · (-4 · 3) = (5.6 · -4) · 3].		

6.15~~20~~ The student will

a) represent a practical situation with a~~graph inequalities~~ linear inequality in one variable; and

b) solve one-step linear inequalities in one variable and graph the solution on a number line.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– Subtraction and division are neither commutative nor associative. – The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true. Distributive property examples: • $-5.2 \cdot (3 + 7) = (-5.2 \cdot 3) + (-5.2 \cdot 7)$, or • $-5.2 \cdot (3 - 7) = (-5.2 \cdot 3) - (-5.2 \cdot 7)$ Converse of the distributive property examples: • $(-5.2 \cdot 3) + (-5.2 \cdot 7) = -5.2 \cdot (3 + 7)$, or • $(-5.2 \cdot 3) - (-5.2 \cdot 7) = -5.2 \cdot (3 - 7)$ – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division. – The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real number (e.g., $\frac{5}{3} + 0 = \frac{5}{3}$). – The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number (e.g., $\sqrt{8} \cdot 1 = \sqrt{8}$). – Inverses are numbers that combine with other numbers and result in identity elements [e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$].		

6.15~~20~~ The student will

a) represent a practical situation with a~~graph inequalities~~ linear inequality in one variable; and

b) solve one-step linear inequalities in one variable and graph the solution on a number line.

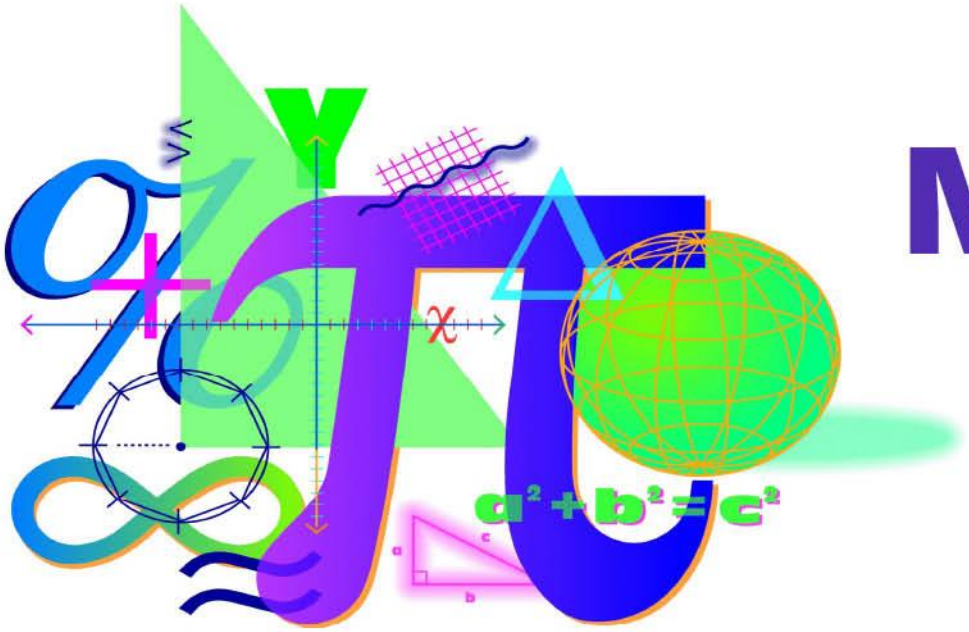
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., 5.6 + (–5.6) = 0). – The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., $-4 \cdot -\frac{1}{4} = 1$). – Zero has no multiplicative inverse. – The multiplicative property of zero states that the product of any real number and zero is zero (e.g., $7 \cdot 0 = 0$ and $0 \cdot 7 = 0$). – Division by zero is not a possible mathematical operation. It is undefined. – The addition property of inequality states that any number must be added to both sides of an inequality to maintain the inequality. – The subtraction property of inequality states that any number must be subtracted from both sides of an inequality to maintain inequality. – The multiplication property of inequality states that if an inequality is multiplied on both sides by the same positive number the inequality symbol remains the same. But if an inequality is multiplied on both sides by the same negative number, the inequality symbol reverses. For example, to solve $\frac{1}{3}x > 9$, multiply both sides of the inequality by 3, so $x > 27$. To solve $-\frac{1}{3}x > 9$, multiply both sides of the inequality by -3, so $x < -27$.		

6.15~~20~~ The student will

a) represent a practical situation with a~~graph inequalities~~ linear inequality in one variable; and

b) solve one-step linear inequalities in one variable and graph the solution on a number line.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– The division property of inequality states that if an inequality is divided on both sides by the same positive number the inequality symbol remains the same. But if an inequality is divided on both sides by the same negative number, the inequality symbol reverses. For example, $-4y < 20$. Divide both sides of the inequality by -4. So $y > -5$. – The substitution property of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality. (e.g., if $x = -5 + 8$ then $x = 3$. Thus, 3 was substituted for -5 + 8 since they are equivalent).		



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Grade 7

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students in the middle grades focus on mastering rational numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to most middle school mathematics topics.~~
- ~~Students develop an understanding of integers and rational numbers by using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.~~
- ~~Students develop an understanding of the properties of operations on real numbers through experiences with rational numbers and by applying the order of operations.~~
- ~~Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.~~

Mathematics instruction in grades 6 – 8 continues to focus on the development of number sense, with emphasis on rational and real numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to many middle school mathematics topics.

Students develop an understanding of integers and rational numbers using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation.
Flexible thinking about rational number representations is encouraged when students solve problems.

Students develop an understanding of real numbers and the properties of operations on real numbers through experiences with rational and irrational numbers and apply the order of operations.

Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

7.1 The student will

a) investigate and describe the concept of negative exponents for powers of ten;
b) determine scientific notation for numbers greater than zero;*
c) compare and order ~~fractions, decimals, percents and numbers written in scientific notation~~rational numbers;*
d) determine square roots of perfect squares;* and
e) identify and describe absolute value ~~for~~ of rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Negative exponents for powers of 10 are used to represent numbers between 0 and 1. [Same as middle column] (e.g., $10^{-3} = \frac{1}{10^3} = 0.001$). • Negative exponents for powers of 10 can be investigated through patterns such as: $10^2 = 100$ $10^1 = 10$ $10^0 = 1$ $10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1$ • Percent means "per 100" or how many "out of 100"; percent is another name for hundredths. • A percent is a ratio in which the denominator is 100. A number followed by a percent symbol (%) is equivalent to that number with a denominator of 100 (e.g., $\frac{3}{5} = \frac{60}{100} = 0.60 = 60\%$). • Scientific notation ~~is~~ should be used whenever the situation calls for use of ~~to represent~~ very large or very small numbers. [Moved from middle column] • A number written in scientific notation is the product of two factors — a decimal greater than or equal to 1 but less than 10, and a power of 10 (e.g., $3.1 \times 10^5 = 310{,}000$ and $2.85 \times 10^{-4} = 0.000285$).	~~• When should scientific notation be used? Scientific notation should be used whenever the situation calls for use of very large or very small numbers.~~ [Moved to US] ~~• How are fractions, decimals and percents related? Any rational number can be represented in fraction, decimal and percent form.~~ [Moved to US] ~~• What does a negative exponent mean when the base is 10? A base of 10 raised to a negative exponent represents a number between 0 and 1.~~ [Same as in US] ~~• How is taking a square root different from squaring a number? Squaring a number and taking a square root are inverse operations.~~ [Moved to US] ~~• Why is the absolute value of a number positive? The absolute value of a number represents distance from zero on a number line regardless of direction.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize powers of 10 with negative exponents by examining patterns. (a) • ~~Write~~ Represent a power of 10 with a negative exponent in fraction and decimal form. (b) ~~• Write a number greater than 0 in scientific notation.~~ ~~• Recognize a number greater than 0 in scientific notation.~~ ~~• Compare and determine equivalent relationships between numbers larger than 0 written in scientific notation.~~ • Represent a number in scientific notation and as a fraction, decimal, and/or percent ~~forms~~. (b) • Compare~~,~~ and order ~~, and determine equivalent relationships among~~rational numbers expressed as integers, fractions, decimals, and percents. Fractions may be expressed as proper, improper, or mixed numbers. Decimals are limited to the thousandths place, and percents are limited to the tenths place. Ordering is limited to no more than ~~4~~ five numbers. (c) • Order no more than ~~3~~ four numbers greater than 0 written in scientific notation. (c)

7.1 The student will

a) investigate and describe the concept of negative exponents for powers of ten;
b) determine scientific notation for numbers greater than zero;*
c) compare and order ~~fractions, decimals, percents and numbers written in scientific notation~~rational numbers;*
d) determine square roots of perfect squares;* and
e) identify and describe absolute value ~~for~~ of rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The set of integers includes the set of whole numbers and their opposites, {…-2, -1, 0, 1, 2…}. Zero has no opposite and is neither positive nor negative. • The opposite of a positive number is negative and the opposite of a negative number is positive. • The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of rational numbers are $\sqrt{25}$, $\frac{1}{4}$, -2.3, 82, 75%, $4.\overline{59}$. [Moved from 8.1] • Any rational number can be represented in fraction, decimal, and percent form. [Moved from middle column] • A proper fraction is a fraction whose numerator is less than the denominator. A proper fraction is a fraction that is always less than one. • An improper fraction is a fraction whose numerator is greater than or equal to the denominator. An improper fraction is a fraction that is equal to or greater than one. • Equivalent relationships among fractions, decimals, and percents ~~can~~ may be determined by using ~~manipulatives~~ concrete materials and pictorial representations (e.g., fraction bars, Base-10 blocks, fraction circles, colored counters, cubes, decimal squares, ~~graph paper~~shaded figures, shaded grids, number lines and calculators).	~~Distance is positive.~~ [Moved to US]	• Identify the perfect squares from 0 to 400. (d) [Moved from 8.5a] • Determine the square root of a perfect square less than or equal to 400. (d) • Demonstrate absolute value using a number line. (e) • Determine the absolute value of a rational number. (e) • Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle to solve practical problems.‡ (e) ~~‡Revised March 2011~~

7.1 The student will

a) investigate and describe the concept of negative exponents for powers of ten;
b) determine scientific notation for numbers greater than zero;*
c) compare and order ~~fractions, decimals, percents and numbers written in scientific notation~~rational numbers;*
d) determine square roots of perfect squares;* and
e) identify and describe absolute value ~~for~~ of rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Negative numbers lie to the left of zero and positive numbers lie to the right of zero on a number line. • Smaller numbers always lie to the left of larger numbers on the number line. • A perfect square is a whole number whose square root is an integer. • A square root of a number is a number which, when multiplied by itself, produces the given number (e.g., $\sqrt{121}$ is 11 since 11 ~~·~~ $\times$ ~~·~~ 11 = 121). • The square root of a number can be represented geometrically as the length of a side of ~~the~~ a square. • Squaring a number and taking a square root are inverse operations. [Moved from middle column] • The absolute value of a number is the distance from 0 on the number line regardless of direction. Distance is positive. [Moved from middle column] (e.g., $\left\lvert\frac{-1}{2}\right\rvert = \frac{1}{2}$). • The absolute value of zero is zero.		

~~7.2 The student will describe and represent arithmetic~~ [Included in AI.7]~~and geometric sequences~~ [Included in AFDA.1 and AII.5]~~using variable expressions.~~

~~UNDERSTANDING THE STANDARD~~ ~~**(Background Information for Instructor Use Only)**~~	~~**ESSENTIAL UNDERSTANDINGS**~~	~~**ESSENTIAL KNOWLEDGE AND SKILLS**~~
• ~~In the numeric pattern of an arithmetic sequence, students must determine the difference, called the *common difference*, between each succeeding number in order to determine what is added to each previous number to obtain the next number.~~ • ~~In geometric sequences, students must determine what each number is multiplied by in order to obtain the next number in the geometric sequence. This multiplier is called the *common ratio*. Sample geometric sequences include~~ – ~~2, 4, 8, 16, 32, …; 1, 5, 25, 125, 625, …; and 80, 20, 5, 1.25, ….~~ • ~~A variable expression can be written to express the relationship between two consecutive terms of a sequence~~ – ~~If n represents a number in the sequence 3, 6, 9, 12…, the next term in the sequence can be determined using the variable expression *n* + 3.~~ – ~~If n represents a number in the sequence 1, 5, 25, 125…, the next term in the sequence can be determined by using the variable expression 5*n*.~~	• ~~When are variable expressions used? Variable expressions can express the relationship between two consecutive terms in a sequence.~~	~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~ • ~~Analyze arithmetic and geometric sequences to discover a variety of patterns.~~ • ~~Identify the common difference in an arithmetic sequence.~~ • ~~Identify the common ratio in a geometric sequence.~~ • ~~Given an arithmetic or geometric sequence, write a variable expression to describe the relationship between two consecutive terms in the sequence.~~

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense.~~
- ~~Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate.~~
- ~~Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonableness of answers.~~
- ~~Students reinforce skills with operations with whole numbers, fractions, and decimals through problem solving and application activities.~~

Computation and estimation in grades 6-8 focus on developing conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring an understanding as to why procedures work and make sense.

Students develop and refine estimation strategies based on an understanding of number concepts, properties and relationships. The development of problem solving, using operations with integers and rational numbers, builds upon the strategies developed in the elementary grades. Students will reinforce these skills and build on the development of proportional reasoning and more advanced mathematical skills.

Students learn to make sense of the mathematical tools available by making valid judgments of the reasonableness of answers. Students will balance the ability to make precise calculations through the application of the order of operations with knowing when calculations may require estimation to obtain appropriate solutions to practical problems.

7.2~~3~~ The student will

a) ~~model addition, subtraction, multiplication and division of integers~~ [Moved to 6.6a] simplify numerical expressions involving rational numbers using the order of operations;* [Moved from 8.1a] and

b) ~~add, subtract, multiply, and divide integers~~ [Moved to 6.6b] solve practical problems involving operations with rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The set of integers is the set of whole numbers and their opposites (e.g., … –3, –2, –1, 0, 1, 2, 3, …). • Integers are used in practical situations, such as temperature changes (above/below zero), balance in a checking account (deposits/withdrawals), and changes in altitude (above/below sea level). • ~~Concrete experiences in formulating rules for adding and subtracting integers should be explored by examining patterns using calculators, along a number line and using manipulatives, such as two-color counters, or by using algebra tiles.~~ [Moved to 6.6] • ~~Concrete experiences in formulating rules for multiplying and dividing integers should be explored by examining patterns with calculators, along a number line and using manipulatives, such as two-color counters, or by using algebra tiles.~~[Moved to 6.6] • The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of rational numbers are $\sqrt{25}$, $\frac{1}{4}$, -2.3, 82, 75%, $4.\overline{59}$. [Moved from 8.1] • Some numbers can belong to more than one subset of the real numbers (e.g., 9 is a natural number, a whole number, an integer, and a rational number.)	• ~~The sums, differences, products and quotients of integers are either positive, zero, or negative. How can this be demonstrated?~~ ~~This can be demonstrated through the use of patterns and models.~~[Moved to 6.6]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Model addition, subtraction, multiplication and division of integers using pictorial representations of concrete manipulatives.~~[Moved to 6.6] • ~~Add, subtract, multiply, and divide integers.~~[Moved to 6.6] • Use the order of operations and apply the properties of real numbers to simplify numerical expressions involving ~~addition, subtraction, multiplication and division of integers~~rational numbers expressed as integers, fractions, decimals, and percents Exponents are limited to 1, 2, 3, and 4 and bases are limited to positive integers. Expressions should not include braces { } but may include brackets [ ] and absolute value | |. (a) [Included from 8.1] • Solve practical problems involving addition, subtraction, multiplication, and division with ~~integers~~rational numbers expressed as integers, fractions, decimals, and percents. (b)

7.2~~3~~ The student will

a) ~~model addition, subtraction, multiplication and division of integers~~ [Moved to 6.6a] simplify numerical expressions involving rational numbers using the order of operations;* [Moved from 8.1a] and

b) ~~add, subtract, multiply, and divide integers~~ [Moved to 6.6b] solve practical problems involving operations with rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The order of operations is a convention that defines the computation order to follow in simplifying an expression. Having an established convention ensures that there is only one correct result when simplifying an expression. The order of operations is as follows: – First, complete all operations within grouping symbols[1]. If there are grouping symbols within other grouping symbols, do the innermost operations first. – Second, evaluate all exponential expressions. – Third, multiply and /or divide in order from left to right. – Fourth, add and /or subtract in order from left to right. [1] Parentheses ( ), brackets [ ], braces { }, absolute value $\lvert\ \rvert$ (e.g., $\lvert 3(-5+2)\rvert$ – 7), and the division bar ~~—as in~~ (e.g., $\frac{3+4}{5+6}$) should be treated as grouping symbols. • Expressions are simplified using the order of operations and applying the properties of real numbers. – The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 5.6 + -4 = -4 + 5.6). – The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 5.6 · -4 = -4 · 5.6). – The associative property of addition states that regrouping the addends does not change the sum [e.g., 5.6 + (-4 + 3) = (5.6 + -4) + 3].		

7.2~~3~~ The student will

a) ~~model addition, subtraction, multiplication and division of integers~~ [Moved to 6.6a] simplify numerical expressions involving rational numbers using the order of operations;* [Moved from 8.1a] and

b) ~~add, subtract, multiply, and divide integers~~ [Moved to 6.6b] solve practical problems involving operations with rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5.6 · (-4 · 3) = (5.6 · -4) · 3]. – Subtraction and division are neither commutative nor associative. – The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true . Distributive property examples: • -5.2 · (3 + 7) = (-5.2 · 3) + (-5.2 · 7), or • -5.2 · (3 – 7) = (-5.2 · 3) – (-5.2 · 7) Converse of the distributive property examples: • (-5.2 · 3) + (-5.2 · 7) = -5.2 · (3 + 7)~~)~~, or • (-5.2 · 3) – (-5.2 · 7) = -5.2 · (3 – 7) – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division. – The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real number (e.g., $\frac{5}{3} + 0 = \frac{5}{3}$). – The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number (e.g., $\sqrt{8} \cdot 1 = \sqrt{8}$).		

7.2~~3~~ The student will

a) ~~model addition, subtraction, multiplication and division of integers~~ [Moved to 6.6a] simplify numerical expressions involving rational numbers using the order of operations;* [Moved from 8.1a] and

b) ~~add, subtract, multiply, and divide integers~~ [Moved to 6.6b] solve practical problems involving operations with rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– Inverses are numbers that combine with other numbers and result in identity elements [e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$]. – The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., $5.6 + (-5.6) = 0$). – The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., $-4 \cdot -\frac{1}{4} = 1$). – Zero has no multiplicative inverse. – The multiplicative property of zero states that the product of any real number and zero is zero (e.g., $7 \cdot 0 = 0$ and $0 \cdot 7 = 0$). – Division by zero is not a possible mathematical operation. It is undefined. – The substitution property of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality. (e.g., if $x = -5 + 8$ then $x = 3$. Thus, 3 was substituted for $-5 + 8$ since they are equivalent).		

7.~~3~~4 The student will solve single-step and multistep practical problems, using proportions~~al reasoning~~.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A proportion is a statement of equality between two ratios. • A proportion can be written as $\frac{a}{b} = \frac{c}{d}$, a:b = c:d, or a is to b as c is to d. [Combined bullets] • Equivalent ratios arise by multiplying each value in a ratio pair by the same positive number. For example, the ratio of 3:2 would be equivalent to the ratio 6:4 because each of the values in 3:2 can be multiplied by 2 to get 6:4. • A proportion can be solved by ~~finding~~ determining the product of the means and the product of the extremes. For example, in the proportion a:b = c:d, a and d are the extremes and b and c are the means. If values are substituted for a, b, c, and d such as 5:12 = 10:24, then the product of extremes ($5 \cdot \cancel{\times} 24$) is equal to the product of the means ($12 \cdot \cancel{\times} 10$). • ~~In a proportional situation, quantities increase or decrease together.~~ [Combined with bullet below] • In a proportional ~~situation~~relationship, two quantities increase multiplicatively. One quantity is a constant ~~Both are~~ multiple~~ied by the same factor~~of the other. [Included from US] • A proportion is an equation which states that two ratios are equal. When solving a proportion, the ratios may first be written as fractions. Example: A recipe for oatmeal cookies calls for 2 cups of flour for every 3 cups of oatmeal. How much flour is needed for a larger batch of cookies that uses 9 cups of oatmeal? To solve this problem, the ratio of flour to oatmeal could be written as a fraction in the proportion used to determine the amount of flour needed when 9 cups of oatmeal is used. To use a proportion to solve for the unknown cups of flour needed, solve the proportion: $\frac{2}{3} = \frac{x}{9}$.	• ~~What makes two quantities proportional?~~ ~~Two quantities are proportional when one quantity is a constant multiple of the other.~~[Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write proportions that represent equivalent relationships between two sets or quantities. • Create and use a table of equivalent ratios to determine missing values. • Solve a proportion to find a missing ~~term~~value. • Apply proportions to convert units of measurement within and between the U.S. Customary System and the metric system when given the conversion factor. Calculators may be used. • Apply proportions to solve practical problems, including scale drawings. Scale factors shall have denominators no greater than 12 and decimals no less than tenths**.** Calculators may be used. • Using 10% as a benchmark, compute 5%, 10%, 15%, or 20% of a given whole number. • Using 10% as a benchmark, ~~mentally~~ compute 5%, 10%, 15%, or 20% in a practical situation such as tips, tax and discounts. • Solve problems involving tips, tax, and discounts. Limit problems to only one percent computation per problem.

7.~~3~~4 The student will solve single-step and multistep practical problems, using proportion~~al reasoning~~.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
To use a table of equivalent ratios to find the unknown amount, create the table: flour (cups): 2, 4, ? oatmeal (cups): 3, 6, 9 To complete the table, we must create an equivalent ratio to 2:3, Just as 4:6 is equivalent to 2:3, then 6 cups of flour to 9 cups of oatmeal would create an equivalent ratio. • A proportion can be solved by ~~finding~~ determining equivalent ~~fractions~~ratios. • A rate is a ratio that compares two quantities measured in different units. A unit rate is a rate with a denominator of 1. Examples of rates include miles/hour and revolutions/minute. • Proportions are used in everyday contexts, such as speed, recipe conversions, scale drawings, map reading, reducing and enlarging, comparison shopping, tips, tax, and discounts, and monetary conversions. • A multistep problem is a problem that requires two or more steps to solve. • Proportions can be used to convert length, weight (mass), and volume (capacity) within and between measurement systems. For example: if ~~2~~ 1 inch~~es~~ is about ~~5~~ 2.54 cm, how many inches are in 16 cm? $\frac{1\ inch}{x} = \frac{2.54\ cm}{16\ cm}$ • Examples of conversions may include, but are not limited to: – Length: between feet and miles; miles and kilometers – Weight: between ounces and pounds; pounds and kilograms – Volume: between cups and fluid ounces; gallons and liters		

7.~~3~~4 The student will solve single-step and multistep practical problems, using proportion~~al reasoning~~.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • When converting measurement units in practical situations, the precision of the conversion factor used will be based on the accuracy required within the context of the problem. For example, when converting from miles to kilometers, we may use a conversion factor of 1 mile ≈ 1.6 km or 1 mile ≈ 1.609 km, depending upon the accuracy needed. • Estimation may be used prior to calculating conversions to evaluate the reasonableness of a solution. • A percent is a ~~special~~ ratio in which the denominator is 100. • Proportions can be used to represent percent problems as follows: $\frac{percent}{100} = \frac{part}{whole}$		

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students develop the measurement skills that provide a natural context and connection among many mathematics concepts. Estimation skills are developed in determining length, weight/mass, liquid volume/capacity, and angle measure. Measurement is an essential part of mathematical explorations throughout the school year.~~
- ~~Students continue to focus on experiences in which they measure objects physically and develop a deep understanding of the concepts and processes of measurement. Physical experiences in measuring various objects and quantities promote the long-term retention and understanding of measurement. Actual measurement activities are used to determine length, weight/mass, and liquid volume/capacity.~~
- ~~Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, pyramids, and cones.~~

[Moved from Geometry Strand Introduction and Rewritten]

Measurement and geometry in the middle grades provide a natural context and connection among many mathematical concepts. Students expand informal experiences with geometry and measurement in the elementary grades and develop a solid foundation for further exploration of these concepts in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.

Students develop measurement skills through exploration and estimation. Physical exploration to determine length, weight/mass, liquid volume/capacity, and angle measure are essential to develop a conceptual understanding of measurement. Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, square-based pyramids, and cones.

Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provide experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.

Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, square-based pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.

Students explore and develop an understanding of the Pythagorean Theorem. Understanding how the Pythagorean Theorem can be applied in practical situations has a far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

Level 0: Pre-recognition. Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

Level 2: Analysis. Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)

Level 4: Deduction. Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)

7.4~~5~~ The student will

a) describe and determine the volume and surface area of rectangular prims and cylinders; and

b) solve problems, including practical problems, involving the volume and surface area of rectangular prisms and cylinders.~~; and~~

~~c) describe how changing one measured attribute of a rectangular prism affects its volume and surface area.~~[Included in 8.6b]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• The area of a rectangle is computed by multiplying the lengths of two adjacent sides.~~ ~~• The area of a circle is computed by squaring the radius and multiplying that product by π ($A = \pi r^2$, where π ≈ 3.14 or $\frac{22}{7}$).~~ • A polyhedron is a solid figure whose faces are all polygons. • A rectangular prism is a polyhedron in which all six faces are rectangles. A rectangular prism has 8 vertices and 12 edges. • A cylinder is a solid figure formed by two congruent parallel faces called bases joined by a curved surface. In this grade level, cylinders are limited to right circular cylinders. • A face is any flat surface of a solid figure. A base is a special face of a solid figure • The surface area of a prism is the sum of the areas of all of its faces. • The volume of a three-dimensional figure is a measure of capacity and is measured in cubic units. • Nets are two-dimensional representations of a three-dimensional figure that can be folded into a model of the three-dimensional figure. • A rectangular prism can be represented on a flat surface as a net that contains six rectangles — two that have measures of the length and width of the base, two others that have measures of the length and height, and two others that have measures of the width and height. The surface area of a rectangular prism is the sum of the areas of all six faces ($SA = 2lw + 2lh + 2wh$).	~~• How are volume and surface area related?~~ ~~Volume is a measure of the amount a container holds while surface area is the sum of the areas of the surfaces on the container.~~[Same as US] ~~• How does the volume of a rectangular prism change when one of the attributes is increased?~~ ~~There is a direct relationship between the volume of a rectangular prism increasing when the length of one of the attributes of the prism is changed by a scale factor.~~ [Moved to 8.6]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** ~~• Determine if a practical problem involving a rectangular prism or cylinder represents the application of volume or surface area.~~ [Reordered] • ~~Find~~ Determine the surface area of ~~a~~ rectangular prisms and cylinders using concrete objects, nets, diagrams, and formulas. (a) • Determine the volume of rectangular prisms and cylinders using concrete objects, diagrams, and formulas. (a) • Determine if a practical problem involving a rectangular prism or cylinder represents the application of volume or surface area. (b) [Reordered] • Solve practical problems that require ~~finding~~ determining the surface area of ~~a~~ rectangular prisms and cylinders. (b) ~~• Find the surface area of a cylinder.~~[Combined with another bullet] ~~• Solve practical problems that require finding the surface area of a cylinder.~~[Combined with another bullet] ~~• Find the volume of a rectangular prism.~~[Combined with another bullet]

7.4~~5~~ The student will

a) describe and determine the volume and surface area of rectangular prims and cylinders; and

b) solve problems, including practical problems, involving the volume and surface area of rectangular prisms and cylinders.~~; and~~

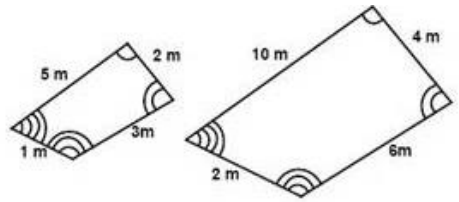
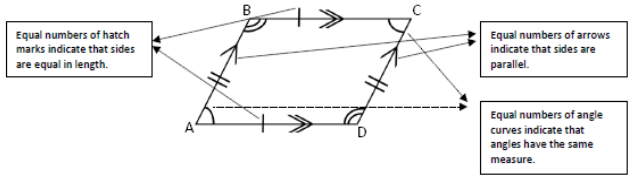
~~c) describe how changing one measured attribute of a rectangular prism affects its volume and surface area.~~[Included in 8.6b]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A cylinder can be represented on a flat surface as a net that contains two circles (the bases ~~for~~ of the cylinder) and one rectangular region (the curved surface of the cylinder) whose length is the circumference of the circular base and whose width is the height of the cylinder. The surface area of the cylinder is the sum of the area of the two circles and the rectangle representing the curved surface ($SA = 2\pi r^2 + 2\pi rh$). • The volume of a rectangular prism is computed by multiplying the area of the base, *B*, (length times width) by the height of the prism ($V = lwh = Bh$). • The volume of a cylinder is computed by multiplying the area of the base, *B*, (πr^2) by the height of the cylinder ($V = \pi r^2 h = Bh$). ~~• There is a direct relationship between changing one measured attribute of a rectangular prism by a scale factor and its volume. For example, doubling the length of a prism will double its volume. This direct relationship does not hold true for surface area.~~[Moved to 8.6]		• Solve practical problems that require ~~finding~~ determining the volume of ~~a~~ rectangular prisms and cylinders. (b). ~~• Find the volume of a cylinder.~~[Combined with another bullet] ~~• Solve practical problems that require finding the volume of a cylinder.~~[Combined with another bullet] ~~• Describe how the volume of a rectangular prism is affected when one measured attribute is multiplied by a scale factor. Problems will be limited to changing attributes by scale factors only.~~ [Included in 8.6] ~~• Describe how the surface area of a rectangular prism is affected when one measured attribute is multiplied by a scale factor. Problems will be limited to changing attributes by scale factors only.~~ [Included in 8.6]

7.5~~6~~ The student will ~~determine whether plane figures – quadrilaterals and triangles – are similar and write proportions to express the relationships between~~solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar ~~figures~~quadrilaterals and triangles.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Two~~ Similar polygons ~~are similar if~~have corresponding ~~(matching)~~sides that are proportional and corresponding interior angles that are congruent~~ and the lengths of corresponding sides are proportional~~. [Included from middle column] • Similarity has practical applications in a variety of areas, including art, architecture, and the sciences. • Similarity does not depend on the position or orientation of the figures. • Congruent polygons have the same size and shape. Corresponding angles and sides are congruent. [Included from middle column] • Congruent polygons are similar polygons for which the ratio of the corresponding sides is 1:1. However, similar polygons are not necessarily congruent. [Included from middle column] • The symbol ~ is used to represent similarity. For example, $\Delta ABC \sim \Delta DEF$. • Similarity statements can be used to determine corresponding parts of similar figures such as: Given: $\Delta ABC \sim \Delta DEF$ $\angle A$ corresponds to $\angle D$ $\overline{AB}$ corresponds to $\overline{DE}$ [Moved from EKS] • A proportion representing corresponding sides of similar figures can be written as $\frac{a}{b} = \frac{c}{d}$. (e.g., Given two similar quadrilaterals with corresponding angles labeled, write a proportion involving corresponding sides.	~~• How do polygons that are similar compare to polygons that are congruent?~~ ~~Congruent polygons have the same size and shape. Similar polygons have the same shape, and corresponding angles between the similar figures are congruent. However, the lengths of the corresponding sides are proportional. All congruent polygons are considered similar with the ratio of the corresponding sides being 1:1.~~[Included in US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify corresponding sides and corresponding ~~and~~ congruent angles of similar ~~figures~~ quadrilaterals and triangles~~using the traditional notation of curved lines for the angles~~. • Given two similar quadrilaterals or triangles, write similarity statements using symbols. [Reordered] • Write proportions to express the relationships between the lengths of corresponding sides of similar ~~figures~~quadrilaterals and triangles. • Solve a proportion to determine a missing side length of similar quadrilaterals or triangles. • Given the angle measure in a quadrilateral or triangle, determine the angle measure of the corresponding angle in a similar quadrilateral or triangle. ~~• Determine if quadrilaterals or triangles are similar by examining congruence of corresponding angles and proportionality of corresponding sides.~~[Included in G.7] ~~• Given two similar figures, write similarity statements using symbols such as $\Delta ABC \sim \Delta DEF$, $\angle A$ corresponds to $\angle D$, and $\overline{AB}$ corresponds to $\overline{DE}$~~ .[Reordered and example moved to US]

7.5~~6~~ The student will ~~determine whether plane figures – quadrilaterals and triangles – are similar and write proportions to express the relationships between~~solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar ~~figures~~quadrilaterals and triangles.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
 $\frac{5}{10} = \frac{2}{4}$ or $\frac{5}{10} = \frac{3}{6}$ or $\frac{1}{2} = \frac{2}{4}$ are some of the ways to express the proportional relationships that exist.) • The traditional notation for marking congruent angles is to use a curve on each angle. Denote which angles are congruent with the same number of curved lines. For example, if $\angle A$ is congruent to $\angle C$, then both angles will be marked with the same number of curved lines.  • Congruent sides are denoted with the same number of hatch (or hash) marks on each congruent side. For example, a side on a polygon with 2 hatch marks is congruent to the side with 2 hatch marks on a congruent polygon or within the same polygon.		

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students expand the informal experiences they have had with geometry in the elementary grades and develop a solid foundation for the exploration of geometry in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.~~

- ~~Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provides experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.~~

- ~~Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.~~

- ~~Students explore and develop an understanding of the Pythagorean Theorem. Mastery of the use of the Pythagorean Theorem has far-reaching impact on subsequent mathematics learning and life experiences.~~

~~The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.~~

- ~~**Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.~~

- ~~**Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)~~

- ~~**Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)~~

- ~~**Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)~~

- ~~**Level 4: Deduction.** Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)~~

[Included in Measurement and Geometry Strand Introduction]

7.6~~7~~ The student will

a) compare and contrast ~~the following~~ quadrilaterals based on their properties~~:~~; and ~~parallelogram, rectangle, square, rhombus, and trapezoid.~~

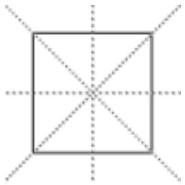
b) solve problems to determine unknown side lengths or angle measures of quadrilaterals.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A polygon is a closed plane figure composed of at least three line segments that do not cross. • A quadrilateral is a ~~closed plane (two-dimensional) figure~~ polygon with four sides ~~that are line segments~~. • A diagonal is a segment in a polygon that connects two vertices but is not a side. • To bisect means to divide into two equal parts. • A line of symmetry divides a figure into two congruent parts each of which is the mirror image of the other. Lines of symmetry are not limited to horizontal and vertical lines. • A parallelogram is a quadrilateral ~~whose~~ with both pairs of opposite sides ~~are~~ parallel ~~and opposite angles are congruent~~. Properties of a parallelogram include the following: – opposite sides are congruent; – opposite angles are congruent; – diagonals bisect each other and one diagonal divides the figure into two congruent triangles. • Parallelograms, with the exception of rectangles and rhombi, have no lines of symmetry. A rectangle has two lines of symmetry. A rhombus has two lines of symmetry. • A rectangle is a ~~parallelogram~~ quadrilateral with four right angles. Properties of a rectangle include the following: – opposite sides are parallel and congruent; – ~~The~~ diagonals ~~of a rectangle~~ are ~~the same length~~congruent and bisect each other.	~~• Why can some quadrilaterals be classified in more than one category?~~ ~~Every quadrilateral in a subset has all of the defining attributes of the subset. For example, if a quadrilateral is a rhombus, it has all the attributes of a rhombus. However, if that rhombus also has the additional property of 4 right angles, then that rhombus is also a square.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast ~~attributes~~ properties of the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid. (a) • Sort and classify polygons as quadrilaterals, parallelograms, rectangles, trapezoids, rhombi, and/or squares based on their properties. Properties include: number of parallel sides, angle measures, number of congruent sides, and the relationship between the diagonals. (a) [Moved from 6.13] • Identify the classification(s) to which a quadrilateral belongs, using deductive reasoning and inference. (a) • Identify the sum of the measures of the interior angles of a quadrilateral as 360° and use that relationship to determine an unknown angle measure in a quadrilateral. (b) [Included from 6.13] • Determine an unknown side length in a quadrilateral. (b)

7.6~~7~~ The student will

a) compare and contrast ~~the following~~ quadrilaterals based on their properties~~:~~; and ~~parallelogram, rectangle, square, rhombus, and trapezoid.~~

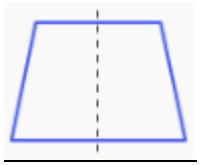
b) solve problems to determine unknown side lengths or angle measures of quadrilaterals.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A square is a ~~rectangle~~ quadrilateral that is a regular polygon with four congruent sides ~~whose diagonals are perpendicular. A square is a rhombus with~~ and four right angles. Properties of a square include the following: – opposite sides are parallel; – diagonals are congruent and bisect each other at right angles. • A rhombus is a ~~parallelogram~~ quadrilateral with four congruent sides ~~whose diagonals bisect each other and intersect at right angles~~. Properties of a rhombus include the following: – all sides are congruent; – opposite sides are parallel; – opposite angles are congruent; – diagonals bisect each other at right angles. • A square has four lines of symmetry. The diagonals of a square coincide with two of the lines of symmetry that can be drawn. Example: Square with lines of symmetry shown:  • A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called bases. The nonparallel sides of a trapezoid are called legs.		

7.6~~7~~ The student will

a) compare and contrast ~~the following~~ quadrilaterals based on their properties~~:~~; and ~~parallelogram, rectangle, square, rhombus, and trapezoid.~~

b) solve problems to determine unknown side lengths or angle measures of quadrilaterals.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• An isosceles trapezoid ~~with congruent, nonparallel sides is called an *isosceles trapezoid*~~has legs of equal length and congruent base angles. An isosceles trapezoid has one line of symmetry. Example: Isosceles trapezoid with line of symmetry shown:  • Quadrilaterals can be sorted according to ~~common attributes~~their properties, using a variety of materials. A chart, graphic organizer, or Venn diagram can be made to organize quadrilaterals according to properties such as sides and/or angles. • ~~A chart, graphic organizer, or Venn diagram can be made to organize quadrilaterals according to attributes such as sides and/or angles.~~[Combined with bullet above] • Quadrilaterals can be classified by the number of parallel sides: parallelogram, rectangle, rhombus, and square each have two pairs of parallel sides; a trapezoid has one pair of parallel sides; other quadrilaterals have no parallel sides. • Quadrilaterals can be classified by the measures of their angles: a rectangle has four 90° angles; a trapezoid may have zero or two 90° angles. • Quadrilaterals can be classified by the number of congruent sides: a rhombus has four congruent sides; a square has four congruent sides; a parallelogram and a rectangle each have two pairs of congruent sides.		

7.6~~7~~ The student will

a) compare and contrast ~~the following~~ quadrilaterals based on their properties~~:~~; and ~~parallelogram, rectangle, square, rhombus, and trapezoid.~~

b) solve problems to determine unknown side lengths or angle measures of quadrilaterals.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A square is a special type of both a rectangle and a rhombus, which are special types of parallelograms, which are special types of quadrilaterals. • Any figure that has the properties of more than one subset of quadrilaterals can belong to more than one subset. For example, rectangles have opposite sides of equal length. Squares have all four sides of equal length thereby satisfying the properties of both subsets. [Moved from middle column] • The sum of the measures of the interior angles of a quadrilateral is 360°.		

7.7~~8~~ The student~~, given a polygon in the coordinate plane,~~ will apply~~represent transformations~~ (translations and reflections of right triangles or rectangles~~, dilations, rotations, and translations) by graphing~~ in the coordinate plane.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A transformation of a figure called the pre-image changes the size, shape, or position of the figure to a new figure called the image. • ~~A rotation of a geometric figure is a turn of the figure around a fixed point. The point may or may not be on the figure. The fixed point is called the *center of rotation*.~~[Included in 8.7] • Translations and reflections do not change the size or shape of a figure (e.g., the preimage and image are congruent figures). Translations and reflections change the position of a figure. [Included from middle column] • A translation ~~of a geometric figure~~ is a ~~slide of the figure~~ transformation in which an image is formed by moving ~~all the~~every point~~s~~ on the ~~figure~~ preimage ~~move~~ the same distance in the same direction. • A reflection is a transformation ~~that~~ in which an image is formed by reflect~~s~~ing ~~a figure~~the preimage ~~across~~ over a line ~~in the plane~~called the line of reflection. All corresponding points in the image and preimage are equidistant from the line of reflection. • ~~A dilation of a geometric figure is a transformation that changes the size of a figure by scale factor to create a similar figure.~~[Included in 8.7] • The image of a polygon is the resulting polygon after the transformation. The preimage is the polygon before the transformation. • A transformation of preimage point A ~~-~~can be denoted as the image A' (read as "A prime").	• ~~How does the transformation of a figure affect the size, shape and position of that figure? Translations, rotations and reflections do not change the size or shape of a figure.~~ [Included in US and 8.7]]~~A dilation of a figure and the original figure are similar.~~ [Moved to 8.7] ~~Reflections, translations and rotations usually change the position of the figure.~~[Included in US and 8.7]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given a preimage in the coordinate plane, ~~I~~identify the coordinates of the image of a right triangle or rectangle that has been translated either vertically, horizontally, or a combination of a vertical and horizontal translation. • ~~Identify the coordinates of the image of a right triangle or rectangle that has been rotated 90° or 180° about the origin.~~[Included in 8.7 and G.3] • Given a preimage in the coordinate plane, ~~I~~identify the coordinates of the image of a right triangle or a rectangle that has been reflected over the *x*- or *y*-axis. • ~~Identify the coordinates of a right triangle or rectangle that has been dilated. The center of the dilation will be the origin.~~[Included in 8.7 and G.3] • Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or rectangle that has been translated and reflected over the *x*- or *y*-axis. • Sketch the image of a right triangle or rectangle that has been translated vertically, ~~or~~ horizontally, or a combination of both. • ~~Sketch the image of a right triangle or rectangle that has been rotated 90° or 180° about the origin.~~[Moved ~~to~~Included in 8.7 and G.3] • Sketch the image of a right triangle or rectangle that has been reflected over the *x*- or *y*-axis.

7.7~~8~~ The student~~, given a polygon in the coordinate plane,~~ will apply~~represent transformations~~ (translations and reflections of right triangles or rectangles~~, dilations, rotations, and translations) by graphing~~ in the coordinate plane.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
		• Sketch the image of a right triangle or rectangle that has been translated and reflected over the *x*- and *y*-axis. • ~~Sketch the image of a dilation of a right triangle or rectangle limited to a scale factor of~~ $\cancel{\frac{1}{4}, \frac{1}{2}, 2, 3 \text{ or } 4.}$ [Moved to 8.7]

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students develop an awareness of the power of data analysis and probability by building on their natural curiosity about data and making predictions.~~
- ~~Students explore methods of data collection and use technology to represent data with various types of graphs. They learn that different types of graphs represent different types of data effectively. They use measures of center and dispersion to analyze and interpret data.~~
- ~~Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability.~~
- ~~Students explore experimental and theoretical probability through experiments and simulations by using concrete, active learning activities.~~

In the middle grades, students develop an awareness of the power of data analysis and the application of probability through fostering their natural curiosity about data and making predictions.

The exploration of various methods of data collection and representation allows students to become effective at using different types of graphs to represent different types of data. Students use measures of center and dispersion to analyze and interpret data.

Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability. Through experiments and simulations, students build on their understanding of the Fundamental Counting Principle from elementary mathematics to learn more about probability in the middle grades.

7.8~~9~~ The student will

a) determine the theoretical and experimental probabilities of an event; and

b) investigate and describe the difference between the experimental probability and theoretical probability of an event.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes (sample space). [Moved from 6.16]. • The probability of an event occurring can be represented as a ratio or the equivalent fraction, decimal, or percent. [Moved from 6.16] • The probability of an event occurring is a ratio between 0 and 1. [Moved from 6.16] – A probability of 0 means the event will never occur. – A probability of 1 means the event will always occur. • The ~~T~~theoretical probability of an event is the expected probability and can be ~~found~~ determined with a ~~formula~~ratio. • Theoretical probability of an event = $\frac{\text{number of possible favorable outcomes}}{\text{total number of possible outcomes}}$ • The experimental probability of an event is determined by carrying out a simulation or an experiment. • The experimental probability of an event = $\frac{\text{number of times desired outcomes occur}}{\text{number of trials in the experiment}}$ • In experimental probability, as the number of trials increases, the experimental probability gets closer to the theoretical probability (Law of Large Numbers).	~~• What is the difference between the theoretical and experimental probability of an event? Theoretical probability of an event is the expected probability and can be found with a formula. The experimental probability of an event is determined by carrying out a simulation or an experiment. In experimental probability, as the number of trials increases, the experimental probability gets closer to the theoretical probability.~~ [Included in US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the theoretical probability of an event. (a) • Determine the experimental probability of an event. (a) • Describe changes in the experimental probability as the number of trials increases. (b) • Investigate and describe the difference between the probability of an event found through experiment or simulation versus the theoretical probability of that same event. (b)

~~7.10 The student will determine the probability of compound events, using the Fundamental (Basic) Counting Principle.~~ [Moved to 5.15]

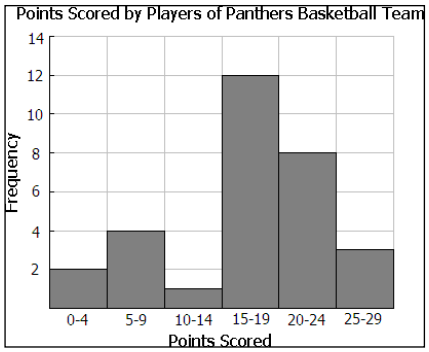
~~UNDERSTANDING THE STANDARD (Teacher Notes)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~The Fundamental (Basic) Counting Principle is a computational procedure to determine the number of possible outcomes of several events. It is the product of the number of outcomes for each event that can be chosen individually (e.g., the possible outcomes or outfits of four shirts, two pants, and three shoes is 4 · 2 · 3 or 24).~~ • ~~Tree diagrams are used to illustrate possible outcomes of events. They can be used to support the Fundamental (Basic) Counting Principle.~~ • ~~A compound event combines two or more simple events. For example, a bag contains 4 red, 3 green and 2 blue marbles. What is the probability of selecting a green and then a blue marble?~~	• ~~What is the Fundamental (Basic) Counting Principle?~~ ~~The Fundamental (Basic) Counting Principle is a computational procedure used to determine the number of possible outcomes of several events.~~ • ~~What is the role of the Fundamental (Basic) Counting Principle in determining the probability of compound events?~~ ~~The Fundamental (Basic) Counting Principle is used to determine the number of outcomes of several events. It is the product of the number of outcomes for each event that can be chosen individually.~~	~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~ • ~~Compute the number of possible outcomes by using the Fundamental (Basic) Counting Principle.~~ • ~~Determine the probability of a compound event containing no more than 2 events.~~

7.9~~11~~ The student, given data in a practical situation, will

a) ~~construct and analyze~~ represent data in a histograms~~; and~~

b) make observations and inferences about data represented in a histogram; and

c) compare ~~and contrast~~ histograms with ~~other types of graphs presenting information from~~ the same data ~~set~~represented in stem-and-leaf plots, line plots, and circle graphs.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A histogram is a graph that provides a visual interpretation of numerical data by indicating the number of data points that lie within a range of values, called a class or a bin. The frequency of the data that falls in each class is depicted by the use of a bar. • All graphs ~~tell a story and~~must include a title and labels that describe the data. • Numerical data that can be characterized using consecutive intervals are best displayed in a histogram. [Moved from middle column] • A histogram is a form of bar graph in which the categories are consecutive and equal intervals. The length or height of each bar is determined by the number of data elements (frequency) falling into a particular interval.  • A frequency distribution shows how often an item, a number, or range of numbers occurs. It can be used to construct a histogram.	~~• What types of data are most appropriate to display in a histogram?~~ ~~Numerical data that can be characterized using consecutive intervals are best displayed in a histogram.~~[Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect, ~~analyze~~organize, ~~display,~~ and ~~interpret~~ represent ~~a~~ data in ~~set using~~ a histogram~~s~~. Data should be limited to 20 items or less.~~For collection and display of raw data, limit the data to 20 items.~~ (a) ~~Determine patterns and relationships within data sets (e.g., trends).~~ • Make observations and inferences~~, conjectures, and predictions based on analysis of a set of~~about data represented in a histogram. (b) • Describe the distribution of data represented in a histogram as skewed right, skewed left, or symmetric. (b) • Compare ~~and contrast~~data represented in histograms with the same data represented in line plots, circle graphs, and stem-and-leaf plots~~ presenting information from the same data set~~. (c)

7.9~~11~~ The student, given data in a practical situation, will

a) ~~construct and analyze~~ represent data in a histograms~~; and~~

b) make observations and inferences about data represented in a histogram; and

c) compare ~~and contrast~~ histograms with ~~other types of graphs presenting information from~~ the same data ~~set~~represented in stem-and-leaf plots, line plots, and circle graphs.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS

STUDENTS WHO READ GARFIELD

Age Group	Tally	Frequency	Cumulative Frequency	
7-10	𝍸 II	7	7	
11-14	𝍸 II	7	14	← 7 + 7
15-18	III	3	17	← 14 + 3
19-22	III	3	20	← 17 + 3

To construct a histogram:

- Organize collected data into a table. Create one column for data range categories, divided into equal intervals that will include all of your data (for example, 0-10, 11-20, 21-30), and another column for frequency.
- Create a graph. Mark the data range intervals on the *x*-axis (horizontal axis) with no space between the categories. Mark frequency on the *y*-axis (vertical axis), also in equal intervals.
- Plot your data. For each data range category, draw a horizontal line at the appropriate frequency or marker. Then, create a vertical bar for that category reaching up to the marked frequency. Do this for each data range category.

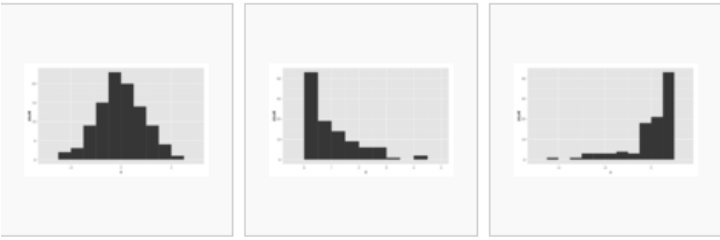
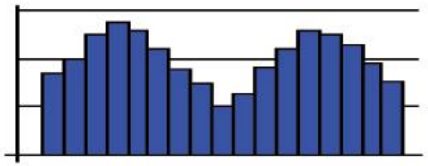
- Histograms give a sense of the density of the underlying distribution of the data.
- Comparisons, predictions and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions.

7.9~~11~~ The student, given data in a practical situation, will

a) ~~construct and analyze~~ represent data in a histograms~~; and~~

b) make observations and inferences about data represented in a histogram; and

c) compare ~~and contrast~~ histograms with ~~other types of graphs presenting information from~~ the same data ~~set~~represented in stem-and-leaf plots, line plots, and circle graphs.

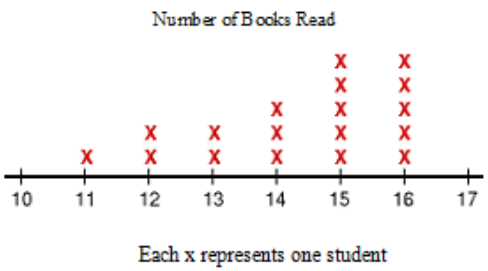
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The words used to describe the patterns in a histogram are: "symmetric", "skewed right" or "skewed left".  Symmetric Skewed right Skewed left • When describing a histogram, it is important to talk about its shape or distribution, its center (such as mean or median) and its spread (including peaks). A symmetric histogram indicates that the mode, median, and mean are the same or are together in the center. A skewed histogram indicates the mean and median have different values and do not lie in the center. Skewed histograms with two modes or "peaks" are known as bi-modal.  • Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data.		

7.9~~11~~ The student, given data in a practical situation, will

a) ~~construct and analyze~~ represent data in a histograms~~; and~~

b) make observations and inferences about data represented in a histogram; and

c) compare ~~and contrast~~ histograms with ~~other types of graphs presenting information from~~ the same data ~~set~~represented in stem-and-leaf plots, line plots, and circle graphs.

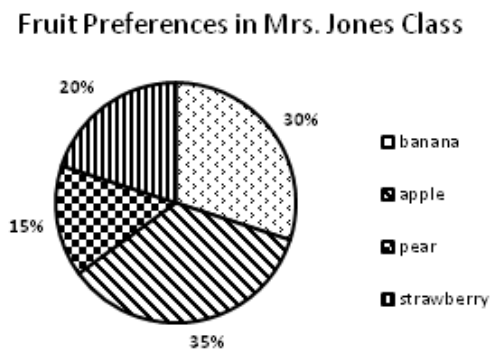
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Different types of graphs can be used to display categorical data. The way data is displayed is often dependent on what someone is trying to communicate. – A line plot shows the frequency of data on a number line. Line plots are used to show the spread of the data and quickly identify the range, mode, and any outliers.  		

7.9~~11~~ The student, given data in a practical situation, will

a) ~~construct and analyze~~ represent data in a histograms~~; and~~

b) make observations and inferences about data represented in a histogram; and

c) compare ~~and contrast~~ histograms with ~~other types of graphs presenting information from~~ the same data ~~set~~represented in stem-and-leaf plots, line plots, and circle graphs.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– A circle graph is used for categorical and discrete numerical data. Circle graphs are used to show a relationship of the parts to a whole.  – A stem and leaf plot is used for discrete numerical data and is used to show frequency of data distribution. Stem | Leaf 0 | 3 6 1 | 2 5 7 8 2 | 4 6 3 | 1 3 7 7 7 4 | 0 4 8 5 | 6 | 2 2 3 8 3|5 = 35		

7.9~~11~~ The student, given data in a practical situation, will

a) ~~construct and analyze~~ represent data in a histograms~~; and~~

b) make observations and inferences about data represented in a histogram; and

c) compare ~~and contrast~~ histograms with ~~other types of graphs presenting information from~~ the same data ~~set~~represented in stem-and-leaf plots, line plots, and circle graphs.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The information displayed in different graphs may be examined to determine how data are or are not related, ~~ascertaining~~ differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inference).		

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct through active learning experiences a more advanced understanding of mathematics;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

- ~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~
- ~~Students extend their knowledge of patterns developed in the elementary grades and through practical experiences by investigating and describing functional relationships.~~
- ~~Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable*, *term*, *coefficient*, *exponent*, *expression*, *equation*, *inequality*, *domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.~~
- ~~Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.~~
- ~~Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.~~

Patterns, functions and algebra become a larger mathematical focus in the middle grades as students extend their knowledge of patterns developed in the elementary grades.

Students make connections between the numeric concepts of ratio and proportion and the algebraic relationships that exist within a set of equivalent ratios. Students use variable expressions to represent proportional relationships between two quantities and begin to connect the concept of a constant of proportionality to rate of change and slope. Representation of relationships between two quantities using tables, graphs, equations, or verbal descriptions allow students to connect their knowledge of patterns to the concept of functional relationships. Graphing linear equations in two variables in the coordinate plane is a focus of the study of functions which continues in high school mathematics.

Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable*, *term*, *coefficient*, *exponent*, *expression*, *equation*, *inequality*, *domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades. Students learn to solve equations by using concrete materials. They expand their skills from one-step to multistep equations and inequalities through their application in practical situation.

7.10 The student will

a) determine if a proportional relationship exists between two quantities, given a table of values or a graph;

b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations;

c) determine the slope as rate of change in a proportional relationship, given two ordered pairs; and

d) graph a line representing a proportional relationship given the slope and an ordered pair, or given the equation in $y = mx$ form where m represents the slope as rate of change.

UNDERSTANDING THE STANDARD		ESSENTIAL KNOWLEDGE AND SKILLS
• When two quantities, x and y, vary in such a way that one of them is a constant multiple of the other, the two quantities are "proportional". A model for that situation is $y = kx$ where k is the constant of proportionality or the constant ratio of y to x. • A table of values representing a proportional relationship includes pairs of values that represent equivalent rates or ratios. All ratios are equivalent and a constant exists that can be multiplied by the measure of the one quantity to get the measure of the other quantity for every ratio pair. An example of a proportional relationship: Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges $8 for each medium pizza. This table represents the cost per number of pizza ordered. x number of pizzas: 1, 2, 3, 4 y total cost: 8, 16, 24, 32 x8 x8 This relationship can be represented by $y = 8x$ where y is the cost and x is the number of medium pizzas ordered. The constant of proportionality is evident with the table as the multiplicative relationship that exists within each x:y ratio. In this example, it is 8. An example of a non-proportional relationship: Uptown Pizza sells medium pizzas for $7 each but charges a $3 delivery fee per order. This table represents the cost per number of pizzas ordered.		**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine if a proportional relationship exists between two quantities given a table of values. (a) • Determine if a proportional relationship exists between two quantities given a graph. (a) • Identify the unit rate (constant of proportionality) of a proportional relationship represented by a table, graph, equation, or verbal description, including those represented in a practical situation. (b) • Given two ordered pairs on a line representing a proportional relationship, determine the slope as rate of change. Ordered pairs are limited to coordinates that are positive integers. (c) • Graph a line representing a proportional relationship, given an ordered pair on the line and the slope. Ordered pairs are limited to coordinates that are positive integers. (d) • Graph a line representing a proportional relationship given the equation of the line in the form $y = mx$, where m represents the slope or rate of change. (d)

7.10 The student will

a) determine if a proportional relationship exists between two quantities, given a table of values or a graph;

b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations;

c) determine the slope as rate of change in a proportional relationship, given two ordered pairs; and

d) graph a line representing a proportional relationship given the slope and an ordered pair, or given the equation in $y = mx$ form where m represents the slope as rate of change.

UNDERSTANDING THE STANDARD		ESSENTIAL KNOWLEDGE AND SKILLS

x number of pizzas	1	2	3	4
y total cost	10	17	24	31

The ratios represented in the table above are not equivalent. Other non-proportional relationships that are not linear will be studied in later courses.

- A graph representing a proportional relationship is a straight line that passes through the origin (0, 0).
- A unit rate (constant of proportionality) is a ratio (represented as a fraction) with a denominator of 1. For example, the ratio of 3 cups of apple juice for every 2 cups of grape juice has a unit rate of $\frac{3}{2}$ cups of apple juice for every 1 cup of grape juice. A unit rate can be described as "for each 1" or "for every 1."
- In grade 6, proportional relationships were represented by the equation $y = kx$, where k represents the constant of proportionality. For example: Cecil walks 2 meters every second (verbal description). If x represents the number of seconds and y represents the number of meters he walks, this proportional relationship can be represented using a table:

x seconds	1	2	3	4
y meters	2	4	6	8

7.10 The student will

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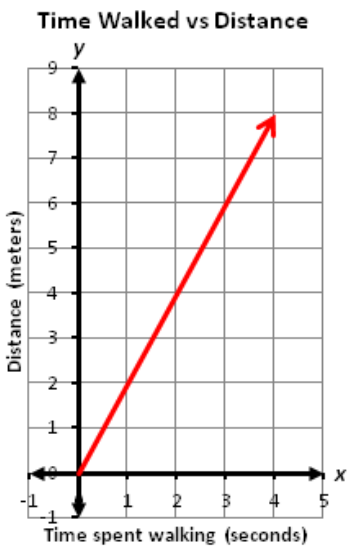
b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations;

c) determine the slope as rate of change in a proportional relationship, given two ordered pairs; and

d) graph a line representing a proportional relationship given the slope and an ordered pair, or given the equation in $y = mx$ form where m represents the slope as rate of change.

UNDERSTANDING THE STANDARD		ESSENTIAL KNOWLEDGE AND SKILLS
This proportional relationship could be represented using the equation $y = 2x$, since he walks 2 meters for each second of time (the constant of proportionality is 2 or $\frac{2}{1}$). The same constant of proportionality exists between the x and y for every ratio pair. This proportional relationship could be represented by the following graph:  		

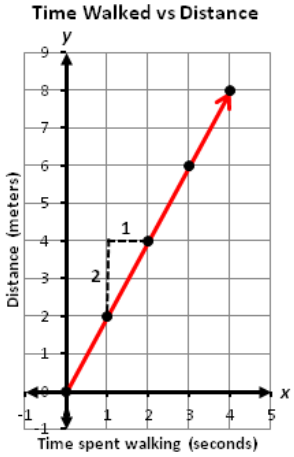
7.10 The student will

a) determine if a proportional relationship exists between two quantities, given a table of values or a graph;

b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations;

c) determine the slope as rate of change in a proportional relationship, given two ordered pairs; and

d) graph a line representing a proportional relationship given the slope and an ordered pair, or given the equation in $y = mx$ form where m represents the slope as rate of change.

UNDERSTANDING THE STANDARD		ESSENTIAL KNOWLEDGE AND SKILLS
• The slope of the graph of a proportional relationship is the constant of proportionality or the constant ratio of y to x. The equation $y = kx$ can be expressed as $y = mx$ where m represents the slope or the constant of proportionality. Slope is the rate of change described by the multiplicative relationship between the y-coordinate and the x-coordinate. Two points are necessary to determine the value for m to describe the direction and steepness of the line. • The slope of a line is a rate of change, a ratio describing the vertical change to the horizontal change. Slope = $\frac{change\ in\ y}{change\ in\ x} = \frac{vertical\ change}{horizontal\ change}$ For example, using this graph, determine the slope as a rate of change from two points on the graph.  		

7.10 The student will

a) determine if a proportional relationship exists between two quantities, given a table of values or a graph;

b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations;

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d) graph a line representing a proportional relationship given the slope and an ordered pair, or given the equation in $y = mx$ form where m represents the slope as rate of change.

UNDERSTANDING THE STANDARD		ESSENTIAL KNOWLEDGE AND SKILLS
The rate of change from (1,2) to (2,4) is 2 units up (the change in y) and 1 unit to the right (the change in x), $\frac{2}{1}$ or 2. Thus, the slope of this line is 2.		

~~7.12 The student will represent relationships with tables, graphs, rules, and words.~~ [Included in 7.10]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Rules that relate elements in two sets can be represented by word sentences, equations, tables of values, graphs, or illustrated pictorially.~~ • ~~A relation is any set of ordered pairs. For each first member, there may be many second members.~~ • ~~A function is a relation in which there is one and only one second member for each first member.~~ • ~~As a table of values, a function has a unique value assigned to the second variable for each value of the first variable.~~ • ~~As a graph, a function is any curve (including straight lines) such that any vertical line would pass through the curve only once.~~ • ~~Some relations are functions; all functions are relations.~~	• ~~What are the different ways to represent the relationship between two sets of numbers?~~ ~~Rules that relate elements in two sets can be represented by word sentences, equations, tables of values, graphs or illustrated pictorially.~~	~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~ • ~~Describe and represent relations and functions, using tables, graphs, rules, and words. Given one representation, students will be able to represent the relation in another form.~~

7.11~~13~~ The student will
~~a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and~~[Included in 7.12 EKS]
~~b)~~ evaluate algebraic expressions for given replacement values of the variables.

UNDERSTANDING THE STANDARD ~~(Teacher Notes)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~• An expression is a name for a number.~~ ~~• An expression that contains a variable is a variable expression.~~ ~~• An expression that contains only numbers is a numerical expression.~~ ~~• A verbal expression is a word phrase (e.g., "the sum of two consecutive integers").~~ ~~• A verbal sentence is a complete word statement (e.g., "The sum of two consecutive integers is five."~~ ~~• An algebraic expression is a variable expression that contains at least one variable (e.g., 2x – 5).~~ ~~An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., 2x + 1 = 5).~~ • To evaluate an algebraic expression, substitute a given replacement value for a variable and apply the order of operations. For example, if a = 3 and b = -2 then 5a + b can be evaluated as: 5(3) + (-2) and simplified using the order of operations to equal 15 + (-2) = which equals 13. • Expressions are simplified by using the order of operations. • The order of operations is a convention that defines the computation order to follow in simplifying an expression. It ensures that there is only one correct value. The order of operations is as follows: – First, complete all operations within grouping symbols[1]. If there are grouping symbols within other grouping symbols, do the innermost operations first. – Second, evaluate all exponential expressions. – Third, multiply and /or divide in order from left to right. – Fourth, add and /or subtract in order from left to right.	~~• How can algebraic expressions and equations be written?~~ ~~Word phrases and sentences can be used to represent algebraic expressions and equations.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** ~~• Write verbal expressions as algebraic expressions. Expressions will be limited to no more than 2 operations.~~ ~~• Write verbal sentences as algebraic equations. Equations will contain no more than 1 variable term.~~ ~~• Translate algebraic expressions and equations to verbal expressions and sentences. Expressions and equations will be limited to no more than two operations.~~ ~~• Identify examples of expressions and equations.~~ • ~~Apply~~ Use the order of operations and apply the properties of real numbers to evaluate expressions for given replacement values of the variables. Exponents are limited to whole numbers and bases are limited to positive integers. Limit the number of replacements to no more than ~~3~~ three per expression.

7.11~~13~~ The student will

~~a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and~~[Included in 7.12 EKS]

~~b)~~ evaluate algebraic expressions for given replacement values of the variables.

UNDERSTANDING THE STANDARD ~~(Teacher Notes)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
[1] Parentheses ( ), brackets [ ], braces { }, and the division bar should be treated as grouping symbols. • Expressions are simplified using the order of operations and applying the properties of real numbers. – The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 5.6 + -4 = -4 + 5.6). – The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 5.6 · -4 = -4 · 5.6). – The associative property of addition states that regrouping the addends does not change the sum [e.g., 5.6 + (-4 + 3) = (5.6 + -4) + 3]. – The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5.6 · (-4 · 3) = (5.6 · -4) · 3]. – Subtraction and division are neither commutative nor associative. – The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true . Distributive property examples: • -5.2 · (3 + 7) = (-5.2 · 3) + (-5.2 · 7), or • -5.2 · (3 – 7) = (-5.2 · 3) – (-5.2 · 7) Converse of the distributive property examples: • (-5.2 · 3) + (-5.2 · 7) = -5.2 · (3 + 7), or • (-5.2 · 3) – (-5.2 · 7) = -5.2 · (3 – 7) – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1)		

7.11~~13~~ The student will

~~a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and~~[Included in 7.12 EKS]

~~b)~~ evaluate algebraic expressions for given replacement values of the variables.

UNDERSTANDING THE STANDARD ~~(Teacher Notes)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
because any number multiplied by 1 is the number. There are no identity elements for subtraction and division. – The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real number (e.g., $\frac{5}{3} + 0 = \frac{5}{3}$). – The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number (e.g., $\sqrt{8} \cdot 1 = \sqrt{8}$). – Inverses are numbers that combine with other numbers and result in identity elements [e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$]. – The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., $5.6 + (-5.6) = 0$). – The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., $-4 \cdot -\frac{1}{4} = 1$). – Zero has no multiplicative inverse. – The multiplicative property of zero states that the product of any real number and zero is zero (e.g., $7 \cdot 0 = 0$ and $0 \cdot 7 = 0$). – Division by zero is not a possible mathematical operation. It is undefined.		

7.11~~13~~ The student will

~~a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and~~[Included in 7.12 EKS]

~~b)~~ evaluate algebraic expressions for given replacement values of the variables.

UNDERSTANDING THE STANDARD ~~(Teacher Notes)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– The substitution property of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality. (e.g., if x = -5 + 8 then x = 3. Thus, 3 was substituted for -5 + 8 since they are equivalent).		

7.12~~14~~ The student will
~~a)~~ solve ~~one- and~~ [Included in 6.14]two-step linear equations in one variable~~; and~~, including
~~b) solve~~ practical problems that require~~ing~~ the solution of a ~~one- and~~ two-step linear equation~~s~~ in one variable.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• An equation is a mathematical sentence that states that two expressions are equal. • ~~A one-step equation is defined as an equation that requires the use of one operation to solve (e.g., $x + 3 = -4$).~~ • The solution to an equation is the value(s) that make it a true statement. Many equations have one solution and can be represented as a point on a number line. • A variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable. • The inverse operation for addition is subtraction, and the inverse operation for multiplication is division. • A two-step equation ~~is defined as an equation that requires the use of two operations to solve~~may include, but not be limited to equations such as the following: ~~(e.g.,~~ $2x + \frac{1}{2}$~~1~~ = -5; -25 = 7.2x + 1; $\frac{x-7}{-3}$ = 4; 2(x + 4) = 8; $\frac{3}{4}x - 2 = 10$ $\frac{x-7}{3}$ ~~= 4)~~. • An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an equal sign (e.g., $\frac{3}{4}$, 5x, 140 - 38.2, 18 · 21, 5 + x.) • An expression that contains a variable is a variable expression. A variable expression is like a phrase: as a phrase does not have a verb, so an expression does not have an equal sign (=). An expression cannot be solved. • A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used when the numbers are unknown. For example, the verbal	• ~~When solving an equation, why is it important to perform identical operations on each side of the equal sign?~~ ~~An operation that is performed on one side of an equation must be performed on the other side to maintain equality.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent and ~~demonstrate steps for solving one- and~~solve two-step linear equations in one variable using a variety of concrete materials~~,~~ and pictorial representations~~ and algebraic sentences~~. • Apply properties of real numbers and properties of equality to ~~Solve~~ solve ~~one- and~~ two-step linear equations in one variable. Coefficients and numerical terms may be rational. • Confirm algebraic solutions to linear equations in one variable. • Write verbal expressions and sentences as algebraic expressions and equations. • Write algebraic expressions and equations as verbal expressions and sentences. • Solve practical problems that require the solution of a one- or two-step linear equation.

7.12~~14~~ The student will
~~a) solve one- and~~ [Included in 6.14]two-step linear equations in one variable~~; and~~, including
~~b) solve~~ practical problems that require~~ing~~ the solution of a ~~one- and~~ two-step linear equations in one variable.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
expression "a number multiplied by 5" could be represented by the variable expression "$n \cdot 5$" or "$5n$". • An algebraic expression is a variable expression that contains at least one variable (e.g., $2x - 3$). • A verbal sentence is a complete word statement (e.g., "The sum of twice a number and two is fifteen." could be represented by "$2n + 2 = 15$"). • An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., $2x - 8 = 7$). • Properties of real numbers and properties of equality can be used to solve equations, justify solutions, and express simplification. – The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 5.6 + -4 = -4 + 5.6). – The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 5.6 · -4 = -4 · 5.6). – The associative property of addition states that regrouping the addends does not change the sum [e.g., 5.6 + (-4 + 3) = (5.6 + -4) + 3]. – The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5.6 · (-4 · 3) = (5.6 · -4) · 3]. – Subtraction and division are neither commutative nor associative. – The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true.		

7.12~~14~~ The student will

~~a)~~ solve ~~one- and~~ [Included in 6.14]two-step linear equations in one variable~~; and~~, including

~~b) solve~~ practical problems that require~~ing~~ the solution of a ~~one- and~~ two-step linear equations in one variable.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
Distributive property examples: • -5.2 · (3 + 7) = (-5.2 · 3) + (-5.2 · 7), or • -5.2 · (3 – 7) = (-5.2 · 3) – (-5.2 · 7) Converse of the distributive property examples: • (-5.2 · 3) + (-5.2 · 7) = -5.2 · (3 + 7), or • (-5.2 · 3) – (-5.2 · 7) = -5.2 · (3 – 7) – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division. – The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real number (e.g., $\frac{5}{3} + 0 = \frac{5}{3}$). – The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number (e.g., $\sqrt{8} \cdot 1 = \sqrt{8}$). – Inverses are numbers that combine with other numbers and result in identity elements [e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$]. – The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., 5.6 + (–5.6) = 0). – The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., $-4 \cdot -\frac{1}{4} = 1$).		

7.12~~14~~ The student will
~~a) solve one- and~~ [Included in 6.14]two-step linear equations in one variable~~; and~~, including
~~b) solve~~ practical problems that require~~ing~~ the solution of a ~~one- and~~ two-step linear equations in one variable.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– Zero has no multiplicative inverse. – The multiplicative property of zero states that the product of any real number and zero is zero (e.g., $7 \cdot 0 = 0$ and $0 \cdot 7 = 0$). – Division by zero is not a possible mathematical operation. It is undefined. – The addition property of equality states that any number must be added to both sides of an equation to maintain equality. – The subtraction property of equality states that any number must be subtracted from both sides of an equation to maintain equality. – The multiplication property of equality states that an equation must be multiplied on both sides by the same number to maintain equality. – The division property of equality states that an equation must be divided on both sides by the same number to maintain equality. • The substitution property of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality. (e.g., if $x = -5 + 8$ then $x = 3$. Thus, 3 was substituted for -5 + 8 since they are equivalent).		

7.13~~15~~ The student will
~~a)~~ solve one- and two-step **[two-step moved from 8.15b]**linear inequalities in one variable, including practical problems~~;~~, and
~~**b)**~~ **graph the solutions ~~to inequalities~~ on ~~the~~ a number line.**

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A one-step inequality may include, but not be limited to inequalities such as the following: $2x > 5$; $y - 3 \leq -6$; $\frac{1}{5}x < -3$; $a - (-4) \geq 11$.~~is defined as an inequality that requires the use of one operation to solve (e.g., $x - 4 > 9$).~~ • A two-step inequality may include, but not be limited to inequalities such as the following: $2x + 1 < -25$; $2x + \frac{1}{2} \geq -5$; $-25 > 7.2x + 1$; $\frac{x-7}{-3} \leq 4$; $2(x + 4) > 8$; $\frac{3}{4}x - 2 \leq 10$. • The solution set to an inequality is the set of all numbers that make the inequality true. • The inverse operation for addition is subtraction, and the inverse operation for multiplication is division. • The procedures for solving inequalities are the same as those to solve equations except for the case when an inequality is multiplied or divided on both sides by a negative number. Then the inequality sign is changed from less than to greater than, or greater than to less than. [Moved from middle column] • When both expressions of an inequality are multiplied or divided by a negative number, the inequality symbol reverses (e.g., $-3x < 15$ is equivalent to $x > -5$). • Solutions to inequalities can be represented using a number line. • In an inequality, there can be more than one value for the variable that makes the inequality true. There can be many solutions. (i.e., $x + 4 > -3$ then the solutions is $x > -7$. This means that x can be any number greater than -7. A few solutions might be -6.5, -3, 0, 4, 25, etc.)	~~• How are the procedures for solving equations and inequalities the same? The procedures are the same except for the case when an inequality is multiplied or divided on both sides by a negative number. Then the inequality sign is changed from less than to greater than, or greater than to less than.~~ [Moved to US] ~~• How is the solution to an inequality different from that of a linear equation? In an inequality, there can be more than one value for the variable that makes the inequality true.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent and solve one- and two-step~~demonstrate steps in solving~~ inequalities in one variable, using a variety of concrete materials~~,~~ and pictorial representations~~, and algebraic sentences~~. • Apply properties of real numbers and properties of inequality to solve one- and two-step inequalities in one variable. Coefficients and numeric terms may be rational. • ~~Graph~~ Represent solutions to inequalities algebraically and graphically ~~on~~ using a ~~the~~ number line. • Solve practical problems that require the solution of a one- or two-step inequality. • Identify a numerical value(s) that ~~satisfies the~~is part of the solution set of a given inequality.

7.13~~15~~ The student will
~~a)~~ solve one- and two-step **[two-step moved from 8.15b]**linear inequalities in one variable, including practical problems~~;~~, and
~~**b)**~~ **graph the solution~~s to inequalities~~ on ~~the~~ a number line.**

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Properties of real numbers and properties of inequality can be used to solve inequalities, justify solutions, and express simplification. – The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 5.6 + -4 = -4 + 5.6). – The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 5.6 · -4 = -4 · 5.6). – The associative property of addition states that regrouping the addends does not change the sum [e.g., 5.6 + (-4 + 3) = (5.6 + -4) + 3]. – The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5.6 · (-4 · 3) = (5.6 · -4) · 3]. – Subtraction and division are neither commutative nor associative. – The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true. Distributive property examples: • -5.2 · (3 + 7) = (-5.2 · 3) + (-5.2 · 7), or • -5.2 · (3 – 7) = (-5.2 · 3) – (-5.2 · 7) Converse of the distributive property examples: • (-5.2 · 3) + (-5.2 · 7) = -5.2 · (3 + 7), or • (-5.2 · 3) – (-5.2 · 7) = -5.2 · (3 – 7)		

7.13~~15~~ The student will
~~a)~~ solve one- and two-step **[two-step moved from 8.15b]**linear inequalities in one variable, including practical problems~~;~~, and
~~**b)**~~ **graph the solution~~s to inequalities~~ on ~~the~~ a number line.**

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division. – The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real number (e.g., $\frac{5}{3} + 0 = \frac{5}{3}$). – The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number (e.g., $\sqrt{8} \cdot 1 = \sqrt{8}$). – Inverses are numbers that combine with other numbers and result in identity elements [e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$]. – The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., $5.6 + (-5.6) = 0$). – The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., $-4 \cdot -\frac{1}{4} = 1$). – Zero has no multiplicative inverse. – The multiplicative property of zero states that the product of any real number and zero is zero (e.g., $7 \cdot 0 = 0$ and $0 \cdot 7 = 0$). – Division by zero is not a possible mathematical operation. It is undefined.		

7.13~~15~~ The student will
~~a)~~ solve one- and two-step **[two-step moved from 8.15b]**linear inequalities in one variable, including practical problems~~;~~, and
~~**b)**~~ **graph the solution~~s to inequalities~~ on ~~the~~ a number line.**

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– The addition property of inequality states that any number must be added to both sides of an inequality to maintain the inequality. – The subtraction property of inequality states that any number must be subtracted from both sides of an inequality to maintain inequality. – The multiplication property of inequality states that if an inequality is multiplied on both sides by the same positive number the inequality symbol remains the same. But if an inequality is multiplied on both sides by the same negative number, the inequality symbol reverses. For example, to solve $\frac{1}{3}x > 9$, multiply both sides of the inequality by 3, so $x > 27$. To solve $-\frac{1}{3}x > 9$, multiply both sides of the inequality by -3, so $x < -27$. – The division property of inequality states that if an inequality is divided on both sides by the same positive number, the inequality symbol remains the same. But if an inequality is divided on both sides by the same negative number, the inequality symbol reverses. For example, $-4y < 20$. Divide both sides of the inequality by -4. So $y > -5$. – The substitution property of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality. (e.g., if $x = -5 + 8$ then $x = 3$. Thus, 3 was substituted for -5 + 8 since they are equivalent).		

~~7.16~~ ~~**The student will apply the following properties of operations with real numbers:**~~

~~a) the commutative and associative properties for addition and multiplication;~~
~~b) the distributive property;~~
~~c) the additive and multiplicative identity properties;~~
~~**d) the additive and multiplicative inverse properties; and**~~
~~e) the multiplicative property of zero.~~

[Incorporated into 7.2, 7.12, and 7.13]

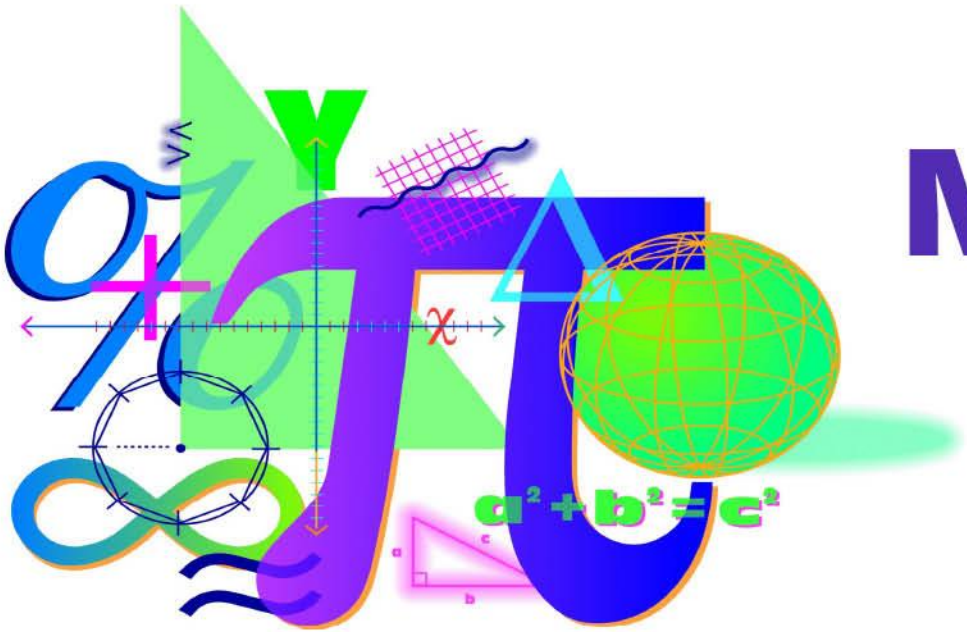
~~UNDERSTANDING THE STANDARD~~ ~~**(Background Information for Instructor Use Only)**~~	~~**ESSENTIAL UNDERSTANDINGS**~~	~~**ESSENTIAL KNOWLEDGE AND SKILLS**~~
• ~~The commutative property for addition states that changing the order of the addends does not change the sum (e.g., 5 + 4 = 4 + 5).~~ • ~~The commutative property for multiplication states that changing the order of the factors does not change the product (e.g., 5 · 4 = 4 · 5).~~ • ~~The associative property of addition states that regrouping the addends does not change the sum [e.g., 5 + (4 + 3) = (5 + 4) + 3].~~ • ~~The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5 · (4 · 3) = (5 · 4) · 3].~~ • ~~Subtraction and division are neither commutative nor associative.~~ • ~~The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number [e.g., 5 · (3 + 7) = (5 · 3) + (5 · 7), or 5 · (3 − 7) = (5 · 3) − (5 · 7)].~~ • ~~Identity elements are numbers that combine with other numbers without changing the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). There are no identity elements for subtraction and division.~~	• ~~Why is it important to apply properties of operations when simplifying expressions? Using the properties of operations with real numbers helps with understanding mathematical relationships.~~	~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~ • ~~Identify properties of operations used in simplifying expressions.~~ • ~~Apply the properties of operations to simplify expressions.~~

~~**7.16 The student will apply the following properties of operations with real numbers:**~~

~~a) the commutative and associative properties for addition and multiplication;~~
~~b) the distributive property;~~
~~c) the additive and multiplicative identity properties;~~
~~**d) the additive and multiplicative inverse properties; and**~~
~~e) the multiplicative property of zero.~~

[Incorporated into 7.2, 7.12, and 7.13]

~~UNDERSTANDING THE STANDARD~~ ~~**(Background Information for Instructor Use Only)**~~	~~**ESSENTIAL UNDERSTANDINGS**~~	~~**ESSENTIAL KNOWLEDGE AND SKILLS**~~
~~• The additive identity property states that the sum of any real number and zero is equal to the given real number (e.g., $5 + 0 = 5$).~~ ~~• The multiplicative identity property states that the product of any real number and one is equal to the given real number (e.g., $8 \cdot 1 = 8$).~~ ~~• Inverses are numbers that combine with other numbers and result in identity elements [e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$].~~ ~~• The additive inverse property states that the sum of a number and its additive inverse always equals zero [e.g., $5 + (-5) = 0$].~~ ~~• The multiplicative inverse property states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., $4 \cdot \frac{1}{4} = 1$).~~ ~~• Zero has no multiplicative inverse.~~ ~~• The multiplicative property of zero states that the product of any real number and zero is zero.~~ ~~• Division by zero is not a possible arithmetic operation. Division by zero is undefined.~~		



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Grade 8

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students in the middle grades focus on mastering rational numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to most middle school mathematics topics.~~
- ~~Students develop an understanding of integers and rational numbers by using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.~~
- ~~Students develop an understanding of the properties of operations on real numbers through experiences with rational numbers and by applying the order of operations.~~
- ~~Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.~~

Mathematics instruction in grades 6 – 8 continues to focus on the development of number sense, with emphasis on rational and real numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to many middle school mathematics topics.

Students develop an understanding of integers and rational numbers using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation.
Flexible thinking about rational number representations is encouraged when students solve problems.

Students develop an understanding of real numbers and the properties of operations on real numbers through experiences with rational and irrational numbers and apply the order of operations.

8.1 The student will

~~a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and~~[Combined with 7.2]

~~b)~~ compare and order real numbers~~decimals, fractions, percents, and numbers written in scientific notation~~. [Moved to EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~*Expression* is a word used to designate any symbolic mathematical phrase that may contain numbers and/or variables. Expressions do not contain equal or inequality signs.~~ • ~~The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where a and b are integers and b does not equal zero (e.g., $\sqrt{25}$, $\frac{1}{4}$, 2.3, 75%, $4.\overline{59}$).~~ [Included in 7.2 and 8.2] • ~~A rational number is any number that can be written in fraction form.~~ [Included in 8.2] • ~~A numerical expression contains only numbers and the operations on those numbers.~~ [Moved to 8.14] • ~~Expressions are simplified using the order of operations and the properties for operations with real numbers, i.e., associative, commutative, and distributive and inverse properties.~~ [Moved to 8.14] • ~~The order of operations, a mathematical convention, is as follows: Complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols (embedded), do the innermost operation first. Evaluate all exponential expressions. Multiply and/or divide in order from left to right. Add and/or subtract in order from left to right.~~ ~~*Parentheses ( ), brackets [ ], braces { }, the absolute value $\lvert\ \rvert$, division/fraction bar –, and the square root symbol $\sqrt{\ }$ should be treated as grouping symbols.~~ [Moved to 7.2 and 8.14] • ~~A power of a number represents repeated multiplication of the number. For example, $(-5)^4$ means $(-5)\cdot(-5)\cdot(-5)\cdot(-5)$. The base is the number that is multiplied, and the exponent~~	• ~~What is the role of the order of operations when simplifying numerical expressions?~~ ~~The order of operations prescribes the order to use to simplify a numerical expression.~~ [Included in 7.2] • ~~How does the different ways rational numbers can be represented help us compare and order rational numbers?~~ ~~Numbers can be represented as decimals, fractions, percents, and in scientific notation. It is often useful to convert numbers to be compared and/or ordered to one representation (e.g., fractions, decimals or percents).~~ [Moved to US] • ~~What is a rational number?~~ ~~A rational number is any number that can be written in fraction form.~~ [Included in 8.2] • ~~When are numbers written in scientific notation?~~ ~~Scientific notation is used to represent very large and very small numbers.~~ [Included in US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Simplify numerical expressions containing: 1) exponents (where the base is a rational number and the exponent is a positive whole number); 2) fractions, decimals, integers and square roots of perfect squares; and 3) grouping symbols (no more than 2 embedded grouping symbols). Order of operations and properties of operations with real numbers should be used.~~ [Combined with 7.2 and 8.14] • Compare and order no more than five real numbers, to include integers, fractions, decimals, mixed numbers, percents, ~~and~~ numbers written in scientific notation, radicals, and π. ~~using positive and negative exponents~~ Ordering may be in ascending or descending order. • Use rational approximations (to the nearest hundredth) of irrational numbers to compare and order, locating values on a number line. Values may be positive or negative.

8.1 The student will

~~a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and~~[Combined with 7.2]

~~b)~~ compare and order real numbers~~decimals, fractions, percents, and numbers written in scientific notation~~. [Moved to EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
~~represents the number of times the base is used as a factor. In this example, (−5) is the base, and 4 is the exponent. The product is 625. Notice that the base appears inside the grouping symbols. The meaning changes with the removal of the grouping symbols. For example, -5^4 means 5·5·5·5 negated which results in a product of −625. The expression $-(5)^4$ means to take the opposite of 5·5·5·5 which is −625. Students should be exposed to all three representations.~~ [Moved to 8.14] • All state approved scientific calculators use algebraic logic (follow the order of operations). [Reordered] • Numbers can be represented as integers, fractions, decimals, percents, and in scientific notation. It is often useful to convert numbers to be compared and/or ordered to one representation (e.g., fractions, decimals or percents).[Moved from middle column] • The density property states that between any two real numbers lies another real number. • Scientific notation is used to represent very large or very small numbers. [Same as middle column] • A number written in scientific notation is the product of two factors: a decimal greater than or equal to one but less than 10 multiplied by a power of 10 (e.g., $3.1 \times 10^5 = 310{,}000$ and $3.1 \times 10^{-5} = 0.000031$). • Any real number raised to the zero power is 1. The only exception to this rule is zero itself. Zero raised to the zero power is undefined. • ~~All state approved scientific calculators use algebraic logic (follow the order of operations).~~ [Reordered]		

8.2 The student will describe ~~orally and in writing~~ the relationships between the subsets of the real number system.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The subsets of real numbers include~~s~~ natural numbers, counting numbers, whole numbers, integers, rational and irrational numbers. • Some numbers can belong to more than one subset of the real numbers (e.g. 4 is a natural number, a whole number, an integer, and a rational number. The attributes of one subset can be contained in whole or in part in another subset. [Moved from the middle column] • The set of natural numbers is the set of counting numbers {1, 2, 3, 4, ...}. • The set of whole numbers includes the set of all the natural numbers or counting numbers and zero {0, 1, 2, 3…}. • The set of integers includes the set of whole numbers and their opposites {…-2, -1, 0, 1, 2…}. Zero has no opposite and is neither positive nor negative. • The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* does not equal zero [Moved from middle column 8.1] The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of rational numbers are ~~(e.g.,~~ $\sqrt{25}$, $\frac{1}{4}$, -2.3, 75%, $4.\overline{59}$~~)~~. • The set of irrational numbers is the set of all nonrepeating, nonterminating decimals. An irrational number cannot be written in fraction form {e.g., π, $\sqrt{2}$, 1.232332333…}. • The real number system is comprised of all rational and irrational numbers.	~~• How are the real numbers related? Some numbers can appear in more than one subset, e.g., 4 is an integer, a whole number, a counting or natural number and a rational number. The attributes of one subset can be contained in whole or in part in another subset.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Describe orally and in writing the relationships among the sets of natural or counting numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers.~~ [Included in bullet below] • Describe and ~~I~~illustrate the relationships among the subsets of the real number system by using representations (graphic organizers, number lines, etc.) ~~such as Venn diagrams~~. Subsets include rational numbers, irrational numbers, integers, whole numbers, and natural or counting numbers. • ~~Identify the subsets of the real number system to which a given number belongs.~~ • ~~Determine~~ Classify ~~whether~~ a given number ~~is~~ as a member of a particular subset or subsets of the real number system, and explain why. • Describe each subset of the set of real numbers and include examples and nonexamples. • Recognize that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.[‡] [‡]~~Revised March 2011~~

8.3 The student will

a) estimate and determine the two consecutive integers between which a square root lies; and [Moved from 8.5b]

b) determine both the positive and negative square roots of a given perfect square. [Moved from 8.5 EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A perfect square is a whole number whose square root is an integer. [Moved from 8.5] • The square root of a given number is any number which, when multiplied times itself, equals the given number. [Moved from 8.5] • Whole numbers, except zero, have both positive and negative roots. [Moved from 8.5] • Any whole number other than a perfect square has a square root that lies between two consecutive whole numbers [Moved from 8.5] (e.g.,$\sqrt{57}$ lies between 7 and 8 since $7^2 = 49$ and $8^2 = 64$.) • The symbol $\sqrt{}$ may be used to represent a positive (principal) root and $-\sqrt{}$ may be used to represent a negative root. • The square root of a whole number that is not a perfect square is an irrational number (e.g., $\sqrt{2}$ is an irrational number). An irrational number cannot be expressed exactly as a fraction $\frac{a}{b}$ where *b* does not equal 0. [Moved from 8.5] • Square root symbols may be used to represent solutions to equations of the form $x^2 = p$. Examples may include: – If $x^2 = 36$, then *x* is $\sqrt{36} = 6$ or $-\sqrt{36} = -6$. – If $x^2 = 5$, then *x* is $\sqrt{5}$ or $-\sqrt{5}$. • The square root of zero is zero. • Students can use grid paper and estimation to determine what is needed to build a perfect square. [Moved from 8.5] The square root of a positive number is usually defined as the side length of a square with the area equal to the given number. If it is not a perfect square, the area provides a means for estimation. [Moved from middle column 8.5]		**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the two consecutive integers between which the positive or negative square root of a given whole number from 1 to 400 lies. (a) [Moved from 8.5] • Determine the positive or negative square root of a given perfect square from 1 to 400. (b) [Moved from 8.5]

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct through active learning experiences a more advanced understanding of mathematics;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense.~~
- ~~Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate.~~
- ~~Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonableness of answers.~~
- ~~Students reinforce skills with operations with whole numbers, fractions, and decimals through problem solving and application activities.~~

Computation and estimation in grades 6-8 focus on developing conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring an understanding as to why procedures work and make sense.

Students develop and refine estimation strategies based on an understanding of number concepts, properties and relationships. The development of problem solving, using operations with integers and rational numbers, builds upon the strategies developed in the elementary grades. Students will reinforce these skills and build on the development of proportional reasoning and more advanced mathematical skills.

Students learn to make sense of the mathematical tools available by making valid judgments of the reasonableness of answers. Students will balance the ability to make precise calculations through the application of the order of operations with knowing when calculations may require estimation to obtain appropriate solutions to practical problems.

8.4~~3~~ The student will

a) solve practical problems involving rational numbers, percents, ratios, and proportions; and

b) determine the percent increase or decrease for a given situation.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Rational numbers may be expressed as whole numbers, integers, fractions, percents, and ratios. • Practical problems may include, but not be limited to, those related to economics, sports, science, social sciences, transportation, and health. Some examples include problems involving the amount of a pay check per month, the discount price on a product, temperature, simple interest, sales tax and installment buying. • A percent is a ~~special~~ ratio with a denominator of 100. [Included from middle column] • Reconciling an account is a process used to verify that two sets of records (usually the balances of two accounts) are in agreement. Reconciliation is used to ensure that the money leaving an account matches the actual money spent. • A discount is a percent of the original price. The discount price is the original price minus the discount. • Simple interest for a number of years is determined by ~~multiplying~~ finding the product of the ~~principle~~ principal, ~~by~~ the annual rate of interest, ~~by~~and the number of years of the loan or investment-$(I = prt)$. • The total value of an investment is equal to the sum of the original investment and the interest earned. • The total cost of a loan is equal to the sum of the original cost and the interest paid. • Percent increase and percent decrease are both percents of change measuring the percent a quantity increases or decreases. [Moved from middle column] ~~• Percent of change is the percent that a quantity increases or decreases.~~ [Combined with bullet above]	~~• What is the difference between percent increase and percent decrease?~~ ~~Percent increase and percent decrease are both percents of change measuring the percent a quantity increases or decreases. Percent increase shows a growing change in the quantity while percent decrease shows a lessening change.~~ [Moved to US] • ~~What is a percent?~~ ~~A percent is a special ratio with a denominator of 100.~~ [Included in US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write a proportion given the relationship of equality between two ratios. (a) • Solve practical problems by using computation procedures for ~~whole numbers, integers, fractions~~rational numbers, percents, ratios, and proportions. Some problems may require the application of a formula. (a) • ~~Maintain a checkbook and check registry for~~Reconcile an account balance given a statement with five or fewer transactions. (a) • Compute a discount or markup and the resulting sale price for one discount or markup. (a) ~~• Compute the percent increase or decrease for a one-step equation found in a real life situation.~~ [Reworded and Reordered] • Compute the sales tax or tip and resulting total. (a) ~~• Substitute values for variables in given formulas. For example, use the simple interest formula $I = prt$ to determine the value of any missing variable when given specific information.~~ • Compute the simple interest and new balance earned in an investment or on a loan ~~for a given number of years~~given the principal amount, interest rate, and time period in years. (a) • Compute the percent increase or decrease found in a practical situation. (b) [Reordered]

8.4~~3~~ The student will

a) solve practical problems involving rational numbers, percents, ratios, and proportions; and

b) determine the percent increase or decrease for a given situation.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Percent increase may determine~~s~~ ~~the~~ a rate of growth and may be calculated using a ratio. $\frac{\text{c}\cancel{\text{C}}\text{hange (new – original)}}{\text{original}}$ • For percent increase, the change will result in a positive number. • Percent decrease may determine~~s~~ the rate of decline and may be calculated using the same ratio as percent increase. However, the change will result in a negative number.		

~~8.4~~ **~~The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables.~~** **[Moved to 8.14a]**

~~UNDERSTANDING THE STANDARD~~ ~~**(Background Information for Instructor Only)**~~	~~**ESSENTIAL UNDERSTANDINGS**~~	~~**ESSENTIAL KNOWLEDGE AND SKILLS**~~
• ~~Algebraic expressions use operations with algebraic symbols (variables) and numbers.~~ • ~~Algebraic expressions are evaluated by substituting numbers for variables and applying the order of operations to simplify the resulting expression.~~ • ~~The order of operations is as follows: Complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols (embedded), do the innermost operation first. Evaluate all exponential expressions. Multiply and/or divide in order from left to right. Add and/or subtract in order from left to right.~~ ~~* Parentheses ( ), brackets [ ], braces { }, the absolute value $\lvert\ \rvert$, division/fraction bar –, and the square root symbol $\sqrt{\ }$ should be treated as grouping symbols.~~	• ~~What is the role of the order of operations when evaluating expressions?~~ ~~Using the order of operations assures only one correct answer for an expression.~~	~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~ • ~~Substitute numbers for variables in algebraic expressions and simplify the expressions by using the order of operations. Exponents are positive and limited to whole numbers less than 4. Square roots are limited to perfect squares.~~ • ~~Apply the order of operations to evaluate formulas. Problems will be limited to positive exponents. Square roots may be included in the expressions but limited to perfect squares.~~

~~8.5~~ **~~The student will~~**

~~a) determine whether a given number is a perfect square; and~~ [Included in 7.1]

~~b) find the two consecutive whole numbers between which a square root lies~~. [Included in 8.3a]

~~UNDERSTANDING THE STANDARD~~ ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
• ~~A perfect square is a whole number whose square root is an integer (e.g., The square root of 25 is 5 and -5; thus, 25 is a perfect square).~~ • ~~The square root of a number is any number which when multiplied by itself equals the number.~~ • ~~Whole numbers have both positive and negative roots.~~ • ~~Any whole number other than a perfect square has a square root that lies between two consecutive whole numbers.~~ • ~~The square root of a whole number that is not a perfect square is an irrational number (e.g., $\sqrt{2}$ is an irrational number). An irrational number cannot be expressed exactly as a ratio.~~ • ~~Students can use grid paper and estimation to determine what is needed to build a perfect square.~~	• ~~How does the area of a square relate to the square of a number?~~ ~~The area determines the perfect square number. If it is not a perfect square, the area provides a means for estimation.~~ • ~~Why do numbers have both positive and negative roots?~~ ~~The square root of a number is any number which when multiplied by itself equals the number. A product, when multiplying two positive factors, is always the same as the product when multiplying their opposites (e.g., 7 · 7 = 49 and -7 · -7 = 49).~~	~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~ • ~~Identify the perfect squares from 0 to 400.~~ • ~~Identify the two consecutive whole numbers between which the square root of a given whole number from 0 to 400 lies (e.g., $\sqrt{57}$ lies between 7 and 8 since $7^2 = 49$ and $8^2 = 64$).~~ • ~~Define a perfect square.~~ • ~~Find the positive or positive and negative square roots of a given whole number from 0 to 400. (Use the symbol $\sqrt{\ }$ to ask for the positive root and $-\sqrt{\ }$ when asking for the negative root.)~~

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students develop the measurement skills that provide a natural context and connection among many mathematics concepts. Estimation skills are developed in determining length, weight/mass, liquid volume/capacity, and angle measure. Measurement is an essential part of mathematical explorations throughout the school year.~~
- ~~Students continue to focus on experiences in which they measure objects physically and develop a deep understanding of the concepts and processes of measurement. Physical experiences in measuring various objects and quantities promote the long-term retention and understanding of measurement. Actual measurement activities are used to determine length, weight/mass, and liquid volume/capacity.~~
- ~~Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, pyramids, and cones.~~

[Moved from Geometry Strand Introduction and Rewritten]

Measurement and geometry in the middle grades provide a natural context and connection among many mathematical concepts. Students expand informal experiences with geometry and measurement in the elementary grades and develop a solid foundation for further exploration of these concepts in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.

Students develop measurement skills through exploration and estimation. Physical exploration to determine length, weight/mass, liquid volume/capacity, and angle measure are essential to develop a conceptual understanding of measurement. Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, square-based pyramids, and cones.

Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provide experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.

Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, square-based pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.

Students explore and develop an understanding of the Pythagorean Theorem. Understanding how the Pythagorean Theorem can be applied in practical situations has a far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

Level 0: Pre-recognition. Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

Level 2: Analysis. Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)

Level 4: Deduction. Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)

8.5~~6~~ The student will use

~~a) verify by measuring and describe~~ the relationships among pairs of angles that are vertical angles, adjacent angles, supplementary angles, and complementary angles~~; and~~

~~b)~~ to determine the measure of unknown angles~~ of less than 360°~~.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Vertical angles are a pair of ~~(all~~ nonadjacent angles~~)~~ formed by two intersecting lines. Vertical angles are congruent and share a common vertex. • Complementary angles are any two angles such that the sum of their measures is 90°. • Supplementary angles are any two angles such that the sum of their measures is 180°. • Complementary and supplementary angles may or may not be adjacent. [Moved from middle column] ~~• Reflex angles measure more than 180°.~~ • Adjacent angles are any two non-overlapping angles that share a common ~~side~~ ray and a common vertex.	~~• How are vertical, adjacent, complementary and supplementary angles related?~~ ~~Adjacent angles are any two non-overlapping angles that share a common side and a common vertex. Vertical angles will always be nonadjacent angles. Supplementary and complementary angles may or may not be adjacent.~~ [Included in US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** ~~• Measure angles of less than 360° to the nearest degree, using appropriate tools.~~ • Identify and describe the relationships between angles formed by two intersecting lines. • Identify and describe the relationship between pairs of angles that are vertical, adjacent, supplementary, and complementary. [Combined bullets below] ~~• Identify and describe the relationship between pairs of angles that are supplementary.~~ [Included in bullet above] ~~• Identify and describe the relationship between pairs of angles that are complementary.~~ [Included in bullet above] ~~• Identify and describe the relationship between pairs of angles that are adjacent.~~ [Included in bullet above] • Use the relationships among supplementary, complementary, vertical, and adjacent angles to solve practical problems.[‡] [‡]~~Revised March 2011~~

8.6~~7~~ The student will

a) ~~investigate and~~ solve problems, including practical problems, involving volume and surface area of rectangular prisms, cylinders, cones, and square-based pyramids; and

b) describe how changing one measured attribute of a rectangular prism~~figure~~ affects the volume and surface area.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A polyhedron is a solid figure whose faces are all polygons. • ~~A pyramid is a polyhedron with a base that is a polygon and other faces that are triangles with a common vertex.~~ [Reordered] • ~~The area of the base of a pyramid is the area of the polygon which is the base.~~ • ~~The total surface area of a pyramid is the sum of the areas of the triangular faces and the area of the base.~~ [Reordered] • Nets are two-dimensional representations of a three-dimensional figure that can be folded into a model of the three-dimensional figure. [Reordered] • Surface area of a solid figure is the sum of the areas of the surfaces of the figure. [Moved from middle column] • Volume is the amount a container holds. [Moved from middle column] • A rectangular prism is a polyhedron that has a congruent pair of parallel rectangular bases and four faces that are rectangles. A rectangular prism has eight vertices and twelve edges. [Reordered] In this course, prisms are limited to right prisms with bases that are rectangles. • The surface area of a rectangular prism is the sum of the areas of the faces and bases, found by using the formula $S.A. = 2lw + 2lh + 2wh$. All six faces are rectangles.[Reordered] • The volume of a rectangular prism is calculated by multiplying the length, width and height of the prism or by using the formula $V = lwh$. [Reordered]	• ~~How does the volume of a three-dimensional figure differ from its surface area?~~ ~~Volume is the amount a container holds.~~ ~~Surface area of a figure is the sum of the area on surfaces of the figure.~~ [Moved to US] • ~~How are the formulas for the volume of prisms and cylinders similar?~~ ~~For both formulas you are finding the area of the base and multiplying that by the height.~~ [Moved to US] • ~~How are the formulas for the volume of cones and pyramids similar?~~ ~~For cones you are finding $\frac{1}{3}$ of the volume of the cylinder with the same size base and height.~~ ~~For pyramids you are finding $\frac{1}{3}$ of the volume of the prism with the same size base and height.~~ [Moved to US] • ~~In general what effect does changing one attribute of a prism by a scale factor have on the volume of the prism?~~ ~~When you increase or decrease the~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Distinguish between situations that are applications of surface area and those that are applications of volume. (a) • ~~Investigate and compute~~ Determine the surface area of rectangular prisms, cylinders, cones, and a square-based ~~or triangular~~ pyramid by ~~finding the sum of the areas of the triangular faces and the base~~ using concrete objects, nets, diagrams and formulas. (a) [Combined bullets below] • ~~Investigate and compute the surface area of a cone by calculating the sum of the areas of the side and the base, using concrete objects, nets, diagrams and formulas.~~ [Included in bullet above] • ~~Investigate and compute the surface area of a right cylinder using concrete objects, nets, diagrams and formulas.~~ [Included in bullet above] • ~~Investigate and compute the surface area of a rectangular prism using concrete objects, nets, diagrams and formulas.~~ [Included in bullet above] • ~~Investigate and compute~~ Determine the volume of rectangular prisms, cylinders, cones, and square-based pyramids, using concrete objects, ~~nets,~~ diagrams, and formulas. (a)

8.6~~7~~ The student will

a) ~~investigate and~~ solve problems, including practical problems, involving volume and surface area of rectangular prisms, cylinders, cones, and square-based pyramids; and

b) describe how changing one measured attribute of a rectangular prism~~figure~~ affects the volume and surface area.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A cube is a polyhedron with six congruent, square faces. All edges are the same length. A cube has eight vertices and twelve edges. • A cylinder is a solid figure formed by two congruent parallel faces called bases joined by a curved surface. In this course, cylinders are limited to right circular cylinders. • The surface area of a right circular cylinder is found by using the formula: $S.A. = 2\pi r^2 + 2\pi rh$, where r is the radius of a base, and h is the height. • The volume of a cylinder is the area of the base of the cylinder multiplied by the height, found by using the formula, $V = \pi r^2 h$, where h is the height and πr^2 is the area of the base. [Reordered] • A cone is a solid figure formed by a face called a base that is joined to a vertex (apex) by a curved surface. In this grade level, cones are limited to right circular cones. • The surface area of a right circular cone is found by using the formula, $S.A. = \pi r^2 + \pi rl$, where l represents the slant height of the cone. The area of the base of a circular cone is πr^2. [Combined bullets] • The volume of a cone is found by using $V = \frac{1}{3}\pi r^2 h$, where h is the height and πr^2 is the area of the base. [Reordered] • A square-based pyramid is a polyhedron with a square base and four faces that are triangles with a common vertex (or apex) above the base. In this grade level, pyramids are limited to right regular pyramids with a square base. • The volume of a pyramid is $\frac{1}{3}Bh$, where B is the area of the base and h is the height.	~~length, width or height of a prism by a factor greater than 1, the volume of the prism is also increased by that factor.~~ [Included in US]	• Solve practical problems involving volume and surface area of rectangular prisms, cylinders, cones, and square-based pyramids. (a) • ~~Compare and contrast the volume and surface area of a prism with a given set of attributes with the volume of a prism where one of the attributes has been increased by a factor of 2, 3, 5 or 10.~~ • Describe how the volume of a rectangular prism is affected when one measured attribute is multiplied by a factor of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, 2, 3, or 4. (b) • Describe how the surface area of a rectangular prism is affected when one measured attribute is multiplied by a factor of $\frac{1}{2}$ or 2. (b) • ~~Describe the two-dimensional figures that result from slicing three-dimensional figures parallel to the base (e.g., as in plane sections of right rectangular prisms and right rectangular pyramids).~~ ‡ ‡~~Revised March 2011~~

8.6~~7~~ The student will

a) ~~investigate and~~ solve problems, including practical problems, involving volume and surface area of rectangular prisms, cylinders, cones, and square-based pyramids; and

b) describe how changing one measured attribute of a rectangular prism~~figure~~ affects the volume and surface area.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The area of the base of a circular cone is πr^2. • ~~The surface area of a right circular cone is $\pi r^2 + \pi rl$, where l represents the slant height of the cone.~~ [Reordered] • ~~The volume of a cone is $\frac{1}{3}\pi r^2 h$, where h is the height and πr^2 is the area of the base.~~ [Reordered] • ~~The surface area of a right circular cylinder is $2\pi r^2 + 2\pi rh$.~~ • The volume of a cylinder is the area of the base of the cylinder multiplied by the height. • ~~The surface area of a rectangular prism is the sum of the areas of the six faces.~~ [Reordered] • ~~The volume of a rectangular prism is calculated by multiplying the length, width and height of the prism.~~ [Reordered] • The surface area of a pyramid is the sum of the areas of the triangular faces and the area of the base, found by using the formula $S.A. = \frac{1}{2}lp + B$ where l is the slant height, p is the perimeter of the base and B is the area of the base. [Reordered] [Moved from middle column] • The volume of a pyramid is found by using the formula $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height. [Same as middle column] • The volume of prisms and cylinders are found by determining the area of the base and multiplying that by the height. [Moved from middle column]		

8.6~~7~~ The student will

a) ~~investigate and~~ solve problems, including practical problems, involving volume and surface area of rectangular prisms, cylinders, cones, and square-based pyramids; and

b) describe how changing one measured attribute of a rectangular prism~~figure~~ affects the volume and surface area.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The formula for determining the volume of cones and cylinders are similar. For cones you are determining $\frac{1}{3}$ of the volume of the cylinder with the same size base and height. [Moved from middle column] • ~~A prism is a solid figure that has a congruent pair of parallel bases and faces that are parallelograms. The surface area of a prism is the sum of the areas of the faces and bases.~~ [Reordered] • When the measurement of one attribute of a prism is changed through multiplication or division the volume increases by the same factor ~~that the~~by which the attribute increased ~~by~~. [Included from middle column] For example, if a prism has a volume of 2 x 3 x 4, the volume is 24 cubic units. However, if one of the attributes ~~are~~ is doubled, the volume doubles. That is, 2 x 3 x 8, the volume is 48 cubic units or 24 doubled. • When one attribute of a rectangular prism is changed through multiplication or division, the surface area is affected differently than the volume. The formula for surface area of a prism is $2(lw) + 2(lh) + 2(wh)$ when the width is doubled then four faces are affected. For example, a prism with length = 7 in., width = 4 in., and height = 3 in. would have a surface area of $2(7 \cdot 4) + 2(7 \cdot 3) + 2(4 \cdot 3)$ or 122 square inches. If the height is doubled to 6 inches then the surface area would be found by $2(7 \cdot 4) + 2(7 \cdot 6) + 2(4 \cdot 6)$ or 188 square inches • ~~The volume of a prism is *Bh*, where *B* is the area of the base and h is the height of the prism.~~ • ~~Nets are two-dimensional representations that can be folded into three-dimensional figures.~~ [Reordered]		

[Included in Measurement and Geometry Strand Introduction]

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students expand the informal experiences they have had with geometry in the elementary grades and develop a solid foundation for the exploration of geometry in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.~~
- ~~Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provides experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.~~
- ~~Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.~~
- ~~Students explore and develop an understanding of the Pythagorean Theorem. Mastery of the use of the Pythagorean Theorem has far-reaching impact on subsequent mathematics learning and life experiences.~~

~~The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.~~

- ~~**Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.~~
- ~~**Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)~~
- ~~**Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)~~
- ~~**Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)~~

- ~~**Level 4: Deduction.** Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)~~

8.7~~8~~ The student will

a) given a polygon, apply transformations, to include translations, rotations, reflections, and dilations, in the coordinate ~~to~~ plane ~~figures~~; and

b) identify practical applications of transformations.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• Translations, rotations and reflections maintain congruence between the preimage and image but change location. Dilations by a scale factor other than 1 produce an image that is not congruent to the pre-image but is similar. Rotations and reflections change the orientation of the image. [Moved from middle column] • A transformation of a figure, called preimage, changes the size, shape, and/or position of the figure to a new figure, called the image. • A transformation of preimage point *A* can be denoted as the image *A'*(read as "*A* prime"). [Reordered] • A rotation ~~of a geometric figure is a clockwise or counterclockwise turn of the figure around~~is a transformation in which an image is formed by rotating the preimage about a fixed point, called the center of rotation. The ~~point~~ center of rotation may or may not be a point on the ~~figure~~preimage. ~~The fixed point is called the *center of rotation*.~~ • A reflection ~~of a geometric figure moves all of the points of the figure across an axis~~is a transformation in which an image is formed by reflecting the preimage over a line called the line of reflection. Each point on the ~~reflected figure~~image is the same distance from the ~~axis~~ line of reflection as the corresponding point in the ~~original figure~~preimage. • A translation ~~of a geometric figure~~is a transformation in which an image is formed by moving every ~~moves all the~~ points~~s~~ on the ~~figure~~ preimage the same distance in the same direction.	• ~~How does the transformation of a figure on the coordinate grid affect the congruency, orientation, location and symmetry of an image? Translations, rotations and reflections maintain congruence between the preimage and image but change location. Dilations by a scale factor other than 1 produce an image that is not congruent to the pre-image but is similar. Rotations and reflections change the orientation of the image.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Demonstrate the reflection of a polygon over the vertical or horizontal axis on a coordinate grid.~~ • ~~Demonstrate 90°, 180°, 270°, and 360°clockwise and counterclockwise rotations of a figure on a coordinate grid. The center of rotation will be limited to the origin.~~ • ~~Demonstrate the translation of a polygon on a coordinate grid.~~ • ~~Demonstrate the dilation of a polygon from a fixed point on a coordinate grid~~ • Given a pre-image in the coordinate plane, identify the coordinates of the image of a right triangle or a rectangle that has been rotated 90° clockwise or counterclockwise about the origin. (a) [Moved from 7.8] • Given a pre-image in the coordinate plane, identify the coordinates of the image of a polygon that has been reflected over the *x*- or *y*-axis. (a) • Given a pre-image in the coordinate plane, identify the coordinates of the image of a right triangle or a rectangle that has been dilated. The center of the dilation will be the origin. (a) [Moved from 7.8] • Given a pre-image in the coordinate plane, identify the coordinates of the image of a polygon that has been translated and reflected over the *x*-or *y*-axis. (a)

8.7~~8~~ The student will

a) given a polygon, apply transformations, to include translations, rotations, reflections, and dilations, in the coordinate ~~to~~ plane ~~figures~~; and

b) identify practical applications of transformations.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A dilation ~~of a geometric figure~~ is a transformation ~~that changes the size of a figure by a scale factor to create a similar figure~~in which an image is formed by enlarging or reducing the preimage proportionally by a scale factor from the center of dilation. A dilation of a figure and the original figure are similar. The center of dilation may or may not be on the preimage. • Practical applications may include, but are not limited to, the following: – A rotation of the hour hand of a clock from 2:00 to 3:00 shows a turn of 30° clockwise; – A reflection of a boat in water shows an image of the boat flipped upside down with the water line being the line of reflection; – A translation of a figure on a wallpaper pattern shows the same figure slid the same distance in the same direction; and – A dilation of a model airplane is the production model of the airplane. ~~• The image of a polygon is the resulting polygon after a transformation. The preimage is the original polygon before the transformation.~~ • ~~A transformation of preimage point *A* can be denoted as the image *A′* (read as "A prime").~~ [Reordered]		• Sketch the image of a polygon that has been translated vertically, horizontally, or a combination of both. (a) • Sketch the image of a right triangle or a rectangle that has been rotated 90° clockwise or counterclockwise about the origin. (a) [Moved from 7.8] • Sketch the image of a polygon that has been reflected over the *x*- or *y*-axis. (a) • Sketch the image of a dilation of a right triangle or a rectangle limited to a scale factor of $\frac{1}{4}$, $\frac{1}{2}$, 2, 3, or 4. The center of the dilation will be the origin. (a) [Moved from 7.8] • Sketch the image of a polygon that has been translated and reflected over the *x*- or *y*-axis. (a) • Identify the type of translation in a given example. (a, b) • Identify practical applications of transformations including, but not limited to, tiling, fabric, and wallpaper designs, art and scale drawings. (b) ~~• Identify the type of transformation in a given example.~~

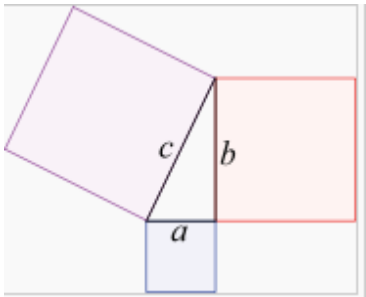
8.8~~9~~ The student will construct a three-dimensional model, given the top or bottom, side, and front views.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A three-dimensional object can be represented as a two-dimensional model with views of the object from different perspectives. [Moved from middle column.] • Three-dimensional models of geometric solids can be used to understand perspective and provide tactile experiences in determining two-dimensional perspectives. • Three-dimensional models of geometric solids can be represented on isometric paper. • The top view is a mirror image of the bottom view.	• ~~How does knowledge of two-dimensional figures inform work with three-dimensional objects?~~ ~~It is important to know that a three-dimensional object can be represented as a two-dimensional model with views of the object from different perspectives.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct three-dimensional models, given the top or bottom, side, and front views. • Identify three-dimensional models given a two-dimensional perspective. • Identify the two-dimensional perspective from the top or bottom, side and front view, given a three-dimensional model.

8.9~~10~~ The student will

a) verify the Pythagorean Theorem; and

b) apply the Pythagorean Theorem.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The Pythagorean Theorem is essential for solving problems involving right triangles. • The relationship between the sides and angles of right triangles are useful in many applied fields. • In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the legs (altitude and base). This relationship is known as the Pythagorean Theorem: $a^2 + b^2 = c^2$.  • The Pythagorean Theorem is used to ~~find~~ determine the measure of any one of the three sides of a right triangle if the measures of the other two sides are known. • Whole number triples that are the measures of the sides of right triangles, such as (3,4,5), (6,8,10), (9,12,15), and (5,12,13), are commonly known as Pythagorean triples. • The hypotenuse of a right triangle is the side opposite the right angle.	• ~~How can the area of squares generated by the legs and the hypotenuse of a right triangle be used to verify the Pythagorean Theorem?~~ ~~For a right triangle, the area of a square with one side equal to the measure of the hypotenuse equals the sum of the areas of the squares with one side each equal to the measures of the legs of the triangle.~~ [Included in US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Identify the parts of a right triangle (the hypotenuse and the legs).~~ [Moved to 5.13] • Verify a triangle is a right triangle given the measures of its three sides. (a) • Verify the Pythagorean Theorem, using diagrams, concrete materials, and measurement. (a) • ~~Find~~ Determine the measure of a side of a right triangle, given the measures of the other two sides. (b) • Solve practical problems involving right triangles by using the Pythagorean Theorem. (b)

8.9~~10~~ The student will

a) verify the Pythagorean Theorem; and

b) apply the Pythagorean Theorem.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The hypotenuse of a right triangle is always the longest side of the right triangle. • The legs of a right triangle form the right angle.		

8.10~~11~~ The student will solve ~~practical~~ area and perimeter problems, including practical problems, involving composite plane figures.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A plane figure is any two-dimensional shape that can be drawn in a plane. • A polygon is a ~~simple,~~ closed plane ~~figure with sides that are~~comprised of at least three line segments that do not cross. • The perimeter ~~of a polygon~~ is the path or distance around ~~the~~ any plane figure. • Circumference is the distance around or "perimeter" of a circle. • The area of a composite figure can be found by subdividing the figure into triangles, rectangles, squares, trapezoids, circles, and semi-circles, calculating their areas, and combining the areas together by addition and/or subtraction based upon the given composite figure. [Moved from middle column] • ~~The area of any composite figure is based upon knowing how to find the area of the composite parts such as triangles and rectangles.~~ [Included in bullet above] • The area of a rectangle is computed by multiplying the lengths of two adjacent sides ($A = lw$). • The area of a triangle is computed by multiplying the measure of its base by the measure of its height and dividing the product by 2 or multiplying by $\frac{1}{2}$ ($A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$). • The area of a parallelogram is computed by multiplying the measure of its base by the measure of its height ($A = bh$). • The area of a trapezoid is computed by taking the average of the measures of the two bases and multiplying this average by the height [$A = \frac{1}{2}h(b_1 + b_2)$]. • The area of a circle is computed by multiplying ~~Pi~~ pi times the radius squared ($A = \pi r^2$).	• ~~How does knowing the areas of polygons assist in calculating the areas of composite figures?~~ • ~~The area of a composite figure can be found by subdividing the figure into triangles, rectangles, squares, trapezoids and semi-circles, calculating their areas, and adding the areas together.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Subdivide a figure into triangles, rectangles, squares, trapezoids and semicircles. ~~Estimate~~ Determine the area of subdivisions and combine to determine the area of the composite figure. • Use the attributes of the subdivisions to determine the perimeter and circumference of a figure. • Apply perimeter, circumference and area formulas to solve practical problems involving composite plane figures.

8.10~~11~~ The student will solve ~~practical~~ area and perimeter problems, including practical problems, involving composite plane figures.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• The circumference of a circle is found by multiplying ~~Pi~~ pi by the diameter or multiplying ~~Pi~~ pi by 2 times the radius ($C = \pi d$ or $C = 2\pi r$). • The area of a semicircle is half the area of a circle with the same diameter or radius. • ~~An estimate of the area of a composite figure can be made by subdividing the polygon into triangles, rectangles, squares, trapezoids and semicircles, estimating their areas, and adding the areas together.~~		

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students develop an awareness of the power of data analysis and probability by building on their natural curiosity about data and making predictions.~~
- ~~Students explore methods of data collection and use technology to represent data with various types of graphs. They learn that different types of graphs represent different types of data effectively. They use measures of center and dispersion to analyze and interpret data.~~
- ~~Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability.~~
- ~~Students explore experimental and theoretical probability through experiments and simulations by using concrete, active learning activities.~~

In the middle grades, students develop an awareness of the power of data analysis and the application of probability through fostering their natural curiosity about data and making predictions.

The exploration of various methods of data collection and representation allows students to become effective at using different types of graphs to represent different types of data. Students use measures of center and dispersion to analyze and interpret data.

Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability. Through experiments and simulations, students build on their understanding of the Fundamental Counting Principle from elementary mathematics to learn more about probability in the middle grades.

8.11~~12~~ The student will

a) compare and contrast the probability of independent and dependent events; and [Moved from 6.16]~~with and without replacement.~~

b) determine probabilities~~y~~ for independent and dependent events.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A simple event is one event (e.g., pulling one sock out of a drawer and examining the probability of getting one color). [Moved from 6.16] • The probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes (sample space). [Moved from 6.16] • The probability of an event occurring can be represented as a ratio or the equivalent fraction, decimal, or percent.). [Moved from 6.16] • The probability of an event occurring is a ratio between 0 and 1. A probability of 0 means the event will never occur. A probability of 1 means the event will always occur. • Two events are either dependent or independent. • If the outcome of one event does not influence the occurrence of the other event, they are called independent. If events are independent, then the second event occurs regardless of whether or not the first occurs. For example, the first roll of a number cube does not influence the second roll of the number cube. Other examples of independent events are, but not limited to: flipping two coins; spinning a spinner and rolling a number cube; flipping a coin and selecting a card; and choosing a card from a deck, replacing the card and selecting again. • The probability of ~~three~~ two independent events is found by using the following formula:	~~• How are the probabilities of dependent and independent events similar? Different?~~ ~~If events are dependent then the second event is considered only if the first event has already occurred. If events are independent, then the second event occurs regardless of whether or not the first occurs.~~ [Moved to US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine whether two events are independent or dependent. (a) [Moved from 6.16] • Compare and contrast the probability of independent and dependent events. (a) [Moved from 6.16] • Determine the probability of no more than two ~~three~~ independent events. (b) • Determine the probability of ~~no more than~~ two dependent events ~~without replacement~~. (b) ~~• Compare the outcomes of events with and without replacement.~~

8.11~~12~~ The student will

a) compare and contrast the probability of independent and dependent events; and [Moved from 6.16]~~with and without replacement.~~

b) determine probabilities~~y~~ for independent and dependent events.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
P(A and B) = P(A)·P(B) ~~$P(A and B and C) = P(A) \cdot P(B) \cdot P(C)$~~ Example: When rolling a six-sided~~three~~ number cube~~s~~ and flipping a coin, simultaneously, what is the probability of rolling a 3 on one cube and getting a heads on the coin~~, a 4 on one cube, and a 5 on the third~~? *P(3 and heads)* = $\frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$ ~~$P(3 and 4 and 5) = P(3) \cdot P(4) \cdot P(5) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$~~ • If the outcome of one event has an impact on the outcome of the other event, the events are called dependent. If events are dependent then the second event is considered only if the first event has already occurred. For example, if you choose a blue card from a set of nine different colored cards that has a total of four blue cards and you do not place that blue card back in the set before selecting a second card, the chance of selecting a blue card the second time is diminished because there are now only three blue cards remaining in the set~~ if you are dealt a King from a deck of cards and you do not place the King back into the deck before selecting a second card, the chance of selecting a King the second time is diminished because there are now only three Kings remaining in the deck~~. Other examples of dependent events include~~are~~, but not limited to: choosing two marbles from a bag but not replacing the first after selecting it; and picking a sock out of a drawer and then picking a second sock without replacing the first. • The probability of two dependent events is found by using the following formula: *P(A and B) = P(A)·P(B after A)* ~~$P(A and B) = P(A) \cdot P(B after A)$~~ Example: You have a bag holding a blue ball, a red ball,		

8.11~~12~~ **The student will**

a) compare and contrast the probability of independent and dependent events; and [Moved from 6.16]~~with and without replacement.~~

b) determine probabilities~~y~~ for independent and dependent events.

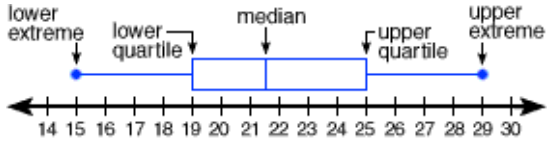
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
and a yellow ball. What is the probability of picking a blue ball out of the bag on the first pick then *without* replacing the blue ball in the bag, picking a red ball on the second pick? $P(\text{blue } \textit{and} \text{ red}) = P(\text{blue}) \cdot P(\text{red } \textit{after} \text{ blue}) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$ ~~$P(\text{blue } \textit{and} \text{ red}) = P(\text{blue}) \cdot P(\text{red } \textit{after} \text{ blue}) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$~~		

8.12 The student will

a) represent data in boxplots; [Moved from A.10]

b) make observations and inferences about data represented in boxplots; and [Moved from A.10]

c) compare boxplots with the same data represented in histograms, stem and leaf plots, and line plots. [Moved from A.10]

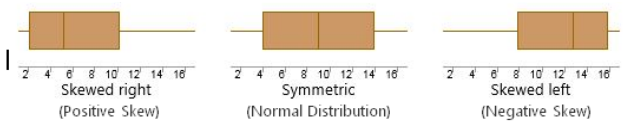
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A boxplot (box-and-whisker plot) is a convenient and informative way to represent single-variable (univariate) data. • Boxplots are effective at giving an overall impression of the shape, center, and spread of the data. It does not show a distribution in as much detail as a stem and leaf plot or a histogram. • A boxplot will allow you to quickly identify key statistical measures (median and range) and major concentrations of data and can be used to analyze a set of data. • A boxplot uses a rectangle to represent the middle half of a set of data and lines (whiskers) at both ends to represent the remainder of the data. The median is marked by a vertical line inside the rectangle. • The five critical points in a boxplot are lower extreme (minimum), lower quartile, median, upper quartile, and upper extreme (maximum). Each of these points represents the bounds for the four quartiles. In the example below, the lower extreme is 15, the lower quartile is 19, the median is 21.5, the upper quartile is 25, and the upper extreme is 29.  • The range is the difference between the upper extreme and the lower extreme. The interquartile range is the difference between the upper quartile and the lower quartile. Using the example above, the range is 14 or 29-15. The interquartile range is 6 or 25 – 19.		**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect, analyze, display, and interpret a data set of no more than 20 items, using boxplots. (a) • Identify and describe the lower extreme (minimum), upper extreme (maximum), median, upper quartile, lower quartile, range, and interquartile range. (a) • Given a data set the student will make inferences about the distribution (dispersion) of the data using a boxplot. (b) • Compare data represented in a boxplot with the same data represented in a histogram, a stem and leaf plot and/or a line plot. (c)

8.12 **The student will**

a) represent data in boxplots; [Moved from A.10]

b) make observations and inferences about data represented in boxplots; and [Moved from A.10]

c) compare boxplots with the same data represented in histograms, stem and leaf plots, and line plots. [Moved from A.10]

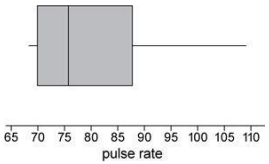
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• When there is an odd number of data values in a set of data, the median will not be considered when calculating the lower and upper quartiles. Example: Calculate the median, lower extreme, and upper extreme for the following data values: 3 5 6 7 8 9 11 13 13 Median: 8 Lower Quartile: 5.5 Upper Quartile: 12 • If all four sections of a boxplot are about the same length then the data is symmetric. A longer section indicates the data are more widely spread than those in shorter sections. If the median does not lie in the center of the box, the distribution can be considered skewed. For example,  • Each of the above boxplots illustrates a different skewness pattern. If most of the observations are concentrated on the low end of the scale, the distribution is skewed right. If most of the observations are concentrated on the high end of the scale, the distribution is skewed left. If a distribution is symmetric, the observations will be evenly split at the median, as shown above in the middle figure.		

8.12 **The student will**

a) represent data in boxplots; [Moved from A.10]

b) make observations and inferences about data represented in boxplots; and [Moved from A.10]

c) compare boxplots with the same data represented in histograms, stem and leaf plots, and line plots. [Moved from A.10]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• In the pulse rate example, many students incorrectly interpret that longer sections contain more data and shorter ones contain less. It is important to remember that roughly the same amount of data is in each section. The numbers in the left whisker (lowest of the data) are spread less widely than those in the right whisker.  • Comparisons, predictions, and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. • The information displayed in different graphs may be examined to determine how data are or are not related, differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inferences).		

8.13 The student will

a) represent data in scatterplots~~make comparisons, predictions, and inferences, using information displayed in graphs~~; and

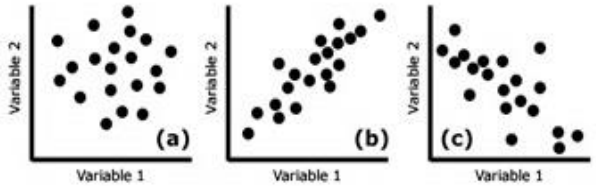
b) make observations and inferences about data represented in~~construct and analyze~~ scatterplots.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A scatterplot illustrates the relationship between two sets of numerical data represented by two variables (bivariate data). A scatterplot consists of points on the coordinate plane. The coordinates of the point represent the measures of the two attributes of the point. [Reordered] • In a scatterplot, each point may represent an independent and dependent variable. The independent variable is graphed on the horizontal axis and the dependent is graphed on the vertical axis. [Reordered] • Scatterplots can be used to predict linear trends and estimate a line of best fit. [Reordered] • A line of best fit helps in making interpretations and predictions about the situation modeled in the data set [Moved from middle column] ~~• Comparisons, predictions, and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions.~~ ~~• The information displayed in different graphs may be examined to determine how data are or are not related, ascertaining differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inferences).~~ • A scatter plot can suggest various kinds of linear correlations between variables. For example, weight and height, where weight would be on *y*-axis and height would be on the *x*-axis. Linear correlations may be positive (rising), negative (falling), or null (uncorrelated). If the pattern of points slopes from lower left to upper right, it indicates a positive correlation between the variables being studied. If the pattern of points slopes from upper left to lower right, it indicates a negative correlation. [Similar to middle column]	~~• Why do we estimate a line of best fit for a scatterplot?~~ ~~A line of best fit helps in making interpretations and predictions about the situation modeled in the data set.~~ [Moved to US] ~~• What are the inferences that can be drawn from sets of data points having a positive relationship, a negative relationship, and no relationship?~~ ~~Sets of data points with positive relationships demonstrate that the values of the two variables are increasing. A negative relationship indicates that as the value of the independent variable increases, the value of the dependent variable decreases.~~ [Included in US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect, organize, and ~~interpret~~ represent a data set of no more than 20 items using scatterplots. (a) • Interpret data in a scatterplot. Predict from the trend an estimate of the line of best fit with a drawing. (b) • Interpret a set of data points in a scatterplot as having a positive relationship, a negative relationship, or no relationship. (b)

8.13 The student will

a) represent data in scatterplots~~make comparisons, predictions, and inferences, using information displayed in graphs~~; and

b) make observations and inferences about data represented in~~construct and analyze~~ scatterplots.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
For example: The following scatterplots illustrate how patterns in data values may indicate linear correlation. No correlation Positive correlation Negative correlation  • Correlation between variables does not necessarily imply causation. For example, as the temperature at the beach increases, the sales at an ice cream store increase. If data were collected for these two variables, a positive correlation would exist, however, there is no causal relationship between the variables. (i.e., the temperature outside does not cause ice cream sales to increase, but there is a relationship between the two) • ~~A scatterplot illustrates the relationship between two sets of data. A scatterplot consists of points. The coordinates of the point represent the measures of the two attributes of the point.~~ • ~~Scatterplots can be used to predict trends and estimate a line of best fit.~~ • ~~In a scatterplot, each point is represented by an independent and dependent variable. The independent variable is graphed on the horizontal axis and the dependent is graphed on the vertical axis.~~ [Reordered] • Correlation between variables is not always linear, and may model other functions that are studied in high school and college level mathematics.		

~~In the middle grades, the focus of mathematics learning is to~~

- ~~build on students' concrete reasoning experiences developed in the elementary grades;~~
- ~~construct a more advanced understanding of mathematics through active learning experiences;~~
- ~~develop deep mathematical understandings required for success in abstract learning experiences; and~~
- ~~apply mathematics as a tool in solving practical problems.~~

~~Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.~~

- ~~Students extend their knowledge of patterns developed in the elementary grades and through life experiences by investigating and describing functional relationships.~~
- ~~Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable*, *term*, *coefficient*, *exponent*, *expression*, *equation*, *inequality*, *domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.~~
- ~~Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.~~
- ~~Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.~~

Patterns, functions and algebra become a larger mathematical focus in the middle grades as students extend their knowledge of patterns developed in the elementary grades.

Students make connections between the numeric concepts of ratio and proportion and the algebraic relationships that exist within a set of equivalent ratios. Students use variable expressions to represent proportional relationships between two quantities and begin to connect the concept of a constant of proportionality to rate of change and slope. Representation of relationships between two quantities using tables, graphs, equations, or verbal descriptions allow students to connect their knowledge of patterns to the concept of functional relationships. Graphing linear equations in two variables in the coordinate plane is a focus of the study of functions which continues in high school mathematics.

Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable*, *term*, *coefficient*, *exponent*, *expression*, *equation*, *inequality*, *domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades. Students learn to solve equations by using concrete materials. They expand their skills from one-step to multistep equations and inequalities through their application in practical situation.

8.14~~4~~ The student will

a) ~~apply the order of operations to~~ evaluate an algebraic expression~~s~~ for given replacement values of the variables[Moved from Computation and Estimation strand 8.4]; and

b) simplify expressions in one variable.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• An expression is a representation of a quantity. It contains numbers, variables, and/or operation symbols. It does not have an equal sign (e.g., $\frac{3}{4}$, 5x, 140 - 38.2, -18 · 21, (5 + 2x) · 4). An expression cannot be solved. [Moved from 8.1] • A numerical expression contains only numbers, the operations symbols, and grouping symbols.[Moved from 8.1] • Expressions are simplified using the order of operations. [Included from 8.1] • The order of operations is as follows: – First, complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols, do the innermost operation first. – Second, evaluate all exponential expressions. – Third, multiply and/or divide in order from left to right. – Fourth, add and/or subtract in order from left to right. * Parentheses ( ), brackets [ ], braces {}, absolute value $\lvert\ \ \rvert$ (i.e., $\lvert 3(-5+2)\rvert - 7$), and the division bar (i.e., $\frac{3+4}{5+6}$) should be treated as grouping symbols. [Reordered and rewritten] • Properties of real numbers can be used to express simplification. – The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 5.6 + -4 = -4 + 5.6). – The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 5.6 · -4 = -4 · 5.6). – The associative property of addition states that regrouping the addends does not change the sum [e.g., 5.6 + (-4 + 3) = (5.6 + -4) + 3].	~~• What is the role of the order of operations when evaluating expressions?~~ ~~Using the order of operations assures only one correct answer for an expression.~~ [Moved to US]	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to • Use the order of operations and apply the properties of real numbers to evaluate expressions for the given replacement values of the variables. ~~Substitute numbers for variables in algebraic expressions and simplify the expressions by using the order of operations.~~ Exponents are ~~positive and~~ limited to whole numbers- and bases are limited to integers~~less than 4~~. Square roots are limited to perfect squares. Limit the number of replacements to no more than three per expression. (a) • Represent algebraic expressions using concrete materials and pictorial representations. Concrete materials may include colored chips or algebra tiles. (a) • Simplify algebraic expressions in one variable. Expressions may need to be expanded (using the distributive property) or require combining like terms to simplify. Expressions will include only linear and numeric terms. Coefficients and numeric terms may be rational. (b) ~~• Apply the order of operations to evaluate formulas. Problems will be limited to positive exponents. Square roots may be included in the expressions but limited to perfect squares.~~ [Incorporated into bullet above]

8.14~~4~~ **The student will**

a) ~~apply the order of operations to~~ evaluate an algebraic expressions for given replacement values of the variables[Moved from Computation and Estimation strand 8.4]; and

b) simplify expressions in one variable.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5.6 · (-4 · 3) = (5.6 · -4) · 3]. – Subtraction and division are neither commutative nor associative. – The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true. ~~and vice versa.~~ Distributive property examples: • -5 (x + 7) = (-5 · x) + (-5· 7) = -5x - 35 • -5 (x – 7) = (-5 · x) – (-5 · 7) = -5x + 35 Converse of the distributive property examples: • -5x – 35 = (-5 · x) + (-5 · 7) = -5 (x + 7), or • -5x + 35 = (-5 · x) – (-5 · 7) = -5 (x – 7) – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division. – The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real number (e.g., $\frac{5}{3} + 0 = \frac{5}{3}$). – The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number (e.g., $\sqrt{8} \cdot 1 = \sqrt{8}$).		

8.14~~4~~ The student will

a) ~~apply the order of operations to~~ evaluate an algebraic expressions for given replacement values of the variables[Moved from Computation and Estimation strand 8.4]; and

b) simplify expressions in one variable.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– Inverses are numbers that combine with other numbers and result in identity elements [e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$]. – The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., 5.6 + (–5.6) = 0). – The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., $-4 \cdot -\frac{1}{4} = 1$). – Zero has no multiplicative inverse. – The multiplicative property of zero states that the product of any real number and zero is zero (e.g., 7 · 0 = 0 and 0 · 7 = 0). • Division by zero is not a possible mathematical operation. It is undefined. • A power of a number represents repeated multiplication of the number. For example, $(-5)^4$ means (–5) · (–5) · (–5) · (−5). The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In this example, (–5) is the base, and 4 is the exponent. The product is 625. Notice that the base appears inside the grouping symbols. The meaning changes with the removal of the grouping symbols. For example, -5^4 means 5 · 5 · 5 · 5 negated which results in a product of -625. The expression $-(5)^4$ means to take the opposite of 5 · 5 · 5 · 5 which is -625. Students should be exposed to all three representations.[Moved from 8.1]		

8.14~~4~~ **The student will**

a) ~~apply the order of operations to~~ evaluate an algebraic expression~~s~~ for given replacement values of the variables[Moved from Computation and Estimation strand 8.4]; and

b) simplify expressions in one variable.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• An ~~A~~algebraic expression~~s~~ is an expression that contains ~~use operations with algebraic symbols (~~variables~~)~~ and numbers. • Algebraic expressions are evaluated by substituting numbers for variables and applying the order of operations to simplify the resulting numeric expression. • ~~The order of operations is as follows: Complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols (embedded), do the innermost operation first. Evaluate all exponential expressions. Multiply and/or divide in order from left to right. Add and/or subtract in order from left to right.~~ • ~~* Parentheses ( ), brackets [ ], braces { }, the absolute value~~ $\lvert\ \rvert$~~, division/fraction bar –, and the square root symbol~~ $\sqrt{\ }$ ~~should be treated as grouping symbols.~~ [Reordered and rewritten]		

8.15~~14~~ The student will ~~make connections between any two representations (tables, graphs, words, and rules) of a given relationship.~~
a) determine whether a given relation is a function; and
b) determine the domain and range of a function. [Moved from 8.17]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A relation is any set of ordered pairs. For each first member, there may be many second members. • ~~A function is a relation in which there is one and only one second member for each first member.~~ • A function is a relation between a set of inputs, called the domain, and a set of outputs, called the range, with the property that each input is related to exactly one output. • As a table of values, a function has a unique value assigned to the second variable for each value of the first variable. In the "not a function" example, the input value 1 has two output values assigned to it, so the example is not a function. function x / y: 2, 3; 1, 5; 0, 3; -1, -3 not a function x / y: 2, 3; 1, 5; 0, 4; 1, -3 • As a set of ordered pairs, a function has a unique or different y-value assigned to each x-value. For example, the set of ordered pairs, {(1, 2), (2, 4), (3, 2), (4, 8)} is a function. This set of ordered pairs, {(1, 2), (2, 4), (3, 2), (2, 3)}, is not a function since the x-value of two has two different y-values. • As a graph, a function is any curve (including straight lines) such that any vertical line would pass through the curve only once. • Some relations are functions; all functions are relations. • Graphs of functions can be discrete or continuous.	• ~~What is the relationship among tables, graphs, words, and rules in modeling a given situation? Any given relationship can be represented by all four.~~ [Included in US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine whether a relation, represented by a set of no more than 10 ordered pairs, a table, or a graph of discrete points is a function. (a) • Identify the domain and range of a function represented as a set of ordered pairs, a table, or a graph of discrete points. (b) • ~~Graph in a coordinate plane ordered pairs that represent a relation.~~ • ~~Describe and represent relations and functions, using tables, graphs, words, and rules. Given one representation, students will be able to represent the relation in another form.~~ • ~~Relate and compare different representations for the same relation.~~

8.15~~14~~ The student will ~~make connections between any two representations (tables, graphs, words, and rules) of a given relationship.~~
a) determine whether a given relation is a function; and
b) determine the domain and range of a function. [Moved from 8.17]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Use Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• In a discrete function graph there are separate, distinct points. You would not use a line to connect these points on a graph. The points between the plotted points have no meaning and cannot be interpreted. For example, the number of pets per household represents a discrete function since you cannot have a fraction of a pet. • In a graph of continuous function every point in the domain can be interpreted therefore it is possible to connect the points on the graph with a continuous line as every point on the line answers the original question being asked. For example, the temperature of a liquid represents a continuous function. • Functions ~~can~~ may be represented as ordered pairs, tables, graphs, equations, physical models, or in words. Any given relationship can be represented using multiple representations. • As a graph, a function in grade 8 mathematics is any curve (including straight lines) such that any vertical line would pass through the function only once (vertical line test). • A discussion about determining whether a continuous graph of a relation is a function using the vertical line test should occur in grade 8, but will be explored further in Algebra I. • The domain is the set of all the input values for the independent variable or *x*-values (first number in an ordered pair). The domain of a continuous linear function is all real numbers. • The range is the set of all the output values for the dependent variable or *y*-values (second number in an ordered pair). The range of a continuous linear function is all real numbers. In the examples that follow, the domain is {-1, 0, 1, 2} and the range is {-3, 3, 5}.		

8.16 The student will

a) identify the slope and *y*-intercept of a linear function given the graph or given the equation in *y = mx + b* form;

b) graph a linear function given the equation in y = *mx + b* form; and~~two variables~~.

c) distinguish between proportional and non-proportional situations modeled by linear functions.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A linear function~~equation~~ is an equation in two variables whose graph is a straight line, a type of continuous function~~ (see SOL 8.14)~~. • A linear function ~~equation~~ represents a situation with a constant rate. For example, when driving at a rate of 35 mph, the distance increases as the time increases, but the rate of speed remains the same. • Graphing a linear function ~~equation~~ requires determining a table of ordered pairs by substituting into the equation values for one variable and solving for the other variable, plotting the ordered pairs in the coordinate plane, and connecting the points to form a straight line. • The axes of a coordinate plane are generally labeled *x* and *y*; however, any letters may be used that are appropriate for the function. • In a graph of a continuous function every point in the domain can be interpreted therefore it is possible to connect the points on the graph with a continuous line as every point on the line answers the original question being asked. [Moved from 8.14] • The context of a problem may determine if it is appropriate for ordered pairs representing a linear relationship to be connected by a straight line. If the independent variable (*x*) represents a discrete quantity (e.g., number of people, number of tickets, etc.) then it is not appropriate to connect the ordered pairs with a straight line when graphing. If the independent variable (*x*) represents a continuous quantity (e.g., amount of time, temperature, etc.) then it is appropriate to connect the ordered pairs with a straight line when graphing.	• ~~What types of real life situations can be represented with linear equations? Any situation with a constant rate can be represented by a linear equation.~~ [Included in US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Construct a table of ordered pairs by substituting values for *x* in a linear equation to find values for *y*.~~ • ~~Plot in the coordinate plane ordered pairs (*x*, *y*) from a table.~~ • ~~Connect the ordered pairs to form a straight line (a continuous function).~~ • Given a function in the form *y = mx + b*, identify the slope and *y*-intercept. (a) • Given a graph of a function, identify the slope and *y*-intercept. (a) • Identify the dependent and independent variable, domain and range of a linear function. (b)[Moved from 8.17] • Graph in a coordinate plane ordered pairs that represent a linear function. (b) • Relate and compare different representations for the same linear function. (a, b) • Given a linear function in the form *y = mx + b*, graph the function. (b) • Recognize and describe a line with a slope that is positive, negative, or 0. (a, b) • Identify~~Interpret the unit rate of the~~ proportional and non-proportional relationships, given its representation as a linear function. (c) ~~graphed as the slope of the~~

8.16 The student will

a) identify the slope and *y*-intercept of a linear function given the graph or given the equation in *y* = *mx* + *b* form;

b) graph a linear function given the equation in y = *mx* + *b* form; and~~two variables~~.

c) distinguish between proportional and non-proportional situations modeled by linear functions.

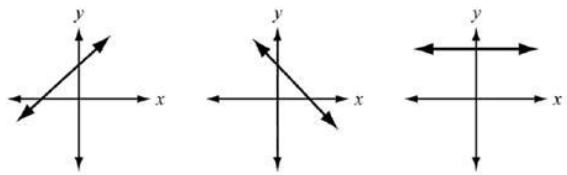
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
Example: The function $y = 7x$ represents the cost in dollars (y) for x tickets to an event. The domain of this function would be discrete and would be represented by discrete points on a graph. Not all values for x could be represented. and connecting the points would not be appropriate. Example: The function $y = -2.5x + 20$ represents the number of gallons of water (y) remaining in a 20-gallon tank being drained for x number of minutes. The domain in this function would be continuous, There would be an x-value representing any point in time until the tank is drained so connecting the points to form a straight line would be appropriate (note: the context of the problem limits the values that x can represent). • Functions can be represented as ordered pairs, tables, graphs, equations, physical models, or in words. Any given relationship can be represented using multiple representations. [Moved from 8.14] • Interpret the equation $y = mx + b$ as defining a linear function whose graph (solution) is a straight line. • The y-intercept (b) is the point at which the graph of a function intersects with the y-axis. It is represented as an ordered pair, (0, b). For example, in the function $y = -3x + 2$, the slope is -3 or $\frac{-3}{1}$ and the y-intercept is 2 or (0, 2), a point on the y-axis. • Slope (m) represents the rate of change in a linear function or the "steepness" and direction of the line. The slope of a line is a rate of change, a ratio describing the vertical change to the horizontal change. slope = $\frac{change\ in\ y}{change\ in\ x} = \frac{vertical\ change}{horizontal\ change}$		~~graph, and compare two different proportional relationships represented in different ways.~~[‡] [‡]~~Revised March 2011~~

8.16 The student will

a) identify the slope and *y*-intercept of a linear function given the graph or given the equation in *y* = *mx* + *b* form;

b) graph a linear function given the equation in y = *mx* + *b* form; and~~two variables.~~

c) distinguish between proportional and non-proportional situations modeled by linear functions.

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– A line is increasing if it goes up from left to right. The slope is positive, i.e., $m > 0$. – A line is decreasing if it goes down from left to right. The slope is negative, i.e., $m < 0$. – A horizontal line has zero slope, i.e., $m = 0$.  A line with a ***positive slope*** slants up to the right. A line with a ***negative slope*** slants down to the right. A line with a ***slope of 0*** is horizontal. • A discussion about lines with undefined slope (vertical lines) should occur with students in grade 8 mathematics to compare undefined slope to lines with a defined slope. Further exploration of this concept will occur in Algebra I. • A proportional relationship between two variables can be represented by a linear function $y = mx$ that passes through the point (0, 0) and thus has a *y*-intercept of 0. The variable *y* results from *x* being multiplied by *m*, the rate of change or slope. • The linear function $y = x + b$ represents a linear function that is non-proportional. The variable *y* results from the value *b* being added to *x*. In this linear relationship, there is a y-intercept of *b* and the constant rate of change or slope would be 1. When there a slope other than 1, there is a coefficient in front of the *x* term, which represents the constant rate of change, or slope.		

8.17~~15~~ The student will

~~a)~~ solve multistep linear equations in one variable with the variable on one and both ~~two~~ sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable.~~;~~

~~b) solve two-step linear inequalities and graph the results on a number line; and~~ [Moved to 8.18]

~~c) identify properties of operations used to solve an equation.~~ [Incorporated into 8.14, 8.17 and 8.18 EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A multistep equation may include, but not be limited to equations such as the following:~~is an equation that requires more than one different mathematical operation to solve.~~ $2x + 1 = \frac{-x}{4}$; $-3(2x + 7) = \frac{1}{-2}x$; $2x + 7 - 5x = 27$; $-5x - (x + 3) = -12$. • ~~A two-step inequality is defined as an inequality that requires the use of two different operations to solve (e.g., $3x - 4 > 9$).~~ [Moved to 8.17] • An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an equal sign (e.g., $\frac{3}{4}$, $5x$, 140 - 38.2, 18 · 21, 5 + x.) • An expression that contains a variable is a variable expression. A variable expression is like a phrase: as a phrase does not have a verb, so an expression does not have an equal sign (=). An expression cannot be solved. • A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used when the numbers are unknown. For example, the verbal expression "a number multiplied by 5" could be represented by the variable expression "$n \cdot 5$" or "$5n$". • An algebraic expression is a variable expression that contains at least one variable (e.g., $2x - 3$). • A verbal sentence is a complete word statement (e.g., "The sum of two consecutive integers is thirty-five." could be represented by "$n + (n + 1) = 35$"). • An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., $2x + 3 = -4x + 1$).	• ~~How does the solution to an equation differ from the solution to an inequality?~~ • ~~While a linear equation has only one replacement value for the variable that makes the equation true, an inequality can have more than one.~~ [Moved to 8.18 US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Solve two- to four-step linear equations in one variable using concrete materials, pictorial representations, and paper and pencil illustrating the steps performed.~~ • Apply properties of real numbers and properties of equality to solve multistep linear equations in one variable (up to four steps). Coefficients and numeric terms may be rational. Equations may contain expressions that need to be expanded (using the distributive property) or require collecting like terms to solve. • ~~Solve two-step inequalities in one variable by showing the steps and using algebraic sentences.~~ [Moved to 8.18] • ~~Graph solutions to two-step linear inequalities on a number line.~~ [Moved to 8.18] • Write verbal expressions and sentences as algebraic expressions and equations. • Write algebraic expressions and equations as verbal expressions and sentences. • Solve practical problems that require the solution of a multistep linear equation. • Confirm algebraic solutions to linear equations in one variable. • ~~Identify properties of operations used to solve an~~

8.17~~15~~ The student will

~~a)~~ solve multistep linear equations in one variable with the variable on one and both ~~two~~ sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable.~~;~~

~~b) solve two-step linear inequalities and graph the results on a number line; and~~ [Moved to 8.18]

~~c) identify properties of operations used to solve an equation.~~ [Incorporated into 8.14, 8.17 and 8.18 EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• In an equation, the equal sign indicates that the value of the expression on the left is ~~the same as~~equivalent to the value of the expression on the right. • ~~To maintain equality, an operation that is performed on one side of an equation must be performed on the other side.~~ • Like terms are terms that have the same variables and exponents. The coefficients do not need to match (e.g., 12x and -5x; 45 and -5 $\frac{2}{3}$; 9y, -51y and $\frac{4}{9}y$.) [Reordered] • Like terms may be added or subtracted using the distributive and other properties. For example, – $4.6y - 5y = (-4.6 - 5)y = -9.6y$ – $w + w - 2w = (1 + 1)\,w - 2w = 2w - 2w = (2 - 2)\,w = 0$ $w = 0$ • ~~When both expressions of an inequality are multiplied or divided by a negative number, the inequality sign reverses.~~ [Moved to 8.18] • Real-world problems can be interpreted, represented, and solved using linear equations in one variable. • Properties of real numbers and properties of equality can be used to solve equations, justify solutions and express simplification. – The commutative property of~~for~~ addition states that changing the order of the addends does not affect~~change~~ the sum (e.g., 5.6 + -4 = -4 + 5.6). – The commutative property of~~for~~ multiplication states that changing the order of the factors does not affect~~change~~ the product (e.g., 5.6 · -4 = -4 · 5.6). – The associative property of addition states that regrouping		~~equation from among:~~ • ~~the commutative properties of addition and multiplication;~~ • ~~the associative properties of addition and multiplication;~~ • ~~the distributive property;~~ • ~~the identity properties of addition and multiplication;~~ • ~~the zero property of multiplication;~~ • ~~the additive inverse property; and~~ • ~~the multiplicative inverse property.~~ [Moved to US]

8.17~~15~~ The student will

~~a)~~ solve multistep linear equations in one variable with the variable on one and both ~~two~~ sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable.~~;~~

~~b) solve two-step linear inequalities and graph the results on a number line; and~~ [Moved to 8.18]

~~c) identify properties of operations used to solve an equation.~~ [Incorporated into 8.14, 8.17 and 8.18 EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
the addends does not change the sum [e.g., 5 + (4 + 3) = (5 + 4) + 3]. – The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5.6 · (-4 · 3) = (5.6 · -4) · 3]. – Subtraction and division are neither commutative nor associative. – The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true. ~~and vice versa.~~ Distributive property examples: • -5 (x + 7) = (-5 · x) + (-5· 7) = -5x - 35 • -5 (x – 7) = (-5 · x) – (-5 · 7) = -5x + 35 Converse of the distributive property examples: • -5x – 35 = (-5 · x) + (-5 · 7) = -5 (x + 7), or • -5x + 35 = (-5 · x) – (-5 · 7) = -5 (x – 7) – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division. – The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real number (e.g., $\frac{5}{3}$ ~~5~~ + 0 = $\frac{5}{3}$~~5~~). – The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number		

8.17~~15~~ The student will

~~a)~~ solve multistep linear equations in one variable with the variable on one and both ~~two~~ sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable.~~;~~

~~b) solve two-step linear inequalities and graph the results on a number line; and~~ [Moved to 8.18]

~~c) identify properties of operations used to solve an equation.~~ [Incorporated into 8.14, 8.17 and 8.18 EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
(e.g., $\sqrt{8} \cdot 1 = \sqrt{8}$). – Inverses are numbers that combine with other numbers and result in identity elements [e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$]. – The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero [e.g., 5.6 + (–5.6) = 0]. – The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (~~or~~ reciprocal) ~~always~~ equals one (e.g., $4 \cdot \frac{1}{4} = 1$). – Zero has no multiplicative inverse. – The multiplicative property of zero states that the product of any real number and zero is zero. (e.g., $7 \cdot 0 = 0$ and $0 \cdot 7 = 0$). – Division by zero is not a possible mathematical ~~arithmetic~~ operation. It is undefined. – The addition property of equality states that any number must be added to both sides of an equation to maintain equality. – The subtraction property of equality states that any number must be subtracted from both sides of an equation to maintain equality. – The multiplication property of equality states that an equation must be multiplied on both sides by the same number to maintain equality.		

8.17~~15~~ The student will

~~a)~~ solve multistep linear equations in one variable with the variable on one and both ~~two~~ sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable.~~;~~

~~b) solve two-step linear inequalities and graph the results on a number line; and~~ [Moved to 8.18]

~~c) identify properties of operations used to solve an equation.~~ [Incorporated into 8.14, 8.17 and 8.18 EKS]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– The division property of equality states that an equation must be divided on both sides by the same number to maintain equality. – The substitution property of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality. (e.g., if x = -5 + 8 then x = 3. Thus, 3 was substituted for -5 + 8 since they are equivalent). • ~~Combining like terms means to combine terms that have the same variable and the same exponent (e.g., 8x + 11 – 3x can be 5x +11 by combining the like terms of 8x and –3x).~~ [Reordered and combined in other bullets]		

8.18 The student will solve multistep linear inequalities in one variable with the variable on one and both sides of the inequality symbol, including practical problems, and graph the results on a number line. [Moved from 8.15b]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
• A multistep inequality may include, but not be limited to inequalities such as the following: $2x + 1 > \frac{-x}{4}$; $-3(2x + 7) \leq \frac{1}{-2}x$; $2x + 7 - 5x < 27$; $-5x - (x + 3) > -12$. • When both expressions of an inequality are multiplied or divided by a negative number, the inequality sign reverses. [Moved from 8.15] • A solution to an inequality is the value or set of values that can be substituted to make the inequality true. • In an inequality, there can be more than one value for the variable that makes the inequality true. There can be many solutions. (i.e., $x + 4 > -3$ then the solutions is $x > -7$. This means that x can be any number greater than -7. A few solutions might be -6.5, -3, 0, 4, 25, etc.) • Real-world problems can be modeled and solved using linear inequalities. • The properties of real numbers and properties of inequality can be used to solve inequalities, justify solutions, and express simplification. – The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 5.6 + -4 = -4 + 5.6). – The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 5.6 · -4 = -4 · 5.6). – The associative property of addition states that regrouping the addends does not change the sum [e.g., 5.6 + (-4 + 3) = (5.6 + -4) + 3].		**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Apply properties of real numbers and properties of inequality to solve linear inequalities (up to four steps) in one variable. Coefficients and numeric terms may be rational. Inequalities may contain expressions that need to be expanded (using the distributive property) or require collecting like terms to solve. • Graph solutions to multistep linear inequalities on a number line. • Translate verbal statements, including practical situations, into algebraic inequalities. • Solve practical problems involving multistep linear inequalities in one variable. • Identify a numerical value(s) that is part of the solution set of a given inequality.

8.18 The student will solve multistep linear inequalities in one variable with the variable on one and both sides of the inequality symbol, including practical problems, and graph the results on a number line. [Moved from 8.15b]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5.6 · (-4 · 3) = (5.6 · -4) · 3]. – Subtraction and division are neither commutative nor associative. – The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true. Distributive property examples: • -5 (x + 7) = (-5 · x) + (-5· 7) = -5x - 35 • -5 (x – 7) = (-5 · x) – (-5 · 7) = -5x + 35 Converse of the distributive property examples: • -5x – 35 = (-5 · x) + (-5 · 7) = -5 (x + 7), or • -5x + 35 = (-5 · x) – (-5 · 7) = -5 (x – 7) – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division. – The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real number (e.g., $\frac{5}{3} + 0 = \frac{5}{3}$). – The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number (e.g., $\sqrt{8} \cdot 1 = \sqrt{8}$). – Inverses are numbers that combine with other numbers and result in identity elements [e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$].		

8.18 The student will solve multistep linear inequalities in one variable with the variable on one and both sides of the inequality symbol, including practical problems, and graph the results on a number line. [Moved from 8.15b]

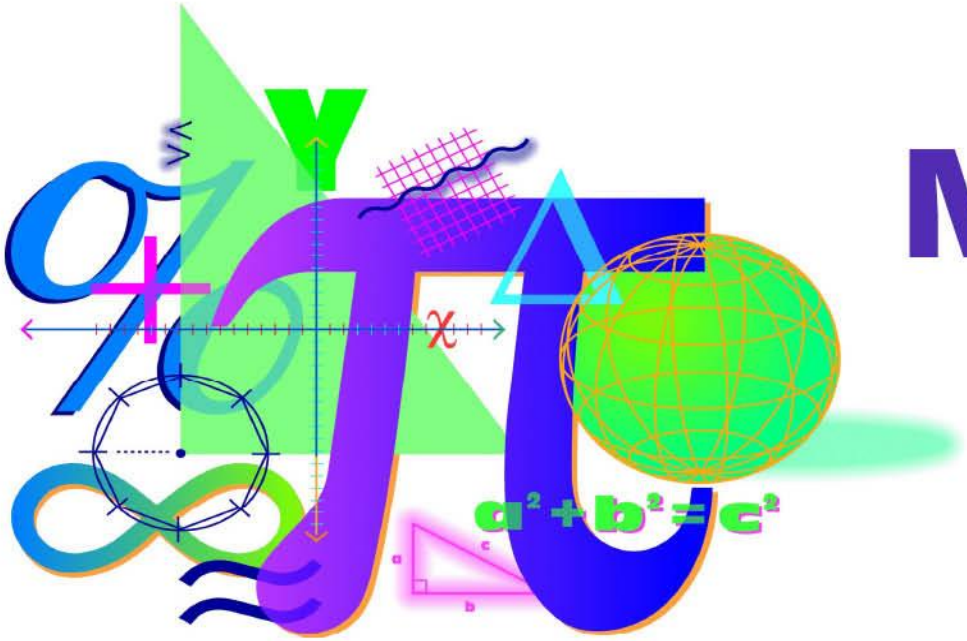
UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., 5.6 + (–5.6) = 0). – The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., $-4 \cdot -\frac{1}{4} = 1$). – Zero has no multiplicative inverse. – The multiplicative property of zero states that the product of any real number and zero is zero (e.g., $7 \cdot 0 = 0$ and $0 \cdot 7 = 0$). – Division by zero is not a possible mathematical operation. It is undefined. – The addition property of inequality states that any number must be added to both sides of an inequality to maintain the inequality. – The subtraction property of inequality states that any number must be subtracted from both sides of an inequality to maintain inequality. – The multiplication property of inequality states that if an inequality is multiplied on both sides by the same positive number the inequality symbol remains the same. But if an inequality is multiplied on both sides by the same negative number, the inequality symbol reverses. For example, to solve $\frac{1}{3}x \geq 9$, multiply both sides of the inequality by 3, so $x \geq 27$. To solve $-\frac{1}{3}x \geq 9$, multiply both sides of the inequality by -3, so $x \leq -27$.		

8.18 The student will solve multistep linear inequalities in one variable with the variable on one and both sides of the inequality symbol, including practical problems, and graph the results on a number line. [Moved from 8.15b]

UNDERSTANDING THE STANDARD ~~(Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	ESSENTIAL KNOWLEDGE AND SKILLS
– The division property of inequality states that if an inequality is divided on both sides by the same positive number the inequality symbol remains the same. But if an inequality is divided on both sides by the same negative number, the inequality symbol reverses. For example, $-4y < 20$. Divide both sides of the inequality by -4. So $y > -5$. • The substitution property of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality (e.g., if $x = -5 + 8$ then $x = 3$. Thus, 3 was substituted for -5 + 8 since they are equivalent).		

~~8.17 The student will identify the domain, range,~~ [Moved to 8.15b]~~independent variable, or dependent~~ [Moved to 8.16 EKS]~~variable in a given situation.~~

~~UNDERSTANDING THE STANDARD (Background Information for Instructor Only)~~	~~ESSENTIAL UNDERSTANDINGS~~	~~ESSENTIAL KNOWLEDGE AND SKILLS~~
~~• The domain is the set of all the input values for the independent variable in a given situation.~~ ~~• The range is the set of all the output values for the dependent variable in a given situation.~~ ~~• The independent variable is the input value.~~ ~~• The dependent variable depends on the independent variable and is the output value.~~ ~~• Below is a table of values for finding the circumference of circles, $C = \pi d$, where the value of π is approximated as 3.14.~~ Diameter / Circumference 1 in. / 3.14 in. 2 in. / 6.28 in. 3 in. / 9.42 in. 4 in. / 12.56 in. ~~• The independent variable, or input, is the diameter of the circle. The values for the diameter make up the domain.~~ ~~• The dependent variable, or output, is the circumference of the circle. The set of values for the circumference makes up the range.~~	~~• What are the similarities and differences among the terms domain, range, independent variable and dependent variable?~~ ~~The value of the dependent variable changes as the independent variable changes. The domain is the set of all input values for the independent variable. The range is the set of all possible values for the dependent variable.~~	~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~ ~~• Apply the following algebraic terms appropriately: *domain*, *range*, *independent variable*, and *dependent variable*.~~ ~~• Identify examples of domain, range, independent variable, and dependent variable.~~ ~~• Determine the domain of a function.~~ ~~• Determine the range of a function.~~ ~~• Determine the independent variable of a relationship.~~ ~~• Determine the dependent variable of a relationship.~~



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Algebra I

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> "Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

~~TOPIC~~STRAND: EXPRESSIONS AND OPERATIONS

ALGEBRA I
STANDARD A.1

The student will

a) represent verbal quantitative situations algebraically; and
b) evaluate ~~these~~ algebraic expressions for given replacement values of the variables.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
~~• Algebra is a tool for reasoning about quantitative situations so that relationships become apparent.~~[Moved to introduction] ~~• Algebra is a tool for describing and representing patterns and relationships.~~[Moved to introduction] • Mathematical modeling involves creating algebraic representations of quantitative ~~real-world~~practical situations. • The numerical value of an expression is dependent upon the values of the replacement set for the variables. • There are a variety of ways to compute the value of a numerical expression and evaluate an algebraic expression using order of operations. • The operations and the magnitude of the numbers in an expression impact the choice of an appropriate computational technique (mental mathematics, calculator, or paper and pencil). ~~An appropriate computational technique could be mental mathematics, calculator, or paper and pencil.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Translate between verbal quantitative situations ~~into~~ and algebraic expressions and equations ~~and vice versa~~. (a) • ~~Model real-world~~Represent practical situations with algebraic expressions in a variety of representations (concrete, pictorial, symbolic, verbal). (a) • Evaluate algebraic expressions using the order of operations that include absolute value, square roots, and cube roots for ~~a~~ given replacement ~~set~~values to include rational numbers, without rationalizing the denominator. (b) ~~• Evaluate expressions that contain absolute value, square roots, and cube roots.~~

~~TOPIC~~STRAND: EXPRESSIONS AND OPERATIONS

ALGEBRA I
STANDARD A.2

The student will perform operations on polynomials, including

a) applying the laws of exponents to perform operations on expressions;
b) adding, subtracting, multiplying, and dividing polynomials; and
c) factoring completely first- and second-degree binomials and trinomials in one ~~or two~~ variables. ~~Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.~~[Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
~~• The laws of exponents can be investigated using inductive reasoning.~~ ~~• A relationship exists between the laws of exponents and scientific notation.~~ [Rewritten and reordered] • Operations with polynomials can be represented concretely, pictorially, and symbolically. • Polynomial expressions can be used to model ~~real-world~~practical situations. ~~• The distributive property is the unifying concept for polynomial operations.~~ • Factoring reverses polynomial multiplication. • Trinomials may be factored by various methods including factoring by grouping. • ~~Some polynomials are p~~Prime polynomials ~~and~~ cannot be factored over the set of real numbers into two or more factors, each of lesser degree than the original polynomial. • Polynomial expressions can be used to define functions and these functions can be represented graphically. ~~• There is a relationship between the factors of any polynomial and~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Simplify monomial expressions and ratios of monomial expressions in which the exponents are integers, using the laws of exponents. (a) • Model sums, differences, products, and quotients of polynomials with concrete objects and their related pictorial and symbolic representations. (b) ~~• Relate concrete and pictorial manipulations that model polynomial operations to their corresponding symbolic representations.~~ • ~~Find~~ Determine sums and differences of polynomials. (b) • ~~Find~~ Determine products of polynomials. The factors ~~will have no more than five total~~should be limited to five or fewer terms (i.e., $(4x+2)(3x+5)$ represents four terms and $(x+1)(2x^2+x+3)$ represents five terms). (b) • ~~Find~~ Determine the quotient of polynomials, using a monomial or binomial divisor, or a completely factored divisor. (b)

~~TOPIC~~STRAND: EXPRESSIONS AND OPERATIONS

ALGEBRA I
STANDARD A.2

The student will perform operations on polynomials, including

a) **applying the laws of exponents to perform operations on expressions;**
b) **adding, subtracting, multiplying, and dividing polynomials; and**
c) **factoring completely first- and second-degree binomials and trinomials in one ~~or two~~ variables. ~~Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.~~[Moved to EKS]**

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
~~the *x*-intercepts of the graph of its related function.~~ • The laws of exponents can be applied to perform operations involving numbers written in scientific notation.[Rewritten and reordered] • For division of polynomials in this standard, instruction on the use of long or synthetic division is not required, but students may benefit from experiences with these methods, which become more useful and prevalent in the study of advanced levels of algebra.	• Factor completely first- and second-degree polynomials in one variable with integral coefficients. After factoring out the greatest common factor (GCF), leading coefficients should have no more than four factors. (c) • Factor and verify algebraic factorizations of polynomials with a graphing utility. (c) [Moved from standard] • ~~Identify prime polynomials.~~ • ~~Use the *x*-intercepts from the graphical representation of the polynomial to determine and confirm its factors.~~[Moved to A.7d EKS]

~~TOPIC~~STRAND: EXPRESSIONS AND OPERATIONS

ALGEBRA I
STANDARD A.3

The student will simplify~~express the~~

a) square roots of non-negative rational numbers and monomial algebraic expressions;
b) cube roots of ~~whole~~ rational numbers; and ~~the square root of a monomial algebraic expression in simplest radical form~~
c) numerical expressions containing square or cube roots.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A radical expression in Algebra I contains the square root symbol ($\sqrt{}$) or the cube root symbol ($\sqrt[3]{}$). • A square root of a number a is a number y such that $y^2 = a$. • A cube root of a number b is a number y such that $y^3 = b$. • A square root in simplest form is one in which the radicand ~~(argument)~~ has no perfect square factors other than one. • The inverse of squaring a number is determining the square root. • Any non-negative number other than a perfect square has a square root that lies between two consecutive whole numbers. • A cube root in simplest form is one in which the radicand~~argument~~ has no perfect cube factors other than one. • The cube root of a perfect cube is an integer. • The cube root of a nonperfect cube lies between two consecutive integers. • The inverse of cubing a number is determining the cube root. ~~In the real number system, the argument of a square root must be nonnegative while the argument of a cube root may be any real number.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Express square roots of a ~~whole~~ non-negative rational number in simplest form. (a) • Express the principal square root of a monomial algebraic expression in simplest form where variables are assumed to have positive values. (a) [Reordered] • Express the cube root of a ~~whole~~ rational number in simplest form. (b) • Simplify a numerical expression containing square or cube roots. (c) • Add, subtract, multiply, and divide two radical expressions limited to a numerical radicand without requiring rationalizing the denominator. (c) • ~~Express the principal square root of a monomial algebraic expression in simplest form where variables are assumed to have positive values.~~[Reordered]

~~TOPIC~~STRAND: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.4

The student will solve ~~multistep linear and quadratic equations in two variables, including~~

a) multistep linear equations in one variable algebraically[Reordered]~~solving literal equations (formulas) for a given variable~~ [Reordered];

~~b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets~~[Moved to EKS];

~~c~~b) ~~solving~~ quadratic equations in one variable algebraically~~ and graphically~~;

~~d~~c) ~~solving~~ literal equations for a specified variable[Reordered]~~multistep linear equations algebraically and graphically~~ [Reordered];

~~e~~d) ~~solving~~ systems of two linear equations in two variables algebraically and graphically; and

~~f~~e) ~~solving real-world~~practical problems involving equations and systems of equations.

~~Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.~~ [Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A solution to an equation is the value or set of values that can be substituted to make the equation true. • Each point on the graph of a linear or quadratic equation in two variables is a solution of the equation. • ~~The solution of an equation in one variable can be found by graphing the expression on each side of the equation separately and finding the *x*-coordinate of the point of intersection.~~ • ~~Real-world~~Practical problems can be interpreted, represented, and solved using linear and quadratic equations. • The process of solving linear and quadratic equations can be modeled in a variety of ways, using concrete, pictorial, and symbolic representations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine if a linear equation in one variable has one, an infinite number, or no solutions. (a) • ~~Solve a literal equation (formula) for a specified variable.~~[Reordered] • ~~Simplify expressions and solve equations, using~~Apply the field properties of ~~the~~ real numbers and properties of equality to ~~justify simplification~~ simplify expressions and solve~~ution~~ equations. (a,b) [Moved from A.4b] • Solve multistep linear equations in one variable algebraically. (b)[Reordered] • Solve quadratic equations in one variable algebraically. Solutions may be rational or irrational and given in simplest form. (b)

~~TOPIC~~STRAND: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.4

The student will solve ~~multistep linear and quadratic equations in two variables, including~~

a) multistep linear equations in one variable algebraically[Reordered]~~solving literal equations (formulas) for a given variable~~ [Reordered];

~~b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets~~[Moved to EKS];

~~c~~b) ~~solving~~ quadratic equations in one variable algebraically~~ and graphically~~;

~~d~~c) ~~solving~~ literal equations for a specified variable[Reordered]~~multistep linear equations algebraically and graphically~~ [Reordered];

~~e~~d) ~~solving~~ systems of two linear equations in two variables algebraically and graphically; and

~~f~~e) ~~solving real-world~~practical problems involving equations and systems of equations.

~~Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.~~ [Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Properties of real numbers and properties of equality ~~can be used to justify equation solutions and expression simplification~~are applied to simplify expressions and solve equations. • Field Properties of Real Numbers: – Associative Property of Addition – Associative Property of Multiplication – Commutative Property of Addition – Commutative Property of Multiplication – Identity Property of Addition (Additive Identity) – Identity Property of Multiplication (Multiplicative Identity) – Inverse Property of Addition (Additive Inverse) – Inverse Property of Multiplication (Multiplicative Inverse) – Distributive Property	• Confirm algebraic solutions to quadratic equations graphically. (b) [Reordered] • ~~Identify the roots or zeros of a quadratic function over the real number system as the solution(s) to the quadratic equation that is formed by setting the given quadratic expression equal to zero.Solve multistep linear equations in one variable.~~ [Reordered] • ~~Confirm algebraic solutions to linear and quadratic equations, using a graphing calculator.~~ [Reordered] • Solve a literal equation for a specified variable. (c) • Given a system of two linear equations in two variables that has a unique solution, solve the system by substitution or elimination to ~~find~~ identify the ordered pair which satisfies both equations. (d)

~~TOPIC~~STRAND: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.4

The student will solve ~~multistep linear and quadratic equations in two variables, including~~

a) multistep linear equations in one variable algebraically[Reordered]~~solving literal equations (formulas) for a given variable~~ [Reordered];

~~b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets~~[Moved to EKS];

~~c~~b) ~~solving~~ quadratic equations in one variable algebraically~~ and graphically~~;

~~d~~c) ~~solving~~ literal equations for a specified variable[Reordered]~~multistep linear equations algebraically and graphically~~ [Reordered];

~~e~~d) ~~solving~~ systems of two linear equations in two variables algebraically and graphically; and

~~f~~e) ~~solving real-world~~practical problems involving equations and systems of equations.

~~Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.~~ [Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Properties of Equality – Multiplicative Property of Zero – Zero Product Property – Reflexive Property – Symmetric Property – Transitive Property of Equality – Addition Property of Equality – Subtraction Property of Equality – Multiplication Property of Equality – Division Property of Equality – Substitution • Quadratic equations in one variable may be solved algebraically by factoring and applying properties of equality or by using the quadratic formula over the set of real numbers (Algebra I) or set of complex numbers (Algebra II).	• Given a system of two linear equations in two variables that has a unique solution, solve the system graphically by identifying the point of intersection. (d) • Solve and confirm algebraic solutions to a system of two linear equations using a graphing utility. (d) • Determine whether a system of two linear equations has one ~~solution~~, an infinite number, or no solutions~~, or infinite solutions~~. (d) • Write a system of two linear equations that models a ~~real-world~~practical situation. (e) • Interpret and determine the reasonableness of the algebraic or graphical solution of a system of two linear equations that models a ~~real-world~~practical situation. (e) • ~~Determine if a linear equation in one variable has one, an infinite~~

~~TOPIC~~STRAND: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.4

The student will solve ~~multistep linear and quadratic equations in two variables, including~~

a) multistep linear equations in one variable algebraically[Reordered]~~solving literal equations (formulas) for a given variable~~ [Reordered];

~~b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets~~[Moved to EKS];

~~c~~b) ~~solving~~ quadratic equations in one variable algebraically~~ and graphically~~;

~~d~~c) ~~solving~~ literal equations for a specified variable[Reordered]~~multistep linear equations algebraically and graphically~~ [Reordered];

~~e~~d) ~~solving~~ systems of two linear equations in two variables algebraically and graphically; and

~~f~~e) ~~solving real-world~~practical problems involving equations and systems of equations.

~~Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.~~ [Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The zero(s), root(s), or ~~the~~ *x*-intercept(s) of a ~~the quadratic~~ function are the ~~real root(s) or~~ solution(s) of ~~the~~ an ~~quadratic~~ equation ~~that is~~ formed by setting the ~~given~~ algebraic ~~quadratic~~ expression of the function equal to zero. • The zero(s), root(s), or *x*-intercept(s) of a function are the values of *x* when $f(x) = 0$. • Literal equations include formulas. • A system of linear equations with exactly one solution is characterized by the graphs of two lines whose intersection is a single point, and the coordinates of this point satisfy both equations. • A system of two linear equations with no solution is characterized by the graphs of two lines that are parallel that do not intersect.	~~number, or no solutions.~~ [Reordered]‡ ~~‡Revised March 2011~~

~~TOPIC~~STRAND: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.4

The student will solve ~~multistep linear and quadratic equations in two variables, including~~

a) multistep linear equations in one variable algebraically[Reordered]~~solving literal equations (formulas) for a given variable~~ [Reordered];

~~b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets~~[Moved to EKS];

~~c~~b) ~~solving~~ quadratic equations in one variable algebraically~~ and graphically~~;

~~d~~c) ~~solving~~ literal equations for a specified variable[Reordered]~~multistep linear equations algebraically and graphically~~ [Reordered];

~~e~~d) ~~solving~~ systems of two linear equations in two variables algebraically and graphically; and

~~f~~e) ~~solving real-world~~practical problems involving equations and systems of equations.

~~Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.~~ [Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A system of two linear equations having infinite solutions is characterized by two ~~graphs~~lines that coincide (the ~~graphs~~ lines ~~will~~ appear to be the graph of one line), and the coordinates of all points on the line satisfy both equations. These lines will have the same slope. • Systems of two linear equations can be used to model two ~~real-world~~practical conditions that must be satisfied simultaneously. • Equations and systems of equations can be used as mathematical models for ~~real-world~~practical situations. • The domain of a function may be restricted algebraically, graphically, or by the practical situation modeled by a function.	

~~TOPIC~~STRAND: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.4

The student will solve ~~multistep linear and quadratic equations in two variables, including~~

a) multistep linear equations in one variable algebraically[Reordered]~~solving literal equations (formulas) for a given variable~~ [Reordered];

~~b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets~~[Moved to EKS];

~~c~~b) ~~solving~~ quadratic equations in one variable algebraically~~ and graphically~~;

~~d~~c) ~~solving~~ literal equations for a specified variable[Reordered]~~multistep linear equations algebraically and graphically~~ [Reordered];

~~e~~d) ~~solving~~ systems of two linear equations in two variables algebraically and graphically; and

~~f~~e) ~~solving real-world~~practical problems involving equations and systems of equations.

~~Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.~~ [Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Solutions and intervals may be expressed in different formats, including set notation or using equations and inequalities. Examples may include:	

Equation/ Inequality	Set Notation
$x = 3$	$\{3\}$
$x = 3$ or $x = 5$	$\{3,5\}$
$0 \le x < 3$	$\{x \mid 0 \le x < 3\}$
$y \ge 3$	$\{y: y \ge 3\}$
Empty (null) set $\emptyset$	$\{\ \}$

~~TOPIC~~STRAND: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.5

The student will ~~solve multistep linear inequalities in two variables, including~~

a) solve~~ing~~ multistep linear inequalities in one variable algebraically and represent the solution graphically;

b) represent the solution of linear inequalities in two variables algebraically and graphically [Moved from SOL stem]~~justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets~~ [Included in EKS];

c) solve~~ing real-world~~ practical problems involving inequalities; and

d) solve~~ing~~ systems of inequalities algebraically and graphically.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A solution to an inequality is the value or set of values that can be substituted to make the inequality true. • The graph of the solutions of a linear inequality is a half-plane bounded by the graph of its related linear equation. Points on the boundary are included unless it is a strict inequality. • ~~Real-world~~Practical problems can be modeled and solved using linear inequalities. • Properties of inequality and order can be used to solve inequalities. • ~~Set builder notation may be used to represent solution sets of inequalities.~~ • The domain of a function may be restricted algebraically, graphically, or by the practical situation modeled by a function.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve multistep linear inequalities in one variable algebraically and represent the solution graphically. (a) • Apply the field properties of real numbers and properties of inequality to solve multistep linear inequalities in one variable algebraically. (a) ~~Justify steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers.~~ • Represent the solution of a linear inequality in two variables algebraically and graphically. (b) • Solve ~~real-world~~practical problems involving inequalities. (c) • Solve systems of two linear inequalities algebraically and graphically, including practical problems. (d) • Determine and verify algebraic solutions using a graphing utility. (a,b,c,d)

~~TOPIC~~STRAND: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.5

The student will ~~solve multistep linear inequalities in two variables, including~~

a) **solve~~ing~~ multistep linear inequalities in one variable algebraically and represent the solution graphically;**
b) **represent the solution of linear inequalities in two variables algebraically and graphically [Moved from SOL stem]~~justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets~~ [Included in EKS];**
c) **solve~~ing real world~~ practical problems involving inequalities; and**
d) **solve~~ing~~ systems of inequalities algebraically and graphically.**

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Solutions and intervals may be expressed in different formats, including set notation or using equations and inequalities. Examples may include:	

Equation/ Inequality	Set Notation
$x = 3$	$\{3\}$
$x = 3$ or $x = 5$	$\{3,5\}$
$0 \le x < 3$	$\{x \mid 0 \le x < 3\}$
$y \ge 3$	$\{y: y \ge 3\}$
Empty (null) set $\emptyset$	$\{\ \}$

- Field Properties of Real Numbers:
  - Associative Property of Addition
  - Associative Property of Multiplication
  - Commutative Property of Addition
  - Commutative Property of Multiplication
  - Identity Property of Addition (Additive Identity)
  - Identity Property of Multiplication (Multiplicative Identity)
  - Inverse Property of Addition (Additive Inverse)
  - Inverse Property of Multiplication (Multiplicative Inverse)
  - Distributive Property

~~TOPIC~~STRAND: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.5

The student will ~~solve multistep linear inequalities in two variables, including~~

a) **solve~~ing~~ multistep linear inequalities in one variable algebraically and represent the solution graphically;**
b) **represent the solution of linear inequalities in two variables algebraically and graphically [Moved from SOL stem]~~justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets~~ [Included in EKS];**
c) **solve~~ing real-world~~ practical problems involving inequalities; and**
d) **solve~~ing~~ systems of inequalities algebraically and graphically.**

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Properties of Inequality – Transitive Property of Inequality – Addition Property of Inequality – Subtraction Property of Inequality – Multiplication Property of Inequality – Division Property of Inequality – Substitution	

~~TOPIC~~STRAND: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.6

The student will ~~graph linear equations and linear inequalities in two variables, including~~ [Linear equations moved to A.6c and linear inequalities moved to A.5b]

a) determin~~ing~~e the slope of a line when given an equation of the line, the graph of the line, or two points on the line~~. Slope will be described as rate of change and will be positive, negative, zero, or undefined~~ [Included in US and EKS]; ~~and~~
b) write~~ing~~ the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line; and
c) graph linear equations in two variables. [Moved from SOL stem]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Changes in slope may be described by dilations or reflections or both. • Changes in the *y*-intercept may be described by translations. • Linear equations can be graphed using slope, *x*- and *y*-intercepts, and/or transformations of the parent function. • The slope of a line represents a constant rate of change in the dependent variable when the independent variable changes by a constant amount. • The equation of a line defines the relationship between two variables. • The graph of a line represents the set of points that satisfies the equation of a line. • A line can be represented by its graph or by an equation written in Standard Form, Slope-Intercept Form, or Point-Slope Form. • ~~The graph of the solutions of a linear inequality is a half-plane bounded by the graph of its related linear equation. Points on the boundary are included unless it is a strict inequality.~~ [Moved to A.5 US]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Graph linear equations and inequalities in two variables, including those that arise from a variety of real-world situations.~~ [Reordered] • ~~Use the parent function $y = x$ and describe transformations defined by changes in the slope or y-intercept.~~ [Reordered] • ~~Find~~ Determine the slope of the line, given the equation of a linear function. (a) • ~~Find~~ Determine the slope of a line, given the coordinates of two points on the line. (a) • ~~Find~~ Determine the slope of a line, given the graph of a line. (a) • Recognize and describe a line with a slope or rate of change that is positive, negative, zero, or undefined. (a) • ~~Use transformational graphing to investigate effects of changes in equation parameters on the graph of the equation.~~ • Write an equation of a line when given the graph of a line. (b)

~~TOPIC~~STRAND: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.6

The student will ~~graph linear equations and linear inequalities in two variables, including~~ [Linear equations moved to A.6c and linear inequalities moved to A.5b]

a) determin~~ing~~e the slope of a line when given an equation of the line, the graph of the line, or two points on the line~~. Slope will be described as rate of change and will be positive, negative, zero, or undefined~~ [Included in US and EKS]; ~~and~~
b) write~~ing~~ the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line; and
c) graph linear equations in two variables. [Moved from SOL stem]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Parallel lines have equal slopes. • The product of the slopes of perpendicular lines is -1 unless one of the lines has an undefined slope. • Slope can be described as a rate of change and will be positive, negative, zero, or undefined. [Moved from A.6a]	• Write an equation of a line when given two points on the line whose coordinates are integers. (b) • Write an equation of a line when given the slope and a point on the line whose coordinates are integers. (b) • Write an equation of a vertical line as $x = a$. (b) • Write the equation of a horizontal line as $y = c$. (b) • Write the equation of a line parallel or perpendicular to a given line through a given point. (b) • Graph a linear equation in two variables, including those that arise from a variety of practical situations. (c) [Reordered] • Use the parent function $y = x$ and describe transformations defined by changes in the slope or y-intercept. (c) [Reordered]

~~TOPIC~~STRAND: FUNCTIONS

ALGEBRA I
STANDARD A.7

The student will investigate and analyze ~~function (~~linear and quadratic ~~)~~function families and their characteristics both algebraically and graphically, including

a) **determining whether a relation is a function;**
b) **domain and range;**
c) **zeros ~~of a function~~;**
d) **~~*x*- and *y*-~~intercepts;**
e) **~~finding the~~ values of a function for elements in its domain; and**
f) **~~making~~ connections between any two ~~and among multiple~~ representations of functions including concrete, verbal, numeric, graphic, and algebraic.**

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~A set of data may be characterized by patterns, and those patterns can be represented in multiple ways.~~ • A relation is a function if and only if each element in the domain is paired with a unique element of the range. [Reordered] • Functions describe the relationship between two variables where each input is paired to a unique output. • Function families consist of a parent function and all transformations of the parent function. • The domain of a function is the set of all possible values of the independent variable. • The range of a function is the set of all possible values of the dependent variable. • For each *x* in the domain of *f*, *x* is a member of the input of the function *f*, *f(x)* is a member of the output of *f*, and the ordered pair [*x*, *f(x)*] is a member of *f*.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine whether a relation, represented by a set of ordered pairs, a table, mapping, or a graph is a function. (a) • Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically. Domains may be limited by problem context or in graphical representations. (b, c, d) • Use the *x*-intercepts from the graphical representation of a quadratic function to determine and confirm its factors. (d) [Moved from A.2 EKS] • For ~~each~~ any value, *x*, in the domain of *f*, ~~find~~ determine *f*(*x*). (e) • Represent relations and functions using concrete, verbal, numeric, graphic, and algebraic forms. Given one representation, ~~students will be able to~~ represent the relation in another form. (f)

~~TOPIC~~STRAND: FUNCTIONS

ALGEBRA I
STANDARD A.7

The student will investigate and analyze ~~function (~~linear and quadratic ~~)~~function families and their characteristics both algebraically and graphically, including

a) determining whether a relation is a function;
b) domain and range;
c) zeros ~~of a function~~;
d) ~~*x*- and *y*-~~intercepts;
e) ~~finding the~~ values of a function for elements in its domain; and
f) ~~making~~ connections between any two ~~and among multiple~~ representations of functions including concrete, verbal, numeric, graphic, and algebraic.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A value *x* in the domain of *f* is an *x*-intercept or a zero of a function *f* if and only if *f(x)* = 0.[Reordered] • ~~Graphs can be used as visual representations to investigate relationships between quantitative data.~~ • ~~Inductive reasoning may be used to make conjectures about characteristics of function families.~~ • ~~Each element in the domain of a relation is the abscissa of a point of the graph of the relation.~~ • ~~Each element in the range of a relation is the ordinate of a point of the graph of the relation.~~ • ~~A relation is a function if and only if each element in the domain is paired with a unique element of the range.~~ [Reordered] • ~~The values of *f*(*x*) are the ordinates of the points of the graph of *f*.~~ • ~~The object *f*(*x*) is the unique object in the range of the function *f* that is associated with the object *x* in the domain of *f*.~~	• Detect patterns in data and represent arithmetic ~~and geometric~~ [Included in AFDA.1 and AII.5] patterns algebraically. (f) • Investigate and analyze characteristics and multiple representations of functions with a graphing utility. (a,b,c,d,e,f)

~~TOPIC~~STRAND: FUNCTIONS

ALGEBRA I
STANDARD A.7

The student will investigate and analyze ~~function (~~linear and quadratic ~~)~~function families and their characteristics both algebraically and graphically, including

a) determining whether a relation is a function;
b) domain and range;
c) zeros ~~of a function~~;
d) ~~*x*- and *y*-~~intercepts;
e) ~~finding the~~ values of a function for elements in its domain; and
f) ~~making~~ connections between any two ~~and among multiple~~ representations of functions including concrete, verbal, numeric, graphic, and algebraic.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~For each *x* in the domain of *f*, *x* is a member of the input of the function *f*, *f*(*x*) is a member of the output of *f*, and the ordered pair [*x*, *f*(*x*)] is a member of *f*.~~ • ~~An object *x* in the domain of *f* is an *x*-intercept or a zero of a function *f* if and only if *f*(*x*) = 0.~~ [Reordered] • ~~Set builder notation may be used to represent domain and range of a relation.~~ • Given a polynomial function $f(x)$, the following statements are equivalent for any real number such that $f(k) = 0$: – *k* is a zero of the polynomial function $f(x)$, located at (*k*,0); – (*x* – *k*) is a factor of $f(x)$; – *k* is a solution or root of the polynomial equation $f(x) = 0$; and – the point (*k*,0) is an *x*-intercept for the graph of $y = f(x)$.	

~~TOPIC~~STRAND: FUNCTIONS

ALGEBRA I
STANDARD A.7

The student will investigate and analyze ~~function (~~linear and quadratic ~~)~~function families and their characteristics both algebraically and graphically, including

a) determining whether a relation is a function;
b) domain and range;
c) zeros ~~of a function~~;
d) ~~*x*- and *y*-~~intercepts;
e) ~~finding the~~ values of a function for elements in its domain; and
f) ~~making~~ connections between any two ~~and among multiple~~ representations of functions including concrete, verbal, numeric, graphic, and algebraic.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The *x*-intercept is the point at which the graph of a relation or function intersects with the *x*-axis. It can be expressed as a value or a coordinate. • The *y*-intercept is the point at which the graph of a relation or function intersects with the *y*-axis. It can be expressed as a value or a coordinate. • Solutions and intervals may be expressed in different formats, including set notation or using equations and inequalities. Examples may include:	

Equation/ Inequality	Set Notation
$x = 3$	$\{3\}$
$x = 3$ or $x = 5$	$\{3,5\}$
$0 \le x < 3$	$\{x \mid 0 \le x < 3\}$
$y \ge 3$	$\{y: y \ge 3\}$
Empty (null) set $\emptyset$	$\{\ \}$

• In an arithmetic pattern, the common difference is the value that is added to obtain the next value in the pattern.

~~TOPIC~~STRAND: ~~FUNCTIONS~~STATISTICS

ALGEBRA I
STANDARD A.8

The student, given a data set or practical situation, ~~in a real-world context~~, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Practical problems may be represented and solved by using direct variation or inverse variations. • A direct variation represents a proportional relationship between two quantities. The statement "*y* is directly proportional to *x*" is translated as $y = kx$. • The constant of proportionality (*k*) in a direct variation is represented by the ratio of the dependent variable to the independent variable and can be referred to as the constant of variation. • A direct variation can be represented by a line passing through the origin.[Reordered] • An inverse variation represents an inversely proportional relationship between two quantities. The statement "*y* is inversely proportional to *x*" is translated as $y = \frac{k}{x}$. • The constant of proportionality (*k*) in an inverse variation is represented by the product of the dependent variable and the independent variable and can be referred to as the constant of variation. • The value of the constant of proportionality is typically positive when applied in practical situations. • ~~A direct variation can be represented by a line passing through the~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given a data set or practical situation, ~~including a real-world situation~~, determine whether a direct variation exists. • Given a data set or practical situation, ~~including a real-world situation~~, determine whether an inverse variation exists. • Given a data set or practical situation, ~~W~~write an equation for a direct variation~~, given a set of data~~. • Given a data set or practical situation, ~~W~~write an equation for an inverse variation~~, given a set of data~~. • Given a data set or practical situation, ~~G~~graph an equation representing a direct variation~~, given a set of data~~.

~~TOPIC~~STRAND: ~~FUNCTIONS~~STATISTICS

ALGEBRA I
STANDARD A.8

The student, given a data set or practical situation, ~~in a real-world context~~, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
~~origin.~~[Reordered] • ~~Real-world problems may be modeled using direct and/or inverse variations.~~[Reordered]	

~~**TOPIC: STATISTICS**~~

~~**ALGEBRA I**~~
~~**STANDARD A.9**~~ **[**Included in AFDA.7 and Algebra II.11**]**

~~**The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.**~~

~~**ESSENTIAL UNDERSTANDINGS**~~	~~**ESSENTIAL KNOWLEDGE AND SKILLS**~~
• ~~Descriptive statistics may include measures of center and dispersion.~~ • ~~Variance, standard deviation, and mean absolute deviation measure the dispersion of the data.~~ • ~~The sum of the deviations of data points from the mean of a data set is 0.~~ • ~~Standard deviation is expressed in the original units of measurement of the data.~~ • ~~Standard deviation addresses the dispersion of data about the mean.~~ • ~~Standard deviation is calculated by taking the square root of the variance.~~ • ~~The greater the value of the standard deviation, the further the data tend to be dispersed from the mean.~~ • ~~For a data distribution with outliers, the mean absolute deviation may be a better measure of dispersion than the standard deviation or variance.~~ • ~~A z-score (standard score) is a measure of position derived from the mean and standard deviation of data.~~ • ~~A z-score derived from a particular data value tells how many standard deviations that data value is above or below the mean of the data set. It is positive if the data value lies above the mean and negative if the data value lies below the mean.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Analyze descriptive statistics to determine the implications for the real-world situations from which the data derive.~~ • ~~Given data, including data in a real-world context, calculate and interpret the mean absolute deviation of a data set.~~ • ~~Given data, including data in a real-world context, calculate variance and standard deviation of a data set and interpret the standard deviation.~~ • ~~Given data, including data in a real-world context, calculate and interpret z-scores for a data set.~~ • ~~Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.~~ • ~~Compare and contrast mean absolute deviation and standard deviation in a real-world context.~~

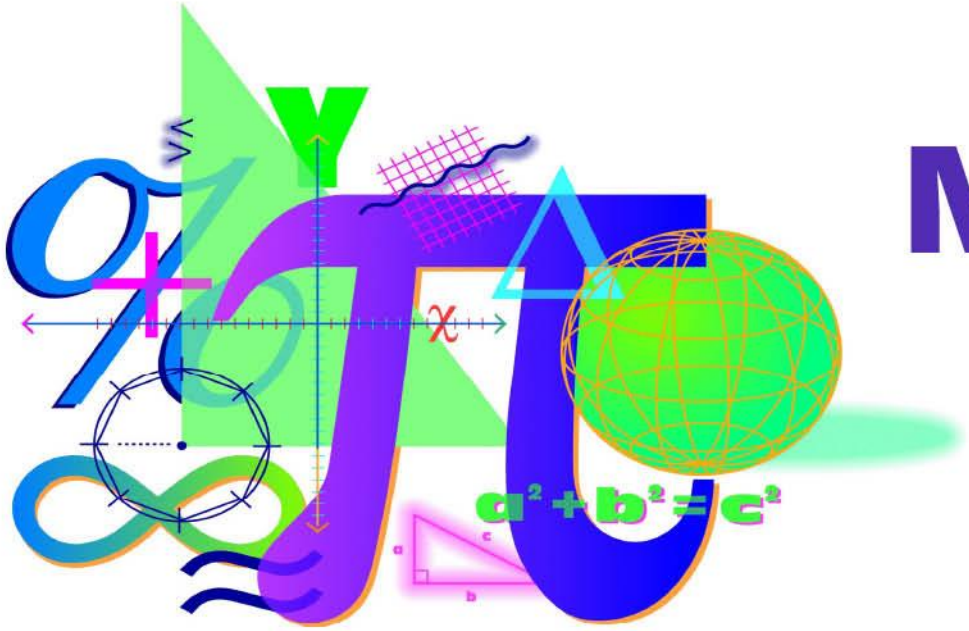
~~TOPIC: STATISTICS~~ **~~ALGEBRA I~~** **~~STANDARD A.10~~ [Moved to 8.12]** **~~The student will compare and contrast multiple univariate data sets, using box-and-whisker plots.~~**	
~~ESSENTIAL UNDERSTANDINGS~~	**~~ESSENTIAL KNOWLEDGE AND SKILLS~~**
• ~~Statistical techniques can be used to organize, display, and compare sets of data.~~ • ~~Box-and-whisker plots can be used to analyze data.~~	**~~The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to~~** • ~~Compare, contrast, and analyze data, including data from real-world situations displayed in box-and-whisker plots.~~

~~TOPIC~~STRAND: STATISTICS

ALGEBRA I
STANDARD A.9~~11~~

The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve ~~real-world~~practical problems, using mathematical models of linear and quadratic functions. ~~Mathematical models will include linear and quadratic functions.~~

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Data and scatterplots may indicate patterns that can be modeled with an algebraic equation. • The curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate. • Knowledge of transformational graphing using parent functions can be used to generate a mathematical model from a scatterplot that approximates the data. • ~~The g~~Graphing ~~calculator~~ utilities can be used to collect, organize, represent, and generate ~~determine the~~an equation of a curve of best fit for a set of data. • ~~The curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate.~~ [Reordered] • Many problems can be solved by using a mathematical model as an interpretation of a ~~real-world~~practical situation. The solution must then refer to the original ~~real-world~~practical situation. • ~~Considerations such as sample size, randomness, and bias should affect experimental design.~~ • Data that fit linear ($y = mx + b$) and quadratic ($y = ax^2 + bx + c$) functions arise from practical situations. • Rounding that occurs during intermediate steps of problem solving may reduce the accuracy of the final answer.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Write~~ Determine an equation ~~for~~ of a curve of best fit, using a graphing utility, given a set of no more than twenty data points in a table, a graph, or ~~real-world~~practical situation. • Make predictions ~~about unknown outcomes~~, using data, scatterplots, or the equation of the curve of best fit. • Solve practical problems involving an equation of the curve of best fit. • ~~Design experiments and collect data to address specific, real-world questions.~~ [Included in AFDA.8] • ~~Evaluate the reasonableness of a mathematical model of a real-world situation.~~



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Geometry

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

~~TOPIC~~STRAND: REASONING, LINES, AND TRANSFORMATIONS

GEOMETRY
STANDARD G.1

The student will use deductive reasoning to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include

a) **identifying the converse, inverse, and contrapositive of a conditional statement;**
b) **translating a short verbal argument into symbolic form; and**
c) **determining the validity of a logical argument. [Added from EKS] ~~using Venn diagrams to represent set relationships~~ [Moved to DM.12]~~; and~~**
~~d) using deductive reasoning.~~

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Inductive reasoning, deductive reasoning, and proof are critical in establishing general claims. • Deductive reasoning is the method that uses logic to draw conclusions based on definitions, postulates, and theorems. • Valid forms of deductive reasoning include the law of syllogism, the law of contrapositive, the law of detachment, and the identification of a counterexample. [Moved from EKS] • Symbolic notation is used to represent logical arguments, including the use of →, ↔, ~, ∴, ∧, and ∨. • The law of syllogism states that if $p \rightarrow q$ is true and $q \rightarrow r$ is true, then $p \rightarrow r$ is true. • The law of contrapositive states that if $p \rightarrow q$ is true and $\sim q$ is true, then $\sim p$ is true. • The law of detachment states that if $p \rightarrow q$ is true and p is true, then q is true. • A counterexample is used to show an argument is false.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the converse, inverse, and contrapositive of a conditional statement. (a) • Translate verbal arguments into symbolic form~~, such as $(p \rightarrow q)$ and $(\sim p \rightarrow \sim q)$~~ using the symbols of formal logic~~∴ ∧ ∨~~ .(b) [Reordered] ~~• Determine the validity of a logical argument.~~ [Reordered] • Determine the validity of a logical argument using ~~Use~~ valid forms of deductive reasoning~~, including the law of syllogism, the law of the contrapositive, the law of detachment, and counterexamples~~. (c) [Moved to US] • Determine that an argument is false using a counterexample. (c) ~~• Select and use various types of reasoning and methods of proof, as appropriate.~~ [Moved to US] ~~• Use Venn diagrams to represent set relationships, such as intersection and union.~~ [Moved to DM.12]

~~TOPIC~~STRAND: REASONING, LINES, AND TRANSFORMATIONS

GEOMETRY
STANDARD G.1

The student will use deductive reasoning to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include

a) identifying the converse, inverse, and contrapositive of a conditional statement;
b) translating a short verbal argument into symbolic form; and
c) determining the validity of a logical argument. [Added from EKS] ~~using Venn diagrams to represent set relationships~~ [Moved to DM.12]~~; and~~
~~d) using deductive reasoning.~~

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Inductive reasoning is the method of drawing conclusions from a limited set of observations. • Proof is a justification that is logically valid and based on initial assumptions, definitions, postulates, ~~and~~ theorems, and/or properties of equality. • Logical arguments consist of a set of premises or hypotheses and a conclusion. ~~• Euclidean geometry is an axiomatic system based on undefined terms (point, line and plane), postulates, and theorems.~~ • When a conditional ($p \rightarrow q$) and its converse ($q \rightarrow p$) are true, the statements can be written as a biconditional, ~~i.e.,~~*p iff q*; or *p if and only if q*; or $p \leftrightarrow q$. • Logical arguments that are valid may not be true. Truth and validity are not synonymous. • Conditional statements can be represented with Venn diagrams. • Formal proofs utilize symbols of formal logic to determine validity of a logical argument. [Moved from EKS]	~~• Interpret Venn diagrams.~~ [Moved to DM.12] • ~~Recognize and use the symbols of formal logic~~ [Reordered]~~, which include $\rightarrow$, $\leftrightarrow$, $\sim$, $\therefore$, $\wedge$, and $\vee$.~~[Moved to US]

~~TOPIC~~STRAND: REASONING, LINES, AND TRANSFORMATIONS

GEOMETRY
STANDARD G.2

The student will use the relationships between angles formed by two lines ~~cut~~ intersected by a transversal to

a) ~~determine whether~~prove two or more lines are parallel; and
b) ~~verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and~~
~~c)~~ solve ~~real-world~~ problems, including practical problems involving angles formed when parallel lines are ~~cut~~ intersected by a transversal.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Deductive or inductive reasoning is used in mathematical proofs. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods. • Parallel lines intersected by a transversal form angles with specific relationships. • Some angle relationships may be used when proving two lines intersected by a transversal are parallel. • If two parallel lines are intersected by a transversal, then: – corresponding angles are congruent; – alternate interior angles are congruent; – alternate exterior angles are congruent; – same-side (consecutive) interior angles are supplementary; and – same-side (consecutive) exterior angles are supplementary. • Deductive proofs can be used to show that two or more lines are parallel.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Use algebraic and coordinate methods as well as deductive proofs to verify whether two lines are parallel.~~[Included in G.3b] • Prove two or more lines are parallel given angle measurements expressed numerically or algebraically. (a) • Prove two lines parallel using deductive proofs given relationships between and among angles. (a) • Solve problems by using the relationships between pairs of angles formed by the intersection of two parallel lines and a transversal including corresponding angles, alternate interior angles, alternate exterior angles, ~~and~~ same-side (consecutive) interior angles, and same-side (consecutive) exterior angles. (b) • Solve ~~real-world~~ problems, including practical problems, involving intersecting and parallel lines ~~in a plane~~. (b)

~~TOPIC~~STRAND: REASONING, LINES, AND TRANSFORMATIONS

GEOMETRY
STANDARD G.2

The student will use the relationships between angles formed by two lines ~~cut~~ intersected by a transversal to

a) ~~determine whether~~prove two or more lines are parallel; and
b) ~~verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and~~
~~c)~~ solve ~~real-world~~ problems, including practical problems involving angles formed when parallel lines are ~~cut~~ intersected by a transversal.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The construction of the line parallel to a given line through a point not on the line can be justified using the angle relationships formed when two lines are intersected by a transversal. ~~The Parallel Postulate differentiates Euclidean from non-Euclidean geometries such as spherical geometry and hyperbolic geometry.~~	

~~TOPIC~~STRAND: REASONING, LINES, AND TRANSFORMATIONS

GEOMETRY
STANDARD G.3

The student will ~~use pictorial representations, including computer software, constructions, and coordinate methods, to~~ [Moved to US] solve problems involving symmetry and transformation. This will include

a) investigating and using formulas for ~~find~~determining distance, midpoint, and slope;
b) applying slope to verify and determine whether lines are parallel or perpendicular;
c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Transformations and combinations of transformations can be used to describe movement of objects in a plane.~~ [Reordered] • Symmetry and transformations can be explored with computer software, paper folding, and coordinate methods. • The distance formula is an application of the Pythagorean Theorem. • Geometric figures can be represented in the coordinate plane. • ~~Techniques for investigating symmetry may include paper folding, coordinate methods, and dynamic geometry software.~~ [Combined] • Parallel lines have the same slope. • The product of the slopes of perpendicular lines is -1 unless one of the lines has an undefined slope. • A transformation of a figure called a preimage changes the size, shape, and/or position of the figure to a new figure called the image. [Reordered]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the coordinates of the midpoint or endpoint of a segment, using the midpoint formula. (a) • Use a formula to ~~find~~determine the slope of a line. (a) • Apply the distance formula to determine the length of a line segment when given the coordinates of the endpoints. (a) [Reordered] • Compare the slopes to determine whether two lines are parallel, perpendicular, or neither. (b) • Determine whether a figure has point symmetry, line symmetry, both, or neither. (c)

~~TOPIC~~STRAND: REASONING, LINES, AND TRANSFORMATIONS

GEOMETRY
STANDARD G.3

The student will ~~use pictorial representations, including computer software, constructions, and coordinate methods, to~~ [Moved to US] solve problems involving symmetry and transformation. This will include

a) investigating and using formulas for ~~find~~determining distance, midpoint, and slope;
b) applying slope to verify and determine whether lines are parallel or perpendicular;
c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Transformations and combinations of transformations can be used to describe movement of objects in a plane. [Reordered] • The image of an object or function graph after an isomorphic transformation is congruent to the preimage of the object. • A rotation is an isomorphic transformation in which an image is formed by rotating the preimage about a point called the center of rotation. The center of rotation may or may not be on the preimage. Rotations may be more than 180^0. • A reflection is an isomorphic transformation in which an image is formed by reflecting the preimage over a line called the line of reflection. All corresponding points in the image are equidistant from the line of reflection. • A translation is an isomorphic transformation in which an image is formed by moving every point on the preimage the same distance in the same direction. • A dilation is a transformation in which an image is formed by enlarging or reducing the preimage proportionally by a scale factor from the center of dilation. The center of dilation may or may not be on the preimage. The image is similar to the preimage.	• Given an image and preimage, identify the transformation or combination of transformations that have~~s~~ ~~taken place as a reflection, rotation, dilation, or translation~~occurred:~~.~~ – a translation; – a reflection over any horizontal or vertical line or the lines $y = x$ or $y = -x$; – a clockwise or counter clockwise rotation of 90° 180°, 270° or 360° on a coordinate grid where the center of rotation is limited to the origin; and – a dilation from a fixed point on a coordinate grid. (d) ~~• Apply the distance formula to find the length of a line segment when given the coordinates of the endpoints.~~ [Reordered]

~~TOPIC~~STRAND: REASONING, LINES, AND TRANSFORMATIONS

GEOMETRY
STANDARD G.4

The student will construct and justify the constructions of

a) a line segment congruent to a given line segment;
b) the perpendicular bisector of a line segment;
c) a perpendicular to a given line from a point not on the line;
d) a perpendicular to a given line at a given point on the line;
e) the bisector of a given angle;
f) an angle congruent to a given angle; ~~and~~
g) a line parallel to a given line through a point not on the ~~given~~ line; and
h) an equilateral triangle, a square, and a regular hexagon inscribed in a circle. [Moved from EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Construction techniques are used to solve ~~real-world~~practical problems in engineering, architectural design, and building construction. • Construction techniques include using a straightedge and compass, paper folding, and dynamic geometry software. • Geometric constructions assist in justifying, verifying, and visually reinforcing geometric relationships. • There are multiple methods to most geometric constructions. Students would benefit from experiences with more than one method and should be able to justify each step of geometric constructions. • Individual steps of constructions can be justified using properties of quadrilaterals, congruent triangles, and/or circles. • The construction for a line segment congruent to a given line segment can be justified using properties of a circle.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct and justify the constructions of – a line segment congruent to a given line segment; (a) – the perpendicular bisector of a line segment; (b) – a perpendicular to a given line from a point not on the line; (c) – a perpendicular to a given line at a given point on the line; (d) – the bisector of a given angle; (e) – an angle congruent to a given angle; ~~and~~(f) – a line parallel to a given line through a point not on the given line; (g) and – an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (h) ~~• Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.~~[Reordered] ‡ ~~• Construct the inscribed and circumscribed circles of a triangle.~~‡ • ~~Construct a tangent line from a point outside a given circle to the~~

~~TOPIC~~STRAND: REASONING, LINES, AND TRANSFORMATIONS

GEOMETRY
STANDARD G.4

The student will construct and justify the constructions of

a) a line segment congruent to a given line segment;
b) the perpendicular bisector of a line segment;
c) a perpendicular to a given line from a point not on the line;
d) a perpendicular to a given line at a given point on the line;
e) the bisector of a given angle;
f) an angle congruent to a given angle; ~~and~~
g) a line parallel to a given line through a point not on the ~~given~~ line; and
h) an equilateral triangle, a square, and a regular hexagon inscribed in a circle. [Moved from EKS]

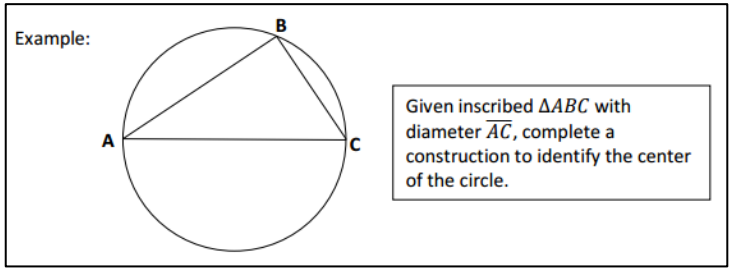
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The construction for the perpendicular bisector of a line segment can be justified using the properties of quadrilaterals or congruent triangles. • The constructions for a perpendicular to a given line from a point on, or not on, the line can be justified using the properties of quadrilaterals or congruent triangles. • The constructions for the bisector of a given angle and an angle congruent to a given angle can be justified using the properties of quadrilaterals or congruent triangles. • The construction for a line parallel to a given line through a point not on the line can be justified using the angle relationships formed when two lines are intersected by a transversal. • The constructions for an equilateral triangle, square, or regular hexagon inscribed in a circle can be justified using properties of circles.	~~circle.~~[‡] ~~[‡]Revised March 2011~~

~~TOPIC~~STRAND: REASONING, LINES, AND TRANSFORMATIONS

GEOMETRY
STANDARD G.4

The student will construct and justify the constructions of

a) a line segment congruent to a given line segment;
b) the perpendicular bisector of a line segment;
c) a perpendicular to a given line from a point not on the line;
d) a perpendicular to a given line at a given point on the line;
e) the bisector of a given angle;
f) an angle congruent to a given angle; ~~and~~
g) a line parallel to a given line through a point not on the ~~given~~ line; and
h) an equilateral triangle, a square, and a regular hexagon inscribed in a circle. [Moved from EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Constructions can be completed within the context of complex figures.  	

~~TOPIC~~STRAND: TRIANGLES

GEOMETRY
STANDARD G.5

The student, given information concerning the lengths of sides and/or measures of angles in triangles, will solve problems, including practical problems. This will include

a) ordering the sides by length, given ~~the~~ angle measures;
b) ordering the angles by degree measure, given ~~the~~ side lengths;
c) determininge whether a triangle exists; and
d) determininge the range in which the length of the third side must lie.

~~These concepts will be considered in the context of real-world situations.~~

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The longest side of a triangle is opposite the largest angle of the triangle and the shortest side is opposite the smallest angle. • In a triangle, the lengths of two sides and the included angle determine the length of the side opposite the angle. • In order for a triangle to exist, the length of each side must be within a range that is determined by the lengths of the other two sides.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given information about the lengths of sides and/or measures of angles in triangles, solve problems, including practical problems. (a,b,c,d) [Reordered] • Order the sides of a triangle by their lengths when given information about the measures of the angles. (a) • Order the angles of a triangle by their measures when given information about the lengths of the sides. (b) • Given the lengths of three segments, determine whether a triangle could be formed. (c) • Given the lengths of two sides of a triangle, determine the range in which the length of the third side must lie. (d) • ~~Solve real-world problems given information about the lengths of sides and/or measures of angles in triangles.~~ [Reordered]

~~TOPIC~~STRAND: TRIANGLES

GEOMETRY
STANDARD G.6

The student, given information in the form of a figure or statement, will prove two triangles are congruent~~, using algebraic and coordinate methods as well as deductive proofs~~.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Deductive or inductive reasoning is used in mathematical proofs. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods. [Moved from EKS] • Congruence has ~~real-world~~practical applications in a variety of areas, including art, architecture, and the sciences. • Congruence does not depend on the position of the triangles. • Congruent triangles are a result of rigid isomorphic transformations. • Concepts of logic can demonstrate congruence or similarity. • Congruent figures are also similar, but similar figures are not necessarily congruent. • Corresponding parts of congruent triangles are congruent. • Two triangles can be proven congruent using: – Side-Angle-Side (SAS); – Side-Side-Side (SSS); – Angle-Angle-Side (AAS); and – Angle-Side-Angle (ASA).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Prove two triangles congruent given relationships among angles and sides of triangles expressed numerically or algebraically. [Rewritten and reordered] • Prove two triangles congruent given representations in the coordinate plane and using coordinate methods (distance formula and slope formula). [Rewritten and reordered] • Use direct proofs~~definitions, postulates, and theorems~~ to prove two triangles congruent. • ~~Use coordinate methods, such as the distance formula and the slope formula, to prove two triangles are congruent.~~ [Reordered] • ~~Use algebraic methods to prove two triangles are congruent.~~ [Reordered]

~~TOPIC~~STRAND: TRIANGLES

GEOMETRY
STANDARD G.6

The student, given information in the form of a figure or statement, will prove two triangles are congruent~~, using algebraic and coordinate methods as well as deductive proofs~~.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Two right triangles can be proven congruent using Hypotenuse-Leg (HL). • Triangle congruency can be explored using geometric constructions such as an angle congruent to a given angle or a line segment congruent to a given line segment. • The construction for the bisector of a given angle can be justified using congruent triangles. • The construction for an angle congruent to a given angle can be justified using congruent triangles. • The construction of the perpendicular to a given line from a point on the line can be justified using congruent triangles. • The construction of the perpendicular to a given line from a point not on the line can be justified using congruent triangles.	

~~TOPIC~~STRAND: TRIANGLES

GEOMETRY
STANDARD G.7

The student, given information in the form of a figure or statement, will prove two triangles are similar~~, using algebraic and coordinate methods as well as deductive proofs~~.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Deductive or inductive reasoning is used in mathematical proofs. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods. [Moved from EKS] • Similarity has ~~real-world~~practical applications in a variety of areas, including art, architecture, and the sciences. • Similarity does not depend on the position of the triangles. • Similar triangles are created using dilations. • Congruent figures are also similar, but similar figures are not necessarily congruent. • Corresponding sides of similar triangles are proportional. • Corresponding angles of similar triangles are congruent. • The altitude in a right triangle creates three similar right triangles. [picture will be added] • Two triangles can be proven similar using: – Side-Angle-Side (SAS); – Side-Side-Side (SSS); and – Angle-Angle (AA).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Prove two triangles similar given relationships among angles and sides of triangles expressed numerically or algebraically. [Rewritten and reordered] • Prove two triangles similar given representations in the coordinate plane and using coordinate methods (distance formula and slope formula). [Rewritten and reordered] • Use direct proofs ~~definitions, postulates, and theorems~~ to prove triangles similar. • ~~Use algebraic methods to prove that triangles are similar.~~ [Reordered] • ~~Use coordinate methods, such as the distance formula, to prove two triangles are similar.~~ [Reordered]

~~TOPIC~~STRAND: TRIANGLES

GEOMETRY
STANDARD G.8

The student will solve ~~real-world~~ problems, including practical problems, involving right triangles. This will include applying ~~by using the~~

a) Pythagorean Theorem and its converse;~~,~~
b) properties of special right triangles;~~,~~ and
c) ~~right triangle trigonometry~~trigonometric ratios.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
~~• The Pythagorean Theorem is essential for solving problems involving right triangles.~~ ~~• Many historical and algebraic proofs of the Pythagorean Theorem exist.~~ ~~• The relationships between the sides and angles of right triangles are useful in many applied fields.~~ ~~• Some practical problems can be solved by choosing an efficient representation of the problem.~~ ~~• Another formula for the area of a triangle is $A = \frac{1}{2}ab\sin C$.~~ • The converse of the Pythagorean Theorem can be used to determine if a triangle is a right triangle. • 45°-45°-90° and 30°-60°-90° triangles are special right triangles because their side lengths can be specified as exact values using radicals rather than decimal approximations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve problems, including practical problems, using right triangle trigonometry and properties of special right triangles. (a,b,c) [Reordered] • Determine whether a triangle formed with three given lengths is a right triangle. (a) • Solve for missing lengths in geometric figures, using properties of 45°-45°-90° triangles where rationalizing denominators may be necessary. (b) • Solve for missing lengths in geometric figures, using properties of 30°-60°-90° triangles where rationalizing denominators may be necessary. (b). • Solve problems, including practical problems, involving right triangles with missing side lengths or angle measurements, using sine, cosine, and tangent ratios. (c) ~~• Solve real-world problems, using right triangle trigonometry and properties of right triangles.~~ [Reordered]

~~TOPIC~~STRAND: TRIANGLES

GEOMETRY
STANDARD G.8

The student will solve ~~real-world~~ problems, including practical problems, involving right triangles. This will include applying ~~by using the~~

a) Pythagorean Theorem and its converse;~~,~~
b) properties of special right triangles;~~,~~ and
c) ~~right triangle trigonometry~~trigonometric ratios.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~The ratios of side lengths in similar right triangles (adjacent/hypotenuse or opposite/hypotenuse) are independent of the scale factor and depend only on the angle the hypotenuse makes with the adjacent side, thus justifying the definition and calculation of trigonometric functions using the ratios of side lengths for similar right triangles.~~The sine of an acute angle in a right triangle is equal to the cosine of its complement. [Moved from EKS]	• ~~Explain and use the relationship between the sine and cosine of complementary angles.~~[‡] [Moved to US] [‡]~~Revised March 2011~~

~~TOPIC~~STRAND: POLYGONS AND CIRCLES

GEOMETRY
STANDARD G.9

The student will verify ~~characteristics of quadrilaterals~~ and use properties of quadrilaterals to solve ~~real-world~~ problems, including practical problems.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~The terms characteristics and properties can be used interchangeably to describe quadrilaterals. The term characteristics is used in elementary and middle school mathematics.~~ • Deductive or inductive reasoning is used in mathematical proofs. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods. • Quadrilaterals have a hierarchical nature based on the relationships between their sides, angles, and diagonals. • ~~Characteristics~~ Properties of quadrilaterals can be used to identify the quadrilateral and to ~~find~~determine the measures of sides and angles. • Given coordinate representations of quadrilaterals, the distance, slope, and midpoint formulas may be used to verify that quadrilaterals have specific properties. • The angle relationships formed when parallel lines are intersected by a transversal can be used to prove properties of quadrilaterals. • Congruent triangles can be used to prove properties of quadrilaterals.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve problems, including ~~real-world~~practical problems, using the properties specific to parallelograms, rectangles, rhombi, squares, isosceles trapezoids, and trapezoids. • Prove that quadrilaterals have specific properties, using coordinate and algebraic methods, such as the distance formula, slope, and midpoint formula. • Prove the ~~characteristics~~ properties of quadrilaterals, using direct proofs~~deductive reasoning, algebraic, and coordinate methods~~. • ~~Prove properties of angles for a quadrilateral inscribed in a circle.~~[†] [Moved to G.11 US] [†]~~Revised March 2011~~

~~TOPIC~~STRAND: POLYGONS AND CIRCLES

GEOMETRY
STANDARD G.9

The student will verify ~~characteristics of quadrilaterals~~ and use properties of quadrilaterals to solve ~~real-world~~ problems, including practical problems.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Properties of a parallelogram include the following: – opposite sides are congruent; – opposite angles are congruent; – consecutive angles are supplementary; and – diagonals bisect each other. • A rectangle is a quadrilateral with four right angles. Properties of rectangle include the following: – opposite sides are parallel and congruent; and – diagonals are congruent and bisect each other. • A rhombus is a quadrilateral with four congruent sides. Properties of a rhombus include the following: – all sides are congruent; – opposite sides are parallel; – opposite angles are congruent; – diagonals are perpendicular bisectors of each other; – diagonals bisect opposite angles; and – diagonals divide the rhombus into four congruent right triangles.	

~~TOPIC~~STRAND: POLYGONS AND CIRCLES

GEOMETRY
STANDARD G.9

The student will verify ~~characteristics of quadrilaterals~~ and use properties of quadrilaterals to solve ~~real-world~~ problems, including practical problems.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A square is a quadrilateral that is a regular polygon with four congruent sides and four right angles. Properties of a square include the following: – opposite sides are parallel; – diagonals are congruent; – diagonals are perpendicular bisectors of each other; and – diagonals divide the square into four congruent 45°-45°-90° triangles. • A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called bases. The nonparallel sides of a trapezoid are called legs. • An isosceles trapezoid has the following properties: – nonparallel sides are congruent; – diagonals are congruent; and – base angles are congruent. • The construction of the perpendicular bisector of a line segment can be justified using the properties of quadrilaterals. • The construction of the perpendicular to a given line from a point on, or not on, the line can be justified using the properties of quadrilaterals. • The construction of the perpendicular to a given line from a point on the line can be justified using the properties of quadrilaterals.	

~~TOPIC~~STRAND: POLYGONS AND CIRCLES

GEOMETRY
STANDARD G.9

The student will verify ~~characteristics of quadrilaterals~~ and use properties of quadrilaterals to solve ~~real-world~~ problems, including practical problems.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The construction of a bisector of a given angle can be justified using the properties of quadrilaterals. • The construction of a square inscribed in a circle can be justified using the properties of squares.	

~~TOPIC~~STRAND: POLYGONS AND CIRCLES

GEOMETRY
STANDARD G.10

The student will solve ~~real-world~~ problems, including practical problems, involving angles of convex polygons. This will include

a) determining the sum of the interior and/or exterior angles;
b) determining the measure of an interior and/or exterior angle; and
c) determining the number of sides of a regular polygon.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~A regular polygon will tessellate the plane if the measure of an interior angle is a factor of 360.~~ [Reordered] • ~~Both regular and nonregular polygons can tessellate the plane.~~ [Reordered] • In convex polygons, each interior angle has a measure less than 180°. • In concave polygons, one or more interior angles have a measure greater than 180°. • Two intersecting lines form angles with specific relationships. • An exterior angle is formed by extending a side of a polygon. • The exterior angle and the corresponding interior angle form a linear pair. • The sum of the measures of the interior angles of a convex polygon may be found by dividing the interior of the polygon into nonoverlapping triangles. • Both regular and nonregular polygons can tessellate the plane. [Reordered] • A regular polygon will tessellate the plane if the measure of an interior angle is a factor of 360. [Reordered]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve ~~real-world~~ problems, including practical problems, involving ~~the measures of interior and exterior~~ angles of convex polygons. (a,b,c) • ~~Identify tessellations in art, construction, and nature.~~ [Moved to US] • ~~Find~~Determine the sum of the measures of the interior and exterior angles of a convex polygon. (a) • ~~Find~~Determine the measure of each interior and exterior angle of a regular polygon. (b) • Determine angle measures of a regular polygon in a tessellation. (b) • ~~Find~~Determine the number of sides of a regular polygon, given the measures of interior or exterior angles of the polygon. (c)

~~TOPIC~~STRAND: POLYGONS AND CIRCLES **GEOMETRY** **STANDARD G.10** **The student will solve ~~real-world~~ problems, including practical problems, involving angles of convex polygons. This will include** **a) determining the sum of the interior and/or exterior angles;** **b) determining the measure of an interior and/or exterior angle; and** **c) determining the number of sides of a regular polygon.**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The sum of the measures of the angles around a point in a tessellation is 360°. • Tessellations can be found in art, construction and nature. [Moved from EKS]	

~~TOPIC~~STRAND: POLYGONS AND CIRCLES

GEOMETRY
STANDARD G.11

The student will solve problems, including practical problems, by applying properties of circles.~~use angles, arcs, chords, tangents, and secants to~~ This will include determining

a) ~~investigate, verify, and apply properties of circles~~[Moved to stem]angle measures formed by intersecting chords, secants, and/or tangents;
b) ~~solve real-world problems involving properties of circles~~[Moved to stem]lengths of segments formed by intersecting chords, secants, and/or tangents; ~~and~~
c) ~~find~~ arc lengths~~;~~ and
d) and areas of a sectors ~~in circles~~.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• All circles are similar. • A chord is a line segment that joins any two points on a circle. A chord is ~~part~~ a segment of a secant. • Arcs can be measured in degrees or in units of length. • ~~Real-world a~~Applications of the properties of circles may be drawn from architecture, art, and construction. • Properties of circles can be verified using deductive reasoning, algebraic, and coordinate methods. [Moved from EKS] • Inscribed quadrilaterals have opposite angles that are supplementary. [Moved from G.9 EKS] • Properties associated with segment lengths can be verified using similar triangles. • The ratio of the central angle to 360° is proportional to the ratio of the arc length to the circumference of the circle. • The ratio of the central angle to 360° is proportional to the ratio of the area of the sector to the area of the circle.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve problems, including practical problems, by applying properties of circles. (a,b,c,d) [Reordered] • ~~Find~~ Determine ~~lengths,~~ angle measures~~, and arc measures~~ associated with – two intersecting chords; – two intersecting secants; – an intersecting secant and tangent; – two intersecting tangents; and – central and inscribed angles. (a) • Determine segment lengths associated with: – two intersecting chords; – two intersecting secants; – an intersecting secant and tangent; and – two intersecting tangents. (b)

~~TOPIC~~STRAND: POLYGONS AND CIRCLES

GEOMETRY
STANDARD G.11

The student will solve problems, including practical problems, by applying properties of circles.~~use angles, arcs, chords, tangents, and secants to~~ This will include determining

a) ~~investigate, verify, and apply properties of circles~~[Moved to stem]angle measures formed by intersecting chords, secants, and/or tangents;
b) ~~solve real-world problems involving properties of circles~~[Moved to stem]lengths of segments formed by intersecting chords, secants, and/or tangents; ~~and~~
c) ~~find~~ arc lengths~~;~~ and
d) and area~~s~~ of a sector~~s in circles~~.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The construction for an inscribed equilateral triangle, square and regular hexagon can be justified using properties of a circle.	• Calculate the ~~area of a sector and the~~ length of an arc of a circle in degrees or units of length.~~, using proportions~~. (c) • Calculate the area of a sector. (d) • ~~Solve real-world problems associated with circles, using properties of angles, lines, and arcs.~~ [Reordered] • ~~Verify properties of circles, using deductive reasoning, algebraic, and coordinate methods.~~ [Moved to US]

~~TOPIC~~STRAND: POLYGONS AND CIRCLES

GEOMETRY
STANDARD G.12

The student will solve problems involving ~~, given the coordinates of the center of a circle and a point on the circle, will write the~~ equations of ~~the~~ circles.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A circle is a locus of points equidistant from a given point, the center. • The distance between any point on the circle and the center is the length of the radius. [Moved from EKS] • Standard form for the equation of a circle is $(x-h)^2+(y-k)^2=r^2$, where the coordinates of the center of the circle are (h,k) and r is the length of the radius. • The equation of a circle gives the coordinates of every point, (x, y), on the circle. • The midpoint formula and distance formula are important when determining the equation of a circle. • The equation of a circle with a given center and radius can be derived using the Pythagorean Theorem. [Moved from EKS] • The midpoint of the diameter is the center of the circle. ~~The circle is a conic section.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given a graph or the equation of a circle in standard form, identify the coordinates of the center. • Given the coordinates of the endpoints of a diameter of a circle, determine the coordinates of the center of the circle. • Given a graph or the equation of a circle in standard form, identify the length of the radius or diameter. • Given the coordinates of the endpoints of the diameter of a circle, determine the length of the radius or diameter. • Given the coordinates of the center and the coordinates of a point on the circle, determine the length of the radius or diameter. • ~~Use the distance formula to find the radius of a circle.~~ [Moved to US] • Given the coordinates of the center and length of the radius of ~~the~~ a circle, identify the coordinates of a point(s) on the circle.

~~TOPIC~~STRAND: POLYGONS AND CIRCLES

GEOMETRY
STANDARD G.12

The student will solve problems involving ~~, given the coordinates of the center of a circle and a point on the circle, will write the~~ equations of ~~the~~ circles.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
	• Determine the equation of a circle given: – a graph of a circle with a center with coordinates that are integers; – coordinates of the center and a point on the circle; – coordinates of the center and the length of the radius or diameter; or – coordinates of the endpoints of a diameter. • ~~Given the equation of a circle in standard form, identify the coordinates of the center and find the radius of the circle.~~ • ~~Given the coordinates of the endpoints of a diameter, find the equation of the circle.~~ [Reordered] • ~~Given the coordinates of the center and a point on the circle, find the equation of the circle.~~ [Combined] • ~~Recognize that the equation of a circle of given center and radius is derived using the Pythagorean Theorem.~~[†] [Moved to US] ~~[†]Revised March 2011~~

~~TOPIC~~STRAND: THREE-DIMENSIONAL FIGURES

GEOMETRY
STANDARD G.13

The student will use ~~formulas for~~ surface area and volume of three-dimensional objects to solve ~~real-world~~practical problems.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A cylinder is a solid figure formed by two congruent parallel faces called bases joined by a curved surface. In this course, cylinders are limited to right circular cylinders. • A cone is a solid figure formed by a face called a base that is joined to a vertex (apex) by a curved surface. In this course, cones are limited to right circular cones. • A prism is a polyhedron that has a congruent pair of parallel bases and faces that are parallelograms. In this course, prisms are limited to right prisms with bases that are triangles, rectangles, or regular hexagons. • A rectangular prism is a polyhedron in which all six faces are rectangles. • A triangular prism is a polyhedron that has a congruent pair of parallel triangular bases and faces that are parallelograms. • A pyramid is a polyhedron with a base that is a polygon and three or more faces that are triangles with a common vertex. • A regular pyramid is a pyramid with a base that is a regular polygon. In this course, pyramids are limited to right regular pyramids with triangular, square, or hexagonal bases. • A regular polygon has congruent sides and congruent interior angles.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the ~~total~~ surface area of cylinders, prisms, pyramids, cones, hemispheres, and spheres, using the appropriate formulas. • ~~Calculate~~ Determine the volume of cylinders, prisms, pyramids, cones, hemispheres, and spheres, using the appropriate formulas. • Solve problems~~, including real-world problems,~~ involving ~~total~~ surface area and volume of cylinders, prisms, pyramids, cones, hemispheres, and spheres, as well as ~~combinations of~~composite three-dimensional figures. • Solve problems involving the lateral surface area of circular cylinders, prisms, and regular pyramids. • Given information about a three-dimensional figure such as length of a side, area of a face, or volume, determine missing information. • ~~Calculators may be used to find decimal approximations for results.~~ [Moved to US]

~~TOPIC~~STRAND: THREE-DIMENSIONAL FIGURES

GEOMETRY
STANDARD G.13

The student will use ~~formulas for~~ surface area and volume of three-dimensional objects to solve ~~real-world~~practical problems.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Subdivision of polygons may assist in determining the area of regular polygons. • The surface area of a prism or pyramid~~three-dimensional object~~ is the sum of the areas of all its faces. • The surface area of a cylinder, cone, or hemisphere is the sum of the areas of the curved surface and bases. • The surface area of a sphere is the area of the curved surface. • The lateral surface area of a cylinder is the area of the curved surface of the cylinder, not including the parallel bases. • The lateral surface area of a rectangular-based prism is the sum of the areas of all faces, not including the parallel bases. • The lateral surface area of a triangular-based prism is the sum of the areas of all faces, not including the triangular-shaped, parallel bases. • The volume of a three-dimensional ~~object~~ figure is the number of unit cubes that would fill the ~~object~~figure. • Composite figures consist of two or more three-dimensional figures. The surface area of a composite figure may not be equal to the sum of the surface areas of the individual figures. • Volume and surface area of spheres, cones and cylinders should be considered in terms of π or as a decimal representation. • Calculators may be used to determine decimal approximations for results. [Moved from EKS]	

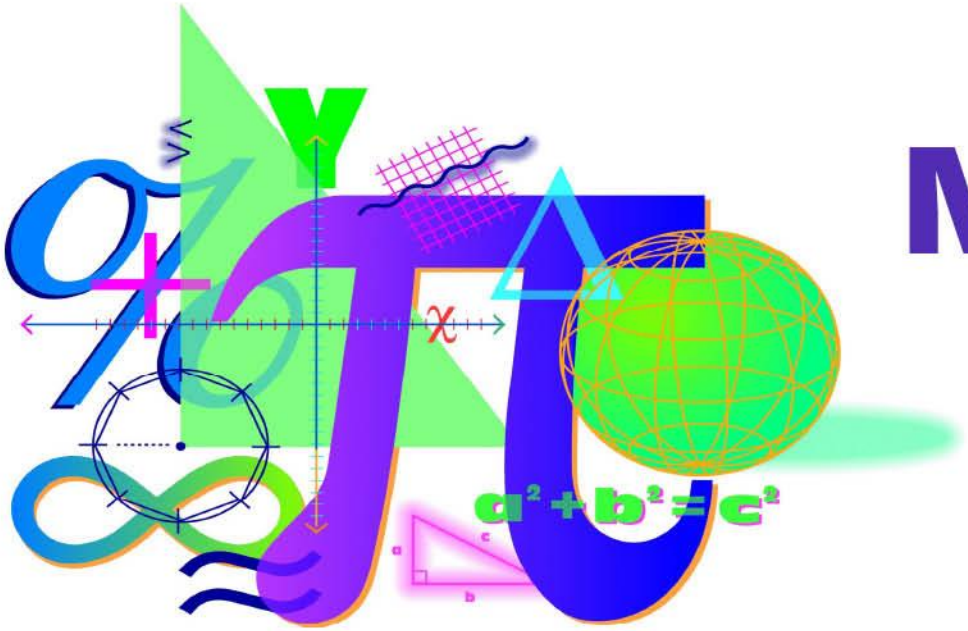
~~TOPIC~~STRAND: THREE-DIMENSIONAL FIGURES

GEOMETRY
STANDARD G.14

The student will ~~use~~apply the concepts of similarity ~~geometric objects in~~to two- or three-dimensionals geometric figures. This will include ~~to~~

a) **comparinge ratios between ~~side~~ lengths, perimeters, areas, and volumes of similar figures;**
b) **determininge how changes in one or more dimensions of a~~n object~~ figure affect area and/or volume of the ~~object~~figure;**
c) **determininge how changes in area and/or volume of a~~n object~~ figure affect one or more dimensions of the ~~object~~figure; and**
d) **solvinge ~~real-world~~ problems, including practical problems, about similar geometric ~~objects~~figures.**

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A change in one dimension of a~~n object~~ figure results in predictable changes in area and/or volume. The resulting figure may or may not be similar to the original figure. • A constant ratio, the scale factor, exists between corresponding ~~lengths of sides~~dimensions of similar figures. • If the ratio between dimensions of similar figures is a:b then: – The ratio of their areas is $a^2:b^2$. – The ratio of their volumes is $a^3:b^3$. • Proportional reasoning is ~~integral~~ important when~~to~~ comparing attribute measures in similar ~~objects~~figures.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare ratios between side lengths, perimeters, areas, and volumes, given two similar figures. (a) • Describe how changes in one or more dimensions affect other derived measures (perimeter, area, total surface area, and volume) of a~~n object~~figure. (b) • Describe how changes in one or more measures (perimeter, area, total surface area, and volume) affect other measures of a~~n object~~figure. (c) • Solve real-world problems involving measured attributes of similar ~~object~~figures. (d)



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Algebra, Functions, and Data Analysis

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.1

The student will investigate and analyze ~~function (~~linear, quadratic, exponential, and logarithmic~~)~~ function families and their characteristics. Key concepts include~~:~~

a) domain and range~~continuity~~; [Reordered]
b) intervals on which a function is increasing or decreasing~~local and absolute maxima and minima~~ [Reordered];
c) ~~domain and range~~maximum and minimum [Reordered];
d) zeros;
e) intercepts;
f) values of a function for elements in its domain~~intervals in which the function is increasing/decreasing~~ [Reordered];
g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic;
h) end behaviors; and
~~h~~i) vertical and horizontal asymptotes.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A relation is a function if and only if each element in the domain is paired with a unique element of the range. • Functions are used to model practical phenomena. [Reordered] • Functions describe the relationship between two variables where each input is paired to a unique output. [Reordered] • Function families consist of a parent function and all transformations of the parent function. [Reordered] • The domain of a function is the set of all possible values of the independent variable. • The range of a function is the set of all possible values of the dependent variable. ~~• The domain of a function consists of the first coordinates of the ordered pairs that are elements of a function. Each element in the~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** ~~• Identify the domain and range for a relation, given a set of ordered pairs, a table, or a graph.~~ ~~• For each *x* in the domain of *f*, find *f*(*x*).~~ [Reordered] ~~• Identify the zeros of the function algebraically and confirm them, using the graphing calculator.~~ [Combined] • Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically. Domains may be limited by problem context or in graphical representations. (a,d,e) • Identify intervals on which the function is increasing or decreasing. (b) [Reordered]

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.1

The student will investigate and analyze ~~function (~~linear, quadratic, exponential, and logarithmic~~)~~ function families and their characteristics. Key concepts include~~:~~

a) domain and range~~continuity~~; [Reordered]
b) intervals on which a function is increasing or decreasing~~local and absolute maxima and minima~~ [Reordered];
c) ~~domain and range~~maximum and minimum [Reordered];
d) zeros;
e) intercepts;
f) values of a function for elements in its domain~~intervals in which the function is increasing/decreasing~~ [Reordered];
g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic;
h) end behaviors; and
~~h~~i) vertical and horizontal asymptotes.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
~~domain is an input into the independent variable of the function.~~ ~~• The range of a function consists of the second coordinates of the ordered pairs that are elements of a function. Each element in the range is an output in the dependent variable of a function.~~ • For each x in the domain of f, x is a member of the input of the function f, $f(x)$ is a member of the output of f, and the ordered pair $[x, f(x)]$ is a member of f. ~~• A value x in the domain of f is an x-intercept or a zero of a function f if and only if $f(x) = 0$.~~[Reordered] ~~• Functions describe the relationship between two variables where each input is paired to a unique output.~~ [Reordered] ~~• Functions are used to model real-world phenomena.~~ [Reordered] • The domain of a function may be restricted algebraically, graphically, or by the practical situation modeled by the function.	• Identify the location and value of the maximum and minimum of a function over the domain of the function graphically or by using a graphing utility. (c) • For any x value in the domain of f, determine $f(x)$. (f) [Reordered] • Represent relations and functions using concrete, verbal, numeric, graphic, and algebraic forms. Given one representation, represent the relation in another form. (g) • Detect patterns in data and represent arithmetic and geometric patterns algebraically. (g) • Describe the end behavior of a function. (h) • Determine the equations of the horizontal asymptote of an exponential function and the vertical asymptote of a logarithmic function. (i)

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.1

The student will investigate and analyze ~~function (~~linear, quadratic, exponential, and logarithmic~~)~~ function families and their characteristics. Key concepts include~~:~~

a) domain and range~~continuity~~; [Reordered]
b) intervals on which a function is increasing or decreasing~~local and absolute maxima and minima~~ [Reordered];
c) ~~domain and range~~maximum and minimum [Reordered];
d) zeros;
e) intercepts;
f) values of a function for elements in its domain~~intervals in which the function is increasing/decreasing~~ [Reordered];
g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic;
h) end behaviors; and
~~h~~i) vertical and horizontal asymptotes.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A function can be described on an interval as increasing, decreasing, or constant over a specified interval or over the entire domain of the function. • A function, *f(x)*, is increasing over an interval if the values of *f(x)* consistently increase over the interval as the *x* values increase. • A function, *f(x)*, is decreasing over an interval if the values of *f(x)* consistently decrease over the interval as the *x* values increase. • A function, *f(x)*, is constant over an interval if the values of *f(x)* remain constant over the interval as the *x* values increase. • ~~A function is increasing on an interval if its graph, as read from left to right, is rising in that interval.~~ • ~~A function is decreasing on an interval if its graph, as read from left to right, is going down in that interval.~~ • ~~Exponential and logarithmic functions are either strictly~~	• Investigate and analyze characteristics and multiple representations of functions with a graphing utility. (a,b,c,d,e,f,g,h,i) • ~~Recognize restricted/discontinuous domains and ranges.~~ • ~~Recognize graphs of parent functions for linear, quadratic, exponential and logarithmic functions.~~ [Included in AFDA.2] • ~~Identify *x* intercepts (zeros), *y* intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, points of discontinuity, end behavior, and maximum and minimum points, given a graph of a function.~~ [Reordered and split] • ~~Describe continuity of a function on its domain or at a point.~~ • ~~Express intervals using correct interval notation and/or a compound inequality.~~ [Moved to US]

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.1

The student will investigate and analyze ~~function (~~linear, quadratic, exponential, and logarithmic~~)~~ function families and their characteristics. Key concepts include~~:~~

a) domain and range~~continuity~~; [Reordered]
b) intervals on which a function is increasing or decreasing~~local and absolute maxima and minima~~ [Reordered];
c) ~~domain and range~~maximum and minimum [Reordered];
d) zeros;
e) intercepts;
f) values of a function for elements in its domain~~intervals in which the function is increasing/decreasing~~ [Reordered];
g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic;
h) end behaviors; and
~~h~~i) vertical and horizontal asymptotes.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
~~increasing or strictly decreasing.~~ • ~~A function is continuous on an interval if the function is defined for every value in the interval and there are no breaks in the graph. A continuous function can be drawn without lifting the pencil.~~ • A turning point is a point on a continuous interval where the graph changes from increasing to decreasing or from decreasing to increasing. • A function, *f*, has a ~~local~~ maximum ~~in some interval~~ at $x = a$ if $f(a)$ is the largest value of *f* ~~in that interval~~over its domain. • A function, *f*, has a ~~local~~ minimum in some interval at $x = a$ if $f(a)$ is the smallest value of *f* ~~in that interval~~over its domain.	

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.1

The student will investigate and analyze ~~function (~~linear, quadratic, exponential, and logarithmic~~)~~ function families and their characteristics. Key concepts include~~:~~

a) domain and range~~continuity~~; [Reordered]
b) intervals on which a function is increasing or decreasing~~local and absolute maxima and minima~~ [Reordered];
c) ~~domain and range~~maximum and minimum [Reordered];
d) zeros;
e) intercepts;
f) values of a function for elements in its domain~~intervals in which the function is increasing/decreasing~~ [Reordered];
g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic;
h) end behaviors; and
~~h~~i) vertical and horizontal asymptotes.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation. Examples may include:	

Equation/ Inequality	Set Notation	Interval Notation
$x = 3$	$\{3\}$	
$x = 3$ or $x = 5$	$\{3,5\}$	
$0 \leq x < 3$	$\{x \mid 0 \leq x < 3\}$	$[0, 3)$
$y \geq 3$	$\{y: y \geq 3\}$	$[3, \infty)$
Empty (null) set $\emptyset$	$\{\ \}$	

- ~~Asymptotes can be used to describe local behavior and end behavior of graphs. They are lines or other curves that approximate the graphical behavior of a function.~~ [Reordered]
- A value x in the domain of f is an x-intercept or a zero of a function f if and only if $f(x) = 0$.

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.1

The student will investigate and analyze ~~function (~~linear, quadratic, exponential, and logarithmic~~)~~ function families and their characteristics. Key concepts include~~:~~

a) domain and range~~continuity~~; [Reordered]
b) intervals on which a function is increasing or decreasing~~local and absolute maxima and minima~~ [Reordered];
c) ~~domain and range~~maximum and minimum [Reordered];
d) zeros;
e) intercepts;
f) values of a function for elements in its domain~~intervals in which the function is increasing/decreasing~~ [Reordered];
g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic;
h) end behaviors; and
~~h~~i) vertical and horizontal asymptotes.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The *x*-intercept is the point at which the graph of a relation or function intersects with the *x*-axis. It can be expressed as a value or a coordinate. • The *y*-intercept is the point at which the graph of a relation or function intersects with the *y*-axis. It can be expressed as a value or a coordinate. • Given a polynomial function *f(x)*, ~~T~~the following statements are equivalent for any real number *k*, such that *f(k)* = 0: • *k* is a zero of the polynomial function *f(x)* located at (*k*, 0); • *k* is a solution or root of the polynomial equation *f*(*x*) = 0; • the point (*k*, 0) is an *x*-intercept for the graph of ~~the polynomial~~polynomial *f(x)* = 0; and • *(x – k)* is a factor of ~~the polynomial~~polynomial *f(x)*.	

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.1

The student will investigate and analyze ~~function (~~linear, quadratic, exponential, and logarithmic~~)~~ function families and their characteristics. Key concepts include~~:~~

a) domain and range~~continuity~~; [Reordered]
b) intervals on which a function is increasing or decreasing~~local and absolute maxima and minima~~ [Reordered];
c) ~~domain and range~~maximum and minimum [Reordered];
d) zeros;
e) intercepts;
f) values of a function for elements in its domain~~intervals in which the function is increasing/decreasing~~ [Reordered];
g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic;
h) end behaviors; and
~~h~~i) vertical and horizontal asymptotes.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Connections between multiple representations (graphs, tables, and equations) of a function can be made. • In an arithmetic pattern, the common difference is the value that is added to obtain the next value in the pattern. • In geometric number patterns, the common ratio is a multiplier used to obtain the next value in the pattern. • End behavior describes a function's values as *x* approaches positive or negative infinity. • Asymptotes can be used to describe local behavior and end behavior of graphs. They are lines or other curves that approximate the graphical behavior of a function. [Reordered]	

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.2

The student will use knowledge of transformations to write an equation, given the graph of a ~~function (~~linear, quadratic, exponential, and logarithmic~~)~~ function.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Knowledge of transformational graphing using parent functions can be used to generate a mathematical model from a scatterplot that approximates the data. • The graph of a parent function is an anchor graph from which other graphs are derived using transformations. • Transformations of graphs include: – Translations (horizontal and vertical shifting of a graph); – Reflections (over the *x*- and *y*-axis); and – Dilations (stretching and compressing graphs). ~~and Rotations~~ • The equation of a line can be determined by two points on the line or by the slope and a point on the line.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write an equation of a line when given the graph of a line. • Recognize graphs of parent functions for linear, quadratic, exponential and logarithmic functions. • Write the equation of a linear, quadratic, exponential, or logarithmic function in ~~*(h, k)*~~vertex form given the graph of the parent function and transformation information. • Describe the transformation from the parent function given the equation written in ~~*(h, k)*~~vertex form or the graph of the function. • Given the equation of a function, recognize the parent function and transformation to graph the given function. • Recognize the vertex of a parabola given a quadratic equation in ~~*(h, k)*~~vertex form or graphed. • Describe the parent function represented by a scatterplot.

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.3

The student will collect and analyze data, ~~and generate an~~determine the equation ~~for~~ of the curve of best fit in order to make predictions, and solve ~~(linear, quadratic, exponential, and logarithmic) of best fit to model real-world~~practical problems ~~or applications~~ using models of linear, quadratic, and exponential functions. ~~Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.~~

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
~~• The regression equation modeling a set of data points can be used to make predictions where appropriate.~~[Rewritten and reordered] • Data and scatterplots may indicate patterns that can be modeled with a function. • The curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate. • Knowledge of transformational graphing using parent functions can be used to generate a mathematical model from a scatterplot that approximates the data. • Graphing ~~calculators~~ utilities can be used to collect, organize, ~~picture~~represent, and ~~create~~generate an equation of a curve of best fit ~~algebraic model of the~~for a set of data. • Data that fit linear ($y = mx + b$), quadratic ($y = ax^2 + bx + c$), and exponential ($y = ab^x$), ~~and logarithmic~~ models arise from practical situations. ~~• Two variables may be strongly associated without a cause-and-effect relationship existing between them.~~ ~~• Each data point may be considered to be comprised of two parts: fit (the part explained by the model) and residual (the result of chance variation or of variables not measured).~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Write~~Determine an equation for the ~~line~~curve of best fit, given a set of no more than twenty data points in a table, on a graph, or ~~from a~~ practical situation. • Make predictions ~~about unknown outcomes~~, using data, scatterplots, or the equation of the ~~a line~~ curve of best fit. • Solve practical problems involving an equation of the curve of best fit. ~~• Collect and analyze data to make decisions and justify conclusions.~~[Combined] ~~• Investigate scatterplots to determine if patterns exist, and identify the patterns.~~[Combined] ~~• Find an equation for the curve of best fit for data, using a graphing calculator. Models will include linear, quadratic, exponential, and logarithmic functions.~~[Combined] • ~~Make predictions, using data, scatterplots, or equation of curve of best fit.~~[Combined] • ~~Given a set of data, determine the model that would best describe~~

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.3

The student will collect and analyze data, ~~and generate an~~determine the equation ~~for~~ of the curve of best fit in order to make predictions, and solve ~~(linear, quadratic, exponential, and logarithmic) of best fit to model real-world~~practical problems ~~or applications~~ using models of linear, quadratic, and exponential functions. ~~Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.~~

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Residual = Actual – Fitted~~ • ~~Least squares regression generates the equation of the line that minimizes the sum of the squared distances between the data points and the line.~~ • A correlation coefficient measures the degree of association between two variables that are related linearly. • Rounding that occurs during intermediate steps of problem solving may reduce the accuracy of the final answer.	~~the data.~~[Combined] • ~~Describe the errors inherent in extrapolation beyond the range of the data.~~ • ~~Estimate the correlation coefficient when given data and/or scatterplots.~~

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.4

The student will ~~transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and~~ use ~~appropriate~~multiple representations of functions for analysis, interpretation, and prediction.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The most appropriate representation of a function depends on the questions to be answered and/or the analysis to be done. • Given data may be represented as discrete points or as a continuous graph with respect to the ~~real-world~~practical context. • ~~Real-world~~Practical data may best be represented as a table, a graph, or as a formula.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given an equation, graph a linear, quadratic, exponential or logarithmic function~~ with the aid of a graphing calculator~~. • Make predictions given a table of values, a graph, or an algebraic formula. • Describe relationships between data represented in a table, in a scatterplot, and as elements of a function. • Determine the appropriate representation of data derived from real-world situations. • Analyze and interpret the data in context of the ~~real-world~~practical situation. • Use a graphing utility to graph, analyze, interpret, and make predictions.

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.5

The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Linear programming models an optimization process. • A linear programming model consists of a system of constraints and an objective quantity that can be maximized or minimized. • Any maximum or minimum value will occur at ~~a corner point~~the vertices of a feasible region.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model practical problems with systems of linear inequalities. • Solve systems of no more than four linear inequalities with pencil and paper and using a graphing ~~calculator~~utility. • Solve systems of no more than four equations algebraically and graphically. • Identify the feasible~~ility~~ region of a system of linear inequalities. • Identify the coordinates of the ~~corner points~~vertices of a feasibile~~ity~~ region. • ~~Find~~ Determine the maximum or minimum value for the function defined over ~~the~~ a feasible~~ility~~ region. • ~~Describe the meaning of~~Determine the absolute maximum or absolute minimum value ~~within its~~in context.

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.6

The student will calculate probabilities. Key concepts include~~:~~

a) **conditional probability;**
b) **dependent and independent events;**
c) **~~addition and multiplication rules~~mutually exclusive events;**
d) **counting techniques (permutations and combinations); and**
e) **Law of Large Numbers.**

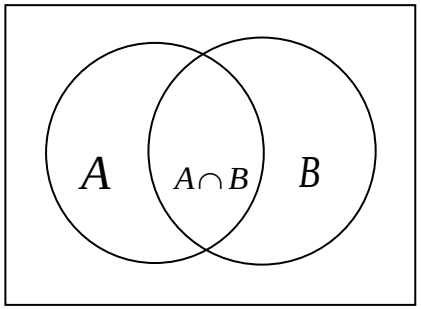
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The Fundamental Counting Principle states that if one decision can be made *n* ways and another can be made *m* ways, then the two decisions can be made *nm* ways. • ~~*Permutations* are used to calculate the number of possible arrangements of objects.~~ [Reordered] • ~~*Combinations* are used to calculate the number of possible selections of objects without regard to the order selected.~~ [Reordered] • A sample space is the set of all possible mutually exclusive outcomes of a random experiment. • An event is a subset of the sample space. • *P(E)* is a way to represent the probability that the event *E* occurs. • Mutually exclusive events are events that cannot both occur simultaneously.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Analyze, interpret and make predictions based on theoretical probability within practical context. (a,b,c,e) [Reordered] • Determine conditional probabilities for dependent, independent, and mutually exclusive events. (a,b,c) [Reordered] • Represent and calculate probabilities using Venn diagrams and probability trees. (a) [Reordered] • ~~Compare and contrast permutations and combinations.~~ [Reordered] • ~~Calculate the number of permutations of *n* objects taken *r* at a time.~~ [Reordered] • ~~Calculate the number of combinations of *n* objects taken *r* at a time.~~ [Reordered]

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.6

The student will calculate probabilities. Key concepts include~~:~~

a) conditional probability;
b) dependent and independent events;
c) ~~addition and multiplication rules~~mutually exclusive events;
d) counting techniques (permutations and combinations); and
e) Law of Large Numbers.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Mutually exclusive events are calculated using the addition or multiplication rules. • If A and B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$. • The complement of event A consists of all outcomes in which event A does not occur. • $P(B\|A)$ is the probability that B will occur given that A has already occurred. $P(B\|A)$ is called the conditional probability of B given A. • Venn diagrams may be used to examine conditional probabilities.  $P(B\|A) = \frac{P(A \cap B)}{P(A)}$ $\Rightarrow P(A \cap B) = P(A)P(B\|A)$	• Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events. (b,c) • Given two or more events in a problem setting, determine if the events are complementary, dependent, independent, and/or mutually exclusive. (b,c) • ~~Find conditional probabilities for dependent, independent, and mutually exclusive events.~~ [Reordered] • ~~Represent and calculate probabilities using Venn diagrams and probability trees.~~ [Reordered] • ~~Analyze, interpret and make predictions based on theoretical probability within real-world context.~~ [Reordered] • Compare and contrast permutations and combinations. (d) [Reordered] • Calculate the number of permutations of n objects taken r at a time, without repetition. (d) [Reordered]

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.6

The student will calculate probabilities. Key concepts include~~:~~

a) conditional probability;
b) dependent and independent events;
c) ~~addition and multiplication rules~~mutually exclusive events;
d) counting techniques (permutations and combinations); and
e) Law of Large Numbers.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Two events, *A* and *B*, are independent if the occurrence of one does not affect the probability of the occurrence of the other. If *A* and *B* are not independent, then they are said to be dependent. • If *A* and *B* are independent events, then $P(A \cap B) = P(A)P(B)$. • Permutations are used to calculate the number of possible arrangements of objects when the order matters. [Reordered] • Combinations are used to calculate the number of possible selections of objects without regard to the order selected. [Reordered] • A permutation is the number of possible was to arrange a group of objects when order matters (e.g., the outcome 1,2,3 is different from the outcome 3,2,1 when order matters; therefore, both arrangements would be included in the possible outcomes).	• Calculate the number of combinations of *n* objects taken *r* at a time, without repetition. (d) [Reordered] • Given a ~~real-world~~practical situation, determine when to use permutations or combinations. (d)

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.6

The student will calculate probabilities. Key concepts include~~:~~

a) conditional probability;
b) dependent and independent events;
c) ~~addition and multiplication rules~~mutually exclusive events;
d) counting techniques (permutations and combinations); and
e) Law of Large Numbers.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A combination is the number of possible ways to select or arrange objects when there is no repetition and order does not matter (e.g., the outcome 1,2,3 is the same as the outcome 3,2,1 when order does not matter; therefore, both arrangements would not be included in the possible outcomes). • The Law of Large Numbers states that as a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.	

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.7

The student will ~~analyze the normal distribution. Key concepts include:~~

a) describe the distribution of a univariate data set using descriptive statistics~~characteristics of normally distributed data~~;
b) identify and describe properties of a normal distribution~~percentiles~~;
c) interpret and compare *z*-scores for normally distributed data~~normalizing data using z-scores~~; and
d) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve~~area under the standard normal curve and probability~~.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Analysis of the descriptive statistical information generated by a univariate data set includes the relationships between central tendency, dispersion, and position.~~ • ~~The normal distribution curve is a family of symmetrical curves defined by the mean and the standard deviation.~~ [Reordered] • ~~Areas under the curve represent probabilities associated with continuous distributions.~~ • The focus of this standard is on the interpretation of descriptive statistics, *z*-scores, probabilities, and their relationship to the normal curve in the context of a data set. • Descriptive statistics include measures of center (mean, median, mode) and dispersion or spread (variance and standard deviation). • Variance and standard deviation measure the spread of data about the mean in a data set. • Standard deviation is expressed in the original units of measurement of the data.	• Describe the distribution of a univariate data set using descriptive statistics including measures of center and dispersion (spread). (a) • ~~Interpret mean, median, mode, range, interquartile range, variance, and standard deviation of a univariate data set in terms of the problem's context.~~ • ~~Explain the influence of outliers on a univariate data set.~~ • ~~Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.~~ • Identify the properties of a normal ~~probability~~ distribution. (b) • Describe how the standard deviation and the mean affect the graph of the normal distribution. (b) • Given standard deviation and mean, calculate and interpret *z*-score for a data point. (c)

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.7

The student will ~~analyze the normal distribution. Key concepts include:~~

a) describe the distribution of a univariate data set using descriptive statistics~~characteristics of normally distributed data~~;
b) identify and describe properties of a normal distribution~~percentiles~~;
c) interpret and compare *z*-scores for normally distributed data~~normalizing data using z-scores~~; and
d) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve~~area under the standard normal curve and probability~~.

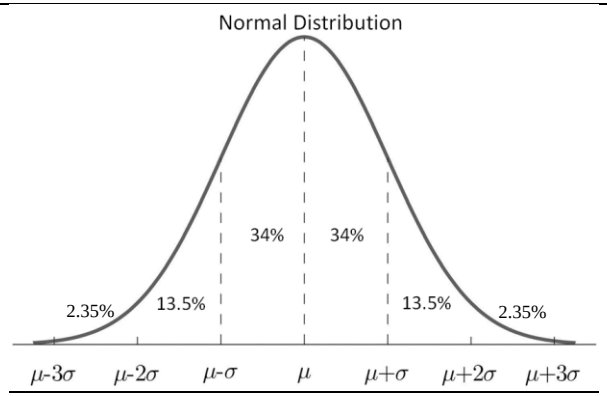
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The greater the value of the standard deviation, the further the data tend to be dispersed from the mean. • The normal distribution curve is the family of symmetrical, bell-shaped curves defined by the mean and the standard deviation of a data set. The arithmetic mean is located on the line of symmetry of the curve and is equivalent to the median and mode of the data set. • The normal curve is a probability distribution and the total area under the curve is 1. • For a normal distribution, approximately 68 percent of the data fall within one standard deviation of the mean, approximately 95 percent of the data fall within two standard deviations of the mean, and approximately 99.7 percent of the data fall within three standard deviations of the mean. This is often referred to as the Empirical Rule or the 68-95-99.7 rule.	• Compare two sets of normally distributed data using a standard normal distribution and *z*-scores, given mean and standard deviation. (c) • Represent probability as area under the curve of a standard normal distribution. (d) • Use a graphing utility or a table of Standard Normal Probabilities to determine probabilities based on *z*-scores. (d) • Use a graphing utility to represent curves of a normally distributed data set. (a,b,c,d) • Use a graphing utility to investigate and determine relationships between a data set and its descriptive statistics. (a,b,c,d) ~~Determine the probability of a given event, using the normal distribution.~~

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.7

The student will ~~analyze the normal distribution. Key concepts include:~~

a) describe the distribution of a univariate data set using descriptive statistics~~characteristics of normally distributed data~~;
b) identify and describe properties of a normal distribution~~percentiles~~;
c) interpret and compare *z*-scores for normally distributed data~~normalizing data using z-scores~~; and
d) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve~~area under the standard normal curve and probability~~.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
 • The mean and standard deviation of a normal distribution affect the location and shape of the curve. The vertical line of symmetry of the normal distribution falls at the mean. The greater the standard deviation, the wider ("flatter" or "less peaked") the distribution of the data. • A *z*-score derived from a particular data value tells how many standard deviations that data value falls above or below the mean of the data set. It is positive if the data value lies above the mean and negative if the data value lies below the mean.	

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.7

The student will ~~analyze the normal distribution. Key concepts include:~~

a) describe the distribution of a univariate data set using descriptive statistics~~characteristics of normally distributed data~~;
b) identify and describe properties of a normal distribution~~percentiles~~;
c) interpret and compare *z*-scores for normally distributed data~~normalizing data using z-scores~~; and
d) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve~~area under the standard normal curve and probability~~.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A standard normal distribution is the set of all *z*-scores. The mean of the data in a standard normal distribution ~~density function~~ is 0 and the standard deviation is 1. This allows for the comparison of unlike normal data. • The table of Standard Normal Probabilities and graphing utilities may be used to determine normal distribution probabilities. • Given a *z*-score (*z*), one table of Standard Normal Probabilities (*z*-table) shows the area under the curve to the left of *z*. This area represents the proportion of observations with a *z*-score less than the one specified. Table rows show the *z*-score's whole number and tenths place. Table columns show the hundredths place. ~~• The amount of data that falls within 1, 2, or 3 standard deviations of the mean is constant and the basis of *z*-score data normalization.~~	

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.8

The student will design and conduct an experiment/survey. Key concepts include~~:~~

a) sample size;
b) sampling technique;
c) controlling sources of bias and experimental error;
d) data collection; and
e) data analysis and reporting.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The value of a sample statistic may vary from sample to sample, even if the simple random samples are taken repeatedly from the population of interest. • Poor data collection can lead to misleading and meaningless conclusions. • Considerations such as sample size, randomness, and bias affect experimental design. • The purpose of sampling is to provide sufficient information so that population characteristics may be inferred. • Inherent bias diminishes as sample size increases. • Experiments must be carefully designed in order to detect a cause-and-effect relationship between variables. • Principles of experimental design include comparison with a control group, randomization, and blindness. • The precision, accuracy and reliability of data collection can be analyzed and described.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling. (a,b) [Reordered] • Determine which sampling technique is best, given a particular context. (b) [Reordered] • ~~Compare and contrast controlled experiments and observational studies and the conclusions one may draw from each.~~ [Reordered] • Identify biased sampling methods. (c) • Given a plan for a survey, identify possible sources of bias, and describe ways to reduce bias. (c) [Reordered] • Compare and contrast controlled experiments and observational studies and the conclusions one may draw from each. (e) [Reordered] • Write a report describing the experiment/survey and the resulting data and analysis. (e) [Reordered] • ~~Select a data collection method appropriate for a given context.~~

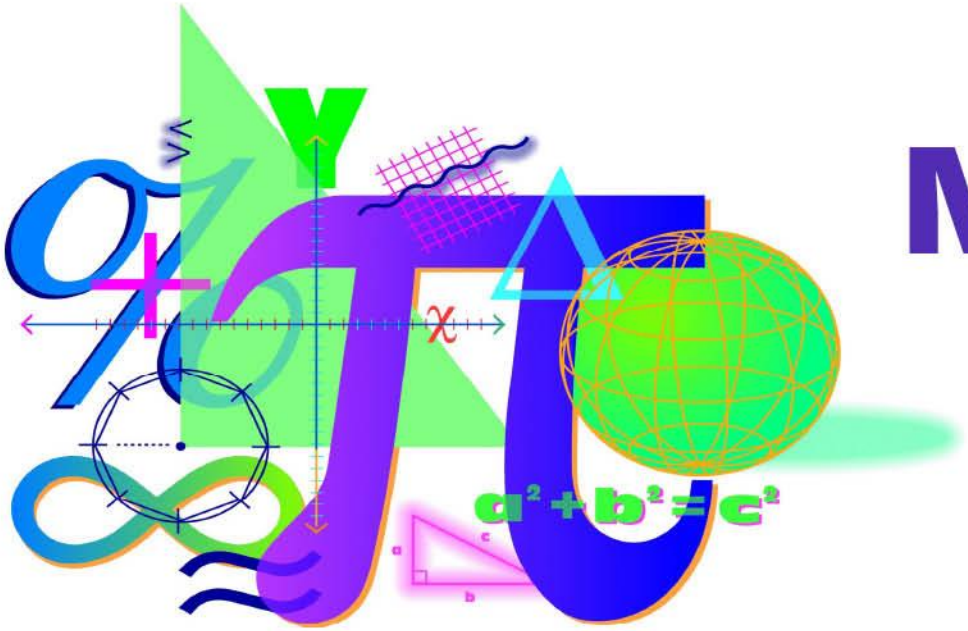
TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.8

The student will design and conduct an experiment/survey. Key concepts include~~:~~

a) sample size;
b) sampling technique;
c) controlling sources of bias and experimental error;
d) data collection; and
e) data analysis and reporting.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
	~~• Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling.~~ [Reordered] ~~• Determine which sampling technique is best, given a particular context.~~ [Reordered] • Plan and conduct an experiment or survey. The experimental design should address control, randomization, and minimization of experimental error. (a,b,c,d) ~~• Design a survey instrument.~~ • ~~Given a plan for a survey, identify possible sources of bias, and describe ways to reduce bias.~~ [Reordered] • ~~Write a report describing the experiment/survey and the resulting data and analysis.~~ [Reordered]



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Algebra II

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

~~TOPIC~~STRAND: EXPRESSIONS AND OPERATIONS

ALGEBRA II
STANDARD AII.1

The student, given rational, radical, or polynomial expressions, will

a) add, subtract, multiply, divide, and simplify rational algebraic expressions;

b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;

~~c) write radical expressions as expressions containing rational exponents and vice versa~~[Moved to EKS]~~;~~ and

~~d~~c) factor polynomials completely in one or two variables.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Computational skills applicable to numerical fractions also apply to rational expressions involving variables. • Radical expressions can be written and simplified using rational exponents. • Only radicals with a common radicand and index can be added or subtracted. • A relationship exists among arithmetic complex fractions, algebraic complex fractions, and rational numbers. • The complete factorization of polynomials has occurred when each factor is a prime polynomial. • Pattern recognition can be used to determine complete factorization of a polynomial. • Polynomials may be factored in various ways, including, but not limited to grouping or applying general patterns including difference of squares, sum and difference of cubes, and perfect square trinomials.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Add, subtract, multiply, and divide rational algebraic expressions. (a) • Simplify a rational algebraic expression with ~~common~~ monomial or binomial factors. Algebraic expressions should be limited to linear and quadratic expressions. (a) • Recognize a complex algebraic fraction, and simplify it as a quotient or product of simple algebraic fractions. (a) • Simplify radical expressions containing positive rational numbers and variables. (b) • Convert ~~from~~ between radical expressions ~~notation to~~and expressions containing rational exponents~~ial notation, and vice versa~~. (b) • Add and subtract radical expressions. (b) • Multiply and divide radical expressions not requiring rationalizing the denominators. (b)

~~TOPIC~~STRAND: EXPRESSIONS AND OPERATIONS

ALGEBRA II
STANDARD AII.1

The student, given rational, radical, or polynomial expressions, will

a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;
~~c) write radical expressions as expressions containing rational exponents and vice versa~~[Moved to EKS]~~;~~ and
~~d~~c) factor polynomials completely in one or two variables.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
	• Factor polynomials in one or two variables with no more than four terms completely over the set of integers ~~by applying general patterns including difference of squares, sum and difference of cubes, and perfect square trinomials~~. Factors of the polynomial should be constant, linear, or quadratic. (c) • ~~Factor polynomials completely over the integers.~~ [Combined] • Verify polynomial identities including the difference of squares, sum and difference of cubes, and perfect square trinomials. (c)[‡] [‡]~~Revised March 2011~~

~~TOPIC~~STRAND: FUNCTIONS

ALGEBRA II
STANDARD AII.5~~2~~ [Moved to "functions" strand]

The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve ~~real-world~~practical problems, including writing the first *n* terms, ~~finding~~ determining the nth term, and evaluating summation formulas. Notation will include $\sum$ and a_n.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Sequences and series arise from ~~real-world~~practical situations. • The study of sequences and series is an application of ~~-~~the investigation of patterns. • A sequence is a function whose domain is the set of natural numbers. • Sequences can be defined explicitly and recursively.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Distinguish between a sequence and a series. • Generalize patterns in a sequence using explicit and recursive formulas. • Use and interpret the notations $\sum$, n, n^{th} term, and a_n. • Given the formula, ~~find~~ determine a_n (the nth term) for an arithmetic or a geometric sequence. • Given formulas, write the first n terms and ~~find~~ determine the sum, S_n, of the first n terms of an arithmetic or geometric series. • Given the formula, ~~find~~ determine the sum of a convergent infinite series. • Model ~~real-world~~practical situations using sequences and series.

~~TOPIC~~STRAND: EXPRESSIONS AND OPERATIONS

ALGEBRA II
STANDARD AII.2~~3~~

The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of *i*~~, and identify field properties that are valid for the complex numbers~~.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Complex numbers can be classified as~~are organized into a hierarchy of subsets~~ either real or imaginary numbers. • A complex number multiplied by its conjugate is a real number. • Equations having no real number solutions may have solutions in the set of complex numbers. • Field properties apply to complex numbers as well as real numbers. • All complex numbers can be written in the form $a + bi$ where a and b are real numbers and i is $\sqrt{-1}$ ~~.~~ (e.g., $\pm\sqrt{-9} = 0 \pm 3i; 5 = 5 + 0i$).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize that the square root of –1 is represented as *i*. • ~~Determine which field properties apply to the complex number system.~~ • Simplify radical expressions containing negative rational numbers and express in $a + bi$ form. • Simplify powers of *i*. • Add, subtract, and multiply complex numbers. • ~~Place the following sets of numbers in a hierarchy of subsets: complex, pure imaginary, real, rational, irrational, integers, whole, and natural.~~ • ~~Write a real number in a+b*i* form.~~ • ~~Write a pure imaginary number in a+b*i* form.~~

~~TOPIC~~STRAND: EQUATIONS AND INEQUALITIES

ALGEBRA II
STANDARD AII.34

The student will solve, algebraically and graphically,

a) **absolute value linear equations and inequalities;**
b) **quadratic equations over the set of complex numbers;**
c) **equations containing rational algebraic expressions; and**
d) **equations containing radical expressions.**

~~Graphing calculators will be used for solving and for confirming the algebraic solutions.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A quadratic function whose graph does not intersect the *x*-axis has roots with imaginary components. • The quadratic formula can be used to solve any quadratic equation. • The quadratic formula can be derived by applying the completion of squares to any quadratic equation in standard form. [Moved from EKS] • The value of the discriminant of a quadratic equation can be used to describe the number and type of ~~real and complex~~ solutions. • Solutions of quadratic equations are real or imaginary. • Imaginary solutions occur in conjugate pairs. • Quadratic equations with exactly one real root can be referred to as having one distinct root with a multiplicity of two. For instance, the quadratic equation, $x^2 - 4x + 4$, has two identical factors, giving one real root with a multiplicity of two.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve absolute value linear inequalities in one variable algebraically. (a) • Represent solutions to absolute value linear inequalities in one or two variables graphically.(a) • Solve absolute value linear equations ~~and inequalities~~ algebraically and graphically. (a) • Solve a quadratic equation over the set of complex numbers using an appropriate strategy. (b) • Calculate the discriminant of a quadratic equation to determine the number and type of ~~real and complex~~ solutions. (b) • Solve rational equations with real solutions containing ~~rational~~factorable algebraic expressions ~~with monomial or binomial denominators~~ algebraically and graphically. Algebraic expressions should be limited to linear and quadratic expressions. (c)

~~TOPIC~~STRAND: EQUATIONS AND INEQUALITIES

ALGEBRA II
STANDARD AII.34

The student will solve, algebraically and graphically,

a) absolute value linear equations and inequalities;
b) quadratic equations over the set of complex numbers;
c) equations containing rational algebraic expressions; and
d) equations containing radical expressions.

~~Graphing calculators will be used for solving and for confirming the algebraic solutions.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The definition of absolute value (for any real numbers *a* and *b*, where $b \geq 0$, if $\lvert a \rvert = b$, then $a = b$ or $a = -b$) is used in solving absolute value equations and inequalities. • Absolute value inequalities in one variable can be solved algebraically~~graphically or by~~ using a compound statement. • Compound statements representing solutions of an inequality in one variable can be represented graphically on a number line. • ~~Real-world~~Practical problems can be interpreted, represented, and solved using equations and inequalities. • The process of solving ~~radical or rational~~ equations can lead to extraneous solutions. • An extraneous solution is a solution of the simplified form of an equation that does not satisfy the original equation. • Equations can be solved in a variety of ways. • The zeros of the function are the root(s) or solution(s) of the equation that is formed by setting the given expression equal to zero.	• Solve an equation containing ~~a~~ no more than one radical expression algebraically and graphically. (d) • Solve equations and ~~V~~verify ~~possible~~algebraic solutions ~~to an equation~~using a graphing utility ~~containing rational or radical expressions~~. (a,b,c,d) [Moved from standard] • ~~Apply an appropriate equation to solve a real-world problem.~~ • ~~Recognize that the quadratic formula can be derived by applying the completion of squares to any quadratic equation in standard form.~~[‡] [Moved to US] [‡]~~Revised March 2011~~

~~TOPIC~~STRAND: EQUATIONS AND INEQUALITIES

ALGEBRA II
STANDARD AII.34

The student will solve, algebraically and graphically,

a) absolute value linear equations and inequalities;
b) quadratic equations over the set of complex numbers;
c) equations containing rational algebraic expressions; and
d) equations containing radical expressions.

~~Graphing calculators will be used for solving and for confirming the algebraic solutions.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The zeros, roots, or solutions of a function are the values of x that make $f(x) = 0$ • The real zeros of a function are the x-intercepts of that function. • Radical expressions may be converted to expressions using rational exponents. • The equation of an inverse variation is a rational function. • ~~Set builder notation may be used to represent solution sets of equations and inequalities.~~ • Solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation. Examples may include:	

Equation/ Inequality	Set Notation	Interval Notation
$x = 3$	$\{3\}$	
$x = 3$ or $x = 5$	$\{3,5\}$	
$0 \le x < 3$	$\{x \mid 0 \le x < 3\}$	$[0, 3)$
$y \ge 3$	$\{y: y \ge 3\}$	$[3, \infty)$
Empty (null) set $\emptyset$	$\{\ \}$	

~~TOPIC~~STRAND: EQUATIONS AND INEQUALITIES

ALGEBRA II
STANDARD AII.4~~5~~

The student will solve ~~nonlinear~~ systems of ~~equations, including~~ linear-quadratic and quadratic-quadratic equations, algebraically and graphically. ~~Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Quadratic equations included in this standard will only include those that can be represented as parabolas of the form $y = ax^2 + bx + c$ where $a \neq 0$. • Solutions of a nonlinear system of equations are numerical values that satisfy every equation in the system. • A linear-quadratic system of equations may have 0, 1, or 2 solutions. • A quadratic-quadratic system of equations may have 0, 1, 2, or infinite number of solutions. • The coordinates of points of intersection in any system of equations are solutions to the system. • ~~Real-world~~Practical problems can be interpreted, represented, and solved using systems of equations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Predict~~ Determine the number of solutions to a ~~nonlinear~~ linear-quadratic and quadratic-quadratic system of ~~two~~ equations. • Solve a linear-quadratic system of two equations algebraically and graphically. • Solve a quadratic-quadratic system of two equations algebraically and graphically. • Solve systems of equations and verify solutions of systems of equations with a graphing utility. [Moved from standard]

~~TOPIC~~STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.6

For absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions, ~~T~~the student will

a) recognize the general shape of function ~~(absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic)~~ families; and ~~will~~

b) use knowledge of transformations to convert between graphic and symbolic forms of functions.

~~A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The transformation of a function called a pre-image changes the size, shape, and/or position of the function to a new function called the image. • The graphs/equations for a family of functions can be determined using a transformational approach. • The graph of a parent function is an anchor graph from which other graphs are derived using transformations. [Reordered] • Transformations of functions may require the domain to be restricted. • Transformations of graphs include – Translations (horizontal and vertical shifting of a graph); – Reflections (over the *x*-axis and *y*-axis);~~,~~ and – Dilations (stretching and compressing graphs). • ~~A parent graph is an anchor graph from which other graphs are derived with transformations.~~ [Reordered] • The reflection of a function over the line $y = x$ represents the inverse of a function.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize the general shape of function families. (a) • Recognize graphs of parent functions. (a) • Identify the equation from a graph of a function. (b) • Identify the graph of a function from the equation. (b) • Write the equation of a function given the graph. (b) • Graph a transformation of a parent function, given the equation. (b) • Identify the transformation(s) of a function, from a graph or from the equation. Transformations of exponential and logarithmic functions should be limited to a single transformation. (b) • Investigate and verify transformations of functions using a graphing utility. (a, b) [Moved from standard] • ~~Given a transformation of a parent function, identify the graph of the transformed function.~~ • ~~Given the equation and using a transformational approach, graph~~

~~TOPIC~~STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.6

For absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions, ~~T~~the student will

a) recognize the general shape of function ~~(absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic)~~ families; and ~~will~~

b) use knowledge of transformations to convert between graphic and symbolic forms of functions.

~~A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
	~~a function.~~ • ~~Given the graph of a function, identify the parent function.~~ • ~~Given the graph of a function, identify the transformations that map the preimage to the image in order to determine the equation of the image.~~ • ~~Using a transformational approach, write the equation of a function given its graph.~~

~~TOPIC~~STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions~~s~~ families algebraically and graphically. Key concepts include

- **a) domain, ~~and~~ range, and continuity~~including limited and discontinuous domains and ranges~~;**
- **b) intervals in which a function is increasing or decreasing; [Reordered]**
- **c) maxima and minima;**
- **d) zeros;**
- **~~c~~e) ~~*x*- and *y*-~~intercepts;**
- **~~d~~f) ~~intervals in which a function is increasing or decreasing~~values of a function for elements in its domain;**
- **~~e~~g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic~~asymptotes~~;**
- **~~f~~h) end behavior;**
- **i) vertical and horizontal asymptotes; [Reordered]**
- **~~g~~j) inverse of a function; and**
- **~~h~~k) composition of ~~multiple~~ functions algebraically.**

~~Graphing calculators will be used as a tool to assist in investigation of functions.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Functions may be used to model practical situations. [reordered] • Functions describe the relationship between two variables where each input is paired to a unique output. • Function families consist of a parent function and all transformations of the parent function. • The domain of a function is the set of all possible values of the independent variable. • The range of a function is the set of all possible values of the dependent variable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the domain, range, zeros, and intercepts of a function (linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) presented algebraically or graphically, including graphs with discontinuities. (a,d,e) • Describe a function as continuous or discontinuous. (a) • ~~Describe restricted/discontinuous domains and ranges.~~

~~TOPIC~~STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions families algebraically and graphically. Key concepts include

- **a) domain, ~~and~~ range, and continuity~~including limited and discontinuous domains and ranges~~;**
- **b) intervals in which a function is increasing or decreasing; [Reordered]**
- **c) maxima and minima;**
- **d) zeros;**
- **~~c~~e) ~~*x*- and *y*-~~intercepts;**
- **~~d~~f) ~~intervals in which a function is increasing or decreasing~~values of a function for elements in its domain;**
- **~~e~~g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic~~asymptotes~~;**
- **~~f~~h) end behavior;**
- **i) vertical and horizontal asymptotes; [Reordered]**
- **~~g~~j) inverse of a function; and**
- **~~h~~k) composition of ~~multiple~~ functions algebraically.**

~~Graphing calculators will be used as a tool to assist in investigation of functions.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• For each x in the domain of *f*, *x* is a member of the input of the function *f*, *f(x)* is a member of the output of *f*, and the ordered pair [*x*, *f(x)*] is a member of *f*. • ~~Functions may be used to model real-world situations.~~ [Reordered] • A function is said to be continuous on an interval if its graph has no breaks, jumps, or holes in that interval. • The domain ~~and range~~ of a function may be restricted algebraically, graphically, or by the ~~real-world~~practical situation modeled by ~~the~~a function.	• Given the graph of a function, identify intervals on which the function (linear, quadratic, absolute value, square root, cube root, polynomial, exponential, and logarithmic) is increasing ~~and~~or decreasing. (b) • Identify the location and value of absolute maxima and absolute minima of a function over the domain of the function graphically or by using a graphing utility. (c) • Identify the location and value of relative maxima or relative minima of a function over some interval of the domain graphically or by using a graphing utility. (c) • For any *x* value in the domain of *f*, determine *f(x)*. (f)

~~TOPIC~~STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions~~s~~ families algebraically and graphically. Key concepts include

- **a) domain, ~~and~~ range, and continuity~~including limited and discontinuous domains and ranges~~;**
- **b) intervals in which a function is increasing or decreasing; [Reordered]**
- **c) maxima and minima;**
- **d) zeros;**
- **~~c~~e) ~~*x*- and *y*-~~intercepts;**
- **~~d~~f) ~~intervals in which a function is increasing or decreasing~~values of a function for elements in its domain;**
- **~~e~~g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic~~asymptotes~~;**
- **~~f~~h) end behavior;**
- **i) vertical and horizontal asymptotes; [Reordered]**
- **~~g~~j) inverse of a function; and**
- **~~h~~k) composition of ~~multiple~~ functions algebraically.**

~~Graphing calculators will be used as a tool to assist in investigation of functions.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Discontinuous domains and ranges include those with removable (holes) and nonremovable (asymptotes) discontinuities. • A function can be described on an interval as increasing, decreasing, or constant over a specified interval or over the entire domain of the function. • A function, *f(x)*, is increasing over an interval if the values of *f(x)* consistently increase over the interval as the *x* values increase. • A function, *f(x)*, is decreasing over an interval if the values of *f(x)* consistently decrease over the interval as the *x* values increase. • A function, *f(x)*, is constant over an interval if the values of *f(x)* remain constant over the interval as the *x* values increase.	• Represent relations and functions using concrete, verbal, numeric, graphic, and algebraic forms. Given one representation, represent the relation in another form. (g) • Describe the end behavior of a function. (h) [Reordered] • ~~Find~~ Determine the equations of vertical and horizontal asymptotes of functions (rational, exponential, and logarithmic). (i) • ~~Describe the end behavior of a function.~~ [Reordered] • ~~Find~~ Determine the inverse of a function (linear, quadratic, cubic, square root, and cube root). (j)

~~TOPIC~~STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions families algebraically and graphically. Key concepts include

- **a) domain, ~~and~~ range, and continuity~~including limited and discontinuous domains and ranges~~;**
- **b) intervals in which a function is increasing or decreasing; [Reordered]**
- **c) maxima and minima;**
- **d) zeros;**
- **~~c~~e) ~~*x*- and *y*-~~intercepts;**
- **~~d~~f) ~~intervals in which a function is increasing or decreasing~~values of a function for elements in its domain;**
- **~~e~~g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic~~asymptotes~~;**
- **~~f~~h) end behavior;**
- **i) vertical and horizontal asymptotes; [Reordered]**
- **~~g~~j) inverse of a function; and**
- **~~h~~k) composition of ~~multiple~~ functions algebraically.**

~~Graphing calculators will be used as a tool to assist in investigation of functions.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation. Examples may include: Equation/ Inequality | Set Notation | Interval Notation $x = 3$ | $\{3\}$ | $x = 3$ or $x = 5$ | $\{3,5\}$ | $0 \le x < 3$ | $\{x \mid 0 \le x < 3\}$ | $[0, 3)$ $y \ge 3$ | $\{y: y \ge 3\}$ | $[3, \infty)$ Empty (null) set $\emptyset$ | $\{\ \}$ | • A function, *f*, has an absolute maximum located at $x = a$ if $f(a)$ is the largest value of *f* over its domain.	• Graph the inverse of a function as a reflection ~~across~~ over the line $y = x$. (j) • ~~Investigate exponential and logarithmic functions, using the graphing calculator.~~ • ~~Convert between logarithmic and exponential forms of an equation with bases consisting of natural numbers.~~ • ~~Find~~ Determine the composition of two functions algebraically. (k) • Use algebraic composition of functions to verify two functions are inverses. (k)

~~TOPIC~~STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions~~s~~ families algebraically and graphically. Key concepts include

- **a) domain, ~~and~~ range, and continuity~~including limited and discontinuous domains and ranges~~;**
- **b) intervals in which a function is increasing or decreasing; [Reordered]**
- **c) maxima and minima;**
- **d) zeros;**
- **~~c~~e) ~~*x*- and *y*-~~intercepts;**
- **~~d~~f) ~~intervals in which a function is increasing or decreasing~~values of a function for elements in its domain;**
- **~~e~~g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic~~asymptotes~~;**
- **~~f~~h) end behavior;**
- **i) vertical and horizontal asymptotes; [Reordered]**
- **~~g~~j) inverse of a function; and**
- **~~h~~k) composition of ~~multiple~~ functions algebraically.**

~~Graphing calculators will be used as a tool to assist in investigation of functions.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A function, *f*, has an absolute minimum located at $x = a$ if $f(a)$ is the smallest value of *f* over its domain. • Relative maximum points occur where the function changes from increasing to decreasing. • A function, *f*, has a relative maximum located at $x = a$ over some interval of the domain if $f(a)$ is the largest value of *f* on the interval. • Relative minimum points occur where the function changes from decreasing to increasing.	• Investigate and analyze characteristics and multiple representations of functions with a graphing utility. (a,b,c,d,e,f,g,h,i,j,k) [Moved from standard]

~~TOPIC~~STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions families algebraically and graphically. Key concepts include

- **a) domain, ~~and~~ range, and continuity~~including limited and discontinuous domains and ranges~~;**
- **b) intervals in which a function is increasing or decreasing; [Reordered]**
- **c) maxima and minima;**
- **d) zeros;**
- **~~c~~e) ~~*x*- and *y*-~~intercepts;**
- **~~d~~f) ~~intervals in which a function is increasing or decreasing~~values of a function for elements in its domain;**
- **~~e~~g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic~~asymptotes~~;**
- **~~f~~h) end behavior;**
- **i) vertical and horizontal asymptotes; [Reordered]**
- **~~g~~j) inverse of a function; and**
- **~~h~~k) composition of ~~multiple~~ functions algebraically.**

~~Graphing calculators will be used as a tool to assist in investigation of functions.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A function, *f*, has a relative minimum located at $x = a$ over some interval of the domain if $f(a)$ is the smallest value of *f* on the interval. • A value *x* in the domain of *f* is an *x*-intercept or a zero of a function *f* if and only if $f(x) = 0$. • Given a polynomial function $f(x)$, the following statements are equivalent for any real number such that $f(k) = 0$: – *k* is a zero of the polynomial function $f(x)$ located at $(k, 0)$; – *k* is a solution or root of the polynomial equation $f(x) = 0$; – the point $(k, 0)$ is an *x*-intercept for the graph of $f(x) = 0$; and – $(x – k)$ is a factor of $f(x)$.	

~~TOPIC~~STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions~~s~~ families algebraically and graphically. Key concepts include

- **a) domain, ~~and~~ range, and continuity~~including limited and discontinuous domains and ranges~~;**
- **b) intervals in which a function is increasing or decreasing; [Reordered]**
- **c) maxima and minima;**
- **d) zeros;**
- **~~c~~e) ~~*x*- and *y*-~~intercepts;**
- **~~d~~f) ~~intervals in which a function is increasing or decreasing~~values of a function for elements in its domain;**
- **~~e~~g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic~~asymptotes~~;**
- **~~f~~h) end behavior;**
- **i) vertical and horizontal asymptotes; [Reordered]**
- **~~g~~j) inverse of a function; and**
- **~~h~~k) composition of ~~multiple~~ functions algebraically.**

~~Graphing calculators will be used as a tool to assist in investigation of functions.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Connections between multiple representations (graphs, tables, and equations) of a function can be made. • End behavior describes the values of a function as *x* approaches positive or negative infinity. [reordered] • If (*a*, *b*) is an element of a function, then (*b*, *a*) is an element of the inverse of the function. [reordered] • The reflection of a function over the line $y = x$ represents the inverse of the reflected function. • A function is invertible if its inverse is also a function. For an inverse of a function to be a function, the domain of the function may need to be restricted.	

~~TOPIC~~STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions families algebraically and graphically. Key concepts include

- **a) domain, ~~and~~ range, and continuity~~including limited and discontinuous domains and ranges~~;**
- **b) intervals in which a function is increasing or decreasing; [Reordered]**
- **c) maxima and minima;**
- **d) zeros;**
- **~~c~~e) ~~*x*- and *y*-~~intercepts;**
- **~~d~~f) ~~intervals in which a function is increasing or decreasing~~values of a function for elements in its domain;**
- **~~e~~g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic~~asymptotes~~;**
- **~~f~~h) end behavior;**
- **i) vertical and horizontal asymptotes; [Reordered]**
- **~~g~~j) inverse of a function; and**
- **~~h~~k) composition of ~~multiple~~ functions algebraically.**

~~Graphing calculators will be used as a tool to assist in investigation of functions.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Exponential and logarithmic functions are inverses of each other. [reordered] • Functions can be combined using composition of functions. • Two functions, *f(x)* and *g(x)*, are inverses of each other if *f(g(x)) = g(f(x)) = x*. • ~~Asymptotes may describe both local and global behavior of functions.~~ [Reordered] • ~~End behavior describes a function as *x* approaches positive and negative infinity.~~ [Reordered] • ~~A zero of a function is a value of *x* that makes f(x) equal zero.~~ [Combined]	

~~TOPIC~~STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions~~s~~ families algebraically and graphically. Key concepts include

- **a) domain, ~~and~~ range, and continuity~~including limited and discontinuous domains and ranges~~;**
- **b) intervals in which a function is increasing or decreasing; [Reordered]**
- **c) maxima and minima;**
- **d) zeros;**
- **~~c~~e) ~~*x*- and *y*-~~intercepts;**
- **~~d~~f) ~~intervals in which a function is increasing or decreasing~~values of a function for elements in its domain;**
- **~~e~~g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic~~asymptotes~~;**
- **~~f~~h) end behavior;**
- **i) vertical and horizontal asymptotes; [Reordered]**
- **~~g~~j) inverse of a function; and**
- **~~h~~k) composition of ~~multiple~~ functions algebraically.**

~~Graphing calculators will be used as a tool to assist in investigation of functions.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~If (a, b) is an element of a function, then (b, a) is an element of the inverse of the function.~~ [Reordered] • ~~Exponential ($y = a^x$) and logarithmic ($y = \log_a x$) functions are inverses of each other.~~ [Reordered] • ~~Functions can be combined using composition of functions.~~ [Reordered]	

~~TOPIC~~STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.8

The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The *Fundamental Theorem of Algebra* states that, including complex and repeated solutions, an nth degree polynomial equation has exactly *n* roots (solutions). • Solutions of polynomial equations may be real, imaginary, or a combination of real and imaginary. • Imaginary solutions occur in conjugate pairs. • Given a polynomial function *f(x)*, the following statements are equivalent for any real number *k*, such that *f(k)* = 0: – *k* is a zero of the polynomial function *f(x)* located at (*k*, 0); – *k* is a solution or root of the polynomial equation *f(x)* = 0; – the point (*k*, 0) is an *x*-intercept for the graph of polynomial *f(x)* = 0; and – *(x – k)* is a factor of polynomial *f(x)*. • Polynomial equations may have fewer distinct roots than the order of the polynomial. In these situations, a root may have "multiplicity." For instance, the polynomial equation $x^3 - 4x^2 + 4x + x$ has two identical factors, $(x - 2)$, and one other factor, x. This polynomial equation has two distinct, real roots, one with a multiplicity of 2.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.~~ • Define a polynomial function in factored form, given its zeros. • Determine a factored form of a polynomial expression from the *x*-intercepts of the graph of its corresponding function. • For a function, identify zeros of multiplicity greater than 1 and describe the effect of those zeros on the graph of the function. • Given a polynomial equation, determine the number ~~of real~~and type of solutions~~ and nonreal solutions~~.

~~TOPIC~~STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.9

The student will collect and analyze data, determine the equation of the curve of best fit~~,~~ in order to make predictions, and solve ~~real world~~practical problems, using mathematical models of linear, quadratic, and exponential functions. ~~Mathematical models will include polynomial, exponential, and logarithmic functions.~~

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Data and scatterplots may indicate patterns that can be modeled with an algebraic equation. • The curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate. • Knowledge of transformational graphing using parent functions can be used to generate a mathematical model from a scatterplot that approximates the data. • Graphing ~~calculators~~ utilities can be used to collect, organize, ~~picture~~represent, and ~~create~~ generate an equation of a curve of best fit ~~algebraic model of the~~for a set of data. • Data that fit ($y = mx + b$), quadratic ($y = ax^2 + bx + c$), and exponential ($y = ab^x$) ~~polynomial ($f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, where n is a nonnegative integer, and the coefficients are real numbers), exponential ($y = b^x$), and logarithmic ($y = \log_b x$)~~ models arise from ~~real-world~~practical situations. • Rounding that occurs during intermediate steps of problem solving may reduce the accuracy of the final answer.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Collect and analyze data.~~ • ~~Investigate scatterplots to determine if patterns exist and then identify the patterns.~~ • ~~Find~~ Determine an equation ~~for~~ of the curve of best fit ~~for data~~, using a graphing ~~calculator~~utility, given a set of no more than twenty data points in a table, graph, or practical situation. ~~Models will include polynomial, exponential, and logarithmic functions.~~ • Make predictions, using data, scatterplots, or the equation of the curve of best fit. • Solve practical problems involving an equation of the curve of best fit. • ~~Given a set of data, determine the model that would best describe the data.~~

~~TOPIC~~STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.10

The student will ~~identify~~represent~~, create,~~ and solve ~~real-world~~problems, including practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Real-world~~Practical problems can be ~~modeled~~ represented and solved by using direct variation, inverse variation, joint variation, and a combination of direct and inverse variations. • A direct variation represents a proportional relationship between two quantities. The statement "y is directly proportional to x" is translated as $y = kx$. • The constant of proportionality (k) in a direct variation is represented by the ratio of the dependent variable to the independent variable and can be referred to as the constant of variation. • A direct variation can be represented by a line passing through the origin. • An inverse variation represents an inversely proportional relationship between two quantities. The statement "y is inversely proportional to x" is translated as $y = \frac{k}{x}$. • The constant of proportionality (k) in an inverse variation is represented by the product of the dependent variable and the independent variable and can be referred to as the constant of variation.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Translate "y varies jointly as x and z" as $y = kxz$.~~[Moved to US] • ~~Translate "y is directly proportional to x" as $y = kx$.~~[Moved to US] • ~~Translate "y is inversely proportional to x" as $y = \frac{k}{x}$.~~[Moved to US] • ~~Given a situation, determine the value of the constant of proportionality.~~[Combined] • Given a data set or practical situation, write the equation for an inverse variation. • Given a data set or practical situation, write the equation for a joint variation. • ~~Set up and s~~Solve problems, including ~~real-world~~practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.

~~TOPIC~~STRAND: FUNCTIONS AND STATISTICS **ALGEBRA II** **STANDARD AII.10** **The student will ~~identify~~represent~~, create,~~ and solve ~~real-world~~problems, including practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The graph of an inverse variation is a rational function. • Joint variation is a combination of direct variations. The statement "*y* varies jointly as *x* and *z*" is translated as $y = kxz$. • The value of the constant of proportionality is typically positive when applied in practical situations.	

STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.11

The student will

a) describe the distribution of a univariate data set using descriptive statistics;
b) identify and describe properties of a normal distribution
c) interpret and compare z-scores for normally distributed data; and
d) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

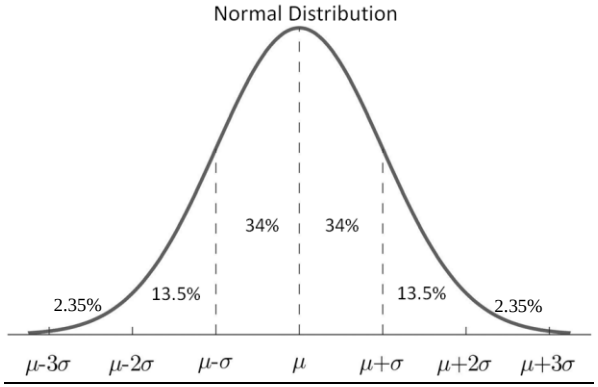
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The focus of this standard is on the interpretation of descriptive statistics, *z*-scores, probabilities, and their relationship to the normal curve in the context of a data set. • Descriptive statistics include measures of center (mean, median, mode) and dispersion or spread (variance and standard deviation). • Variance and standard deviation measure the spread of data about the mean in a data set. • Standard deviation is expressed in the original units of measurement of the data. • The greater the value of the standard deviation, the further the data tend to be dispersed from the mean. • A normal distribution curve is the family of~~a~~ symmetrical, bell-shaped curves defined by the mean and the standard deviation of a data set. The arithmetic mean is located on the line of symmetry of the curve and is equivalent to the median and mode of the data set. ~~• Areas under the curve represent probabilities associated with continuous distributions.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the distribution of a univariate data set using descriptive statistics including measures of center and dispersion (spread). (a) • Identify the properties of a normal ~~probability~~ distribution. (b) • Describe how the standard deviation and the mean affect the graph of the normal distribution. (b) • Solve problems involving the relationship of the mean, standard deviation, and z-score of a normally distributed data set. (c) • Compare two sets of normally distributed data using a standard normal distribution and *z*-scores, given the mean and standard deviation. (c) • Represent probability as area under the curve of a standard normal ~~probability~~ distribution. (d)

STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.11

The student will

a) describe the distribution of a univariate data set using descriptive statistics;
b) identify and describe properties of a normal distribution
c) interpret and compare z-scores for normally distributed data; and
d) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The normal curve is a probability distribution and the total area under the curve is 1. • For a normal distribution, approximately 68 percent of the data fall within one standard deviation of the mean, approximately 95 percent of the data fall within two standard deviations of the mean, and approximately 99.7 percent of the data fall within three standard deviations of the mean. This is often referred to as the Empirical Rule or the 68-95-99.7 rule.  	• Use the graphing ~~calculator~~utility or a table of ~~s~~Standard ~~n~~Normal ~~p~~Probabilities~~y table~~ to determine probabilities- associated with areas under the standard normal curve~~or percentiles based on z-scores~~. (d) • Use a graphing utility to represent curves of a normally distributed data set. (a,b,c,d) • Use a graphing utility to investigate, represent, and determine relationships between a normally distributed data set and its descriptive statistics. (a,b,c,d)

STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.11

The student will

a) describe the distribution of a univariate data set using descriptive statistics;
b) identify and describe properties of a normal distribution
c) interpret and compare z-scores for normally distributed data; and
d) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The mean and standard deviation of a normal distribution affect the location and shape of the curve. The vertical line of symmetry of the normal distribution falls at the mean. The greater the standard deviation, the wider ("flatter" or "less peaked") the distribution of the data. • A *z*-score derived from a particular data value tells how many standard deviations that data value falls above or below the mean of the data set. It is positive if the data value lies above the mean and negative if the data value lies below the mean. • A standard normal distribution is the set of all z-scores. The mean of the data in a standard normal distribution is 0 and the standard deviation is 1. This allows for the comparison of unlike normal data. • The table of Standard Normal Probabilities and graphing utilities may be used to determine normal distribution probabilities. • Given a *z*-score (*z*), one table of Standard Normal Probabilities (*z*-table) shows the area under the curve to the left of *z*. This area represents the proportion of observations with a *z*-score less than the one specified. Table rows show the *z*-score's whole number and tenths place. Table columns show the hundredths place. • ~~The standard normal curve allows for the comparison of data~~	

STRAND: FUNCTIONS AND STATISTICS

ALGEBRA II
STANDARD AII.11

The student will

a) describe the distribution of a univariate data set using descriptive statistics;
b) identify and describe properties of a normal distribution
c) interpret and compare z-scores for normally distributed data; and
d) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

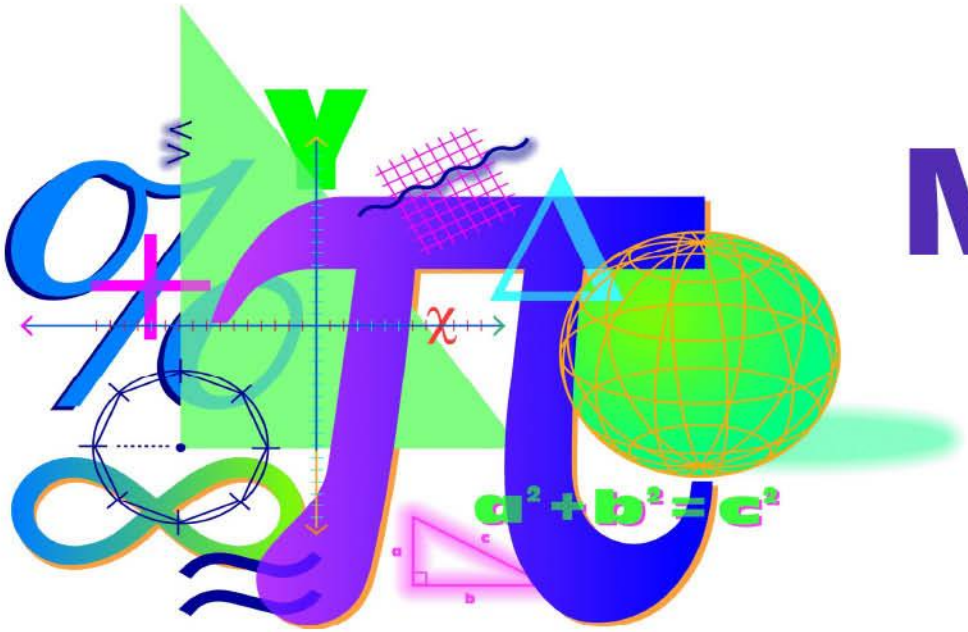
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
~~from different normal distributions.~~ • ~~A z-score is a measure of position derived from the mean and standard deviation of data.~~ [Combined] • ~~A z-score expresses, in standard deviation units, how far an element falls from the mean of the data set.~~ [Combined] • ~~A z-score is a derived score from a given normal distribution.~~ [Combined] • ~~A standard normal distribution is the set of all z-scores.~~ [Combined]	

~~TOPIC~~STRAND: STATISTICS

ALGEBRA II
STANDARD AII.12

The student will compute and distinguish between permutations and combinations ~~and use technology for applications~~.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The *Fundamental Counting Principle* states that if one decision can be made *n* ways and another can be made *m* ways, then the two decisions can be made *nm* ways. • *Permutations* are used to calculate the number of possible arrangements of objects. • *Combinations* are used to calculate the number of possible selections of objects without regard to the order selected. • A permutation is the number of possible was to arrange a group of objects when order matters (e.g., the outcome 1,2,3 is different from the outcome 3,2,1 when order matters; therefore, both arrangements would be included in the possible outcomes). • A combination is the number of possible ways to select or arrange objects when there is no repetition and order does not matter (e.g., the outcome 1,2,3 is the same as the outcome 3,2,1 when order does not matter; therefore, both arrangements would not be included in the possible outcomes).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast permutations and combinations. • Calculate the number of permutations of *n* objects taken *r* at a time. • Calculate the number of combinations of *n* objects taken *r* at a time. • Use permutations and combinations as counting techniques to solve ~~real-world~~practical problems. • Calculate and verify permutations and combinations using the graphing utility. [Moved from standard]



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Trigonometry

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

~~TOPIC~~STRAND: TRIANGULAR AND CIRCULAR TRIGONOMETRIC FUNCTIONS

TRIGONOMETRY
STANDARD T.1

The student, given a point ~~other than the origin~~ on the terminal side of an~~the~~ angle in standard position, or the value of the trigonometric function of the angle, will ~~use~~determine ~~the definitions of the six trigonometric functions to find~~ the sine, cosine, tangent, cotangent, secant, and cosecant of the angle ~~in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.~~

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Triangular trigonometric function definitions are related to circular trigonometric function definitions. • Both degrees and radians are units for measuring angles. • Drawing an angle in standard position will force the terminal side to lie in a specific quadrant or axis. • A point on the terminal side of an angle determines a reference triangle from which the values of the six trigonometric functions may be derived. • If one trigonometric function value is known, then a triangle can be formed to use in determining the other five trigonometric function values. [Moved from T.2]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Define the six triangular trigonometric functions of an angle in a right triangle. • ~~Define the six circular trigonometric functions of an angle in standard position.~~ [Moved to T.2 EKS] • Draw a reference right triangle when given a point on the terminal side of the angle in standard position. • Draw a reference right triangle when given the value of a trigonometric function of the angle. • Determine the value of any trigonometric function when given a point on the terminal side of an angle in standard position. [reordered] • Given one trigonometric function value, determine the other five trigonometric function values. [Moved from T.2] • ~~Make the connection between the triangular and circular~~

~~TOPIC~~STRAND: TRIANGULAR AND CIRCULAR TRIGONOMETRIC FUNCTIONS

TRIGONOMETRY
STANDARD T.1

The student, given a point ~~other than the origin~~ on the terminal side of an~~the~~ angle in standard position, or the value of the trigonometric function of the angle, will ~~use~~determine ~~the definitions of the six trigonometric functions to find~~ the sine, cosine, tangent, cotangent, secant, and cosecant of the angle ~~in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.~~

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
	~~trigonometric functions.~~ • ~~Recognize and draw an angle in standard position.~~ [Combined] • ~~Show how a point on the terminal side of an angle determines a reference triangle.~~ [Combined]

~~TOPIC~~STRAND: TRIANGULAR AND CIRCULAR TRIGONOMETRIC FUNCTIONS

TRIGONOMETRY
STANDARD T.2

~~The student, given the value of one trigonometric function, will find the values of the other trigonometric functions, using the definitions and properties of the trigonometric functions.~~[Combined into T.1]

The student will develop and apply the properties of the unit circle in degrees and radians. [Moved and rewritten from T.3]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Triangular trigonometric function definitions are related to circular trigonometric function definitions. • ~~If one trigonometric function value is known, then a triangle can be formed to use in finding the other five trigonometric function values.~~[Moved to T.1 US] • Knowledge of the unit circle is a useful tool for ~~find~~determining all six trigonometric values for special angles (30°, 45°, 60°, and 90°). • Angles of special right triangles (30°-60°-90° and 45°-45°-90°) are widely used in mathematics. • Special right triangles may be used to develop the unit circle. • Unit circle properties will allow special angle and related angle trigonometric values to be found without the aid of a calculator. • Degrees and radians are units of angle measure. • A radian is the measure of the central angle that is determined by an arc whose length is the same as the radius of the circle. • There is a connection between sides and angles of a special right triangles, the unit circle, and the coordinate plane.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Define the six circular trigonometric functions of an angle in standard position. [Moved from T.1] • Apply the properties of the unit circle to determine trigonometric function values of special angles and their related angles in both degrees and radians without using a calculator. [Moved from T.3 EKS] • Apply the properties of the unit circle to convert between special angles expressed in radians and degrees, without using a calculator. • ~~Given one trigonometric function value, find the other five trigonometric function values.~~ [Moved to T.1 EKS] • ~~Develop the unit circle, using both degrees and radians.~~ [Reordered] • ~~Solve problems, using the circular function definitions and the properties of the unit circle.~~ • ~~Recognize the connections between the coordinates of points on a unit circle and~~ ~~– coordinate geometry;~~

~~TOPIC~~STRAND: TRIANGULAR AND CIRCULAR TRIGONOMETRIC FUNCTIONS **TRIGONOMETRY STANDARD T.2** ~~**The student, given the value of one trigonometric function, will find the values of the other trigonometric functions, using the definitions and properties of the trigonometric functions.**~~**[Combined into T.1]** **The student will develop and apply the properties of the unit circle in degrees and radians. [Moved and rewritten from T.3]**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
	~~– cosine and sine values; and~~ ~~– lengths of sides of special right triangles (30°-60°-90° and 45°-45°-90°).~~

~~TOPIC~~STRAND: ~~TRIANGULAR AND CIRCULAR~~APPLICATIONS OF TRIGONOMETRIC FUNCTIONS
TRIGONOMETRY
STANDARD T.9~~3~~

~~The student will find, without the aid of a calculator, the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting angle measures from radians to degrees and vice versa.~~[Moved to T.2]

The student will solve problems, including practical problems, involving
a) arc length and area of sectors in circles using radians and degrees; and
b) linear and angular velocity.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Special angles are widely used in mathematics.~~ [Moved to T.2 US] • ~~Unit circle properties will allow special angle and related angle trigonometric values to be found without the aid of a calculator.~~ [Moved to T.2 US] • Degrees and radians are units of angle measure. • A radian is the measure of the central angle that is determined by an arc whose length is the same as the radius of the circle. • The relationship between the radian measure of an angle and the length of the intercepted arc can be represented by $s = r\theta$, where s is the arc length, r is the length of the radius, and θ is the measure of the angle.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Convert between any angle expressed in radians and degrees without using a calculator. (a) [reordered] • Derive the relationship between the radian measure of an angle and the length of the intercepted arc. (b) • Calculate the length of an arc in radians. (a) • Calculate the area of sectors in circles. (b) • Solve practical problems involving linear and angular velocity. (c) • ~~Find trigonometric function values of special angles and their related angles in both degrees and radians.~~ [Moved to T.2 EKS] • ~~Apply the properties of the unit circle without using a calculator.~~ [Combined and moved to T.2 EKS] • ~~Use a conversion factor to convert from radians to degrees and vice versa without using a calculator.~~ [Reordered]

~~TOPIC~~STRAND: ~~INVERSE TRIGONOMETRIC FUNCTIONS~~EQUATIONS AND IDENTITES TRIGONOMETRY STANDARD T.74 **The student will ~~find~~determine~~, with the aid of a calculator,~~ the value of any trigonometric function and inverse trigonometric function.**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The trigonometric function values of any angle can be found by using a calculator. • The inverse trigonometric functions can be used to ~~find~~ determine angle measures whose trigonometric function values are known. • Calculations of inverse trigonometric function values can be related to the triangular definitions of the trigonometric functions. • Trigonometric functions are not invertible, because they are periodic. Domain restrictions on trigonometric functions are necessary in order to determine the inverse trigonometric function.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use a calculator to ~~find~~ determine the trigonometric function values of any angle in either degrees or radians. • Define inverse trigonometric functions. • ~~Find~~ Determine angle measures by using the inverse trigonometric functions when the trigonometric function values are given.

~~TOPIC~~STRAND: ~~TRIGONOMETRIC~~ EQUATIONS AND IDENTITIES **TRIGONOMETRY** **STANDARD T.5** **The student will verify basic trigonometric identities and make substitutions, using the basic identities.**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Trigonometric identities can be used to simplify trigonometric expressions, equations, or identities. • Trigonometric identity substitutions can help solve trigonometric equations, verify another identity, or simplify trigonometric expressions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use trigonometric identities to make algebraic substitutions to simplify and verify trigonometric identities. The basic trigonometric identities include – reciprocal identities; – Pythagorean identities; – sum and difference identities; – double-angle identities; and – half-angle identities.

~~TOPIC~~STRAND: ~~TRIGONOMETRIC EQUATIONS,~~ GRAPHS~~, AND PRACTICAL PROBLEMS~~ OF TRIGONOMETRIC FUNCTIONS

TRIGONOMETRY
STANDARD T.3~~6~~

The student, given one of the six trigonometric functions in standard form, will

a) state the domain and the range of the function;
b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;
c) sketch the graph of the function by using transformations for at least a two-period interval; and
d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The domain and range of a trigonometric function determine the scales of the axes for the graph of the trigonometric function. • The amplitude, period, phase shift, and vertical shift are important characteristics of the graph of a trigonometric function, and each has a specific purpose in applications using trigonometric equations. • The graph of a trigonometric function can be used to display information about the periodic behavior of a real-world situation, such as wave motion or the motion of a Ferris wheel. • Standard form of the trigonometric functions may be written in multiple ways {e.g., $y = A \sin (Bx + C) + D$ or $y = A \sin [B(x + C)] + D$}. [Moved from EKS]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • State the domain and the range of a trigonometric function written in standard form. [Reordered] • Determine the amplitude, period, phase shift, ~~and~~ vertical shift, and asymptotes of a trigonometric function from the equation of the function and from the graph of the function. • Describe the effect of changing A, B, C, or D in the standard form of a trigonometric equation ~~{e.g., $y = A \sin (Bx + C) + D$ or $y = A \cos [B(x + C)] + D$}~~. [Moved to US] • Sketch the graph of a function written in standard form by using transformations for at least a two-period interval, including both positive and negative values for the domain. [Reordered] ~~• State the domain and the range of a function written in standard form {e.g., $y = A \sin (Bx + C) + D$ or $y = A \cos [B(x + C)] + D$}.~~ [Reordered, Moved to US]

~~TOPIC~~STRAND: ~~TRIGONOMETRIC EQUATIONS,~~ GRAPHS~~, AND PRACTICAL PROBLEMS~~ OF TRIGONOMETRIC FUNCTIONS **TRIGONOMETRY** **STANDARD T.3~~6~~** **The student, given one of the six trigonometric functions in standard form, will** **a) state the domain and the range of the function;** **b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;** **c) sketch the graph of the function by using transformations for at least a two-period interval; and** **d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
	• ~~Sketch the graph of a function written in standard form {e.g., $y = A \sin (Bx + C) + D$ or $y = A \cos [B(x + C)] + D$} by using transformations for at least one period or one cycle.~~ [Reordered, Moved to US]

~~TOPIC~~STRAND: ~~INVERSE~~ GRAPHS OF TRIGONOMETRIC FUNCTIONS **TRIGONOMETRY STANDARD T.4~~7~~** **The student will graph the six inverse trigonometric functions.~~identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.~~ [Included in EKS]**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Trigonometric functions are not invertible, because they are periodic. Domain restrictions on trigonometric functions are necessary in order to determine the inverse trigonometric function. • ~~Restrictions on the domains of some inverse trigonometric functions exist.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Find~~ Determine the domain and range of the inverse trigonometric functions. • Use the restrictions on the domains of the inverse trigonometric functions in ~~finding~~ determining the values of the inverse trigonometric functions. • ~~Identify the g~~Graph~~s of the~~ inverse trigonometric functions.

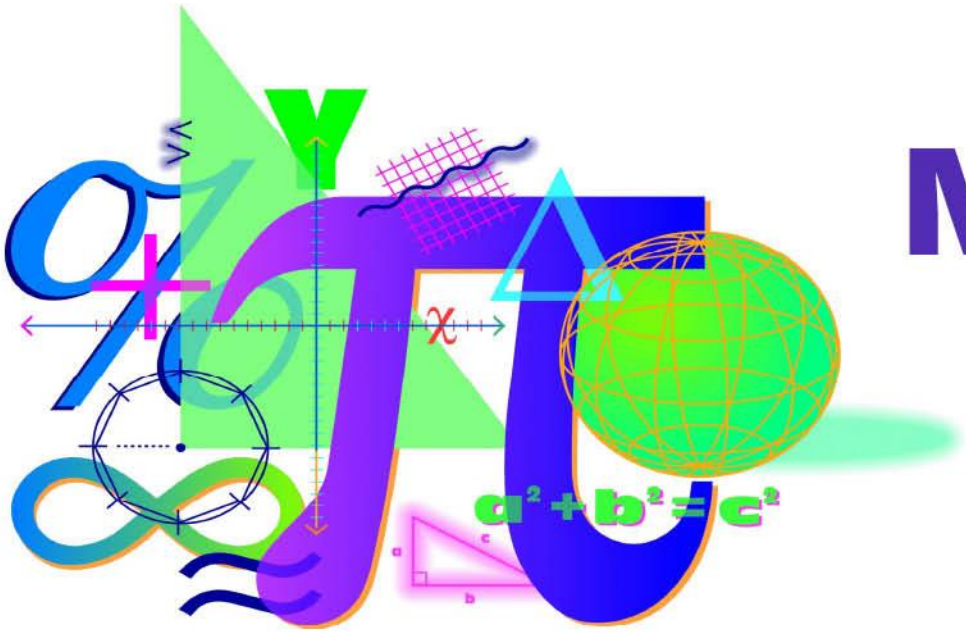
~~TOPIC~~STRAND: ~~TRIGONOMETRIC~~ EQUATIONS~~, GRAPHS,~~ AND IDENTITIES~~PRACTICAL PROBLEMS~~ TRIGONOMETRY **STANDARD T.6~~8~~** **The student will solve trigonometric equations and inequalities~~that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities~~.**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Solutions for trigonometric equations will depend on the domains. • A calculator can be used to ~~find~~ determine the solution of a trigonometric equation as the points of intersection of the~~ graphs when one side of the equation is entered in the calculator as Y_1 and the other side is entered as Y_2~~.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve trigonometric equations with and without restricted domains algebraically and ~~by using a~~ graphically~~ng utility~~. • Solve trigonometric inequalities~~equations with infinite solutions~~ algebraically and ~~by using a~~ graphically~~ng utility~~. • ~~Check for reasonableness of results, and v~~Verify algebraic solutions, using a graphing utility.

~~TOPIC~~STRAND: APPLICATIONS OF TRIGONOMETRIC FUNCTIONS~~EQUATIONS, GRAPHS, AND PRACTICAL PROBLEMS~~

TRIGONOMETRY
STANDARD T.8~~9~~

The student will ~~identify,~~ create~~,~~ and solve ~~real-world~~practical problems involving triangles.~~ Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.~~ [Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A ~~real-world~~practical problem may be solved by using one of a variety of techniques associated with triangles.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Write~~ Create and solve ~~a real-world~~practical problems involving triangles. • ~~Solve real-world problems involving triangles.~~ [Combined] • Use the trigonometric functions, Pythagorean Theorem, Law of Sines, and Law of Cosines to solve ~~real-world~~practical problems. • Use the trigonometric functions to model ~~real-world~~practical situations. • Identify a solution technique that could be used with a given problem. • ~~Prove~~ Apply the sum and difference identities ~~addition and subtraction formulas~~ for sine, cosine, and tangent ~~and use them~~ to solve problems.[‡] [‡]~~Revised March 2011~~



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Computer Mathematics

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

~~TOPIC~~STRAND: PROBLEM SOLVING

COMPUTER MATHEMATICS
STANDARD COM.1

The student will design and apply computer programs~~ming techniques and skills~~ to solve practical ~~real-world~~ problems in mathematics arising from ~~consumer,~~ business~~,~~ and ~~other~~ applications in mathematics. ~~Problems will include opportunities for students to analyze data in charts, graphs, and tables and to use their knowledge of equations, formulas, and functions to solve these problems.~~

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~The computer is an essential tool for mathematical problem solving in consumer-related problems.~~ • ~~A subtask that has been solved previously may be reused again and again.~~[Moved to COM.5] • Programming languages require the use of particular structures to express algorithms as programs. • Designing algorithms is a~~the~~ problem solving phase of computer programming. • ~~Real-world~~ Practical problems that can be modeled mathematically can be solved with a computer program. • ~~Data arising from probability and statistics applications can be displayed in tables and graphs and analyzed within the structure of a computer program.~~ • Sample mathematical problems may include: – the bisection method for a solution to a polynomial equation; – use of recursion to solve the Tower of Hanoi problem; – implementing a cellular automaton such as Conway's Game of Life; – implementing a maze-traversing or map-coloring algorithm; and – determining if an input number is prime.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Design and implement computer programs to solve ~~consumer~~ practical problems. • Analyze and interpret graphs, charts, and tables in the design and implementation of a computer program. • Design and implement computer programs to – solve practical problems arising from business; and – solve mathematical problems, using formulas, equations, and functions.~~;~~ ~~solve mathematical problems, using equations;~~ ~~solve mathematical problems, using functions;~~ ~~solve problems related to geometry, business, and leisure;~~ ~~solve probability problems;~~ ~~solve data-analysis problems; and~~ ~~solve statistical problems.~~[Combined with previous bullet]

~~TOPIC~~STRAND: PROGRAM DESIGN

COMPUTER MATHEMATICS
STANDARD COM.2

The student will design, write, document, test, and debug, ~~and document~~ a computer program. ~~Programming documentation will include preconditions and postconditions of program segments, input/output specifications, the step-by-step plan, the test data, a sample run, and the program listing with appropriately placed comments.~~[Moved to US]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~The successful completion of a structured program requires problem solving skills.~~Programming documentation includes the step-by-step plan, the test data, a sample run, and the program listing with appropriately placed comments. • Documentation enables programmers to use modules written by others without having to read and understand their code. • Comments communicate a programmer's original intent and are useful for debugging incorrect code.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe ~~what~~ a computer program ~~is~~. • Design, write, document, test, and debug a complete computer program. • List and describe the ~~stages~~ processes involved in writing a computer program. • Describe the function of an algorithm. • ~~Describe the interplay between hardware and software in program execution.~~ [Moved to COM.16 EKS] • ~~Compare and contrast compiling and executing a program.~~ • Provide required documentation for a program. • Determine what a given output statement will print. • Debug a program. • ~~Provide required documentation for a program.~~

~~TOPIC~~STRAND: PROGRAM DESIGN

COMPUTER MATHEMATICS
STANDARD COM.3

The student will write program specifications that define the constraints of a given problem.~~ These specifications will include descriptions of preconditions, postconditions, the desired output, analysis of the available input, and an indication as to whether or not the problem is solvable under the given conditions.~~[Moved to US]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A programmer begins the programming process by analyzing the problem and developing a general solution (algorithm). • The successful completion of a ~~structured~~ computer program requires problem solving skills. • Program specifications include descriptions of preconditions, postconditions, the desired input and output, analysis of the available input, and an indication as to whether or not the problem is solvable using a computer program.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write program specifications that define the constraints of a given problem.[Reordered] • ~~For a given problem, d~~Describe the preconditions, postconditions, and desired input and output of a given problem. • Determine whether or not a given problem is solvable using a computer program. • ~~Write program specifications that define the constraints of a problem.~~[Reordered]

~~TOPIC~~STRAND: PROGRAM DESIGN

COMPUTER MATHEMATICS
STANDARD COM.4

The student will design a step-by-step ~~plan (~~algorithm~~)~~ to solve a given problem. ~~The plan will be in the form of a program flowchart, pseudo code, hierarchy chart, and/or data-flow diagram.~~[Moved to US]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• All programs are implementations of algorithms. • Algorithms to solve problems may include flowcharts, pseudo code, and/or data-flow diagrams.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Design a step-by-step ~~plan~~ algorithm to solve a problem. ~~Utilize the following problem solving formats:~~ ~~flowchart;~~ ~~pseudo code;~~ ~~hierarchy chart; and~~ ~~data-flow diagram.~~

~~TOPIC~~STRAND: PROGRAM DESIGN

COMPUTER MATHEMATICS
STANDARD COM.5

The student will divide a given problem into ~~manageable sections (~~modules~~)~~ by task and implement the solution. ~~The modules will include an appropriate user-defined functions, subroutines, and procedures. Enrichment topics might include user-defined libraries (units) and object-oriented programming.~~

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Functional decomposition is a way to develop a program in which the problem is divided into sub-problems whose solutions comprise the solution to the original problem. • A subtask that has been solved previously may be reused in other programs.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Subd~~Divide a problem into modules by task. • ~~Implement the solution of the problem.~~[Reordered] • Write task-oriented modules, ~~including~~which may include – a user-defined function; – subroutines; ~~and~~ or – procedures. • Determine the need for a subroutine or user-defined function. • Determine the difference between and the need for internal and external subroutines and functions. • Implement the solution of the problem. [Reordered]

~~TOPIC~~STRAND: PROGRAM DESIGN

COMPUTER MATHEMATICS
STANDARD COM.6~~12~~

The student will translate ~~a~~ mathematical expressions into ~~a computer~~ programming ~~statement~~expressions~~, which involves~~ by declaring variables, writing assignment statements, and using the order of operations.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A variable identifies a location in memory where a data value that can be changed is stored. • An assignment statement stores the value of an expression into a variable. • ~~The order of operations is~~ ~~Parentheses;~~ ~~exponents;~~ ~~multiplication and division in order from left to right; and~~ ~~addition and subtraction in order from left to right.~~ • ~~Variable assignment statements will differ depending upon the programming language used.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Translate ~~a~~ mathematical expressions into a ~~computer~~ programming expression~~statement~~. • ~~Use the order of operations to simplify expressions.~~[Reordered] • Declare appropriately-named variables to store values used in computations. • Write variable assignment statements. • Use the order of operations to simplify expressions.[Reordered] • Construct and evaluate expressions that include multiple arithmetic operations.

~~TOPIC~~STRAND: PROGRAM DESIGN **COMPUTER MATHEMATICS** **STANDARD COM.7~~13~~** **The student will select and ~~implement~~ call ~~built-in (~~library~~)~~ functions ~~in~~to process~~ing~~ data, as appropriate.**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The argument of a library function is a value or expression associated with the independent variable. • A library function is a subroutine.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use library functions in designing programs to process data. • Use library functions that are arithmetic or string operations~~ors~~. • Invoke a value-returning library function.

~~TOPIC~~STRAND: PROGRAM DESIGN

COMPUTER MATHEMATICS
STANDARD COM.8~~14~~

The student will implement conditional statements that include "if/then" statements, "if/then/else" statements, case statements, and Boolean logic.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Boolean logic is a system using variables with only two values: TRUE and FALSE. • The "if" statement is the fundamental control structure that allows branches in the flow of control. • Case statements may be used to simplify a series of "if/then/else" statements.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct a simple logical (Boolean) expression to evaluate a given condition. • Construct an "if/then" statement to perform a specific task. • Construct an "if/then/else" statement to perform a specific task. • Construct a case statement to perform a specific task. • Use conditional statements to incorporate decision making into programs.

~~TOPIC~~STRAND: PROGRAM DESIGN **COMPUTER MATHEMATICS** **STANDARD COM.9~~17~~** **The student will implement pre-~~existing~~defined algorithms, including sort routines, search routines, and simple animation routines.**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Arranging values into an order is known as sorting. • ~~A sequential search algorithm starts at the beginning of a list and examines each data value in sequence.~~ • A search algorithm finds an item with specified properties among a collection of items. • ~~Implementation of animation routines will differ depending upon the programming language used.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Implement pre-~~existing~~ defined algorithms into a program. • Implement a sort routine on a one-dimensional array. • Implement a sequential search routine on a one-dimensional array. • Implement a binary search routine on a one-dimensional array. • Implement a simple animation routine.

~~TOPIC~~STRAND: PROGRAM IMPLEMENTATION

COMPUTER MATHEMATICS
STANDARD COM.10~~6~~

The student will design and implement the input phase of a program, which will include designing screen layout, ~~and~~ getting information into the program by way of user interaction~~, data statements,~~ and/or file input, and validating input. ~~The input phase will also include methods of filtering out invalid data (error trapping).~~

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A program needs data on which to operate. • A file is a named area in secondary storage that holds a collection of information.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Design a screen layout to facilitate input. • Design program information input by user interaction and file input. – ~~user interaction;~~ – ~~data statements (BASIC); and~~ – ~~file input.~~ • ~~Filter out invalid~~Validate data, using a variety of methods ~~(error trapping)~~. • Construct input statements to read values into a program. • Determine the contents of variables that have been assigned values by input statements.

~~TOPIC~~STRAND: PROGRAM IMPLEMENTATION

COMPUTER MATHEMATICS
STANDARD COM.11~~7~~

The student will design and implement the output phase of a computer program, which will include designing output layout, accessing ~~a variety of~~ available output devices, using output statements, and labeling results.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Output is dependent on input and processing. • Implementation of the output portion of a program includes designing the output layout and displaying ~~it~~ the output in the desired format.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Design an output layout. • Access ~~various~~ available output devices. • Use output statements. • Label results.

~~TOPIC~~STRAND: PROGRAM IMPLEMENTATION

COMPUTER MATHEMATICS
STANDARD COM.12~~8~~

The student will design and implement computer graphics to enhance output~~, which will include topics appropriate for the available programming environment as well as student background~~. ~~Students will use graphics as an end in itself, as an enhancement to other output, and as a vehicle for reinforcing programming techniques.~~

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Work with computer graphics is specific to the ~~computer operating system~~programming language.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Design and implement computer graphics using various techniques such as~~.~~ – plotting points or shapes; – determining and set window or screen dimensions; – determining and set screen or background colors; – using box commands. ~~• Implement computer graphics.~~ ~~– Plot points and areas.~~ ~~– Determine and set window or screen dimensions.~~ ~~– Determine and set screen and background colors.~~ ~~– Use box commands.~~ • Describe the role of graphics in the computer environment.

~~TOPIC~~STRAND: PROGRAM IMPLEMENTATION

COMPUTER MATHEMATICS
STANDARD COM.13~~15~~

The student will implement various mechanisms for performing iteration with an algorithm~~loops, including iterative loops~~. ~~Other topics will include single entry point, single exit point, preconditions, and postconditions.~~

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• An ~~loop~~ iterative algorithm executes a sequence of statements repeatedly. • Iterative algorithms execute a fixed number of times. • Iterative algorithms can be nested. • Iterative algorithms include – iterative loops; – indefinite loops; – pretest loops; – posttest loops; and – nested loops.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine when ~~a loop~~iterative algorithm is needed in a computer program. ~~• Implement loops into programs. Include~~ ~~– iterative loops;~~ ~~– pretest loops; and~~ ~~– posttest loops.~~[Moved to US] • Incorporate single entry point, single exit point, pre-conditions, and postconditions into ~~loops~~iterative algorithms.

~~TOPIC~~STRAND: PROGRAM IMPLEMENTATION

COMPUTER MATHEMATICS
STANDARD COM.14~~16~~

The student will select and implement appropriate data structures, including arrays (one- and/or two-dimensional ~~and/or multidimensional~~), ~~files,~~ and ~~records~~objects. ~~Implementation will include creating the data structure, putting information into the structure, and retrieving information from the structure.~~

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Structured data types~~ Data structures are collections of components that are given a single name and whose organization is characterized by the method used to access the individual components. • ~~Multi~~Two-dimensional arrays may be viewed as arrays of one-dimensional arrays. • Data files may be used to store information needed each time the program is executed. • An object is a basic data structure used to organize data of different types and referenced by name.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Declare~~ Implement a one-dimensional or two-dimensional array for a given problem. – Choose an appropriate ~~component~~ data type for an array. – Assign a value to an array ~~component~~element. – Fill an array with data, and process the data in the array. – Access a particular ~~component~~ element of a two-dimensional array. – Process a two-dimensional array by rows and by columns. – Retrieve data from an array. • Use data files in computer programs, both as a source of input data and as a way to save data for the next program execution. • Implement objects to consolidate related information of different data types.

~~TOPIC~~STRAND: DATA MANIPULATION

COMPUTER MATHEMATICS
STANDARD COM.15~~9~~

The student will define ~~simple~~and use appropriate variable data types that include integer, real (fixed and scientific notation), character, string, ~~and~~ Boolean, and object.

~~STANDARD COM.10~~

~~The student will use appropriate variable data types, including integer, real (fixed and scientific notation), character, string, and Boolean. This will also include variables representing structured data types.~~ [Combined with COM.15]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A data type ~~is~~defines a set of values and a set of operations on the values. • Boolean data has only two literal constants, and they represent TRUE and FALSE. • A string is an array of character data for which there exists an aggregate name~~constant.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Define ~~and use~~ variables using data types, including – integer; – real (fixed and scientific notation); – character; – string; ~~and~~ – Boolean; and – object. • ~~Write numeric and string variables, using valid names.~~ • Use standard naming conventions to create variable names.

~~TOPIC~~STRAND: DATA MANIPULATION

COMPUTER MATHEMATICS
STANDARD COM.16~~11~~

The student will describe the way the computer stores, accesses, and processes variables, including the following topics: the use of variables versus constants, ~~variables' addresses, pointers,~~ parameter passing, scope of variables, and local versus global variables.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Computers consist of hardware components that interact with software. • Program design must identify each piece of data as a constant, local variable, or global variable depending on the scope of the data in the program. • Parameters are used to provide information to other parts of the program (functions, methods, subroutines, or procedures). • Parameter passing may include using variables' addresses or pointers or equivalent features. • Variable addresses and pointers are used to access data stored in computer memory.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine when the use of a variable is appropriate. • Describe how a computer stores, accesses, and processes variables. • Incorporate ~~variable addresses, pointers, and~~ parameter passing into programs. • Differentiate between local and global variables, and describe their appropriate use. • Compare and contrast variables and constants. • Describe the basic interplay between hardware and software in program execution. [Moved from COM.2 EKS]

~~TOPIC~~STRAND: PROGRAM TESTING **COMPUTER MATHEMATICS** **STANDARD COM.17~~18~~** **The student will test a program~~,~~ using an appropriate set of data. The ~~set of~~ test data should include boundary cases and test all branches of a program. ~~be appropriate and complete for the type of program being tested.~~**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A program test can reveal problems (bugs) in the program. • Testing a program for bugs is part of problem solving. • Various forms of data can be used to debug a program.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Test a program, using an appropriate and complete set of data. • Demonstrate that a set of data tests all branches of a program.

~~TOPIC~~STRAND: PROGRAM TESTING

COMPUTER MATHEMATICS
STANDARD COM.18~~19~~

The student will debug a program~~,~~ using appropriate techniques (e.g., appropriately placed controlled breaks, the printing of intermediate results, and other debugging tools available in the programming environment), and identify the difference ~~between~~ among syntax errors, runtime errors, and logic errors.

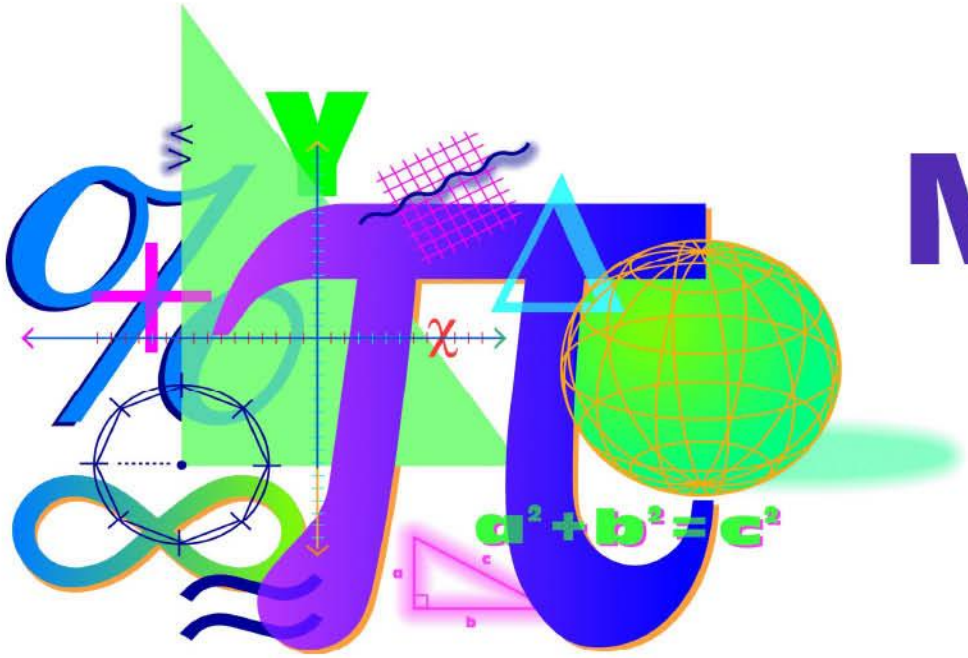
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Debugging ~~a program is problem solving~~is a methodical process of finding and correcting errors so the computer program executes as intended.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Debug a program, using controlled breaks, the printing of intermediate results, and other debugging tools. • Identify the differences among syntax errors, runtime errors, and logic errors.

~~**TOPIC: PROGRAM TESTING**~~

~~**COMPUTER MATHEMATICS**~~
~~**STANDARD COM.20**~~

~~**The student will design, write, test, debug, and document a complete structured program that requires the synthesis of many of the concepts contained in previous standards.**~~ **[Included in COM.2]**

~~**ESSENTIAL UNDERSTANDINGS**~~	~~**ESSENTIAL KNOWLEDGE AND SKILLS**~~
• ~~The successful completion of a structured program requires problem solving skills.~~	~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Design, write, test, debug, and document a complete structured program.~~



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Probability and Statistics

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

~~TOPIC~~STRAND: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.1[†]

The student will analyze graphical displays of univariate data, including dotplots, stemplots, boxplots, cumulative frequency graphs, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. ~~Appropriate technology will be used to create graphical displays.~~ [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning in a context. • Measures of central tendency describe how the data cluster or group. • Measures of dispersion describe how the data spread (disperse) around the center of the data. • Graphical displays of data may be analyzed informally. • Data analysis must take place within the context of the problem.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Create and interpret graphical displays of data, including dotplots, ~~stem-and-leaf~~ stemplots, boxplots, cumulative frequency graphs, and histograms, using appropriate technology. • Examine graphs of data for clusters and gaps, and relate those phenomena to the data in context. • Examine graphs of data for outliers, and explain the outlier(s) within the context of the data. • Examine graphs of data and identify the central tendency of the data as well as the spread. • Explain the central tendency and the spread of the data within the context of the data.

[†] Standard should be included in a one-semester course in Probability and Statistics.

~~TOPIC~~STRAND: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.2†

The student will analyze numerical characteristics of univariate data sets to describe patterns and departure from patterns, using mean, median, mode, variance, standard deviation, interquartile range, range, and outliers.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning within a context. • Analysis of the descriptive statistical information generated by a univariate data set should include the interplay between central tendency and dispersion as well as among specific measures. • Data points identified algorithmically as outliers should not be excluded from the data unless sufficient evidence exists to show them to be in error.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Interpret mean, median, mode, range, interquartile range, variance, and standard deviation of a univariate data set in terms of the problem's context. • Identify possible outliers, using an algorithm. • Explain the influence of outliers on a univariate data set. • Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.

† Standard should be included in a one-semester course in Probability and Statistics.

PROBABILITY AND STATISTICS **STANDARD PS.3[†]** **The student will compare distributions of two or more univariate data sets, numerically and graphically, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.**	**~~TOPIC~~STRAND: DESCRIPTIVE STATISTICS**
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Data are collected for a purpose and have meaning in a context. • Statistical tendency refers to typical cases but not necessarily to individual cases.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast two or more univariate data sets, numerically and graphically, by analyzing measures of center and spread within a contextual framework. • Describe any unusual features of the data, such as clusters, gaps, or outliers, within the context of the data. • Analyze ~~in context kurtosis and~~ skewness in conjunction with measures of center and spread in a contextual framework~~with other descriptive measures~~.

[†] Standard should be included in a one-semester course in Probability and Statistics.

~~TOPIC~~STRAND: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.4[†]

The student will analyze scatterplots to identify and describe the relationship between two variables, using shape; strength of relationship; clusters; positive, negative, or no association; outliers; and influential points.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A scatterplot serves two purposes: – to determine if there is a useful relationship between two variables, and – to determine the family of equations that describes the relationship. • Data are collected for a purpose and have meaning in a context. • Association between two variables considers both the direction and strength of the association. • The strength of an association between two variables reflects how accurately the value of one variable can be predicted based on the value of the other variable. • Outliers are observations with large residuals and do not follow the pattern apparent in the other data points.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Examine scatterplots of data, and describe skewness, ~~kurtosis,~~ and correlation within the context of the data. • Describe and explain any unusual features of the data, such as clusters, gaps, or outliers, within the context of the data. • Identify influential data points (observations that have great effect on a line of best fit because of extreme *x*-values) and describe the effect of the influential points.

[†] Standard should be included in a one-semester course in Probability and Statistics.

~~TOPIC~~STRAND: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.5

The student will ~~find~~determine and interpret linear correlation, use the method of least squares regression to model the linear relationship between two variables, and use the residual plots to assess linearity.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning in a context. • Least squares regression generates the equation of the line that minimizes the sum of the squared distances from the data points to the line. • Each data point may be considered to be comprised of two parts: fit (the part explained by the model) and residual (the result of chance variation or of variables not measured). • Residual = Actual – Fitted • A correlation coefficient measures the degree of association between two variables that are related linearly. • Two variables may be strongly associated without a cause-and-effect relationship existing between them.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Calculate a correlation coefficient, *r*. • Explain how the correlation coefficient, *r*, measures association by looking at its formula. • Interpret the coefficient of determination, r^2, in a contextual framework. • Use regression lines to make predictions, and identify the limitations of the predictions. • Use residual plots to determine if a linear model is satisfactory for describing the relationship between two variables. • Describe the errors inherent in extrapolation beyond the range of the data. • Use least squares regression to ~~find~~determine the equation of the line of best fit for a set of data. • Interpret the slope and *y*-intercept of the least squares regression line in a contextual framework. • Explain how least squares regression generates the equation of the line of best fit by examining the formulas used in computation.

PROBABILITY AND STATISTICS **STANDARD PS.6** **The student will make logarithmic and power transformations to achieve linearity.**	**~~TOPIC~~STRAND: DESCRIPTIVE STATISTICS**
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A logarithmic transformation reduces positive skewness because it compresses the upper tail of the distribution while stretching the lower tail. • Nonlinear transformations do not preserve relative spacing between data points.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Apply a logarithmic transformation to data. • Explain how a logarithmic transformation works to achieve a linear relationship between variables. • Apply a power transformation to data. • Explain how a power transformation works to achieve a linear relationship between variables.

~~TOPIC~~STRAND: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.7[†]

The student, using two-way tables and other graphical displays, will analyze categorical data to describe patterns and departure from patterns and to ~~find~~determine marginal frequency and relative frequencies, including conditional frequencies.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning in a context. • Simpson's paradox refers to the fact that aggregate proportions can reverse the direction of the relationship seen in the individual parts. • Two categorical variables are independent if the conditional frequencies of one variable are the same for every category of the other variable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Produce a two-way table as a summary of the information obtained from two categorical variables. • Create and interpret graphical displays of categorical data including bar charts. • Calculate marginal, relative, and conditional frequencies in a two-way table. • Use marginal, relative, and conditional frequencies to analyze data in two-way tables within the context of the data.

[†] Standard should be included in a one-semester course in Probability and Statistics.

PROBABILITY AND STATISTICS **STANDARD PS.8**[†] **The student will describe the methods of data collection in a census, sample survey, experiment, and observational study and identify an appropriate method of solution for a given problem setting.**	**~~TOPIC~~STRAND: DATA COLLECTION**
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The value of a sample statistic varies from sample to sample if the simple random samples are taken repeatedly from the population of interest. • Poor data collection can lead to misleading and meaningless conclusions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast controlled experiments and observational studies and the conclusions one can draw from each. • Compare and contrast population and sample and parameter and statistic. • Identify biased sampling methods. • Describe simple random sampling. • Select a data collection method appropriate for a given context.

[†] Standard should be included in a one-semester course in Probability and Statistics.

~~TOPIC~~STRAND: DATA COLLECTION

PROBABILITY AND STATISTICS
STANDARD PS.9†

The student will plan and conduct a survey. The plan will address sampling techniques ~~(e.g., simple random and stratified)~~[Moved to EKS] and methods to reduce bias.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The purpose of sampling is to provide sufficient information so that population characteristics may be inferred. • Inherent bias diminishes as sample size increases.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Distinguish between a population and a sample. [Moved from standard] • Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling. • Determine which sampling technique is best, given a particular context. • Plan a survey to answer a question or address an issue. • Given a plan for a survey, identify possible sources of bias, and describe ways to reduce bias. • Design a survey instrument. • Conduct a survey.

† Standard should be included in a one-semester course in Probability and Statistics.

~~TOPIC~~STRAND: DATA COLLECTION

PROBABILITY AND STATISTICS
STANDARD PS.10[†]

The student will plan and conduct a~~n~~ well-designed experiment. The plan will address control, randomization, replication, blinding, and measurement of experimental error.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Experiments must be ~~carefully~~ well-designed in order to detect a cause-and-effect relationship between variables. • Principles of experimental design include comparison with a control group, randomization, and blindness.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Plan and conduct ~~an~~ a well-designed experiment. The experimental design should address control, randomization, replication, blinding and minimization of experimental error. • Identify treatments, levels, factors, control groups, and experimental units in an experimental design. • Identify sources of bias and confounding, including the placebo effect. • Identify a situation when a block design, including matched pairs, would reduce the effects of confounding variables.

[†] Standard should be included in a one-semester course in Probability and Statistics.

PROBABILITY AND STATISTICS **STANDARD PS.11†**	**~~TOPIC~~STRAND: PROBABILITY**
The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive.	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The complement of event A consists of all outcomes in which event A does not occur. • Two events, A and B, are independent if the occurrence of one does not affect the probability of the occurrence of the other. If A and B are not independent, then they are said to be dependent. • Events A and B are mutually exclusive if they cannot occur simultaneously.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events. • Given two or more events in a problem setting, determine if the events are complementary, dependent, independent, and/or mutually exclusive.

† Standard should be included in a one-semester course in Probability and Statistics.

~~TOPIC~~STRAND: PROBABILITY **PROBABILITY AND STATISTICS** **STANDARD PS.12[†]** **The student will ~~find~~determine probabilities (relative frequency and theoretical), including conditional probabilities for events that are either dependent or independent, by applying the Law of Large Numbers concept, the addition rule, and the multiplication rule.**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Data are collected for a purpose and have meaning in a context. • Venn diagrams may be used to ~~find~~determine conditional probabilities. • The Law of Large Numbers states that as a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Calculate relative frequency and expected frequency. • ~~Find~~Determine conditional probabilities for dependent, independent, and mutually exclusive events.

[†] Standard should be included in a one-semester course in Probability and Statistics.

~~TOPIC~~STRAND: PROBABILITY

PROBABILITY AND STATISTICS
STANDARD PS.13

The student will develop, interpret, and apply the binomial and geometric probability distributions for discrete random variables, including computing the mean and standard deviation for the binomial and geometric variables.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A probability distribution is a complete listing of all possible outcomes of an experiment together with their probabilities. • In a binomial distribution, ~~T~~the procedure has a fixed number of independent trials. • In a geometric distribution, the procedure is repeated until the first success. • A random variable assumes different values depending on the event outcome. • A probability distribution combines descriptive statistical techniques and probabilities to form a theoretical model of behavior.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Develop the binomial and geometric probability distributions within a ~~real-world~~practical context. • Calculate the mean and standard deviation for the binomial and geometric variables. • Use the binomial and geometric distributions to calculate probabilities associated with experiments for which there are only two possible outcomes.

PROBABILITY AND STATISTICS **STANDARD PS.14** **The student will simulate probability distributions, including binomial and geometric.**	**~~TOPIC~~STRAND: PROBABILITY**
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A probability distribution combines descriptive methods and probabilities to form a theoretical model of behavior. • A probability distribution gives the probability for each value of the random variable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Design and conduct ~~an experiment~~a simulation of a binomial distribution~~that simulates a binomial distribution~~. • Design and conduct a simulation of ~~an experiment that simulates~~ a geometric distribution. • Calculate probabilities resulting from simulations.

~~TOPIC~~STRAND: PROBABILITY

PROBABILITY AND STATISTICS
STANDARD PS.15

The student will identify random variables as independent or dependent and ~~find~~ determine the mean and standard deviations for random variables and sums and differences of independent random variables.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A random variable is a variable that has a single numerical value, determined by chance, for each outcome of a procedure.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast independent and dependent random variables. • Determine the mean (expected value) and standard deviation for a random variable and linear transformation of a random variable. • Determine the mean (expected value) for sums and differences of random variables. • ~~Find~~ Determine the standard deviation for sums and differences of independent random variables.

~~TOPIC~~STRAND: PROBABILITY

PROBABILITY AND STATISTICS
STANDARD PS.16†

The student will identify properties of a normal distribution and apply the normal distribution to determine probabilities~~, using a table or graphing calculator~~[Moved to EKS].

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The normal distribution curve is a family of symmetrical curves defined by the mean and the standard deviation. • Areas under the curve represent probabilities associated with continuous distributions. • The normal curve is a probability distribution and the total area under the curve is 1.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the properties of a normal ~~probability~~ distribution. • Describe how the standard deviation and the mean affect the graph of the normal distribution. • Calculate and interpret the z-score of a given data value from a normal distribution. • Determine the probability of a given event, using the normal distribution. • Use a graphing calculator and a table of Standard Normal Probabilities to determine probabilities. [Moved from standard]

† Standard should be included in a one-semester course in Probability and Statistics.

~~TOPIC~~STRAND: INFERENTIAL STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.17

The student, given data from a large sample, will ~~find~~determine and interpret appropriate point estimates and confidence intervals for parameters. The parameters will include proportion and mean, difference between two proportions, ~~and~~ difference between two means (independent and paired), and slope of a least-squares regression line.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A primary goal of sampling is to estimate the value of a parameter based on a statistic. • Confidence intervals use the sample statistic to construct an interval of values that one can be reasonably certain contains the true (unknown) parameter. • Confidence intervals and tests of significance are complementary procedures. • Paired comparisons experimental design allows control for possible effects of extraneous variables.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct confidence intervals to estimate a population parameter, such as a proportion or the difference between two proportions; ~~or~~ a mean or the difference between two means; or slope of a least-squares regression line. • Select a value for ~~alpha (Type I error) for a~~ the confidence level of a confidence interval. • Interpret confidence intervals and confidence levels in the context of the data. • Explain the importance of random sampling for confidence intervals. • Explain how changes in confidence level and sample size effect width of the confidence interval and margin of error. • Calculate point estimates for parameters and discuss the limitations of point estimates.

~~TOPIC~~STRAND: INFERENTIAL STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.18

The student will apply and interpret the logic of an appropriate hypothesis-testing procedure. Tests will include large sample test for proportion, mean, difference between two proportions, ~~and~~ difference between two means (independent and paired); ~~and~~ Chi-squared tests for goodness of fit, homogeneity of proportions, and independence; and slope of a least-squares regression line.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Confidence intervals and tests of significance are complementary procedures. • Paired comparisons experimental design allows control for possible effects of extraneous variables. • Tests of significance assess the extent to which sample data support a hypothesis about a population parameter. • The purpose of a goodness of fit test is to decide if the sample results are consistent with results that would have been obtained if a random sample had been selected from a population with a known distribution. • Practical significance and statistical significance are not necessarily congruent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use the Chi-squared test for goodness of fit to decide if the population being analyzed fits a particular distribution pattern. • Use hypothesis-testing procedures to determine whether or not to reject the null hypothesis. The null hypothesis may address proportion, mean, difference between two proportions or two means, goodness of fit, homogeneity of proportions, ~~and~~ independence, and the slope of a least-squares regression line. • Compare and contrast Type I and Type II errors. • Explain how and why the hypothesis-testing procedure allows one to reach a statistical decision.

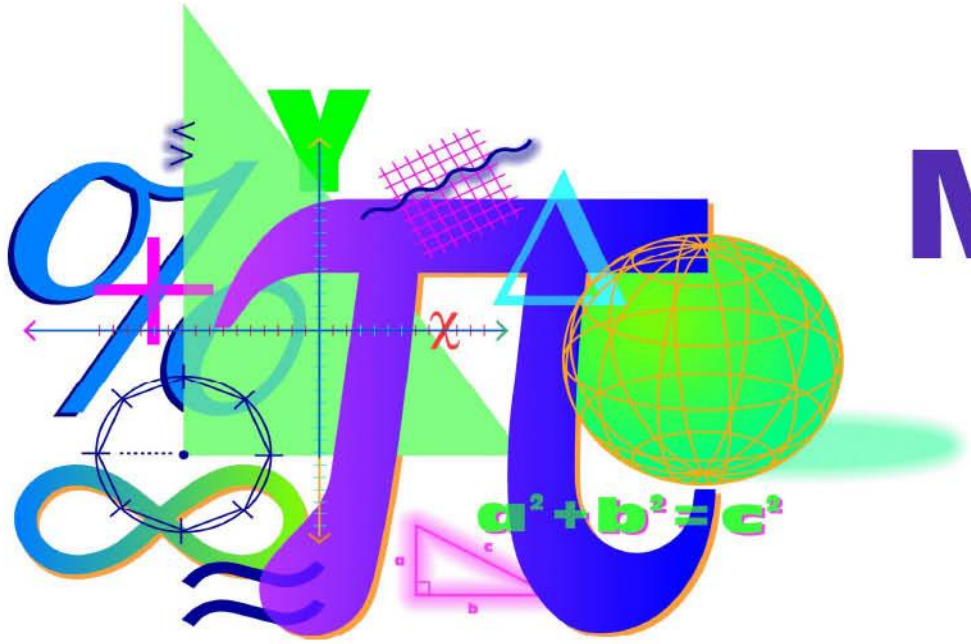
~~TOPIC~~STRAND: INFERENTIAL STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.19

The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The Central Limit Theorem states: – The mean of the sampling distribution of means is equal to the population mean. – If the sample size is sufficiently large, the sampling distribution approximates the normal probability distribution. – If the population is normally distributed, the sampling distribution is normal regardless of sample size. • Sampling distributions have less variability with larger sample sizes.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the use of the Central Limit Theorem for drawing inferences about a population parameter based on a sample statistic. • Describe the effect of sample size on the sampling distribution and on related probabilities. • Use the normal approximation to calculate probabilities of sample statistics falling within a given interval. • Identify and describe the characteristics of a sampling distribution of a sample proportion, mean, difference between two sample proportions, or difference between two sample means.

~~TOPIC~~STRAND: INFERENTIAL STATISTICS **PROBABILITY AND STATISTICS** **STANDARD PS.20** **The student will identify properties of a t-distribution and apply t-distributions to single-sample and two-sample (independent and matched pairs) t-procedures~~, using tables or graphing calculators~~.**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Paired comparisons experimental design allows control for possible effects of extraneous variables. • The sampling distribution of means with a small sample size follows a t-distribution.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the properties of a t-distribution. • Compare and contrast a t-distribution and a normal distribution. • Use a t-test for single-sample and two-sample data.



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Discrete Mathematics

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

~~TOPIC~~STRAND: GRAPHS

DISCRETE MATHEMATICS
STANDARD DM.1†

The student will model problems, using vertex-edge graphs. The concepts of valence, connectedness, paths, planarity, and directed graphs will be investigated. ~~Adjacency matrices and matrix operations will be used to solve problems (e.g., food chains, number of paths).~~[Included in US]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A tournament is a digraph that results from giving directions to the edges of a complete graph. • Adjacent vertices are connected by an edge. • In a connected graph, every pair of vertices is adjacent. • Graphs can be used to solve problems including food chains and number of paths. [Moved from standard]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Find~~Determine the valence of each vertex in a graph. • Use graphs to model situations in which the vertices represent objects, and edges (drawn between vertices) represent a particular relationship between objects. • Represent the vertices and edges of a graph as an adjacency matrix, and use the matrix to solve problems. • Investigate and describe valence and connectedness. • Determine whether a graph is planar or nonplanar. • Use directed graphs (digraphs) to represent situations with restrictions in traversal possibilities.

† Standard should be included in a one-semester course in Discrete Mathematics.

~~TOPIC~~STRAND: GRAPHS

DISCRETE MATHEMATICS
STANDARD DM.2†

The student will solve problems through investigation and application of circuits, cycles, Euler Paths, Euler Circuits, Hamilton Paths, and Hamilton Circuits. Optimal solutions will be sought using existing algorithms and student-created algorithms.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• ~~Euler's Theorem states:~~ If G is a connected graph and all its valences are even, then G has an Euler Circuit. • Pairs of routes (circuits) correspond to the same Hamilton Circuit because one route can be obtained from the other by traversing the vertices in reverse order. There are $\frac{(n-1)!}{2}$ Hamilton Circuits. • A multigraph is connected if there is a path between every pair of vertices.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine if a graph has an Euler Circuit or Path, and ~~find~~ determine it, if it exists. • Determine if a graph has a Hamilton Circuit or Path, and ~~find~~ determine it, if it exists. • Count the number of Hamilton Circuits for a complete graph with n vertices. • Use an ~~the~~ Euler Circuit algorithm to solve optimization problems.

† Standard should be included in a one-semester course in Discrete Mathematics.

~~TOPIC~~STRAND: GRAPHS **DISCRETE MATHEMATICS** **STANDARD DM.3†** **The student will apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization. ~~Graph coloring and chromatic number will be used.~~[Moved to EKS]**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Every planar graph has a chromatic number that is less than or equal to four (the four-color-map theorem). • A graph can be colored with two colors if and only if it contains no cycle of odd length. • The chromatic number of a graph cannot exceed one more than the maximum number of degrees of the vertices of the graph.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model projects consisting of several subtasks, using a graph. • Use graphs to resolve conflicts that arise in scheduling. • Determine the chromatic number of a graph. [Moved from standard]

† Standard should be included in a one-semester course in Discrete Mathematics.

~~TOPIC~~STRAND: GRAPHS

DISCRETE MATHEMATICS
STANDARD DM.4

The student will apply algorithms~~, such as Kruskal's, Prim's, or Dijkstra's,~~ [Included in EKS] relating to trees, networks, and paths. Appropriate technology will be used to determine the number of possible solutions and generate solutions when a feasible number exists.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A spanning tree of a connected graph *G* is a tree that is a subgraph of *G* and contains every vertex of *G*.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use Kruskal's Algorithm to ~~find~~determine the shortest spanning tree of a connected graph. • Use Prim's Algorithm to ~~find~~determine the shortest spanning tree of a connected graph. • Use Dijkstra's Algorithm to ~~find~~determine the shortest spanning tree of a connected graph.

DISCRETE MATHEMATICS **STANDARD DM.5~~7~~†**	**~~TOPIC~~STRAND: ELECTION THEORY AND FAIR DIVISION**
The student will analyze and describe the issue of fair division in discrete and continuous cases. ~~(e.g., cake cutting, estate division[Included in EKS]).~~ ~~Algorithms for continuous and discrete cases will be applied.~~	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Group decision making combines the wishes of many to yield a single fair result. • A fair division problem may be discrete or continuous. • The success of the estate division algorithm requires that each heir be capable of placing a value on each object in the estate. • A fair division problem consists of *n* individuals (players) who must partition some set of goods, *s*, into *n* disjoint sets.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and describe situations involving discrete division (e.g., estate division). • Use an algorithm for fair division for a group of indivisible objects. • Investigate and describe situations involving continuous division of an infinitely divisible set (e.g., cake cutting). • Use an algorithm for fair division of an infinitely divisible set.

† Standard should be included in a one-semester course in Discrete Mathematics.

~~TOPIC~~STRAND: ELECTION THEORY AND FAIR DIVISION

DISCRETE MATHEMATICS
STANDARD DM.6~~8~~[‡]

The student will investigate and describe weighted voting and the results of various election methods. These may include approval and preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Historically, popular voting methods have often led to counterintuitive results. • A candidate who wins over every other candidate in a one-on-one ballot is a Condorcet winner. • A Borda count assigns points in descending order to each voter's subsequent ranking and then adds these points to arrive at a group's final ranking. • To select a voting system is to compromise between the shortcomings inherent in each system.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine in how many different ways a voter can rank choices. • Investigate and describe the following voting procedures: – weighted voting; – plurality; – majority; – sequential (winners run off); – sequential (losers are eliminated); – Borda count; and – Condorcet winner. • Compare and contrast different voting procedures. • Describe the possible effects of approval voting, insincere and sincere voting, a preference schedule, and strategic voting on the election outcome.

[†] Standard should be included in a one-semester course in Discrete Mathematics.

DISCRETE MATHEMATICS **STANDARD DM.7~~9~~** **The student will identify apportionment inconsistencies that apply to issues such as salary caps in sports and allocation of representatives to Congress. Historical and current methods will be compared.**	**~~TOPIC~~STRAND: ELECTION THEORY AND FAIR DIVISION**
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The apportionment of Congressional representatives is based on the latest census.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast the Hamilton and Jefferson methods of political apportionment with the Hill-Huntington method (currently in use in the U.S. House of Representatives) and the Webster-Willcox method. • Solve allocation problems, using apportionment methods. • Investigate and describe how salary caps affect apportionment.

~~TOPIC~~STRAND: COMPUTER MATHEMATICS

DISCRETE MATHEMATICS
STANDARD DM.8~~11~~

The student will describe and apply sorting algorithms and coding algorithms used in sorting, processing, and communicating information. ~~These will include~~

~~a) bubble sort, merge sort, and network sort; and~~
~~b) ISBN, UPC, Zip, and banking codes.~~[Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• A bubble sort orders elements of an array by comparing adjacent elements. • A merge sort combines two sorted lists into a single sorted list. • Coding algorithms must account for the number of possible codes within the constraints of the coding system.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Select and apply a sorting algorithm, such as a – bubble sort; – merge sort; and – network sort. • Describe and apply a coding algorithm, such as – ISBN numbers; – UPC codes; – Zip codes; and – banking codes.

~~TOPIC~~STRAND: COMPUTER MATHEMATICS **DISCRETE MATHEMATICS** **STANDARD DM.9~~12~~†** **The student will select, justify, and apply an appropriate technique to solve a logic problem. ~~Techniques will include Venn diagrams, truth tables, and matrices.~~[Included in EKS]**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Two-valued (Boolean) algebra serves as a workable method for interpreting the logical truth and falsity of compound statements. • Venn diagrams provide pictures of topics in set theory, such as intersection and union, mutually exclusive sets, and the empty set.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Generate truth tables that encode the truth and falsity of two or more statements. • Use Venn diagrams to represent set relationships, such as intersection and union. [Moved from G.1] • Interpret Venn diagrams. [Moved from G.1] • Use Venn diagrams to codify and solve logic problems. • Use matrices as arrays of data to solve logic problems.

† Standard should be included in a one-semester course in Discrete Mathematics.

~~TOPIC~~STRAND: RECURSION AND OPTIMIZATION **DISCRETE MATHEMATICS** **STANDARD DM.10~~5~~** **The student will use algorithms to schedule tasks in order to determine a minimum project time. The algorithms will include critical path analysis, the list-processing algorithm, and student-created algorithms.**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Critical path scheduling sometimes yields optimal solutions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Specify in a digraph the order in which tests are to be performed. • Identify the critical path to determine the earliest completion time (minimum project time). • Use the list-processing algorithm to determine an optimal schedule. • Create and test scheduling algorithms.

~~TOPIC~~STRAND: RECURSION AND OPTIMIZATION

DISCRETE MATHEMATICS
STANDARD DM.11~~6~~

The student will solve linear programming problems. ~~Appropriate technology will be used to facilitate the use of matrices, graphing techniques, and the Simplex method of determining solutions.~~[Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Linear programming models an optimization process. • A linear programming model consists of a system of constraints and an objective quantity that can be maximized or minimized. • Any maximum or minimum value for a system of inequalities will occur at a corner point of a feasible region.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model ~~real-world~~practical problems with systems of linear inequalities. • Identify the feasibility region of a system of linear inequalities with no more than four constraints. • Identify the coordinates of the corner points of a feasibility region. • ~~Find~~Determine the maximum or minimum value of the system. • Describe the meaning of the maximum or minimum value in terms of the original problem.

~~TOPIC~~STRAND: RECURSION AND OPTIMIZATION

DISCRETE MATHEMATICS
STANDARD DM.12~~10~~

The student will use the recursive process and difference equations with the aid of appropriate technology to generate

a) compound interest;
b) sequences and series;
c) fractals;
d) population growth models; and
e) the Fibonacci sequence.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Recursion is a process that creates new objects from existing objects that were created by the same process. • A fractal is a figure whose dimension is not a whole number. • Fractals are self-similar.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use finite differences and recursion to model compound interest and population growth situations. • Model arithmetic and geometric sequences and series recursively. • Compare and contrast the recursive process, and create fractals. • Compare and contrast the recursive process and the Fibonacci sequence. • ~~Find~~Determine a recursive relationship that generates the Fibonacci sequence.

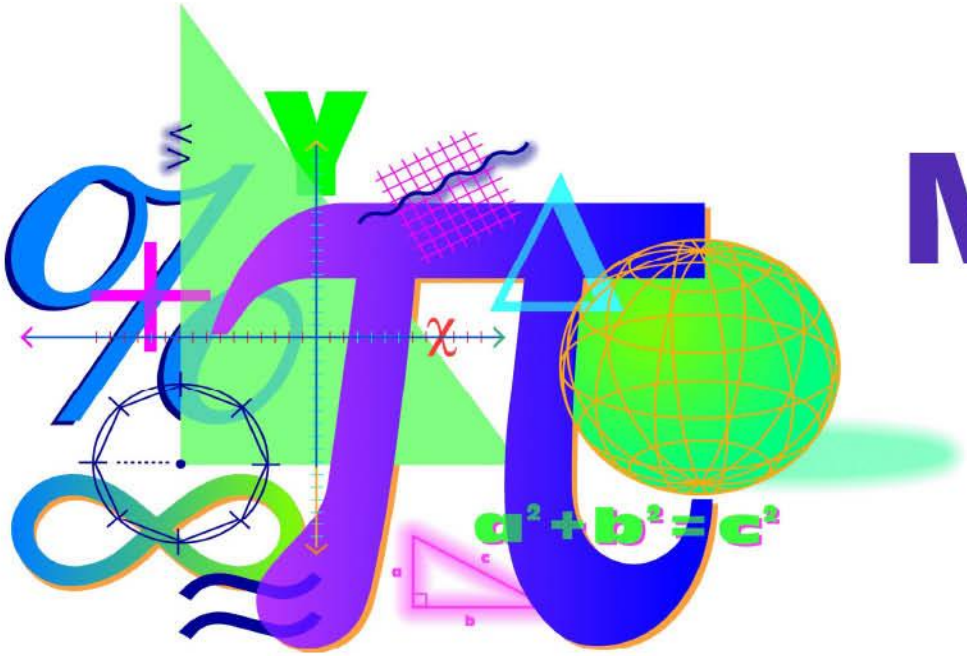
~~TOPIC~~STRAND: RECURSION AND OPTIMIZATION

DISCRETE MATHEMATICS
STANDARD DM.13

The student will apply the formulas of combinatorics in the areas of

a) the Fundamental (Basic) Counting Principle;
b) knapsack and bin-packing problems;
c) permutations and combinations; and
d) the pigeonhole principle.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The branch of mathematics that addresses the number of ways objects can be arranged or combined is combinatorics. • If n and r are positive integers and $n \geq r$, $n\,P\,r = \frac{n!}{(n-r)!}$ and $n\,C\,r = \frac{n!}{r!\,(n-r)!}$. • A bin-packing problem determines the minimum number of containers of fixed volume (bins) required to hold a set of objects. • A knapsack problem determines the most valuable set of objects that fit into a container (knapsack) of fixed volume. • Bin packing and knapsack packing are optimization techniques.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Find~~Determine the number of combinations possible when subsets of r elements are selected from a set of n elements without regard to order. • Use the Fundamental (Basic) Counting Principle to determine the number of possible outcomes of an event. • Use the knapsack and bin-packing algorithms to solve ~~real-world~~practical problems. • ~~Find~~Determine the number of permutations possible when r objects selected from n objects are ordered. • Use the pigeonhole principle to solve packing problems to facilitate proofs.



Mathematics
Standards of Learning

Curriculum Framework ~~2009~~ 2016

Mathematical Analysis

Board of Education
Commonwealth of Virginia

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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

~~The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.~~

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework* ~~2009~~
Introduction

The ~~2009~~ 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the ~~2009~~ 2016 *Mathematics Standards of Learning*, ~~and~~ amplifies the *Mathematics Standards of Learning* ~~by~~ and further defines~~ing~~ the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.~~

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.[Reordered and rewritten]

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: ~~Essential~~ Understanding~~s~~ the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

~~*Essential Understandings*~~
~~This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.~~

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction~~background information for the teacher (K-8). It contains content~~ The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s)~~that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard~~.

Essential Knowledge and Skills
~~Each standard is expanded in the Essential Knowledge and Skills column.~~ This section provides a detailed expansion of the mathematics knowledge and skills that ~~What~~ each student should know and be able to demonstrate~~do in each standard is outlined~~. This is not meant to be an exhaustive list of student expectations~~nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.~~

~~The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.~~[Reordered and rewritten]

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

~~TOPIC~~STRAND: FUNCTIONS

MATHEMATICAL ANALYSIS
STANDARD MA.1

The student will investigate and identify the ~~characteristics~~ properties of polynomial, rational, piecewise, and step ~~and rational~~ functions and ~~use these to~~ sketch their graphs ~~of the functions~~. ~~This will include determining zeros, upper and lower bounds, *y* intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics.~~[Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The graph~~s~~ of polynomial, ~~and~~ rational, piecewise, and step functions can be determined by exploring ~~characteristics~~ properties ~~and components~~ of the functions. • A variety of notations can be used such as set builder notation and interval notation.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify a polynomial, rational, piecewise, and step function, given an equation or graph. • ~~Identify rational functions, given an equation or graph.~~ • Given a graph or equation of a polynomial, rational, piecewise, or step function, identify:~~Identify~~ – d~~D~~omain and ~~,~~ range, – zeros, ~~upper and lower bounds,~~ – ~~*y*~~ intercepts, – ~~-~~symmetry~~,~~ – asymptotes (horizontal, vertical and oblique/slant); – ~~,~~ points of discontinuity; – intervals for which the function is increasing, decreasing or constant~~decreasing;~~, ~~points of discontinuity,~~ – end behavior;~~,~~ and – relative and/or absolute maximum and minimum points.~~, given a graph of a function.~~ • Sketch the graph of a polynomial, rational, piecewise, or step function. • ~~Sketch the graph of a rational function.~~

~~TOPIC~~STRAND: FUNCTIONS

MATHEMATICAL ANALYSIS
STANDARD MA.1

The student will investigate and identify the ~~characteristics~~ properties of polynomial, rational, piecewise, and step ~~and rational~~ functions and ~~use these to~~ sketch their graphs ~~of the functions~~. ~~This will include determining zeros, upper and lower bounds, *y* intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics.~~[Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
	• Investigate and verify characteristics of a polynomial, ~~or~~ rational, ~~function~~ piecewise, and step function, using a graphing ~~calculator~~utility. • Rationalize the denominator of a rational function.

~~TOPIC~~STRAND: FUNCTIONS

MATHEMATICAL ANALYSIS
STANDARD MA.2~~9~~

The student will investigate and identify the characteristics of exponential and logarithmic functions to graph the function, ~~in order to graph these functions and~~ solve equations, and solve ~~real-world~~practical problems. ~~This will include the role of e, natural and common logarithms, laws of exponents and logarithms, and the solution of logarithmic and exponential equations.~~ [Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Exponential and logarithmic functions are inverse functions. • Some examples of appropriate models or situations for exponential and logarithmic functions are: – Population growth; – Compound interest; – Depreciation/appreciation; – Richter scale; and – Radioactive decay.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify exponential functions from an equation or a graph. • Identify logarithmic functions from an equation or a graph. • Define *e*, and know its approximate value. • ~~Write~~ Convert between ~~logarithmic~~ equations written in logarithmic and ~~in~~ exponential form ~~and vice versa~~. • Identify common and natural logarithms, given an equation or practical situation. • Use laws of exponents and logarithms to solve equations and simplify expressions. • Model ~~real-world~~practical problems, using exponential and logarithmic functions. • Graph exponential and logarithmic functions ~~, using a graphing utility,~~ and identify asymptotes, end behavior, intercepts, domain, and range.

~~TOPIC~~STRAND: FUNCTIONS

MATHEMATICAL ANALYSIS
STANDARD MA.3~~2~~

The student will apply compositions of functions and inverses of functions to ~~real-world~~practical situations~~. Analytical methods and graphing utilities will be used~~ [Included in EKS] ~~to~~ and investigate and verify the domain and range of resulting functions.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• In composition of functions, a function serves as input for another function. • The composition of functions $f(x)$ and $g(x)$ can be determined using the graphs of $f(x)$ and $g(x)$. • A graph of a function and its inverse are symmetric about the line $y = x$. • $\left(f \circ f^{-1}\right)(x) = \left(f^{-1} \circ f\right)(x) = x$	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Find~~Determine the composition of functions algebraically and graphically. • ~~Find~~Determine the inverse of a function algebraically and graphically. • Determine the domain and range of ~~the~~ composite functions algebraically and graphically. • Determine the domain and range of the inverse of a function algebraically and graphically. • ~~Verify the accuracy of sketches of functions, using a graphing utility.~~[Included in MA.1]

~~TOPIC~~STRAND: FUNCTIONS

MATHEMATICAL ANALYSIS
STANDARD MA.4~~7~~

The student will ~~find~~determine the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. ~~A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution.~~[Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The limit of a function is the value approached by $f(x)$ as x approaches a given value or infinity.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Verify ~~intuitive reasoning~~estimates about the limit of a function~~,~~ using a graphing utility. • ~~Find~~Determine the limit of a function algebraically, and verify with a graphing utility. • ~~Find~~Determine the limit of a function numerically, and verify with a graphing utility. • Use limit notation when describing end behavior of a function.

~~TOPIC~~STRAND: FUNCTIONS

MATHEMATICAL ANALYSIS
STANDARD MA.5~~3~~

The student will investigate and describe the continuity of functions~~, using graphs and algebraic methods~~. [Moved to EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Continuous and discontinuous functions can be identified by their equations or graphs. • Discontinuity can be described as point (removable), jump or infinite. • A function, *f*, is continuous at a point *c* if and only if – $f(c)$ exists – $\lim_{x\to c} f(x)$ exists – $f(c) = \lim_{x\to c} f(x)$	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe continuity of a function. • Investigate the continuity of functions including absolute value, step, rational, and piece-wise~~-defined~~ functions, using graphical and algebraic methods. • Classify types of discontinuity. • Prove continuity at a point using the definition of limits. • ~~Use transformations to sketch absolute value, step, and rational functions.~~[Included in MA2.6] • ~~Verify the accuracy of sketches of functions, using a graphing utility.~~ [Moved to MA.1]

~~TOPIC~~STRAND: ANALYTIC~~AL~~ GEOMETRY

MATHEMATICAL ANALYSIS
STANDARD MA.6~~8~~

The student will investigate, graph and identify the ~~characteristics~~ properties of conic sections from equations in ~~(*h*, *k*)~~vertex and standard form~~s~~. ~~Transformations in the coordinate plane will be used to graph conic sections.~~ [Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Matrices can be used to represent transformations of figures in the plane. • A conic section is a figure formed by the intersection of a plane with a right circular cone. Conic sections include the parabola, circle, ellipse, and hyperbola.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given a translation or rotation matrix, ~~find~~determine an equation for the transformed function or conic section. • Investigate and verify graphs of transformed conic sections, using a graphing utility. • Graph conic sections from equations written in vertex or standard form using transformations. • Identify properties of conic sections.

~~TOPIC~~STRAND: ANALYTIC~~AL~~ GEOMETRY

MATHEMATICAL ANALYSIS
STANDARD MA.7~~11~~

The student will perform operations with vectors in the coordinate plane and solve ~~real-world~~ practical problems using vectors. ~~This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.~~[Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Every vector has an equal vector that has its initial point at the origin. • The magnitude and direction of a vector with the origin as the initial point are completely determined by the coordinates of its terminal point. • Every nonzero vector has a corresponding unit vector, which has the same direction as the vector, but a magnitude of 1. [Moved from EKS]	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use vector notation. • Perform the operations of addition, subtraction, scalar multiplication, and inner (dot) product on vectors. • Graph vectors and resultant vectors. • Express complex numbers in vector notation. • ~~Define *unit vector*, and find the unit vector in the same direction as a given vector.~~ [Moved to US] • Identify properties of vector addition, scalar multiplication, and dot product. • ~~Find~~Determine the components of a vector ~~components~~. • ~~Find~~Determine the norm (magnitude) of a vector. • Use vectors in simple geometric proofs. • Solve ~~real-world~~ problems using vectors.

~~TOPIC~~STRAND: ANALYTIC GEOMETRY

MATHEMATICAL ANALYSIS
STANDARD MA.8~~13~~

The student will identify, create, and solve ~~real-world~~practical problems involving triangles. ~~Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.~~[Included in the EKS.]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Real-world~~Practical problems can be modeled using trigonometry and vectors. • Triangles can be formed from vectors.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve and create problems~~,~~ using trigonometric functions. • Solve and create problems~~,~~ using the Pythagorean Theorem. • Solve and create problems~~,~~ using the Law of Sines and the Law of Cosines. • Solve ~~real-world~~ problems where triangles are formed from ~~using~~ vectors.

~~TOPIC~~STRAND: EQUATIONS

MATHEMATICAL ANALYSIS
STANDARD MA.9~~10~~

The student will investigate and identify the characteristics of the graphs of polar equations.~~, using graphing utilities. This will include classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.~~[Included in EKS]

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• The real number system is represented geometrically on the number line, and the complex number system is represented geometrically on the plane where $a + bi$ corresponds to the point (a, b) in the plane.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Recognize~~ Classify polar equations (rose, cardioid, limaçon, lemniscate, spiral, and circle), given the graph or the equation. • Determine the effects of changes in the parameters of polar equations on the graph, using a graphing utility. • Convert between complex numbers ~~from~~ written in rectangular form ~~to~~and polar form ~~and vice versa~~. • ~~Find~~Determine and verify the intersection of the graphs of two polar equations, using a graphing utility.

~~TOPIC~~STRAND: EQUATIONS

MATHEMATICAL ANALYSIS
STANDARD MA.10~~12~~

The student will use parametric equations to model and solve ~~application~~ practical problems.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Parametric equations are used to express two dependent variables, *x* and *y*, in terms of an independent variable (parameter), *t*. • Some curves cannot be represented as a function, *f*(*x*). Parametric graphing enables the representation of these curves in terms of functions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Graph parametric equations.~~, using a graphing utility~~. • Use parametric equations to model motion over time. • Determine solutions to parametric equations graphically.~~, using a graphing utility.~~ • ~~Compare and contrast traditional solution methods with parametric methods.~~ • Use a graphing utility to graph and analyze parametric equations.

~~TOPIC~~STRAND: EQUATIONS

MATHEMATICAL ANALYSIS
STANDARD MA.11~~14~~

The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Matrices ~~are a convenient shorthand for~~may be used to solve~~ing~~ systems of equations. • Matrices can model a variety of linear systems. • Solutions of a linear system are values that satisfy every equation in the system. • Matrices can be used to model and solve real-world problems.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Multiply matrices by a scalar. [Reordered] • Add, subtract, and multiply matrices. ~~and multiply matrices by a scalar.~~ • Model problems with a system of no more than three linear equations. • Express a system of linear equations as a matrix equation. • Solve a system of equations using matrices~~matrix equation~~. • ~~Find~~Determine the inverse of a 2 x 2 or 3 x 3 matrix using paper-pencil. • Verify two matrices are inverses using matrix multiplication. • Verify the commutative and associative properties for matrix addition and multiplication.~~.~~

~~TOPIC~~STRAND: DISCRETE MATHEMATICS **MATHEMATICAL ANALYSIS** **STANDARD MA.124** **The student will expand binomials having positive integral exponents~~ through the use of the Binomial Theorem, the formula for combinations, and Pascal's Triangle~~.[Included in EKS]**	
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The Binomial Theorem provides a formula for calculating the product $(a + b)^n$ for any positive integer n. • Pascal's Triangle is a triangular array of binomial coefficients.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Expand binomials having positive integral exponents. • Use the Binomial Theorem, the formula for combinations, and Pascal's Triangle to expand binomials.

~~TOPIC~~STRAND: DISCRETE MATHEMATICS

MATHEMATICAL ANALYSIS
STANDARD MA.13~~5~~

The student will ~~find~~determine the sum ~~(sigma notation included)~~ of finite and infinite convergent series~~, which will lead to an intuitive approach to a limit~~.

~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	ESSENTIAL KNOWLEDGE AND SKILLS
• Examination of infinite sequences and series may lead to a limiting process. • Arithmetic sequences have a common difference between any two consecutive terms. • Geometric sequences have a common ~~factor~~ ratio between any two consecutive terms.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use and interpret the notation: $\sum$, n, nth, and a_n. • Derive the formulas associated with arithmetic and geometric sequences and series. • Given the formula, ~~find~~determine the nth term, a_n, for an arithmetic or geometric sequence. • Given the formula, ~~find~~determine the sum, S_n, if it exists, of an arithmetic or geometric series. • Model and solve problems, using sequence and series information. • Distinguish between a convergent and divergent series. • Discuss convergent series in relation to the concept of a limit.

MATHEMATICAL ANALYSIS **STANDARD MA.14~~6~~** **The student will use mathematical induction to prove formulas and mathematical statements.**	**~~TOPIC~~STRAND: DISCRETE MATHEMATICS**
~~ESSENTIAL UNDERSTANDINGS~~UNDERSTANDING THE STANDARD	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Mathematical induction is a method of proof that depends on a recursive process. • Mathematical induction allows reasoning from specific true values of the variable to general values of the variable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare inductive and deductive reasoning. • Prove formulas ~~/~~and mathematical statements, using mathematical induction.