Z-Scores and NFL Quarterback Salaries

Strand: Data Analysis
Topic: Interpreting and Comparing Z-scores
Primary SOL: AFDA.7
b) The student will interpret and compare z-scores for normally distributed data.

Materials
- 2007 Quarterback Salary Data Table activity sheet (attached)
- Z-scores and NFL Quarterback Salaries activity sheet (attached)
- Laptops, tablets, or other smart devices
- NFL salary data website

Vocabulary
mean, median, mode, range, standard deviation, variance, z-score

Student/Teacher Actions: What should students be doing? What should teachers be doing?
Time: 90 minutes

1. Review with students z-scores, how to convert the raw score into a z-score, and how to interpret a z-score equivalent of a raw score.
2. Distribute copies of the 2007 Quarterback Salary Data Table activity sheet. Assign students to work in groups. Groups can be heterogeneous or homogeneous. Model to students how to use a graphing utility to make the task of dealing with a large data set more efficient. Facilitate a class discussion on students’ responses to the questions that follow.
3. Students may question the use of data from 2007. Challenge students to research the current salary data of NFL quarterbacks. Distribute copies of the Z-Scores and NFL Quarterback Salaries activity sheet. If laptops, tablets, or other smart devices are not available to students, show the data provided by clicking on the link (Sportrac.com NFL quarterback salaries). Students can work in pairs, threes, or small groups to complete the activity. Each student should record their answer on their individual sheet.

Assessment
• Questions
  o What is the relationship between z-scores and standard deviation?
  o What does a z-score tell you about the position of an element with respect to the mean?
  o What does it mean if the z-score is negative?

• Journal/writing prompts
  o Given the standard deviation, the mean, and the value of an element of a data set, explain how you would find the associated z-score.
  o How does a z-score relate to its associated element’s value?
Other Assessments
- Normal curves are symmetric and bell-shaped. They are centered on the mean. Pulse rates are normally distributed with a mean of 72 and a standard deviation of 12. Construct a model of the normal distribution curve for the data.
- Amy took the ACT and scored a 25 on the mathematics portion of the test. Her friend Stephanie scored a 610 on the mathematics portion of her SAT. Both tests have scores that are normally distributed. For the SAT, the mean was 515 and the standard deviation was 106. For the ACT, the mean was 21 and the standard deviation was 4.7. Whose achievement was higher on the mathematics portion?

Extensions and Connections
- Have students conduct research on NFL quarterback salaries in 2011 and 2017 using two or more web search engines. Once students have collected the data, they can compare individual salary changes for quarterbacks who played each year during this time period. Then, ask students to compare the overall salary total for quarterbacks for each year (2011 and 2017). How much have salaries increased from 2007 to 2017? What is the standard deviation of salaries in 2011 and 2017? Compare to the standard deviations calculated in the original activity.
- Students can complete the enhanced scope and sequence lesson AII.11 on z-scores.

Strategies for Differentiation
- Student data can be reduced to first-string quarterbacks, to limit the number of data points.
- Use vocabulary cards for related vocabulary listed above.
- Students can use a spreadsheet to enter data and calculate statistical points.
- Provide students with representations of a normal curve so they can construct a graph depicting the data points they analyzed.
- Once a line plot is made and standard deviation is computed, have students measure one standard deviation, using the same scale as their line plot. Students can cut out the length of the standard deviation from an index card or sticky note and then physically measure the number of standard deviations, in addition to mathematically computing it.

Note: The following pages are intended for classroom use for students as a visual aid to learning.
### 2007 Quarterback Salary Data Table

Data from USA Today

<table>
<thead>
<tr>
<th>Team Name</th>
<th>Quarterback Salary</th>
<th>Deviation from the Mean</th>
<th>Square each Deviation from the mean</th>
<th>$z$- score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bears</td>
<td>$2,039,800</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bengals</td>
<td>$7,250,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bills</td>
<td>$1,804,560</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Broncos</td>
<td>$8,253,020</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Browns</td>
<td>$1,147,000</td>
<td></td>
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<tr>
<td>Buccaneers</td>
<td>$5,003,360</td>
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</tr>
<tr>
<td>Cardinals</td>
<td>$10,339,920</td>
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</tr>
<tr>
<td>Chargers</td>
<td>$5,005,760</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Chiefs</td>
<td>$3,753,600</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Colts</td>
<td>$11,003,840</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cowboys</td>
<td>$1,506,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dolphins</td>
<td>$6,000,000</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Eagles</td>
<td>$5,504,080</td>
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<tr>
<td>Falcons</td>
<td>$3,503,720</td>
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<tr>
<td>Forty-niners</td>
<td>$1,675,000</td>
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<tr>
<td>Giants</td>
<td>$6,450,000</td>
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<tr>
<td>Jaguars</td>
<td>$1,105,280</td>
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<tr>
<td>Jets</td>
<td>$4,000,000</td>
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<tr>
<td>Lions</td>
<td>$1,500,000</td>
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<td>Packers</td>
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<td>Panthers</td>
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<td>Patriots</td>
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<td>Raiders</td>
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<tr>
<td>Rams</td>
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<tr>
<td>Ravens</td>
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<td>Redskins</td>
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<td>Seahawks</td>
<td>$6,004,320</td>
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<td>Steelers</td>
<td>$1,009,840</td>
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<tr>
<td>Texans</td>
<td>$8,000,000</td>
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<tr>
<td>Titans</td>
<td>$13,143,000</td>
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<tr>
<td>Vikings</td>
<td>$1,485,000</td>
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</tr>
<tr>
<td>TOTALS</td>
<td>$\sum x =$</td>
<td>$\sum (x - \mu) =$</td>
<td>$\sum (x - \mu)^2 =$</td>
<td>$\sum (x - \mu) / \sigma =$</td>
</tr>
</tbody>
</table>
Mathematics Instructional Plan—AFDA

Using the 2007 Quarterback Salary Data, answer the following questions:

a. The number of teams, \( N \), is equal to __________

b. Find the sum of all of the quarterback salaries, \( \sum x = \) __________

c. Calculate the mean \( \mu = \frac{\sum x}{N} = \) ______________
   
   - What information does the mean provide?
   
   - How many salaries are above the mean? ______
   
   - How many salaries are below the mean? ______

d. Calculate the variance \( \sigma^2 = \frac{\sum (x-\mu)^2}{N} = \) ______________

e. Calculate the standard deviation: \( \sigma = \sqrt{\sigma^2} \) ______________
   
   - Which quarterback salaries are within one standard deviation of the mean?
   
   - What percentage of salaries is that? _____
   
   - Which quarterback salaries are not within two standard deviations of the mean?
2. Calculate the z-score for each salary, \( z = \frac{x - \mu}{\sigma} \), and complete that column of the table.

   a. Find which team’s quarterback has a salary farthest from the mean. What was the corresponding z-score?

   b. Find which team’s quarterback has a salary closest to the mean. What was the corresponding z-score?

   c. What information does each z-score provide?

   d. Determine how many of the z-scores are positive and how many are negative. Then, explain why that makes sense.

   e. What is the sum of all of the z-scores? Explain why this makes sense.
Z-Scores and NFL Quarterback Salaries

Use the data from Over the Cap NFL Quarterback salaries to answer the following questions:
1. Using the quarterback salary data from the link provided above, determine the following:
   a. The number of teams, \( N \), is equal to __________
   b. Find the sum of all of the quarterback salaries, \( \sum x = \) __________
   c. Calculate the mean \( \mu = \frac{\sum x}{N} \) : ______________
      ▪ What information does the mean provide?
      ▪ How many salaries are above the mean? ______
      ▪ How many salaries are below the mean? ______
      ▪ Was that what you expected? Explain.

   d. Calculate the variance \( \sigma^2 = \frac{\sum (x - \mu)^2}{N} \) : ______________
   e. Calculate the standard deviation \( \sigma = \sqrt{\sigma^2} \) : ______________
      ▪ Which quarterback salaries are within one standard deviation of the mean?
      ▪ What percentage of salaries is that? ____
      ▪ Which quarterback salaries are not within two standard deviations of the mean?

2. Calculate the z-score for each salary, \( z\text{-score} = \frac{x - \mu}{\sigma} \), and complete that column of the table.
   a. Find which team’s quarterback has a salary farthest from the mean. What was the corresponding z-score?
b. Find which team’s quarterback has a salary closest to the mean. What was the corresponding z-score?

c. What information does each z-score provide?

d. Determine how many of the z-scores are positive and how many are negative. Then, explain why that makes sense.

e. What is the sum of all of the z-scores? Explain why this makes sense.

3. Ask students to pretend that a typo was made when entering the Rams’ quarterback salary. Instead of $17,502,040, it should have been $1,750,204. How would this affect the data?

4. Repeat the process with salaries of other position players.

5. What are the significant changes between the 2007 data and the 2017 data? Explain.