
Mathematics Standards of Learning for Virginia Public Schools

**Adopted in October 2001 by the
Board of Education**

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Foreword

The Standards of Learning in this publication represent a major development in public education in Virginia. The revised Mathematics Standards of Learning were adopted in October 2001 by the state Board of Education.

The Standards of Learning provide a framework for instructional programs designed to raise the academic achievement of all students in Virginia and are an important part of Virginia's efforts to provide challenging educational programs in the public schools.

The Standards of Learning set reasonable targets and expectations for what teachers need to teach and students need to learn. The standards are not intended to encompass the entire curriculum for a given grade level or course or to prescribe how the content should be taught; the standards are to be incorporated into a broader, locally designed curriculum. Teachers are encouraged to go beyond the standards and select instructional strategies and assessment methods appropriate for their students.

The Standards of Learning are recognized as a model for other states. They were developed through a series of public hearings and the efforts of parents, teachers, education officials, and representatives of business and industry. The standards set clear, concise, and measurable academic expectations for young people. Parents are encouraged to work with their children to help them achieve these academic standards.

A major objective of Virginia's educational agenda is to give the citizens of the commonwealth a program of public education that is among the best in the nation and that meets the needs of all young people in the commonwealth. These Standards of Learning chart the course for achieving that objective.

Mathematics

Standards of Learning

The Standards of Learning for mathematics identify academic content for essential components of the mathematics curriculum at different grade levels for Virginia's public schools. Standards are identified for kindergarten through grade eight and for a core set of high school courses. Throughout a student's mathematics schooling from kindergarten through grade eight, specific content strands or topics are included. These content strands are Number and Number Sense; Computation and Estimation; Measurement; Geometry; Probability and Statistics; and Patterns, Functions, and Algebra. The Standards of Learning for each strand progress in complexity at each grade level and throughout the high school courses.

The Standards of Learning are not intended to encompass the entire curriculum for a given grade level or course or to prescribe how the content should be taught. Teachers are encouraged to go beyond the standards and to select instructional strategies and assessment methods appropriate for their students.

Goals

Students today require stronger mathematical knowledge and skills to pursue higher education, to compete in a technologically oriented workforce, and to be informed citizens. Students must gain an understanding of fundamental ideas in arithmetic, measurement, geometry, probability, data analysis and statistics, and algebra and functions, and develop proficiency in mathematical skills. In addition, students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving in science, engineering, business and industry, government, and practical affairs. Hence, the use of technology must be an integral part of teaching and learning. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skills should be the shared responsibility of teachers of all disciplines.

The content of the mathematics standards is intended to support the following five goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-life data and situations within and outside mathematics and then apply appropriate strategies to find an acceptable solution. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problem types. A major goal of the mathematics program is to help students become competent mathematical problem solvers.

Mathematical Communication

Students will use the language of mathematics, including specialized vocabulary and symbols, to express mathematical ideas precisely. Representing, discussing, reading, writing, and listening to mathematics will help students to clarify their thinking and deepen their understanding of the mathematics being studied.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will learn to apply proportional and spatial reasoning and to reason from a variety of representations such as graphs, tables, and charts.

Mathematical Connections

Students will relate concepts and procedures from different topics in mathematics to one another and see mathematics as an integrated field of study. Through the application of content and process skills, students will make connections between different areas of mathematics and between mathematics and other disciplines, especially science. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships with a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should move easily among different representations—graphical, numerical, algebraic, verbal, and physical—and recognize that representation is both a process and a product.

Kindergarten

The kindergarten standards place emphasis on developing the concept of number by counting; combining, sorting, and comparing sets of objects; recognizing and describing simple repeating patterns; and recognizing shapes and sizes of figures and objects. Students will investigate nonstandard measurement, collect data, and create graphs.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

- K.1 The student, given two sets containing 10 or fewer concrete items, will identify and describe one set as having more, fewer, or the same number of members as the other set, using the concept of one - to - one correspondence.
- K.2 The student, given a set containing 10 or fewer concrete items, will
 - a) tell how many are in the set by counting the number of items orally;
 - b) select the corresponding numeral from a given set; and
 - c) write the numeral to tell how many are in the set.
- K.3 The student, given an ordered set of three objects and/or pictures, will indicate the ordinal position of each item, first through third, and the ordered position of each item from left-to-right, right-to-left, top-to-bottom, and/or bottom-to-top.
- K.4 The student will investigate and recognize patterns from counting by fives and tens to 30, using concrete objects and a calculator.
- K.5 The student will count forward to 30 and backward from 10.

Computation and Estimation

K.6 The student will add and subtract whole numbers, using up to 10 concrete items.

Measurement

K.7 The student will recognize a penny, nickel, dime, and quarter and will determine the value of a collection of pennies and/or nickels whose total value is 10 cents or less.

K.8 The student will identify the instruments used to measure length (ruler), weight (scale), time (clock: digital and analog; calendar: day, month, and season), and temperature (thermometer).

K.9 The student will tell time to the hour, using an analog or digital clock.

K.10 The student will compare two objects or events, using direct comparisons or nonstandard units of measure, according to one or more of the following attributes: length (shorter, longer), height (taller, shorter), weight (heavier, lighter), temperature (hotter, colder). Examples of nonstandard units include foot length, hand span, new pencil, paper clip, block.

Geometry

K.11 The student will identify, describe, and draw two-dimensional (plane) geometric figures (circle, triangle, square, and rectangle).

K.12 The student will describe the location of one object relative to another (above, below, next to) and identify representations of plane geometric figures (circle, triangle, square, and rectangle) regardless of their position and orientation in space.

K.13 The student will compare the size (larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle).

Probability and Statistics

K.14 The student will gather data relating to familiar experiences by counting and tallying.

K.15 The student will display objects and information, using objects graphs, pictorial graphs and tables.

K.16 The student will investigate and describe the results of dropping a two-colored counter or using a multicolored spinner.

Patterns, Functions, and Algebra

- K.17 The student will sort and classify objects according to similar attributes (size, shape, and color).
- K.18 The student will identify, describe, and extend a repeating relationship (pattern) found in common objects, sounds, and movements.

Grade One

The first-grade standards place emphasis on counting, sorting, and comparing sets of up to 100 objects; recognizing and describing simple repeating and growing patterns; and drawing, sorting, and describing certain two-dimensional figures. Students' understanding of number is expanded through learning and applying the basic addition facts through the fives table and the corresponding subtraction facts; using nonstandard units to measure; and organizing and interpreting data. The idea of fractions is introduced.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

- 1.1 The student will count objects in a given set containing between 1 and 100 objects and write the corresponding numeral.
- 1.2 The student will group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.
- 1.3 The student will count forward by ones, fives, and tens to 100, by twos to 20, and backward by ones from 20.
- 1.4 The student will recognize and write numerals 0 through 100.
- 1.5 The student will identify the ordinal positions first through tenth, using an ordered set of objects.
- 1.6 The student will identify and represent the concepts of one-half and one-fourth, using appropriate materials or a drawing.

Computation and Estimation

- 1.7 The student, given a familiar problem situation involving magnitude, will
- a) select a reasonable magnitude from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, and 500); and
 - b) explain the reasonableness of his/her choice.
- 1.8 The student will recall basic addition facts - i.e., sums to 10 or less - and the corresponding subtraction facts.
- 1.9 The student will create and solve story and picture problems involving one-step solutions, using basic addition and subtraction facts.

Measurement

- 1.10 The student will
- a) identify the number of pennies equivalent to a nickel, a dime, and a quarter; and
 - b) determine the value of a collection of pennies, nickels, and dimes whose total value is 100 cents or less.
- 1.11 The student will tell time to the half-hour, using an analog or digital clock.
- 1.12 The student will use nonstandard units to measure length and weight.
- 1.13 The student will compare the volumes of two given containers by using concrete materials (e.g., jelly beans, sand, water, rice).
- 1.14 The student will compare the weights of two objects, using a balance scale.

Geometry

- 1.15 The student will describe the proximity of objects in space (*near, far, close by, below, above, up, down, beside, and next to*).
- 1.16 The student will draw, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, corners, and square corners.
- 1.17 The student will identify and describe objects in his/her environment that depict plane geometric figures (triangle, rectangle, square, and circle).

Probability and Statistics

- 1.18 The student will investigate, identify, and describe various forms of data collection in his/her world (e.g., recording daily temperature, lunch count, attendance, and favorite ice cream), using tables, picture graphs, and object graphs.
- 1.20 The student will interpret information displayed in a picture or object graph, using the vocabulary *more*, *less*, *fewer*, *greater than*, *less than*, and *equal to*.

Patterns, Functions, and Algebra

- 1.20 The student will sort and classify concrete objects according to one or more attributes, including color, size, shape, and thickness.
- 1.21 The student will recognize, describe, extend, and create a wide variety of patterns, including rhythmic, color, shape, and numerical. Patterns will include both growing and repeating patterns. Concrete materials and calculators will be used by students.

Grade Two

The second-grade standards extend the study of number and spatial sense to include three-digit numbers and three-dimensional figures. Students will continue to learn, use, and gain proficiency in the basic addition facts through the nines table and the corresponding subtraction facts. Students will begin to use standard U.S. Customary and metric units of measurement; predict, using simple probability; and create and interpret picture and bar graphs. Students will work with a variety of patterns and will develop knowledge of equality by identifying missing numbers in addition and subtraction facts.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

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Number and Number Sense

- 2.1 The student will
 - a) read, write, and identify the place value of each digit in a three-digit numeral, using numeration models; and
 - b) round two-digit numbers to the nearest ten.
- 2.2 The student will compare two whole numbers between 0 and 999, using symbols ($>$, $<$, or $=$) and words (*greater than*, *less than*, or *equal to*).
- 2.3 The student will identify the ordinal positions first through twentieth, using an ordered set of objects
- 2.4 The student will identify the part of a set and/or region that represents fractions for one-half, one-third, one-fourth, one-eighth, and one-tenth and write the corresponding fraction.

- 2.5 The student will
- a) count forward by twos, fives, and tens to 100, starting at various multiples of 2, 5, or 10, using mental mathematics, paper and pencil, hundred chart, calculators, and/or concrete objects, as appropriate;
 - b) count backward by tens from 100;
 - c) group objects by threes and fours; and
 - d) recognize even and odd numbers, using objects.

Computation and Estimation

- 2.6 The student will recall basic addition facts - i.e., sums to 18 or less - and the corresponding subtraction facts.
- 2.7 The student, given two whole numbers whose sum is 99 or less, will
- a) estimate the sum; and
 - b) find the sum, using various methods of calculation (mental computation, concrete materials, and paper and pencil).
- 2.8 The student, given two whole numbers, each of which is 99 or less, will
- a) estimate the difference; and
 - b) find the difference, using various methods of calculation (mental computation, concrete materials, and paper and pencil).
- 2.9 The student will create and solve one-step addition and subtraction problems using data from simple tables, picture graphs, bar graphs, and practical situations.
- 2.10 The student, given a simple addition or subtraction fact, will recognize and describe the related facts which represent and describe the inverse relationship between addition and subtraction (e.g., $3 + \underline{\quad} = 7$, $\underline{\quad} + 3 = 7$; $7 - 3 = \underline{\quad}$, and $7 - \underline{\quad} = 3$).

Measurement

- 2.11 The student will
- a) count and compare a collection of pennies, nickels, dimes, and quarters whose total value is \$2.00 or less; and
 - b) identify the correct usage of the cent symbol (¢), dollar symbol ($\text{\$}$), and decimal point (\.).
- 2.12 The student will estimate and then use a ruler to make linear measurements to the nearest centimeter and inch, including measuring the distance around a polygon in order to determine perimeter.
- 2.13 The student, given grid paper, will estimate and then count the number of square units needed to cover a given surface in order to determine area.

- 2.14 The student will estimate and then count the number of cubes in a rectangular box in order to determine volume.
- 2.15 The student will estimate and then determine weight/mass of familiar objects in pounds and/or kilograms, using a scale.
- 2.16 The student will tell and write time to the quarter hour, using analog and digital clocks.
- 2.17 The student will use actual measuring devices to compare metric and U.S. Customary units (cups, pints, quarts, gallons, and liters) for measuring liquid volume, using the concepts of *more*, *less*, and *equivalent*.
- 2.18 The student will
- use calendar language appropriately (e.g., months, *today*, *yesterday*, *next week*, *last week*);
 - determine past and future days of the week; and
 - identify specific dates on a given calendar.
- 2.19 The student will read the temperature on a Celsius and/or Fahrenheit thermometer to the nearest 10 degrees.

Geometry

- 2.20 The student will identify, describe, and sort three-dimensional (solid) concrete figures, including a cube, rectangular solid (prism), square pyramid, sphere, cylinder, and cone, according to the number and shape of the solid's faces, edges, and corners.
- 2.21 The student will identify and create figures, symmetric along a line, using various concrete materials.
- 2.22 The student will compare and contrast plane and solid geometric shapes (circle/sphere, square/cube, and rectangle/rectangular solid).

Probability and Statistics

- 2.23 The student will read, construct, and interpret a simple picture and bar graph.
- 2.24 The student will record data from experiments, using spinners and colored tiles/cubes, and use the data to predict which of two events is more likely to occur if the experiment is repeated.

Patterns, Functions, and Algebra

- 2.25 The student will identify, create, and extend a wide variety of patterns, using numbers, concrete objects, and pictures.

2.26 The student will solve problems by completing a numerical sentence involving the basic facts for addition and subtraction. Examples include: $3 + \underline{\quad} = 7$, or $9 - \underline{\quad} = 2$. Students will create story problems, using the numerical sentences.

Grade Three

The third-grade standards place emphasis on learning multiplication and division facts through the nines table. Concrete materials and two-dimensional representations will be used to introduce addition and subtraction with fractions and decimals and the concept of probability as chance. Students will use standard units (U.S. Customary and metric) for temperature, length, liquid volume, and weight and identify relevant properties of shapes, line segments, and angles.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

- 3.1 The student will read and write six-digit numerals and identify the place value for each digit.
- 3.2 The student will round a whole number, 9,999 or less, to the nearest ten, hundred, and thousand.
- 3.3 The student will compare two whole numbers between 0 and 9,999, using symbols ($>$, $<$, or $=$) and words (*greater than*, *less than*, or *equal to*).
- 3.4 The student will recognize and use the inverse relationships between addition/subtraction and multiplication/division to complete basic fact sentences. Students will use these relationships to solve problems such as $5 + 3 = 8$ and $8 - 3 = \underline{\quad}$.
- 3.5 The student will
 - a) divide regions and sets to represent a fraction; and
 - b) name and write the fractions represented by a given model (area/region, length/measurement, and set). Fractions (including mixed numbers) will include halves, thirds, fourths, eighths, and tenths.

- 3.6 The student will compare the numerical value of two fractions having like and unlike denominators, using concrete or pictorial models involving areas/regions, lengths/measurements, and sets.
- 3.7 The student will read and write decimals expressed as tenths and hundredths, using concrete materials and models.

Computation and Estimation

- 3.8 The student will solve problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping, using various computational methods, including calculators, paper and pencil, mental computation, and estimation.
- 3.9 The student will recall the multiplication and division facts through the nines table.
- 3.10 The student will represent multiplication and division, using area and set models, and create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 5 or less.
- 3.11 The student will add and subtract with proper fractions having like denominators of 10 or less, using concrete materials and pictorial models representing areas/regions, lengths/measurements, and sets.
- 3.12 The student will add and subtract with decimals expressed as tenths, using concrete materials, pictorial representations, and paper and pencil.

Measurement

- 3.13 The student will determine by counting the value of a collection of bills and coins whose total value is \$5.00 or less, compare the value of the coins or bills, and make change.
- 3.14 The student will estimate and then use actual measuring devices with metric and U.S. Customary units to measure
- a) length—inches, feet, yards, centimeters, and meters;
 - b) liquid volume—cups, pints, quarts, gallons, and liters; and
 - c) weight/mass—ounces, pounds, grams, and kilograms.
- 3.15 The student will tell time to the nearest five-minute interval and to the nearest minute, using analog and digital clocks.
- 3.16 The student will identify equivalent periods of time, including relationships among days, months, and years, as well as minutes and hours.
- 3.17 The student will read temperature to the nearest degree from a Celsius thermometer and a Fahrenheit thermometer. Real thermometers and physical models of thermometers will be used.

Geometry

- 3.18 The student will analyze two-dimensional (plane) and three-dimensional (solid) geometric figures (circle, square, rectangle, triangle, cube, rectangular solid [prism], square pyramid, sphere, cone, and cylinder) and identify relevant properties, including the number of corners, square corners, edges, and the number and shape of faces, using concrete models.
- 3.19 The student will identify and draw representations of line segments and angles, using a ruler or straightedge.
- 3.20 The student, given appropriate drawings or models, will identify and describe congruent and symmetrical, two-dimensional (plane) figures, using tracing procedures.

Probability and Statistics

- 3.21 The student, given grid paper, will
- collect and organize data on a given topic of his/her choice, using observations, measurements, surveys, or experiments; and
 - construct a line plot, a picture graph, or a bar graph to represent the results. Each graph will include an appropriate title and key.
- 3.22 The student will read and interpret data represented in line plots, bar graphs, and picture graphs and write a sentence analyzing the data.
- 3.23 The student will investigate and describe the concept of probability as chance and list possible results of a given situation.

Patterns, Functions, and Algebra

- 3.24 The student will recognize and describe a variety of patterns formed using concrete objects, numbers, tables, and pictures, and extend the pattern, using the same or different forms (concrete objects, numbers, tables, and pictures).
- 3.25 The student will
- investigate and create patterns involving numbers, operations (addition and multiplication), and relations that model the identity and commutative properties for addition and multiplication; and
 - demonstrate an understanding of equality by recognizing that the equal sign (=) links equivalent quantities, such as $4 \cdot 3 = 2 \cdot 6$.

Grade Four

The fourth-grade standards place emphasis on multiplication and division with whole numbers and solving problems involving addition and subtraction of fractions and decimals. Students will continue to learn and use the basic multiplication facts as they become proficient in multiplying larger numbers. Students also will refine their estimation skills for computations and measurements and investigate relationships between and among simple two-dimensional (plane) figures and three-dimensional (solid) figures. Students will identify and draw representations of points, lines, line segments, and rays. Students will graph points in the first quadrant in the coordinate plane and extend and duplicate patterns. Concrete materials and two-dimensional representations will be used to solve problems involving perimeter, patterns, probability, and equivalence of fractions and decimals. Students will recognize a geometric transformation, such as reflection (flip), translation (slide), and rotation (turn).

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

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Number and Number Sense

- 4.1 The student will
- a) identify (orally and in writing) the place value for each digit in a whole number expressed through millions;
 - b) compare two whole numbers expressed through millions, using symbols ($>$, $<$, or $=$); and
 - c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.
- 4.2 The student will
- a) identify, model, and compare rational numbers (fractions and mixed numbers), using concrete objects and pictures;
 - b) represent equivalent fractions; and
 - c) relate fractions to decimals, using concrete objects.

- 4.3 The student will compare the numerical value of fractions (with like and unlike denominators) having denominators of 12 or less, using concrete materials.
- 4.4 The student will
- read, write, represent, and identify decimals expressed through thousandths;
 - round to the nearest whole number, tenth, and hundredth; and
 - compare the value of two decimals, using symbols ($<$, $>$, or $=$), concrete materials, drawings, and calculators.

Computation and Estimation

- 4.5 The student will estimate whole-number sums and differences and describe the method of estimation. Students will refine estimates, using terms such as *closer to*, *between*, and *a little more than*.
- 4.6 The student will add and subtract whole numbers written in vertical and horizontal form, choosing appropriately between paper and pencil methods and calculators.
- 4.7 The student will find the product of two whole numbers when one factor has two digits or fewer and the other factor has three digits or fewer, using estimation and paper and pencil. For larger products (a two-digit numeral times a three-digit numeral), estimation and calculators will be used.
- 4.8 The student will estimate and find the quotient of two whole numbers, given a one-digit divisor.
- 4.9 The student will
- add and subtract with fractions having like and unlike denominators of 12 or less, using concrete materials, pictorial representations, and paper and pencil;
 - add and subtract with decimals through thousandths, using concrete materials, pictorial representations, and paper and pencil; and
 - solve problems involving addition and subtraction with fractions having like and unlike denominators of 12 or less and with decimals expressed through thousandths, using various computational methods, including calculators, paper and pencil, mental computation, and estimation.

Measurement

- 4.10 The student will
- estimate and measure weight/mass, using actual measuring devices, and describe the results in U.S. Customary/metric units as appropriate, including ounces, pounds, grams, and kilograms;
 - identify equivalent measurements between units within the U.S. Customary system (ounces and pounds) and between units within the metric system (grams and kilograms); and
 - estimate the conversion of ounces and grams and pounds and kilograms,

using approximate comparisons (1 ounce is about 28 grams, or 1 gram is about the weight of a paper clip; 1 kilogram is a little more than 2 pounds). *

** The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U.S. Customary and metric units.*

4.11 The student will

- a) estimate and measure length, using actual measuring devices, and describe the results in both metric and U.S. Customary units, including part of an inch ($\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$), inches, feet, yards, millimeters, centimeters, and meters;
- b) identify equivalent measurements between units within the U.S. Customary system (inches and feet; feet and yards; inches and yards) and between units within the metric system (millimeters and centimeters; centimeters and meters; and millimeters and meters); and
- c) estimate the conversion of inches and centimeters, yards and meters, and miles and kilometers, using approximate comparisons (1 inch is about 2.5 centimeters, 1 meter is a little longer than 1 yard, 1 mile is slightly farther than 1.5 kilometers, or 1 kilometer is slightly farther than half a mile). *

** The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U.S. Customary and metric units.*

4.12 The student will

- a) estimate and measure liquid volume, using actual measuring devices and using metric and U.S. Customary units, including cups, pints, quarts, gallons, milliliters, and liters;
- b) identify equivalent measurements between units within the U.S. Customary system (cups, pints, quarts, and gallons) and between units within the metric system (milliliters and liters); and
- c) estimate the conversion of quarts and liters, using approximate comparisons (1 quart is a little less than 1 liter, 1 liter is a little more than 1 quart).*

** The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U. S. Customary and metric unit*

4.13 The student will

- a) identify and describe situations representing the use of perimeter and area; and
- b) use measuring devices to find perimeter in both standard and nonstandard units of measure.

Geometry

4.14 The student will investigate and describe the relationships between and among points, lines, line segments, and rays.

- 4.15 The student will
- identify and draw representations of points, lines, line segments, rays, and angles, using a straightedge or ruler; and
 - describe the path of shortest distance between two points on a flat surface.
- 4.16 The student will identify and draw representations of lines that illustrate intersection, parallelism, and perpendicularity.
- 4.17 The student will
- analyze and compare the properties of two-dimensional (plane) geometric figures (circle, square, rectangle, triangle, parallelogram, and rhombus) and three-dimensional (solid) geometric figures (sphere, cube, and rectangular solid [prism]);
 - identify congruent and noncongruent shapes; and
 - investigate congruence of plane figures after geometric transformations such as reflection (flip), translation (slide) and rotation (turn), using mirrors, paper folding, and tracing.
- 4.18 The student will identify the ordered pair for a point and locate the point for an ordered pair in the first quadrant of a coordinate plane.

Probability and Statistics

- 4.19 The student will
- predict the likelihood of outcomes of a simple event, using the terms *certain*, *likely*, *unlikely*, *impossible*; and
 - determine the probability of a given simple event, using concrete materials.
- 4.20 The student will collect, organize, and display data in line and bar graphs with scale increments of one or greater than one and use the display to interpret the results, draw conclusions, and make predictions.

Patterns, Functions, and Algebra

- 4.21 The student will recognize, create, and extend numerical and geometric patterns, using concrete materials, number lines, symbols, tables, and words.
- 4.22 The student will recognize and demonstrate the meaning of equality, using symbols representing numbers, operations, and relations [e.g., $3 + 5 = 5 + 3$ and $15 + (35 + 16) = (15 + 35) + 16$].

Grade Five

The fifth-grade standards place emphasis on developing proficiency in using whole numbers, fractions, and decimals to solve problems. Students will collect, display, and analyze data in a variety of ways and solve probability problems, using a sample space or tree diagram. Students also will solve problems involving volume, area, and perimeter. Students will be introduced to variable expressions and open sentences.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

- 5.1 The student will
- read, write, and identify the place values of decimals through thousandths;
 - round decimal numbers to the nearest tenth or hundredth; and
 - compare the values of two decimals through thousandths, using the symbols $>$, $<$, or $=$.
- 5.2 The student will
- recognize and name commonly used fractions (halves, fourths, fifths, eighths, and tenths) in their equivalent decimal form and vice versa; and
 - order a given set of fractions and decimals from least to greatest. Fractions will include like and unlike denominators limited to 12 or less, and mixed numbers.

Computation and Estimation

- 5.3 The student will create and solve problems involving addition, subtraction, multiplication, and division of whole numbers, using paper and pencil, estimation, mental computation, and calculators.

- 5.4 The student will find the sum, difference, and product of two numbers expressed as decimals through thousandths, using an appropriate method of calculation, including paper and pencil, estimation, mental computation, and calculators.
- 5.5 The student, given a dividend of four digits or fewer and a divisor of two digits or fewer, will find the quotient and remainder.
- 5.6 The student, given a dividend expressed as a decimal through thousandths and a single-digit divisor, will find the quotient.
- 5.7 The student will add and subtract with fractions and mixed numbers, with and without regrouping, and express answers in simplest form. Problems will include like and unlike denominators limited to 12 or less.

Measurement

- 5.8 The student will describe and determine the perimeter of a polygon and the area of a square, rectangle, and right triangle, given the appropriate measures.
- 5.9 The student will identify and describe the diameter, radius, chord, and circumference of a circle.
- 5.10 The student will differentiate between perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation.
- 5.11 The student will choose an appropriate measuring device and unit of measure to solve problems involving measurement of
- a) length—part of an inch ($\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$), inches, feet, yards, miles, millimeters, centimeters, meters, and kilometers;
 - b) weight/mass—ounces, pounds, tons, grams, and kilograms;
 - c) liquid volume—cups, pints, quarts, gallons, milliliters, and liters;
 - d) area—square units; and
 - e) temperature—Celsius and Fahrenheit units.
- Problems also will include estimating the conversion of Celsius and Fahrenheit units relative to familiar situations (water freezes at 0°C and 32°F , water boils at 100°C and 212°F , normal body temperature is about 37°C and 98.6°F).
- 5.12 The student will determine an amount of elapsed time in hours and minutes within a 24-hour period.
- 5.13 The student will measure and draw right, acute, and obtuse angles and triangles, using appropriate tools.

Geometry

- 5.14 The student will classify angles and triangles as right, acute, or obtuse.
- 5.15 The student, using two-dimensional (plane) figures (square, rectangle, triangle, parallelogram, rhombus, kite, and trapezoid) will
- recognize, identify, describe, and analyze their properties in order to develop definitions of these figures;
 - identify and explore congruent, noncongruent, and similar figures;
 - investigate and describe the results of combining and subdividing shapes;
 - identify and describe a line of symmetry; and
 - recognize the images of figures resulting from geometric transformations such as translation (slide), reflection (flip), or rotation (turn).
- 5.16 The student will identify, compare, and analyze properties of three-dimensional (solid) geometric shapes (cylinder, cone, cube, square pyramid, and rectangular prism).

Probability and Statistics

- 5.17 The student will
- solve problems involving the probability of a single event by using tree diagrams or by constructing a sample space representing all possible results;
 - predict the probability of outcomes of simple experiments, representing it with fractions or decimals from 0 to 1, and test the prediction; and
 - create a problem statement involving probability and based on information from a given problem situation. Students will not be required to solve the created problem statement.
- 5.18 The student will, given a problem situation, collect, organize, and display a set of numerical data in a variety of forms, using bar graphs, stem-and-leaf plots, and line graphs, to draw conclusions and make predictions.
- 5.19 The student will find the mean, median, mode, and range of a set of data.

Patterns, Functions, and Algebra

- 5.20 The student will analyze the structure of numerical and geometric patterns (how they change or grow) and express the relationship, using words, tables, graphs, or a mathematical sentence. Concrete materials and calculators will be used.
- 5.21 The student will
- investigate and describe the concept of variable;
 - use a variable expression to represent a given verbal quantitative expression involving one operation ; and
 - write an open sentence to represent a given mathematical relationship, using a variable.

5.22 The student will create a problem situation based on a given open sentence using a single variable.

Grade Six

The sixth-grade standards place continued emphasis on the study of whole numbers, decimals, and rational numbers (fractions). Students will use ratios to compare data sets; make conversions within a given measurement system; classify three-dimensional figures; collect, analyze, display, and interpret data, using a variety of graphical and statistical methods; begin using integers and percents; find the probability of an event; and investigate numerical and geometric patterns. Students will be introduced to algebraic terms and solving algebraic equations in one variable.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as fraction calculators, computers, spreadsheets, laser discs, and videos. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students will also identify real-life applications of the mathematical principles they are learning and apply these to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

- 6.1 The student will identify representations of a given percent and describe orally and in writing the equivalence relationships among fractions, decimals, and percents.
- 6.2 The student will describe and compare two sets of data, using ratios, and will use appropriate notations, such as a/b , a to b , and $a:b$.
- 6.3 The student will
 - a) find common multiples and factors, including least common multiple and greatest common factor;
 - b) identify and describe prime and composite numbers; and
 - c) identify and describe the characteristics of even and odd integers.
- 6.4 The student will compare and order whole numbers, fractions, and decimals, using concrete materials, drawings or pictures, and mathematical symbols.

6.5 The student will identify, represent, order, and compare integers.

Computation and Estimation

- 6.6 The student will
- a) solve problems that involve addition, subtraction, multiplication, and/or division with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less, and express their answers in simplest form; and
 - b) find the quotient, given a dividend expressed as a decimal through thousandths and a divisor expressed as a decimal to thousandths with exactly one non-zero digit.
- 6.7 The student will use estimation strategies to solve multistep practical problems involving whole numbers, decimals, and fractions (rational numbers).
- 6.8 The student will solve multistep consumer-application problems involving fractions and decimals and present data and conclusions in paragraphs, tables, or graphs. Planning a budget will be included.

Measurement

- 6.9 The student will compare and convert units of measure for length, area, weight/mass, and volume within the U.S. Customary system and the metric system and estimate conversions between units in each system:
- a) length—part of an inch ($\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$), inches, feet, yards, miles, millimeters, centimeters, meters, and kilometers;
 - b) weight/mass—ounces, pounds, tons, grams, and kilograms;
 - c) liquid volume—cups, pints, quarts, gallons, milliliters, and liters; and
 - d) area—square units. *
- * The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U.S. Customary and metric units.*
- 6.10 The student will estimate and then determine length, weight/mass, area, and liquid volume/capacity, using standard and nonstandard units of measure.
- 6.11 The student will determine if a problem situation involving polygons of four or fewer sides represents the application of perimeter or area and apply the appropriate formula.
- 6.12 The student will
- a) solve problems involving the circumference and/or area of a circle when given the diameter or radius; and
 - b) derive approximations for pi (π) from measurements for circumference and diameter, using concrete materials or computer models.

- 6.13 The student will
- a) estimate angle measures, using 45° , 90° , and 180° as referents, and use the appropriate tools to measure the given angles; and
 - b) measure and draw right, acute, and obtuse angles and triangles.

Geometry

- 6.14 The student will identify, classify, and describe the characteristics of plane figures, describing their similarities, differences, and defining properties.
- 6.15 The student will determine congruence of segments, angles, and polygons by direct comparison, given their attributes. Examples of noncongruent and congruent figures will be included.
- 6.16 The student will construct the perpendicular bisector of a line segment and an angle bisector.
- 6.17 The student will sketch, construct models of, and classify solid figures (rectangular prism, cone, cylinder, and pyramid).

Probability and Statistics

- 6.18 The student, given a problem situation, will collect, analyze, display, and interpret data in a variety of graphical methods, including
- a) line, bar, and circle graphs;
 - b) stem-and-leaf plots; and
 - c) box-and-whisker plots.
- Circle graphs will be limited to halves, fourths, and eighths.
- 6.19 The student will describe the mean, median, and mode as measures of central tendency, describe the range, and determine their meaning for a set of data.
- 6.20 The student will
- a) make a sample space for selected experiments and represent it in the form of a list, chart, picture, or tree diagram; and
 - b) determine and interpret the probability of an event occurring from a given sample space and represent the probability as a ratio, decimal or percent, as appropriate for the given situation.

Patterns, Functions, and Algebra

- 6.21 The student will investigate, describe, and extend numerical and geometric patterns, including triangular numbers, patterns formed by powers of 10, and arithmetic sequences.
- 6.22 The student will investigate and describe concepts of positive exponents, perfect squares, square roots, and, for numbers greater than 10, scientific notation. Calculators will be used to develop exponential patterns.
- 6.23 The student will
- a) model and solve algebraic equations, using concrete materials;
 - b) solve one-step linear equations in one variable, involving whole number coefficients and positive rational solutions; and
 - c) use the following algebraic terms appropriately: *variable*, *coefficient*, *term*, and *equation*.

Grade Seven

The seventh-grade standards place emphasis on solving problems involving consumer applications, using proportional reasoning, and gaining proficiency in computations with integers. The students will gain an understanding of the properties of real numbers, solve one-step linear equations and inequalities, and use data analysis techniques to make inferences, conjectures, and predictions. Two- and three-dimensional representations, graphing transformations in the coordinate plane, and probability will be extended.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as fraction calculators, computers, spreadsheets, laser discs, and videos. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students will also identify real-life applications of the mathematical principles they are learning and apply these to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

- 7.1 The student will compare, order, and determine equivalent relationships between fractions, decimals, and percents, including use of scientific notation for numbers greater than 10.
- 7.2 The student will simplify expressions that contain rational numbers (whole numbers, fractions, and decimals) and positive exponents, using order of operations, mental mathematics, and appropriate tools.
- 7.3 The student will identify and apply the following properties of operations with real numbers:
 - a) the commutative and associative properties for addition and multiplication;
 - b) the distributive property;
 - c) the additive and multiplicative identity properties;
 - d) the additive and multiplicative inverse properties; and
 - e) the multiplicative property of zero.

Computation and Estimation

- 7.4 The student will
- solve practical problems using rational numbers (whole numbers, fractions, decimals) and percents; and
 - solve consumer–application problems involving tips, discounts, sales tax, and simple interest.
- 7.5 The student will formulate rules for and solve practical problems involving basic operations (addition, subtraction, multiplication, and division) with integers.
- 7.6 The student will use proportions to solve practical problems, which may include scale drawings, that contain rational numbers (whole numbers, fractions, and decimals), and percents.

Measurement

- 7.7 The student, given appropriate dimensions, will
- estimate and find the area of polygons by subdividing them into rectangles and right triangles; and
 - apply perimeter and area formulas in practical situations.
- 7.8 The student will investigate and solve problems involving the volume and surface area of rectangular prisms and cylinders, using concrete materials and practical situations to develop formulas.

Geometry

- 7.9 The student will compare and contrast the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid. Deductive reasoning and inference will be used to classify quadrilaterals.
- 7.10 The student will identify and draw the following polygons: pentagon, hexagon, heptagon, octagon, nonagon, and decagon
- 7.11 The student will determine if geometric figures – quadrilaterals or triangles – are similar and write proportions to express the relationships between corresponding parts of similar figures.
- 7.12 The student will identify and graph ordered pairs in the four quadrants of a coordinate plane.
- 7.13 The student, given a polygon in the coordinate plane, will represent transformations – rotation and translation – by graphing the coordinates of the vertices of the transformed polygon and sketching the resulting figure.

Probability and Statistics

- 7.14 The student will investigate and describe the difference between the probability of an event found through simulation versus the theoretical probability of that same event.
- 7.15 The student will identify and describe the number of possible arrangements of several objects, using a tree diagram or the Fundamental (Basic) Counting Principle.
- 7.16 The student will create and solve problems involving the measures of central tendency (mean, median, mode) and the range of a set of data.
- 7.17 The student, given a problem situation, will collect, analyze, display, and interpret data, using a variety of graphical methods, including
- frequency distributions;
 - line plots;
 - histograms;
 - stem-and-leaf plots;
 - box-and-whisker plots; and
 - scattergrams.
- 7.18 The student will make inferences, conjectures, and predictions based on analysis of a set of data.

Patterns, Functions, and Algebra

- 7.19 The student will represent, analyze, and generalize a variety of patterns, including arithmetic sequences and geometric sequences, with tables, graphs, rules, and words in order to investigate and describe functional relationships.
- 7.20 The student will write verbal expressions as algebraic expressions and sentences as equations.
- 7.21 The student will use the following algebraic terms appropriately: *equation*, *inequality*, and *expression*.
- 7.22 The student will
- solve one-step linear equations and inequalities in one variable with strategies involving inverse operations and integers, using concrete materials, pictorial representations, and paper and pencil; and
 - solve practical problems requiring the solution of a one-step linear equation.

Grade Eight

The eighth-grade standards contain both content that reviews or extends concepts and skills learned in previous grades and new content that prepares students for more abstract concepts in algebra and geometry. Students will gain proficiency in computation with rational numbers (positive and negative fractions, positive and negative decimals, whole numbers, and integers) and use proportions to solve a variety of problems. New concepts include solving two-step equations and inequalities, graphing linear equations, visualizing three-dimensional shapes represented in two-dimensional drawings, applying transformations to geometric shapes in the coordinate plane, and using matrices to organize and interpret data. Students will verify and apply the Pythagorean Theorem and represent relations and functions using tables, graphs, and rules.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as fraction calculators, computers, spreadsheets, laser discs, and videos. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students will also identify real-life applications of the mathematical principles they are learning that can be applied to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

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Number and Number Sense

- 8.1 The student will
- a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers;
 - b) recognize, represent, compare, and order numbers expressed in scientific notation; and
 - c) compare and order decimals, fractions, percents, and numbers written in scientific notation.
- 8.2 The student will describe orally and in writing the relationship between the subsets of the real number system.

Computation and Estimation

- 8.3 The student will solve practical problems involving rational numbers, percents, ratios, and proportions. Problems will be of varying complexities and will involve real-life data, such as finding a discount and discount prices and balancing a checkbook.
- 8.4 The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables. Problems will be limited to positive exponents.
- 8.5 The student, given a whole number from 0 to 100, will identify it as a perfect square or find the two consecutive whole numbers between which the square root lies.

Measurement

- 8.6 The student will verify by measuring and describe the relationships among vertical angles, supplementary angles, and complementary angles and will measure and draw angles of less than 360° .
- 8.7 The student will investigate and solve practical problems involving volume and surface area of rectangular solids (prisms), cylinders, cones, and pyramids.

Geometry

- 8.8 The student will apply transformations (rotate or turn, reflect or flip, translate or slide, and dilate or scale) to geometric figures represented on graph paper. The student will identify applications of transformations, such as tiling, fabric design, art, and scaling.
- 8.9 The student will construct a three-dimensional model, given the top, side, and/or bottom views.
- 8.10 The student will
- verify the Pythagorean Theorem, using diagrams, concrete materials, and measurement; and
 - apply the Pythagorean Theorem to find the missing length of a side of a right triangle when given the lengths of the other two sides.

Probability and Statistics

- 8.11 The student will analyze problem situations, including games of chance, board games, or grading scales, and make predictions, using knowledge of probability.
- 8.12 The student will make comparisons, predictions, and inferences, using information displayed in frequency distributions; box-and-whisker plots; scattergrams; line, bar, circle, and picture graphs; and histograms.
- 8.13 The student will use a matrix to organize and describe data.

Patterns, Functions, and Algebra

- 8.14 The student will
- a) describe and represent relations and functions, using tables, graphs, and rules; and
 - b) relate and compare tables, graphs, and rules as different forms of representation for relationships.
- 8.15 The student will solve two-step equations and inequalities in one variable, using concrete materials, pictorial representations, and paper and pencil.
- 8.16 The student will graph a linear equation in two variables, in the coordinate plane, using a table of ordered pairs.
- 8.17 The student will create and solve problems, using proportions, formulas, and functions.
- 8.18 The student will use the following algebraic terms appropriately: *domain*, *range*, *independent variable*, and *dependent variable*.

Algebra I

The standards below outline the content for a one-year course in Algebra I. All students are expected to achieve the Algebra I standards. When planning for instruction, consideration will be given to the sequential development of concepts and skills by using concrete materials to assist students in making the transition from the arithmetic to the symbolic. Students should be helped to make connections and build relationships between algebra and arithmetic, geometry, and probability and statistics. Connections also should be made to other subject areas through practical applications. This approach to teaching algebra should help students attach meaning to the abstract concepts of algebra.

These standards require students to use algebra as a tool for representing and solving a variety of practical problems. Tables and graphs will be used to interpret algebraic expressions, equations, and inequalities and to analyze functions. Matrices will be used to organize and manipulate data.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of functions; they provide a powerful tool for solving and verifying solutions to equations and inequalities.

Throughout the course, students should be encouraged to talk about mathematics, use the language and symbols of mathematics in representations and communication, discuss problems and problem solving, and develop their confidence in mathematics.

- A.1 The student will solve multistep linear equations and inequalities in one variable, solve literal equations (formulas) for a given variable, and apply these skills to solve practical problems. Graphing calculators will be used to confirm algebraic solutions.
- A.2 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables. Students will choose an appropriate computational technique, such as mental mathematics, calculator, or paper and pencil.
- A.3 The student will justify steps used in simplifying expressions and solving equations and inequalities. Justifications will include the use of concrete objects; pictorial representations; and the properties of real numbers, equality, and inequality.
- A.4 The student will use matrices to organize and manipulate data, including matrix addition, subtraction, and scalar multiplication. Data will arise from business, industrial, and consumer situations.
- A.5 The student will create and use tabular, symbolic, graphical, verbal, and physical representations to analyze a given set of data for the existence of a pattern, determine the domain and range of relations, and identify the relations that are functions.

- A.6 The student will select, justify, and apply an appropriate technique to graph linear functions and linear inequalities in two variables. Techniques will include slope-intercept, x - and y -intercepts, graphing by transformation, and the use of the graphing calculator.
- A.7 The student will determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined. The graphing calculator will be used to investigate the effect of changes in the slope on the graph of the line.
- A.8 The student will write an equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.
- A.9 The student will solve systems of two linear equations in two variables both algebraically and graphically and apply these techniques to solve practical problems. Graphing calculators will be used both as a primary tool for solution and to confirm an algebraic solution.
- A.10 The student will apply the laws of exponents to perform operations on expressions with integral exponents, using scientific notation when appropriate.
- A.11 The student will add, subtract, and multiply polynomials and divide polynomials with monomial divisors, using concrete objects, pictorial and area representations, and algebraic manipulations.
- A.12 The student will factor completely first- and second-degree binomials and trinomials in one or two variables. The graphing calculator will be used as a tool for factoring and for confirming algebraic factorizations.
- A.13 The student will express the square root of a whole number in simplest radical form and approximate square roots to the nearest tenth.
- A.14 The student will solve quadratic equations in one variable both algebraically and graphically. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.
- A.15 The student will, given a rule, find the values of a function for elements in its domain and locate the zeros of the function both algebraically and with a graphing calculator. The value of $f(x)$ will be related to the ordinate on the graph.
- A.16 The student will, given a set of data points, write an equation for a line of best fit and use the equation to make predictions.
- A.17 The student will compare and contrast multiple one-variable data sets, using statistical techniques that include measures of central tendency, range, and box-and-whisker graphs.

A.18 The student will analyze a relation to determine whether a direct variation exists and represent it algebraically and graphically, if possible.

Geometry

This course is designed for students who have successfully completed the standards for Algebra I. The course includes, among other things, properties of geometric figures, trigonometric relationships, and reasoning to justify conclusions. Methods of justification will include paragraph proofs, two-column proofs, indirect proofs, coordinate proofs, and verbal arguments. A gradual development of formal proof is encouraged. Inductive and intuitive approaches to proof as well as deductive axiomatic methods should be used.

This set of standards includes emphasis on two- and three-dimensional reasoning skills, coordinate and transformational geometry, and the use of geometric models to solve problems. A variety of applications and some general problem-solving techniques including algebraic skills, should be used to implement these standards. Calculators, computers, graphing utilities (graphing calculators or computer graphing simulators), dynamic geometry software, and other appropriate technology tools will be used to assist in teaching and learning. Any technology that will enhance student learning should be used.

- G.1 The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
- identifying the converse, inverse, and contrapositive of a conditional statement;
 - translating a short verbal argument into symbolic form;
 - using Venn diagrams to represent set relationships; and
 - using deductive reasoning, including the law of syllogism.
- G.2 The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include
- investigating and using formulas for finding distance, midpoint, and slope;
 - investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
 - determining whether a figure has been translated, reflected, or rotated.
- G.3 The student will solve practical problems involving complementary, supplementary, and congruent angles that include vertical angles, angles formed when parallel lines are cut by a transversal, and angles in polygons.
- G.4 The student will use the relationships between angles formed by two lines cut by a transversal to determine if two lines are parallel and verify, using algebraic and coordinate methods as well as deductive proofs.

- G.5 The student will
- a) investigate and identify congruence and similarity relationships between triangles; and
 - b) prove two triangles are congruent or similar, given information in the form of a figure or statement, using algebraic and coordinate as well as deductive proofs.
- G.6 The student, given information concerning the lengths of sides and/or measures of angles, will apply the triangle inequality properties to determine whether a triangle exists and to order sides and angles. These concepts will be considered in the context of practical situations.
- G.7 The student will solve practical problems involving right triangles by using the Pythagorean Theorem, properties of special right triangles, and right triangle trigonometry. Solutions will be expressed in radical form or as decimal approximations.
- G.8 The student will
- a) investigate and identify properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals;
 - b) prove these properties of quadrilaterals, using algebraic and coordinate methods as well as deductive reasoning; and
 - c) use properties of quadrilaterals to solve practical problems.
- G.9 The student will use measures of interior and exterior angles of polygons to solve problems. Tessellations and tiling problems will be used to make connections to art, construction, and nature.
- G.10 The student will investigate and solve practical problems involving circles, using properties of angles, arcs, chords, tangents, and secants. Problems will include finding arc length and the area of a sector, and may be drawn from applications of architecture, art, and construction.
- G.11 The student will construct a line segment congruent to a given line segment, the bisector of a line segment, a perpendicular to a given line from a point not on the line, a perpendicular to a given line at a point on the line, the bisector of a given angle, and an angle congruent to a given angle.
- G.12 The student will make a model of a three-dimensional figure from a two-dimensional drawing and make a two-dimensional representation of a three-dimensional object. Models and representations will include scale drawings, perspective drawings, blueprints, or computer simulations.

- G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve practical problems. Calculators will be used to find decimal approximations for results.
- G.14 The student will
- a) use proportional reasoning to solve practical problems, given similar geometric objects; and
 - b) determine how changes in one dimension of an object affect area and/or volume of the object.

Algebra II

The standards below outline the content for a one-year course in Algebra II. Students enrolled in Algebra II are assumed to have mastered those concepts outlined in the Algebra I standards. A thorough treatment of advanced algebraic concepts is provided through the study of functions, “families of functions,” equations, inequalities, systems of equations and inequalities, polynomials, rational expressions, complex numbers, matrices, and sequences and series. Emphasis will be placed on practical applications and modeling throughout the course of study. Oral and written communication concerning the language of algebra, logic of procedures, and interpretation of results also should permeate the course.

These standards include a transformational approach to graphing functions. Transformational graphing uses translation, reflection, dilation, and rotation to generate a “family of graphs” from a given graph and builds a strong connection between algebraic and graphic representations of functions. Students will vary the coefficients and constants of an equation, observe the changes in the graph of the equation, and make generalizations that can be applied to many graphs.

Graphing utilities (graphing calculators or computer graphing simulators), computers, spreadsheets, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through mathematical modeling and aid in the investigation and study of functions. They also provide an effective tool for solving/verifying equations and inequalities. Any other available technology that will enhance student learning should be used.

- AII.1 The student will identify field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets, complex numbers, and matrices.
- AII.2 The student will add, subtract, multiply, divide, and simplify rational expressions, including complex fractions.
- AII.3 The student will
 - a) add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents; and
 - b) write radical expressions as expressions containing rational exponents and vice versa.
- AII.4 The student will solve absolute value equations and inequalities graphically and algebraically. Graphing calculators will be used as a primary method of solution and to verify algebraic solutions.
- AII.5 The student will identify and factor completely polynomials representing the difference of squares, perfect square trinomials, the sum and difference of cubes, and general trinomials.

- AII.6 The student will select, justify, and apply a technique to solve a quadratic equation over the set of complex numbers. Graphing calculators will be used for solving and for confirming the algebraic solutions.
- AII.7 The student will solve equations containing rational expressions and equations containing radical expressions algebraically and graphically. Graphing calculators will be used for solving and for confirming the algebraic solutions.
- AII.8 The student will recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.
- AII.9 The student will find the domain, range, zeros, and inverse of a function; the value of a function for a given element in its domain; and the composition of multiple functions. Functions will include exponential, logarithmic, and those that have domains and ranges that are limited and/or discontinuous. The graphing calculator will be used as a tool to assist in investigation of functions.
- AII.10 The student will investigate and describe through the use of graphs the relationships between the solution of an equation, zero of a function, x -intercept of a graph, and factors of a polynomial expression.
- AII.11 The student will use matrix multiplication to solve practical problems. Graphing calculators or computer programs with matrix capabilities will be used to find the product.
- AII.12 The student will represent problem situations with a system of linear equations and solve the system, using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.
- AII.13 The student will solve practical problems, using systems of linear inequalities and linear programming, and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.
- AII.14 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.
- AII.15 The student will recognize the general shape of polynomial, exponential, and logarithmic functions. The graphing calculator will be used as a tool to investigate the shape and behavior of these functions.

- AII.16 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including writing the first n terms, finding the n th term, and evaluating summation formulas. Notation will include Σ and a_n .
- AII.17 The student will perform operations on complex numbers and express the results in simplest form. Simplifying results will involve using patterns of the powers of i .
- AII.18 The student will identify conic sections (circle, ellipse, parabola, and hyperbola) from his/her equations. Given the equations in (h, k) form, the student will sketch graphs of conic sections, using transformations.
- AII.19 The student will collect and analyze data to make predictions and solve practical problems. Graphing calculators will be used to investigate scatterplots and to determine the equation for a curve of best fit. Models will include linear, quadratic, exponential, and logarithmic functions.
- AII.20 The student will identify, create, and solve practical problems involving inverse variation and a combination of direct and inverse variations.

Trigonometry

The standards below outline the content for a one-semester course in trigonometry. Students enrolled in trigonometry are assumed to have mastered those concepts outlined in the Algebra II standards. A thorough treatment of trigonometry is provided through the study of trigonometric definitions, applications, graphing, and solving trigonometric equations and inequalities. Emphasis should also be placed on using connections between right triangle ratios, trigonometric functions, and circular functions. In addition, applications and modeling should be included throughout the course of study. Emphasis should also be placed on oral and written communication concerning the language of mathematics, logic of procedure, and interpretation of results.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of trigonometric functions and their inverses. They also provide a powerful tool for solving/verifying trigonometric equations and inequalities.

- T.1 The student will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of an angle in standard position, given a point, other than the origin, on the terminal side of the angle. Circular function definitions will be connected with trigonometric function definitions.
- T.2 The student, given the value of one trigonometric function, will find the values of the other trigonometric functions. Properties of the unit circle and definitions of circular functions will be applied.
- T.3 The student will find without the aid of a calculating utility the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting radians to degrees and vice versa.
- T.4 The student will find with the aid of a calculator the value of any trigonometric function and inverse trigonometric function.
- T.5 The student will verify basic trigonometric identities and make substitutions, using the basic identities.
- T.6 The student, given one of the six trigonometric functions in standard form [e.g., $y = A \sin (Bx + C) + D$, where A , B , C , and D are real numbers], will
- state the domain and the range of the function;
 - determine the amplitude, period, phase shift, and vertical shift; and
 - sketch the graph of the function by using transformations for at least a one-period interval.
- The graphing calculator will be used to investigate the effect of changing A , B , C , and D on the graph of a trigonometric functions.

- T.7 The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.
- T.8 The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities. Graphing utilities will be used to solve equations, check for reasonableness of results, and verify algebraic solutions.
- T.9 The student will identify, create, and solve practical problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Algebra II and Trigonometry

The standards for this combined course in Algebra II and Trigonometry include all of the standards listed for Algebra II and Trigonometry. This course is designed for advanced students who are capable of a more rigorous course at an accelerated pace. The standards listed for this course provide the foundation for students to pursue a sequence of advanced mathematical studies from Mathematical Analysis to Advanced Placement Calculus.

- AII/T.1 The student will identify field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets, complex numbers, and matrices.
- AII/T.2 The student will add, subtract, multiply, divide, and simplify rational expressions, including complex fractions.
- AII/T.3 The student will
- a) add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents; and
 - b) write radical expressions as expressions containing rational exponents and vice versa.
- AII/T.4 The student will solve absolute value equations and inequalities graphically and algebraically. Graphing calculators will be used as a primary method of solution and to verify algebraic solutions.
- AII/T.5 The student will identify and factor completely polynomials representing the difference of squares, perfect square trinomials, the sum and difference of cubes, and general trinomials.
- AII/T.6 The student will select, justify, and apply a technique to solve a quadratic equation over the set of complex numbers. Graphing calculators will be used for solving and for confirming the algebraic solutions.
- AII/T.7 The student will solve equations containing rational expressions and equations containing radical expressions algebraically and graphically. Graphing calculators will be used for solving and for confirming the algebraic solutions.
- AII/T.8 The student will recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.

- AII/T.9 The student will find the domain, range, zeros, and inverse of a function; the value of a function for a given element in its domain; and the composition of multiple functions. Functions will include exponential, logarithmic, and those that have domains and ranges that are limited and/or discontinuous. The graphing calculator will be used as a tool to assist in investigation of functions.
- AII/T.10 The student will investigate and describe through the use of graphs the relationships between the solution of an equation, zero of a function, x -intercept of a graph, and factors of a polynomial expression.
- AII/T.11 The student will use matrix multiplication to solve practical problems. Graphing calculators or computer programs with matrix capabilities will be used to find the product.
- AII/T.12 The student will represent problem situations with a system of linear equations and solve the system, using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.
- AII/T.13 The student will solve practical problems, using systems of linear inequalities and linear programming, and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.
- AII/T.14 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.
- AII/T.15 The student will recognize the general shape of polynomial, exponential, and logarithmic functions. The graphing calculator will be used as a tool to investigate the shape and behavior of these functions.
- AII/T.16 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including writing the first n terms, finding the n th term, and evaluating summation formulas. Notation will include Σ and a_n .
- AII/T.17 The student will perform operations on complex numbers and express the results in simplest form. Simplifying results will involve using patterns of the powers of i .
- AII/T.18 The student will identify conic sections (circle, ellipse, parabola, and hyperbola) from his/her equations. Given the equations in (h, k) form, the student will sketch graphs of conic sections, using transformations.

- AII/T.19 The student will collect and analyze data to make predictions and solve practical problems. Graphing calculators will be used to investigate scatterplots and to determine the equation for a curve of best fit. Models will include linear, quadratic, exponential, and logarithmic functions.
- AII/T.20 The student will identify, create, and solve practical problems involving inverse variation and a combination of direct and inverse variations.
- AII/T.21 The student will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of an angle in standard position, given a point, other than the origin, on the terminal side of the angle. Circular function definitions will be connected with trigonometric function definitions.
- AII/T.22 The student, given the value of one trigonometric function, will find the values of the other trigonometric functions. Properties of the unit circle and definitions of circular functions will be applied.
- AII/T.23 The student will find without the aid of a calculating utility the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting radians to degrees and vice versa.
- AII/T.24 The student will find with the aid of a calculator the value of any trigonometric function and inverse trigonometric function.
- AII/T.25 The student will verify basic trigonometric identities and make substitutions, using the basic identities.
- AII/T.26 The student, given one of the six trigonometric functions in standard form [e.g., $y = A \sin(Bx + C) + D$, where A , B , C , and D are real numbers], will
- state the domain and the range of the function;
 - determine the amplitude, period, phase shift, and vertical shift; and
 - sketch the graph of the function by using transformations for at least a one-period interval.
- The graphing calculator will be used to investigate the effect of changing A , B , C , and D on the graph of a trigonometric functions.
- AII/T.27 The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.
- AII/T.28 The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities. Graphing utilities will be used to solve equations, check for reasonableness of results, and verify algebraic solutions.

AII/T.29

The student will identify, create, and solve practical problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Computer Mathematics

This course is intended to provide students with experiences in using computer programming techniques and skills to solve problems that can be set up as mathematical models. Students enrolled in Computer Mathematics are assumed to have studied the concepts and skills in Algebra I and beginning geometry. Students who successfully complete the standards for this course may earn credit toward meeting the mathematics graduation requirement. It is recognized that many students will gain computer skills in other mathematics courses or in a separate curriculum outside of mathematics and prior to high school. In such cases, the standards indicated by an asterisk (*) should be included in the student's course of study and treated as a review.

Even though computer ideas should be introduced in the context of mathematical concepts, problem solving per se should be developed in the most general sense, making the techniques applicable by students in many other environments. Strategies include defining the problem; developing, refining, and implementing a plan; and testing and revising the solution. Programming, ranging from simple programs involving only a few lines to complex programs involving subprograms, should permeate the entire course and may include programming a graphing calculator or scripting a problem solution in a database or spreadsheet. Programming concepts, problem-solving strategies, and mathematical applications should be integrated throughout the course.

These standards identify fundamental principles and concepts in the field of computer science that will be used within the context of mathematical problem solving in a variety of applications. As students develop and refine skills in logic, organization, and precise expression, they will apply those skills to enhance learning in all disciplines.

- COM.1 The student will apply programming techniques and skills to solve practical problems in mathematics arising from consumer, business, other applications in mathematics. Problems will include opportunities for students to analyze data in charts, graphs, and tables and to use their knowledge of equations, formulas, and functions to solve these problems.

- *COM.2 The student will design, write, test, debug, and document a program. Programming documentation will include pre-conditions and post-conditions of program segments, input/output specifications, the step-by-step plan, the test data, a sample run, and the program listing with appropriately placed comments.

- *COM.3 The student will write program specifications that define the constraints of a given problem. These specifications will include descriptions of pre-conditions, post-conditions, the desired output, analysis of the available input, and an indication as to whether or not the problem is solvable under the given conditions.

- *COM.4 The student will design a step-by-step plan (algorithm) to solve a given problem. The plan will be in the form of a program flowchart, pseudo code, hierarchy chart, and/or data-flow diagram.
- *COM.5 The student will divide a given problem into manageable sections (modules) by task and implement the solution. The modules will include an appropriate user-defined function, subroutines, and procedures. Enrichment topics might include user-defined libraries (units) and object-oriented programming.
- *COM.6 The student will design and implement the input phase of a program, which will include designing screen layout and getting information into the program by way of user interaction, data statements, and/or file input. The input phase also will include methods of filtering out invalid data (error trapping).
- *COM.7 The student will design and implement the output phase of a computer program, which will include designing output layout, accessing a variety of output devices, using output statements, and labeling results.
- COM.8 The student will design and implement computer graphics, which will include topics appropriate for the available programming environment as well as student background. Students will use graphics as an end in itself, as an enhancement to other output, and as a vehicle for reinforcing programming techniques.
- COM.9 The student will define simple variable data types that include integer, real (fixed and scientific notation), character, string, and Boolean. The student will use appropriate variable data types, including integer, real (fixed and scientific notation), character, string, and Boolean. This will also include variables representing structured data types.
- COM.10 The student will describe the way the computer stores, accesses, and processes variables, including the following topics: the use of variables versus constants, variables addresses, pointers, parameter passing, scope of variables, and local versus global variables.
- *COM.11 The student will translate a mathematical expression into a computer statement, which involves writing assignment statements and using the order of operations.
- COM.12 The student will select and implement built-in (library) functions in processing data.
- COM.13 The student will implement conditional statements that include "if/then" statements, "if/then/else" statements, case statements, and Boolean logic.
- COM.14 The student will implement loops, including iterative loops. Other topics will include single entry point, single exit point, pre-conditions, and post-conditions.

- COM.15 The student will select and implement appropriate data structures, including arrays (one-dimensional and/or multidimensional), files, and records. Implementation will include creating the data structure, putting information into the structure, and retrieving information from the structure.
- *COM.16 The student will implement pre-existing algorithms, including sort routines, search routines, and simple animation routines.
- *COM.17 The student will test a program, using an appropriate set of data. The set of test data should be appropriate and complete for the type of program being tested.
- COM.18 The student will debug a program, using appropriate techniques (e.g., appropriately placed controlled breaks, the printing of intermediate results, and other debugging tools available in the programming environment), and identify the difference between syntax errors and logic errors.
- COM.19 The student will design, write, test, debug, and document a complete structured program that requires the synthesis of many of the concepts contained in previous standards.
- COM.20 The student will design, write, test, debug, and document a complete structured program that requires the synthesis of many of the concepts contained in previous standards.

Probability and Statistics

The following standards outline the content of a one-year course in Probability and Statistics. If a one-semester course is desired, the standards with an asterisk (*) would apply. Students enrolled in this course are assumed to have mastered the concepts identified in the Standards of Learning for Algebra II. The purpose of the course is to present basic concepts and techniques for collecting and analyzing data, drawing conclusions, and making predictions.

A graphing calculator is essential for every student taking the Probability and Statistics course and is required for the Advanced Placement Statistics Examination. The calculator may not fully substitute for a computer, however. In the absence of a computer for student use, teachers may provide students with examples of computer output generated by a statistical software package.

*PS.1 The student will analyze graphical displays of data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

*PS.2 The student will analyze numerical characteristics of univariate data sets to describe patterns and departure from patterns, using mean, median, mode, variance, standard deviation, interquartile range, range, and outliers. Appropriate technology will be used to calculate statistics.

*PS.3 The student will compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features. Appropriate technology will be used to generate graphical displays.

*PS.4 The student will analyze scatterplots to identify and describe the relationship between two variables, using shape; strength of relationship; clusters; positive, negative, or no association; outliers; and influential points. Appropriate technology will be used to generate scatterplots and identify outliers and influential points.

PS.5 The student will find and interpret linear correlation, use the method of least squares regression to model the linear relationship between two variables, and use the residual plots to assess linearity. Appropriate technology will be used to compute correlation coefficients and residual plots.

PS.6 The student will make logarithmic and power transformations to achieve linearity. Appropriate technology will be used.

- PS.7 The student, using two-way tables, will analyze categorical data to describe patterns and departure from patterns and to find marginal frequency and relative frequencies, including conditional frequencies.
- *PS.8 The student will describe the methods of data collection in a census, sample survey, experiment, and observational study and identify an appropriate method of solution for a given problem setting.
- *PS.9 The student will plan and conduct a survey. The plan will address sampling techniques (e.g., simple random and stratified) and methods to reduce bias.
- PS.10 The student will plan and conduct an experiment. The plan will address control, randomization, and measurement of experimental error.
- *PS.11 The student will compute and distinguish between permutations and combinations and use technology for applications.
- *PS.12 The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive.
- *PS.13 The student will find probabilities (relative frequency and theoretical), including conditional probabilities for events that are either dependent or independent, by applying the “law of large numbers” concept, the addition rule, and the multiplication rule.
- *PS.14 The student will develop, interpret, and apply the binomial probability distribution for discrete random variables, including computing the mean and standard deviation for the binomial variable.
- PS.15 The student will simulate probability distributions, including binomial and geometric.
- PS.16 The student will identify random variables as independent or dependent and find the mean and standard deviations for sums and differences of independent random variables.
- *PS.17 The student will identify properties of a normal distribution and apply the normal distribution to determine probabilities, using a table or graphing calculator.
- *PS.18 The student, given data from a large sample, will find and interpret point estimates and confidence intervals for parameters. The parameters will include proportion and mean, difference between two proportions, and difference between two means (independent and paired).

- PS.19 The student will apply and interpret the logic of a hypothesis-testing procedure. Tests will include large sample test for proportion, mean, difference between two proportions, and difference between two means (independent and paired) and Chi-squared test for goodness of fit, homogeneity of proportions, and independence.
- PS.20 The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means.
- PS.21 The student will identify properties of a t-distribution and apply t-distributions to single-sample and two-sample (independent and matched pairs) t-procedures, using tables or graphing calculators.

Discrete Mathematics

The following standards outline the content of a one-year course in Discrete Mathematics. If a one-semester course is desired, the standards with an asterisk (*) would apply. Students enrolled in Discrete Mathematics are assumed to have mastered the concepts outlined in the Standards of Learning for Algebra II.

Discrete mathematics may be described as the study of mathematical properties of sets and systems that have a countable (discrete) number of elements. With the advent of modern technology, discrete (discontinuous) models have become as important as continuous models. In this course, the main focus is problem solving in a discrete setting. Techniques that are not considered in the current traditional courses of algebra, geometry, and calculus will be utilized. As students solve problems, they will analyze and determine whether or not a solution exists (existence problems), investigate how many solutions exist (counting problems), and focus on finding the best solution (optimization problems). Connections will be made to other disciplines. The importance of discrete mathematics has been influenced by computers. Modern technology (graphing calculators and/or computers) will be an integral component of this course.

- *DM.1 The student will model problems, using vertex-edge graphs. The concepts of valence, connectedness, paths, planarity, and directed graphs will be investigated. Adjacency matrices and matrix operations will be used to solve problems (e.g., food chains, number of paths).
- *DM.2 The student will solve problems through investigation and application of circuits, cycles, Euler Paths, Euler Circuits, Hamilton Paths, and Hamilton Circuits. Optimal solutions will be sought using existing algorithms and student-created algorithms.
- *DM.3 The student will apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization. Graph coloring and chromatic number will be used.
- *DM.4 The student will apply algorithms, such as Kruskal's, Prim's, or Dijkstra's, relating to trees, networks, and paths. Appropriate technology will be used to determine the number of possible solutions and generate solutions when a feasible number exists.
- *DM.5 The student will use algorithms to schedule tasks in order to determine a minimum project time. The algorithms will include critical path analysis, the list-processing algorithm, and student-created algorithms.
- *DM.6 The student will solve linear programming problems. Appropriate technology will be used to facilitate the use of matrices, graphing techniques, and the Simplex method of determining solutions.

*DM.7 The student will analyze and describe the issue of fair division (e.g., cake cutting, estate division). Algorithms for continuous and discrete cases will be applied.

DM.8 The student will investigate and describe weighted voting and the results of various election methods. These may include approval and preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners.

DM.9 The student will identify apportionment inconsistencies that apply to issues such as salary caps in sports and allocation of representatives to Congress. Historical and current methods will be compared.

DM.10 The student will use the recursive process and difference equations with the aid of appropriate technology to generate

- a) compound interest;
- b) sequences and series;
- c) fractals;
- d) population growth models; and
- e) the Fibonacci sequence.

DM.11 The student will describe and apply sorting algorithms and coding algorithms used in storing, processing, and communicating information. These will include

- a) bubble sort, merge sort, and network sort; and
- b) ISBN, UPC, Zip, and banking codes.

DM.12 The student will select, justify, and apply an appropriate technique to solve a logic problem. Techniques will include Venn diagrams, truth tables, and matrices.

DM.13 The student will apply the formulas of combinatorics in the areas of

- a) the Fundamental (Basic) Counting Principle;
- b) knapsack and bin-packing problems;
- c) permutations and combinations; and
- d) the pigeonhole principle.

Mathematical Analysis

The standards below outline the content for a one-year course in Mathematical Analysis. Students enrolled in Mathematical Analysis are assumed to have mastered Algebra II concepts and have some exposure to trigonometry. Mathematical Analysis develops students' understanding of algebraic and transcendental functions, parametric and polar equations, sequences and series, and vectors. The content of this course serves as appropriate preparation for a calculus course.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of functions and their inverses. They also provide a powerful tool for solving and verifying equations and inequalities.

- MA.1 The student will investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. This will include determining zeros, upper and lower bounds, y-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics.
- MA.2 The student will find compositions of functions and inverses of functions. Analytical methods and graphing utilities will be used to investigate and verify the domain and range of resulting functions.
- MA.3 The student will investigate and describe the continuity of functions, using graphs. The functions will include absolute value, piecewise, and step functions.
- MA.4 The student will expand binomials having positive integral exponents through the use of the Binomial Theorem, the formula for combinations, and Pascal's Triangle.
- MA.5 The student will solve practical problems involving arithmetic and geometric sequences and series. This will include finding the sum (sigma notation included) of finite and infinite convergent series that will lead to an intuitive approach to a limit.
- MA.6 The student will use mathematical induction to prove formulas/statements.
- MA.7 The student will find the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution.
- MA.8 The student will investigate and identify the characteristics of conic section equations in (h, k) and standard forms. The techniques of translation and rotation of axes in the coordinate plane will be used to graph conic sections.

- MA.9 The student will investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and practical problems. This will include the role of e , natural and common logarithms, laws of exponents and logarithms, and the solution of logarithmic and exponential equations. Graphing utilities will be used to investigate and verify the graphs and solutions.
- MA.10 The student will investigate and identify the characteristics of the graphs of polar equations, using graphing utilities. This will include classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.
- MA.11 The student will perform operations with vectors in the coordinate plane and solve practical problems using vectors. This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.
- MA.12 The student will use parametric equations to model and solve application problems. Graphing utilities will be used to develop an understanding of the graph of parametric equations.
- MA.13 The student will identify, create, and solve practical problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Advanced Placement Calculus

This course is intended for students who have a thorough knowledge of analytic geometry and elementary functions in addition to college preparatory algebra, geometry, and trigonometry. The purpose of the course is to prepare students for advanced placement in college calculus. These standards incorporate The College Board Advanced Placement Course Description Syllabus. Teachers should update course content as changes occur in future College Board publications.

As mandated by The College Board, graphing calculators are required for this course. Computers should be used when feasible by students and by teachers. Any other available technology tools that will enhance student learning should be used. Instructional activities that engage students in solving application problems of varying complexities are encouraged.

- APC.1 The student will define and apply the properties of elementary functions, including algebraic, trigonometric, exponential, and composite functions and their inverses, and graph these functions, using a graphing calculator. Properties of functions will include domains, ranges, combinations, odd, even, periodicity, symmetry, asymptotes, zeros, upper and lower bounds, and intervals where the function is increasing or decreasing.
- APC.2 The student will define and apply the properties of limits of functions. Limits will be evaluated graphically and algebraically. This will include
- limits of a constant;
 - limits of a sum, product, and quotient;
 - one-sided limits; and
 - limits at infinity, infinite limits, and non-existent limits. *
- *AP Calculus BC will include l'Hopital's Rule, which will be used to find the limit of functions whose limits yield the indeterminate forms: $0/0$ and ∞/∞ .
- APC.3 The student will use limits to define continuity and determine where a function is continuous or discontinuous. This will include
- continuity in terms of limits;
 - continuity at a point and over a closed interval;
 - application of the Intermediate Value Theorem and the Extreme Value Theorem; and
 - geometric understanding and interpretation of continuity and discontinuity.
- APC.4 The student will investigate asymptotic and unbounded behavior in functions. This will include
- describing and understanding asymptotes in terms of graphical behavior and limits involving infinity; and
 - comparing relative magnitudes of functions and their rates of change.

- APC.5 The student will investigate derivatives presented in graphic, numerical, and analytic contexts and the relationship between continuity and differentiability. The derivative will be defined as the limit of the difference quotient and interpreted as an instantaneous rate of change.
- APC.6 The student will investigate the derivative at a point on a curve. This will include
- finding the slope of a curve at a point, including points at which the tangent is vertical and points at which there are no tangents;
 - using local linear approximation to find the slope of a tangent line to a curve at the point;
 - defining instantaneous rate of change as the limit of average rate of change; and
 - approximating rate of change from graphs and tables of values.
- APC.7 The student will analyze the derivative of a function as a function in itself. This will include
- comparing corresponding characteristics of the graphs of f , f' , and f'' ;
 - defining the relationship between the increasing and decreasing behavior of f and the sign of f' ;
 - translating verbal descriptions into equations involving derivatives and vice versa;
 - analyzing the geometric consequences of the Mean Value Theorem;
 - defining the relationship between the concavity of f and the sign of f'' ; and
 - identifying points of inflection as places where concavity changes and finding points of inflection.
- APC.8 The student will apply the derivative to solve problems. This will include
- analysis of curves and the ideas of concavity and monotonicity;
 - optimization involving global and local extrema;
 - modeling of rates of change and related rates;
 - use of implicit differentiation to find the derivative of an inverse function;
 - interpretation of the derivative as a rate of change in applied contexts, including velocity, speed, and acceleration; and
 - differentiation of nonlogarithmic functions, using the technique of logarithmic differentiation. *
- * AP Calculus BC will also apply the derivative to solve problems. This will include
- analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration vectors;
 - numerical solution of differential equations, using Euler's method;
 - l'Hopital's Rule to test the convergence of improper integrals and series; and

- d) geometric interpretation of differential equations via slope fields and the relationship between slope fields and the solution curves for the differential equations.

APC.9 The student will apply formulas to find derivatives. This will include

- derivatives of algebraic, trigonometric, exponential, logarithmic, and inverse trigonometric functions;
- derivations of sums, products, quotients, inverses, and composites (chain rule) of elementary functions;
- derivatives of implicitly defined functions; and
- higher order derivatives of algebraic, trigonometric, exponential, and logarithmic, functions. *

* AP Calculus BC will also include finding derivatives of parametric, polar, and vector functions.

APC.10 The student will use Riemann sums and the Trapezoidal Rule to approximate definite integrals of functions represented algebraically, graphically, and by a table of values and will interpret the definite integral as the accumulated rate of change of a quantity over an interval interpreted as the change of the quantity over the interval

$$\int_a^b f'(x) dx = f(b) - f(a).$$

Riemann sums will use left, right, and midpoint evaluation points over equal subdivisions.

APC.11 The student will find antiderivatives directly from derivatives of basic functions and by substitution of variables (including change of limits for definite integrals). *

* AP Calculus BC will also include finding antiderivatives by parts and simple partial fractions (nonrepeating linear factors only), and finding improper integrals as limits of definite integrals.

* AP Calculus BC will also solve logistic differential equations and use them in modeling.

APC.12 The student will identify the properties of the definite integral. This will include additivity and linearity, the definite integral as an area, and the definite integral as a limit of a sum as well as the fundamental theorem:

$$\frac{d}{dx} \int_a^x f(t) d(t) = f(x).$$

APC.13 The student will use the Fundamental Theorem of Calculus to evaluate definite integrals, represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

- APC.14 The student will find specific antiderivatives, using initial conditions (including applications to motion along a line). Separable differential equations will be solved and used in modeling (in particular, the equation $y'=ky$ and exponential growth).
- APC.15 The student will use integration techniques and appropriate integrals to model physical, biological, and economic situations. The emphasis will be on using the integral of a rate of change to give accumulated change or on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. Specific applications will include
- the area of a region;
 - the volume of a solid with known cross-section;
 - the average value of a function; and
 - the distance traveled by a particle along a line. *
- * AP Calculus BC will include finding the area of a region (including a region bounded by polar curves) and finding the length of a curve (including a curve given in parametric form).
- APC.16 The student will define a series and test for convergence of a series in terms of the limit of the sequence of partial sums. This will include
- geometric series with applications;
 - harmonic series;
 - alternating series with error bound;
 - terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series; and
 - ratio test for convergence and divergence. *
- * For those students who are enrolled in AP Calculus BC.
- APC.17 The student will define, restate, and apply Taylor series. This will include
- Taylor polynomial approximations with graphical demonstration of convergence;
 - Maclaurin series and the general Taylor series centered at $x = a$;
 - Maclaurin series for the functions e^x , $\sin x$, $\cos x$, and $1/(1 - x)$;
 - formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series;
 - functions defined by power series;
 - radius and interval of convergence of power series; and
 - Lagrange error bound of a Taylor polynomial. *
- * For those students who are enrolled in AP Calculus BC.