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# Mathematics Standards of Learning

for  
Virginia  
Public Schools



Board of Education  
Commonwealth of Virginia

June 1995

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Commonwealth of Virginia  
Board of Education  
Post Office Box 2120  
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# Mathematics Standards of Learning

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The Standards of Learning for Mathematics identify academic content for essential components of the mathematics curriculum at different grade levels for Virginia's public schools. Standards are identified for kindergarten through grade eight and for a core set of high school courses. Throughout a student's mathematics schooling from kindergarten through grade eight, specific content strands or topics are included. These content strands are Number and Number Sense; Computation and Estimation; Measurement; Geometry; Probability and Statistics; and Patterns, Functions, and Algebra. The Standards of Learning for each strand progress in complexity at each grade level and throughout the high school courses.

The Standards of Learning are not intended to encompass the entire curriculum for a given grade level or course or to prescribe how the content should be taught. Teachers are encouraged to go beyond the standards and to select instructional strategies and assessment methods appropriate for their students.

## Goals

Students today require stronger mathematical knowledge and skills to pursue higher education, to compete in a technologically oriented workforce, and to be informed citizens. Students must gain an understanding of fundamental ideas in arithmetic, measurement, geometry, probability, data analysis and statistics, and algebra and functions, and develop proficiency in mathematical skills. In addition, students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving in science, engineering, business and industry, government, and practical affairs. Hence, the use of technology must be an integral part of teaching and learning. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Please note the computer/technology standards following the grade five and grade eight standards, respectively. The teaching of these skills should be the shared responsibility of teachers of all disciplines.

The content of the mathematics standards is intended to support the following four goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, and making mathematical connections.

## Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-life data and situations within and outside mathematics and then apply appropriate strategies to find an acceptable solution. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problem types. A major goal of the mathematics program is to help students become competent mathematical problem solvers.

## Mathematical Communication

Students will use the language of mathematics, including specialized vocabulary and symbols, to represent and describe mathematical ideas, generalizations, and relationships. Representing, discussing, reading, writing, and listening to mathematics will help students to clarify their thinking and deepen their understanding of the mathematics being studied.

## Mathematical Reasoning

Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will learn to apply proportional and spatial reasoning and to reason from graphs.

## Mathematical Connections

Students will relate concepts and procedures from different topics in mathematics to one another, using a variety of representations—graphical, numerical, algebraic, verbal, and physical. Through the application of

content, students will make connections between different areas of mathematics and between mathematics and other disciplines, especially science. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that reinforce each other.

# Kindergarten

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The kindergarten standards place emphasis on counting; combining, sorting, and comparing sets of objects; recognizing and describing simple patterns; and recognizing shapes and sizes of figures and objects. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

## Number and Number Sense

- K.1 The student, given two sets containing 10 or fewer concrete items, will identify and describe one set as having more, fewer, or the same number of members as the other set, using the concept of 1 to 1 correspondence.
- K.2 The student, given a set containing nine or fewer concrete items, will
- tell how many are in the set by counting the number of items orally;
  - select the corresponding numeral from a given set; and
  - trace over the numeral using tactile materials (e.g., sand, sandpaper, carpeting, or finger paint).
- K.3 The student, given an ordered set of three objects and/or pictures, will indicate the ordered position of each item, from left-to-right, right-to-left, top-to-bottom, and/or bottom-to-top.
- K.4 The student will investigate and recognize patterns from counting by fives and tens, using concrete objects and a calculator.
- K.5 The student will count forward to 20 and backward from 10.
- K.6 The student will determine the value of a collection of pennies, using pennies or models.

## Computation and Estimation

- K.7 The student will add and subtract whole numbers using up to 10 concrete items.
- K.8 The student, given a familiar problem situation involving magnitude, will
- select a reasonable magnitude from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, and 500); and
  - explain the reasonableness of his/her choice.

## Measurement

- K.9 The student will recognize a penny, nickel, dime, and quarter.
- K.10 The student will identify the instruments used to measure length (ruler), weight (scale), time (clock: digital and analog; calendar: day, month, and season), and temperature (thermometer).
- K.11 The student will tell time to the hour using an analog or digital clock.
- K.12 The student will compare two objects or events, using direct comparisons or nonstandard units of measure, according to one or more of the following attributes: length (shorter, longer), height (taller, shorter), weight

(heavier, lighter), temperature (hotter, colder). Examples of nonstandard units include foot length, hand span, new pencil, paper clip, block, etc.

### Geometry

- K.13 The student will identify, describe, and make plane geometric figures (circle, triangle, square, and rectangle).
- K.14 The student will identify representations of plane geometric figures (circle, triangle, square, and rectangle), regardless of their position and orientation in space.
- K.15 The student will compare the size (larger/smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle).

### Probability and Statistics

- K.16 The student will gather data relating to familiar experiences by counting and tallying.
- K.17 The student will display objects and information, using object and pictorial graphs and tables.
- K.18 The student will investigate and describe the results of dropping a two-colored counter or using a multicolored spinner.

### Patterns, Functions, and Algebra

- K.19 The student will sort and classify objects according to similar attributes (size, shape, and color).
- K.20 The student will identify, describe, and extend a repeating relationship (pattern) found in common objects, sounds, and movements.

## Grade One

The first-grade standards introduce the idea of fractions and continue the development of sorting and patterning skills. In first grade, students will learn the basic addition facts through the fives table and the corresponding subtraction facts. Students also will draw and describe certain two-dimensional figures and use nonstandard units to measure length and weight. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

### Number and Number Sense

- 1.1 The student will count objects in a given set containing between 10 and 100 objects and write the corresponding numeral.
- 1.2 The student will group concrete objects by ones and tens to develop an understanding of place value.
- 1.3 The student will count by twos, fives, and tens to 100.
- 1.4 The student will recognize and write numerals 0 through 100.
- 1.5 The student will identify the ordinal positions first through tenth, using an ordered set of objects.
- 1.6 The student will identify and represent the concepts of one-half and one-fourth, using appropriate materials or a drawing.

- 1.7 The student will count a collection of pennies, a collection of nickels, and a collection of dimes whose total value is 100 cents or less.

### Computation and Estimation

- 1.8 The student will recall basic addition facts, sums to 10 or less, and the corresponding subtraction facts.
- 1.9 The student will solve story and picture problems involving one-step solutions, using basic addition and subtraction facts.

### Measurement

- 1.10 The student will identify the number of pennies equivalent to a nickel, a dime, and a quarter.
- 1.11 The student will tell time to the half-hour, using an analog or digital clock.
- 1.12 The student will use nonstandard units to measure length and weight.
- 1.13 The student will compare the volumes of two given containers by using concrete materials (e.g., jelly beans, sand, water, and rice).
- 1.14 The student will compare the weight of two objects using a balance scale.

### Geometry

- 1.15 The student will describe the proximity of objects in space (near, far, close by, below, up, down, beside, and next to).
- 1.16 The student will draw and describe triangles, squares, rectangles, and circles according to number of sides, corners, and square corners.

- 1.17 The student will identify and describe objects in his/her environment that depict geometric figures: triangle, rectangle, square, and circle.

### Probability and Statistics

- 1.18 The student will investigate, identify, and describe various forms of data collection in his/her world (e.g., recording daily temperature, lunch count, attendance, and favorite ice cream).
- 1.19 The student will interpret information displayed in a picture or object graph using the vocabulary: more, less, fewer, greater than, and less than.

### Patterns, Functions, and Algebra

- 1.20 The student will sort and classify concrete objects according to one or more attributes, including color, size, shape, and thickness.
- 1.21 The student will recognize, describe, extend, and create a wide variety of patterns, including rhythmic, color, shape, and numeric. Patterns will include both growing and repeating patterns. Concrete materials and calculators will be used by students.

## Grade Two

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The second-grade standards extend the study of number and spatial sense to include three-digit numbers and three-dimensional figures. Students will continue to learn and use the basic addition facts through the nines table and the corresponding subtraction facts. Students will also begin to estimate and make measurements. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

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### Number and Number Sense

- 2.1 The student will identify the place value of each digit in a three-digit numeral, using numeration models.
- 2.2 The student will compare two whole numbers between 0 and 999, using symbols ( $>$ ,  $<$ , or  $=$ ) and words ("greater than," "less than," or "equal to").
- 2.3 The student will identify the positions first through twentieth, using an ordered set of objects.
- 2.4 The student will identify the part of a set and/or region that represents one-half, one-third, one-fourth, one-eighth, and one-tenth and write the corresponding fraction.

- 2.5 The student will count by twos and fives to 100 and by threes and fours to 96, using mental mathematics, paper and pencil, hundred chart, calculators, and/or concrete objects.

### Computation and Estimation

- 2.6 The student will recall basic addition facts, sums to 18 or less, and the corresponding subtraction facts.
- 2.7 The student, given two whole numbers whose sum is 99 or less, will
- estimate the sum; and
  - find the sum using various methods of calculation (mental computation, concrete materials, and paper and pencil).
- 2.8 The student, given two whole numbers each 99 or less, will
- estimate the difference; and
  - find the difference using various methods of calculation (mental computation, concrete materials, and paper and pencil).
- 2.9 The student will solve addition and subtraction problems using data from simple charts and picture graphs. Problems will require a one-step solution.
- 2.10 The student, given a simple addition or subtraction fact, will recognize and describe the related facts which represent and describe the inverse relationship between addition and subtraction (e.g.,  $3 + \underline{\quad} = 7$ ,  $\underline{\quad} + 3 = 7$ ,  $7 - 3 = \underline{\quad}$ , and  $7 - \underline{\quad} = 3$ ).
- 2.11 The student will
- count, compare, and make change, using a collection of coins and one-dollar bills; and
  - identify the correct usage of the cent symbol ( $\text{¢}$ ), dollar symbol ( $\text{\$}$ ), and decimal point ( $\text{\.}$ ).

### Measurement

- 2.12 The student will estimate and then use a ruler to make linear measurements to the nearest centimeter and inch, including the distance around a polygon (determine perimeter).
- 2.13 The student, given grid paper, will estimate and then count the number of square units needed to cover a given surface (determine area).
- 2.14 The student will estimate and then count the number of cubes in a rectangular box (determine volume).

- 2.15 The student will estimate and then determine weight/mass of familiar objects in pounds and/or kilograms, using a scale.
- 2.16 The student will tell and write time to the quarter hour, using analog and digital clocks.
- 2.17 The student will use actual measuring devices to compare metric and U.S. Customary units (cups, pints, quarts, gallons, and liters) for measuring liquid volume, using the concepts of more, less, and equivalent.

### Geometry

- 2.18 The student will identify and describe a cube, rectangular solid, sphere, cylinder, and cone, according to the number and shape of faces, edges, bases, and corners.
- 2.19 The student will identify and create figures, symmetric along a line, using various concrete materials.
- 2.20 The student will compare and contrast plane and solid geometric shapes (circle/sphere, square/cube, triangle/pyramid, and rectangle/rectangular solid).

### Probability and Statistics

- 2.21 The student will read, construct, and interpret a simple picture and bar graph.
- 2.22 The student, given a calendar, will determine past and future days of the week and identify specific dates.
- 2.23 The student will record data from experiments using spinners and colored tiles/cubes and use the data to predict which of two events is more likely to occur if the experiment is repeated.

### Patterns, Functions, and Algebra

- 2.24 The student will complete a sequence of 10 or fewer consecutive whole numbers 0 through 999.
- 2.25 The student will identify, create, and extend a wide variety of patterns using symbols and objects.
- 2.26 The student will solve problems by completing a numerical sentence involving the basic facts for addition and subtraction. Examples include:  $3 + \underline{\quad} = 7$ , or  $9 - \underline{\quad} = 2$ . Students will create story problems using the numerical sentences.

# Grade Three

The third-grade standards place emphasis on using a variety of methods to solve problems involving addition and subtraction of whole numbers. Students also will learn the multiplication and division facts through the nines table. Concrete materials will be used to introduce addition and subtraction with fractions and decimals and the concept of probability as chance. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative concepts and relationships or for proficiency in basic computations. Students also will identify real-life applications of the mathematical principles they are learning that can be applied to science and other disciplines they are studying.

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## Number and Number Sense

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| <p>3.1 The student will read and write six-digit numerals and identify the place value for each digit.</p> <p>3.2 The student will round a whole number, 999 or less, to the nearest ten and hundred.</p> <p>3.3 The student will compare two whole numbers between 0 and 9,999, using symbols (<math>&gt;</math>, <math>&lt;</math>, or <math>=</math>) and words (“greater than,” “less than,” or “equal to”).</p> <p>3.4 The student will recognize and use the inverse relationships between addition/subtraction and multiplication/division to complete basic fact sentences. Students will use these relationships to solve problems such as <math>5 + 3 = 8</math> and <math>8 - 3 = \underline{\quad}</math>.</p> <p>3.5 The student will name and write the fractions represented by drawings or concrete materials and represent a given fraction, using concrete materials and symbols.</p> <p>3.6 The student will compare the numerical value of two fractions having like and unlike denominators, using concrete materials.</p> <p>3.7 The student will read and write decimals expressed as tenths and hundredths, using concrete materials.</p> | <p>3.9 The student will recall the multiplication and division facts through the nines table.</p> <p>3.10 The student will create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 5 or less.</p> <p>3.11 The student will add and subtract with proper fractions having like denominators of 10 or less, using concrete materials.</p> <p>3.12 The student will add and subtract with decimals expressed as tenths, using concrete materials and paper and pencil.</p> <p>3.13 The student will determine by counting the value of a collection of bills and coins up to \$5.00, compare the value of the coins or bills, and make change.</p> |
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## Measurement

## Computation and Estimation

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| <p>3.8 The student will solve problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping, using various computational methods, including calculators, paper and pencil, mental computation, and estimation.</p> | <p>3.14 The student will estimate and then use actual measuring devices with metric and U.S. Customary units to measure</p> <ul style="list-style-type: none"><li>• length—_inches, feet, yards, centimeters, and meters;</li><li>• liquid volume—cups, pints, quarts, gallons, and liters; and</li><li>• weight/mass—ounces, pounds, grams, and kilograms.</li></ul> <p>3.15 The student will tell time to the nearest five-minute interval and to the nearest minute, using analog and digital clocks.</p> |
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- 3.16 The student will identify equivalent periods of time, including relationships among days, months, and years, as well as minutes and hours.
- 3.17 The student will read temperature, to the nearest degree, from a Celsius thermometer and a Fahrenheit thermometer. Real thermometers and physical models of thermometers will be used.

**Geometry**

- 3.18 The student will analyze plane and solid geometric figures (square, rectangle, triangle, cube, rectangular solid, and cylinder) and identify relevant properties, including the number of corners, square corners, the shape of faces, and edges.
- 3.19 The student will identify and draw representations of line segments and angles, using a ruler or straightedge.
- 3.20 The student, given appropriate drawings or models, will identify and describe congruent and symmetrical two-dimensional figures, using tracing procedures.

**Probability and Statistics**

- 3.21 The student, given grid paper, will collect data on a given topic of his/her choice and construct a bar graph showing the results. A title and key will be included.
- 3.22 The student will read and interpret data represented in bar and picture graphs.
- 3.23 The student will investigate and describe the concept of probability as chance, and list possible results of a given situation.

**Patterns, Functions, and Algebra**

- 3.24 The student will recognize and describe patterns formed using concrete objects, tables, and pictures and extend the pattern.
- 3.25 The student will analyze a given pattern formed using concrete objects and pictures and then create a pattern with the same attributes.

# Grade Four

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The fourth-grade standards place emphasis on division with whole numbers and solving problems involving addition and subtraction of fractions and decimals. Students will continue to learn and use the basic multiplication facts as they become proficient in multiplying larger numbers. Students also will refine their estimation skills for computations and measurements and investigate the relationships between and among points, lines, segments, and rays. Concrete materials will be used to solve problems involving perimeter, patterns, and probability. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative concepts and relationships or for proficiency in basic computations. Students also will identify real-life applications of the mathematical principles they are learning that can be applied to science and other disciplines they are studying.

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**Number and Number Sense**

- 4.1 The student will
  - identify, orally and in writing, the place value for each digit in a whole number expressed through millions;
  - compare two whole numbers, expressed through millions, using symbols ( $>$ ,  $<$ , or  $=$ ); and
  - round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.
- 4.2 The student will identify and represent equivalent fractions and relate fractions to decimals, using concrete objects.

- 4.3 The student will compare the numerical value of fractions having denominators of 12 or less.
- 4.4 The student will read, write, represent, and identify decimals expressed through thousandths, and round to the nearest tenth and hundredth, using concrete materials, drawings, calculators, and symbols.

### Computation and Estimation

- 4.5 The student will create and solve problems involving addition and subtraction of money amounts using various computational methods, including calculators, paper and pencil, mental computation, and estimation.
- 4.6 The student will estimate whole-number sums and differences and describe the method of estimation. Students will refine estimates, using terms such as closer to, between, and a little more than.
- 4.7 The student will add and subtract whole numbers written in vertical and horizontal form, choosing appropriately between paper and pencil methods and calculators.
- 4.8 The student will find the product of two whole numbers when one factor has two digits or less and the other factor has three digits or less, using estimation and paper and pencil. For larger products (a two-digit numeral times a three-digit numeral), estimation and calculators will be used.
- 4.9 The student will estimate and find the quotient of two whole numbers given a one-digit divisor.
- 4.10 The student will
- add and subtract with fractions having like and unlike denominators of 12 or less and with decimals through thousandths, using concrete materials and paper and pencil; and
  - solve problems involving addition and subtraction with fractions having like and unlike denominators of 12 or less and decimals expressed through thousandths.

### Measurement

- 4.11 The student will
- estimate and measure weight/mass using actual measuring devices and express the results in both metric and U.S. Customary units, including ounces, pounds, grams, and kilograms; and
  - estimate the conversion of ounces and grams and pounds and kilograms, using approximate compar-

isons (1 ounce is about 28 grams, or 1 gram is about the weight of a paper clip; 1 kilogram is a little more than 2 pounds).\*

\* *The intent of this standard is for students to make “ballpark” comparisons and not to memorize conversion factors between U.S. and metric units.*

- 4.12 The student will
- estimate and measure length using actual measuring devices and describe the results in both metric and U.S. Customary units, including part of an inch ( $\frac{1}{2}$ ,  $\frac{1}{4}$ , and  $\frac{1}{8}$ ), inches, feet, yards, millimeters, centimeters, and meters; and
  - estimate the conversion of inches and centimeters, yards and meters, and miles and kilometers, using approximate comparisons (1 inch is about 2.5 centimeters, 1 meter is a little longer than 1 yard, 1 mile is slightly farther than 1.5 kilometers, or 1 kilometer is slightly farther than half a mile).\*
- \* *The intent of this standard is for students to make “ballpark” comparisons and not to memorize conversion factors between U.S. and metric units.*
- 4.13 The student will
- estimate and measure liquid volume using actual measuring devices and using metric and U.S. Customary units, including cups, pints, quarts, gallons, milliliters, and liters; and
  - estimate the conversion of quarts and liters, using approximate comparisons (1 quart is a little less than 1 liter, 1 liter is a little more than 1 quart).\*
- \* *The intent of this standard is for students to make “ballpark” comparisons and not to memorize conversion factors between U.S. and metric units.*
- 4.14 The student will identify and describe situations representing the use of perimeter and will use measuring devices to find perimeter in both standard and non-standard units of measure.

### Geometry

- 4.15 The student will investigate and describe the relationships between and among points, lines, line segments, and rays.
- 4.16 The student will identify and draw representations of points, lines, line segments, rays, and angles, using a straightedge or ruler.
- 4.17 The student will identify lines which illustrate intersection, parallelism, and perpendicularity.

**Probability and Statistics**

- 4.18 The student will determine the probability of a given simple event, using concrete materials.
- 4.19 The student will collect, organize, and display data in line and bar graphs with scale increments of one or greater than one.

**Patterns, Functions, and Algebra**

- 4.20 The student will identify and locate missing whole numbers on a given number line.
- 4.21 The student will extend a given pattern, using concrete materials and tables.
- 4.22 The student will solve problems involving pattern identification and completion of patterns.

# Grade Five

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The fifth-grade standards place emphasis on developing proficiency in using whole numbers, fractions, and decimals to solve problems. Students will collect, display, and analyze data in a variety of ways and solve probability problems, using a sample space or tree diagram. Students also will solve problems involving area and perimeter, classify triangles, and plot points in the coordinate plane. Variables, expressions, and open sentences will be introduced. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative concepts and relationships or for proficiency in basic computations. Students also will identify real-life applications of the mathematical principles they are learning that can be applied to science and other disciplines they are studying.

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**Number and Number Sense**

- 5.1 The student will read, write, and identify the place values of decimals through ten-thousandths.
- 5.2 The student will compare the value of two decimals through ten-thousandths using the symbols  $>$ ,  $<$ , or  $=$ .

- 5.6 The student, given a dividend expressed as a decimal through ten-thousandths and a single-digit divisor, will find the quotient.

- 5.7 The student will add and subtract with fractions and mixed numerals, with and without regrouping, and express answers in simplest form. Problems will include like and unlike denominators, limited to 12 or less.

**Computation and Estimation**

- 5.3 The student will create and solve problems involving addition, subtraction, multiplication, and division of whole numbers, using paper and pencil, estimation, mental computation, and calculators.
- 5.4 The student will find the product of two numbers expressed as decimals through thousandths, using an appropriate method of calculation, including paper and pencil, estimation, mental computation, and calculators.
- 5.5 The student, given a dividend of four digits or less and a divisor of two digits or less, will find the quotient and remainder.

**Measurement**

- 5.8 The student will describe and determine the perimeter of a polygon and the area of a square, rectangle, and triangle, given the appropriate measures.
- 5.9 The student will identify and describe the diameter, radius, chord, and circumference of a circle.
- 5.10 The student will differentiate between area and perimeter and identify whether the application of the concept of perimeter or area is appropriate for a given situation.

- 5.11 The student will choose an appropriate measuring device and unit of measure to solve problems involving measurement of
- length—part of an inch ( $\frac{1}{2}$ ,  $\frac{1}{4}$ , and  $\frac{1}{8}$ ), inches, feet, yards, miles, millimeters, centimeters, meters, and kilometers;
  - weight/mass—ounces, pounds, tons, grams, and kilograms;
  - liquid volume—cups, pints, quarts, gallons, milliliters, and liters;
  - area—square units; and
  - temperature—Celsius and Fahrenheit units.
- Problems also will include estimating the conversion of Celsius and Fahrenheit units relative to familiar situations (water freezes at  $0^{\circ}\text{C}$  and  $32^{\circ}\text{F}$ , water boils at  $100^{\circ}\text{C}$  and  $212^{\circ}\text{F}$ , normal body temperature is about  $37^{\circ}\text{C}$  and  $98.6^{\circ}\text{F}$ ).
- 5.12 The student will determine an amount of elapsed time in hours and minutes within a 24-hour period.

### Geometry

- 5.13 The student will classify angles and triangles as right, acute, or obtuse.
- 5.14 The student will measure and draw right, acute, and obtuse angles and triangles, using appropriate tools.
- 5.15 The student will identify the ordered pair for a point and locate the point for an ordered pair in the first quadrant of a coordinate plane.

### Probability and Statistics

- 5.16 The student will
- solve problems involving the probability of a single event by using tree diagrams or by constructing a sample space representing all possible results; and
  - create a problem statement involving probability based on information from a given problem situation. Students will not be required to solve the problem created.
- 5.17 The student will collect, organize, and display a set of numerical data in a variety of forms, given a problem situation, using bar graphs, stem-and-leaf plots, and line graphs.
- 5.18 The student will find the mean and mode of a set of data.

### Patterns, Functions, and Algebra

- 5.19 The student will investigate, describe, and extend numerical and geometric patterns, including triangular numbers, perfect squares, patterns formed by powers of 10, and arithmetic sequences. Concrete materials and calculators will be used.
- 5.20 The student will
- investigate and describe the concept of variable;
  - use a variable to represent a given verbal quantitative expression, involving one operation; and
  - write an open sentence, using a variable to represent a given mathematical relationship.
- 5.21 The student will create a problem situation based on a given open sentence using a single variable.

## Computer/Technology Standards by the End of Grade Five

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Computer/Technology skills are essential components of every student's education. In order to maximize opportunities for students to acquire necessary skills for academic success, the teaching of these skills should be the shared responsibility of teachers of all disciplines.

**Minimum** skills that students should acquire by the end of **Grade 5** include the following:

- C/T5.1 The student will demonstrate a basic understanding of computer theory including bits, bytes, and binary logic.
- C/T5.2 The student will develop basic technology skills.
- Develop a basic technology vocabulary that includes cursor, software, memory, disk drive, hard drive, and CD-ROM.
  - Select and use technology appropriate to tasks.
  - Develop basic keyboarding skills.
  - Operate peripheral devices.
  - Apply technologies to strategies for problem solving and critical thinking.

- C/T5.3 The student will process, store, retrieve, and transmit electronic information.
- Use search strategies to retrieve electronic information using databases, CD-ROMs, videodiscs, and telecommunications.
  - Use electronic encyclopedias, almanacs, indexes, and catalogs.
  - Use local and wide-area networks and modem-delivered services to access information from electronic databases.
  - Describe advantages and disadvantages of various computer processing, storage, retrieval, and transmission techniques.

- C/T5.4 The student will communicate through application software.
- Create a 1-2 page document using word processing skills, writing process steps, and publishing programs.
  - Use simple computer graphics and integrate graphics into word-processed documents.
  - Create simple databases and spreadsheets to manage information and create reports.
  - Use local and worldwide network communication systems.

## Grade Six

The sixth-grade standards place continued emphasis on the study of whole numbers, decimals, and fractions. Students will use ratios to compare data sets, make conversions within a given measurement system, make geometric constructions and classify three-dimensional figures, and solve linear equations in one variable. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students also will identify real-life applications of the mathematical principles they are learning that can be applied to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

### Number and Number Sense

- 6.1 The student will identify representations of a given percent and describe orally and in writing the equivalence relationship between fractions, decimals, and percents.
- 6.2 The student will describe and compare two sets of data using ratios and will use appropriate notations such as  $a/b$ ,  $a$  to  $b$ , and  $a:b$ .
- 6.3 The student will explain orally and in writing the concepts of prime and composite numbers.
- 6.4 The student will compare and order whole numbers, fractions, and decimals, using concrete materials, drawings or pictures, and mathematical symbols.
- 6.5 The student will identify and represent integers on a number line.

### Computation and Estimation

- 6.6 The student will
- solve problems that involve addition, subtraction, and/or multiplication with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less and express their answers in simplest form; and
  - find the quotient, given a dividend expressed as a decimal through thousandths and a divisor expressed as a decimal to thousandths with exactly one non-zero digit. For divisors with more than one non-zero digit, estimation and calculators will be used.
- 6.7 The student will use estimation strategies to solve multistep practical problems involving whole numbers, decimals, and fractions.

- 6.8 The student will solve multistep consumer application problems involving fractions and decimals and present data and conclusions in paragraphs, tables, or graphs.

### Measurement

- 6.9 The student will compare and convert units of measures for length, weight/mass, and volume within the U.S. Customary system and within the metric system and estimate conversions between units in each system:\*
- length—part of an inch ( $\frac{1}{2}$ ,  $\frac{1}{4}$ , and  $\frac{1}{8}$ ), inches, feet, yards, miles, millimeters, centimeters, meters, and kilometers;
  - weight/mass—ounces, pounds, tons, grams, and kilograms;
  - liquid volume—cups, pints, quarts, gallons, milliliters, and liters; and
  - area—square units.
- \* *The intent of this standard is for students to make “ballpark” comparisons and not to memorize conversion factors between U.S. and metric units.*
- 6.10 The student will estimate and then determine length, weight/mass, area, and liquid volume/capacity, using standard and nonstandard units of measure.
- 6.11 The student will determine if a problem situation involving polygons of four sides or less represents the application of perimeter or area and apply the appropriate formula.
- 6.12 The student will create and solve problems by finding the circumference and/or area of a circle when given the diameter or radius. Using concrete materials or computer models, the student will derive approximations for pi from measurements for circumference and diameter.
- 6.13 The student will estimate angle measures using  $45^\circ$ ,  $90^\circ$ , and  $180^\circ$  as referents and use the appropriate tools to measure the given angles.

### Geometry

- 6.14 The student will identify, classify, and describe the characteristics of plane figures including similarities and differences.
- 6.15 The student will determine congruence of segments, angles, and polygons by direct comparison, given their attributes. Examples of noncongruent and congruent figures will be included.
- 6.16 The student will construct the perpendicular bisector of a line segment and an angle bisector, using a compass and straightedge.

- 6.17 The student will sketch, construct models, and classify rectangular prisms, cones, cylinders, and pyramids.

### Probability and Statistics

- 6.18 The student, given a problem situation, will collect, analyze, display, and interpret data in a variety of graphical methods, including line, bar, and circle graphs and stem-and-leaf and box-and-whisker plots. Circle graphs will be limited to halves, fourths, and eighths.
- 6.19 The student will describe the mean, median, and mode as measures of central tendency and determine their meaning for a set of data.
- 6.20 The student will determine and interpret the probability of an event occurring from a given sample space.

### Patterns, Functions, and Algebra

- 6.21 The student will recognize, describe, and extend a variety of numerical and geometric patterns.
- 6.22 The student will investigate and describe concepts of exponents, perfect squares, and square roots, using calculators to develop the exponential patterns. Patterns will include zero and negative exponents, which lead to the idea of scientific notation. Investigations will include the binary number system as an application of exponents and patterns.
- 6.23 The student will
- model and solve algebraic equations, using concrete materials; and
  - solve one-step linear equations in one variable, involving whole number coefficients and positive rational solutions.

# Grade Seven

The seventh-grade standards place emphasis on solving problems involving consumer applications and proportional reasoning. The students will gain an understanding of the properties of real numbers, solve linear equations and inequalities, and use data analysis techniques to make inferences and predictions. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as fraction calculators, computers, laser discs, and videos. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students also will identify real-life applications of the mathematical principles they are learning that can be applied to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

## Number and Number Sense

- 7.1 The student will compare, order, and determine equivalent relationships between fractions, decimals, and percents, including scientific notation.
- 7.2 The student will find common multiples and factors, including least common multiple and greatest common factor.
- 7.3 The student will simplify expressions by using order of operations, mental mathematics, and appropriate tools. Exponents will be included.
- 7.4 The student will explain orally and in writing the following properties of operations with real numbers:
- the commutative and associative properties for addition and multiplication;
  - the distributive property;
  - the additive and multiplicative identity properties;
  - the additive and multiplicative inverse properties; and
  - the multiplicative property of zero.

## Computation and Estimation

- 7.5 The student will solve consumer application problems involving tips, discounts, sales tax, and simple interest, using whole numbers, fractions, decimals, and percents.
- 7.6 The student will
- solve practical problems involving basic operations with integers by formulating rules for operating with integers and using a number line to compute; and
  - explain the need for integers, using examples from real-life situations.

- 7.7 The student will use proportions to solve practical problems, including scale drawings that contain whole numbers, fractions, decimals, and percents.

## Measurement

- 7.8 The student, given appropriate dimensions, will estimate and find the area of polygons by subdividing them into rectangles and right triangles.
- 7.9 The student will investigate and solve problems involving the volume and surface area of rectangular prisms and cylinders, using concrete materials and practical situations to develop formulas.

## Geometry

- 7.10 The student will compare and contrast the following quadrilaterals: a parallelogram, rectangle, square, rhombus, and trapezoid. Deductive reasoning and inference will be used to classify quadrilaterals.
- 7.11 The student will identify and draw the following polygons: pentagon, hexagon, heptagon, octagon, nonagon, and decagon.
- 7.12 The student will determine if geometric figures (quadrilaterals and triangles) are similar and write proportions to express the relationships between corresponding parts of similar figures.
- 7.13 The student will construct a three-dimensional model using cubes, given the top, side, and/or bottom views, and determine the volume and surface area of the model.

7.14 The student will inscribe equilateral triangles, squares, and hexagons in circles, using a compass and straight-edge.

### Probability and Statistics

7.15 The student will investigate and describe the difference between the probability of an event found through simulation versus the theoretical probability of that same event.

7.16 The student will make a sample space for selected experiments and represent it in the form of a list, chart, picture, or tree diagram.

7.17 The student will determine the probability of a given simple event and express that probability as a ratio, decimal, or a percent as appropriate for the given situation.

7.18 The student will identify and describe the number of possible arrangements of several objects, using a tree diagram or the Basic Counting Principle.

7.19 The student will create and solve problems involving the mean, median, mode, and range of a set of data.

7.20 The student will display data, using frequency distributions, line plots, stem-and-leaf plots, box-and-whisker plots, and scattergrams.

7.21 The student will make inferences and predictions based on the analysis of a set of data that the student(s) collect.

### Patterns, Functions, and Algebra

7.22 The student will investigate and describe functional relationships, including the number of sides of a regular polygon and the sum of the measures of the interior angles.

7.23 The student will write verbal expressions/sentences as algebraic expressions/equations.

7.24 The student will use the following algebraic terms appropriately in written and/or oral expression: equation, inequality, variable, expression, term, coefficient, domain, and range.

7.25 The student will

- solve two-step linear equations and inequalities in one variable, using strategies involving inverse operations and integers; and
- solve practical problems requiring the solution of a two-step linear equation.

7.26 The student will identify and graph ordered pairs in the four quadrants of a coordinate plane.

## Grade Eight

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The eighth-grade standards are designed to prepare students for Algebra I. The standards contain both content that reviews or extends concepts and skills learned in previous grades and new content that prepares students for more abstract concepts in algebra. New concepts include solving multistep equations, graphing linear equations, applying transformations to geometric figures, and using matrices to organize and interpret data. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as fraction calculators, computers, spreadsheets, laser discs, and videos. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students will also identify real-life applications of the mathematical principles they are learning that can be applied to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

### Number and Number Sense

8.1 The student will use proportions to solve scale-model problems with fractions and decimals.

8.2 The student will simplify numerical expressions involving exponents, using order of operations.

- 8.3 The student will describe orally and in writing the relationship between the subsets of the real number system.

### Computation and Estimation

- 8.4 The student will solve practical problems involving whole numbers, integers, and rational numbers, including percents. Problems will be of varying complexities, involving real-life data.
- 8.5 The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables.
- 8.6 The student, given a whole number from 0 to 100, will identify it as a perfect square or find the two consecutive whole numbers between which the square root lies.

### Measurement

- 8.7 The student will verify by measuring and describe the relationships between vertical angles and angles that are supplementary and complementary.
- 8.8 The student will investigate and solve problems involving volume and surface area of cones and pyramids, using concrete materials and practical situations.

### Geometry

- 8.9 The student will apply transformations (rotate or turn, reflect or flip, translate or slide, and dilate or scale) to geometric figures represented on graph paper. The student will identify applications of transformations such as tiling, fabric design, art, and scaling.

- 8.10 The student will describe, classify, and construct plane figures and solid figures, including prisms, pyramids, cylinders, and cones.
- 8.11 The student will verify the Pythagorean Theorem by measuring and then applying the Pythagorean Theorem to find the missing length of a side of a right triangle when the lengths of the other two sides are given.

### Probability and Statistics

- 8.12 The student will analyze problem situations, such as games of chance, board games, or grading scales, and make predictions, using knowledge of probability.
- 8.13 The student will use information displayed in line, bar, circle, and picture graphs and histograms to make comparisons, predictions, and inferences.
- 8.14 The student will use a matrix to organize and describe data.

### Patterns, Functions, and Algebra

- 8.15 The student will investigate and describe functional relationships, including the number of sides of a regular polygon and the maximum number of possible diagonals, expressing the algebraic concept of the number of diagonals of the  $n$ -sided polygon.
- 8.16 The student will solve multistep equations in one variable.
- 8.17 The student will graph a linear equation in two variables on the coordinate plane, using a table of ordered pairs.
- 8.18 The student will describe and represent relations using tables, graphs, and rules.
- 8.19 The student will create and solve problems using proportions, formulas, and functions.

## Computer/Technology Standards by the End of Grade Eight

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Computer/Technology skills are essential components of every student's education. In order to maximize opportunities for students to acquire necessary skills for academic success, the teaching of these skills should be the shared responsibility of teachers of all disciplines.

**Minimum** skills that students should acquire by the end of **Grade 8** include the following:

- C/T8.1 The student will communicate through application software.
- Compose and edit a multipage document at the keyboard, using word processing skills and the writing process steps.
  - Communicate with spreadsheets by entering data and setting up formulas, analyzing data, and creating graphs or charts to visually represent data.

- Communicate with databases by defining fields and entering data, sorting, and producing reports in various forms.
  - Use advanced publishing software, graphics programs, and scanners to produce page layouts.
  - Integrate databases, graphics, and spreadsheets into word-processed documents.
- C/T8.2 The student will communicate through networks and telecommunication.
- Use local and worldwide network communication systems.
  - Develop hypermedia “home page” documents that can be accessed by worldwide networks.
- C/T8.3 The student will have a basic understanding of computer processing, storing, retrieval and transmission technologies and a practical appreciation of the relevant advantages and disadvantages of various processing, storage, retrieval, and transmission technologies.
- C/T8.4 The student will process, store, retrieve, and transmit electronic information.
- Use search strategies to retrieve electronic information.
  - Use electronic encyclopedias, almanacs, indexes, and catalogs to retrieve and select relevant information.
  - Use laser discs with a computer in an interactive mode.
  - Use local and wide-area networks and modem-delivered services to access and retrieve information from electronic databases.
  - Use databases to perform research.

## Algebra I

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The standards below outline the content for a one-year course in Algebra I. All students are expected to achieve the Algebra I standards. When planning for instruction, consideration should be given to the student’s cognitive level and readiness for dealing with abstract concepts. Students should be helped to make connections and to build relationships between algebra and arithmetic, geometry, and probability and statistics. Connections also should be made to other subject areas through practical applications. This approach to teaching algebra should help students attach meaning to the abstract concepts of algebra.

These standards require students to use algebra as a tool for representing and solving a variety of practical problems. Tables and graphs will be used to interpret algebraic expressions, equations, and inequalities and to analyze functions. Matrices will be used to organize and manipulate data.

Calculators, computers, spreadsheets, and graphing utilities (graphing calculators or computer graphing simulators) should be used as tools to assist in problem solving. Graphing utilities enhance the understanding of functions; they provide a powerful tool for solving and verifying solutions to equations and inequalities.

Throughout the course, students should be encouraged to talk about mathematics, to use the language and symbols of mathematics to communicate, to discuss problems and problem solving, and to develop their confidence in mathematics.

- A.1 The student will solve linear equations and inequalities in one variable, solve literal equations (formulas) for a given variable and apply these skills to solve practical problems. Graphing calculators will be used to confirm algebraic solutions.
- A.2 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables. Students will choose an appropriate computational technique, such as mental mathematics, calculator, or paper and pencil.
- A.3 The student will justify steps used in simplifying expressions and solving equations and inequalities. Justifications will include the use of concrete objects, pictorial representations, and the properties of real numbers.

- A.4 The student will use matrices to organize and manipulate data, including matrix addition, subtraction, and scalar multiplication. Data will arise from business, industrial, and consumer situations.
- A.5 The student will analyze a given set of data for the existence of a pattern, represent the pattern algebraically and graphically, if possible, and determine if the relation is a function.
- A.6 The student will select, justify, and apply an appropriate technique to graph a linear function in two variables. Techniques will include slope-intercept, x- and y-intercepts, graphing by transformation, and the use of the graphing calculator.
- A.7 The student will determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined. The graphing calculator will be used to investigate the effect of changes in the slope on the graph of the line.
- A.8 The student will write an equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.
- A.9 The student will solve systems of two linear equations in two variables, both algebraically and graphically, and apply these techniques to solve practical problems. Graphing calculators will be used as both a primary tool of solution and to confirm an algebraic solution.
- A.10 The student will apply the laws of exponents to perform operations on expressions with integral exponents, using scientific notation when appropriate.
- A.11 The student will add, subtract, and multiply polynomials and divide polynomials with monomial divisors, using concrete objects, pictorial representations, and algebraic manipulations.
- A.12 The student will factor completely first- and second-degree binomials and trinomials in one or two variables. The graphing calculator will be used as both a primary tool for factoring and for confirming an algebraic factorization.
- A.13 The student will estimate square roots to the nearest tenth and use a calculator to compute decimal approximations of radicals.
- A.14 The student will solve quadratic equations in one variable both algebraically and graphically. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.
- A.15 The student will determine the domain and range of a relation given a graph or a set of ordered pairs and will identify the relations that are functions.
- A.16 The student will, given a rule, find the values of a function for elements in its domain and locate the zeros of the function both algebraically and with a graphing calculator. The value of  $f(x)$  will be related to the ordinate on the graph.
- A.17 The student will, given a set of data points, write an equation for a line of best fit, using the median fit method, and use the equation to make predictions.
- A.18 The student will compare multiple one-variable data sets, using statistical techniques that include measures of central tendency, range, stem-and-leaf plots, and box-and-whisker graphs.
- A.19 The student will analyze a relation to determine whether a direct or inverse variation exists and represent it algebraically and graphically, if possible.

## Geometry

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This course is designed for students who have successfully completed the standards for Algebra I. The course, among other things, includes the deductive axiomatic method of proof to justify theorems and to tell whether conclusions are valid. Methods of justification will include paragraph proofs, flow charts, two-column proofs, indirect proofs, coordinate proofs, and verbal arguments. A gradual development of formal proof is encouraged. Inductive and intuitive approaches also should be used.

This set of standards includes emphasis on two- and three-dimensional reasoning skills, coordinate and transformational geometry, and the use of geometric models to solve problems. A variety of applications and some general problem-solving techniques should be used to implement these standards, including algebraic skills. Calculators, computers, and graphing utilities (graphing calculators or computer graphing simulators) should be used by the student where feasible. Any technology that will enhance student learning should be used.

- G.1 The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
- identifying the converse, inverse, and contrapositive of a conditional statement;
  - translating a short verbal argument into symbolic form;
  - diagramming arguments involving quantifiers (all, no, none, some), using Venn diagrams; and
  - using valid forms of deductive reasoning, including the law of syllogism.
- G.2 The student will use pictorial representations, including computer software and coordinate methods to solve problems involving symmetry and transformation. This will include
- using formulas for finding distance, midpoint, and slope;
  - investigating and determining whether a figure is symmetric with respect to a line or a point; and
  - determining whether a figure has been translated, reflected, or rotated.
- G.3 The student will solve practical problems involving complementary, supplementary, and congruent angles that include vertical angles, angles formed when parallel lines are cut by a transversal, and angles in polygons.
- G.4 The student will use the relationships between angles formed by two lines cut by a transversal to determine if two lines are parallel and verify, using algebraic and coordinate methods as well as deductive proofs.
- G.5 The student will
- investigate and identify congruence and similarity relationships between triangles; and
  - prove two triangles are congruent or similar given information in the form of a figure or statement, using algebraic and coordinate as well as deductive proofs.
- G.6 The student, given information concerning the lengths of sides and/or measures of angles, will apply the triangle inequality properties to determine whether a triangle exists and to order sides and angles. These concepts will be considered in the context of practical situations.
- G.7 The student will solve practical problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry. Calculators will be used to solve problems and find decimal approximations for the solutions.
- G.8 The student will
- investigate and identify properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals;
  - prove these properties of quadrilaterals using algebraic and coordinate as well as deductive proofs; and
  - use properties of quadrilaterals to solve practical problems.
- G.9 The student will use measures of interior and exterior angles of polygons to solve problems. Tessellations and tiling problems will be used to make connections to art, construction, and nature.
- G.10 The student will investigate and use the properties of angles, arcs, chords, tangents, and secants to solve problems involving circles. Problems will include finding the area of a sector and applications of architecture, art, and construction.
- G.11 The student will construct, using a compass and straightedge, a line segment congruent to a given line segment, the bisector of a line segment, a perpendicular to a given line from a point not on the line, a perpendicular to a given line at a point on the line, the bisector of a given angle, and an angle congruent to a given angle.
- G.12 The student will make a model of a three-dimensional figure from a two-dimensional drawing and make a two-dimensional representation of a three-dimensional object. Models and representations will include scale drawings, perspective drawings, blueprints, or computer simulations.
- G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve practical problems. Calculators will be used to find decimal approximations for results.
- G.14 The student, given similar geometric objects, will use proportional reasoning to solve practical problems; investigate relationships between linear, square, and cubic measures; and describe how changes in one of the measures of the object affect the others.
- G.15 The student will
- draw a system of vectors and find the resultant graphically, write the components of a vector as a column matrix, and find the resultant by matrix addition; and
  - solve practical problems using a system of vectors.

# Algebra II

The standards below outline the content for a one-year course in Algebra II. Students enrolled in Algebra II are assumed to have mastered those concepts outlined in the Algebra I standards. A thorough treatment of advanced algebraic concepts is provided through the study of functions, polynomials, rational expressions, complex numbers, matrices, and sequences and series. Emphasis should be placed on practical applications and modeling throughout the course of study. Oral and written communication concerning the language of algebra, logic of procedures, and interpretation of results also should permeate the course.

These standards include a transformational approach to graphing functions. Transformational graphing uses translation, reflection, dilation, and rotation to generate a “family of graphs” from a given graph and builds a strong connection between algebraic and graphic representations of functions. Students will vary the coefficients and constants of an equation, observe the changes in the graph of the equation, and make generalizations that can be applied to many graphs.

Graphing utilities (graphing calculators or computer graphing simulators) and spreadsheets will be used by students and teachers. Graphing utilities enhance the understanding of realistic applications through mathematical modeling and aid in the investigation and study of functions and their inverses. They also provide an effective tool for solving/verifying equations and inequalities. Any other available technology that will enhance student learning should be used.

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| <p>AII.1 The student will identify field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets, complex numbers, and matrices.</p> <p>AII.2 The student will add, subtract, multiply, divide, and simplify rational expressions, including complex fractions.</p> <p>AII.3 The student will</p> <ul style="list-style-type: none"> <li>• add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents; and</li> <li>• write radical expressions as expressions containing rational exponents, and vice versa.</li> </ul> <p>AII.4 The student will solve absolute value equations and inequalities graphically and algebraically. Graphing calculators will be used both as a primary method of solution and to verify algebraic solutions.</p> <p>AII.5 The student will identify and factor completely polynomials representing the difference of squares, perfect square trinomials, the sum and difference of cubes, and general trinomials.</p> <p>AII.6 The student will select, justify, and apply a technique to solve a quadratic equation over the set of complex numbers. Graphing calculators will be used for solving and confirming algebraic solutions.</p> <p>AII.7 The student will solve equations containing rational expressions and equations containing radical expressions algebraically and graphically. Graphing cal-</p> | <p>culators will be used for solving and confirming algebraic solutions.</p> <p>AII.8 The student will recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.</p> <p>AII.9 The student will find the domain, range, zeros and inverse of a function, the value of a function for a given element in its domain, and the composition of multiple functions. Functions will include those that have domains and ranges that are limited and/or discontinuous. The graphing calculator will be used as a tool to assist in investigation of functions, including exponential and logarithmic.</p> <p>AII.10 The student will investigate and describe the relationships between the solution of an equation, zero of a function, x-intercept of a graph, and factors of a polynomial expression through the use of graphs.</p> <p>AII.11 The student will use matrix multiplication to solve practical problems. Graphing calculators or computer programs with matrix capabilities will be used to find the product.</p> <p>AII.12 The student will represent problem situations with a system of linear equations and solve the system using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.</p> |
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- AII.13 The student will solve systems of linear inequalities and linear programming problems and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.
- AII.14 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.
- AII.15 The student will recognize the general shape of polynomial functions, locate the zeros, sketch the graphs, and verify graphical solutions algebraically. The graphing calculator will be used as a tool to investigate the shape and behavior of polynomial functions.
- AII.16 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve problems, including writing the first  $n$  terms, finding the  $n$ th term, and evaluating summation formulas. Notation will include  $\sum$  and  $a_n$ .
- AII.17 The student will perform operations on complex numbers and express the results in simplest form. Simplifying results will involve using patterns of the powers of  $i$ .
- AII.18 The student will identify conic sections (circle, ellipse, parabola, and hyperbola) from his/her equations. Given the equations in  $(h, k)$  form, students will sketch graphs of conic sections, using transformations.
- AII.19 The student will collect and analyze data to make predictions, write equations, and solve practical problems. Graphing calculators will be used to investigate scatterplots to determine the equation for a curve of best fit.
- AII.20 The student will identify, create, and solve practical problems involving a combination of direct and inverse variations.

## Trigonometry

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The standards below outline the content for a one-semester course in trigonometry. A thorough treatment of trigonometry is provided through the study of trigonometric definitions, applications, graphing, and solving trigonometric equations and inequalities. Emphasis should be placed on using connections between right triangle ratios, trigonometric functions, and circular functions. In addition, applications and modeling should be included throughout the course of study. Emphasis should be placed on oral and written communication concerning the language of mathematics, logic of procedure, and interpretation of results. Students enrolled in trigonometry are assumed to have mastered those concepts outlined in the Algebra II standards.

Graphing utilities (graphing calculators or computer graphing simulators) will be used by students and teachers. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of trigonometric functions and their inverses. They also provide a powerful tool for solving/verifying trigonometric equations and inequalities. Any other technology that will enhance student learning should be used if available.

- T.1 The student will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of an angle in standard position, given a point, other than the origin, on the terminal side of the angle. Circular function definitions will be connected with trigonometric function definitions.
- T.2 The student, given the value of one trigonometric function, will find the values of the other trigonometric functions. Properties of the unit circle and definitions of circular functions will be applied.
- T.3 The student will find the values of the trigonometric functions of the special angles and their related angles as found in the unit circle without the aid of a calculating utility. This will include converting radians to degrees and vice versa.
- T.4 The student will use a calculator to find the value of any trigonometric function and inverse trigonometric function.
- T.5 The student will verify basic trigonometric identities and make substitutions using the basic identities.

- T.6 The student, given one of the six trigonometric functions in standard form (e.g.,  $y = A \sin(Bx + C) + D$ , where  $A$ ,  $B$ ,  $C$ , and  $D$  are real numbers), will
- state the domain and the range of the function;
  - determine the amplitude, period, phase shift, and vertical shift; and
  - sketch the graph of the function by using transformations for at least a one-period interval.
- The graphing calculator will be used to investigate the effect of changing  $A$ ,  $B$ ,  $C$ , and  $D$  on the graph of a trigonometric function.
- T.7 The student will identify the domain and range of the inverse trigonometric functions and recognize the graph of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.
- T.8 The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities. Graphing utilities will be used to solve equations, to check for reasonableness of results, and to verify algebraic solutions.
- T.9 The student will identify, create, and solve practical problems involving triangles and vectors. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

## Algebra II and Trigonometry

The standards for this combined course in Algebra II and Trigonometry include all of the standards listed for Algebra II and Trigonometry. This course is designed for advanced students who are capable of a more rigorous course at an accelerated pace. The standards listed for this course provide the foundation for students to pursue a sequence of advanced mathematical studies from Mathematical Analysis to Advanced Placement Calculus.

- AII/T.1 The student will identify field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets, complex numbers, and matrices.
- AII/T.2 The student will add, subtract, multiply, divide, and simplify rational expressions, including complex fractions.
- AII/T.3 The student will
- add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents; and
  - write radical expressions as expressions containing rational exponents and vice versa.
- AII/T.4 The student will solve absolute value equations and inequalities graphically and algebraically. Graphing calculators will be used both as a primary method of solution and to verify algebraic solutions.
- AII/T.5 The student will identify and factor completely polynomials representing the difference of squares, perfect square trinomials, the sum and difference of cubes, and general trinomials.
- AII/T.6 The student will select, justify, and apply a technique to solve a quadratic equation over the set of complex numbers. Graphing calculators will be used for solving and confirming algebraic solutions.
- AII/T.7 The student will solve equations containing rational expressions and equations containing radical expressions algebraically and graphically. Graphing calculators will be used both as a primary tool for solving and confirming algebraic solutions.
- AII/T.8 The student will recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.
- AII/T.9 The student will find the domain, range, zeros, and inverse of a function; the value of a function for a given element in its domain; and the composition of multiple functions. Functions will include those that have domains and ranges that are limited and/or discontinuous. The graphing calculator will be used as a tool to assist in investigation of functions, including exponential and logarithmic.
- AII/T.10 The student will investigate and describe the relationships between the solution of an equation, zero of a function, x-intercept of a graph, and factors of a polynomial expression through the use of graphs.

- AII/T.11 The student will use matrix multiplication to solve practical problems. Graphing calculators or computer programs with matrix capabilities will be used to find the product.
- AII/T.12 The student will represent problem situations with a system of linear equations and solve the system, using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.
- AII/T.13 The student will solve systems of linear inequalities and linear programming problems and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.
- AII/T.14 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.
- AII/T.15 The student will recognize the general shape of polynomial functions, locate the zeros, sketch the graphs, and verify graphical solutions algebraically. The graphing calculator will be used as a tool to investigate the shape and behavior of polynomial functions.
- AII/T.16 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve problems, including writing the first  $n$  terms, finding the  $n^{\text{th}}$  term, and evaluating summation formulas. Notation will include  $\Sigma$  and  $a_n$ .
- AII/T.17 The student will perform operations on complex numbers and express the results in simplest form. Simplifying results will involve using patterns of the powers of  $i$ .
- AII/T.18 The student will identify conic sections (circle, ellipse, parabola, and hyperbola) from his/her equations. Given the equations in  $(h, k)$  form, students will sketch graphs, using transformations.
- AII/T.19 The student will collect and analyze data to make predictions, write equations, and solve practical problems. Graphing calculators will be used to investigate scatterplots to determine the equation for a curve of best fit.
- AII/T.20 The student will solve practical problems involving a combination of direct and inverse variations.
- AII/T.21 The student will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of an angle in standard position, given a point, other than the origin, on the terminal side of the angle. Circular function definitions will be connected with trigonometric function definitions.
- AII/T.22 The student, given the value of one trigonometric function, will find the values of the other trigonometric functions. Properties of the unit circle and definitions of circular functions will be applied.
- AII/T.23 The student will find the values of the trigonometric functions of the special angles and their related angles as found in the unit circle without the aid of a calculating utility. This will include converting radians to degrees and vice versa.
- AII/T.24 The student will use a calculator to find the value of any trigonometric function and inverse trigonometric function.
- AII/T.25 The student will verify basic trigonometric identities and make substitutions using the basic identities.
- AII/T.26 The student, given one of the six trigonometric functions in standard form (e.g.,  $y = A \sin(Bx + C) + D$ , where  $A$ ,  $B$ ,  $C$ , and  $D$  are real numbers), will
- state the domain and the range of the function;
  - determine the amplitude, period, phase shift, and vertical shift; and
  - sketch the graph of the function by using transformations for at least a one-period interval.
- The graphing calculator will be used to investigate the effect of changing  $A$ ,  $B$ ,  $C$ , and  $D$  on the graph of a trigonometric function.
- AII/T.27 The student will identify the domain and range of the inverse trigonometric functions and recognize the graph of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.
- AII/T.28 The student will solve trigonometric equations that include both infinite solutions as well as restricted domain solutions and solve basic trigonometric inequalities. Graphing utilities will be used to solve equations, to check for reasonableness of results, and to verify algebraic solutions.
- AII/T.29 The student will identify, create, and solve practical problems involving triangles and vectors. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

# Mathematical Analysis

The standards below outline the content for a one-year course in Mathematical Analysis. Mathematical Analysis is intended not only to extend students' knowledge of function characteristics but also to introduce them to another mode of mathematical reasoning. Students enrolled in Mathematical Analysis are assumed to have mastered Algebra II concepts and have some exposure to trigonometry. The content of this course will serve as appropriate preparation for a calculus course.

Graphing utilities (graphing calculators or computer graphing simulators) will be used by students and teachers. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of functions and their inverses. They also provide a powerful tool for solving and verifying equations and inequalities. Any other technology that will enhance student learning should be used if available.

- MA.1 The student will investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. This will include determining zeros, upper and lower bounds, y-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics.
- MA.2 The student will perform operations, including composition and inversion of functions, and determine the domain and range of results. Continuity of functions and special functions such as absolute value, step functions, and piece-wise, will be included. Curve sketching and transformations will be included. Graphing utilities will be used to investigate and verify the graphs.
- MA.3 The student will use graphs to investigate and describe the continuity of functions. The functions will include piece-wise-defined and step functions.
- MA.4 The student will expand binomials having positive integral exponents through the use of the Binomial Theorem, the formula for combinations, and Pascal's Triangle.
- MA.5 The student will solve problems involving arithmetic and geometric sequences and series. This will include finding the sum (sigma notation included) of finite and infinite convergent series that will lead to an intuitive approach to a limit.
- MA.6 The student will apply the method of mathematical induction to prove formulas/statements.
- MA.7 The student will find the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution.
- MA.8 The student will apply the techniques of translation and rotation of axes in the coordinate plane to graphing functions and conic sections. A graphing utility will be used to investigate and verify the graphs. Matrices will be used to represent transformations.
- MA.9 The student will investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and to solve equations and practical problems. This will include the role of  $e$ , natural and common logarithms, laws of exponents and logarithms, and the solution of logarithmic and exponential equations. Graphing utilities will be used to investigate and verify the graphs and solutions.
- MA.10 The student will investigate and identify the characteristics of the graphs of polar equations using graphing utilities. This will include classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.
- MA.11 The student will perform operations with vectors in the coordinate plane and solve practical problems using vectors. This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.
- MA.12 The student will use parametric equations to model and solve application problems. Graphing utilities will be used to develop an understanding of the graph of parametric equations.
- MA.13 The student will identify, create, and solve practical problems involving triangles and vectors. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

# Advanced Placement Calculus

This course is intended for students who have a thorough knowledge of analytic geometry and elementary functions in addition to college preparatory algebra, geometry, and trigonometry. The purpose of the course is to prepare the student for advanced placement in college calculus. These standards incorporate the 1995-1996 College Board Advanced Placement Course Description Syllabus. Teachers should update course content as changes occur in future College Board publications.

As mandated by The College Board, graphing calculators will be required for this course. Computers should be used where feasible by the student and by the teacher. Any technology that will enhance student learning should be used if available. Instructional activities that engage students in solving application problems of varying complexities are encouraged.

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| <p>APC.1 The student will define and apply the properties of elementary functions, including algebraic, trigonometric, exponential, and composite functions and their inverses, and graph these functions using a graphing calculator. Properties of functions will include domains, ranges, combinations, odd, even, periodicity, symmetry, asymptotes, zeros, upper and lower bounds, and intervals where the function is increasing or decreasing.</p> <p>APC.2 The student will define and apply the properties of limits of functions. This will include limits of a constant, sum, product, quotient, one-sided limits, limits at infinity, infinite limits, and nonexistent limits.<br/>* AP Calculus BC will include the rigorous definitions of a limit.</p> <p>APC.3 The student will state the definition of continuity and determine where a function is continuous or discontinuous. This will include</p> <ul style="list-style-type: none"> <li>• continuity at a point;</li> <li>• continuity over a closed interval;</li> <li>• application of the Intermediate Value Theorem; and</li> <li>• graphical interpretation of continuity and discontinuity.</li> </ul> <p>APC.4 The student will find the derivative of an algebraic function by using the definition of a derivative. This will include investigating and describing the relationship between differentiability and continuity.</p> <p>APC.5 The student will apply formulas to find the derivative of algebraic, trigonometric, exponential, and logarithmic functions and their inverses.</p> <p>APC.6 The student will apply formulas to find the derivative of the sum, product, quotient, inverse, and composite (chain rule) of elementary functions.</p> | <p>APC.7 The student will find the derivative of an implicitly defined function.</p> <p>APC.8 The student will find the higher order derivatives of algebraic, trigonometric, exponential, and logarithmic functions.</p> <p>APC.9 The student will use logarithmic differentiation as a technique to differentiate nonlogarithmic functions.</p> <p>APC.10 The student will state (without proof) the Mean Value Theorem for derivatives and apply it both algebraically and graphically.</p> <p>APC.11 The student will use l'Hopital's rule to find the limit of functions whose limits yield the indeterminate forms:</p> $\frac{0}{0} \quad \text{and} \quad \frac{\infty}{\infty}$ <p>* For AP Calculus BC, these functions will also include functions whose limits yield the indeterminate forms:<br/><math>00</math> , <math>1\infty</math> , <math>\infty^\infty</math> , and <math>\infty - \infty</math></p> <p>APC.12 The student will apply the derivative to solve problems, including tangent and normal lines to a curve, curve sketching, velocity, acceleration, related rates of change, Newton's method, differentials and linear approximations, and optimization problems.</p> <p>APC.13 The student will find the indefinite integral of algebraic, exponential, logarithmic, and trigonometric functions. The special integration techniques of substitution (change of variables) and integration by parts will be included.<br/>*AP Calculus BC will also include integration by trigonometric substitution and integration by partial fractions (only linear factors in the denominator).</p> <p>APC.14 The student will identify the properties of the definite integral. This will include the Fundamental</p> |
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Theorem of Calculus and the definite integral as an area and as a limit of a sum as well as the fundamental theorem:

$$\frac{d}{d(x)} \int_a^x f(t) d(t) = f(x)$$

\* AP Calculus BC will include composite functions defined by integrals, e.g.,

$$f(x) = \int_0^{x^2} e^{-t^2} d(t)$$

APC.15 The student will apply the definite integral to solve problems. These problems will include finding distance traveled on a line and velocity from acceleration with initial conditions, growth and decay problems, solutions of separable differential equations, the average value of a function, area between curves, volumes of solids of revolution about the axes or lines parallel to the axes using disc/washer and shell methods, and volumes of solids with known cross-sectional areas.

\*AP Calculus BC will also include areas bounded by polar curves.

APC.16 The student will compute an approximate value for a definite integral. This will include numerical calculations using Riemann Sums and the Trapezoidal Rule.

\*AP Calculus BC will also utilize Simpson's Rule.

\*APC.17 The student will find the derivatives of vector functions and parametrically defined functions and use them to solve problems. The problems will include tangent and normal lines to parametrically defined curves, velocity and acceleration, and velocity and acceleration vectors for motion on a plane curve.

\*APC.18 The student will use integration to solve problems. This will include areas bounded by polar curves, length of a path (including parametric curves), work (Hooke's law), and improper integrals.

\*APC. 19 The student will define and test for convergence of a series of real numbers and of functions. This will include geometric series, comparison (including limit comparison), ratio, root, and integral tests, absolute and conditional convergence, alternating series and error approximation, and p-series.

\*APC. 20 The student will define, restate, and apply power series. This will include addition, substitution, term-by-term differentiation and integration, interval of convergence, Taylor's series, Maclaurin series expansions, and Taylor polynomials with remainder and Lagrange error approximation.

\* For those students who are enrolled in AP Calculus BC.

## Computer Mathematics

This Computer Mathematics course is intended to provide students with experiences in using the computer to solve problems which can be set up as mathematical models. Students who successfully complete the standards for this course may earn high school mathematics credit. It is recognized that many students will gain computer skills in other mathematics courses or in a separate curriculum outside of mathematics and prior to high school. In such cases, the standards indicated by an asterisk (\*) should be included in the student's course of study and treated as a review for those students who enroll in Computer Mathematics.

Even though computer ideas should be introduced in the context of mathematical concepts, problem solving per se should be developed in the most general sense, making the techniques applicable by students in many other environments. Strategies include defining the problem; developing, refining, and implementing a plan; and testing and revising the solution. Programming, ranging from simple programs involving only a few lines to complex programs involving subprograms, should permeate the entire course.

These standards identify fundamental principles and concepts in the field of computer science. Students will develop and refine skills in logic, organization, and precise expression that will enhance learning in other disciplines.

The standards that follow are separated into two groups: those related to programming concepts—Standards 1 through 21—and those dealing with mathematical applications—Standards 22 and 24. This separation is not intended to suggest that they be treated separately in the instructional program. Programming concepts, problem-solving strategies, and mathematical applications should be integrated throughout the course.

- \*COM.1 The student will describe the program development cycle: defining the problem, planning a solution, carrying out the plan, debugging the program, and providing program documentation.
- \*COM.2 The student will write program specifications that define the constraints of a given problem. These specifications include descriptions of pre-conditions, post-conditions, the desired output, analysis of the available input, and an indication as to whether or not the program is solvable under the given conditions.
- \*COM.3 The student will design a step-by-step plan (algorithm) to solve a given problem. The plan will be in the form of a program flowchart, pseudo code, a hierarchy chart and/or data flow diagram.
- \*COM.4 The student will use operating system commands, which include creating a new file, opening an existing file, saving a file, making a printed copy (hard copy) of the file, and executing a program.
- \*COM.5 The student will divide a given problem into manageable sections (modules) by task and implement the solution. The modules will include an appropriate user-defined function, subroutines, and procedures. Enrichment topics can include user-defined libraries (units) and object-oriented programming.
- \*COM.6 The student will design and implement the input phase of a program, which will include designing screen layout and getting information into the program by way of user interaction, data statements (BASIC), and/or file input. The input phase also will include methods of filtering out invalid data (error trapping).
- \*COM.7 The student will design and implement the output phase of a computer program, which will include designing output layout, accessing a variety of output devices, using output statements, and labeling results.
- COM.8 The student will design and implement computer graphics, which will include topics appropriate for the available programming environment as well as student background. Students will use graphics as an end in itself, as an enhancement to other output, and as a vehicle for reinforcing programming techniques.
- COM.9 The student will define simple variable data types that include integer, real (fixed and scientific notation), character, string, and Boolean.
- COM.10 The student will use appropriate variable data types, including integer, real (fixed and scientific notation), character, string, and Boolean. This will also include variables representing structured data types.
- \*COM.11 The student will describe the way the computer stores, accesses, and processes variables, including the following topics: the use of variables versus constants, variables addresses, pointers, parameter passing, scope of variables, and local versus global variables. This will also include use of terminology, including memory, CPU, RAM, ROM, baud, byte, bits, floppy disc, and hard drive.
- COM.12 The student will translate a mathematical expression into a computer statement, which involves writing assignment statements and using the order of operations.
- COM.13 The student will select and implement built-in (library) functions in processing data, which include trigonometric functions, absolute value functions, random number functions, end of line, end of file, and string.
- COM.14 The student will implement conditional statements that include if/then, if/then/else, case statements, and Boolean logic.
- COM.15 The student will implement a loop, including iterative loops, pretest loops, and post-test loops. Other topics will include single entry point, single exit point, preconditions, post-conditions and loop invariance.
- \*COM.16 The student will select and implement appropriate data structures, including arrays (one-dimensional and/or multidimensional), files, and records. Implementation will include creating the data structure, putting information into the structure, and retrieving information from the structure.
- \*COM.17 The student will implement pre-existing algorithms, including sort routines, search routines, and animation routines.

- COM.18 The student will test a program using an appropriate set of data. The set of test data should be appropriate and complete for the type of program being tested.
- COM.19 The student will debug a program using appropriate techniques (e.g., appropriately placed controlled breaks, the printing of intermediate results, and other debugging tools available in the programming environment), and identify the difference between syntax errors and logic errors.
- COM.20 The student will properly document a program including the preconditions and post-conditions of program segments, input/output specifications, the step-by-step plan, the test data, a sample run, and the program listing with appropriately placed comments.
- COM.21 The student will design, write, test, debug, and document a complete structured program which requires the synthesis of many of the concepts contained in previous standards.
- \*COM.22 The student will solve practical consumer problems that involve analyzing and interpreting graphs, charts, and/or tables.
- COM.23 The student will solve mathematical problems using formulas, equations, and functions. Problems will include those related to geometry, business, and leisure (e.g., sports and recreational activities).
- COM.24 The student will solve probability, data analysis, and statistical problems.

