Mathematics Standards of Learning

for Virginia Public Schools

Secondary Courses

Mathematics
Standards of Learning

The Standards of Learning for Mathematics identify academic content for essential components of the mathematics curriculum at different grade levels for Virginia's public schools. Standards are identified for kindergarten through grade eight and for a core set of high school courses. Throughout a student's mathematics schooling from kindergarten through grade eight, specific content strands or topics are included. These content strands are Number and Number Sense; Computation and Estimation; Measurement; Geometry; Probability and Statistics; and Patterns, Functions, and Algebra. The Standards of Learning for each strand progress in complexity at each grade level and throughout the high school courses.

The Standards of Learning are not intended to encompass the entire curriculum for a given grade level or course or to prescribe how the content should be taught. Teachers are encouraged to go beyond the standards and to select instructional strategies and assessment methods appropriate for their students.

Goals

Students today require stronger mathematical knowledge and skills to pursue higher education, to compete in a technologically oriented workforce, and to be informed citizens. Students must gain an understanding of fundamental ideas in arithmetic, measurement, geometry, probability, data analysis and statistics, and algebra and functions, and develop proficiency in mathematical skills. In addition, students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving in science, engineering, business and industry, government, and practical affairs. Hence, the use of technology must be an integral part of teaching and learning. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Please note the computer/technology standards following the grade five and grade eight standards, respectively. The teaching of these skills should be the shared responsibility of teachers of all disciplines.

The content of the mathematics standards is intended to support the following four goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, and making mathematical connections.

Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-life data and situations within and outside mathematics and then apply appropriate strategies to find an acceptable solution. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problem types. A major goal of the mathematics program is to help students become competent mathematical problem solvers.

Mathematical Communication

Students will use the language of mathematics, including specialized vocabulary and symbols, to represent and describe mathematical ideas, generalizations, and relationships. Representing, discussing, reading, writing, and listening to mathematics will help students to clarify their thinking and deepen their understanding of the mathematics being studied.

Mathematical Reasoning

Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will learn to apply proportional and spatial reasoning and to reason from graphs.

Mathematical Connections

Students will relate concepts and procedures from different topics in mathematics to one another, using a variety of representations—graphical, numerical, algebraic, verbal, and physical. Through the application of content, students will make connections between different areas of mathematics and between mathematics and other disciplines, especially science. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that reinforce each other.

Algebra I

The standards below outline the content for a one-year course in Algebra I. All students are expected to achieve the Algebra I standards. When planning for instruction, consideration should be given to the student's cognitive level and readiness for dealing with abstract concepts. Students should be helped to make connections and to build relationships between algebra and arithmetic, geometry, and probability and statistics. Connections also should be made to other subject areas through practical applications. This approach to teaching algebra should help students attach meaning to the abstract concepts of algebra.

These standards require students to use algebra as a tool for representing and solving a variety of practical problems. Tables and graphs will be used to interpret algebraic expressions, equations, and inequalities and to analyze functions. Matrices will be used to organize and manipulate data.

Calculators, computers, spreadsheets, and graphing utilities (graphing calculators or computer graphing simulators) should be used as tools to assist in problem solving. Graphing utilities enhance the understanding of functions; they provide a powerful tool for solving and verifying solutions to equations and inequalities.

Throughout the course, students should be encouraged to talk about mathematics, to use the language and symbols of mathematics to communicate, to discuss problems and problem solving, and to develop their confidence in mathematics.

A.1 The student will solve linear equations and inequalities in one variable, solve literal equations (formulas) for a given variable and apply these skills to solve practical problems. Graphing calculators will be used to confirm algebraic solutions.

A.2 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables. Students will choose an appropriate computational technique, such as mental mathematics, calculator, or paper and pencil.

A.3 The student will justify steps used in simplifying expressions and solving equations and inequalities. Justifications will include the use of concrete objects, pictorial representations, and the properties of real numbers.

A.4 The student will use matrices to organize and manipulate data, including matrix addition, subtraction, and scalar multiplication. Data will arise from business, industrial, and consumer situations.

A.5 The student will analyze a given set of data for the existence of a pattern, represent the pattern algebraically and graphically, if possible, and determine if the relation is a function.

A.6 The student will select, justify, and apply an appropriate technique to graph a linear function in two variables. Techniques will include slope-intercept, x- and y-intercepts, graphing by transformation, and the use of the graphing calculator.

A.7 The student will determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined. The graphing calculator will be used to investigate the effect of changes in the slope on the graph of the line.

A.8 The student will write an equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.

A.9 The student will solve systems of two linear equations in two variables, both algebraically and graphically, and apply these techniques to solve practical problems. Graphing calculators will be used as both a primary tool of solution and to confirm an algebraic solution.

A.10 The student will apply the laws of exponents to perform operations on expressions with integral exponents, using scientific notation when appropriate.

A.11 The student will add, subtract, and multiply polynomials and divide polynomials with monomial divisors, using concrete objects, pictorial representations, and algebraic manipulations.

A.12 The student will factor completely first- and second-degree binomials and trinomials in one or two variables. The graphing calculator will be used as both a primary tool for factoring and for confirming an algebraic factorization.

A.13 The student will estimate square roots to the nearest tenth and use a calculator to compute decimal approximations of radicals.

A.14 The student will solve quadratic equations in one variable both algebraically and graphically. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

A.15 The student will determine the domain and range of a relation given a graph or a set of ordered pairs and will identify the relations that are functions.

A.16 The student will, given a rule, find the values of a function for elements in its domain and locate the zeros of the function both algebraically and with a graphing calculator. The value of f(x) will be related to the ordinate on the graph.

A.17 The student will, given a set of data points, write an equation for a line of best fit, using the median fit method, and use the equation to make predictions.

A.18 The student will compare multiple one-variable data sets, using statistical techniques that include measures of central tendency, range, stem-and-leaf plots, and box-and-whisker graphs.

A.19 The student will analyze a relation to determine whether a direct or inverse variation exists and represent it algebraically and graphically, if possible.

Geometry

This course is designed for students who have successfully completed the standards for Algebra I. The course, among other things, includes the deductive axiomatic method of proof to justify theorems and to tell whether conclusions are valid. Methods of justification will include paragraph proofs, flow charts, two-column proofs, indirect proofs, coordinate proofs, and verbal arguments. A gradual development of formal proof is encouraged. Inductive and intuitive approaches also should be used.

This set of standards includes emphasis on two- and three-dimensional reasoning skills, coordinate and transformational geometry, and the use of geometric models to solve problems. A variety of applications and some general problem-solving techniques should be used to implement these standards, including algebraic skills. Calculators, computers, and graphing utilities (graphing calculators or computer graphing simulators) should be used by the student where feasible. Any technology that will enhance student learning should be used.

G.1 The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
- identifying the converse, inverse, and contrapositive of a conditional statement;
- translating a short verbal argument into symbolic form;
- diagramming arguments involving quantifiers (all, no, none, some), using Venn diagrams; and
- using valid forms of deductive reasoning, including the law of syllogism.

G.2 The student will use pictorial representations, including computer software and coordinate methods to solve problems involving symmetry and transformation. This will include
- using formulas for finding distance, midpoint, and slope;
- investigating and determining whether a figure is symmetric with respect to a line or a point; and
- determining whether a figure has been translated, reflected, or rotated.

G.3 The student will solve practical problems involving complementary, supplementary, and congruent angles that include vertical angles, angles formed when parallel lines are cut by a transversal, and angles in polygons.

G.4 The student will use the relationships between angles formed by two lines cut by a transversal to determine if two lines are parallel and verify, using algebraic and coordinate methods as well as deductive proofs.

G.5 The student will
- investigate and identify congruence and similarity relationships between triangles; and
- prove two triangles are congruent or similar given information in the form of a figure or statement, using algebraic and coordinate as well as deductive proofs.

G.6 The student, given information concerning the lengths of sides and/or measures of angles, will apply the triangle inequality properties to determine whether a triangle exists and to order sides and angles. These concepts will be considered in the context of practical situations.

G.7 The student will solve practical problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry. Calculators will be used to solve problems and find decimal approximations for the solutions.

G.8 The student will
- investigate and identify properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals;
- prove these properties of quadrilaterals using algebraic and coordinate as well as deductive proofs; and
- use properties of quadrilaterals to solve practical problems.

G.9 The student will use measures of interior and exterior angles of polygons to solve problems. Tessellations and tiling problems will be used to make connections to art, construction, and nature.

G.10 The student will investigate and use the properties of angles, arcs, chords, tangents, and secants to solve problems involving circles. Problems will include finding the area of a sector and applications of architecture, art, and construction.

G.11 The student will construct, using a compass and straightedge, a line segment congruent to a given line segment, the bisector of a line segment, a perpendicular to a given line from a point not on the line, a perpendicular to a given line at a point on the line, the bisector of a given angle, and an angle congruent to a given angle.

G.12 The student will make a model of a three-dimensional figure from a two-dimensional drawing and make a two-dimensional representation of a three-dimensional object. Models and representations will include scale drawings, perspective drawings, blueprints, or computer simulations.

G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve practical problems. Calculators will be used to find decimal approximations for results.

G.14 The student, given similar geometric objects, will use proportional reasoning to solve practical problems; investigate relationships between linear, square, and cubic measures; and describe how changes in one of the measures of the object affect the others.

G.15 The student will
- draw a system of vectors and find the resultant graphically, write the components of a vector as a column matrix, and find the resultant by matrix addition; and
- solve practical problems using a system of vectors.

Algebra II

The standards below outline the content for a one-year course in Algebra II. Students enrolled in Algebra II are assumed to have mastered those concepts outlined in the Algebra I standards. A thorough treatment of advanced algebraic concepts is provided through the study of functions, polynomials, rational expressions, complex numbers, matrices, and sequences and series. Emphasis should be placed on practical applications and modeling throughout the course of study. Oral and written communication concerning the language of algebra, logic of procedures, and interpretation of results also should permeate the course.

These standards include a transformational approach to graphing functions. Transformational graphing uses translation, reflection, dilation, and rotation to generate a "family of graphs" from a given graph and builds a strong connection between algebraic and graphic representations of functions. Students will vary the coefficients and constants of an equation, observe the changes in the graph of the equation, and make generalizations that can be applied to many graphs.

Graphing utilities (graphing calculators or computer graphing simulators) and spreadsheets will be used by students and teachers. Graphing utilities enhance the understanding of realistic applications through mathematical modeling and aid in the investigation and study of functions and their inverses. They also provide an effective tool for solving/verifying equations and inequalities. Any other available technology that will enhance student learning should be used.

AII.1 The student will identify field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets, complex numbers, and matrices.

AII.2 The student will add, subtract, multiply, divide, and simplify rational expressions, including complex fractions.

AII.3 The student will

- add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents; and
- write radical expressions as expressions containing rational exponents, and vice versa.

AII.4 The student will solve absolute value equations and inequalities graphically and algebraically. Graphing calculators will be used both as a primary method of solution and to verify algebraic solutions.

AII.5 The student will identify and factor completely polynomials representing the difference of squares, perfect square trinomials, the sum and difference of cubes, and general trinomials.

AII.6 The student will select, justify, and apply a technique to solve a quadratic equation over the set of complex numbers. Graphing calculators will be used for solving and confirming algebraic solutions.

AII.7 The student will solve equations containing rational expressions and equations containing radical expressions algebraically and graphically. Graphing calculators will be used for solving and confirming algebraic solutions.

AII.8 The student will recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.

AII.9 The student will find the domain, range, zeros and inverse of a function, the value of a function for a given element in its domain, and the composition of multiple functions. Functions will include those that have domains and ranges that are limited and/or discontinuous. The graphing calculator will be used as a tool to assist in investigation of functions, including exponential and logarithmic.

AII.10 The student will investigate and describe the relationships between the solution of an equation, zero of a function, x-intercept of a graph, and factors of a polynomial expression through the use of graphs.

AII.11 The student will use matrix multiplication to solve practical problems. Graphing calculators or computer programs with matrix capabilities will be used to find the product.

AII.12 The student will represent problem situations with a system of linear equations and solve the system using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.

AII.13 The student will solve systems of linear inequalities and linear programming problems and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.

AII.14 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.

AII.15 The student will recognize the general shape of polynomial functions, locate the zeros, sketch the graphs, and verify graphical solutions algebraically. The graphing calculator will be used as a tool to investigate the shape and behavior of polynomial functions.

AII.16 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve problems, including writing the first n terms, finding the nth term, and evaluating summation formulas. Notation will include $\sum$ and a_n.

AII.17 The student will perform operations on complex numbers and express the results in simplest form. Simplifying results will involve using patterns of the powers of i.

AII.18 The student will identify conic sections (circle, ellipse, parabola, and hyperbola) from his/her equations. Given the equations in (h, k) form, students will sketch graphs of conic sections, using transformations.

AII.19 The student will collect and analyze data to make predictions, write equations, and solve practical problems. Graphing calculators will be used to investigate scatterplots to determine the equation for a curve of best fit.

AII.20 The student will identify, create, and solve practical problems involving a combination of direct and inverse variations.

Trigonometry

The standards below outline the content for a one-semester course in trigonometry. A thorough treatment of trigonometry is provided through the study of trigonometric definitions, applications, graphing, and solving trigonometric equations and inequalities. Emphasis should be placed on using connections between right triangle ratios, trigonometric functions, and circular functions. In addition, applications and modeling should be included throughout the course of study. Emphasis should be placed on oral and written communication concerning the language of mathematics, logic of procedure, and interpretation of results. Students enrolled in trigonometry are assumed to have mastered those concepts outlined in the Algebra II standards.

Graphing utilities (graphing calculators or computer graphing simulators) will be used by students and teachers. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of trigonometric functions and their inverses. They also provide a powerful tool for solving/verifying trigonometric equations and inequalities. Any other technology that will enhance student learning should be used if available.

T.1 The student will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of an angle in standard position, given a point, other than the origin, on the terminal side of the angle. Circular function definitions will be connected with trigonometric function definitions.

T.2 The student, given the value of one trigonometric function, will find the values of the other trigonometric functions. Properties of the unit circle and definitions of circular functions will be applied.

T.3 The student will find the values of the trigonometric functions of the special angles and their related angles as found in the unit circle without the aid of a calculating utility. This will include converting radians to degrees and vice versa.

T.4 The student will use a calculator to find the value of any trigonometric function and inverse trigonometric function.

T.5 The student will verify basic trigonometric identities and make substitutions using the basic identities.

T.6 The student, given one of the six trigonometric functions in standard form (e.g., $y = A\sin(Bx + C) + D$, where A, B, C, and D are real numbers), will

- state the domain and the range of the function;
- determine the amplitude, period, phase shift, and vertical shift; and
- sketch the graph of the function by using transformations for at least a one-period interval.

The graphing calculator will be used to investigate the effect of changing A, B, C, and D on the graph of a trigonometric function.

T.7 The student will identify the domain and range of the inverse trigonometric functions and recognize the graph of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.

T.8 The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities. Graphing utilities will be used to solve equations, to check for reasonableness of results, and to verify algebraic solutions.

T.9 The student will identify, create, and solve practical problems involving triangles and vectors. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Algebra II and Trigonometry

The standards for this combined course in Algebra II and Trigonometry include all of the standards listed for Algebra II and Trigonometry. This course is designed for advanced students who are capable of a more rigorous course at an accelerated pace. The standards listed for this course provide the foundation for students to pursue a sequence of advanced mathematical studies from Mathematical Analysis to Advanced Placement Calculus.

AII/T.1 The student will identify field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets, complex numbers, and matrices.

AII/T.2 The student will add, subtract, multiply, divide, and simplify rational expressions, including complex fractions.

AII/T.3 The student will

- add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents; and
- write radical expressions as expressions containing rational exponents and vice versa.

AII/T.4 The student will solve absolute value equations and inequalities graphically and algebraically. Graphing calculators will be used both as a primary method of solution and to verify algebraic solutions.

AII/T.5 The student will identify and factor completely polynomials representing the difference of squares, perfect square trinomials, the sum and difference of cubes, and general trinomials.

AII/T.6 The student will select, justify, and apply a technique to solve a quadratic equation over the set of complex numbers. Graphing calculators will be used for solving and confirming algebraic solutions.

AII/T.7 The student will solve equations containing rational expressions and equations containing radical expressions algebraically and graphically. Graphing calculators will be used both as a primary tool for solving and confirming algebraic solutions.

AII/T.8 The student will recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.

AII/T.9 The student will find the domain, range, zeros, and inverse of a function; the value of a function for a given element in its domain; and the composition of multiple functions. Functions will include those that have domains and ranges that are limited and/or discontinuous. The graphing calculator will be used as a tool to assist in investigation of functions, including exponential and logarithmic.

AII/T.10 The student will investigate and describe the relationships between the solution of an equation, zero of a function, x-intercept of a graph, and factors of a polynomial expression through the use of graphs.

AII/T.11 The student will use matrix multiplication to solve practical problems. Graphing calculators or computer programs with matrix capabilities will be used to find the product.

AII/T.12 The student will represent problem situations with a system of linear equations and solve the system, using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.

AII/T.13 The student will solve systems of linear inequalities and linear programming problems and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.

AII/T. 14 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.

AII/T.15 The student will recognize the general shape of polynomial functions, locate the zeros, sketch the graphs, and verify graphical solutions algebraically. The graphing calculator will be used as a tool to investigate the shape and behavior of polynomial functions.

AII/T.16 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve problems, including writing the first n terms, finding the n^{th} term, and evaluating summation formulas. Notation will include $\sum$ and a_n.

AII/T.17 The student will perform operations on complex numbers and express the results in simplest form. Simplifying results will involve using patterns of the powers of *i*.

AII/T.18 The student will identify conic sections (circle, ellipse, parabola, and hyperbola) from his/her equations. Given the equations in (h, k) form, students will sketch graphs, using transformations.

AII/T.19 The student will collect and analyze data to make predictions, write equations, and solve practical problems. Graphing calculators will be used to investigate scatterplots to determine the equation for a curve of best fit.

AII/T.20 The student will solve practical problems involving a combination of direct and inverse variations.

AII/T.21 The student will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of an angle in standard position, given a point, other than the origin, on the terminal side of the angle. Circular function definitions will be connected with trigonometric function definitions.

AII/T.22 The student, given the value of one trigonometric function, will find the values of the other trigonometric functions. Properties of the unit circle and definitions of circular functions will be applied.

AII/T.23 The student will find the values of the trigonometric functions of the special angles and their related angles as found in the unit circle without the aid of a calculating utility. This will include converting radians to degrees and vice versa.

AII/T.24 The student will use a calculator to find the value of any trigonometric function and inverse trigonometric function.

AII/T.25 The student will verify basic trigonometric identities and make substitutions using the basic identities.

AII/T.26 The student, given one of the six trigonometric functions in standard form (e.g., $y = A\sin(Bx + C) + D$, where A, B, C, and D are real numbers), will

- state the domain and the range of the function;
- determine the amplitude, period, phase shift, and vertical shift; and
- sketch the graph of the function by using transformations for at least a one-period interval.

The graphing calculator will be used to investigate the effect of changing A, B, C, and D on the graph of a trigonometric function.

AII/T.27 The student will identify the domain and range of the inverse trigonometric functions and recognize the graph of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.

AII/T.28 The student will solve trigonometric equations that include both infinite solutions as well as restricted domain solutions and solve basic trigonometric inequalities. Graphing utilities will be used to solve equations, to check for reasonableness of results, and to verify algebraic solutions.

AII/T.29 The student will identify, create, and solve practical problems involving triangles and vectors. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Mathematical Analysis

The standards below outline the content for a one-year course in Mathematical Analysis. Mathematical Analysis is intended not only to extend students' knowledge of function characteristics but also to introduce them to another mode of mathematical reasoning. Students enrolled in Mathematical Analysis are assumed to have mastered Algebra II concepts and have some exposure to trigonometry. The content of this course will serve as appropriate preparation for a calculus course.

Graphing utilities (graphing calculators or computer graphing simulators) will be used by students and teachers. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of functions and their inverses. They also provide a powerful tool for solving and verifying equations and inequalities. Any other technology that will enhance student learning should be used if available.

MA.1 The student will investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. This will include determining zeros, upper and lower bounds, y-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics.

MA.2 The student will perform operations, including composition and inversion of functions, and determine the domain and range of results. Continuity of functions and special functions such as absolute value, step functions, and piece-wise, will be included. Curve sketching and transformations will be included. Graphing utilities will be used to investigate and verify the graphs.

MA.3 The student will use graphs to investigate and describe the continuity of functions. The functions will include piece-wise-defined and step functions.

MA.4 The student will expand binomials having positive integral exponents through the use of the Binomial Theorem, the formula for combinations, and Pascal's Triangle.

MA.5 The student will solve problems involving arithmetic and geometric sequences and series. This will include finding the sum (sigma notation included) of finite and infinite convergent series that will lead to an intuitive approach to a limit.

MA.6 The student will apply the method of mathematical induction to prove formulas/statements.

MA.7 The student will find the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution.

MA.8 The student will apply the techniques of translation and rotation of axes in the coordinate plane to graphing functions and conic sections. A graphing utility will be used to investigate and verify the graphs. Matrices will be used to represent transformations.

MA.9 The student will investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and to solve equations and practical problems. This will include the role of e, natural and common logarithms, laws of exponents and logarithms, and the solution of logarithmic and exponential equations. Graphing utilities will be used to investigate and verify the graphs and solutions.

MA.10 The student will investigate and identify the characteristics of the graphs of polar equations using graphing utilities. This will include classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.

MA.11 The student will perform operations with vectors in the coordinate plane and solve practical problems using vectors. This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.

MA.12 The student will use parametric equations to model and solve application problems. Graphing utilities will be used to develop an understanding of the graph of parametric equations.

MA.13 The student will identify, create, and solve practical problems involving triangles and vectors. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Advanced Placement Calculus

This course is intended for students who have a thorough knowledge of analytic geometry and elementary functions in addition to college preparatory algebra, geometry, and trigonometry. The purpose of the course is to prepare the student for advanced placement in college calculus. These standards incorporate the 1995-1996 College Board Advanced Placement Course Description Syllabus. Teachers should update course content as changes occur in future College Board publications.

As mandated by The College Board, graphing calculators will be required for this course. Computers should be used where feasible by the student and by the teacher. Any technology that will enhance student learning should be used if available. Instructional activities that engage students in solving application problems of varying complexities are encouraged.

APC.1 The student will define and apply the properties of elementary functions, including algebraic, trigonometric, exponential, and composite functions and their inverses, and graph these functions using a graphing calculator. Properties of functions will include domains, ranges, combinations, odd, even, periodicity, symmetry, asymptotes, zeros, upper and lower bounds, and intervals where the function is increasing or decreasing.

APC.2 The student will define and apply the properties of limits of functions. This will include limits of a constant, sum, product, quotient, one-sided limits, limits at infinity, infinite limits, and nonexistent limits.

* AP Calculus BC will include the rigorous definitions of a limit.

APC.3 The student will state the definition of continuity and determine where a function is continuous or discontinuous. This will include

- continuity at a point;
- continuity over a closed interval;
- application of the Intermediate Value Theorem; and
- graphical interpretation of continuity and discontinuity.

APC.4 The student will find the derivative of an algebraic function by using the definition of a derivative. This will include investigating and describing the relationship between differentiability and continuity.

APC.5 The student will apply formulas to find the derivative of algebraic, trigonometric, exponential, and logarithmic functions and their inverses.

APC.6 The student will apply formulas to find the derivative of the sum, product, quotient, inverse, and composite (chain rule) of elementary functions.

APC.7 The student will find the derivative of an implicitly defined function.

APC.8 The student will find the higher order derivatives of algebraic, trigonometric, exponential, and logarithmic functions.

APC.9 The student will use logarithmic differentiation as a technique to differentiate nonlogarithmic functions.

APC.10 The student will state (without proof) the Mean Value Theorem for derivatives and apply it both algebraically and graphically.

APC.11 The student will use l'Hopital's rule to find the limit of functions whose limits yield the indeterminate forms:

$$\frac{0}{0} \quad \text{and} \quad \frac{\infty}{\infty}$$

* For AP Calculus BC, these functions will also include functions whose limits yield the indeterminate forms:

$$0 \cdot \infty \ , \ 1^{\infty} \ , \ \infty^{\infty} \text{, and } \infty - \infty$$

APC.12 The student will apply the derivative to solve problems, including tangent and normal lines to a curve, curve sketching, velocity, acceleration, related rates of change, Newton's method, differentials and linear approximations, and optimization problems.

APC.13 The student will find the indefinite integral of algebraic, exponential, logarithmic, and trigonometric functions. The special integration techniques of substitution (change of variables) and integration by parts will be included.

*AP Calculus BC will also include integration by trigonometric substitution and integration by partial fractions (only linear factors in the denominator).

APC.14 The student will identify the properties of the definite integral. This will include the Fundamental Theorem of Calculus and the definite integral as an area and as a limit of a sum as well as the fundamental theorem:

$$\frac{d}{d(x)} \int_a^x f(t)\, d(t) = f(x)$$

* AP Calculus BC will include composite functions defined by integrals, e.g.,

$$f(x) = \int_0^{x^2} e^{-t^2} \, d(t)$$

APC.15 The student will apply the definite integral to solve problems. These problems will include finding distance traveled on a line and velocity from acceleration with initial conditions, growth and decay problems, solutions of separable differential equations, the average value of a function, area between curves, volumes of solids of revolution about the axes or lines parallel to the axes using disc/washer and shell methods, and volumes of solids with known cross-sectional areas.

*AP Calculus BC will also include areas bounded by polar curves.

APC.16 The student will compute an approximate value for a definite integral. This will include numerical cal-

culations using Riemann Sums and the Trapezoidal Rule.

*AP Calculus BC will also utilize Simpson's Rule.

*APC.17 The student will find the derivatives of vector functions and parametrically defined functions and use them to solve problems. The problems will include tangent and normal lines to parametrically defined curves, velocity and acceleration, and velocity and acceleration vectors for motion on a plane curve.

*APC.18 The student will use integration to solve problems. This will include areas bounded by polar curves, length of a path (including parametric curves), work (Hooke's law), and improper integrals.

*APC. 19 The student will define and test for convergence of a series of real numbers and of functions. This will include geometric series, comparison (including limit comparison), ratio, root, and integral tests, absolute and conditional convergence, alternating series and error approximation, and p-series.

*APC. 20 The student will define, restate, and apply power series. This will include addition, substitution, term-by-term differentiation and integration, interval of convergence, Taylor's series, Maclaurin series expansions, and Taylor polynomials with remainder and Lagrange error approximation.

* For those students who are enrolled in AP Calculus BC.

Computer Mathematics

This Computer Mathematics course is intended to provide students with experiences in using the computer to solve problems which can be set up as mathematical models. Students who successfully complete the standards for this course may earn high school mathematics credit. It is recognized that many students will gain computer skills in other mathematics courses or in a separate curriculum outside of mathematics and prior to high school. In such cases, the standards indicated by an asterisk (*) should be included in the student's course of study and treated as a review for those students who enroll in Computer Mathematics.

Even though computer ideas should be introduced in the context of mathematical concepts, problem solving per se should be developed in the most general sense, making the techniques applicable by students in many other environments. Strategies include defining the problem; developing, refining, and implementing a plan; and testing and revising the solution. Programming, ranging from simple programs involving only a few lines to complex programs involving subprograms, should permeate the entire course.

These standards identify fundamental principles and concepts in the field of computer science. Students will develop and refine skills in logic, organization, and precise expression that will enhance learning in other disciplines.

The standards that follow are separated into two groups: those related to programming concepts—Standards 1 through 21—and those dealing with mathematical applications—Standards 22 and 24. This separation is not intended to suggest that they be treated separately in the instructional program. Programming concepts, problem-solving strategies, and mathematical applications should be integrated throughout the course.

*COM.1 The student will describe the program development cycle: defining the problem, planning a solution, carrying out the plan, debugging the program, and providing program documentation.

*COM.2 The student will write program specifications that define the constraints of a given problem. These specifications include descriptions of pre-conditions, post-conditions, the desired output, analysis of the available input, and an indication as to whether or not the program is solvable under the given conditions.

*COM.3 The student will design a step-by-step plan (algorithm) to solve a given problem. The plan will be in the form of a program flowchart, pseudo code, a hierarchy chart and/or data flow diagram.

*COM.4 The student will use operating system commands, which include creating a new file, opening an existing file, saving a file, making a printed copy (hard copy) of the file, and executing a program.

*COM.5 The student will divide a given problem into manageable sections (modules) by task and implement the solution. The modules will include an appropriate user-defined function, subroutines, and procedures. Enrichment topics can include user-defined libraries (units) and object-oriented programming.

*COM.6 The student will design and implement the input phase of a program, which will include designing screen layout and getting information into the program by way of user interaction, data statements (BASIC), and/or file input. The input phase also will include methods of filtering out invalid data (error trapping).

*COM.7 The student will design and implement the output phase of a computer program, which will include designing output layout, accessing a variety of output devices, using output statements, and labeling results.

COM.8 The student will design and implement computer graphics, which will include topics appropriate for the available programming environment as well as student background. Students will use graphics as an end in itself, as an enhancement to other output, and as a vehicle for reinforcing programming techniques.

COM.9 The student will define simple variable data types that include integer, real (fixed and scientific notation), character, string, and Boolean.

COM.10 The student will use appropriate variable data types, including integer, real (fixed and scientific notation), character, string, and Boolean. This will also include variables representing structured data types.

*COM.11 The student will describe the way the computer stores, accesses, and processes variables, including the following topics: the use of variables versus constants, variables addresses, pointers, parameter passing, scope of variables, and local versus global variables. This will also include use of terminology, including memory, CPU, RAM, ROM, baud, byte, bits, floppy disc, and hard drive.

COM.12 The student will translate a mathematical expression into a computer statement, which involves writing assignment statements and using the order of operations.

COM.13 The student will select and implement built-in (library) functions in processing data, which include trigonometric functions, absolute value functions, random number functions, end of line, end of file, and string.

COM.14 The student will implement conditional statements that include if/then, if/then/else, case statements, and Boolean logic.

COM.15 The student will implement a loop, including iterative loops, pretest loops, and post-test loops. Other topics will include single entry point, single exit point, preconditions, post-conditions and loop invariance.

*COM.16 The student will select and implement appropriate data structures, including arrays (one-dimensional and/or multidimensional), files, and records. Implementation will include creating the data structure, putting information into the structure, and retrieving information from the structure.

*COM.17 The student will implement pre-existing algorithms, including sort routines, search routines, and animation routines.

COM.18 The student will test a program using an appropriate set of data. The set of test data should be appropriate and complete for the type of program being tested.

COM.19 The student will debug a program using appropriate techniques (e.g., appropriately placed controlled breaks, the printing of intermediate results, and other debugging tools available in the programming environment), and identify the difference between syntax errors and logic errors.

COM.20 The student will properly document a program including the preconditions and post-conditions of program segments, input/output specifications, the step-by-step plan, the test data, a sample run, and the program listing with appropriately placed comments.

COM.21 The student will design, write, test, debug, and document a complete structured program which requires the synthesis of many of the concepts contained in previous standards.

*COM.22 The student will solve practical consumer problems that involve analyzing and interpreting graphs, charts, and/or tables.

COM.23 The student will solve mathematical problems using formulas, equations, and functions. Problems will include those related to geometry, business, and leisure (e.g., sports and recreational activities).

COM.24 The student will solve probability, data analysis, and statistical problems.

Optional Mathematics Courses

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Commonwealth of Virginia
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Probability And Statistics

The following standards outline the content for a one-year course in Probability and Statistics. If a semester course is desired, the standards with an asterisk (*) would apply. Students enrolled in this course are assumed to have mastered the concepts identified in the Standards of Learning for Algebra II. The purpose of the course is to present basic concepts and techniques for collecting and analyzing data, drawing conclusions, and making predictions.

A graphing calculator is essential for every student taking the Probability and Statistics course and is required for the Advanced Placement Statistics Examination. The calculator may not fully substitute for a computer, however. In the absence of a computer for student use, teachers may consider providing students with examples of computer output generated by a statistical software package.

The standards for the full-year course are consistent with the 1997-1998 College Board Advanced Placement Course Description Syllabus. Teachers should update course content as changes occur in future College Board publications.

*PS.1 The student will analyze graphical displays of data, including dotplots, stemplots, and histograms to identify and describe patterns and departures from patterns utilizing central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

*PS.2 The student will analyze numerical characteristics of univariate data sets to describe patterns and departure from patterns utilizing mean, median, mode, variance, standard deviation, interquartile range, range, and outliers. Appropriate technology will be used to calculate statistics.

*PS.3 The student will compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features. Appropriate technology will be used to generate graphical displays.

*PS.4 The student will analyze scatterplots to identify and describe the relationship between two variables using shape; strength of relationship; clusters; positive, negative or no association; outliers and influential points. Appropriate technology will be used to generate scatterplots and to identify outliers and influential points.

*PS.5 The student will find and interpret linear correlation and use the method of least squares regression to model the linear relationship between two variables and use the residual plots to assess linearity. Appropriate technology will be used to compute correlation coefficients and residual plots.

PS.6 The student will make logarithmic and power transformations to achieve linearity. Appropriate technology will be used.

PS.7 The student will analyze categorical data, using two-way tables, to describe patterns and departure from patterns and to find marginal frequency and relative frequencies, including conditional frequencies.

*PS.8 The student will describe the methods of data collection in a census, sample survey, experiment, and observational study and identify an appropriate method for a given problem setting.

*PS.9 The student will plan and conduct a survey. The plan will address sampling techniques (e.g., simple random and stratified) and methods to reduce bias.

PS.10 The student will plan and conduct an experiment. The plan will address control, randomization, and measurement of experimental error.

*PS.11 The student will compute and distinguish between permutations and combinations and use technology for applications.

*PS.12 The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive.

*PS.13 The student will find probabilities (relative frequency and theoretical) including conditional probabilities for events that are either dependent or independent, by applying the "law of large numbers" concept, the addition rule, and the multiplication rule.

*PS.14 The student will develop, interpret, and apply the binomial probability distribution for discrete random variables, including computing the mean and standard deviation for the binomial variable.

PS.15 The student will simulate probability distributions, including binomial and geometric.

PS.16 The student will identify random variables as independent or dependent and find the mean and standard deviations for sums and differences of independent random variables.

*PS.17 The student will identify properties of a normal distribution and use a table or graphing calculator to apply the normal distribution to determine probabilities.

PS.18 The student, given data from a large sample, will find and interpret point estimates and confidence interval for parameters. The parameters will include proportion and mean, difference between two proportions, and difference between two means (independent and paired).

PS.19 The student will apply and interpret the logic of an hypothesis testing procedure. Tests will include large sample test for proportion, mean, difference between two proportions, difference between two means (independent and paired) and Chi-square test for goodness of fit, homogeneity of proportions, and independence.

PS.20 The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter, and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means.

PS.21 The student will identify properties of a t-distribution and use tables or graphing calculators to apply t-distributions to single sample and two sample (independent and matched pairs) t-procedures.

Discrete Mathematics

The following standards outline the content for a one-year course in Discrete Mathematics. If a semester course is desired, the standards with an asterisk (*) would apply. Students enrolled in Discrete Mathematics are assumed to have mastered the concepts outlined in the Standards of Learning for Algebra II.

Discrete mathematics may be described as the study of mathematical properties of sets and systems that have a countable (discrete) number of elements. With the advent of modern technology, discrete (discontinuous) models have become as important as continuous models. In this course, the main focus is problem solving in a discrete setting. Techniques that are not considered in the current traditional courses of algebra, geometry, and calculus will be utilized. As students solve problems they will analyze and determine whether or not a solution exists (existence problems), investigate how many solutions exist (counting problems), and focus on finding the best solution (optimization problems). Connections will be made to other disciplines.

The importance of discrete mathematics has been influenced by computers. Modern technology (graphing calculators and/or computers) will be an integral component of this course.

*DM.1 The student will model problems using vertex-edge graphs. The concepts of valence, connectedness, paths, planarity, and directed graphs will be investigated. Adjacency matrices and matrix operations will be used to solve problems (e.g., food chains, number of paths).

*DM.2 The student will solve problems through investigation and application of circuits, cycles, Euler Paths, Euler Circuits, Hamilton Paths, and Hamilton Circuits. Optimal solutions will be sought using existing algorithms and student-created algorithms.

*DM.3 The student will apply graphs to conflict resolution problems such as map coloring, scheduling, matching, and optimization. Graph coloring and chromatic number will be used.

*DM.4 The student will apply algorithms, such as Kruskal's, Prim's, or Dijkstra's, relating to trees, networks, and paths. Appropriate technology will be used to determine the number of possible solutions and to generate solutions when a feasible number exists.

*DM.5 The student will use algorithms to schedule tasks to determine a minimum project time. The algorithms will include critical path analysis, the list-processing algorithm, and student-created algorithms.

*DM.6 The student will solve linear programming problems. Appropriate technology will be used to facilitate the use of matrices, graphing techniques, and the Simplex method of determining solutions.

*DM.7 The student will analyze and describe the issue of fair division (e.g., cake cutting, estate division). Algorithms for continuous and discrete cases will be applied.

DM.8 The student will investigate and describe weighted voting and the results of various election methods. These may include approval and preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Codorcet winners.

DM.9 The student will identify apportionment inconsistencies that apply to issues such as salary caps in sports and allocation of representatives to Congress. Historical and current methods will be compared.

DM.10 The student will use the recursive process and difference equations with the aid of appropriate technology to generate

- compound interest;
- sequences and series;
- fractals;
- population growth models; and
- Fibonacci sequence.

DM.11 The student will describe and apply sorting algorithms and coding algorithms used in storing, processing, and communicating information. These will include

- bubble-sort, merge sort, and network sort; and
- ISBN, UPC, Zip, and banking codes.

DM.12 The student will select, justify, and apply an appropriate technique to solve a logic problem. Techniques will include Venn diagrams, truth tables, and matrices.

DM.13 The student will apply the formulas of combinatorics in the areas of

- the fundamental counting principle;
- knap-sack and bin-packing problems;
- permutations and combinations; and
- pigeon-hole principle.