Modeling Division of Fractions

**Reporting Category**  Number and Number Sense and Computation and Estimation

**Topic**  Modeling division of fractions

**Primary SOL**  
6.4 The student will demonstrate multiple representations of multiplication and division of fractions.

6.6 The student will
   a) multiply and divide fractions and mixed numbers; and
   b) estimate solutions and then solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions.

**Materials**
- Paper
- Writing utensils

**Vocabulary**
- multiplication, division, fraction, rectangle, equivalent (earlier grades)

**Student/Teacher Actions (what students and teachers should be doing to facilitate learning)**

“Models can help students clarify ideas that are often confusing in a purely symbolic mode.” ~ Van de Walle, Lovin

Modeling is an imperative first step in building a long-term retention of the concepts and the connected procedures associated with multiplication and division of fractions. Teachers are encouraged to scaffold instruction multiplication and division of fractions by reviewing the concept of multiplication and division with students. For multiplication, remind students that when developing the concept of 3 x 4 in earlier grades, the words “3 groups of 4” were used to describe the symbolic relationship. For division, \( \frac{12}{3} \) symbolically represents the question “How many groups of 3 are in 12?” The review of these key concepts will assist students in making connections to prior knowledge when developing an understanding of multiplication and division of fractions.

1. Ask students to consider the problem “John has \( 1 \frac{1}{2} \) gallons of liquid fertilizer concentrate.

   Each batch of fertilizer requires \( \frac{3}{4} \) gallon of liquid fertilizer concentrate. How many batches can John make?” The problem is asking how many batches can John make (each batch using \( \frac{3}{4} \) gallon concentrate) from \( 1 \frac{1}{2} \) gallons concentrate.
Have students work in pairs. Ask students to model $1 \frac{1}{2}$ gallons of liquid fertilizer concentrate.

![Diagram of 1.5 gallons]

Have students talk with their partners about this problem and share their thoughts with each other. They should think about how many $\frac{3}{4}$s are in 1 then model their thinking.

![Diagram of 1 gallon divided into quarters]

2. Students might say there is 1 and a little or $\frac{1}{4}$ left over. Knowing that you have to find the number of $\frac{3}{4}$ gallons in $1 \frac{1}{2}$ gallons, ask students how they would find out how many $\frac{3}{4}$ gallons are in the rest of the shaded portion. Students might modify the model to help them think about this problem. Ask students to share their ideas with the whole class. Highlight the idea that draws the model in $\frac{1}{4}$s as shown below.

![Diagram of 1 gallon divided into quarters]

3. Students should see that there is another batch ($\frac{3}{4}$ gallons concentrate) in the model. Have students work with a partner to model their solution.

![Diagram of 1 gallon divided into quarters]

4. Students should answer that John can make 2 batches of fertilizer from $1 \frac{1}{2}$ gallons of liquid fertilizer concentrate.
5. Restate the problem, asking students how many \( \frac{3}{4} \) gallons in \( \frac{6}{4} \) gallons? You can see that there are 2 batches of \( \frac{3}{4} \) gallon groups in \( \frac{6}{4} \) or \( 1 \frac{1}{2} \) gallons which can be symbolically written \( 1 \frac{1}{2} \div \frac{3}{4} \) or \( \frac{3}{2} \div \frac{3}{4} \) or \( \frac{6}{4} \div \frac{3}{4} \). Hold a class discussion.

6. Have students work in groups to represent the following problems on paper. Select a group of students for each problem to demonstrate their representation and their thinking to the class.

\[
\begin{align*}
1 \frac{1}{3} & \div 1 \frac{1}{2} & \frac{3}{4} & \div 1 \frac{1}{3} & \frac{3}{5} & \div \frac{1}{4} & 2 \frac{1}{2} & \div \frac{5}{6}
\end{align*}
\]

Hold a class discussion after everyone has had the opportunity to work on all of the problems.

7. Students can begin to relate the models being used to the algorithm. Have students discuss their models, solutions, and patterns that they noticed. This discussion can lead in to the algorithm or procedure for dividing fractions. Help students to make the connection.

Assessment

- **Questions**
  - How are division of fractions and multiplication of whole numbers alike?
  - How does division of fractions relate to the area model you drew?
  - What other representations might you use to solve a division of fractions problem? (arrays, paper folding, repeated addition, repeated subtraction, fraction strips, pattern blocks)
  - When dividing fractions, what is the meaning of the operation?

- **Journal/Writing Prompts**
  - Explain how division of fractions was used to solve the fertilizer concentrate problem.
  - Describe how you would use an area model to solve the problem \( \frac{2}{3} \div \frac{1}{4} \).

Extensions and Connections (for all students)

- Have students complete additional division of fractions problems, using other representations, such as arrays, paper folding, repeated addition, repeated subtraction, fraction strips, or pattern blocks. Have them compare the models.

- Use index cards with a division of fractions problem written on one card (e.g., \( \frac{3}{4} \div \frac{1}{3} \)) and a pictorial representation of that problem (e.g., a rectangle divided and shaded appropriately) on another card. Make several of these pairs, mix the cards, and lay them
face down on a table. Pair students to play a “memory game” in which they must select two cards and match pairs.

- Discuss real-world situations in which division of fractions is used.
- Invite a contractor who tiles kitchens and bathrooms to visit the classroom and discuss how he or she plans for the number of different tiles when making patterns. Have him/her discuss how multiplication and/or division of fractions is important to this type of work.

**Strategies for Differentiation**

- Use grid paper with premade rectangles for students to use when solving the problems in their groups.
- Have students use fraction tiles or other manipulatives in the activities to help them relate to the pictorial models.
- When pairing students or dividing them into small groups, ensure that students with different abilities are put together.