What Are the Chances?

Reporting Category	Probability and Statistics	
Topic	Investigating and describing the theoretical and experimental probabilities	
Primary SOL	7.9	The student will investigate and describe the difference between the experimental probability and theoretical probability of an event.
Related SOL	7.10	

Materials

- Coins
- Number cubes
- What are the Chances? activity sheet (attached)
- Calculators

Vocabulary

probability, outcome, sample space, impossible event, certain event, equally likely, tree diagram, simple event, independent event, dependent event (earlier grades)

theoretical probability, experimental probability, Law of Large Numbers (7.9)

compound event, Fundamental (Basic) Counting Principle (7.10)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Show students a coin, and ask what the possible outcomes are when a coin is flipped (i.e., heads, tails). Ask students what the chances of flipping heads are. Write their responses on the board, and discuss the different representations (i.e., 1/2, 0.50, 50%). Have students explain their responses. (This should include a discussion of the formula they used for finding probability.)
2. Show students a number cube, and ask what the possible outcomes are when a number cube is rolled (i.e., 1, 2, 3, 4, 5, 6). Ask students what the chances of rolling a 3 are. Write their responses on the board, and discuss the different representations (i.e., 1/6, approx 0.17, and approx 17%). Have students explain their responses.
3. Show students two number cubes and ask them what the possible outcomes are when both cubes are rolled and their sum is taken (there are 36 possible outcomes). Ask students what the chances of rolling a sum of 9 are. Write their responses on the board, and discuss the different representations (i.e., 4/36 = 1/9, approx 0.11, and approx 11%). Have students explain their responses.
4. Ask students to express these probabilities in statements. For example, "If I flip a coin two times, one of those times I should flip heads," or, "When I flip a coin, I should get heads 50% of the time." Emphasize that the probabilities they have identified represent theoretical probability. Discuss the concept, and share with students how theoretical probabilities are calculated:

   $$\text{theoretical probability} = \frac{\text{number of possible favorable outcomes}}{\text{total number of possible outcomes}}$$

5. Ask students if they think the theoretical probability for heads will hold true if we flip a coin 10 times. Demonstrate this and record the results. Ask students what the probability of flipping heads was. Discuss if this was the same or different than the theoretical probability they already established. Explain that the probability they got after flipping the coin 10 times is called experimental probability, which results from calculating probability using the results of an experiment. Discuss how this differs from theoretical probability. Share with students how experimental probabilities are calculated:

   $$\text{experimental probability} = \frac{\text{number of times desired outcomes occur}}{\text{number of trials in the experiment}}$$

6. Distribute the What Are the Chances? activity sheet. Explain that students will be evaluating each of these games of chance. They will first predict what they think will be the most frequent outcome for each game of chance, then perform the experiment 10 times. For each trial they will record the actual outcome and state whether it matches their prediction.
7. When students have finished their experiments, have them answer questions 1-3. Discuss the results as a class. Students should note that their experimental probabilities did not match their theoretical ones very well. Discuss the importance of sample size with students, and have them identify situations in which sample size would be important. Ask students to determine how they could get a better sample size with their games of chance (by combining class data).
8. Gather class data to complete question 4. Have students complete questions 5 and 6 and discuss their responses. Students should see that the more trials that are performed, the closer their experimental probability will be to the theoretical probability (Law of Large Numbers).

Assessment

- **Questions**
  - Why is it useful to know about probability?
  - What is the difference between the theoretical and experimental probability of an event?
  - How does the experimental probability of an event change as the number of trials increases?
- **Journal/Writing Prompts**
  - Explain why some people refer to the number seven as “lucky seven.”
  - Write an explanation of the two types of probability for someone who has never heard of them.
- **Other**
  - Have students design their own experiment and compare theoretical to experimental results.
  - Read students a list of events, and have them decide whether they represent theoretical or experimental probability.

Extensions and Connections (for all students)

- Have students create their own games of chance and have classmates determine whether the games are fair or not, using what they know about theoretical and experimental probability.
- Set up stations with spinners and playing cards and have students explore the theoretical and experimental probability of different events, such as spinning a certain color or number or choosing cards by color, suit, or number and suit.

Strategies for Differentiation

- Allow students to use online versions of the manipulatives in the lesson to explore theoretical and experimental probability.
- Have each game with directions and questions on a separate piece of paper so students only have to focus on one activity at a time.
- Print the What Are the Chances? activity sheet using a landscape format so there is more room for students to write.

What Are the Chances?

Name _____ **Date** _____

For each game of chance, predict what will be the most frequent outcome. Circle your choice. Perform the experiment 10 times. For each trial, record the actual outcome in the "Result" row. If this matches what you predicted, put a check mark in the "Prediction" row.

Game 1: Flip a coin

Prediction for the most frequent outcome: Heads Tails

Result										
Prediction										

Game 2: Roll 1 Number Cube

Prediction for the most frequent outcome: 1 2 3 4 5 6

Result										
Prediction										

Game 3: Sum of 2 Number Cubes

Prediction for the most frequent outcome: 2 3 4 5 6 7 8 9 10 11 12

Result										
Prediction										

Questions

1. In which game of chance were your predictions most accurate?

2. Complete the table below with the theoretical probability for each event. Then use the results from your experiments to calculate the experimental probability for each event.

Game of Chance	**Event**	**Theoretical Probability**	**Experimental Probability**
Flip a Coin	Tails		
Roll 1 Number Cube	4		
Roll 2 Number Cubes	8		

3. Compare the theoretical and experimental probabilities for each game of chance. Were you close in any of the experiments?

4. Collect and record the data from the entire class for each game of chance.

Game of Chance	**Event**	**Class Experimental Probability**
Flip a Coin	Tails	
Roll 1 Number Cube	4	
Roll 2 Number Cubes	8	

5. Are the experimental probabilities different in questions 2 and 4? Why, or why not?

6. How do the theoretical probabilities in question 2 compare to the experimental probabilities in question 4? What do you think would happen if more trials were conducted?