Organizing Topic: Quadratic Modeling

Mathematical Goals:

- Students will model quadratic relationships using data.
- Students will analyze quadratic functions.
- Students will transform the quadratic parent function, making relationships to real-life situations.
- Students will be able to relate graphs to the corresponding tables and quadratic function.
- Students will experiment with changing the elements of quadratic functions and will predict the changes in the outcome of the function that includes obtaining different roots and different maximum or minimum points.
- Students will state the number of roots, the coordinates of the vertex, and how the parabola opens when given an equation in vertex form.
- Students will state any limitations to the domain and range of a quadratic function and how it relates to real-life situations.
- Students will make predictions using the curve of best fit for the modeled data as well as evaluating for specific values in the domain of the data.

Standards Addressed: AFDA.1; AFDA.2; AFDA.3; AFDA.4

Data Used: Data obtained using Web sites, surveys, experiments, and textbook

Materials:

- Graphing calculator applications:
  - **EasyData/Ranger**™
  - **Transformation™**
- CBR™ or CBR2™
- Graphing calculator
- Handout – Quadratic CBR™ Exploration
- Handout – Solutions, Factors, Line Of Reflection, …
- Handout – Quadratic Investigation Vertex Form
- Handout – Triangular Numbers
- Handout – Terry's Time
- Handout – Quadratics Playground Exploration
- Handout – Quadratics Web Page Exploration
- May also need graph paper and concrete objects such as two-color chips or snap blocks

Instructional Activities:

I. Quadratic CBR™ Exploration

The exploration encourages students to relate four different physical movements to a graph that is illustrated using a parabola.

Concepts will include:

- domain and range;
- scatter plots;
- distance-time graphs;
- curve of best fit; and
- relationship between movements and the graphs produced.

II. Solutions, Factors, Line Of Symmetry

The activity relates the factors of *c*, the constant value of a trinomial, to the solution(s) of the equation and the factors of the trinomial. The relationship between the linear coefficient and the line of symmetry will be explored. Lastly, the relationship between minimum points of the family of parabolas with solutions that are factors of *c* are explored.

Concepts will include:

- domain and range;
- intercepts;
- zeros of the function;
- solutions of the equation;
- line of symmetry;
- maximum and minimum points;
- relationship between zeros, factors, and the equation; and
- curve of best fit.

III. Quadratic Investigation Vertex Form

Students will investigate each type of transformation on the parent quadratic graph.

Concepts will include:

- domain and range;
- intercepts;
- zeros;
- line of symmetry;
- intervals in which the graph is increasing and decreasing;
- maxima and minima;
- (h, k) or vertex form of the equation;
- how the graph opens;
- translations;
- reflections;
- dilations; and
- rotations.

IV. Triangular Numbers

Students will explore the quadratic equation that represents the sequence commonly known as ***triangular numbers***.

Concepts will include:

- domain and range;

- scatter plots;
- patterns;
- relationship between the numerical differences in the values of the independent variable and the equation representing the data;
- relationship between written expressions and symbols; and
- systems of equations.

V. Terry's Time

Terry is running a relay race as her coach records her position at various times. Students will explore the rate of change at various time intervals; relate what is happening physically to the graph and the equation that best represents Terry's movement. Graphical analysis, the equation of the curve of best fit, and the associated table of values will be explored.
Concepts will include:

- domain and range;
- rate of change;
- scatter plots;
- maximum and minimum points;
- distance-time graphs;
- curve of best fit;
- line of symmetry;
- intervals in which the function is increasing and decreasing; and
- relating real world situations to graphs.

VI. Quadratics Playground Exploration

Given a specific amount of fencing, students will determine the size of a playground that will maximize the usable area. Analysis of the graph, the curve of best fit, and the table of values representing the situation will be explored.
Concepts will include:

- domain and range;
- rate of change;
- scatter plots;
- area-width and length-width graphs;
- maximum and minimum points (optimization);
- curve of best fit;
- line of symmetry;
- intervals in which the function is increasing and decreasing;
- relating real-life situation to graphs;
- relating graphs of linear data to quadratic data; and
- using data to make decisions.

VII. Quadratics Web Page Exploration

A picture is placed on a Web site, starting as a dot and growing as time elapses. Relationships are made with length versus time and area versus time when the picture is in the shape of a square. The challenge increases as the picture is inserted as a rectangle. Relationships between time and length, time and height, time and area, and length and area are explored.

Concepts will include:

- domain and range;
- rate of change;
- scatter plots;
- area-time and length-time graphs;
- maximum and minimum points (optimization);
- curve of best fit;
- line of symmetry;
- intervals in which the function is increasing or decreasing;
- relating real-life situation to graphs;
- relating graphs of linear data to quadratic data; and
- using data to make decisions.

Activity I: Teacher Notes--Quadratic CBR™ Exploration

Assign students to small groups of two to four. The settings on the CBR™ should be in feet and set to gather a maximum of 20 pieces of data taken in 0.5 second intervals. There are four different activities.

1. Students will hold the CBR™ unit parallel to the floor and as high as they can safely. Two movements may be assigned to each group or different groups can be responsible for one movement and then all groups can compare results. Using a hand or a book, students will move the object, starting near the unit and moving down and then back toward the unit. Then the student will reverse the movements, starting low at the floor, moving upward toward the unit, then back down again. Students will record data in the table of values and graph the results. Questions are on the handout regarding object movements and the equation that represents the scenario.
2. Students will place the CBR™ unit on the floor facing up. Two movements may be assigned to each group or different groups can be responsible for one movement and then all can compare their results. Using their hand or a book, students will move the object, starting near the unit and moving upward and then back toward the unit. Then the student may reverse the movements, starting high, moving downward toward the unit, then back up again. Students will record data in their table of values and graph their results. Questions are on the handout regarding their movements and the equation that represents the scenario.

3. One student will hold the CBR™ perpendicular to the floor facing another student. Nothing should be in the path between the CBR™ and the student that might interfere with the data recording. Two movements may be assigned to each group or different groups may be responsible for one movement and then all can compare their results. The student will stand near the unit, walk away, and then walk back again. The walker does not physically turn around. It is best if students get to a point and then walk backwards toward the unit. The student will stand ***x*** feet away from the unit and, when the data begins to record, will walk toward the unit and then walk back again. The walker does not physically turn around. It is best if they get to a point and then walk backwards toward the unit. Students will record data in their table of values and graph the results. Questions on the handout address student movements and the equation that represents the scenario.
4. The walker will cross the room while the holder of the unit stays stationary, pivoting slowly so that the unit follows the walker. Students will record data in a table of values and graph the results. Questions are on the handout regarding their movements and the equation that represents the scenario.

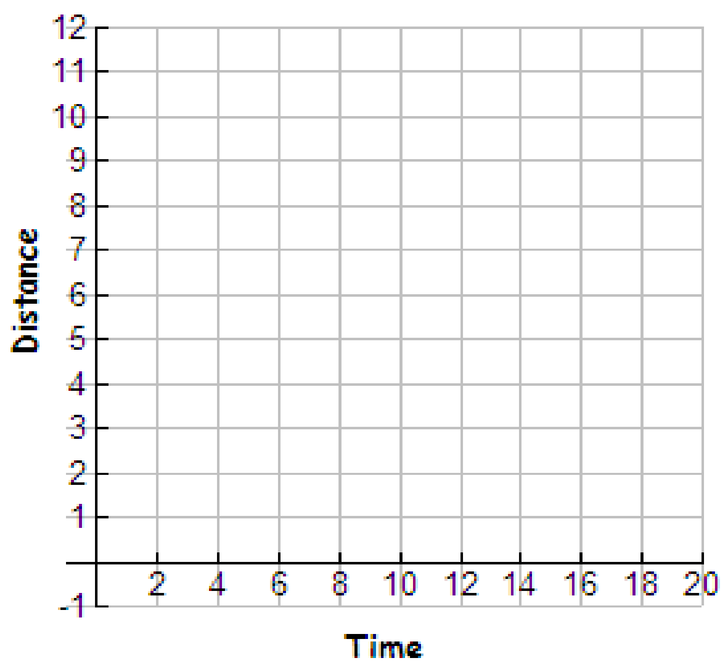
Quadratic CBR™ Exploration

Connect the CBR™ to the graphing calculator and begin the **Ranger**™ program or **EasyData**™. In the setup menu, change the units to feet.

Collecting the data.

1. Aim the CBR™ downward toward the floor.
   - Begin with a ball, a book, or even your hand, on the floor and slowly raise it upward toward the CBR™ and then back again to the floor OR begin with your hand or object close to the CBR™, lower to the floor and then back up again.
   - Attempt to maintain the same speed of movement going downward and upward.
   - Record the values below and plot the points on the graph. Be sure to select points that will enable you to illustrate the curve (points while moving upward and points while moving downward).

Time	Distance



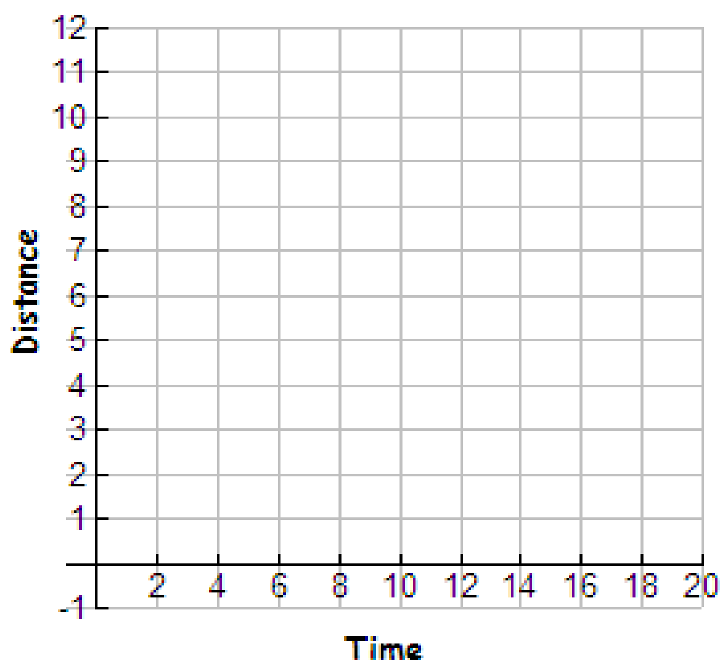
2. Describe the graph created by your movements. Relate movements to the points and the overall shape.

3. Determine the equation of the curve of best fit using **Transformation™** Application.

4. Determine the equation of the curve of best fit using regression on your calculator.

5. Place the CBR™ on the floor facing upward. Begin with a ball, a book, or even your hand, several feet above the CBR™.
   - Lower the object or your hand toward the CBR™ and then back up again OR
   - Begin at the unit and raise the object or your hand and then lower it back toward the unit.
   - Attempt to maintain the same speed of movement going downward and upward.
   - Record the values below and plot the points on the graph. Be sure to select points that will enable you to illustrate the curve (points while moving upward and points while moving downward).

Time	Distance



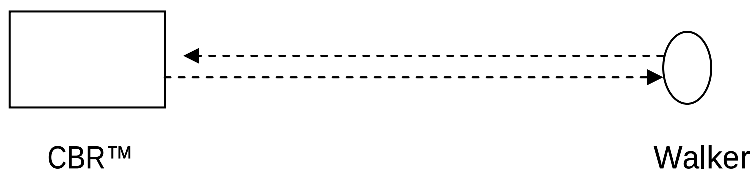
6. Describe the graph created by your movements. Relate movements to the points and the overall shape.

7. Determine the equation of the curve of best fit using the **Transformation™** Application.

8. Determine the equation of the curve of best fit using regression on your calculator.

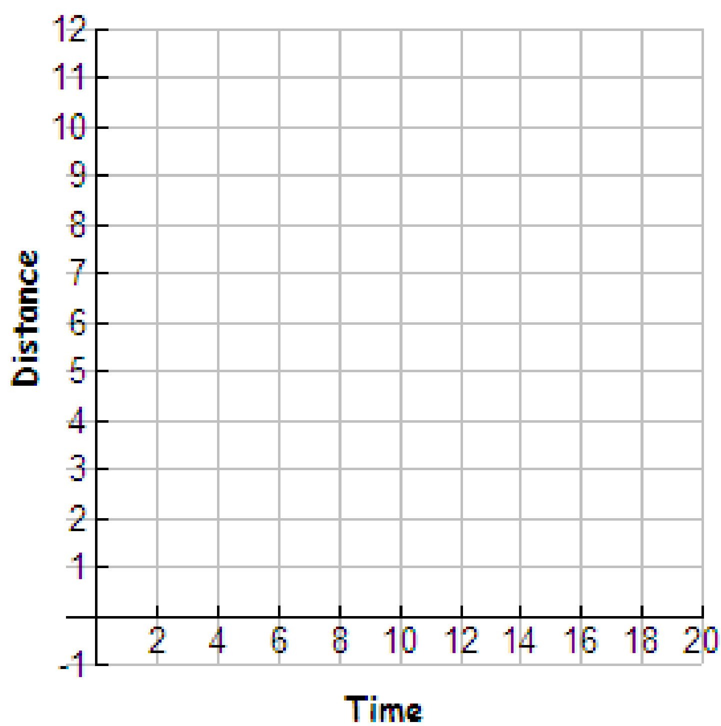
9. Compare the results from questions 1-4 and 5-8.

Collecting the Data



This exploration involves the CBR™ being pointed at a walker. The walker will either move away from the CBR™ and then back toward it or the reverse. When the walker begins moving he/she needs to begin slowing their pace as they get near the point where their moving direction will change and then speed up slowly the farther they get away from the turning point on the return trip. The walker's body does not turn – he/she needs to walk facing the same direction at all times and walk backwards when needed. Walking movements may also be reversed.

Time	Distance

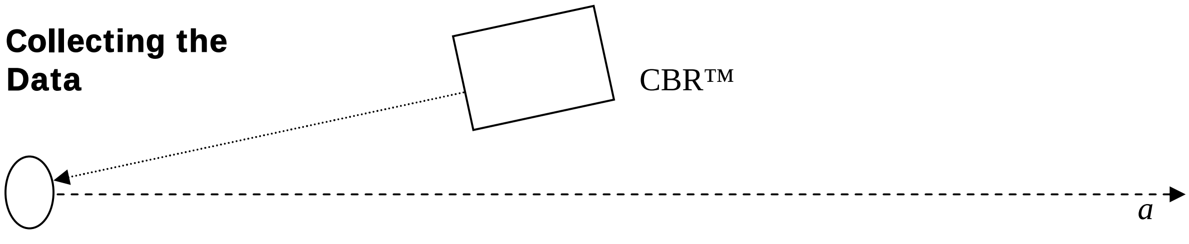


10. Describe the graph created by your movements. Relate your movements to the points and the overall shape.

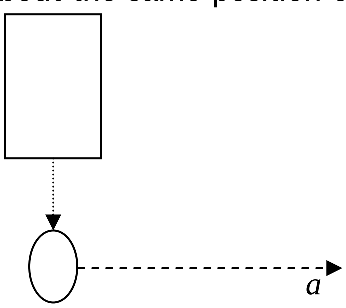
11. Determine the equation of the curve of best fit using **Transformation**™ Application.

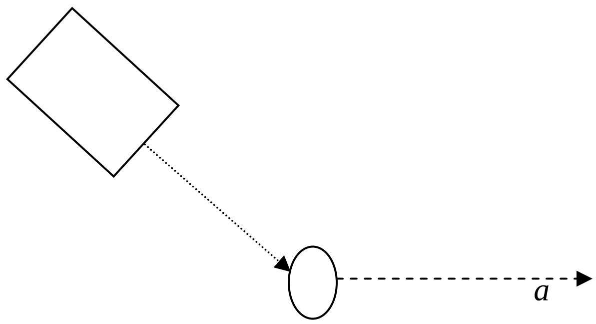
12. Determine the equation of the curve of best fit using regression on your calculator.

Collecting the Data



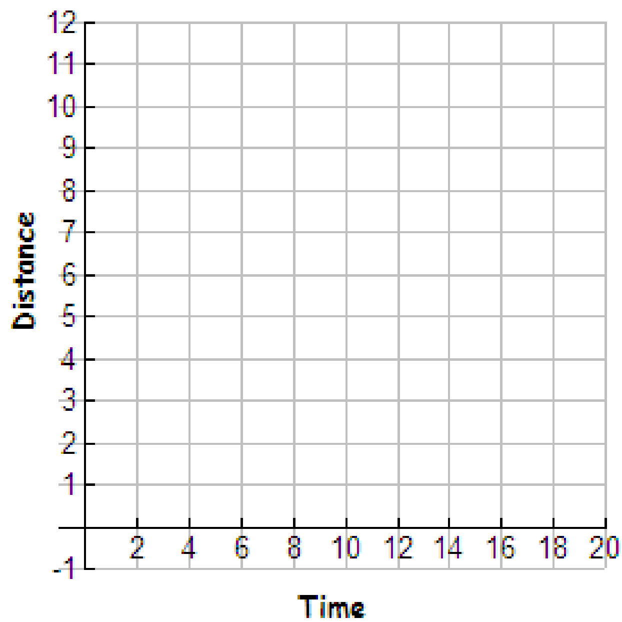
During this exploration an individual will be walking along line *a*. As the person gets closer to being directly in front of the CBR™ their speed should decrease and then increase as they continue on their path. At the same time the person holding the CBR™ will continually rotate so that the CBR™ will always be pointing at the walker at about the same position on their body.





13. Record the values and graph the data.

Time	Distance



14. Describe the graph created by your movements. Relate movements to the points and the overall shape.

15. Determine the equation of the curve of best fit using **Transformation**™ Application.

16. Determine the equation of the curve of best fit using regression on your calculator.

17. Compare your results.

Fitting the Equation

18. Select one or more of the above explorations. Record the data in **L1** and **L2** under **[Stat] [Edit]**. The graphing calculator will automatically store the residuals, **RESID**, under **[2nd] [Stat]** (**Residual** = **Actual** – **Fitted** values). The closer the sum of the residuals is to zero, the better the fit.
You have already determined the equation of the curve of best fit. See the specific exploration for the regression equation. Turn on **Stat Plot 1** to activate the scatter plot for graph. Press **[2nd][Graph]** to view the table of values for the curve of best fit and complete the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

Turn on **Stat Plot 2** to activate another scatter plot showing the residuals. If this was a perfect fit, **Stat Plot 2** values would all be on the x-axis. Why?

Use your calculator and, using linear regression, determine the equation of the curve of best fit for the given data. Repeat the above process recording the values in the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

Compare and contrast the data in the two tables.

Activity II: Teacher Notes--Solutions, Factors, Lines of Reflection

Students will relate solutions of quadratic equations to factors of quadratic expressions as part of quadratic functions, and zeros of quadratic functions. The solutions are all factors of a specific integer. Students will discover the relationship between average value of the solutions to the optimal point and line of reflection. Lastly, students will relate all the quadratic equations that have solutions that are factors of the same integer by plotting all the graphs on the same coordinate plane. Their minimum points will be plotted for each and the equation determined that contain these elements.

Solutions, Factors, Lines of Reflection

Factors

1. Name all the factors of -36.

2. What is the equation of the line that is the x-axis?

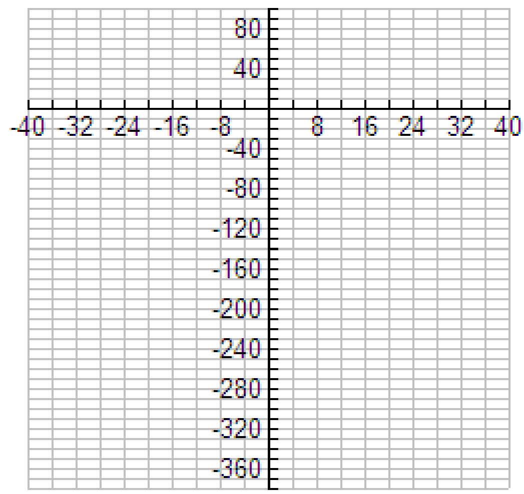
3. Using the graphing calculator, graph the following trinomial given in question **a** as **y1** and your response to question 2 as **y2**.

   a) $\mathbf{y = x^2 - 35x - 36}$

   b) Determine the point abscissas where the graph crosses the x-axis.
   ______ and ______

   c) Use the graphing calculator to determine the coordinates of the minimum point for the quadratic recorded in **y1**. (,)

   d) Graph **y1** on the coordinate plane provided using your answers to questions **b** and **c** as an aid.



y = x^2 - 35x -36

   e) Set each of the values found in question b) equal to ***x***. These ***x*** values are called ***solutions***.

   ______________________ and ______________________

   f) Solve the equation implied in part **e**.

   g) Determine the mean of the two ***x*** values given in part **e**.

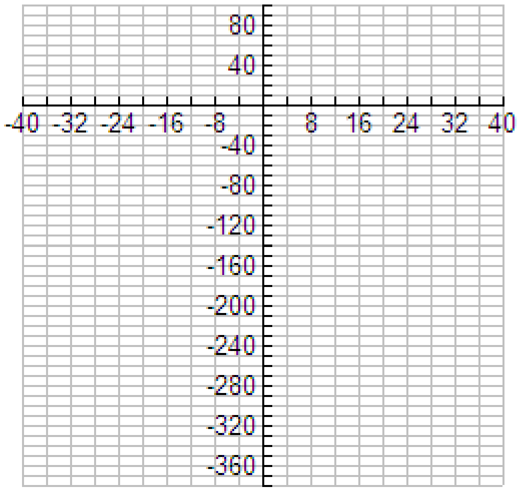
   h) Each of the expressions that are set equal to zero in part **f** are called **factors** of the original graphed polynomial. Multiply the two factors from part **f**.

   i) Recall that the general form of the trinomial is $\mathbf{y = ax^2 + bx + c}$ where **a**, **b**, and **c** are real values. Determine the value of the linear coefficient, ***b*****,** and substitute into the equation $x = \frac{-b}{2}$.

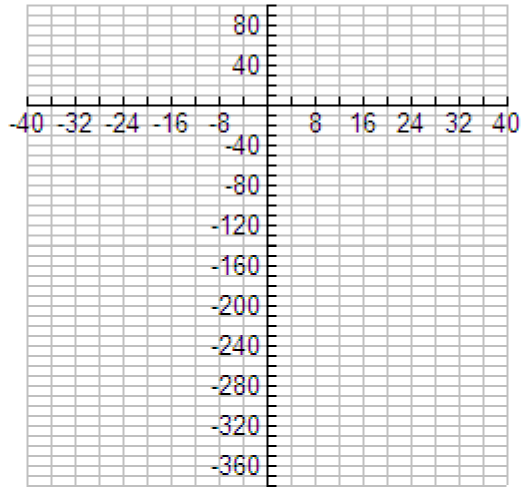
   j) On the graph, draw a dotted line that represents the equation in part **i**.

4. Repeat the activity in question 3 for each of the following by completing the chart and graphing each on the next page.

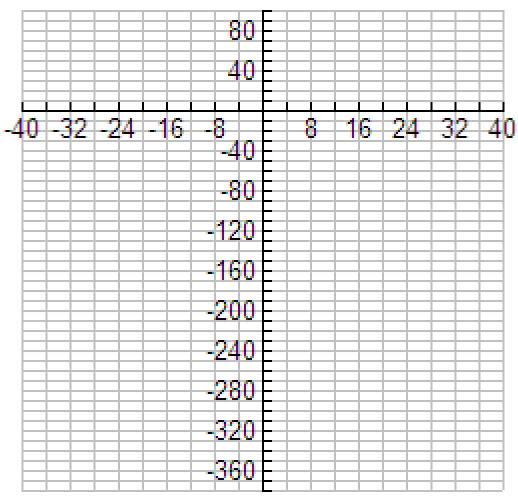
	Trinomial - A	B x - values	C Minimum Point	D See next page	E Solutions	F Set = 0	G Multiply Factors	H x = $\frac{-b}{2}$
1.	$y = x^2 - 35x - 36$							
2.	$y = x^2 - 16x - 36$							
3.	$y = x^2 - 9x - 36$							
4.	$y = x^2 - 5x - 36$							
5.	$y = x^2 - 36$							
6.	$y = x^2 + 35x - 36$							
7.	$y = x^2 + 16x - 36$							
8.	$y = x^2 + 9x - 36$							
9.	$y = x^2 + 5x - 36$							



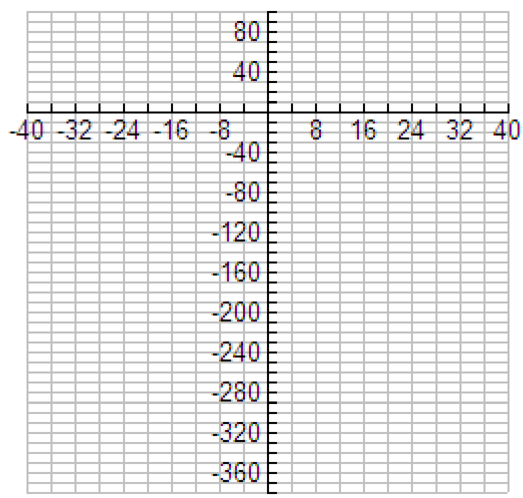
y = x^2 - 16x -36



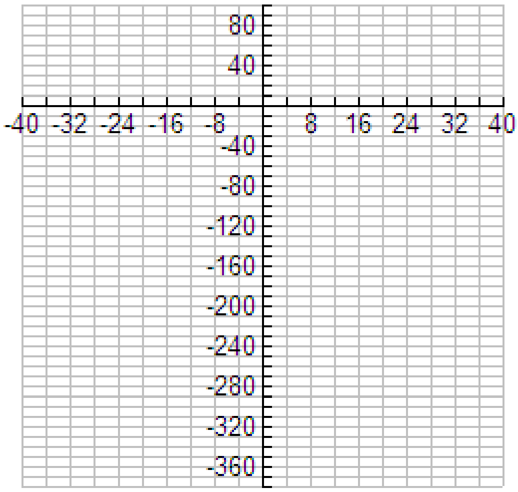
y = x^2 - 9x -36



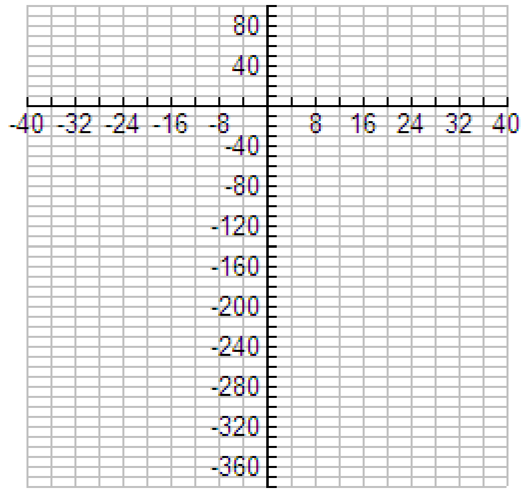
y = x^2 - 5x -36



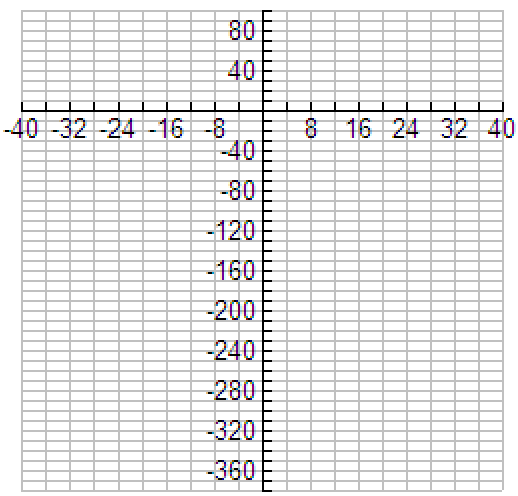
y = x^2 -36



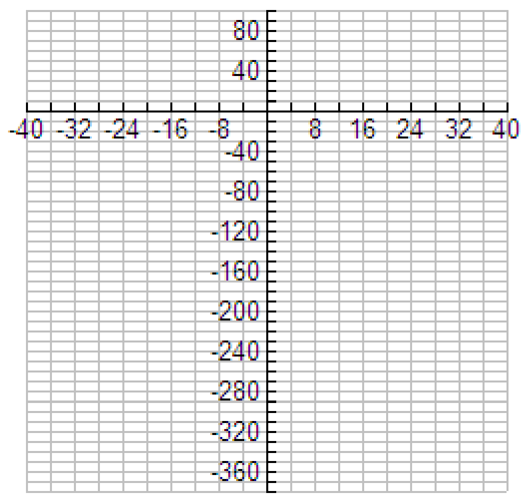
$y = x^2+16x -36$



$y = x^2+35x -36$



$y = x^2+9x -36$



$y = x^2 +5x -36$

Information from Graphs

5. Discuss the answers to part **b** and any relationships to other parts in the exploration.

6. Compare the results and relationships between parts **b, c, e, f,** and **h**.

7. Compare the results to part **c** and their relationship to part **i**.

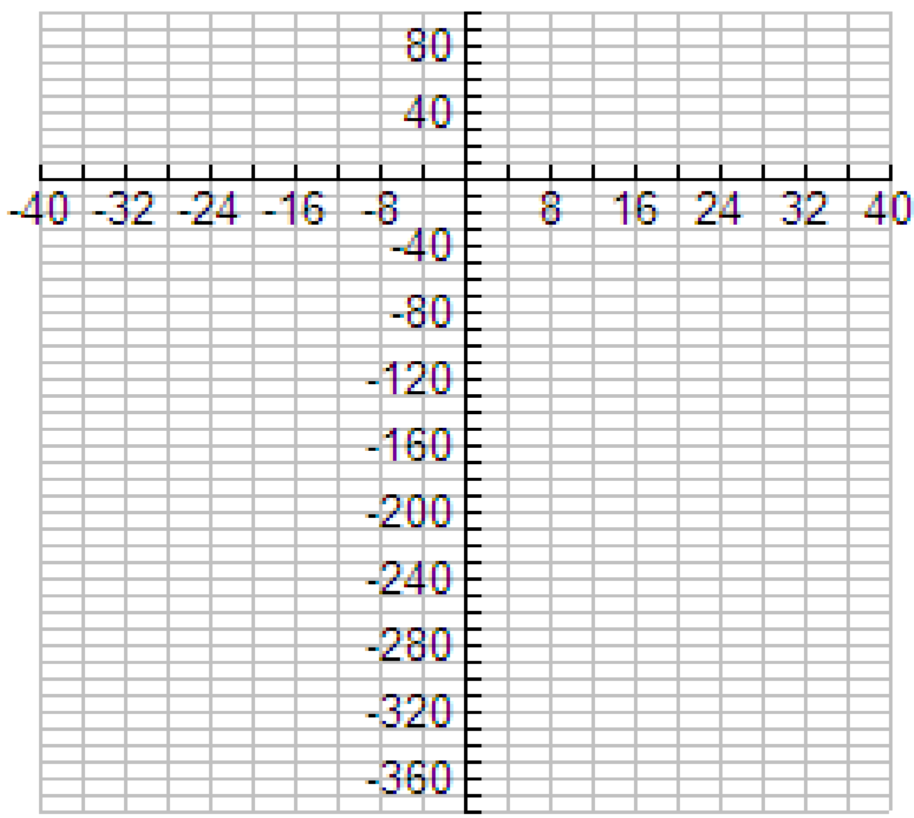
8. Discuss the relationship between parts **g**, **i,** and **j**.

More graphing

9. Draw each of the graphs on the same coordinate plane below.

10. On each of the graphs, plot the minimum point and record the coordinates of the minimum points in the table.

Minx	Miny



11. Draw the curve that connects the minimum points of the graph.

Parent Functions and Minima

12. Using the graphing calculator, record each of the minimum points using the **List** function.

    L1 = minimum x value, **L2** = minimum y value

13. What type of parent function does the curved line formed by connecting the minimum points illustrate?

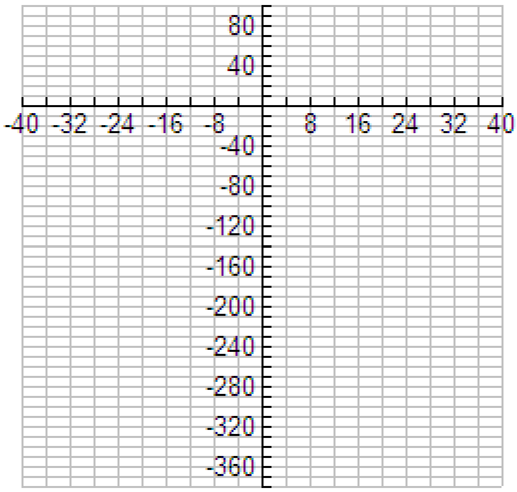
14. Use the **Transformation™** Application and the general equation of a trinomial to determine the best fitting curve that contains all of the minimum points.

    Record the equation of the curve of best fit. y = ________________________

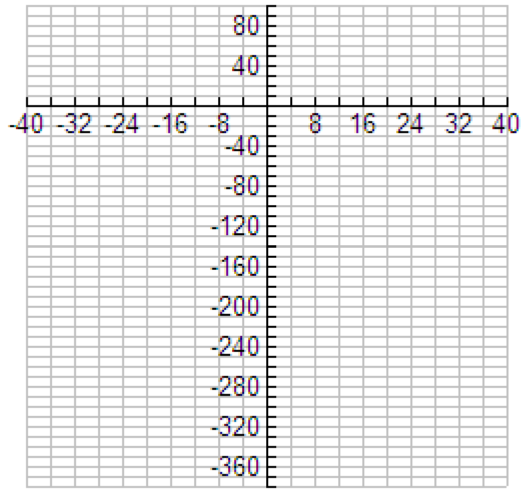
    Summarize this exploration in your own words. Be sure to think about each section and how it might relate to other sections.

15. Repeat the entire exploration using the factors of positive 36.

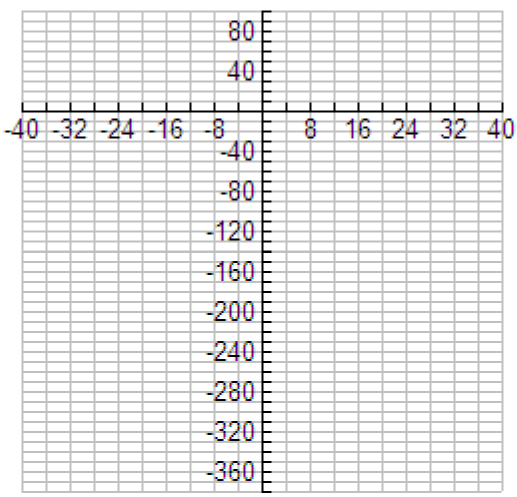
	Trinomial - A	B x - values	C Minimum Pt.	D See next page	E Solutions	F Set = 0	G Multiply Factors	H x = -b/2
1.	$y = x^2 - 37x + 36$							
2.	$y = x^2 - 20x + 36$							
3.	$y = x^2 - 15x + 36$							
4.	$y = x^2 - 13x + 36$							
5.	$y = x^2 - 12x + 36$							
6.	$y = x^2 + 37x + 36$							
7.	$y = x^2 + 20x + 36$							
8.	$y = x^2 + 15x + 36$							
9.	$y = x^2 + 13x + 36$							
10.	$y = x^2 + 12x + 36$							



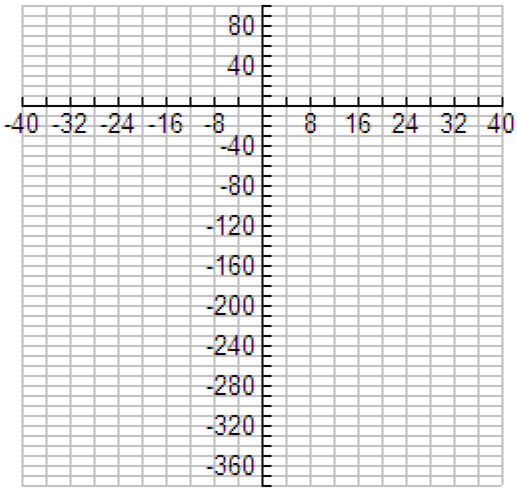
y = x^2 - 37x + 36



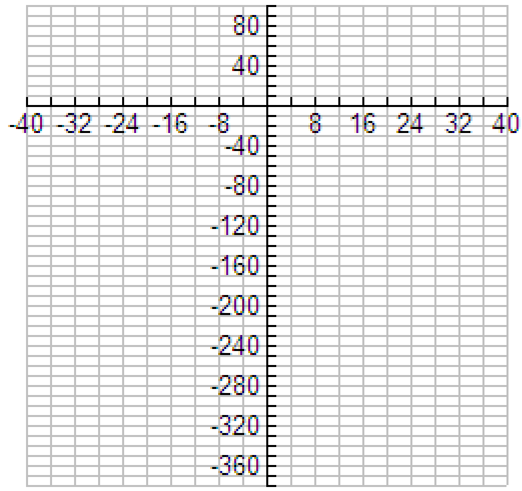
y = x^2 - 20x + 36



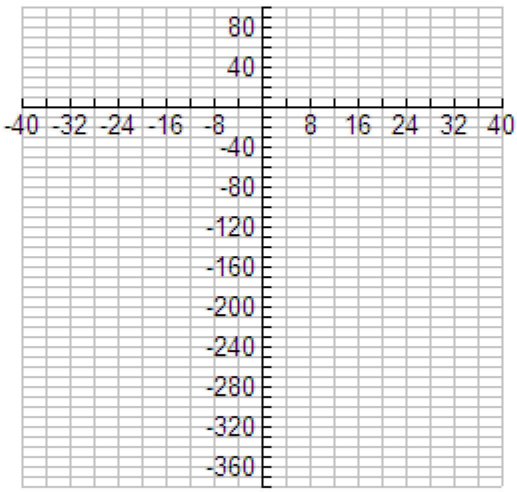
y = x^2 - 15x + 36



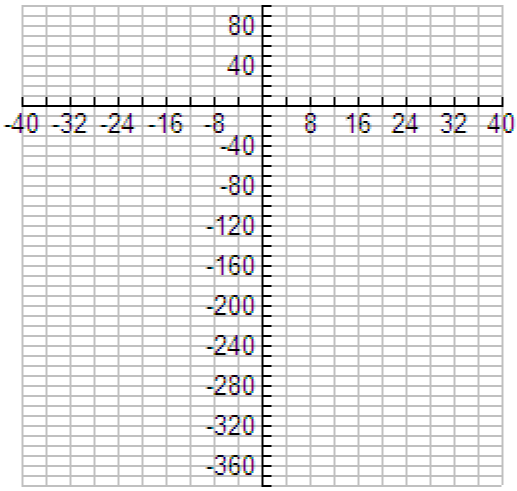
y = x^2 - 13x + 36



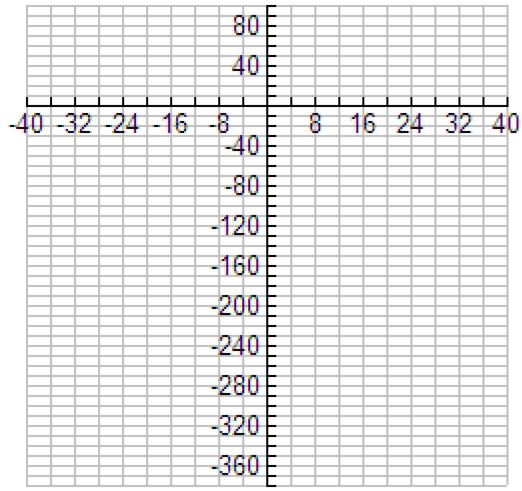
y = x^2 - 12x + 36



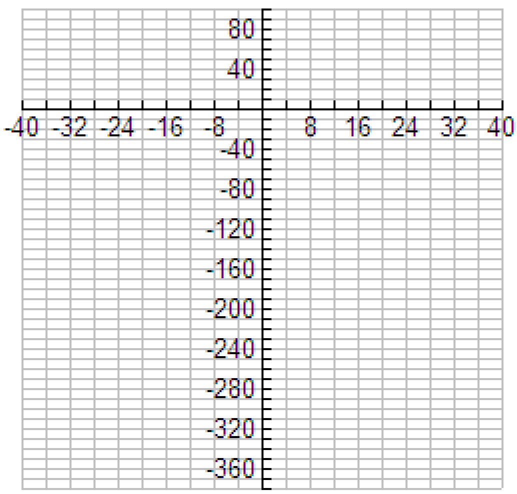
y = x^2 + 37x + 36



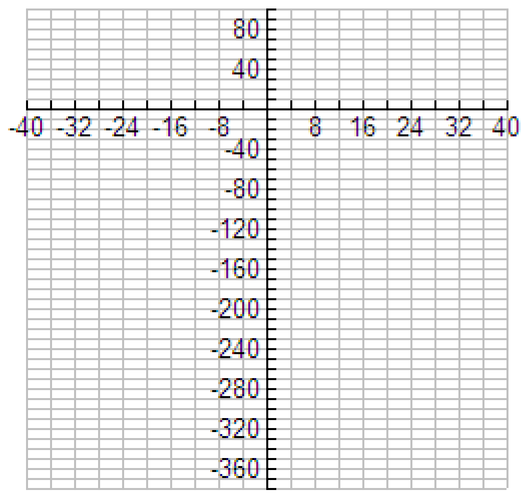
y = x^2 + 20x + 36



y = x^2 + 15x + 36



y = x^2 + 13x + 36



y = x^2 + 12x + 36

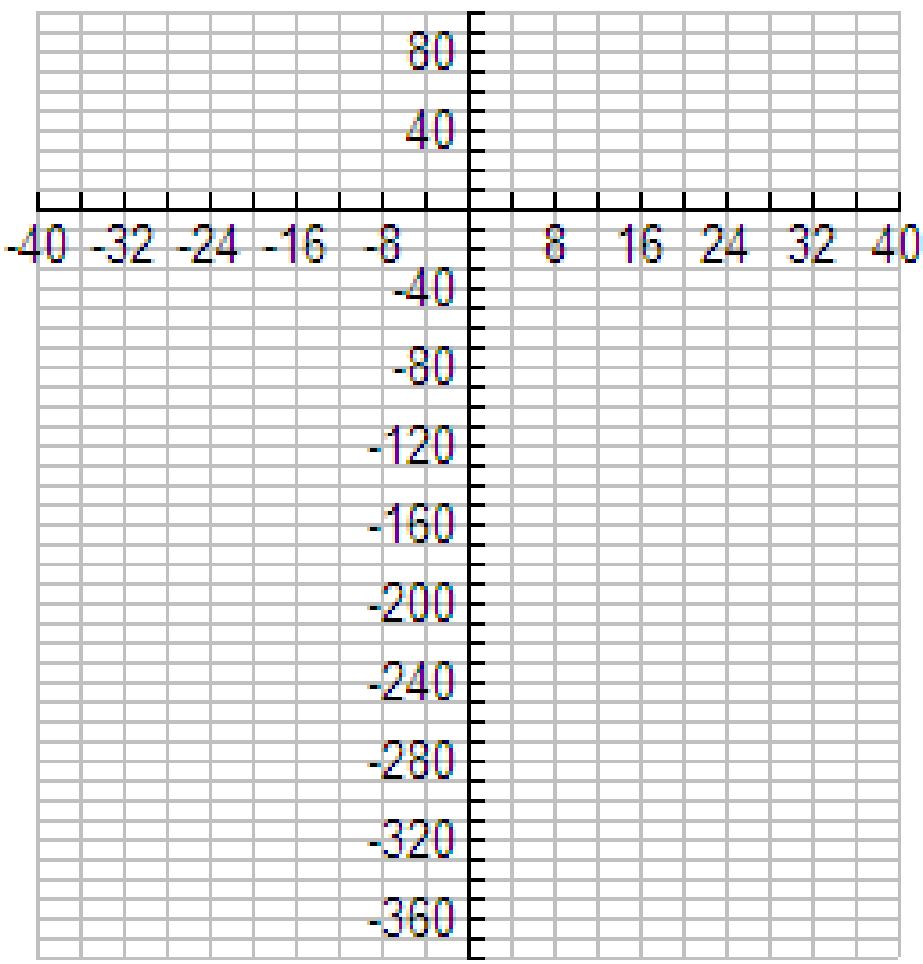
16. Discuss the answers to part **b** and any relationships to other parts in the exploration.

17. Compare the results to part **c** and their relationship to part **i**.

18. Discuss the relationship between parts **g**, **i,** and **j**.

Analysis of Graphs

19. Draw each of the graphs on the same coordinate plane below. On each of the graphs, plot the minimum point and record the minimum points on the table.



Minx	Miny

21. Draw the curved line that connects the minimum points of each graph.

More Analysis of Graphing

22. Using the graphing calculator, record each of the minimum points using the list function.
    L1 = minimum x-value, **L2** = minimum y-value

23. What type of parent function does the curve formed by connecting the minimum points illustrate?

24. Use the **Transformation**™ Application and the general equation of a trinomial to determine the best fitting curve that contains all of the minimum points.
    Record the equation of the curve of best fit. y = ________________________

25. Summarize this exploration in your own words. Be sure to think about each phase and how it might relate to other sections.

Activity III: Teacher Notes--Quadratic Investigation Vertex Form

Students will graph the parent quadratic equation in vertex form. Then, taking the ***a*, *h*,** and ***k*** (one at a time) and changing the value, the student will re-graph the transformed equation. The new graph will be explored and related to the parent function.

Quadratic Investigation Vertex Form

Graphing
Turn on the **Transformation™** Application on the graphing calculator. In **y1,** type in ***a*** **(x -** ***b*****)**^{2} **+** ***c***.

I.

Quadratic Equation	Domain	Range	Zeros	x-intercept	y-intercept	Line of Symmetry	Max/Min Point	Increasing	Decreasing
$y = \boldsymbol{1}(x - \boldsymbol{0})^2 + \boldsymbol{0}$									
$y = \boldsymbol{2}(x - \boldsymbol{0})^2 + \boldsymbol{0}$									
$y = \boldsymbol{5}(x - \boldsymbol{0})^2 + \boldsymbol{0}$									

1. Describe what is happening to the graph when $a > 1$.

2. Describe the graphs by comparing and contrasting the items analyzed above.

Quadratic Equation	Domain	Range	Zeros	x-int.	y-int.	Line of Sym.	Max/Min Point	Increasing	Decreasing
$y = \boldsymbol{-1}\,(x - \boldsymbol{0})^2 + \boldsymbol{0}$									
$y = \boldsymbol{-2}\,(x - \boldsymbol{0})^2 + \boldsymbol{0}$									
$y = \boldsymbol{-5}\,(x - \boldsymbol{0})^2 + \boldsymbol{0}$									

3. Describe what is happening to the graph when $\boldsymbol{a < 0}$.

4. Describe the graphs by comparing and contrasting the items analyzed above.

Quadratic Equation	Domain	Range	Zeros	x-intercept	y-intercept	Line of Symmetry	Max/Min Point	Increasing	Decreasing
$y \quad \frac{1}{2} \ x \quad 0^{2} \quad 0$									

Quadratic Equation	Domain	Range	Zeros	x-intercept	y-intercept	Line of Symmetry	Max/Min Point	Increasing	Decreasing
$y = \boldsymbol{1}\,(x - \boldsymbol{1})^2 + \boldsymbol{0}$									
$y = \boldsymbol{1}\,(x - \boldsymbol{2})^2 + \boldsymbol{0}$									
$y = \boldsymbol{1}\,(x - \boldsymbol{5})^2 + \boldsymbol{0}$									

7. Describe what is happening to the graph when $\boldsymbol{b > 0}$.

8. Describe the graphs by comparing and contrasting the items analyzed above.

Quadratic Equation	Domain	Range	Zeros	x-intercept	y-intercept	Line of Symmetry	Max/Min Point	Increasing	Decreasing
$y = \boldsymbol{1}\,(x - \mathbf{0})^2 + {}^{-}\boldsymbol{1}$									
$y = \boldsymbol{1}\,(x - \mathbf{0})^2 + {}^{-}\boldsymbol{3}$									
$y = \boldsymbol{1}\,(x - \mathbf{0})^2 + {}^{-}\boldsymbol{7}$									

13. Describe what is happening to the graph when $\boldsymbol{c < 0}$.

14. Describe the graphs by comparing and contrasting the items analyzed above.

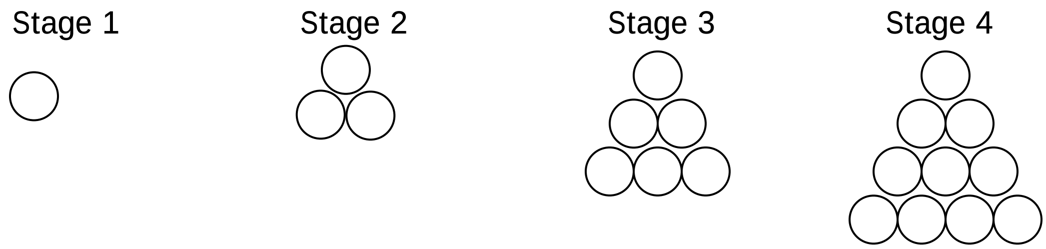
Activity IV: Teacher Notes--Triangular Numbers

Students may either draw or use concrete objects to create representations of triangular numbers through the fourth stage (iteration). Working in groups or individually, students will attempt to determine the number of objects at each stage (iteration). Students will then discover that the difference between the dependent values is not constant. By determining the differences of the last values, students will be able to acknowledge that there are two stages of differences to obtain the constant; therefore, the data will follow a quadratic model.

For enrichment, students may use three of the stage (iteration) values and the general form of the quadratic equation to determine the equation of the curve of best fit using systems of equations.

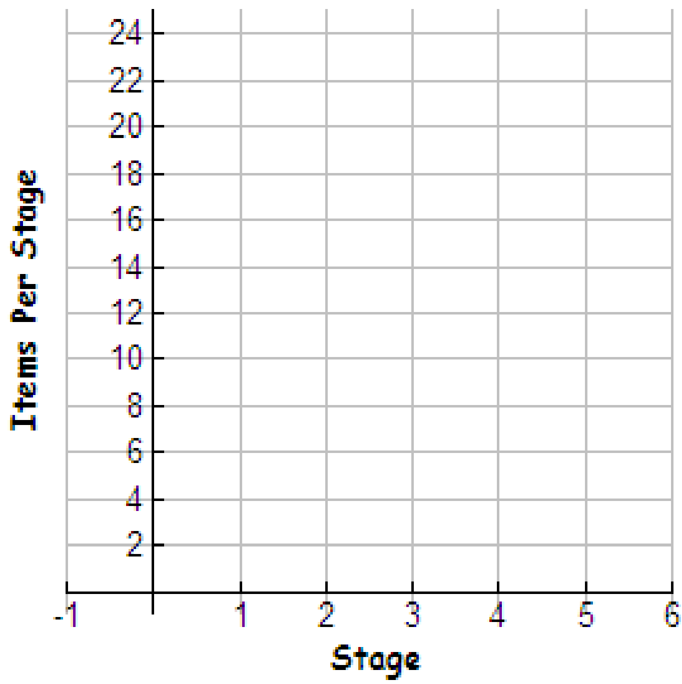
Triangular Numbers

Given the following pattern:



Collecting the Data
Complete the table below and then plot the points on the coordinate plane.

STAGE	ITEMS PER STAGE
1	
2	
3	
4	
n	



Writing About the Pattern

1. Use Stage 4 and fill in the following chart by brainstorming methods to determine the number of items at that stage. Each method must be based on that specific stage. Use the illustration to help you in your efforts.

Written explanation	**Symbols**	**Simplify expression**

STAGE	**ITEMS PER STAGE**
1	
2	
3	
4	
n	

2. Determine the difference in the values in the "Items Per Stage". Are they the same?

3. If not, continue finding the difference of the last results until the values are constant. How many times did you have to find the difference in values before those values were constant?

Graphing the Data

4. Use the **List** function on the graphing calculator and enter the data (Stage, Items Per Stage). Enter into lists as follows:
   L1 = Stage, **L2** = Items Per Stage.

5. Turn on the **[Stat Plot]** and use the **Transformation**™ Application to determine the curve of best fit.

   Record the equation of the curve of best fit y=______________________

Using the Data to Predict

6. How many items will there be at the following stages?

Stage	**Items Per Stage**
5	
10	
15	
55	
n	

The general form of the equation that represents ***triangular numbers*** is $y = ax^2 + bx + c$. The value of **x** stands for the stage and the **y**-value stands for the number of items in that stage.

7. Using the stages and information gathered in the table, substitute a stage for all the x-values and its corresponding y-values (the number of items).

   Stage 2 ____________________

   Stage 3 ____________________

   Stage 4 ____________________

8. Solve the system of equations. Use the simplified equations for stage 2 and stage 3. Eliminate the variable ***c***. Solve another system of equations. Use the equations from #3 and #4 and eliminate the variable ***b***, solving for ***a***.

9. Now that you know ***a***, use one of the equations from #3 or #4 and substitute in ***a***, solve for ***b***.

10. Now you know the values for ***a*** and ***b***. Select one of the original simplified equations from #2 and solve for ***c***.

11. Substitute the values found by substituting into the systems into the equation

    $\boldsymbol{y} = \boldsymbol{a}x^2 + \boldsymbol{b}x + \boldsymbol{c}.$ $\boldsymbol{y} =$ ________________________

12. Compare this result to the curve of best fit found using the **Transformation™** Application.

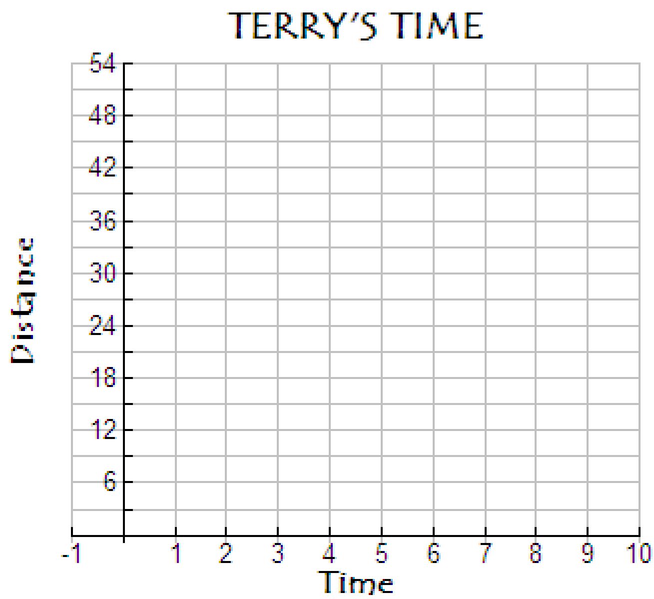
Activity V: Teacher's Notes--Terry's Time

Terry's coach has timed Terry as she runs. The coach has taken time readings at various distances from the starting line as Terry has practiced. Students will graph the data and begin analyzing the data from the table, the graph, and the actual race. The analysis will include acknowledging that the rate of change is not the same in different time intervals. Forward and backward movements and how they relate to the graph will be explored along with what is actually happening when Terry is halfway through the race.

Terry's Time

The coach at the local high school decides to videotape the practice race and time the various members of the team so they can discuss techniques. Terry's time is recorded in seconds for the 50 yard relay.

Time	Distance
0	0
1	19.753
2	34.568
3	44.444
4	49.383
5	48.383
6	44.444
7	34.568
8	19.753
9	0



Collecting and Describing the Data

1. Graph data from the table onto the coordinate plane above.

2. What would be the unit of measure for the average rate of change during the relay race?

3. How fast is Terry running between the 1st and 2nd seconds of the race?

4. How fast is Terry running between the 4th and 5th seconds of the race?

5. How fast is Terry running as she passes the 44.444 yard mark?

6. How fast is Terry running midway into the relay race?

7. When is Terry running away from the starting position?

8. When is Terry running toward the starting position?

9. When is Terry's rate of change the greatest?

10. When is Terry's rate of change the least?

11. What is happening at exactly 4.5 seconds?

12. Determine the following for the given scenario.

    Domain   Range

13. Determine an equation that will best fit the data using the **Transformation™** Application and the vertex form of the parent function typed in **y1**.

    y = ________________________________

    Let the calculator determine the regression equation. y = ___________

14. What is the equation of the line of symmetry for the graph illustrating the scenario?

15. Describe the relationships between the data represented in the table, on the scatter plot, and as elements of the function.

16. Describe in words Terry's race using the average rate of change for the various times above and how they relate to the race and the graph.

Fitting the Equation

17. The graphing calculator will automatically store the residuals, **RESID**, under **[2nd] [Stat].** The closer the sum of the residuals is to zero, the better the fit. At this time **L1** is time and **L2** is the distance traveled. You have already determined the equation of the curve of best fit, recorded in #13. Turn on **Stat Plot 1** to activate the scatter plot for (time, distance) graph. Press **[2nd] [Graph]** to view the table for the curve of best fit. Complete the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

18. Turn on **Stat Plot 2** to activate another scatter plot of (time, **RESID**). See above to enter **RESID** for the **Ylist**. If this was a perfect fit, **Stat Plot 2** values would all be on the x-axis. Why?
Use your calculator and determine the regression for the given data and repeat the process in #17, recording the values in the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

19. Compare and contrast the data in the two tables.

Activity VI: Teacher Notes--Quadratics Playground Exploration

Students are given a scenario in which they are to create a playground by using a specific length of fencing. Students are to determine the dimensions of the sides of the playground when one of the sides is the house so that the play area is maximized. Students will relate the various widths to lengths and the various widths to area graphically and will explain the relationships. An analysis of the graph and the equation representing the data will include domain, range, intervals in which the graph is increasing or decreasing, and the line of symmetry.

Quadratics Playground Exploration

Mary Ellen wants to add a playground to the backyard of her home for her small children. For convenience and safety, she wants the playground to be enclosed with a fence but one side of the play area will be bounded by the house. She went to the store and got a great deal on 60 feet of fencing. She has been trying to determine the best dimensions for the playground but is frustrated. So she decides to create a chart with the data she has already derived. Using the chart and the corresponding graph will enable Mary Ellen to make a wise decision.

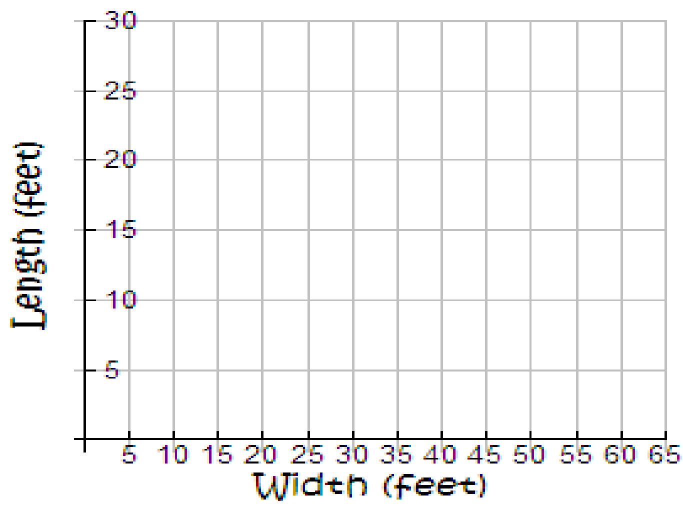
Collecting the Data

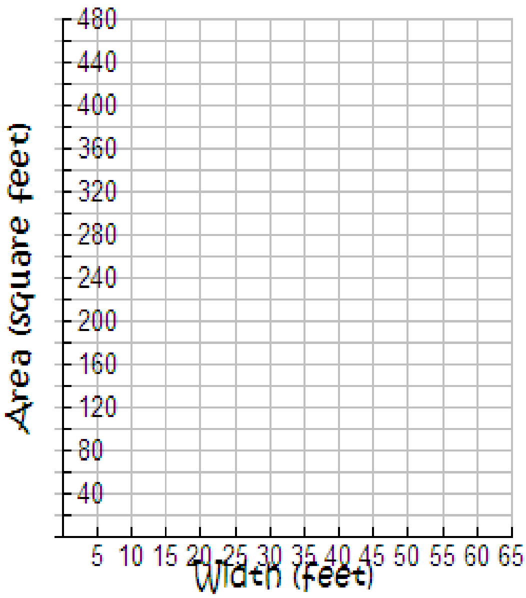
1. Complete the following table:

WIDTH	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**
LENGTH											
PERIMETER											
AREA											

2. a. Graph the relationship between the width in feet and the length in feet of the playground.

   b. Describe the change in the playground by comparing the width in feet to the length in feet.





3. Graph the relationship between width in feet of the playground to its overall area in square feet.

4. Describe the change in the playground by comparing the width in feet to the overall area in square feet.

5. What type of function models the curve relating width to the length of the playground? Explain your reasoning.

6. Determine the function that describes the curve relating width to length. Note: Use the **Transformation**™ Application on the calculator to help you.

   y = ________________________________

7. In your own words, describe how the curve relates width to length.

8. What type of function models the curve relating width to the area of the playground? Explain your reasoning.

9. Determine the function that describes the curve relating width to the area of the playground.
   Note: Use the **Transformation™** Application to help you.

   y = ________________________________

10. In your own words, describe how the curve relates width to the area of the playground.

11. You have made your decision about the best length and width of the playground.

    The maximum area will be ___________________________, which is created using the following dimensions: _______________________.

12. Describe how you might determine the answers to maximize the area and determine the dimensions using the graphs.

13. Determine the following for the given scenario.
    a. Domain Range

    b. Explain and state the equation written in (*h,k*) form that represents the data.

    c. What is the equation for the line of symmetry for the graph illustrating the scenario?

    d. Describe what is happening in the real life situation when the graph is increasing and/or decreasing.

    e. Describe the relationships between the data represented in the table, on the scatter plot, and as elements of the function.

Fitting the Equation

14. Determine the quadratic regression for the (width, area) data. The graphing calculator will automatically store the residuals, RESID, under **[2nd] [Stat].** (**Residual** = **Actual** – **Fitted**) values. The closer the sum of the residuals is to zero, the better the fit. At this time, **L1** is width and **L2** is the area of the playground.
    Turn on **Stat Plot 1** to activate the scatter plot for (width, area) graph. Press **[2nd] [Graph]** to view the table of values for the curve of best fit and complete the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

15. Turn on **Stat Plot 2** to activate another scatter plot of (width, **RESID**).
If this was a perfect fit, **Stat Plot 2** values would all be on the x-axis. Why?

Use your calculator and determine the linear regression for the given data and repeat the above process recording the values in the table below.

TIME L1	**ACTUAL DISTANCE L2**	**FITTED DISTANCE L3**	**ACTUAL – FITTED L4 = L2 – L3**	**(ACTUAL – FITTED)^2 L5 = (L4)^2**
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

16. Compare and contrast the data in the two tables.

Activity VII: Teacher's Notes--Quadratics Web Page Exploration

This activity works well with individuals or with students in small groups. A student is going to insert a picture of a car on a Web site he/she is creating. There are two different scenarios, one with the picture inserted as a square and the next time as a rectangle.

Square: The picture becomes larger (both length and width increase at a constant rate of change). Area for each set of dimensions is determined. Students will graph time versus height, determine the curve of best fit using the **Transformation™** Application by first recognizing the parent function. Comparisons between the graphs are discussed and the domain and range for this scenario is determined.

Rectangle: The picture becomes larger (both length and height increase at a rate in which the length is always twice the height). Area for each set of dimensions is determined. Students will graph time versus height and time versus length; determine the equation of the curve of best fit using the **Transformation™** Application by first recognizing the parent function. Comparisons between the graphs are discussed. Graphs will be created illustrating time versus area and length versus area. The equation of the curve of best fit will be determined using the **Transformation™** Application by first recognizing the parent function. Comparisons between the graphs are discussed. The domain and range for this scenario are determined.

Finally, given a specific size for a computer monitor, the student will determine which quadrilateral (square or rectangle) to use (the maximum dimensions of the quadrilateral that will fill most of the monitor screen).

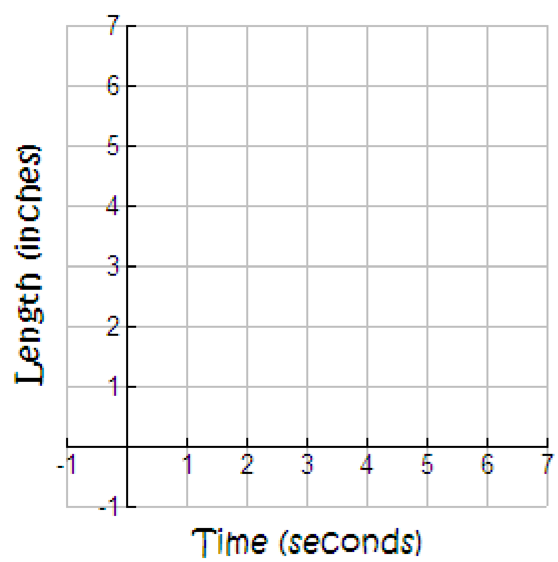
Quadratics Web Page Exploration

You are creating your own Web page and do not want it to be boring. To add some interest you decide to insert a picture of your car but it was to appear and become larger the longer it was seen on the screen. A square would be easier to work with numerically and geometrically. When your Web page comes up, you do not see the car (or the square where it will be). After the first second, the square is visible and for each additional second, the side lengths of the square will increase by an inch.

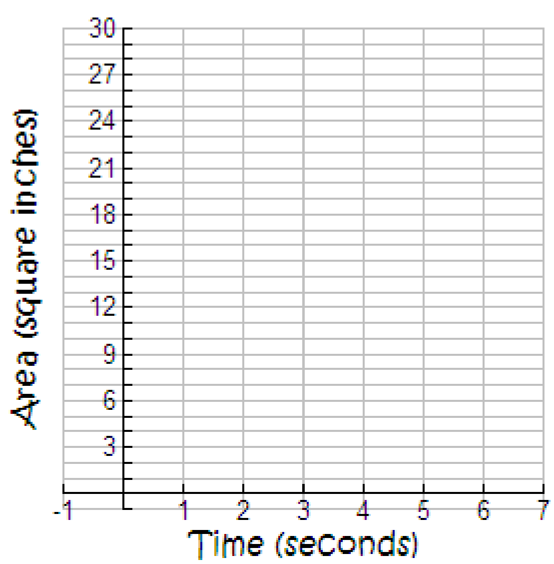
Complete the following table.

TIME	SIDE LENGTH OF IMAGE	AREA OF PICTURE
0		
1		
2		
3		
4		
5		
6		
Expression		

1. Graph the relationship between the time in seconds and the length in inches.



2. Describe the change in the picture size by comparing the time to the length.



3. Graph the relationship between time in seconds and the area in square inches.

4. Describe the change in the picture by comparing the time to the area.

5. What type of function models the curve relating time to side length?

6. Determine the function that describes the curve relating time to side length.

7. In your own words, describe how the curve relates time to side length.

8. What type of function models the curve relating time to the area of the picture?

9. Determine the function that best describes the curve relating time to the area of the picture.

10. In your own words, describe how the curve relates time to the area of the picture.

11. You love the Web page but decide to make one more change. The monitor screen is 12 inches by 9 inches and you want the picture to take up as much of the screen as possible.

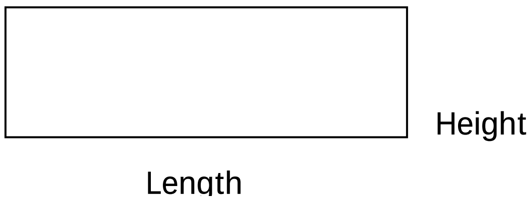
    The maximum side length is ___________________________

    The maximum area will be ____________________________

12. Describe how you might determine the above answers to maximum side length and maximum area using the graphs.

13. Discuss the possible values for the domain and for the range.

14. You still are not satisfied and decide to take it up another level. To add some interest, you decide to insert the same picture of your car but it is to appear and become larger the longer it is seen on the screen. The length will always be twice as long as its height. The rectangle is similar to the original shape of the picture and, therefore, will create a better picture on the screen. When your Web page comes up, you do not see the car (or the rectangle where it will be). After the first second the square is visible and, for each additional second, the height of the rectangle will increase by an inch.

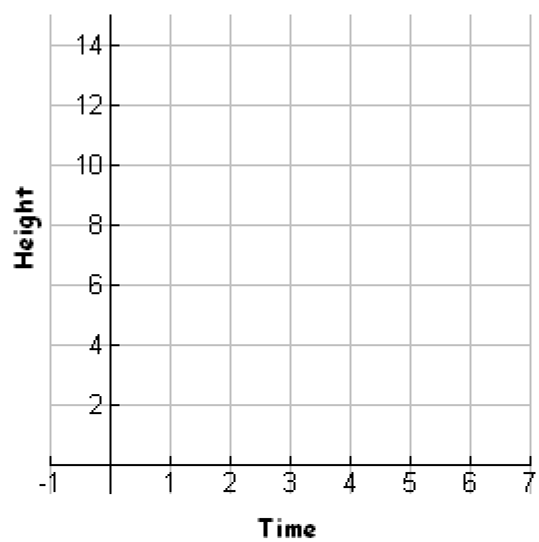


Complete the following table:

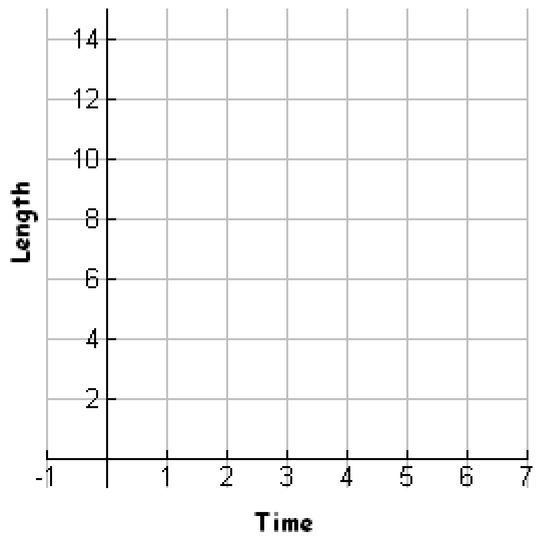
TIME	HEIGHT	LENGTH	AREA OF PICTURE
0			
1			
2			
3			
4			
5			
6			
Expression			

15. Graph the relationship between the time in seconds and the height in inches.

16. Describe the change in the picture size by comparing the time to the height.

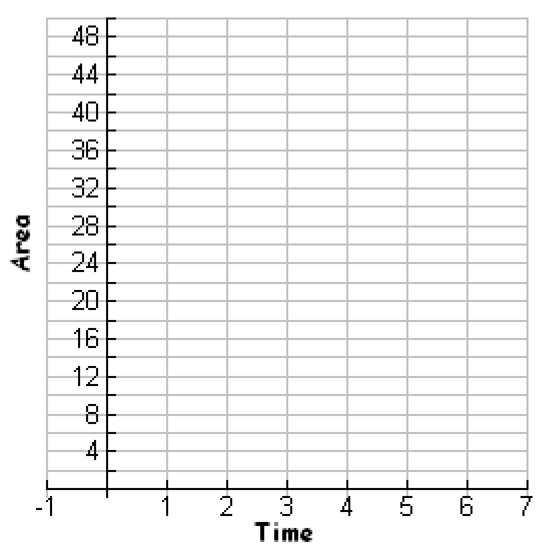


17. Graph the relationship between the time in seconds and the length in inches.



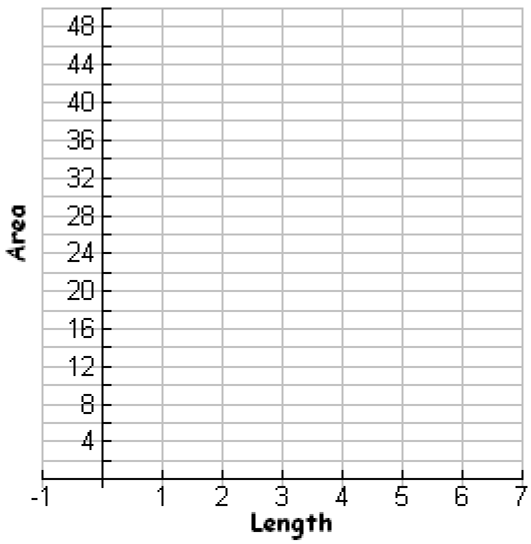
18. Describe the change in the picture size by comparing the time to the length.

19. Graph the relationship between time in seconds and the area in square inches.



20. Describe the change in the picture by comparing the time to the area.

21. Graph the relationship between length and the area in square inches.



22. Describe the change in the picture by comparing the length to the area.

23. What type of function models the curve, relating time to height?

24. Determine the function that describes the curve relating time to height.

25. What type of function models the curve relating time to length?

26. Determine the function that describes the curve relating time to length.

27. In your own words, describe how the curve relating time to height compares to the curve relating time to length.

28. What type of function models the curve relating time to the area of the picture?

29. Determine the function that describes the curve relating time to the area of the picture.

30. What type of function models the curve relating length to the area of the picture?

31. Determine the function that describes the curve relating length to the area of the picture.

32. In your own words, describe how the curve relating time to the area of the picture compares to the curve relating length to the area of the picture.

33. You love the Web page but need to put a limit on how big the picture will get. The monitor screen is 12 inches by 15 inches and you want the picture to take up as much of the screen as possible.

34. Would you use the square or the rectangle to create the picture that best adapts to the screen? Why?

    The maximum length is ______________________________

    The maximum height is ______________________________

    The maximum area will be ____________________________

    Describe how you might determine the above answers to maximum side length and maximum area using the graphs.

35. Discuss the possible values for the domain and for the range.

Fitting the Equation

36. Select either the data from time versus area or length versus area. The graphing calculator will automatically store the residuals, **RESID,** under **[2nd] [Stat].** The closer the sum of the residuals is to zero, the better the fit. At this time, **L1** is time/length and **L2** is the area. You have already determined the equation of the curve of best fit, recorded in #7 or #9.Turn on **Stat Plot 1** to activate the scatter plot for (time/length, area) graph. Press **[2nd] [Graph]** to view the table for the curve of best fit. Complete the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

37. Turn on **Stat Plot 2** to activate another scatter plot of (time, RESID). See above to enter **RESID** for the **Ylist**. If this was a perfect fit, **Stat Plot 2** values would all be on the x-axis. Why?

38. Use your calculator and determine the linear regression equation for the given data and repeat the above process recording the values in the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

39. Compare and contrast the two tables.

Sample Resources:

The Math Forum – Exploring Data: Exploring Data, Courses and Software
http://mathforum.org/workshops/usi/dataproject/usi.genwebsites.html

- The Data Library – Pat Daley

  This site includes collaborative projects - specific data collection projects that teachers and their students may become a part of; data sets that can be downloaded then sorted, manipulated, and graphed; and other sources of data - sites like the Bureau of Labor Statistics and the Chance Database that offer many more data sets in other formats.

Texas Instruments Education Technology Activities
http://education.ti.com/calculators/downloads/US/Activities/

- Find ready to use math activities.

Computer Support Group, CSGNetwork
http://csgnetwork.com/stopdistinfo.html

- Braking and stopping distance chart

The Federal Interagency Forum on Child and Family Statistics (Forum) is a collection of 22 Federal government agencies involved in research and activities related to children and families.
http://www.childstats.gov/americaschildren/tables.asp

- Family and Social Environment

U.S. Department of Labor; Bureau of Labor Statistics; Consumer Price Index
http://stats.bls.gov/cpi/home.htm

- Create customized tables using the Consumer Price Index – Consumer Price Database; Average Price Data

American Factfinder
http://factfinder2.census.gov/faces/nav/jsf/pages/index.xhtml

- Population, Housing, Economic, and Geographic data