Exponents and Radicals

Reporting Category	Expressions and Operations
Topic	Performing operations with and writing radical expressions containing rational exponents
Primary SOL	AII.1 The student, given rational, radical, or polynomial expressions, will b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; and c) write radical expressions as expressions containing rational exponents, and vice versa.

Materials

- "I Have. Who Has?" Cards (attached)
- Cardstock
- Scissors
- Rational Exponents Matching Cards (attached)
- Attached handout

Vocabulary

rational, integer, volume, surface area (earlier grades)

index, radicand, radical (AII.1b and AII.1c); *varies directly* (AII.10); *compound annual interest* (for extension handout)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

DAY 1

1. Demonstrate how to construct all properties of exponents from the definition of $a^n = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n \text{ factors of } a}$, where $n \in N$.
   Stress the importance of solutions having positive integral exponents and rational denominators. Powers with nonintegral exponents should be written as radicals.
2. Distribute the "I Have. Who Has?" Cards, one to each student. If there are extras, continue handing out cards until all are distributed. Ask the student with the starting card to read it aloud. After he or she reads, "Who has $4^{\frac{3}{2}}$?", write that expression on the board. Then, direct all students to write and simplify that expression. When they are finished, ask the student holding the card containing the simplified form of the expression to read his or her card: "I have 8. Who has $125^{\frac{2}{3}}$?" The game continues in this manner until the last card is read.

DAY 2

1. Distribute the Rational Exponents Matching Cards, one to each student. Again, some students may receive more than one card. Explain that the cards will group into sets so that each set of cards shows all the steps in the simplification of an expression. Challenge

students to locate classmates holding cards that show the equivalent expressions. Once all cards are matched up into sets, have students in each group order the cards according to the sequence of steps in the simplification of the expression.

Assessment

- **Questions**
  - When a radical is completely simplified, what must be the relationship between the exponents under the radical and the index?
  - If a power includes a rational exponent, what will the numerator and denominator of that exponent represent if the power is written in radical form?
- **Journal/Writing Prompts**
  - Explain why $(a + b)^n \neq a^n + b^n$.
- **Other**
  - Explain the differences between a^{-n}, $a^{\frac{1}{n}}$, and $a^{-\frac{1}{n}}$.
  - Using the quotient rule, prove that $a^0 = 1$.

Extensions and Connections (for all students)

- Have students complete the attached Rational Exponents handout.

Strategies for Differentiation

- Provide a handout of the problems contained in the "I Have. Who Has?" game for students who need additional time to simplify.
- Give students a sentence frame to support their articulation of exponential and rational expressions.
- Have students create and use flash cards for the properties of exponents.
- Have students create a table of perfect squares and perfect cubes to use as an aide.
- For the "I Have. Who Has?" game, create a set of cards that involves only radicals or exponential expression simplification.
- Provide a list of multiple-choice answer options for the Rational Exponents handout.

"I Have. Who Has?" Cards

Copy cards on cardstock, and cut out.

I have the starting card. Who has $4^{\frac{3}{2}}$?	I have 8. Who has $125^{\frac{2}{3}}$?
I have 25. Who has $\sqrt[3]{32}$?	I have $2\sqrt[3]{4}$. Who has $\sqrt{72}$?
I have $6\sqrt{2}$. Who has $\sqrt{a^6b^4}$?	I have a^3b^2. Who has $\sqrt{27}$?
I have $3\sqrt{3}$. Who has $\frac{10}{\sqrt{2}}$?	I have $5\sqrt{2}$. Who has $\sqrt{a^5b^3}$?

I have $a^2b\sqrt{ab}$. Who has $\sqrt{\frac{25}{2}}$?	I have $\frac{5\sqrt{2}}{2}$. Who has $3\sqrt{48}$?
I have $12\sqrt{3}$. Who has $(3-\sqrt{2})^2$?	I have $11-6\sqrt{2}$. Who has $(a^2b^2)^3$?
I have a^6b^6. Who has $2(a^2b)^3$?	I have $2a^6b^3$. Who has $2b(2a^3b)^2$?
I have $8a^6b^3$. Who has $\sqrt[3]{54}$?	I have $3\sqrt[3]{2}$. Who has $(ab^2)^2$?

I have a^2b^4. Who has $\frac{7}{3+\sqrt{2}}$?	I have $3-\sqrt{2}$. Who has $81^{\frac{2}{3}}$?
I have $9\sqrt[3]{9}$. Who has $\frac{8}{\sqrt{3}}$?	I have $\frac{8\sqrt{3}}{3}$. Who has $(5-\sqrt{6})^2$?
I have $31-10\sqrt{6}$. Who has $\frac{2-\sqrt{3}}{5-\sqrt{7}}$?	I have $\frac{10+2\sqrt{7}-5\sqrt{3}-\sqrt{21}}{18}$. Who has $2(a^4b)^4$?
I have $2a^{16}b^4$. Who has $\frac{8}{\sqrt{2}}$?	I have $4\sqrt{2}$. Who has the last card?

Rational Exponents Matching Cards

Copy cards on cardstock, and cut out.

$\sqrt[3]{x^2} \cdot \sqrt[4]{x^3}$	$9^{\frac{3}{2}}$	$\sqrt[4]{y^2}$
$x^{\frac{2}{3}} \cdot x^{\frac{3}{4}}$	$\left(\sqrt{9}\right)^3$	$y^{\frac{2}{4}}$
$x^{\frac{8}{12}} \cdot x^{\frac{9}{12}}$	3^3	$y^{\frac{1}{2}}$
$x^{\frac{17}{12}}$	27	$\sqrt{y}$
$\sqrt[12]{x^{17}}$	$4x^{\frac{1}{4}}$	$4\sqrt[4]{x}$

$\sqrt{5}\cdot\sqrt[3]{6}$	$\sqrt[6]{5^3\cdot 6^2}$	$\left(16y^4\right)^{\frac{3}{4}}$
$5^{\frac{1}{2}}\cdot 6^{\frac{1}{3}}$	$\sqrt[6]{125\cdot 36}$	$16^{\frac{3}{4}}y^3$
$5^{\frac{3}{6}}\cdot 6^{\frac{2}{6}}$	$\sqrt[6]{4500}$	$y^3\left(\sqrt[4]{16}\right)^3$
$\sqrt[6]{5^3}\cdot\sqrt[6]{6^2}$	$\left(\sqrt[4]{m}\right)^5$	$y^3(2)^3$
$m^{\frac{5}{4}}$	$\sqrt[4]{m^5}$	$8y^3$

$m^{-\frac{1}{3}}$	$4^{-\frac{1}{2}}$	$-(32)^{\frac{1}{5}}$
$\frac{1}{m^{\frac{1}{3}}}$	$\frac{1}{4^{\frac{1}{2}}}$	$-\sqrt[5]{32}$
$\frac{1}{\sqrt[3]{m}}$	$\frac{1}{\sqrt{4}}$	-2
$\frac{\sqrt[3]{m^2}}{m}$	$\frac{1}{2}$	

Rational Exponents

1. The volume, V, of a soap bubble is related to its surface area, A. In cubic centimeters, this relationship is shown by the formula $V = 0.094A^{3/2}$. What is the surface area of a bubble with a volume of 5 cm^3?

2. The number of hours, H, that milk stays fresh is a function of the surrounding temperature, T. In degrees centigrade, this relationship is shown by the formula $H(T) = 180 \cdot 10^{(-.04T)}$. How many hours will newly pasteurized milk stay fresh when stored at 8°C?

3. A fast ship's speed, S, in knots varies directly as the seventh root of the power, P, in horsepower being generated by the engine. This is expressed by the formula $S = 6.492P^{1/7}$. If a ship is traveling at a speed of 25 knots, about how much power is the engine generating?

4. A formula that the police use for finding the speed, S, in miles per hours that a car was going from the length, L, in feet of its skid marks is $S = \sqrt{5L}$. The *Guinness Book of Records* reports that in 1960, a Jaguar in England had the longest skid mark ever recorded: 950 ft.
   a. What was the Jaguar's approximate speed?
   b. About how far will a car travel if it skids from 50 mph to a stop?

5. Mike has $500 he wants to save for a big trip in the summer after he graduates from high school. His goal is to find an investment vehicle that will allow his money to double to $1,000 in 4 years. What compound annual interest rate, R, will make this happen? This can be expressed by the formula $A = P(1 + R)^T$, where P is the original principal, R is the annual interest rate, T is the number of years, and A is the total value of the investment.