Nonlinear Systems of Equations

Reporting Category	Equations and Inequalities	
Topic	Solving nonlinear systems of equations	
Primary SOL	AII.5	The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.
Related SOL	AII.4	

Materials

- Graphing calculators
- Four attached handouts
- Transparency sheets

Vocabulary

circle, parabola, quadratic, linear, linear system, points of intersection, inverse system (earlier grades)

quadratic system, quadratic term, nonlinear system, linear-quadratic system, quadratic-quadratic system (AII.5)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

This lesson on nonlinear systems of equations goes beyond the standard by including conics other than those in the form $y = ax^2 + bx + c$ in examples.

1. Begin by reviewing linear systems of equations and the three approaches students used in Algebra I to solve them—i.e., substitution, addition method, and graphing. Show an example of each method.
2. Distribute scissors and copies of the attached Possible Number of Points of Intersection handout, and have students cut out the figures. Have students work in pairs to determine the possible number of points of intersection (the number of solutions to a corresponding system) for each pair of figures, filling in the accompanying table as they experiment by manipulating the figures.
3. After modeling the algebraic and graphing methods for solving a linear-quadratic system, distribute copies of the attached Linear-Quadratic System Practice handout, and have students work in pairs to solve each problem. Instruct Partner A to solve the problem algebraically and Partner B to solve the same problem graphically. Then, have partners confer and confirm the solution. Circulate and give help as needed.
4. Distribute copies of the attached Quadratic-Quadratic System Exploration, and have students work in small groups to complete it. When they are finished, have the whole class discuss the results. Then, model algebraic methods for solving quadratic-quadratic systems.
5. Distribute copies of the attached Quadratic-Quadratic System Practice handout, and have students work in pairs to solve each problem. Direct Partner A to solve the problem

algebraically and Partner B to solve the same problem graphically. Then, have partners confer and confirm the solution. Circulate and give help as needed.

Assessment

- **Questions**
  - What allows us to use substitution to solve a linear system of equations? What allows us to add two equations together to solve a system?
  - How do you decide which method is best for each type of problem?
- **Journal/Writing Prompts**
  - Write the steps you would instruct someone to use to solve a linear-quadratic system.
  - Explain how you know that you have no solutions when solving a quadratic-quadratic system algebraically.
  - Explain how you know that you have no solutions when solving a quadratic-quadratic system graphically.
- **Other**
  - Provide students with ordered pair solutions to nonlinear systems of equations, and have them create the systems of equations to correspond to the given solutions.

Extensions and Connections (for all students)

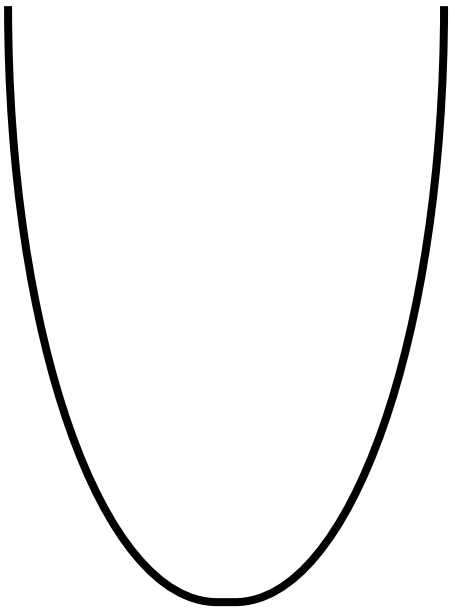
- Have students find a real-world problem that can be solved with a nonlinear system of equations.

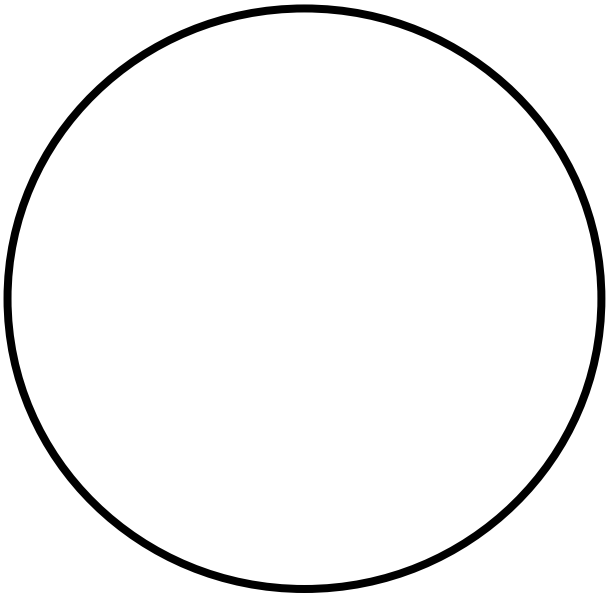
Strategies for Differentiation

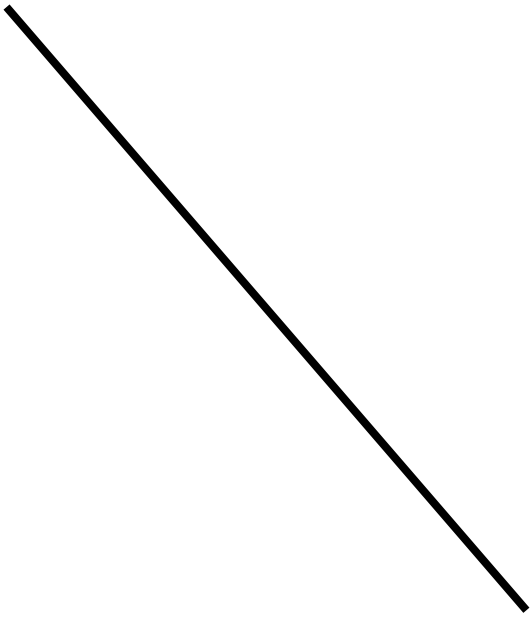
- Use a Conic Section graphing application on a calculator to help students visualize solutions to nonlinear systems of equations.
- Create a matching game to match graphs with nonlinear systems of equations.
- Create a sentence frame to accompany the Possible Number of Points of Intersection handout. The sentence frame might be, "My parabola and my circle intersect at two point(s)."

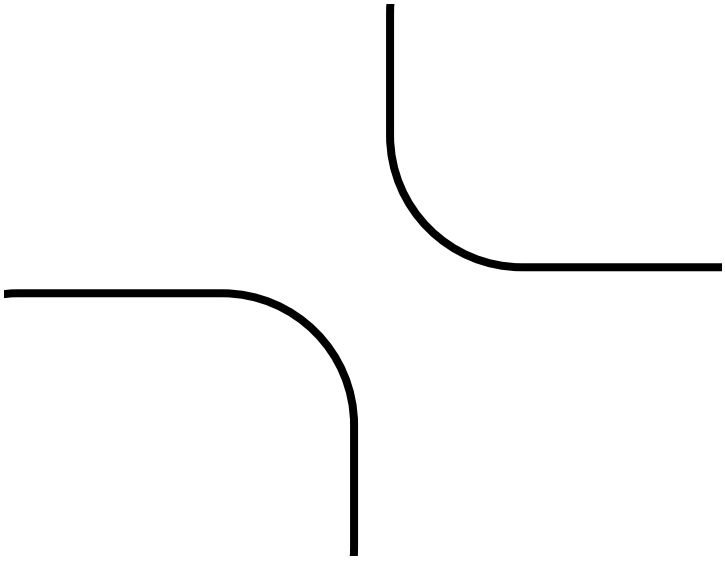
Possible Number of Points of Intersection

Cut out each of the four figures shown below. Explore the possible number of points of intersection when two figures at a time are overlaid, one on top of the other. Fill in the accompanying table.









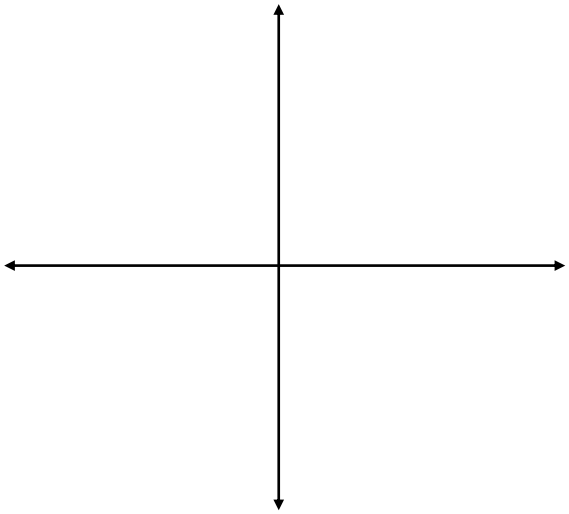
Possible Number of Points of Intersection				
	Line	**Parabola**	**Circle**	**Hyperbola of the form $y = \frac{k}{x}$**
Line	0, 1, or infinitely many	0, 1, or 2		
Parabola				
Circle				
Hyperbola of the form $y = \frac{k}{x}$				

Linear-Quadratic System Practice

1. $\begin{cases} x^2 + y^2 = 25 \\ y = \frac{3}{4}x \end{cases}$

Solve algebraically by substitution.

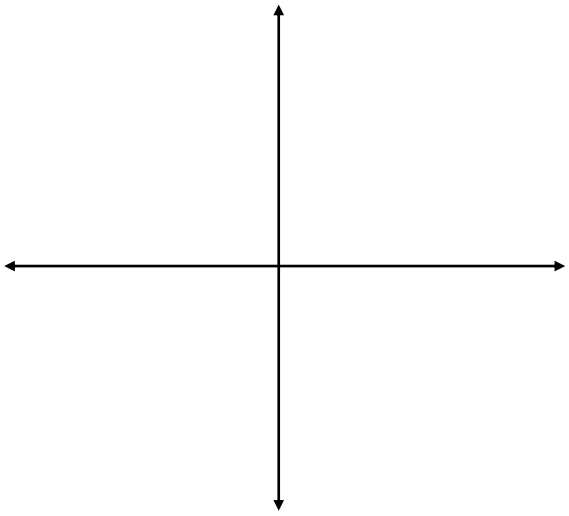
Solve by graphing.



2. $\begin{cases} y = x^2 + 5x - 10 \\ y = 2x + 18 \end{cases}$

Solve algebraically by substitution.

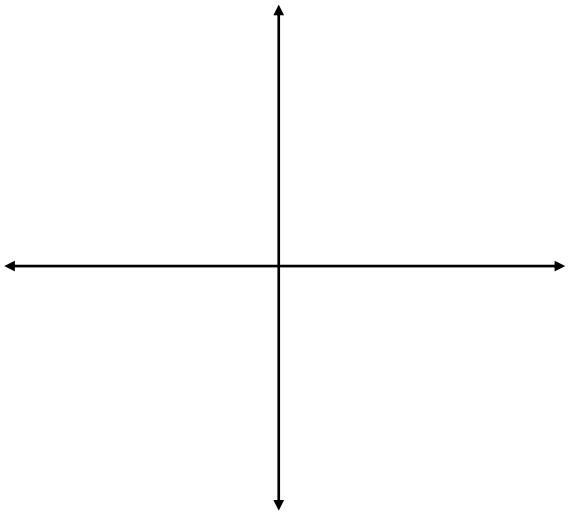
Solve by graphing.



3. $\begin{cases} x^2 + y^2 = 9 \\ y = x - 5 \end{cases}$

Solve algebraically by substitution.

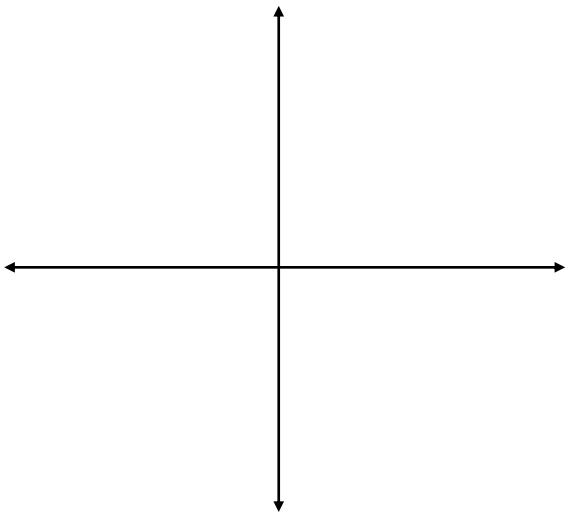
Solve by graphing.



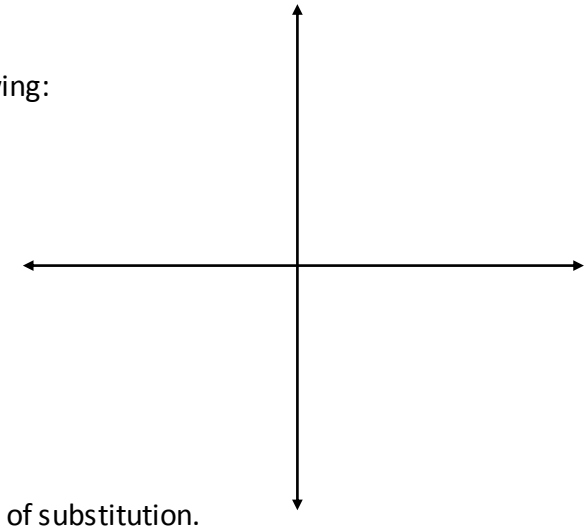
4. $\begin{cases} y = x^2 + 4x - 2 \\ y = 6x - 3 \end{cases}$

Solve algebraically by substitution.

Solve by graphing.



Quadratic-Quadratic System Exploration



1. Given the system $\begin{cases} x^2 + y^2 = 25 \\ y = x^2 - 13 \end{cases}$, complete the following:
   a. Sketch both equations on the coordinate system shown at right.
   b. Describe graphically and algebraically what is meant by a solution to a system of equations.

   c. How many solutions does this system have?

   d. Solve this system algebraically, using the process of substitution.

   e. Graph this system on the graphing calculator. State the calculator window you use to "capture" this graph and the solutions. Describe the procedure you used.

2. If possible, rewrite the system shown above, keeping one of the equations the same but changing the other equation to satisfy each of the following stated requirements. If the change is not possible, explain why.
   a. There are exactly three solutions for the system.

   b. There are exactly two solutions for the system.

   c. There is only one solution for the system.

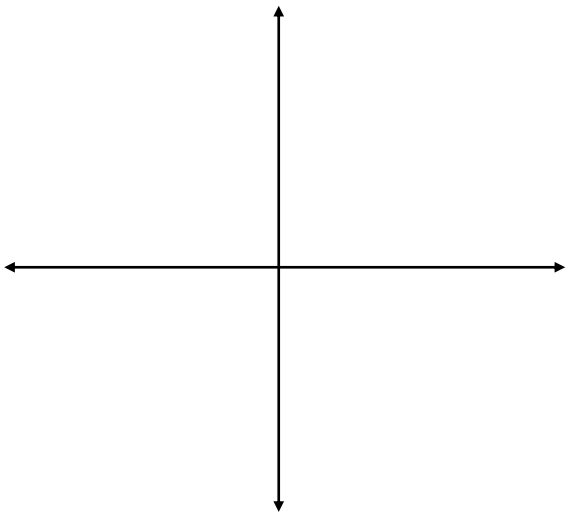
   d. There is no solution for the system.

Quadratic-Quadratic System Practice

1. $\begin{cases} x^2 + y^2 = 25 \\ xy = 12 \end{cases}$

Solve algebraically by substitution.

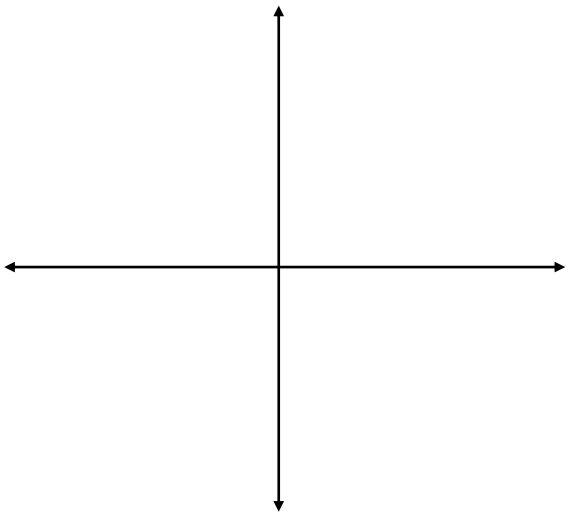
Solve by graphing.



2. $\begin{cases} y = x^2 - 9 \\ x^2 + y^2 = 9 \end{cases}$

Solve algebraically by substitution.

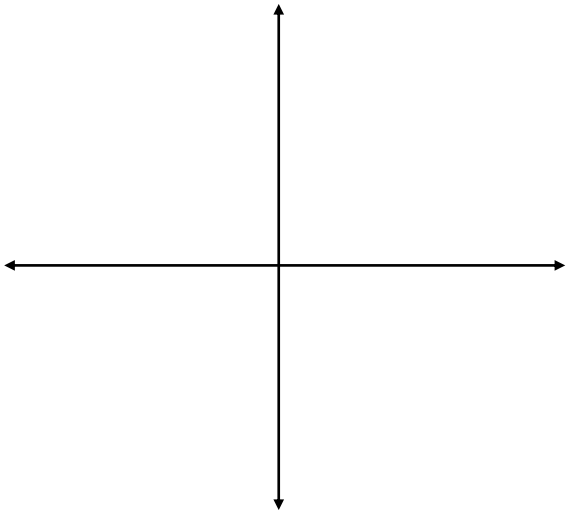
Solve by graphing.



3. $\begin{cases} y = -x^2 + 9 \\ y = x^2 + 1 \end{cases}$

Solve algebraically by substitution.

Solve by graphing.



4. $\begin{cases} x + y^2 = 6 \\ x^2 + y^2 = 26 \end{cases}$

Solve algebraically by substitution.

Solve by graphing.