# Functions: Domain, Range, End Behavior, Increasing or Decreasing

**Reporting Category**  Functions  
**Topic** Finding domain and range; determining whether a function is increasing or decreasing  
**Primary SOL** AII.7  The student will investigate and analyze functions algebraically and graphically. Key concepts include  
  a) domain and range, including limited and discontinuous domains and ranges;  
  d) intervals in which a function is increasing or decreasing;  
  f) end behavior.  
Graphing calculators will be used as a tool to assist in investigation of functions.  
**Related SOL**  AII.6  
**Materials**  
- Graphing calculators  
- Two attached handouts  
**Vocabulary**  
  domain, range (earlier grades)  
  increasing function, decreasing function, end behavior (AII.7)  
**Student/Teacher Actions (what students and teachers should be doing to facilitate learning)**  
1. Have students graph the function $f(x) = x^2$ while you demonstrate the graphing steps. Then, have students discuss with partners the definitions of domain and range and determine the domain and range of this quadratic function.  
2. Discuss the difference between a continuous function and a discontinuous function. Have students determine whether or not this quadratic is a continuous function.  
3. While tracing the graph with your finger, ask students to tell you when the function is increasing and when it is decreasing. Ask them exactly where the graph changes from decreasing to increasing.  
4. Discuss the end behavior of the function, both as $x$ approaches negative infinity and as it approaches positive infinity.  
5. Demonstrate, and have students copy into notes, how to express the domain $\{x \mid x \in \mathbb{R}\}$, the range $\{f(x) \mid f(x) \geq 0\}$, intervals where the function is decreasing $\{x \mid -\infty < x < 0\}$ and increasing $\{x \mid 0 < x < \infty\}$, and end behavior. (As $x$ approaches negative infinity, $f(x)$ approaches positive infinity; as $x$ approaches positive infinity, $f(x)$ approaches positive infinity.)
6. Have students graph on their calculators \( f(x) = \sqrt{2x - 6} \) as you graph it on a display calculator. Ask, “Why do we see the graph in the first quadrant only? How would the domain and range, as well as the end behavior, be different if we were looking at a parabola in the first and fourth quadrants?” Draw what you are referring to on the board. Then, erase it, and refer to the original function again. Direct students to express the domain, range, intervals for increasing and decreasing, and end behavior of this function on their papers. Call volunteers to write one each on the board. Discuss the fact that we cannot describe end behavior as \( x \) approaches negative infinity because \( x \) cannot be less than 3.

7. Distribute copies of the attached Where to Begin and End handout, and have students work in pairs or groups to complete it.

8. Use the attached Domain and Range handout for review on the next day.

Assessment
- Questions
  - What are the domain and range for \( f(x) = \sqrt{x - 4} \) ?
  - How would you describe the end behavior of \( f(x) = 2^x \) ?
- Journal/Writing Prompts
  - Create and describe a function, both algebraically and graphically, that would be increasing in two different intervals and decreasing in one interval.
  - Compare and contrast the end behaviors of a quadratic function and its reflection over the x-axis.

Extensions and Connections (for all students)
- Have students state the domain and range for a circle with center (2,5) and radius 4.
- To prepare students for further study in mathematics, demonstrate the use of interval notation.

Strategies for Differentiation
- Have students model with their arms end behavior of a polynomial function, depending on the sign of the leading coefficient and the degree of the polynomial.
- Use an interactive whiteboard with graphing software that allows students to manipulate graphs.
Where to Begin and End

For each function below, state the domain and range, name the intervals where the function is increasing or decreasing, and describe the end behavior.

1. Domain ________________________
   Range ________________________
   Increasing ____________________
   Decreasing ____________________
   End behavior ___________________

2. Domain ________________________
   Range ________________________
   Increasing ____________________
   Decreasing ____________________
   End behavior ___________________

3. Domain ________________________
   Range ________________________
   Increasing ____________________
   Decreasing ____________________
   End behavior ___________________

4. Domain ________________________
   Range ________________________
   Increasing ____________________
   Decreasing ____________________
   End behavior ___________________
5. \( f(x) = 3x + 5 \)
   Domain ____________________________
   Range ____________________________
   Increasing ____________________________
   Decreasing ____________________________
   End behavior ____________________________

6. \( f(x) = -3x + 5 \)
   Domain ____________________________
   Range ____________________________
   Increasing ____________________________
   Decreasing ____________________________
   End behavior ____________________________

7. \( f(x) = x^2 \)
   Domain ____________________________
   Range ____________________________
   Increasing ____________________________
   Decreasing ____________________________
   End behavior ____________________________

8. \( f(x) = (x + 3)^3 \)
   Domain ____________________________
   Range ____________________________
   Increasing ____________________________
   Decreasing ____________________________
   End behavior ____________________________

9. \( f(x) = -2x^2 - 2 \)
   Domain ____________________________
   Range ____________________________
   Increasing ____________________________
   Decreasing ____________________________
   End behavior ____________________________

10. \( f(x) = x^3 + 6x^2 + 9x \)
    Domain ____________________________
    Range ____________________________
    Increasing ____________________________
    Decreasing ____________________________
    End behavior ____________________________
## Domain and Range

For each function below, graph the function, state the domain and range, name the intervals where the function is increasing or decreasing, and describe the end behavior.

<table>
<thead>
<tr>
<th>Function</th>
<th>Graph</th>
<th>Domain and Range</th>
<th>Intervals Where Increasing or Decreasing</th>
<th>End Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( f(x) = \frac{1}{2}x^2 )</td>
<td></td>
<td>Domain: ( \text{not specified} )</td>
<td></td>
<td></td>
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<tr>
<td>2. ( y = x^2 + 3 )</td>
<td></td>
<td>Domain: ( \text{not specified} )</td>
<td></td>
<td></td>
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<tr>
<td>3. ( y = -3x^2 )</td>
<td></td>
<td>Domain: ( \text{not specified} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. ( y = x(5 - x) )</td>
<td></td>
<td>Domain: ( \text{not specified} )</td>
<td></td>
<td></td>
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<tr>
<td>5. ( m(x) = \left(\frac{1}{3}\right)^x )</td>
<td></td>
<td>Domain: ( \text{not specified} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. ( h(x) = 3^x )</td>
<td></td>
<td>Domain: ( \text{not specified} )</td>
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