Factors, Zeros, and Solutions

**Reporting Category**  
Functions

**Topic**  
Exploring relationships among factors, zeros, and solutions

**Primary SOL**  
AII.8  
The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.

**Related SOL**  
AII.1d, AII.4b, AII.7b

**Materials**
- Graphing calculators
- Graph paper
- Attached handout
- Matching Cards (attached)

**Vocabulary**
- solution, factor, slope, x-intercept, y-intercept (earlier grades)
- zeros of a function (AII.7b)

**Student/Teacher Actions (what students and teachers should be doing to facilitate learning)**

1. Have students solve the equation $3x + 5 = 11$. Then, have them set the left side of the equation equal to zero. Tell them to replace zero with $y$ and consider the line $y = 3x - 6$. Instruct them to graph the line on graph paper. Review the terminology slope, x-intercept, and y-intercept. Then, have them do the same procedure (i.e., solve, set left side equal to zero, replace zero with $y$, and graph) for the equation $-2x + 6 = 4$. They should notice that in both cases, the x-intercept is the same as the solution to the equation.

2. Review the term zeros from SOL AII.7. Remind students that zeros of a function are the x-intercepts of the graph of the function. Be sure students are able to make the connection between the facts that a zero of a function is the value of $x$ that makes $f(x) = 0$ and that a point on the graph where $f(x) = 0$ is an x-intercept.

3. Have students graph $y = (x - 3)(x + 2)$ on a graphing calculator. Have them investigate the x-intercepts and discuss with partners what the x-intercepts can tell us about the equation $y = (x - 3)(x + 2)$. Next, have them replace $y$ with zero and solve the equation.

4. Have students solve the equation $2x^2 - 10 = 8x$, using an algebraic method of factoring, the quadratic formula, or completing the square. Instruct students to rewrite the equation so that the left side is zero, replace the zero with $y$, and graph the quadratic function on their calculators. Make sure they see that the relationship between zeros and solutions is the same for quadratics as it is for linear equations.

5. Have students refer to the previous example and look at the equation written in factored form: $0 = 2(x + 1)(x - 5)$. Discuss the relationship between the factors of the equation and the zeros of the function on the calculator.
6. Review the Fundamental Theorem of Algebra and its corollary, which states in simple terms that a polynomial function of degree \( n \) will have exactly \( n \) zeros, including multiplicities. Emphasize that although a perfect square trinomial function such as \( y = x^2 + 8x + 16 \) has only one zero, it is still a quadratic function due to multiplicities of zeros.

7. Distribute copies of the attached Zeros and Factors handout, and have students complete it.

8. Have students do a matching activity, using the attached sets of Matching Cards, each set containing six cards that relate to each other. Either give small groups copies of all the cards to match up into sets of six cards each, or give each student one card, and have students form six-card sets by searching out their related cards. (Note: Not all equations and functions that go together are the same. The idea is that every card can be connected to another card by commonalities. For example, \( x + 3 = 1 \) could be paired with \( y = -2x - 4 \) because the solution of one is the same as the zero of the other.)

Assessment
- Questions
  - What are the zeros of the function that is graphed at right?
  - What are possible factors of the function that is graphed at right?
  - For the equation \( y = (x - 6)(x + 2) \), what are the zeros of the function?
  - What are the solutions to the equation \( x^2 - 4x - 12 = 0 \)?

- Journal/Writing Prompts
  - Explain why knowing the zeros of a function is not enough to define the function.
  - In your own words, state the Fundamental Theorem of Algebra. How is this theorem important when graphing a function, using the zeros of the function?

Extensions and Connections (for all students)
- Write in expanded form the equation of a function that has a leading coefficient of 3 and zeros that are \(-3\) and \(4\).
- Write the equation of a cubic function that has only two distinct real zeros.
- After students do the matching activity, have them create their own similar matching activity cards. Assign them double zeros, pairs of distinct zeros, and sets of 3 zeros, and have them create a six-card matching set for each.

Strategies for Differentiation
- Create sets of matching cards for students to match functional “solutions to,” zeros, and factors.
- Have students use interactive whiteboards to match zeros, equations, and graphs.
Zeros and Factors

Match each function with the correct x-intercepts.

_____1. \( y = (x - 2)(x + 8) \)  
   a. (4,0) and (-6,0)

_____2. \( y = (x + 4)(x + 6) \)  
   b. (2,0) and (-8,0)

_____3. \( y = x^2 - 2x - 24 \)  
   c. (-2,0) and (8,0)

_____4. \( y = x^2 + 10x + 16 \)  
   d. (-4,0) and (-6,0)

_____5. \( y = (x + 2)(x - 8) \)  
   e. (2,0) and (8,0)

_____6. \( y = 2x^2 + 4x - 48 \)  
   f. (4,0) and (6,0)

_____7. \( y = (x - 2)(x - 8) \)  
   g. (-2,0) and (-8,0)

_____8. \( y = 3x^2 - 30x + 72 \)  
   h. (6,0) and (-4,0)

Write in expanded form an equation of a function that would have the given zeros.

9. \(-3, 9\)

10. \(7\)

11. \(-1, 2, 4\)

12. \(-5, 0, 3\)
### Matching Cards

Copy cards on cardstock, and cut out.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solutions and zeros</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2x^2 = 7x - 3)</td>
<td>(\left{\frac{1}{2}, 3\right})</td>
</tr>
<tr>
<td><strong>Function</strong></td>
<td><strong>x-intercepts</strong></td>
</tr>
<tr>
<td>(y = 2x^2 - 7x + 3)</td>
<td>(\left{\frac{1}{2}, 0\right} \text{ and } (3,0))</td>
</tr>
<tr>
<td><strong>Factored form</strong></td>
<td><strong>Graph</strong></td>
</tr>
<tr>
<td>(y = (2x - 1)(x - 3))</td>
<td><img src="graph1.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>Equation</th>
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</tr>
</thead>
<tbody>
<tr>
<td>(x^2 + x - 6 = 0)</td>
<td>{3, 2}</td>
</tr>
<tr>
<td><strong>Function</strong></td>
<td><strong>x-intercepts</strong></td>
</tr>
<tr>
<td>(y = -2x^2 - 2x + 12)</td>
<td>((-3, 0) \text{ and } (2, 0))</td>
</tr>
<tr>
<td><strong>Factored form</strong></td>
<td><strong>Graph</strong></td>
</tr>
<tr>
<td>(y = -2(x + 3)(x - 2))</td>
<td><img src="graph2.png" alt="Graph" /></td>
</tr>
<tr>
<td>Equation</td>
<td>Solutions and zeros</td>
</tr>
<tr>
<td>----------</td>
<td>---------------------</td>
</tr>
<tr>
<td>(x^2 = 5x - 6)</td>
<td>({2,3})</td>
</tr>
<tr>
<td>Function</td>
<td>x-intercepts</td>
</tr>
<tr>
<td>(y = 2x^2 - 10x + 12)</td>
<td>((2,0) \text{ and } (3,0))</td>
</tr>
<tr>
<td>Factored form</td>
<td>Graph</td>
</tr>
<tr>
<td>(y = 2(x - 2)(x - 3))</td>
<td></td>
</tr>
</tbody>
</table>

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<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>(x^2 = 1 - \frac{3}{2}x)</td>
<td>(\left{-2, \frac{1}{2}\right})</td>
</tr>
<tr>
<td>Function</td>
<td>x-intercepts</td>
</tr>
<tr>
<td>(y = -2x^2 - 3x + 2)</td>
<td>(\left(\frac{1}{2},0\right) \text{ and } (-2,0))</td>
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<td>Factored form</td>
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