Angles, Arcs, and Segments in Circles

Reporting Category Polygons and Circles

Topic Investigating angles and segments of circles

Primary SOL G.11a The student will use angles, arcs, chords, tangents, and secants to investigate, verify, and apply properties of circles.

Related SOL G.7

Materials
- Activity Sheets 1 and 2 (attached)
- Dynamic geometry software package (Lesson can be modified for use without computers.)
- Straightedge
- Pencil

Vocabulary

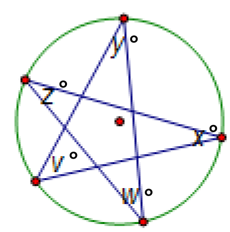
circle, chord, diameter, radius (earlier grades)

secant, tangent, secant segment, tangent segment, arc, minor arc, major arc, angle measure, arc measure, arc angle, arc length, central angle, inscribed angle, interior angle, exterior angle, chord, segment, (G.11), *AA similarity* (G.7)

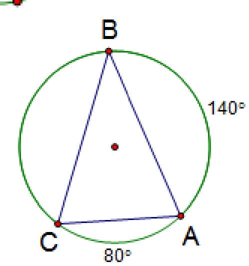
Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Have students complete Activity Sheet 1, using a dynamic geometry software package. If time is limited, assign the later parts to groups.
2. Have students discuss their findings with their partner after each item is completed. Have them record these findings on their own activity sheet.
3. Discuss findings as a whole class. If some parts were assigned to groups, have the groups present their results.
4. Have students complete Activity Sheet 2.
5. Have students discuss their findings with their partner.
6. Discuss findings as a whole class.

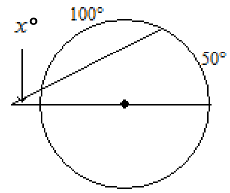
Assessment
- **Questions**
  - What is $v° + w° + x° + y° + z°$? Explain your reasoning.



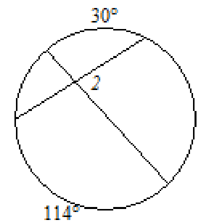
  - Find m$\angle BAC$. What kind of triangle is ΔABC? Explain your reasoning.



- What is the value of *x*? Explain your reasoning.

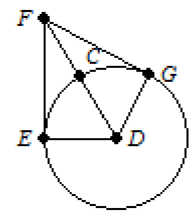


- Find m$\angle 2$. Show and justify your work.

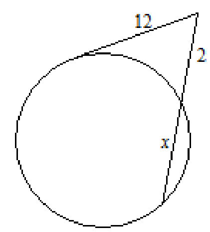


- Which is longer, a chord that is 8 cm from the center of a circle that has radius 10 cm or a chord that is 12 cm from the center of a circle with a radius of 13 cm? Explain your reasoning.

- FG = 16 m and CD = 12 m. Find FC. Explain your reasoning. *Assume that segments that appear tangent are tangent.*



- A tangent segment and a secant segment intersect at a point C exterior to a circle. The tangent is 12 units long, and the exterior part of the secant segment is 2 units long. What is the length *x* of the chord?



- **Journal/Writing Prompts**
  - Complete a journal entry summarizing the activity.
  - Explain how an angle formed by a tangent and a chord is like an inscribed angle.
  - Explain how the formula relating the segments formed by intersecting chords is related to similar triangles.
  - Explain the difference between a tangent and a secant to a circle.
  - Explain how to construct a tangent to a circle, through a given point.
  - Decide whether a central angle is an interior angle. Explain.
- **Other**
  - Have students work in small groups and present findings to whole class.
  - Have students work in small groups to write test questions. Use these questions for assessment purposes.

Extensions and Connections (for all students)

- Assign different teams of students to use a dynamic geometry software package to investigate different circle theorems that have not been covered in this activity. Have the teams present their findings to the class, using their dynamic geometry software package sketch to illustrate the theorem.

- After constructing $\angle BEC$ in the inscribed angle investigation, have students move the point E so that it lies on the arc BC. Have students explore how $\angle BEC$ and $\angle BDC$ are related. Use this construction to explore opposite angles in quadrilaterals inscribed in circles.
- Have students explore circle constructions, such as the following: Given a triangle, construct a circle that passes through all three vertices, or construct a circle that is tangent to all three sides of the triangle.
- Take students to the gymnasium. Have them use the circle at the center of the court and use rope or string to draw tangent lines and secants to the baskets. Have students identify segments, angles, and arcs.
- Have students create quizzes, games, presentations, or graphic organizers.

Strategies for Differentiation

- Give students a software file with the circles and angles already made and measured.
- Provide an alternative format for the directions on the activity sheets (e.g., tape the directions or problems, reword them, divide the information into smaller pieces).
- Use memory strategies that include keywords, first-letter mnemonics, visual imagery, and rhymes. For example, have students sing a song to help them remember the key concepts: (sung to the tune "My Bonnie") "A secant cuts through the circle, inscribed comes from the ending. The central angle is like pizza, but tangents only touch the ending. Cutting, forming, more secants and tangents for me, for me, cutting, forming, more secants and tangents for me."
- Using the software program, have students color-code secants, tangents, arcs, and angles.
- Use visual learning software to create assistive learning tools (such as a webbing activity with a central idea and key concepts tied together to be used for a reference sheet) for students.
- Use presentation software to isolate or highlight key points of the lesson. This may be helpful when breaking down the student activities into smaller steps.
- Create a math glossary or folded graphic organizer with examples, pictures, and definitions.
- Have students create summary statements that compare and contrast the key terms in the lesson.
- A class word wall could be created.

Activity Sheet 1: Angles, Arcs, and Segments in Circles

Name ___________________________________ **Date** _____________________

Complete the following activities, using a dynamic geometry software package.

Central Angle

1. Open a new sketch.
2. Draw a circle. Label the center *A* and a point on the circle *B*.
3. Draw a second point on the circle, and label it *C*.
4. Construct segment $\overline{AB}$.
5. Construct segment $\overline{AC}$.
6. $\angle BAC$ is called a *central angle*, because the vertex is the center of the circle. Find m$\angle BAC$.
7. Construct the arc *BC*. If arc *BC* is a major arc, move *B* or *C* until arc *BC* is a minor arc. Arc *BC* is called the intercepted arc for $\angle BAC$.
8. Find the measure of arc *BC* (the arc *measure*, which may be called the arc angle, *not* the arc length).
9. Gently move one of the points on the circle (without making arc *BC* a major arc), and notice what happens to the measure of the central angle and the measure of its intercepted arc. Describe your findings.

10. Sketch your circle, and write an equation relating m$\angle BAC$ and the measure of arc *BC*.

11. Name your file as directed by your teacher.

Inscribed Angles

1. Open a new sketch.
2. Draw a circle. Label the center *A* and a point on the circle *B*.
3. Draw a second point on the circle, and label it *C*.
4. Draw a third point on the circle, and label it *D*. Place *D* so that neither segment $\overline{BD}$ nor $\overline{CD}$ would go through *A* (the center of the circle).
5. Construct segment $\overline{BD}$ and segment $\overline{CD}$.
6. $\angle BDC$ is called an *inscribed angle* (an angle formed by two chords that intersect ON the circle). Find m$\angle BDC$.
7. Construct arc *BC*. The arc should *not* contain the point *D*, but it is okay if arc *BC* is a major arc.
8. Find the measure of arc *BC* (the arc *measure*, which may be called the arc angle, *not* the arc length).
9. Compare m$\angle BDC$ and the measure of arc *BC*. How are they related?
10. Gently move one of the points on the circle, (without moving D onto arc *BC*) and notice what happens to the measure of the central angle (m$\angle BDC$) and the measure of its intercepted arc. Describe your findings.

11. Sketch your circle, and write an equation relating m$\angle BDC$ and the measure of the intercepted arc.
12. Draw another point on the circle, and label it *E*. Move *E* if necessary so *E* is not on arc *BC*.
13. Construct segment $\overline{EB}$ and segment $\overline{EC}$.
14. $\angle BEC$ is also an inscribed angle. Find m$\angle BEC$.
15. What is the intercepted arc of $\angle BEC$?
16. Compare m$\angle BEC$ to m$\angle BDC$ and the measure of arc *BC*. What do you notice? What can you conclude?

17. Name your file as directed by your teacher.

Chords and Interior Angles

1. Open a new sketch.
2. Draw a circle. Label the center *A* and a point on the circle *B*.
3. Draw a second point on the circle, and label it *C*. Draw the chord $\overline{BC}$.
4. Draw two more points on the circle, label them *D* and *E*, and draw chord $\overline{DE}$.
5. If the two chords do not intersect, move the points *B, C, D,* and *E* until the chords intersect. Label the point of intersection *F*.
6. The four angles formed by the intersecting chords are called *interior angles* (because the vertex is inside the circle). Find m$\angle BFE$.
7. Construct the intercepted arc *BE* for $\angle BFE$.
8. What angle is vertical with m$\angle BFE$? _________ Construct the intercepted arc for this angle as well.
9. Find the measures of the two intercepted arcs.

10. Figure out a formula, using the measures of the intercepted arcs, so that m$\angle BFE$ will be the same as the value of the formula. (This is not as obvious as some other formulas. You may want to use a textbook or other reference source.)

11. Move the points *B, C, D,* and *E* and adjust the circle, keeping $\overline{BC}$ and $\overline{DE}$ as intersecting chords. Does your formula still work?

12. Name your file as directed by your teacher.

Secants and Exterior Angles

1. Open a new sketch.
2. Draw a circle. Label the center *A* and a point on the circle *B*.
3. Draw a second point on the circle, and label it *C*.
4. Draw a point outside the circle, and label it *D*.

5. Construct segment $\overline{BD}$ and segment $\overline{CD}$. If these segments are not secant segments, move point *D* and point *C* until $\overline{BD}$ and $\overline{CD}$ are secant segments.
6. $\angle BDC$ is called an *exterior angle* (because the vertex is outside the circle). Find m$\angle BDC$.
7. Draw the point on segment $\overline{CD}$ where the secant intersects the circle, and label it *E*.
8. Mark the point on segment $\overline{BD}$ where the secant intersects the circle, and label it *F*.
9. Construct arc *BC* and arc *EF*. Find the measures of the two arcs.
10. Figure out a formula, using the measures of arc *BC* and arc *EF,* so that m$\angle BDC$ will be the same as the value of the formula. (This is not as obvious as some other formulas. You may want to use a textbook or other reference source.)

11. Move the point *D*, and adjust the circle, keeping $\overline{BD}$ and $\overline{CD}$ as secant segments. Does your formula still work?

12. Name your file as directed by your teacher.

Tangents
1. Open a new sketch.
2. Draw a circle. Label the center *A* and a point on the circle *B*.
3. Draw a point outside the circle, and label it *C*.
4. Draw $\overline{AC}$.
5. Construct the midpoint of $\overline{AC}$. Name it *D*.
6. Construct a circle with center *D* and radius $\overline{CD}$.
7. The two points where the circles intersect are points of tangency. Label them *E* and *F*.
8. Draw the two tangent segments $\overline{CE}$ and $\overline{CF}$ and the two radii $\overline{AE}$ and $\overline{AF}$. Hide the second circle (with center at *D*).
9. Measure the angles formed by the tangent segments and the radii ($\angle AFC$ and $\angle AEC$). What do you notice?

10. Move the point *C* and adjust the circle. What can you conclude about angles formed by a tangent and a radius drawn to the point of tangency?

11. Measure the lengths $\overline{CE}$ and $\overline{CF}$. What do you notice?
12. Move the point *C* and adjust the circle. What can you conclude about the two tangent segments to a circle drawn from one exterior point?

13. Name your file as directed by your teacher.

Angles formed by a Tangent and a Chord (“Like Inscribed Angles”)
1. Open a new sketch.
2. Draw a circle. Label the center *A* and a point on the circle *B*.
3. Draw $\overline{AB}$.
4. Construct a line through *B*, perpendicular to $\overline{AB}$. This line is tangent to the circle. Label a point *C* on the perpendicular line.

5. Construct a chord from the point *B* to a new point on the circle. Name the new point *D*.
6. Hide *A* and the radius $\overline{AB}$. (Don't hide *B*!)
7. The intercepted arc for $\angle DBC$ is arc *BD*. Why?

8. Construct and measure the arc *BD*.
9. Draw another point *E* on the circle. If *E* is on the intercepted arc *BD* for $\angle DBC$, then move it so it is no longer on the intercepted arc.
10. Draw segments $\overline{EB}$ and $\overline{ED}$.
11. What is the intercepted arc for angle $\angle DEB$?
12. Compare the intercepted arcs for $\angle DBC$ and $\angle DEB$. What do you notice?

13. Measure the angles $\angle DBC$ and $\angle DEB$. What do you notice?

14. What kind on angle (central, inscribed, interior or exterior) is $\angle DBC$? Write a formula relating m$\angle DBC$ and the measure of arc *BD*.

15. Write a formula relating m$\angle DEB$ and the measure of arc *BD*.

16. Move the point *C,* and adjust the circle. What can you conclude about angles formed by a tangent and a chord?

17. How is an angle formed by a tangent and a chord like an inscribed angle?

18. Name your file as directed by your teacher.

Congruent Chords

1. Open a new sketch.
2. Draw a circle. Construct a chord $\overline{BC}$. Draw another point on the circle, and label it *D*. Construct a chord with one endpoint *D* which is congruent to $\overline{BC}$. (Draw a circle with radius $\overline{BC}$ and center *D*. Label one of the points where the two circles intersect as *E*. Create chord $\overline{DE}$. $\overline{DE} \cong \overline{BC}$. Hide the second circle.)
3. Construct the *minor* arc *BC* and *minor* arc *DE*. Each of these is called an arc of the chord. Find the measure of arc *BC* and the measure of arc *DE*. What do you notice? What can you conclude?
4. Name your file as directed by your teacher.

Circles

Draw two circles. Investigate the different ways two circles can intersect. Draw the diagrams here.

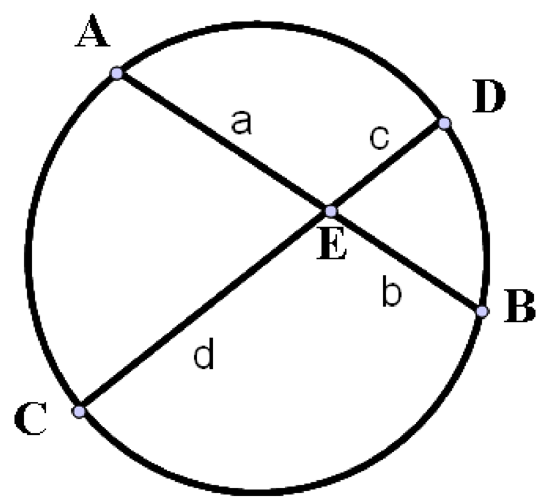
Activity Sheet 2: Segments in Circles

Name ______________________________ **Date** ____________________

In Activity Sheet 1, we learned about the arcs and angles formed when chords, secants, or tangents of a circle intersect. In this activity, we explore the segments that are formed.

Chord-Chord

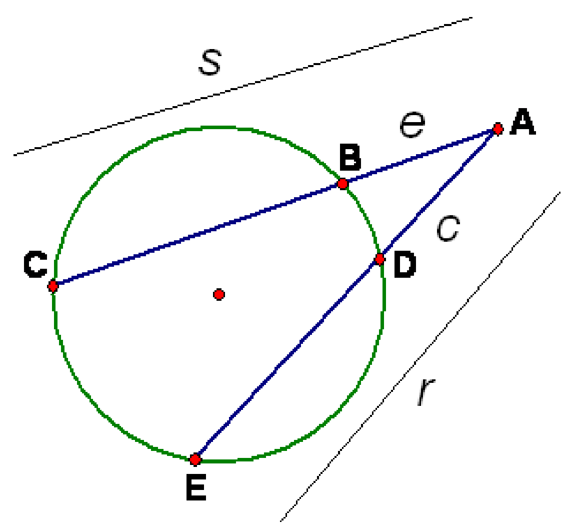
In this series of questions, you will explore the relationship between a, b, c, and d in the diagram at right, and s, e, r, and c in the following diagram.



1. Use a straightedge to draw the segments $\overline{AC}$ and $\overline{BD}$.
2. What can you say about m$\angle$*AEC* and m$\angle$*DEB*? Why? Mark your diagram appropriately.
3. What can you say about m$\angle$*CAE* and m$\angle$*BDE*? Why? (Hint: Consider their intercepted arcs.) Mark your diagram appropriately.
4. What can you say about Δ*CAE* and Δ*BDE*? Why? (State the postulate or theorem you used.)
5. Use what you discovered about the triangles in Step 4 to find a proportion relating a, b, c, and d.
6. Cross-multiply to get an equation in terms of a, b, c, and d that does not involve fractions.

Secant-Secant

Follow the steps to show that $s \cdot e = r \cdot c$ in the diagram at right.



1. Draw the segments $\overline{EC}$ and $\overline{BD}$.
2. What can you say about $\angle$*CAE* and $\angle$*DAB*? Why? Mark your diagram appropriately.
3. What can you say about $\angle$*BDE* and $\angle$*BDA*? Why?
4. What can you say about $\angle$*BDE* and $\angle$*ECB*? Why? (Hint: Consider the sum of their intercepted arcs. The angles are *not* congruent.)
5. What can you say about $\angle$*ECB* and $\angle$*BDA*? Why? Mark your diagram appropriately.

6. What can you say about $\angle ECB$ and $\angle ECA$? Why?

7. What can you say about $\angle ECA$ and $\angle BDA$? Why?

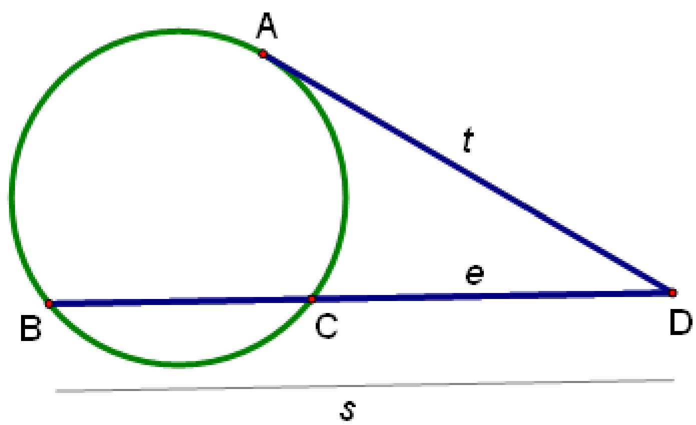
8. What can you say about ΔBDA and ΔECA? Why? (State the postulate or theorem you used.)

9. Use what you discovered about the triangles in Step 14 to find a proportion relating *s*, *e*, *r*, and *c*. (Pay attention to corresponding sides. You may want to sketch the two triangles separately.)

10. Cross-multiply to get an equation in terms of *s*, *e*, *r*, and *c* that does not involve fractions.

Secant-Tangent

$\overline{AD}$ is tangent to the circle. Show $s \cdot e = t^2$.



1. Draw the segments $\overline{AB}$ and $\overline{AC}$.
2. What can you say about $\angle ADB$ and $\angle CDA$? Why? Mark your diagram appropriately.

3. What is the intercepted arc of $\angle ABD$?

4. What is the intercepted arc of $\angle CAD$?

5. What can you say about $\angle ABD$ and $\angle CAD$? Why? (Hint: Consider their intercepted arcs and their angle types [central, inscribed, "like inscribed", interior, or exterior.]) Mark your diagram appropriately.

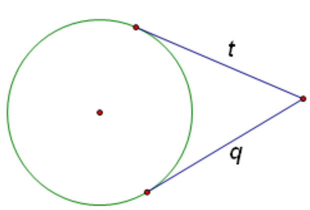
6. What can you say about ΔABD and ΔCAD? Why? (State the postulate or theorem you used.)

7. Use what you discovered about the triangles in Step 6 to find a proportion relating *s*, *e*, and *t*. (Pay attention to corresponding sides. You may want to sketch the two triangles separately.)

8. Cross-multiply to get an equation in terms of *s*, *e*, and *t* that does not involve fractions.

Tangent-Tangent

1. Use the **Tangents** investigation #11–12 from Activity Sheet 1 to get an equation relating *t* and *q*:



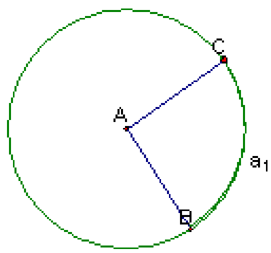
Angles, Arcs, and Segments in Circles

Instructor's Reference

Central Angle

$m\angle CAB = 92.47°$

$a_1 = 92.47°$



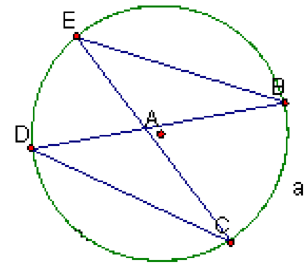
Inscribed Angle

$m\angle BDC = 35.50°$

$a_1 = 71.01°$

$\frac{a_1}{2} = 35.50°$

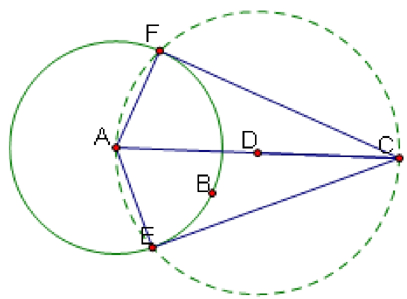
$m\angle BEC = 35.50°$



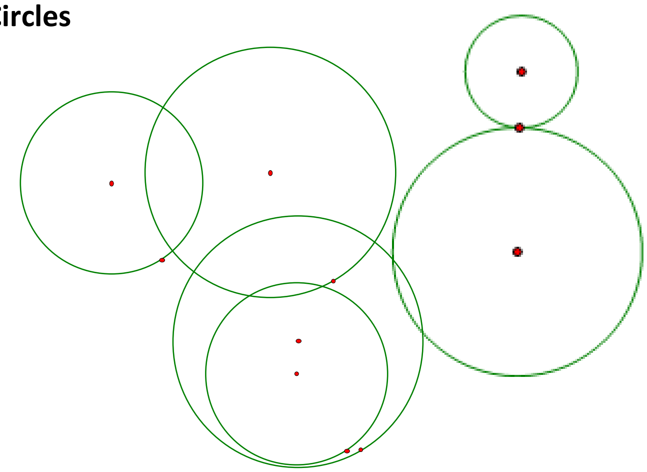
Tangents

$m\angle AFC = 90.00°$
$m\angle AEC = 90.00°$

$m\ \overline{FC} = 3.98$ cm
$m\ \overline{CE} = 3.98$ cm



Circles



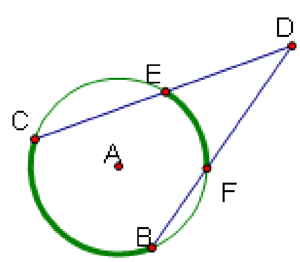
Exterior Angle

$m\angle CDB = 35.30°$

$m\ \overset{\frown}{FE} = 59.30°$

$m\ \overset{\frown}{CB} = 129.91°$

$\frac{m\ \overset{\frown}{CB} - m\ \overset{\frown}{FE}}{2} = 35.30°$



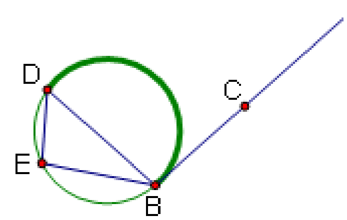
"Like Inscribed" Angle (tangent-chord angle)

$m\angle DBC = 96.76°$

$m\angle DEB = 96.76°$

$m\ \overset{\frown}{BD} = 193.53°$

$\frac{m\ \overset{\frown}{BD}}{2} = 96.76°$



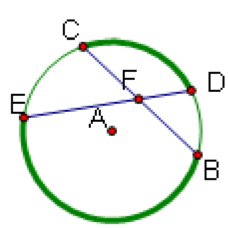
Interior Angle

$m\angle BFE = 127.52°$

$m\ \overset{\frown}{EB} = 173.04°$

$m\ \overset{\frown}{DC} = 81.99°$

$\frac{m\ \overset{\frown}{EB} + m\ \overset{\frown}{DC}}{2} = 127.52°$



Congruent Chords

$m\ \overline{BC} = 1.73$ cm

$m\ \overline{ED} = 1.73$ cm

$m\ \overset{\frown}{DE} = 102.16°$

$m\ \overset{\frown}{CB} = 102.16°$