Circles in the Coordinate Plane

Reporting Category Polygons and Circles

Topic Writing equations of circles

Primary SOL G.12 The student, given the coordinates of the center of a circle and a point on the circle, will write the equation of the circle.

Related SOL G.3a, G.3d, G.11

Materials

- Activity Sheet (attached)
- Graph paper and compass or dynamic geometry software package
- A coordinate plane for graphing at the board (or an overhead projector with coordinate plane transparency)
- Compass and marker for using at the board or with overhead projector

Vocabulary

circle, radius, diameter, center, coordinates, coordinate plane, equation, reflect, image (earlier grades)

tangent circles (G.11), *locus* (G.12), *distance formula* (G.3a)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Using a coordinate plane for graphing on the board, use a compass to graph a circle with center at the origin and radius 5, explaining what you are doing. (You can have students do the same thing using graph paper, compass, and pencil, as they follow along.)
2. Ask individual students to come to the board and mark points on the circle with their coordinates. At this point students will probably identify the points on the axes.
3. List the following ordered pairs on the board: (3, 4), (−4, −4), (−3, 4), (−1, 5), (−3, −4), (3, −4), (2, −4).
4. Ask students which of the points are on the circle. Allow students to graph the points to answer the question.
5. Ask students, “What do all the points that are on the circle have in common?” Remind them about the definition of a circle and the distance formula if necessary. Mark the origin as (0, 0) if you haven’t already. Encourage the class to “discover” that the distance from the origin to all the points on the circle is (exactly) 5 units.
6. Ask students if the points $(2, \sqrt{22})$ and $(-2, \sqrt{21})$ lie on the circle. (You can add them to the list.) If necessary, remind them about what they just discovered about points that lie on the circle.
7. Mark any other point on the circle, and label it (x, y).
8. Write out the distance formula. Ask and write the following: “What is the distance from (x, y) to (0, 0)?”

9. Plug in (*x, y*), (0, 0) and 5 into the formula. Point out to students that you are writing (*x* − 0) rather than (0 − *x*). Simplify and square both sides.
10. Point to the equation and ask, "What is this?" (answer: an equation) If necessary, point out the equal sign as a hint.
11. Explain that this is the equation of a circle with center (0, 0) and radius 5. Show how a couple of points on the circle satisfy the equation.
12. Draw a circle on the same coordinate plane with center (0, 0) and radius 6 on the coordinate plane at the board. (Erase the first circle if the graph is too cluttered.) Mark a point on the circle and label it (*x, y*). Have the class use the distance formula to find the equation of the circle.
13. Draw a circle on the board (not on a coordinate plane). Label the center (0, 0), draw a radius, and label the radius r. Mark a point on the circle, and label it (*x, y*). Have the class find the equation of a circle with center (0, 0) and radius *r.*
14. Optional: If yours is a proofs-based class, you may want to write this derivation as a two-column proof. (The reason for using the distance formula is the Pythagorean Theorem.)
15. Emphasize this equation, and have students write it in their notebook, along with a description.
16. Erase the graph, and graph a circle with radius 5 and center (1, 2). Have students identify a few points on the equation.
17. Mark a point on the circle and label it (*x, y*). Have the class use the distance formula to find the equation of the circle. Have them square both sides of the equation, but tell them not to expand the expressions (*x* − 1)2 and (*y* − 2)2.
18. Explain that this is the equation of a circle with center (1, 2) and radius 5. Show how a couple of points on the circle satisfy the equation.
19. Draw a circle on the board (not on a coordinate plane). Label the center (*h, k*). Point out that these are the variables that are usually used to identify the center of a circle.
20. Draw a radius, and label the radius *r*. Mark a point on the circle and label it (*x, y*). Have the class find the equation of a circle with center (*h, k*) and radius *r*. As before, students should not expand the expressions (*x* − *h*)2 and (*y* − *k*)2, and students should square both sides.
21. Emphasize this equation, and have students write it in their notebook, along with a description.
22. Have students work in pairs to complete the activity sheet. Each student should record his/her own findings. Have students discuss findings with their partners. Discuss findings as a whole group.

Assessment

- **Questions**
  - Use the distance formula to find the equation of a circle with center (−2, 3) and radius 4.
  - Determine whether the point (6, −8) lies on the circle with equation $(x-1)^2+(y+4)^2=169$. Justify your answer.
  - Find the radius and center of the circle with equation $(x-1)^2+(y+4)^2=169$.

  - A circle has diameter with endpoints (3, 4) and (−3, −4). Find the equation of the circle. Justify your answer.
  - P(1, 0) and Q(3, −2) are endpoints of a diameter of a circle. What is the radius of the circle?
- **Journal/Writing Prompts**
  - Explain how the equation of the circle $(x-1)^2+(y+4)^2=169$ is related to the distance formula.
  - Explain how you would find the equation of a circle whose graph is given.
- **Other**
  - Have small groups of students design a circle design of their own like the one in Activity Sheet 1. Students should then write directions using a variety of descriptions (including equations) and create a table, as in the Activity. Students should also provide a key (the graph and completed table). Use these student-created activities for assessment purposes.

Extensions and Connections (for all students)

- Have students explore translations and dilations of circles in the coordinate plane and the effect on the equations.
- Is the graph of a circle a function?

Strategies for Differentiation

- Provide students with circles, paper, or electronic files.
- Encourage students to use color after their initial pencil work.
- Review graphing calculator skills and experiment with manipulating variables other than those listed in the lesson.

Activity Sheet 1: Circles in the Coordinate Plane

Name ________________________________ **Date** ______________________

Graph the following circles on the same coordinate plane, using graph paper and a compass or a dynamic geometry or graphing software package, and complete the table.

1. Circle c_1 has center (0, 0) and radius 2.
2. Circle c_2 has center (0, 0), and (−3, 4) is one point on the circle.
3. Circle c_3 has center (−3, 0), and (−3, 2) is one point on the circle.
4. Circle c_4 has center (3, 0) and is congruent to c_3.
5. (1, 0) and (−1, 0) are two points on a diameter of the circle c_5. (Hint: What is the center of the circle?)
6. Circle c_6 has center (0, 3) and is tangent to c_2 and c_5.
7. Reflect circle c_6 across the *x*-axis. The image is circle c_7.
8. Complete the table below.

	center = (*h, k*)	**radius = *r***	**List four points on the circle.**	**Equation of the Circle**
c_1				
c_2				
c_3				
c_4				
c_5				
c_6				
c_7				

Activity Sheet 2: Circles in the Coordinate Plane

Name ________________________________ **Date** ______________________

Graph the following equations, using a graphing calculator or graphing software. Then, answer the questions.

1. Graph the equations $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ on the same graph. What is the difference between the two graphs? How does this relate to the difference between the two equations? Be specific, and use vocabulary from this class.

2. Graph the equations $x^2 + y^2 = 1$ and $(x-2)^2 + y^2 = 1$ on the same graph. What is the difference between the two graphs? How does this relate to the difference between the two equations? Be specific, and use vocabulary from this class.

3. Graph the equations $x^2 + y^2 = 1$ and $x^2 + (y-2)^2 = 1$ on the same graph. What is the difference between the two graphs? How does this relate to the difference between the two equations? Be specific, and use vocabulary from this class.

4. Graph the equations $x^2 + y^2 = 1$ and $(x-2)^2 + (y-2)^2 = 1$ on the same graph. What is the difference between the two graphs? How does this relate to the difference between the two equations? Be specific, and use vocabulary from this class.