

# Surface Area and Volume

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<b>Reporting Category</b>	Three-Dimensional Figures
<b>Topic</b>	Deriving formulas for surface area and volume
<b>Primary SOL</b>	G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.
<b>Related SOL</b>	G.14, G.8

## Materials

- Activity Sheets 1, 2, and 3 (attached)
- Calculators
- Cans
- Scissors
- Boxes made of light cardboard
- Oranges
- Wax paper
- Knife (to cut oranges)
- Unit cubes
- Milk cartons or other boxes of the same shape but different sizes
- Hollow, solid figures
- Sand, rice, or water
- Rulers
- Graduated cylinders

## Vocabulary

*length, width, height, surface area, volume, prism, cylinder, sphere, scale* (earlier grades)  
*derive, formula, rectangular prism, net, lateral area, base, two-dimensional, three dimensional* (G.13)

## Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Discuss with students: What are the formulas for determining surface area of solid figures? How can these formulas be used to solve problems?
2. Have students work with partners to complete the “Finding Formulas” activity sheet to derive the formulas for surface area and volume of a rectangular prism, a cylinder, and a sphere.
3. Have students complete the second activity, “Making Nets,” to extend their knowledge of surface area of a variety of solid shapes.
4. Have students use the newly learned information to complete the “Solving Problems” activity sheet.
5. Discuss the relationship of surface area to two-dimensional perimeter and area.

6. Distribute milk cartons with the tops cut off, or open boxes of other kinds, along with unit cubes. Ask students to fill the boxes with cubes as completely as they can and estimate the volume of the box based on counting the cubes used to fill the box.
7. Ask students to develop a quicker method than filling and counting for figuring out how many whole cubes will fill the box. (They should decide on  $length \times width \times height$ .)
8. Have students generalize this finding to other prisms/cylinders by relating the area of the base to the height. Use a can as a model of a right circular cylinder. Students should calculate the area of the base and multiply by the height to find the volume. Since  $1 \text{ cm}^3 = 1 \text{ ml}$ , a graduated cylinder can be used to compare the estimated volume with the actual volume.
9. Have students explore the relationship between the area of a pyramid and the area of a cube or prism with the same base and height, using the power solids and sand, water, or rice.
10. Have students complete the “Exploring Volume” activity sheet.

### Assessment

- **Questions**

- List the following three-dimensional figures in order from the least volume to the greatest.
  - a pyramid with a square base with sides measuring 25 cm and height 30 cm
  - a sphere with radius 10 cm
  - a rectangular prism measuring 10 cm by 20 cm by 30 cm
  - a cylinder with radius 10 cm and height 20 cm
- A water tank is 3 m tall and has a diameter of 4 m. The water level is 2 m high. How many more cubic meters of water can be added to the tank? Justify your answer.
- A red cylinder has diameter that is twice that of a blue cylinder. The height of the red cylinder is half that of the blue cylinder. Compare the volumes of the two cylinders. Which holds more?

- **Journal/Writing Prompts**

- The label of a soup can is a rectangular piece of paper. Explain how the radius and height of the soup can are related to the length and width of the rectangle.
- Describe a real-world example that uses surface area or volume.
- Write a practical problem and solution that uses surface area or volume.

- **Other**

- Use the completed activity sheets to assess student understanding.
- Have students bring in an example of packaging that does not form a solid that the class has studied (or allow students to select from a collection you have gathered). Have them determine (or estimate) the solid’s surface area and volume and explain their process and reasoning.

### Extensions and Connections (for all students)

- Discuss the relationship of surface area and volume of three-dimensional solids and their relationship to area and perimeter of two-dimensional figures.

- Have students create a cereal box with maximum volume and minimum surface area and ease of display.
- Create a lesson that includes mock prices of popcorn containers in various three-dimensional shapes. Ask students to analyze the best value with justification for their reasoning.
- Have students search the Internet for more nets of platonic solids and investigate their surface areas and volumes.

### **Strategies for Differentiation**

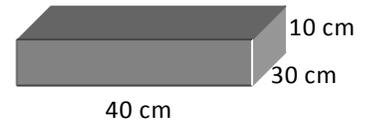
- Use manipulatives to build figures from nets.
- Use enlarged graph paper for nets.
- Use presentation software to post multimedia classroom lectures, student conclusions, and notes online.
- Have students work in groups to make up their own review materials.
- Use presentation software to create a multimedia presentation to highlight key elements of the lesson.
- Have students use the presentation as a review at the end of the lesson.
- Using computer software, create visual representations of the formulas and figures, and randomly distribute them to students. Have students cut and paste to match the formulas to the correct figures.

# Activity Sheet 1: Finding Formulas

1. After you purchase a gift for a friend, you decide to cover the sides and bottom of the gift box with wrapping paper. A diagram of the box with its dimensions appears below.

- How much wrapping paper will you need to cover the sides and bottom of the box?
- Your gift box is called an open box because it has no top surface.

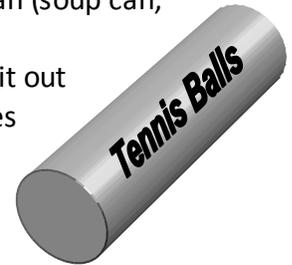
If this were a closed box with a top surface, how much additional paper would be required to cover the top surface?  
How much total paper would be required?



- How can you generalize the process you used to find the surface area of the closed box?
- Let  $l$  = length,  $w$  = width, and  $h$  = height of the box.
- Compare the formula you and your partner developed to that of another group. Did you have the same result? You should be able to justify your formula to your classmates.

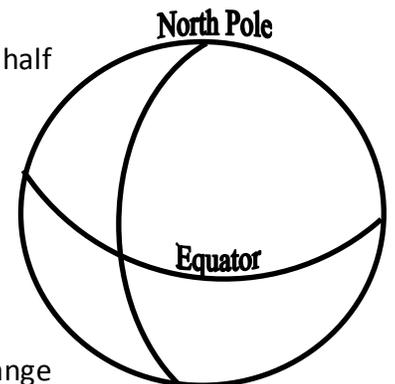
2. If your gift were a can of tennis balls, the surface area would be the surface of the cylinder (the lateral area) plus the areas of the top and bottom (the bases). Use a can (soup can, soda can, tennis ball can) for this activity.

- Wrap a piece of paper around the can, trim it to fit exactly, and spread it out flat. What shape is it? How can you find its area? What relationship does the length of the label have to the can? The height of the label?
- What shape are the bases of the can? Are the two bases congruent? What is the area of each base?
- The surface area of the can = the lateral area + the area of the two bases. For your can, what is the surface area? Use your calculator to find decimal approximations to the nearest tenth.



3. The surface area of a sphere is more difficult to figure out. On a globe, a *great circle* is a circle drawn so that when the sphere is cut along the line, the cut passes through the center of the sphere. The equator is a great circle on a globe.

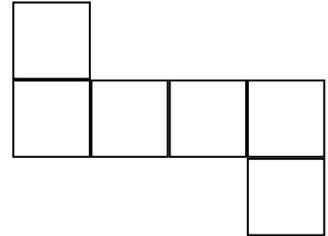
- Draw a great circle on an orange, and carefully cut the orange in half along the line of the great circle. Trace five cut halves on a piece of waxed paper.
- Carefully peel both halves of the orange, and fill in as many circles as you can with the peel. How many circles did your group fill? How does this compare with the findings of other groups? What is the class estimate for the number of great circles that can be filled by the peel?
- Using one of your great circle tracings, find the radius of your orange and the area of one great circle.
- Given the area of one great circle and your estimate of the number of circles that can be filled by the peel, what is the surface area of the orange?
- What is the general formula for the surface area of a sphere in terms of its radius?



# Activity Sheet 2: Making Nets

Name \_\_\_\_\_ Date \_\_\_\_\_

1. A *net* is a flattened paper model of a solid shape. For example, the net shown to the right, when folded, makes a cube. Can you draw a different net which, when folded, will also make a cube? If so, draw it, cut it out, and fold it to test your drawing.



2. A net is helpful because it represents the surface area of a shape. Take a box and cut it into a net. Note whether your box is open or closed. Sketch your box and its net. Use the formula you derived in “Finding Formulas” problem 1 to find the surface area of your box. Explain to a classmate how your net relates to your formula.

3. Now sketch a net of the can you used in “Finding Formulas” problem 2. How does this net relate to the surface area formula you found?

4. Sketch a net of the pyramid shown to the right. Use your net to find the surface area of the pyramid.

