Similar Solids and Proportional Reasoning

Reporting Category Three-Dimensional Figures

Topic Investigating similar solids and proportional reasoning

Primary SOL G.14 The student will use similar geometric objects in two- or three-dimensions to

a) compare ratios between side lengths, perimeters, areas, and volumes;
b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
d) solve real-world problems about similar geometric objects.

Related SOL G.7, G.13

Materials

- Activity Sheets 1 and 2 (attached)
- Plastic cubes
- Graph paper, same grid size as size of cubes

Vocabulary

similar, volume, area, surface area, perimeter, scale factor, ratio, dimension, length, diameter, rectangle, prism, rectangular prism, square, sphere (earlier grades)

proportional (G.14), *dilate, dilation* (G.3d)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Have students work in pairs to complete Activity Sheet 1. Each student should record his/her own findings. Have students discuss findings with their partners. Discuss findings as a whole group.
2. Have students work in pairs to complete Activity Sheet 2. Each student should record his/her own findings. Have students discuss findings with their partners. Discuss findings as a whole group.

Assessment

- **Questions**
  - If you double the height of a cylindrical paint can, what happens to the volume of the resulting cylinder? What happens to the area of the paint can label? Is the resulting cylinder similar to the original paint can? Explain.
  - If the scale factor of two similar cones is 2:3, and the volume of the small cone is 80 cubic inches, what is the volume of the larger cone? Explain and show your work.
  - If the surface areas of two similar solids are 4900π and 6400π, then what is the ratio of the volumes? Explain and show your work.
  - Explain why all cubes are similar to each other.

  - A machine has been developed that can shrink you down to Lilliputian size. If you use this machine, how long would your forearm be?
  - If you placed a basketball in the shrinking machine, what would the volume of the transformed basketball be?
- **Journal/Writing Prompts**
  - Complete a journal entry summarizing one of the activities.
  - Describe a real-world example that uses similar solids.
  - Write a practical problem and solution that uses similar solids.
  - Explain similar solids to someone who has never taken geometry.
- **Other**
  - Draw two spheres with ratio of surface areas 1:4. (To draw a sphere, start by drawing a circle.)
  - Research the size of balls for various sports and determine two balls with ratio of volumes 1:64.
  - Double the length, width, and height of a rectangular prism. Draw a picture to show how many of the smaller prisms you can pack in the larger prism. How is this related to the volumes of the two prisms?
  - Have students read the poem "One Inch Tall" by Shel Silverstein. Have students determine if one of the statements in the poem is true. (For example, could you walk beneath the door?)
  - Have students create a scale model.

Extensions and Connections (for all students)

- Invite a carpenter, builder, or architect to the class to demonstrate the various job applications that use surface area and volume and proportional reasoning.
- Have students create nets of similar solids. (This can be done using graph paper and drawing tools or using a dynamic geometry software package to create a net and dilate it.)
- Explore the series of *Archy and Mehitabel* newspaper columns.
- Have students read the excerpt from *Gulliver's Travels* at the end of this lesson. Lesson notes and supplementary questions are provided as well.

Strategies for Differentiation

- When instruction is presented orally, provide a visual component to support the activity sheet. Use colored markers or chalk, transparencies or slides to present the information in small chunks. Provide a visual for each piece of oral information.
- Have students build the figures in Activity Sheet 1 using cubes before drawing them and completing the tables.
- Use isometric graph paper to help students visualize the three-dimensional component.
- Use colors to mark corresponding parts.
- Put proportions into words, like analogies, such as
  $\frac{radius}{volume} = \frac{radius}{volume}$ or $\frac{small}{small} = \frac{medium}{medium} = \frac{large}{large}$.
- Have students use the rules $\frac{small}{big} = \frac{small}{big}$ and $\frac{big}{small} = \frac{big}{small}$.

Activity Sheet 1: Investigating Similar Solids

Name _____ **Date** _____

1. On the graph paper, draw a rectangle (not a square) with length and width between 2 and 5 units.
2. Leaving at least three rows between the two rectangles, draw another rectangle with length and width that are twice the length and width of the original rectangle.
3. Leaving at least three rows between the rectangle and the other rectangles, draw a third rectangle with length and width that are three times the length and width of the ORIGINAL rectangle.
4. Complete the table below.

	Small Rectangle	**Medium Rectangle**	**Large Rectangle**
Length			
Width			
Perimeter			
Area			

5. How many of the small rectangles fit in the medium rectangle? _____
6. How many of the small rectangles fit in the large rectangle? _____
7. Compute the ratios of the three rectangles' lengths, perimeters, and areas using the data above. Complete the table below. *Be sure to reduce all ratios!*

Ratios:	**Small:Medium**	**Small:Large**	**Medium:Large**
Length			
Perimeter			
Area			

8. How are the ratios of the areas related to your answers to #2 and #3?

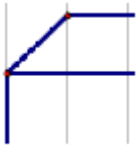


figure 1

9. Use cubes to build a small rectangular prism with length and width the same as the small rectangle and with height 1. How many rectangles do you need? _____ Sketch this prism on your small rectangle by adding diagonals as shown in figure 1.
10. If you have enough cubes, build a medium rectangular prism with length and width the same as the medium rectangle and with height 2. How many rectangles do you need? _____ Sketch this prism on your medium rectangle by adding diagonals as shown in figure 1, only two deep.
11. If you have enough cubes, build a large rectangular prism with length and width the same as the large rectangle and with height 3. How many rectangles do you need? _____ Sketch this prism on your large rectangle by adding diagonals as shown in figure 1, only three deep.

12. Complete the rest of the table below.

	Small Rectangular Prism	**Medium Rectangular Prism**	**Large Rectangular Prism**
Height	1	2	3
Surface Area			
Volume			

13. Compare the volumes of the prisms to your answers to #7, #8, and #9. Explain.

14. Similar solids are solids of the same type (like spheres and rectangular prisms) that have proportional linear measures (like length, width, height, perimeter, and radius). Are your three rectangular prisms similar? How do you know?

15. How many of the small rectangular prisms fit in the medium rectangular prism? _____
16. How many of the small rectangular prisms fit in the large rectangular prism? _____
17. Compute the ratios of the three rectangular prisms' heights, surface areas, and volumes using the data above. Complete the table below. *Be sure to reduce all ratios!*

Ratios:	**Small:Medium**	**Small:Large**	**Medium:Large**
Height			
Surface Area			
Volume			

18. How are the ratios of the volumes related to your answers to #14 and #15?

19. What is the scale factor of the small rectangular prism to the medium rectangular prism? _____
20. What is the scale factor of the small rectangular prism to the large rectangular prism? _____
21. How are the scale factors (#16) related to the corresponding ratios of surface areas and volumes?

Activity Sheet 2: Investigating Similar Solids

Name _____ **Date** _____

You must show your work including proportions or other equations where appropriate (# 4–8). Do not round your answers. (i.e., $1125\sqrt{3}\pi$ cm^3, rather than 6121.6 cm^3)

The ratio of the heights of two similar cones is 2:5. The radius of the smaller cone is 6 cm.

1. What is the scale factor of the two cones?

2. What is the radius of the larger cone?

3. What is the ratio of the lateral areas?

4. The lateral area of the smaller cone is 72π square centimeters. What is the lateral area of the larger cone? (Hint: Use #3)

5. What is the ratio of the volumes?

6. The volume of the larger cone is $1125\sqrt{3}\pi$ cubic centimeters. What is the volume of the smaller cone? (Hint: Use #5)

7. Find the slant heights and heights of the two cones. (You may already have found one or more of these.)

8. Find the angle *x* formed by a radius and the sides of the cones.

9. Complete the table below.

	r	***L.A.***	***V***	***l***	***h***
Small Cone		72π			
Large Cone			$1125\sqrt{3}\pi$		
Ratio	2:5				

10. Compute the volumes of the following solids:
    a) A cylinder with height *r* and radius *r*
    b) A hemisphere with radius *r*
    c) A cone with height *r* and radius *r*

11. Draw diagrams of the three figures. Try to draw them so they have the same radius and height. Compare the volumes. How many cones of water would it take to fill the cylinder? The hemisphere?

Extension: Gulliver's Travels

Teacher Notes

Materials

- Plastic linking cubes
- Excerpt from *Gulliver's Travels* (public domain)
- Calculators
- Objects like those in Gulliver's pockets (handkerchief, comb, folded and tied paper, snuff box, pocket watch, knife, and other objects described)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Have students read the excerpt from *Gulliver's Travels*. Take time to discuss the objects that are described there. Help students identify at least five of the objects described.
2. Using the objects like those found in Gulliver's pockets, have students estimate the size of the Lilliputians. Have students measure the comb, handkerchief, and other objects, and help them to investigate the proportional relationships.
3. At a point in the story before that in the excerpt, the Lilliputians feed Gulliver and are amazed by the amount of food he consumes. This is related to his volume and overall size as much as to his height. While the volume of a human is difficult to calculate, the volume of cubes and spheres is not. Use these shapes to investigate the relationship between changes of side length/diameter and surface area and volume.

Assessment

- **Journal/Writing Prompts**
  - Would you prefer to visit Lilliput, or to be a Lilliputian visiting Gulliver's England? Explain.
- **Other**
  - Have students estimate what size a modern device (such as a cell phone or car) would have to be for a Lilliputian to use it as Gulliver would. Allow students to choose their device. As an extension, students could investigate whether modern technology would allow their device to be made that small. (For example, can cell phone memory be made that small?)

Excerpt from *Gulliver's Travels*

By Jonathan Swift
(public domain)

Gulliver's ship was caught in a storm. He swam to safety on a mysterious island called Lilliput. He was captured and bound, taken to the Emperor's castle, and presented to the Emperor and Empress. When he was released from his rope bindings, the Lilliputians wanted to ensure that he had no weapons. This is the inventory of the contents of his pockets. (From part I, chapter II)

Imprimis. In the right coat-pocket of the Great Man-Mountain (the Lilliputian's name for Gulliver) after the strictest search, we found only one great piece of course cloth, large enough to be a foot-cloth for your Majesty's chief room of state. In the left pocket, we saw a huge silver chest, with a cover of the same metal, which we, the searchers, were not able to lift. We desired it should be opened, and one of us, stepping into it, found himself up to the midleg in a sort of dust, some part whereof, flying up to our faces, set us both a sneezing for several times together. In his right waistcoat-pocket, we found a prodigious bundle of white thin substances, folded one over another, about the bigness of three men, tied with a strong cable and marked with black figures; which we humbly conceive to be writings, every letter almost half as large as the palm of our hand. In the left, there was a sort of engine (a term used for any mechanical device), from the back of which were extended twenty long poles, resembling the palisades before your Majesty's court; wherewith we conjecture the Man-Mountain combs his head, for we did not always trouble him with questions, because we found it a great difficulty to make him understand us. In the large pocket on the right side of his middle cover (so I translate the word ranfu-lo, by which they meant breeches) we saw a hollow pillar of iron, about the length of a man, fastened to a strong piece of timber, larger than the pillar; and upon one side of the pillar were huge pieces of iron sticking out, cut into strange figures, which we know not what to make of. In the left pocket, another engine of the same kind. In the smaller pocket on the right side, were several round flat pieces of white and red metal, of different bulk; some of the white, which seemed to be silver, were so large and heavy, that my comrade and I could hardly lift them. In the left pocket were two black pillars, irregularly shaped: we could not, without difficulty, reach the top of them as we stood at the bottom of his pocket. One of them was covered, and seemed all of a piece; but at the upper end of the other, there appeared a white round substance, about twice the bigness of our heads. Within each of these were enclosed a prodigious plate of steel; which, by our orders, we obliged him to show us, because we apprehended they might be dangerous engines. He took them out of their cases, and told us, that in his own country his practice was to shave his beard with one of these, and to cut his meat with the other. There were two pockets which we could not enter: these he called his fobs; there were two large slits cut into the top of his middle cover, but squeezed closed by the pressure of his belly. Out of the right fob hung a great silver chain, with a wonderful kind of engine at the bottom. We directed him to draw out whatever was at the end of that chain; which appeared to

be a globe, half silver, and half of some transparent metal: for on the transparent side we saw strange figures circularly drawn, and thought we could touch them, until we found our fingers stopped with that lucid substance. He put this engine to our ears, which made an incessant noise like that of a watermill. And we conjecture it is either some unknown animal, or the god that he worships; but we are more inclined to the latter opinion, because he assured us (if we understood him right, for he expressed himself very imperfectly), that he seldom did anything without consulting it. He called it his oracle, and said that it pointed out the time for every action of his life. From the left fob he took out a net almost large enough for a fisherman, but contrived to open and shut like a purse, and served him for the same use: we found therein several massy pieces of yellow metal, which, if they be of real gold, must be of immense value.

Having thus, in obedience to your Majesty's commands, diligently searched all his pockets, we observed a girdle about his waist made of the hide of some prodigious animal; from which, on the left side, hung a sword of the length of five men, and on the right, a bag or pouch divided into two cells, each cell capable of holding three of your Majesty's subjects. In one of these cells were several gloves or balls of a most ponderous metal, about the bigness of our heads, and required a strong hand to lift them: the other cell contained a heap of certain black grains, but of no great bulk or weight, for we could hold above fifty of them in the palms of our hands.

This is an exact inventory of what we found about the body of the Man-Mountain, who used us with great civility, and due respect to your Majesty's commission. Signed and sealed on the fourth day of the eighty-ninth moon of you Majesty's auspicious reign.

CLEFREN FRELOCK, MARSI FRELOCK

Activity Sheet 4: Exploring Volume

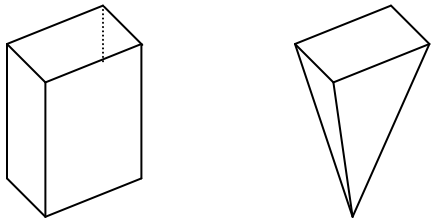
Name _____ **Date** _____

1. Which will carry the most water in a given length—two pipes with one having a 3 dm radius and the other a 4 dm radius, or one pipe with a 5 dm radius? Explain.

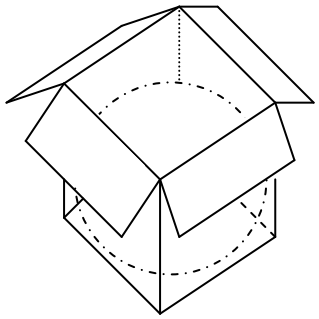
2. A company delivers 36 cartons of paper to your school. Each carton measures 40 cm x 30 cm x 25 cm. Is it possible to fit all cartons in an empty storage closet 1 m x 1 m x 2 m? Justify your conclusion with a visual explanation.

3. You have studied the pyramids and want to make a scale model of a pyramid with a square base and sides that are isosceles triangles. How much clay is required if the base of the actual pyramid is 30 m on each side and the height of the pyramid is 30 m? Your scale is 1 cm = 15 m.

4. A movie theater decides to change the shape of its popcorn holder from a rectangular box to a pyramidal box. The tops of both boxes are the same and the height remains the same. If the rectangular bag of popcorn cost $4.00, what is a fair price for the new box?



5. A manufacturer of globes that are approximately 1 m in diameter packs the globes in 1-cubic-meter boxes for shipping. How much packing material (foam peanuts) is needed for a shipment of 100 globes?



6. Take two sheets of paper the same size. Roll one sheet vertically and tape to form a right circular cylinder. Roll the second sheet horizontally, and tape it to form a second right circular cylinder. Tape each cylinder so that there is no overlap of paper—i.e., the edges should meet exactly. If each cylinder were filled with popcorn, would they contain the same amount? Explain and justify your answer.