

# Inductive and Deductive Reasoning

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**Reporting Category** Reasoning, Lines, and Transformations

**Topic** Practicing inductive and deductive reasoning strategies

**Primary SOL** G.1 The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include

- identifying the converse, inverse, and contrapositive of a conditional statement;
- translating a short verbal argument into symbolic form;
- using Venn diagrams to represent set relationships; and
- using deductive reasoning.

**Related SOL** G.2a, G.6

## Materials

- Activity Sheets 1 and 2 (attached)
- Algebraic Qualities of Equality handout (attached)

## Vocabulary

*intersection, union, Venn diagram* (earlier grades)

*conditional statement, hypothesis, conclusion, negation, negate, converse, inverse, contrapositive, biconditional, Law of the Contrapositive, Law of Detachment, Law of Syllogism, deductive reasoning, inductive reasoning, valid argument, logical argument, conjecture, verify, proof, prove, disprove, counterexample, observation, undefined term, postulate, theorem* (G.1)

## Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

- Distribute copies of the two activity sheets. Review the basic vocabulary included on the sheets. Then, have students work in pairs or small groups to complete the activity sheets.
- Distribute copies of the Algebraic Qualities of Equality handout, and have students use it for matching, playing Concentration, or filling in the steps of a proof in addition to writing.

## Assessment

- Questions**
  - How could you disprove the statement, “All fish fly”?
  - What is the next term in the sequence 1, 2, 3, 8, 16, \_\_\_\_\_? Did you use inductive or deductive reasoning? Explain.
  - The definition of vertical angles says, “If two angles are vertical, then they are congruent.”  $\angle 1$  and  $\angle 2$  are vertical. What can you conclude? What type of reasoning did you use?
- Journal/Writing Prompts**
  - Have students complete a journal entry summarizing inductive and deductive reasoning strategies.

- Have students write about a time when they used inductive or deductive reasoning. Students should explain how they know they used inductive or deductive reasoning.
- Does inductive reasoning always result in a true conjecture? Explain.
- **Other**
  - Give an example of correct deductive reasoning using conditional statements. Explain why the reasoning is correct.
  - Give an example of faulty reasoning using conditional statements.

### Extensions and Connections (for all students)

- Have students investigate Lewis Carroll’s logic puzzles.
- Have students solve the logic puzzle from J. K. Rowling’s *Harry Potter and the Sorcerer’s Stone*, 1998, p. 285.
- Invite a journalist or political analyst to visit the class. Ask the guest speaker to explain the relationships among facts, trends, and educated guesses.

### Strategies for Differentiation

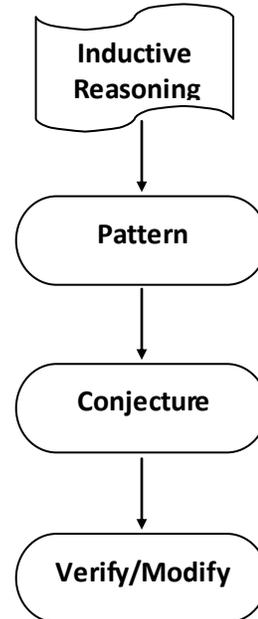
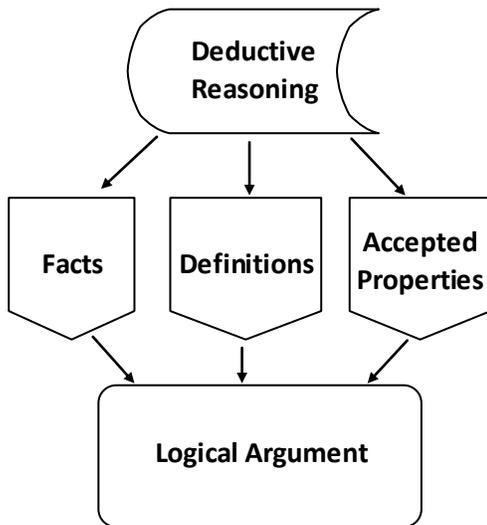
- Have students write their own conditional statement in “if..., then” form. Take four slips of paper. Write “If” on one slip, “then” on another, the hypothesis on the third, and the conclusion on the fourth. Flip the hypothesis (top to bottom), and write the negation of the hypothesis on the back. Do the same for the conclusion. Use these slips to illustrate converse, inverse, and contrapositive. This can also be done in a larger format with students holding the slips in front of the class. When introducing symbols, label the hypothesis, conclusion, and negation statements with  $p$ ,  $\sim p$ ,  $q$ , and  $\sim q$ .
- Use three slips of paper, as above, labeled with  $p$ ,  $\rightarrow$ , and  $q$  to illustrate converse, inverse, and contrapositive using symbols. (Write  $\sim p$  on the back of  $p$  and  $\sim q$  on the back of  $q$ , flipping from top to bottom.)
- Write the names of properties on one set of index cards and their definitions on another set of index cards. Have students pick one or more cards and match themselves with the student who has the corresponding property or definition. Check for correctness, and ask students to post their answers on the board or orally share their answers with the class.
- Have students use presentation software to create presentations of the vocabulary terms. This will allow them to provide their own visual cues and props to reinforce the learning process.
- Post the properties in the room with the statements.
- Have students write the topics and statements on index cards and practice their understanding with one another before and after the lesson.
- Have students draw illustrations to represent the mathematical properties. Illustrations may be groupings of people at work or at play, animals in habitat scenes, and (but not limited to) daily or current events.
- Have students use a diagram to illustrate the information (each student could have his/her own diagram).
- Have students use the same graphic organizer as that found on Activity Sheet 1 to organize material learned in this lesson.
- Have advanced students explore truth tables.
- Use an overhead projector to illustrate the steps in deductive and inductive reasoning.

- Place two extra-large equilateral triangles on the board. One triangle is red; vertex pointing up. The other triangle is blue; place it on its vertex or point. Place four green boxes (called information blocks) inside the triangles, at the top of each triangle, keeping them in a straight line: one in the red and three in the blue. Then place four yellow boxes (called clue blocks) inside the triangles, in the middle of each triangle, keeping them in a straight line: two in the red and two in the blue. Then place four orange boxes (called conclusion blocks) inside the triangles, in the middle of each, keeping them in a straight line: three in the red and one in the blue. After this process, explain to the class that the inductive process begins with few facts but ends with many possible conclusions. Then explain that the deductive process begins with many facts but ends with few possible conclusions.

# Activity Sheet 1: Inductive and Deductive Reasoning

Name \_\_\_\_\_ Date \_\_\_\_\_

Inductive reasoning works from more specific observations to broader generalizations.



Example of Deductive Reasoning	Example of Inductive Reasoning
<ul style="list-style-type: none"> <li>• Tom knows that if he misses the practice the day before a game, then he will not be a starting player in the game.</li> <li>• Tom misses practice on Tuesday.</li> <li>• <i>Conclusion:</i> He will not be able to start in the game on Wednesday.</li> </ul>	<ul style="list-style-type: none"> <li>• <i>Observation:</i> Mia came to class late this morning.</li> <li>• <i>Observation:</i> Mia's hair was uncombed.</li> <li>• <i>Prior Experience:</i> Mia is very fussy about her hair.</li> <li>• <i>Conclusion:</i> Mia overslept.</li> </ul>

Complete the following conjectures based on the patterns you observe in specific cases:

Conjecture: The sum of any two odd numbers is \_\_\_\_\_.

$1 + 1 = 2$	$7 + 11 = 18$
$1 + 3 = 4$	$13 + 19 = 32$
$3 + 5 = 8$	$201 + 305 = 506$

Conjecture: The product of any two odd numbers is \_\_\_\_\_.

Conjecture: The product of a number  $(n - 1)$  and the number  $(n + 1)$  is always equal to \_\_\_\_\_.

Prove or disprove the following conjecture:

Conjecture: For all real numbers  $x$ , the expression  $x^2$  is greater than or equal to  $x$ .

# Activity Sheet 2: Inductive and Deductive Reasoning

Name \_\_\_\_\_ Date \_\_\_\_\_

- John always listens to his favorite radio station, an oldies station, when he drives his car. Every morning he listens to his radio on the way to work. On Monday, when he turns on his car radio, it is playing country music. Make a list of valid conjectures to explain why his radio is playing different music.
- $\angle M$  is obtuse. Make a list of conjectures based on that information.

- Based on the table to the right, Marina concluded that when one of the two addends is negative, the sum is always negative. Write a counterexample for her conjecture.

Addends		Sum
-8	-10	-18
-17	-5	-22
15	-23	-8
-26	22	-4

The Algebraic Properties of Equality (see handout) can be used to solve  $5x - 18 = 3x + 2$  and to write a reason for each step, as shown in the table below.

Statement	Reason
$5x - 18 = 3x + 2$	Given
$2x - 18 = 2$	Subtraction Property of Equality
$2x = 20$	Addition Property of Equality
$x = 10$	Division Property of Equality

Using a table like this one, solve each of the following equations, and state a reason for each step.

- $-2(-w + 3) = 15$
- $p - 1 = 6$
- $2r - 7 = 9$
- $3(2t + 9) = 30$
- Given  $3(4v - 1) - 8v = 17$ , prove  $v = 5$ .

Match each of the following conditional statements with a property:

- |                                   |                              |                                 |
|-----------------------------------|------------------------------|---------------------------------|
| <b>A.</b> Multiplication Property | <b>D.</b> Addition Property  | <b>G.</b> Distributive Property |
| <b>B.</b> Substitution Property   | <b>E.</b> Symmetric Property | <b>H.</b> Subtraction Property  |
| <b>C.</b> Transitive Property     | <b>F.</b> Reflexive Property | <b>I.</b> Division Property     |

- If  $JK = PQ$  and  $PQ = ST$ , then  $JK = ST$ . \_\_\_\_\_
- If  $m\angle S = 30^\circ$ , then  $5^\circ + m\angle S = 35^\circ$ . \_\_\_\_\_
- If  $ST = 2$  and  $SU = ST + 3$ , then  $SU = 5$ . \_\_\_\_\_
- If  $m\angle K = 45^\circ$ , then  $3(m\angle K) = 135^\circ$ . \_\_\_\_\_
- If  $m\angle P = m\angle Q$ , then  $m\angle Q = m\angle P$ . \_\_\_\_\_

# Algebraic Properties of Equality

Note:  $a$ ,  $b$ , and  $c$  are real numbers.

<b>Addition Property</b>	<b>If <math>a = b</math>, then <math>a + c = b + c</math></b>
<b>Subtraction Property</b>	<b>If <math>a = b</math>, then <math>a - c = b - c</math></b>
<b>Multiplication Property</b>	<b>If <math>a = b</math>, then <math>ac = bc</math></b>
<b>Division Property</b>	<b>If <math>a = b</math> and <math>c \neq 0</math>, then <math>a \div c = b \div c</math></b>
<b>Reflexive Property</b>	<b><math>a = a</math></b>
<b>Symmetric Property</b>	<b>If <math>a = b</math>, then <math>b = a</math></b>
<b>Transitive Property</b>	<b>If <math>a = b</math> and <math>b = c</math>, then <math>a = c</math></b>
<b>Substitution Property</b>	<b>If <math>a = b</math>, then <math>a</math> can be substituted for <math>b</math> in any equation or expression.</b>
<b>Distributive Property</b>	<b><math>a(b + c) = ab + ac</math></b>