Constructions

Reporting Category  Reasoning, Lines, and Transformations
Topic  Completing constructions, using a straightedge and compass
Primary SOL  G.4  The student will construct and justify the constructions of
  a) a line segment congruent to a given line segment;
  b) the perpendicular bisector of a line segment;
  c) a perpendicular to a given line from a point not on the line;
  d) a perpendicular to a given line at a given point on the line;
  e) the bisector of a given angle,
  f) an angle congruent to a given angle; and
  g) a line parallel to a given line through a point not on the given line.
Related SOL  G.2, G.6, G.9, G.8, G.11

Materials
  ● Activity Sheets 1 and 2
  ● Straightedges
  ● Compasses
  ● Pencils

Vocabulary
  perpendicular, parallel, ray, angle, segment, intersection, midpoint, congruent, radius (earlier grades)
  arc, perpendicular bisector, angle bisector, construct (G.4)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)
  1. Have students work in pairs to complete the activity sheets using a straightedge, pencil, and compass.
  2. Have students compare and contrast the various constructions with their partners.
  3. Discuss findings as a whole group.
  4. Make sure students understand that it is not okay to estimate or attempt to draw lines or segments freehand without a straightedge or circles or arcs freehand without a compass.

Assessment
  ● Questions
    o Use a straightedge and pencil to draw two angles whose rays do not intersect. Construct an angle whose measure is equal to the sum of the measures of the two angles. How can you check that your construction is correct?
    o Use a straightedge and pencil to draw two segments. Construct a segment whose length is equal to the sum of the lengths of the two segments. How can you check that your construction is correct?
• Use a straightedge and pencil to draw a segment. Construct a perpendicular bisector for the segment. How can you check that your construction is correct?

• Use a straightedge and pencil to draw an angle. Construct an angle bisector for the angle. How can you check that your construction is correct?

• Use a straightedge and pencil to draw a line and a point on the line. Construct a line through the point, perpendicular to the line. How can you check that your construction is correct?

• Use a straightedge and pencil to draw a line and a point not on the line. Construct a line through the point, perpendicular to the line. How can you check that your construction is correct?

• The construction of a line parallel to a line, through a point not on the line uses a postulate about parallel lines cut by a transversal. What postulate is used? How could the construction be changed to use a different postulate or theorem?

• **Journal/Writing Prompts**
  - Complete a journal entry summarizing steps for each construction.
  - Explain how to construct a triangle congruent to another triangle.
  - Explain how to construct an equilateral triangle.
  - Explain how to construct a 30° angle.

• **Other**
  - Have students work in pairs to evaluate strategies.
  - Use activity sheets to help assess student understanding.

**Extensions and Connections (for all students)**

• Constructions can be taught as the vocabulary is taught. For example, when congruent segments and segment bisectors are defined, teach those constructions.

• Have students investigate practical problems involving constructions.

• Have students construct familiar polygons such as equilateral triangles, parallelograms, rhombi, rectangles, and squares.

• Have students construct medians, altitudes, and angle bisectors for triangles.

• Have students complete creative diagrams, using combined constructions.

• Have students use a dynamic geometry software package to perform constructions (without using macros).

• Take students to a football field and have them find examples of line segments, congruent angles, perpendicular lines, etc.

• Invite a surveyor or carpenter to the class to demonstrate how he or she uses bisectors, angles, etc., when surveying land.

• Assign students to work in groups to make illustrations of a town, a home, a school, a mall, or any other scene that includes constructions they have learned. The illustrations should be at least poster-sized.

**Strategies for Differentiation**

• Use dynamic geometry software.
• Provide whiteboard-sized compass and straightedge for students to practice with on the whiteboard.
• Use the compass in the software that comes with interactive whiteboards to demonstrate.
• Provide students with patty paper to fold and/or trace equal segments, angle bisectors, etc.
• Use different color ink/pencils for line segments, congruent angles, bisectors, etc., so that students can physically see the different steps. Also have students use the different colors as they do their constructions.
• Have students work in pairs, with one student giving the other student directions.
• Have students work in pairs, with one student drawing any one of the constructions presented in this unit. Using patty paper, the other student duplicates the construction(s).
• Pair students up and have them look around the classroom for examples of places in the room that illustrate congruent segments, bisectors, perpendicular lines, etc.
Activity Sheet 1: Constructions

Name __________________________ Date __________________

1. Constructing a line segment congruent to a given line segment:
   Given a line segment $\overline{AB}$,
   \[ A \quad B \]
   a. Use a straightedge to draw a line, choose a point on the line, and label it $X$.
   \[ X \]
   b. Use your compass to measure the length of segment $\overline{AB}$, drawing an arc as you measure.
   \[ A \quad B \]
   c. From $X$, draw the exact arc that was drawn on segment $\overline{AB}$, and label it $Y$.
   \[ X \quad Y \]
   d. $\overline{XY} \cong \overline{AB}$: Justification
      We use $A$ as the center of a circle and $B$ as a point on that circle. We keep the same radius and draw a congruent circle from point $X$ so that $Y$ is a point on that circle. Since segments $\overline{AB}$ and $\overline{XY}$ are radii of congruent circles, the segments are congruent.

2. Constructing an angle congruent to a given angle:
   Given $\angle ABC$,
   \[ A \quad B \quad C \]
   a. From point $B$, use your compass to draw an arc that intersects ray $\overrightarrow{BA}$ and ray $\overrightarrow{BC}$.
   \[ A \quad B \quad C \]
   b. Draw a ray, and label it $\overrightarrow{YT}$.
   \[ Y \quad T \]
   c. From point $Y$, draw an arc with the same radius as the radius of arc $DE$. Label the point of intersection $Z$.
d. Use the compass to measure $DE$. From point $Z$, draw an arc with radius $DE$. (This is different than the radius of arc $DE$!)

e. Label the intersection of the two arcs $X$. Draw ray $\overrightarrow{YX}$.

f. Now, $\angle XYZ \cong \angle ABC$.

3. Constructing a perpendicular to a given line at a point on the line:
Given line $|\,$ and point $A$ on $|$,$

a. From $A$, draw two arcs the same distance from $A$ and intersecting line $|\,$, and label the points of intersection $X$ and $Y$.

b. From $X$, draw an arc that intersects line $|\,$ past $A$. Then, draw the same arc from point $Y$ and label the point where the two arcs intersect each other $Z$.

c. Draw the line that passes through points $A$ and $Z$. 
d. Line $AZ \perp \text{line } \mid$ at $A$.

4. Constructing a perpendicular to a given line from a point not on the line:
   Given line $\mid$ and point $A$ not on $\mid$,
   
   a. From point $A$, draw an arc that intersects line $\mid$ in two points. Call these points $X$ and $Y$.
   
   b. From $X$, draw an arc with radius that is more than half the length to point $Y$. Using the same arc radius, draw another arc from $Y$ that intersects the first arc. Call the point of intersection $Z$.
   
   c. Use a straightedge to draw the straight line through points $A$ and $Z$.
   
   d. Line $\overrightarrow{AZ} \perp \text{line } \mid$.

5. Constructing the perpendicular bisector of a given segment:
   Given a line segment, $\overline{AB}$,
   
   a. From $A$, draw an arc that is more than half the length to point $B$. Draw an arc on both sides of the segment $AB$. 
b. Using the same arc length, draw another arc from \( B \) that intersects the first arc on both sides of the segment. Call the points where the arcs intersect \( X \) and \( Y \).

c. Use a straightedge to draw the straight line through points \( X \) and \( Y \).

d. Line \( \overline{XY} \) is the \( \perp \) bisector of segment \( \overline{AB} \).

6. Constructing the bisector of a given angle:
   Given \( \angle ABC \),
a. From $B$, draw an arc that intersects $\overline{BA}$ at $X$ and $\overline{BC}$ at $Y$.

b. From $X$, draw an arc that is large enough to reach past $B$. Using the same compass opening and $Y$ as the circle center, draw another arc that intersects the first arc. Label the intersection $Z$.

c. Draw the ray from $B$ through $Z$. Ray $\overrightarrow{BZ}$ is the angle bisector of $\angle ABC$.

d. $\overrightarrow{BZ}$ bisects $\angle ABC$.

7. Constructing a line parallel to a given line through a point not on the given line:
   Note: You must be familiar with the construction of an angle congruent to a given angle to complete this construction.
   Given line $\overline{\ }$ and point $A$ not on $\overline{\ }$, 
a. From point A, use a straightedge to draw a line that intersects the given line. (This line is called a transversal.) Label the intersection X.

b. Copy one of the angles formed by the transversal and the given line in the corresponding position at A (using the transversal as one side of the angle).
Activity Sheet 2: Constructions

Name ___________________________ Date __________________

Construct a line segment congruent to each given line segment.

1. \[ \overline{AB} \]
2. \[ \overline{CD} \]
3. \[ \overline{WZ} \]

Construct an angle congruent to each given angle.

4. \[ \angle D \]
5. \[ \angle KLM \]
6. \[ \angle QOP \]

Construct a line perpendicular to each given line through the given point on the line.

7. \[ \overline{OV} \]
8. \[ \overline{TO} \]
9. \[ \overline{AM} \]

Construct a line perpendicular to each given line through the given point not on the line.

10. \[ \overline{HU} \]
11. \[ \overline{MC} \]
12. \[ \overline{VE} \]
Construct the perpendicular bisector for each given line segment.

13.  

14.  

15.  

Construct the angle bisector of each given angle.

16.  

17.  

18.  

Construct a line parallel to each given line through the given point not on the line.

19.  

20.  

21.  