

Congruent Triangles

Reporting Category Triangles

Topic Exploring congruent triangles, using constructions, proofs, and coordinate methods

Primary SOL G.6 The student, given information in the form of a figure or statement, will prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.

Related SOL G.4, G.5

Materials

- Straightedges
- Compasses
- Pencils
- Activity Sheets 1, 2, and 3
- Scissors

Vocabulary

line segment, congruent angles (earlier grades)

congruent triangles, included angle, non-included angle, included side, non-included side (G.6), construct (G.4)

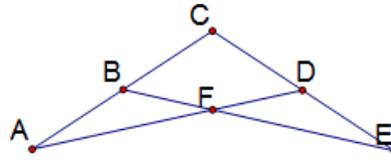
Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Define *congruent triangles* and *corresponding parts of congruent triangles are congruent (CPCTC)*. Explain to the class that according to the definition, you need to show that all three pairs of angles are congruent and all three pairs of sides are congruent.
2. Explain that Activity Sheet 1 will explore which combinations of 3 congruent pairs of parts are enough to show triangles are congruent.
3. Have students work in pairs to complete Activity Sheet 1. Alternatively, applets on the congruence theorems can be found at Web sites for the National Council of Teachers of Mathematics and the National Library of Virtual Manipulatives by searching for congruence theorems. Each student should record his/her own findings. Have students discuss findings with their partners. Discuss findings as a whole group.
4. Have students work in pairs to complete Activity Sheet 2. Be prepared for students to ask about the order of the pairs of corresponding congruent parts in the proof. Acknowledge that there can be some variation. Each student should record his/her own findings. Have students discuss findings with their partners. Discuss findings as a whole group.
5. Assign groups of two-to-four students to a proof from your textbook or other source, and have students complete one copy per group of Activity Sheet 3. Each student should record his/her own findings. Have students discuss findings with their partners. Discuss findings as a whole group.

Assessment

• Questions

- If $\overline{CB} \cong \overline{CD}$ and $\overline{CA} \cong \overline{CE}$, is $\angle A \cong \angle E$? Explain.
- Draw an isosceles triangle



$\triangle ABC$ with $\overline{AB} \cong \overline{AC}$. Label any point D on \overline{BC} , and draw \overline{AD} . List and mark all congruent parts on $\triangle ABD$ and $\triangle ACD$. Do you have enough information to say the two triangles are congruent? Explain.

• Journal/Writing Prompts

- Complete a journal entry summarizing Activity Sheet 1.
- What are the five ways to determine that two triangles are congruent?
- Explain when SSA is enough information to determine whether two triangles are congruent.
- Explain why AAA is not enough information to determine that two triangles are congruent.
- Explain how AAS follows from ASA.
- Describe a real-world example that uses congruent triangles.

• Other

- Have students complete the same activity for different segments and angles.
- Have students construct two triangles that have two pairs of congruent sides and one pair of congruent angles but are not congruent.
- Have students draw pairs of triangles that illustrate each of the five congruence shortcuts. One pair should use a reflexive side, and one pair should have a pair of vertical angles.

Extensions and Connections (for all students)

- Have students draw a segment and two obtuse angles. Have them try to construct a triangle with the segment included by the two angles. Have students discuss any problem with the construction. What theorem or corollary explains the problem?
- Have students draw three angles. Have them try to construct a triangle with these three angle measures. If they are able to do this, have them change only one of the angles and try again. Have students discuss their findings. What theorem or corollary explains the problem?
- Ask students to determine whether it is ALWAYS possible to construct two non-congruent triangles, given two side lengths and a non-included side.
- Ask students to discuss whether they can construct congruent triangles given ANY 3 segment lengths. What theorem can be used to determine when constructing such a triangle is possible?
- Have students explore the use of a carpenter's square and explain why this is an application of congruent triangles.
- Have students explore how congruent triangles are related to why constructions work.
- Research bridge structures and the use of triangles in design and engineering.

Technology

- Use interactive whiteboards and software.

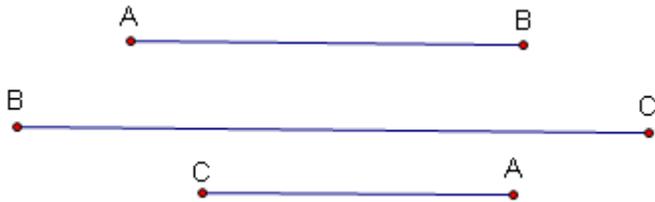
- Provide dynamic geometry software packages for students to use.
- Develop a visual component to support the activity sheet.
- Use colors for Activity Sheet 1, so the sides that are congruent are the same color. Mark the vertices of corresponding angles with dots of the same color. (Mark sides and opposite angles with the same color.)

Activity Sheet 1: Congruent Triangles Shortcuts

Name _____ Date _____

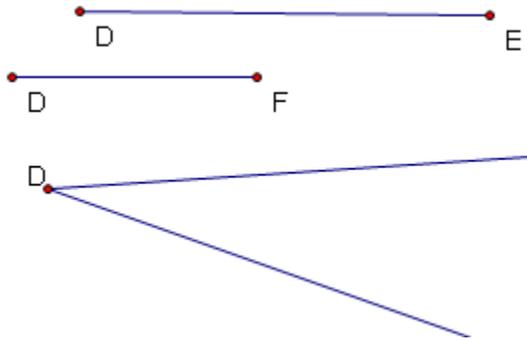
Use a pencil, straightedge, and compass to complete the following tasks and questions:

Part 1: Side-Side-Side (SSS)



1. Construct triangle $\triangle ABC$, with sides congruent to the segments \overline{AB} , \overline{BC} , and \overline{CA} above. Label the vertices A , B , and C , corresponding to the labels above.
2. Compare your triangle to the triangles of other members of your group. How are they the same? How are they different?
3. Is it possible to construct two triangles that are not congruent?
4. Write a conjecture (prediction) about triangles with three pairs of congruent sides.

Part 2: Side-Angle-Side (SAS)



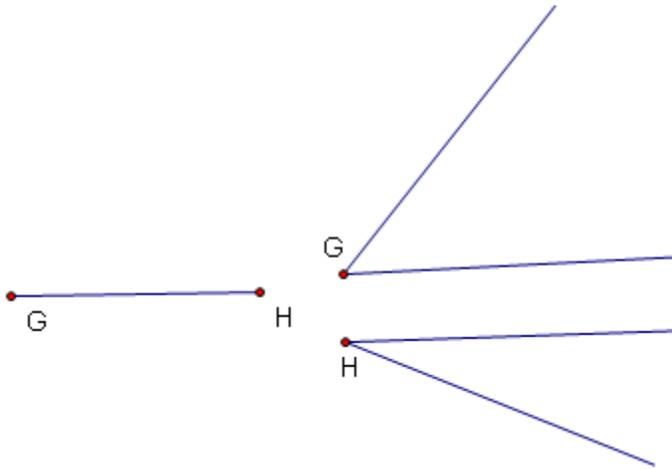
1. Construct triangle $\triangle DEF$, with angle congruent to $\angle D$ above and sides congruent to the segments \overline{DE} and \overline{DF} above. Label the vertices D , E , and F , corresponding to the labels above. Note that $\angle D$ is called the **included angle** because \overline{DE} and \overline{DF} form the sides of $\angle D$.

2. Compare your triangle to the triangles of other members of your group. How are they the same? How are they different?

3. Is it possible to construct two triangles that are not congruent?

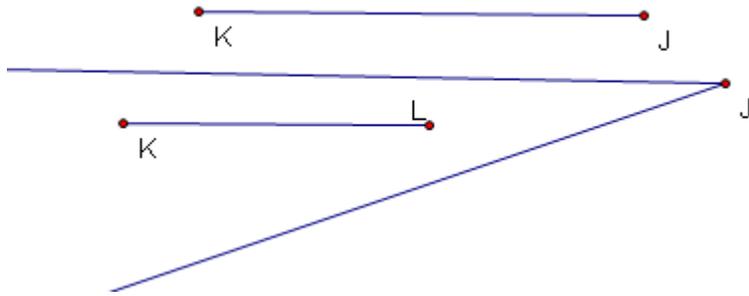
4. Write a conjecture (prediction) about two triangles with two sides and the included angle that are congruent.

Part 3: Angle-Side-Angle (ASA)



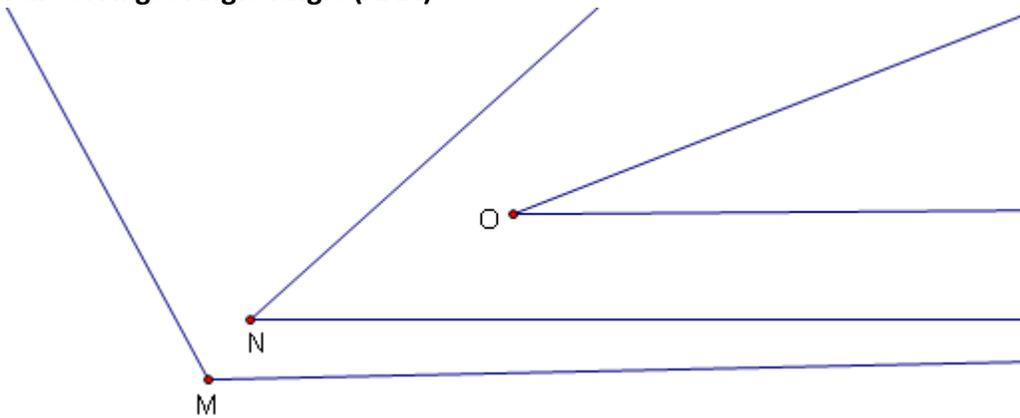
1. Construct triangle $\triangle GHI$, with angles congruent to $\angle G$ and $\angle H$ above and side congruent to the segment \overline{GH} above. Label the vertices G , H , and I , corresponding to the labels above. Note that \overline{GH} is called the **included side** because the vertices of $\angle G$ and $\angle H$ are the endpoints of \overline{GH} .
2. Compare your triangle to the triangles of other members of your group. How are they the same? How are they different?
3. Is it possible to construct two triangles that are not congruent?
4. Write a conjecture (prediction) about two triangles with two angles and the included side that are congruent.

Part 4: Side-Side-Angle (SSA)



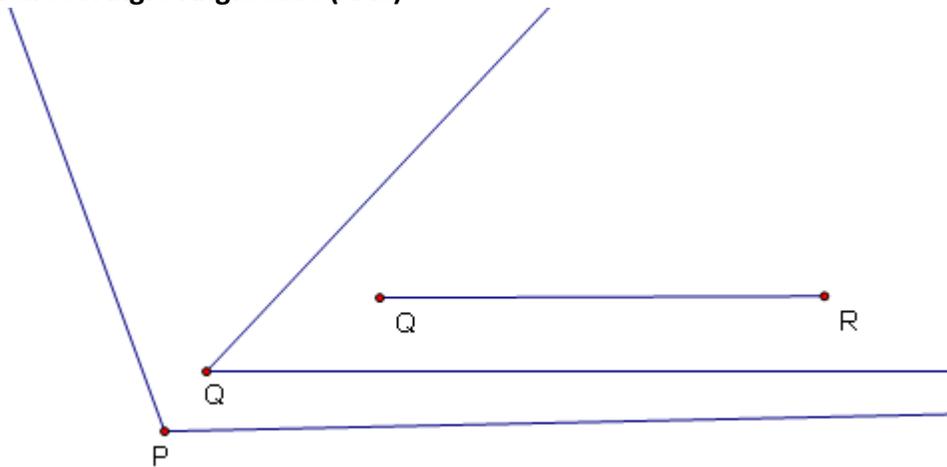
1. Construct triangle $\triangle JKL$, with angle congruent to $\angle J$ above and sides congruent to the segments \overline{JK} and \overline{KL} above. Label the vertices J, K , and L , corresponding to the labels above. Note that $\angle J$ is called a **non-included angle** because \overline{JK} and \overline{KL} do not form the sides of $\angle J$.
2. Compare your triangle to the triangles of other members of your group. How are they the same? How are they different?
3. Is it possible to construct two triangles that are not congruent?
4. Can you say that two triangles with two congruent sides and a pair of congruent non-included angles must be congruent? *Could* they be congruent?

Part 5: Angle-Angle-Angle (AAA)



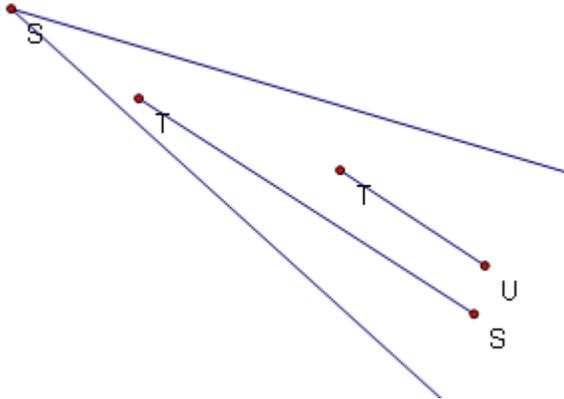
1. Construct triangle $\triangle MNO$, with angles congruent to $\angle M$, $\angle N$ and $\angle O$ above. Label the vertices M , N , and O , corresponding to the labels above.
2. Compare your triangle to the triangles of other members of your group. How are they the same? How are they different?
3. Is it possible to construct two triangles that are not congruent?
4. Can you say that two triangles with three pairs of congruent angles must be congruent? *Could* they be congruent?
5. Can you construct a triangle given *any* three angles? What must be true about the three angle measures?

Part 6: Angle-Angle-Side (AAS)



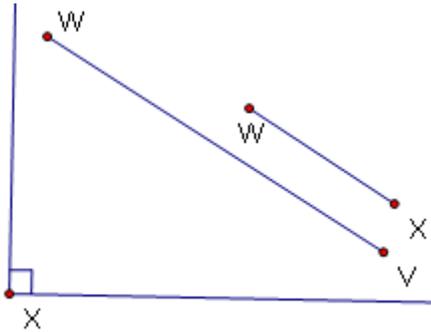
1. Construct triangle $\triangle PQR$, with side congruent to \overline{QR} and angles congruent to $\angle P$ and $\angle Q$ above. Label the vertices P , Q , and R , corresponding to the labels above. Note that \overline{QR} is called a **non-included side**, because the vertices of $\angle P$ and $\angle Q$ are not the endpoints of \overline{QR} .
2. Compare your triangle to the triangles of other members of your group. How are they the same? How are they different?
3. Is it possible to construct two triangles that are not congruent?
4. Write a conjecture (prediction) about two triangles with two angles and a non-included side that are congruent.

Part 7: Another Side-Side-Angle (SSA)



1. Construct triangle ΔSTU , with angle congruent to $\angle S$ above and sides congruent to the segments \overline{ST} and \overline{TU} above. Label the vertices S , T , and U , corresponding to the labels above. Note that $\angle S$ is called a **non-included angle**, because \overline{ST} and \overline{TU} do not form the sides of $\angle S$.
2. Compare your triangle to the triangles of other members of your group. How are they the same? How are they different?
3. Is it possible to construct two triangles that are not congruent?
4. What is the difference between this construction and the construction in Part 4, which was also an SSA construction?

Part 8: Side-Side-Angle (SSA) and Hypotenuse-Leg (HL)



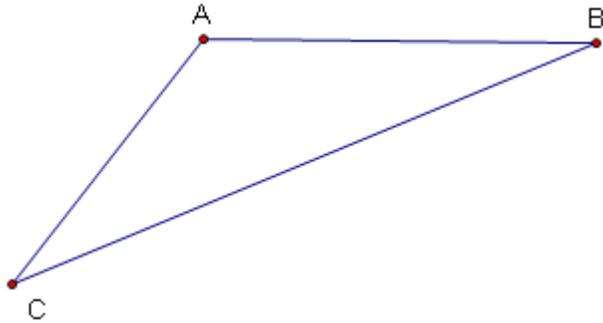
1. Construct triangle $\triangle VWX$, with (right) angle congruent to $\angle X$ above and sides congruent to the segments \overline{VW} and \overline{WX} above. Label the vertices V , W , and X , corresponding to the labels above. Note that $\angle X$ is called a **non-included angle**, because \overline{VW} and \overline{WX} *do not* form the sides of $\angle X$. Note also that \overline{VW} is the **hypotenuse**, and \overline{WX} is a **leg** of a right triangle.
2. Compare your triangle to the triangles of other members of your group. How are they the same? How are they different?
3. Is it possible to construct two triangles that are not congruent?
4. Which of the following applies to your triangle: SSS, SAS, ASA, AAS, SSA, or AAA?
5. Write a conjecture (prediction) about two triangles with right angles, congruent hypotenuses, and one pair of congruent legs.

Activity Sheet 1: Congruent Triangles

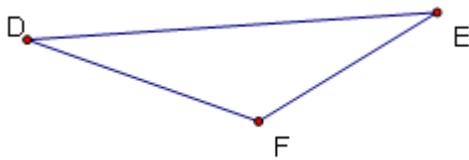
Shortcuts

Instructor's Reference

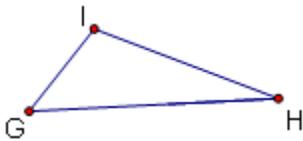
1. The SSS triangle should be congruent to this triangle.



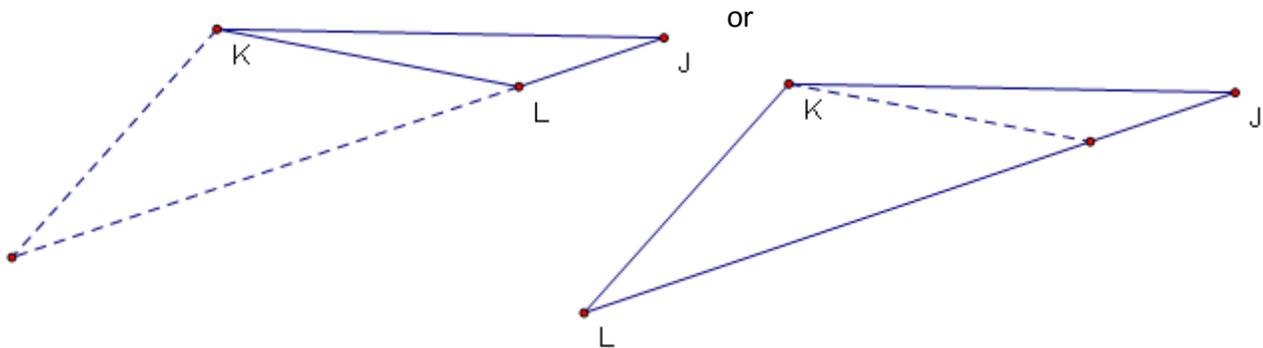
2. The SAS triangle should be congruent to this triangle.



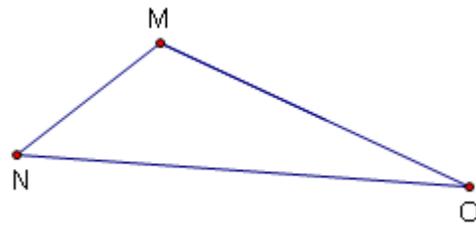
3. The ASA triangle should be congruent to this triangle.



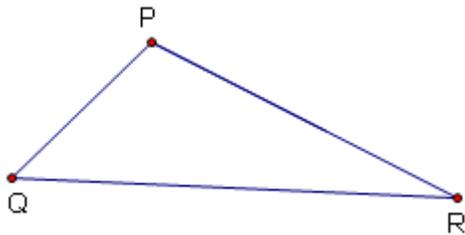
4. There are two possible SSA triangles.



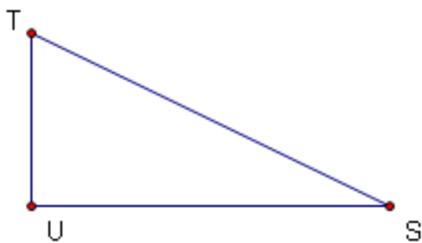
5. There are many different AAA triangles. (They are similar.) Here is one.



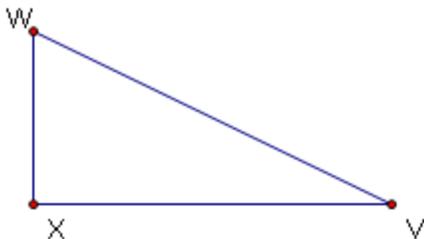
6. The AAS triangle should be congruent to this triangle.



7. This is SSA, but all triangles are congruent to this triangle.



8. The HL triangle should be congruent to this triangle.



Activity Sheet 2: Proofs Scramble

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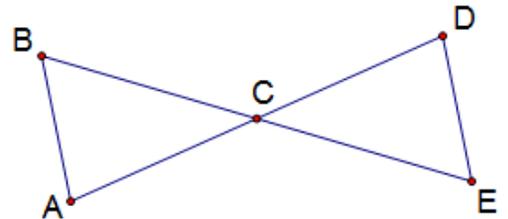
The statements and reasons in the proof below are scrambled.

- Cut apart the proof on the dotted lines.
- Assemble the proof.
- Tape or glue your proof, or rewrite it on a sheet of paper.

Given:

C is the midpoint of \overline{AD} .

$\overline{AB} \parallel \overline{DE}$



Prove: $\overline{AB} \cong \overline{DE}$

Statements	Reasons
$\triangle ABC \cong \triangle DEC$	$\overline{AC} \cong \overline{DC}$
CPCTC	C is the midpoint of \overline{AD} .
given	$\angle ACB \cong \angle DCE$
$\overline{AB} \parallel \overline{DE}$	ASA
$\overline{AB} \cong \overline{DE}$	given
Definition of Midpoint	$\angle A \cong \angle D$
Vertical Angles Theorem	Alternate Interior Angles Theorem

Activity Sheet 3: DIY Proofs Scramble

Name _____ Date _____

Directions:

- Fill in the outline below for the proof of your assigned problem. Include a diagram!
- Cut apart your proof on the dotted lines.
- Mark the back of each piece with the problem number, and place in an envelope. Label the envelope with the page/worksheet and problem #. Swap with another group, assemble their proof, and write it down or check it with your homework, as directed.

Page or worksheet _____ Problem # _____ Diagram:

Given:

Prove:

Statements	Reasons