The Pythagorean Relationship

**Reporting Category**  Triangles

**Topic**  Exploring the Pythagorean Theorem and its converse

**Primary SOL**  G.8 The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

**Related SOL**  G.3a, G.5

**Materials**
- Eleven-pin geoboards (or dot paper or electronic version of eleven-pin geoboard)
- Overhead geoboard
- Ruler
- Protractor
- Activity Sheets 1 and 2 (attached)

**Vocabulary**
- hypotenuse, leg (of a right triangle), Pythagorean Theorem, distance, length, square, acute, obtuse, right triangle (earlier grades)
- converse (G.1)

**Student/Teacher Actions (what students and teachers should be doing to facilitate learning)**
1. Distribute copies of Activity Sheet 1, and have students complete it in small groups. Each student should record his/her own findings. Have students discuss their findings in their group. Discuss findings as a whole group.
2. On a transparent geoboard on the overhead projector, construct a right triangle in which one leg is horizontal and the other is vertical.
3. Ask a student to construct a square on each leg and then on the hypotenuse of the triangle.
4. Ask students to find the area of each square. It may be difficult for some students to recognize a way to find the area of the square on the hypotenuse, so you may need to assist them.
5. Distribute copies of Activity Sheet 2. Have students fill in the data from the example done by the whole class.
6. Have students work to find several other examples and record them in the chart.
7. Have students present their findings to the whole class.

**Assessment**
- **Questions**
  - How can you find the distance from home plate to second base on a square baseball diamond that measures 90 feet on each side?
The shortstop is standing halfway between second base and third base on a baseball diamond (a square that measures 90 feet on each side.) What is the length of a throw from the shortstop to first base?

Which is longer, the diagonal of a rectangle with sides that measure 9 and 11 inches or the diagonal of a square with sides that measure 10 inches?

A door frame that appears to be rectangular has height 213 cm, width 92 cm and one diagonal that measures 231 cm. Is the door frame really rectangular? Explain.

How can you explain that \( \triangle ABC \), shown at right, is a right triangle?

**Journal/Writing Prompts**

Complete a journal entry summarizing one of the activities.

Explain why the Pythagorean Theorem is considered one of the most important theorems in all of mathematics.

Explain what you can determine about a triangle by using the converse of the Pythagorean Theorem, and explain how you would do this.

**Other**

State the Pythagorean Theorem in your own words. Include a diagram.

You can use trigonometry to measure lengths indirectly that you cannot measure directly. Find or take photographs of situations where trigonometry can be used to measure something indirectly.

Have students take turns rolling three dice, and have students determine what kind of triangle has sides with those lengths (if any). Repeat several times. Keep score and give small prizes for top scores.

**Extensions and Connections (for all students)**

Have students investigate the Pythagorean Triples and make generalizations about those lengths.

Have students find a proof of the Pythagorean Theorem other than the proofs investigated here. Have them present the proof to the class and/or write a journal entry about it.

Have students explore similar right triangles.

Have students who are interested in science study parallax.

Invite a carpenter, builder, or city planner to demonstrate to the class how they use the Pythagorean Theorem in their jobs.

Have students construct square roots by constructing the following.

- An isosceles right triangle with leg lengths of 1 unit. (Students will need to start with a segment that is 1 unit long.) The hypotenuse is \( \sqrt{2} \) units.
- A right triangle with one leg the hypotenuse of the last triangle and the other leg 1 unit long. The hypotenuse is \( \sqrt{3} \) units.
- Repeat the last step to construct \( \sqrt{4} \), \( \sqrt{5} \), ...

Have students create their own Pythagorean Theorem Web site.

Provide an enlarged map of your area, and plot key points (e.g., school, park). Use the distance formula or Pythagorean Theorem to calculate the distances between key points. Use spreadsheet software to tabulate results.
Strategies for Differentiation

- Bring various lengths of two-by-fours to the classroom, and lean them up against the walls. Have groups of students go around the room and measure the length of each board and its height on the wall. Then ask them to calculate the distance from the base of the wall to the bottom of the board. Have the groups report back their findings to the large group.
- Use colors for the different sides (i.e., mark the hypotenuse with the same color consistently).
- Allow students to use dynamic geometry software to do the lesson.
- Have students use a calculator (e.g., talking or big-numbered calculator).
- Use Web resources to post classroom lectures and notes online.
- Use software to create assistive learning tools for students.
- Color-code the different squares and different sides of triangles.
- Use presentation software to isolate or highlight key points of the lesson.
- Have students work in groups to make up their own quizzes.
- Have students use presentation software to demonstrate their knowledge and understanding of the key terms (e.g., pictures, sound, and motion).
- Have students use a chart to organize what they know, what they want to learn, and what they have learned to organize information presented in the unit.
Activity Sheet 1: Exploration of Right Triangles

Make a right triangle on an 11-pin geoboard or dot paper. Construct (or draw) a square on each side of the triangle. Label the shortest side a, the middle side b, and the longest side c. Measure the lengths a, b, and c of the sides. Complete the table.

<table>
<thead>
<tr>
<th>Length of side a</th>
<th>Length of side b</th>
<th>Length of side c</th>
<th>Area of square on side a</th>
<th>Area of square on side b</th>
<th>Area of square on side c</th>
<th>$a^2 + b^2$</th>
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1. Across from what angle do you always find side c, the longest side?

2. What other patterns do you see?

3. State the relationship in words, using the letters a, b, and c.

4. When do you think this would be true? Why?

An Algebraic Approach to the Pythagorean Theorem
Fill in expressions for each of the indicated areas.
1. Area of the large square $WXYZ = (a + b)^2 = (a + b)(a + b) = \underline{\hspace{100mm}}$

2. Area of the large square $WXYZ = \text{area of square } STUV + 4(\text{area of triangle } XST) = \underline{\hspace{100mm}} + \underline{\hspace{100mm}}$

3. Set the expressions from #1 and #2 equal to each other and simplify. Where have you seen this before? Shade a right triangle in the drawing for which the relationship is true.
Activity Sheet 2: Exploration of the Converse of the Pythagorean Theorem

Using your geoboard or dot paper, construct different types of triangles. Use a ruler to measure side lengths, if necessary. Use a protractor, if necessary, to determine if the triangle is acute, right, or obtuse. Complete the table. Note that c should be the length of the longest side.

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<tr>
<th>Length of side a</th>
<th>Length of side b</th>
<th>Length of longest side c</th>
<th>Sketch of Triangle</th>
<th>c²</th>
<th>a²</th>
<th>b²</th>
<th>Which one?</th>
<th>Type of triangle: acute, right, or obtuse</th>
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1. What patterns do you see?

2. State the relationship in words, using the letters a, b, and c.

3. What generalizations can you make?