A Mystery to Solve

Reporting Category Equations and Inequalities

Topic Investigating properties with an undefined operation

Primary SOL A.4b The student will solve multistep linear and quadratic equations in two variables, including justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets.

Related SOL A.4d

Materials

(none identified)

Vocabulary

field property, associative property, commutative property, distributive property, identity property, inverse property (earlier grades)

closure property, zero product property, reflexive, transitive, symmetric (A.4)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Display the following list of field properties of real numbers and properties (axioms) of equality, discuss them, and provide examples for students. Have students generate some examples or explain the property in their own words.

[Field] Properties of Real Numbers

Let a, b, and c be real numbers.

Property	Addition	Multiplication
closure	$a + b$ is a real number	ab is a real number
associative	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
commutative	$a + b = b + a$	$ab = ba$
identity	$a + 0 = a = 0 + a$	$a \cdot 1 = a = 1 \cdot a$
inverse	$a + (-a) = 0 = (-a) + a$	$a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a$, if $a \neq 0$
distributive	$a(b + c) = ab + ac$	

Other Properties of Real Numbers

Let a, b, and c be real numbers.

Property	
multiplicative property of zero	$a \cdot 0 = 0 = 0 \cdot a$
zero product	If $ab = 0$, then $a = 0$ or $b = 0$.
reflexive	$a = a$
transitive	If $a = b$ and $b = c$, then $a = c$.
symmetric	If $a = b$, then $b = a$

Properties of Equality and Inequality

Let a, b, and c be real numbers.

Property	Equality	Inequality
addition	If $a = b$, then $a + c = b + c$.	If $a < b$, then $a + c < b + c$. If $a > b$, then $a + c > b + c$.
subtraction	If $a = b$, then $a - c = b - c$.	If $a < b$, then $a - c < b - c$. If $a > b$, then $a - c > b - c$.
multiplication	If $a = b$, then $ac = bc$.	If $a < b$ and $c > 0$, then $ac < bc$. If $a < b$ and $c < 0$, then $ac > bc$. If $a > b$ and $c > 0$, then $ac > bc$. If $a > b$ and $c < 0$, then $ac < bc$.
division	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.	If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$. If $a < b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$. If $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$. If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.

2. Display the table at right, which shows how an operation, ☺, works with the set of numbers {1, 2, 3, 4, 5, 6}. Ask students whether this set has an identity element for this operation. If so, ask what it is and how they determined that it is an identity element.
3. Next, ask whether any number in the set has an inverse for this operation. If so, ask students to identify the inverse for each number that has an inverse. Ask how they determined whether or not a number has an inverse.

☺	**1**	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	3	4	5	6	1
3	3	4	1	6	2	5
4	4	5	6	1	3	2
5	5	6	2	3	1	4
6	6	1	5	2	4	3

4. Direct students to use the table to solve the equation 3 ☺ x = 5. Have them check their solutions by testing whether 3 ☺ "their" x = 5.
5. Tell students that for the equation 3 ☺ x = 5, Steve tried the following:

   3 ☺ x = 5

   3 ☺ 3 ☺ x = 3 ☺ 5

   x = 2

   Have students explain what Steve did at each step and check the solution to see whether it is correct. Ask them to explain what happened.

Assessment

- **Questions**
  - Is the set {−1, 0, 1} closed with respect to addition? Is it closed with respect to multiplication? Show your work, and provide written explanations to support your work.

  - Consider the closed set {−1, 0, 1}. Which of the field properties of real numbers hold for the set? Show your work, and provide written explanations to support your work.
  - Solve $3(x + 6) = 5(x - 4)$, and justify each step of your solution with a field property of real numbers or property of equality.
- **Journal/Writing Prompts**
  - Explain what it means for a set to be "closed" with respect to a specific operation.

Extensions and Connections (for all students)

- Have small groups of students create on index cards examples of the associative property, commutative property, distributive property, identity property, and inverse property. Have students put all their cards into one pile, shuffle the cards, and as a class, sort all of the cards into groups according to the property.
- Divide the class into groups of three or four. Give each group a set of index cards that show examples of the properties being studied. Have each group arrange the cards into categories and then share their findings with the class.

Strategies for Differentiation

- Have students create flash cards listing the properties on one side and illustrations and/or definitions on the other.
- Have students create a graphic organizer for the field properties of real numbers and properties (axioms) of equality.