Types of Variations

Reporting Category	Statistics
Topic	Exploring variations
Primary SOL	AII.10 The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.
Related SOL	AII.1a

Materials

- Graphing calculators
- Two attached handouts

Vocabulary

direct variation, direct proportion (earlier grades)

inverse variation, joint variation, combined variation (AII.10)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Review direct variation and the generalized model used to describe a direct variation ($y = kx$). Provide instruction related to inverse variation, joint variation, and a combination of direct and inverse variations. Include examples of finding the equation from a variation statement and of creating a variation statement from an equation. Show how to obtain the constant of variation, using given data.
2. Distribute copies of the attached Variation Examples handout, and have students complete it. (Note: This will help students transition to confidently translating the various variation types.)
3. Distribute copies of the attached Direct, Inverse, and Joint Variations handout, and have students work individually to complete it in whole or in part. (Note: You may choose to use parts of this handout for assessment or for another activity.)

Assessment

- **Questions**
  - Direct variation, inverse variation, and joint variation are names of specific types of mathematical models. What patterns in given data would lead you to choose one of these to model the data?
  - What are some real-world formulas that are examples of direct, inverse, and joint variation?
- **Journal/Writing Prompts**
  - Explain the information you use to determine whether a variation exists.
  - In your own words, explain what is meant by a "constant of variation."
- **Other**
  - Use part or all of the Direct, Inverse, and Joint Variations handout for assessment.

Extensions and Connections (for all students)

- Ask students to collect over a period of a week examples of variations that they see or that occur in their lives.

Strategies for Differentiation

- Have students create and use flash cards, each with a variation type listed on one side and several examples of that type on the other.
- Provide students with sentence frames of the exact language they should use as they describe the variation relationships listed in the AII.10 standard.
- Create an "I Have. Who Has?" game to provide students with additional practice verbalizing variation relationships.

Variation Examples

Type of Variation	Examples of Variation Statements	Corresponding Equations
Direct	1. G varies directly as *t* squared. 2. 3.	1. $G = kt^2$ 2. 3.
Inverse	1. 2. 3.	1. 2. 3.
Joint	1. 2. 3.	1. 2. 3.
Combined	1. 2. 3.	1. 2. 3.

Direct, Inverse, and Joint Variations

Part I

Identify each of the following statements as a direct, inverse, or joint variation by filling in the blank with the words *directly, inversely,* or *jointly.*

1. Volume of a gas, *V*, at constant temperature varies _____ with its pressure, *P*.
2. Intensity of sound varies _____ with distance away from the object creating the sound.
3. The weight of a body varies _____ with the square of the distance it is from the center of the earth.
4. The power of an electrical circuit varies _____ as the resistance and current.
5. The heat loss through a glass window of a house on a cold day varies _____ as the difference between the inside and outside temperatures and the area of the window. The heat loss varies _____ as the thickness of the window glass.
6. The amount of sales tax paid varies _____ as the total of the goods purchased.
7. The time to complete a job varies _____ as the number of workers working.
8. To balance a seesaw, the distance a person is from the pivot is _____ proportional to his/her weight.
9. The intensity of a light varies _____as the square of the distance from the light source.
10. The time it takes to complete a specific trip varies _____ as the speed of travel.
11. The cost of gas on a trip varies _____ with the length of the trip.
12. The length of a spring varies _____ with the force applied to it.
13. The number of congruent marbles that fits into a box is _____ proportional to the cube of the radius of each marble.
14. The number of people invited to dinner varies _____ as the amount of space each guest has at the table.
15. The number of people invited to dinner varies _____ as the number of pieces of silverware used.
16. The time it takes to harvest a crop varies _____ with the number of people assisting in the harvest.
17. The time it takes a runner to complete a lap on the track varies _____ as the speed of the runner.
18. The cost of a cake varies _____as the cake's thickness and the square of the radius.
19. The number of calories burned during exercise varies _____ with the time spent performing the exercise.
20. The power generated by a windmill is _____ proportional to the cube of the wind speed.

Part II

Write the equation being described by each of the following statements.

21. The volume, *v*, of a balloon is directly proportional to the cube of the balloon's radius, *r*.
    _____
22. The number, *n*, of grapefruit that can fit into a box is inversely proportional to the cube of the diameter, *d*, of each grapefruit. _____
23. The time, *t*, that a plane spends on the runway varies inversely as the take-off speed.
    _____
24. The weight, *w*, that a column of a bridge can support varies directly as the fourth power of its diameter, *d*, and inversely as the square of its length, *l*. _____
25. The radiation, *r*, from the decay of plutonium is directly proportional to the mass, *m*, of the sample tested and inversely proportional to the square of the distance, *d*, from the detector to the sample. _____
26. #20 in Part I _____
27. #3 in Part I _____
28. #8 in Part I _____
29. #9 in Part I _____

Part III

Write an equation for and solve each of the following word problems.

30. The cost, *c*, in cents of lighting a 100-watt bulb varies directly as the time, *t*, in hours, that the light is on. The cost of using the bulb for 1,000 hours is $0.15. Determine the cost of using the bulb for 2,400 hours. _____
31. The power, *P*, in watts of an electrical circuit varies jointly as the resistance, *R*, and the square of the current, *C*. For a 240-watt refrigerator that draws a current of 2 amperes, the resistance is 60 ohms. What is the resistance of a 600-watt microwave oven that draws a current of 5 amperes? _____
32. The force needed to keep a car from skidding on a curve varies directly as the weight of the car and the square of the speed and inversely as the radius of the curve. Suppose a 3,960 lb. force is required to keep a 2,200 lb. car traveling at 30 mph from skidding on a curve of radius 500 ft. How much force is required to keep a 3,000 lb. car traveling at 45 mph from skidding on a curve of radius 400 ft.? _____

Part IV

Write a general equation for each of the following relationships, and sketch it:

33. *Y* varies jointly as *W* and *X.*

34. *Y* varies directly as the square of *X.*

35. *Y* varies inversely as *X.*

36. *Y* varies inversely as the square of *X.*

Part V

Write a variation statement for each of the following models in which *k* is the constant of variation.

37. $V = \frac{k}{t^2}$

38. $R = klwh$

39. $G = \frac{kp_1p_2}{d}$

40. $S = k(b_1 + b_2)h$