Permutations and Combinations

Reporting Category Statistics

Topic Counting using permutations and combinations

Primary SOL AII.12 The student will compute and distinguish between permutations and combinations and use technology for applications.

Materials
- Graphing calculators
- Three attached handouts

Vocabulary

counting, multiplication counting principle (earlier grades)

permutation, combination, factorial (AII.12)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Distribute copies of the attached Counting Exploration handout, and have students work in groups to complete it. Then, hold a class discussion of student responses. (Note: This activity reviews critical concepts necessary for understanding and interpreting permutations and combinations; therefore, a thorough discussion of the results is important. It also includes an introduction to factorials, though you should reinforce some of the simplification methods by expanding factorials.)

2. Describe a *permutation* as an arrangement of objects in which order is important. Present the problem: "There are seven marching bands in a parade, and all of them want to be at or near the beginning of the parade. How many permutations are there for their order in the parade?" Put seven blank spaces, left to right, on the board to represent the seven band positions in the parade. Ask students, "From how many bands can I choose one band for first place? From how many can I choose one for second place? For third place? For fourth place? For fifth, sixth, and seventh places?" As each question is answered, write the number in the appropriate space. (Note: The goal of this activity is to connect factorials to counting permutations.)

3. Next, present this problem: "Twenty bands have applied to march in the parade, but only seven spots are available. How many permutations of seven bands are possible for their order in the parade?" Again, put seven blank spaces on the board, and ask, "From how many bands can I choose one band for first place? From how many can I choose one for second place? The blanks spaces will be filled as follows: 20 19 18 17 16 15 14 . Have students state that the total number of permutations is the product of these seven numbers. Make the connection: $\dfrac{20!}{(20-7)!} = \dfrac{20!}{13!} = 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14$.

4. Distribute copies of the attached Permutations Activity handout, and have students work in pairs to complete it. When they are finished, have them share their work with the class.

5. Describe a *combination* as a collection of objects in which the order is not important. Present the problem: "A student gets to select two of the six gifts in a gift box. How many different pairs of gifts can she select?" Have students list all the possible combinations of two out of the six gifts. Discuss how combinations are like permutations, and how they are different. Some students will understand that a combination of *r* objects selected from a group of *n* is the same thing as $\dfrac{{}_nP_r}{r!}$ because the order of the *r* objects is not important. This expression also describes the combination formula.

6. Distribute copies of the attached Combinations Activity handout, and have students work in pairs to complete it. When they are finished, have them share their work with the class.

Assessment
- **Questions**
  - What are some examples of counting problems for which you would use a permutation? What are some examples for which you would use a combination?
  - What are some words you associate with permutation? What are some words you associate with combination?
- **Journal/Writing Prompts**
  - Explain why a locker combination is called a combination when it is actually a permutation.
  - Explain why 0! is equal to 1.

Extensions and Connections (for all students)
- Several lottery games have odds calculated by using permutations and combinations. Have students investigate one such game and determine whether a permutation or a combination is used to calculate the odds.
- Show students how Pascal's triangle can be used as a combination table.

Strategies for Differentiation
- Have students physically arrange themselves in order to complete questions 1 and 2 on the Counting Exploration handout.
- Provide opportunities for students to model permutations and combinations physically.

Counting Exploration

1. Three students, Andy, Beth, and Cara, must arrange their desks in a row in their classroom. Your job is to figure out how many different arrangements they can make. Make a list of all possible arrangements. How many arrangements are possible?

2. Dana is added to the group of students in #1 above, so you now have to arrange four students in a row. How many arrangements are possible? Justify your answer.

3. Mr. Smith's class wants to elect a president, vice-president, and treasurer. The names of students in his class are: Arlene, Bud, Cathy, David, Eli, Fred, George, Harry, Isaiah, Jack, Kim, Larry, Marlin, Ned, Oprah, Perry, Quentin, Ralph, and Sara. How many different slates of officers are possible?

4. Mr. Smith's class wants to create a dress code committee composed of three students. Using the student names in #3 above, how many different committees are possible?

5. How is the answer to #3 related to the answer to #4?

6. Bill has 3 hats, 5 shirts, 10 pairs of pants, and 6 pairs of shoes. If an outfit consists of a hat, a shirt, a pair of pants and a pair of shoes, how many different outfits does Bill have?

7. Joe's pizza has the following optional toppings: pepperoni, sausage, peppers, and onions. All pizzas come with cheese. How many different combinations of toppings are possible? Show all of them, or describe them in such a way that you show you have accounted for all possibilities.

8. n factorial, written as $n!$, is equal to $n \cdot (n - 1) \cdot (n - 2) \cdot \ldots \cdot 2 \cdot 1$. Complete the factorial table at right, and simplify the factorial expressions.

$n!$	
1!	
2!	
3!	
4!	
5!	
6!	
7!	
8!	
9!	
10!	

a. $\dfrac{5!}{3!}$

b. $\dfrac{7!}{4!}$

c. $\dfrac{8!}{5!3!}$

d. $\dfrac{10!}{5!5!}$

Permutations Activity

Complete the following table by providing missing information.

Problem	Filling in the blanks: How many ways can you fill each space?	Using the permutation formula $_nP_r = \dfrac{n!}{(n-r)!}$	Answer
The manager of a baseball team has named the 9 starters for a game. He needs to determine the batting order. How many batting orders are possible?	___ ___ ___ ___ ___ ___ ___ ___ ___	$_9P_9 = \dfrac{9!}{(9-9)!} = \dfrac{9!}{0!} = 9!$	362,880
There are 33 cars in a car race. The first-, second-, and third-place finishers win a prize. How many different arrangements of the first three positions are possible?	___ ___ ___		
	<u>10</u> <u>9</u> <u>8</u> <u>7</u>		5,040
The license plates in a certain country contain a permutation of exactly 5 letters. If no letter can be repeated, how many letter permutations are possible?			
		$_7P_4 = \dfrac{7!}{(7-4)!} = \dfrac{7!}{4!}$	

Combinations Activity

Complete the following table by providing missing information.

Problem	Example of one possible combination	Using the combination formula $_nC_r = \left(\dfrac{n}{r}\right) = \dfrac{n!}{r!(n-r)!}$	Answer
The manager of a baseball team has to select the 9 starters for a game. She has 11 players to choose from. How many combinations of 9 players can she choose?		$_{11}C_9 = \dfrac{11!}{9!(11-9)!} = \dfrac{11!}{9!2!}$	55
A committee of 5 students is to be selected from a class of 20 students. How many different committees are possible?	Students 1, 5, 7, 19, 20		15,504
		$_9C_3 = \dfrac{9!}{3!(9-3)!} = \dfrac{9!}{3!6!}$	
The special combo pizza at Ciro's Pizza includes any 5 toppings from the list of 16 toppings. How many different combo pizzas are possible?			
		$_7P_4 = \dfrac{7!}{(7-4)!} = \dfrac{7!}{4!}$	