# Rational Expressions

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| **Reporting Category** | Expressions and Operations |
| **Topic** | Performing operations with rational expressions |
| **Primary SOL** | AII.1a The student, given rational, radical, or polynomial expressions, will add, subtract, multiply, divide, and simplify rational algebraic expressions. |
| **Related SOL** | AII.1d |

**Materials**

- Graphing calculators
- Three attached handouts

**Vocabulary**

*simplify, factor, undefined* (earlier grades)

*rational expression* (AII.1a)

**Student/Teacher Actions (what students and teachers should be doing to facilitate learning)**

*DAY 1*

1. Write the following rational expressions across the top of the board: $x^2-4$, $x^2+4x+4$, $x^2-4x+4$, and $x^4-2x^2$. Have volunteers come up and factor the expressions. Next, ask six different volunteers to come up and create fractions out of the factored expressions, e.g., $\frac{(x+2)(x-2)}{(x+2)(x+2)}$. Have pairs of students discuss what can be done to simplify the created fractions. Write the created fractions on the side of the board to be returned to after a review of simplifying fractions.
2. Review simplifying fractions. Stress the fact that a fraction can be simplified only if the numerator and denominator have common factors (*factors*: numbers and/or variables that are being multiplied). For example, $\frac{18}{24}$ can be simplified because $\frac{18}{24}=\frac{6\times 3}{6\times 4}$, which shows that there is a common factor of 6. To simplify $\frac{18}{24}$, we divide by $\frac{6}{6}$ (an expression equal to 1), which results in $\frac{3}{4}$. (Note: You might also show this by writing the prime factorization of each number and crossing out like factors.)
3. Refer back to the rational expressions written across the top of the board and the fractions that were created by the students out of the factored expressions. Stress again that we can simplify only when the numerator and denominator have common factors. (Note: It is important that students understand that addends cannot be simplified; only factors can be simplified.) Have a student read one of the fractions aloud, e.g., "*x* plus two, times *x* minus two, divided by *x* plus two, times *x* plus two." Now, ask the student to repeat this, and ask the rest of the class to count how many times the student says the word *times*. Have

another student read the numerator only, and ask the class how many times the word *times* was said. After they answer "once," ask how many factors are separated by multiplication. Explain that when simplifying $\frac{(x+2)(x-2)}{(x+2)(x+2)}$, neither the $x$'s nor the 2s can be simplified by themselves. Only the *factors* (numbers and/or variables that are being multiplied) can be simplified, and those factors must be identical. Thus, we cannot cancel the $x + 2$ with the $x - 2$.

4. Repeat this exercise with several expressions, making sure to include monomial factors.
5. Distribute copies of the attached Simplifying Rational Expressions handout, and have students work in pairs to complete it. Instruct partners to alternate reading the factored expressions aloud. Stress that they should focus on how many expressions are being multiplied*.* These are the factors that can possibly be simplified.

*Day 2*

1. Review multiplication and division of common fractions. Focus on the fact that we can simplify by canceling common factors in a numerator and a denominator. Review adding and subtracting fractions with like and unlike denominators. Remind students that the reason the least common multiple of 6 and 8 is 24 is because $6 = 3 \cdot 2$ and $8 = 2 \cdot 2 \cdot 2$. The least common multiple must include one of every distinct factor from each of 6 and 8; therefore, the LCM = $3 \cdot 2 \cdot 2 \cdot 2$, which is 24.
2. Have students find the LCM for several sets of numbers.
3. Using the same method of factoring and multiplying distinct factors to determine the LCM, transition into finding the LCM of variable expressions. Advise students to put parentheses around pairs of addends as a reminder that the entire expression is a factor.
4. Now, show examples of adding and subtracting rational expressions. Again, be sure to show examples of addition and subtraction of common fractions. Remind students that if the value of the denominator changes by a factor, the value of the numerator must change by the same factor in order to preserve the value of the expression.

*Day 3*

1. Distribute copies of the attached Rational Expressions: Extension handout. Have students work together in small groups to discuss which operation(s) each expression contains and decide which steps must be taken to simplify each expression. Have groups share ideas with the entire class.
2. Demonstrate two methods for simplifying problems such as #2 on the handout.
   - Method 1: Multiply the numerator and denominator by the LCM of all fractions within the complex expression. Then, simplify the resulting rational expression.

$$\frac{x(2x+1)}{x(2x+1)} \cdot \frac{\frac{1}{x} - \frac{1}{2x+1}}{\frac{4x}{2x+1}} = \frac{(2x+1)-x}{4x^2} = \frac{x+1}{4x^2}$$

- Method 2: Simplify the numerator of the complex fraction; simplify the denominator of the complex fraction; then, simplify the resulting rational expression.

$$\frac{\frac{1}{x}-\frac{1}{2x+1}}{\frac{4x}{2x+1}}=\frac{\frac{2x+1}{x(2x+1)}-\frac{x}{x(2x+1)}}{\frac{4x}{2x+1}}=\frac{\frac{x+1}{x(2x+1)}}{\frac{4x}{2x+1}}=\frac{x+1}{x(2x+1)}\div\frac{4x}{2x+1}=\frac{x+1}{x(\cancel{2x+1})}\cdot\frac{\cancel{2x+1}}{4x}=\frac{x+1}{4x^2}$$

**Assessment**

- **Questions**
  - Why must all rational expressions be factored before they can be simplified?
  - How do we determine the common denominator when adding rational expressions?
  - In a fraction, why must we change the value of the numerator when we change the value of the denominator?
- **Journal/Writing Prompts**
  - Compare and contrast simplifying and performing operations on rational expressions to simplifying and performing operations on non-variable fractions.
  - Explain why the expression $\frac{x^2+6}{x^2+3}$, which seemingly can be simplified, actually cannot be simplified.

**Extensions and Connections (for all students)**

- Explain to students that there are often restricted values for the variables due to the fact that the value of a denominator of a fraction cannot be zero. Revisit the handouts students have completed, and call on individuals to verbally give the restricted value(s) for each expression. Tell students to look at not only their answers, but also the original expressions.
- Give students the function $y=\frac{x+2}{x-1}$, and have them consider what the *x*- and *y*-intercepts will be. After they have discussed their thoughts, have them graph the function on their calculators.
- Have students graph unsimplified rational expressions and then graph the simplification to see whether the two graphs are the same. (Note: They will need to account for removable discontinuities.)
- Have students complete the attached Performing Operations on Rational Expressions set of problems.

**Strategies for Differentiation**

- Provide a sentence frame for students to read this complex fraction aloud. $\frac{(x+2)(x-2)}{(x+2)(x+2)}$
- Create a card activity in which students put in correct order cards containing steps to simplifying an expression. Use an interactive whiteboard for this activity.

# Simplifying Rational Expressions

Simplify the following rational expressions completely.

1. $\dfrac{3m}{6m}$

2. $\dfrac{12ab^2}{15a^2}$

3. $\dfrac{(c+2)(c-1)}{c+2}$

4. $\dfrac{r^2(r+3)}{r(r-3)}$

5. $\dfrac{10v+40}{8v+32}$

6. $\dfrac{x-1}{x^2-1}$

7. $\dfrac{d^2+6d+8}{d^2-d-20}$

8. $\dfrac{h^2-9}{h^2+6h+9}$

9. $\dfrac{f^2+2f-8}{2f^2-8}$

10. $\dfrac{m+2}{m^2+8}$

11. $\dfrac{j^2+7j+6}{12j^3-14j^2-10j}$

12. $\dfrac{2y^2+9y-18}{4y^2-6y}$

# Simplifying Rational Expressions: Extension

Simplify the following rational expressions completely.

1. $$\frac{\dfrac{x+3}{x^2-25}}{\dfrac{x^2-9}{x+5}}$$

2. $$\frac{\dfrac{1}{x}-\dfrac{1}{2x+1}}{\dfrac{4x}{2x+1}}$$

3. $$\frac{\dfrac{2}{4x+12}}{\dfrac{4}{2x+6}+\dfrac{1}{x+3}}$$

4. $$\frac{\dfrac{4}{x-3}+3}{\dfrac{4x-1}{x-3}+4}$$

5. $$\frac{\dfrac{4}{x^2-25}+\dfrac{2}{x+5}}{\dfrac{1}{x+5}+\dfrac{1}{x-5}}$$

# Performing Operations on Rational Expressions

Multiply and divide each of the following rational expressions as indicated, and give the result in simplest form.

1. $\frac{8x-32}{x^3+x^2}\cdot\frac{x^5-x^4-2x^2}{4x^2-16x+16}$

2. $\frac{w^2+2w-3}{3w^2-w-2}\div(w+3)$

3. $\frac{40mn}{18r^2}\div\frac{25m^2n}{27r^2m}$

4. $\frac{k^2+5kn-6n^2}{2k^2}\cdot\frac{12k}{4k-4n}$

5. $\frac{x^2+8x+16}{c^2-6c+9}\div\frac{2x+8}{3c-9}$

6. $\frac{7x+7y}{y-x}\div\frac{21}{4x-4y}$

Add and subtract the following rational expressions, as indicated, and simplify answers.

7. $\frac{5}{a}+\frac{3}{a-1}$

8. $\frac{3-4n}{n^2+3n-10}-\frac{2}{n+5}$

9. $\frac{x}{x^2-x-30}-\frac{1}{x+5}$

10. $\frac{x}{x^2-9}+\frac{3}{x(x-3)}$

11. $\frac{3}{p+1}+\frac{4}{2p-6}-\frac{p^2-5}{p^2-2p-3}$