Complex Numbers

Reporting Category Expressions and Operations

Topic Performing complex number arithmetic

Primary SOL AII.3 The student will perform operations on complex numbers, express the results in simplest form, using patterns of the powers of *i*, and identify field properties that are valid for the complex numbers.

Materials

- Powers of *i* Table (attached)
- Two attached handouts

Vocabulary

real numbers, rational numbers, irrational numbers, integers, whole numbers, natural numbers, commutative, associative, closed (earlier grades)

complex numbers, pure imaginary numbers (AII.3)

ohm, voltage, and amps (for optional extension)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Distribute copies of the attached Powers of *i* Table. Guide students in evaluating $i^0, i^1, \ldots i^4$ by expansion. Then, use the Think-Pair-Share strategy to have students discover the pattern for powers of *i*. Finally, have pairs complete the table. (Think-Pair-Share involves students thinking about the problem individually, then, pairing with another student to discuss ideas, and finally, sharing conclusions with the class.)
2. Distribute copies of the attached Complex Numbers handout to pairs of students, and have them complete it. While pairs work, walk around and check for understanding, asking leading questions as necessary. After they have completed the handout, have each pair compare their answers with those of another pair, discuss any discrepancies, and arrive at a consensus for each answer. Finally, review the answers with the whole class.

Assessment

- **Questions**
  - What properties are valid for all complex numbers?
  - How can you quickly evaluate *i* raised to any power?
- **Journal/Writing Prompts**
  - Compare and contrast field properties of real numbers versus imaginary numbers.
  - Explain why the product of conjugates is always a real and rational number.

Extensions and Connections (for all students)

- Have students complete the attached Complex Numbers – Extension handout.

Strategies for Differentiation

- Give students the option of another graphic organizer with examples of each operation.

- Have students create and use flash cards of properties and sets of real numbers.
- Provide students with extra practice, including variables within the complex numbers.
- Show students how to do complex arithmetic on a graphing calculator.
- Show similarities with complex arithmetic and polynomial arithmetic.
- Allow students to present proofs of complex operations orally.
- Allow students to use a talking graphing calculator to do calculations with complex numbers.

Powers of *i* Table

i^0	1	i^{12}	
i^1	i	i^{15}	
i^2	-1	i^{21}	
i^3		i^{30}	
i^4		i^{35}	
i^5		i^{40}	
i^6		i^{102}	
i^7		i^{441}	
i^8		i^{1003}	

Complex Numbers

1. Place the following sets of numbers in a hierarchy, using a Venn diagram.
   - Complex numbers
   - Pure imaginary numbers
   - Real numbers
   - Rational numbers
   - Irrational numbers
   - Integers
   - Whole numbers
   - Natural numbers

Simplify the following completely over *C:*

2. $\sqrt{-50}$
3. $-\sqrt{-18}$
4. i^{15}
5. i^{50}
6. $i^{13} + i^{23} + i^{40}$
7. $(\sqrt{5})^{2}$
8. $(3 + 2i) + (9 + 7i) + (-2 - i)$
9. $(5 - 3i) - (-1 + 4i)$
10. $-i(5 + 2i)$
11. $-1i(9 - 2i) - (5 + 7i)$
12. $4i(1 + i) + 3(6 - 2i)$
13. $(3 + i)(2 + i)$
14. $(3 + 5i)(8 - i)$
15. $i(4 - i)(3 + 2i)$
16. $(2 - 3i)^{4}$
17. $\dfrac{5-i}{i}$
18. $\dfrac{2+i}{2-i}$
19. $\dfrac{3-4i}{2+5i}$

Identify the complex number described by each of the following descriptions, and verify your answer.

20. The identity for addition on *C*
21. The multiplicative identity on *C*
22. The additive inverse of $a + bi$
23. The multiplicative inverse of $a + bi$

Give the representation of each of the following statements in symbols, and state whether the statement is true or false. If false, give a counterexample. If true, verify (prove) the statement.

24. The sum of two imaginary numbers is imaginary.
25. The product of two imaginary numbers is real.
26. Addition of complex numbers is commutative.
27. Multiplication of complex numbers is associative.
28. The distributive property of multiplication over addition holds for complex numbers.
29. The set of imaginary numbers is closed under addition.
30. The set of complex numbers is closed under multiplication.

Complex Numbers – Extension

Complex numbers are commonly used in electronics when representing voltage, current, and impedance in connection with alternating current (AC).

- **Voltage**, V, (in volts): the electrical potential between two points in an electrical circuit
- **Current**, I, (in amps): the rate of flow of electrical charge through a circuit
- **Impedance**, Z, (in ohms): the opposition to the flow of current caused by resistors, coils, and capacitors

The total impedance in a circuit is represented by a complex number whose real part denotes the opposition to current flow due to resistors and whose imaginary part represents opposition due to coils and capacitors. For example, for a total impedance of $8 - 3i$:

- 8 ohms: resistors' impedance
- 3 ohms: coils' and capacitors' impedance
- $I = \frac{V}{Z}$

The total impedance of two circuits in series (flowing from one to the other) is found by adding the impedances of the two individual circuits. The total voltage of two circuits in series is found by adding the voltages of the two individual circuits.

- Suppose two AC circuits are connected in series, one with impedance of $-5 + 7i$ ohms, the other with impedance $8 - 13i$ ohms.
    - Find the total impedance.
    - If the total voltage is 15V, find the current.
- In an AC circuit, V across resistors is 15V, and total V across the coils and capacitors is 18V.
    - Find the total voltage.
    - Find the impedance.
    - If this circuit is really two circuits in series and the impedance of one is $2.5 - 2i$ ohms, find the impedance of the other.