Radical Equations

Reporting Category	Equations and Inequalities
Topic	Solving equations containing radical expressions.
Primary SOL	AII.4d The student will solve, algebraically and graphically, equations containing radical expressions. Graphing calculators will be used for solving and for confirming the algebraic solutions.
Related SOL	AII.1a, d

Materials

- Graphing calculators
- Three attached handouts

Vocabulary

square, square root, radical, power (earlier grades)

radical algebraic equations, extraneous solution, radicand, index (AII.4d)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Distribute copies of the attached Solving Radical Equations: Introductory Exercise handout, and have students complete it, working individually and then in pairs to share and confirm or revise their responses. Have them place emphasis on the justification of their answers. Follow with a class discussion of each problem. Pay particular attention to #5 in which an untrue statement leads to an extraneous solution. Review the meaning of the term *extraneous solution.*
2. To explore the algebraic and graphical methods for solving rational expressions, begin with the algebraic. Distribute copies of the attached Steps for Solving Radical Equations Algebraically handout. Encourage students to work with their partners to monitor and communicate what is happening as you lead them through the examples. After you work through each example, have a student pair come up and work the similar, accompanying problem. The variety of problems is meant to encompass the scope of typical Algebra II problems.
3. Have students use a graphical method to generate solutions to the equations on the algebraic handout. Direct them to convert each equation to a system in which Y_1 is the left-hand side and Y_2 is the right-hand side, and then to find the points of intersection. Discuss the importance of identifying the domain for each function and using the domain to help determine an appropriate viewing window.
4. Distribute copies of the attached Solving Radical Equations: Practice Problems with Hints handout, and have students complete it. (Note: This set of problems involving radical equations contains some that are a bit more challenging.)

Assessment

- **Questions**
    - How can you determine the solution set to a radical equation algebraically?

    - If you graph one side of a given radical equation as Y_1 and the other as Y_2, how do you determine the solution set to the equation?
    - What would you do to eliminate the radicals in an equation such as $\sqrt[3]{x-1}=\sqrt{8}$?
- **Journal/Writing Prompts**
    - Explain how you can use a graph's points of intersection to solve a radical equation.
    - In your own words, explain what is meant by the term *extraneous solution.* Is it a solution or not? Explain why.
- **Other**
    - Give students solution sets, and ask them to create matching radical equations. (Note: Such open-ended problems allow students to be creative and differentiate the task based upon their own level of understanding.)

Extensions and Connections (for all students)
- Guide students to make connections to graphing functions containing radicals, paying particular attention to restrictions on the domain.

Strategies for Differentiation
- Construct additional introductory problems to reinforce similar concepts.
- Create an additional Steps for Solving Radical Equations Algebraically handout with examples in the left-hand column and similar radical equations in the right-hand column for students to solve.
- Have students create flash cards, each with a radical equation on one side and the first step on the other, to help them take the initial steps.

Solving Radical Equations: Introductory Exercise

Determine which of the following are true, and justify your answers.

1. $x = 2$ is a solution to $\sqrt{3x - 2} = -2$

2. $x = 2$ is a solution to $\sqrt{3x - 2} = 2$

3. $x = 4$ is a solution to $\sqrt[3]{5x^2 - 4x} = x$

4. $a = b \Rightarrow a^n = b^n$

5. $a^n = b^n \Rightarrow a = b$

6. $x^{\frac{1}{2}} = \sqrt{x}$

7. $3\sqrt{2x} + \sqrt[3]{x} = 14$ when $x = 8$

8. $x - \sqrt{x} - 42 = 0$ when 36 or 49

9. $\sqrt[3]{x^2} = \sqrt{x^3}$ for infinitely many values of x

10. $\sqrt{x^2} = |x|$

Steps for Solving Radical Equations Algebraically

Example 1: $\sqrt{3x-5}=5$

Step 1: Square both sides to eliminate the radical.

$$\left(\sqrt{3x-5}\right)^2=(5)^2$$

Step 2: Simplify and solve familiar equation.

$\Rightarrow 3x-5=25$

$\Rightarrow 3x=30$

$\Rightarrow x=10$

Step 3: Verify solution.

Does $\sqrt{3(10)-5}=5$?

Problem 1: Now, you follow the steps to solve $\sqrt{5x+1}=4$.

Step 1:

Step 2:

Step 3:

Example 2: $\sqrt[3]{5x+2}=3$

Step 1: Cube both sides to eliminate the radical.

$$\left(\sqrt[3]{5x+2}\right)^3=3^3$$

Step 2: Simplify and solve familiar equation.

$\Rightarrow 5x+2=27$

$\Rightarrow 5x=25$

$\Rightarrow x=5$

Step 3: Verify solution.

Does $\sqrt[3]{5(5)+2}=3$?

Problem 2: Now, you follow the steps to solve $\sqrt[3]{2x-1}=5$.

Step 1:

Step 2:

Step 3:

Example 3: $\sqrt{x+2}=x$

Step 1: Square both sides to eliminate the radical.

$$\left(\sqrt{x+2}\right)^2=x^2$$

Step 2: Simplify and solve familiar equation.

$\Rightarrow x+2=x^2$

$\Rightarrow 0=x^2-x-2=(x-2)(x+1)$

$\Rightarrow x=2 \ \text{or} \ x=-1$

Step 3: Verify solution.

Check with $x=2$ $\sqrt{2+2}=2$: True

Check with $x=-1$ $\sqrt{-1+2}=-1$: False

So, the only solution is $x=2$.

(−1 is extraneous.)

Problem 3: Now, you follow the steps to solve $\sqrt{2x+8}=x$.

Step 1:

Step 2:

Step 3:

Solving Radical Equations: Practice Problems with Hints

Solve each of the following, and check your solutions.

1. $2\sqrt{x}-5=3$ (Hint: First solve for the $\sqrt{x}$.)
2. $x-2\sqrt{x}-15=0$ (Hint: Let $\sqrt{x}=n \Rightarrow x=n^2$, and then substitute.)
3. $\sqrt{x^2+2}=x+1$ (Hint: Don't forget, when you square a binomial, it becomes a trinomial.)
4. $\sqrt{\frac{3x+1}{x-1}}=\sqrt{2x-1}$ (Hint: Make sure you check for extraneous solutions.)
5. $x^{\frac{1}{2}}=x-2$ (Hint: Remember what $x^{\frac{1}{2}}$ equals.)

Challenge Problems

6. The distance from a point on the curve $y=\sqrt{x}$ to the point (2,0) is equal to 2 at what points on the curve? (Hint: Draw a picture.)
7. Finish the right side of the equation $\sqrt{3x-5}=\sqrt[3]{ax}$ by finding *a* so that *x* = 5 is a solution to the equation. (Hint: If *x* = 5 satisfies the equation, what does that mean?)
8. Solve $\sqrt{2x}+\sqrt{x+1}=\sqrt{x+41}$.
(Hint: Don't forget, when you square a binomial, it becomes a trinomial.)
9. Solve $\sqrt{2x-5}=3-x$, and verify your solution, using a graphing calculator.
(Hint: You will need to use the quadratic formula.)
10. Solve $x^{\frac{2}{3}}-x^{\frac{1}{3}}-2=0$. (Hint: Let $u=x^{\frac{1}{3}}$, and solve for *u* first.)