Curve of Best Fit

Reporting Category	Statistics	
Topic	Collecting and analyzing data, using curve of best fit	
Primary SOL	AII.9	The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions
Related SOL	AII.6	

Materials
- Graphing calculators
- Six attached handouts
- Sets of six card-stock circular regions with radii of 1 cm, 2 cm, 3 cm, 4 cm, 5 cm, and 6 cm
- One-centimeter grid paper
- Paper cups
- Large bag of M&Ms®
- Paper towels

Vocabulary

line of best fit, mathematical model (earlier grades)

radical algebraic equations, extraneous solution, radicand, index

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)
1. Distribute copies of the attached Curve of Best Fit Introductory Exercise handout, and have students complete it. (Note: The goal is to have students reflect on the general shape of each sample graph and use that knowledge to guide them in choosing a model that will likely best fit specific data.)
2. Distribute copies of the attached Curve of Best Fit Exploration handout, and have students work in pairs to draw curves of best fit for the sets of data. For each set of data, have them describe the model that best fits the data as either a quadratic polynomial function, a cubic polynomial function, an exponential function, or a logarithmic function.
3. Distribute sets of six circular regions, grid paper, and copies of the attached Activity 1: Data and Regressions handout. Have students work individually or in pairs to complete the handout. (Note: The more circular regions students have to measure, the better their scattergraphs will look.)
4. Distribute copies of the attached Activity 2: Data and Regressions handout, and have student work individually or in pairs to complete it. (Note: This is a very interesting situation. During certain years, the regression "appears" to be linear, while at other times, it takes on different shapes. This could be used as an introduction to linearization in calculus.)

5. Divide the class into groups of three or fewer. Give each group a large cup filled with M&Ms®, a paper towel, and a copy of the attached Activity 3: M&Ms® Experiment handout. Have each group work to compete the handout. (Note: In the data collection process, the number of M&Ms remaining usually will not go to zero due to the manufacturing process. However, if the number does go to zero, this data point needs to be recorded as 0.01.)
6. Distribute copies of the attached Activity 4: Music and Mathematics handout, and have students complete it. (Note: Make sure that students understand the explanation given at the tope of the handout. They will have studied this in Physical Science class (grade 8), so this should be a review of what they already know.)

Assessment

- **Questions**
  - What needs to be considered when determining a model for best fit?
  - What might you see in data that is best modeled by a logarithmic model? By an exponential model?

Extensions and Connections (for all students)

- Have students solve the Ewok-Jawa Problem handout found on the last page of this document.
- Have students research what kind of analysis a statistician is doing when finding a curve of best fit.
- Discuss the method of least squares as a way for finding linear regression, including how it might apply to other nonlinear models.
- Have students put data in graphing calculators and try a few regression options. Direct them to check the correlation coefficient to determine the model that best fits the data.

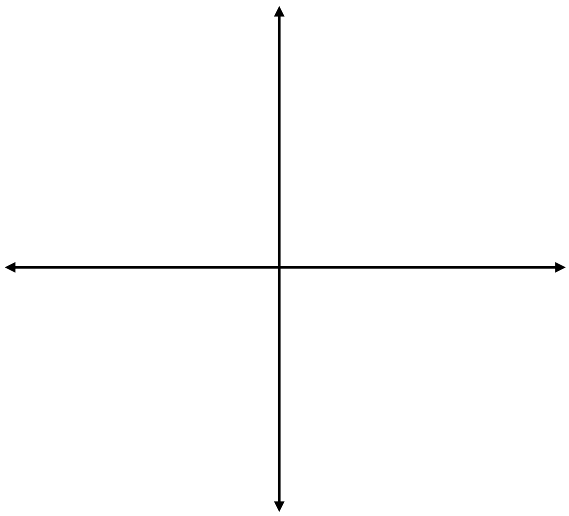
Strategies for Differentiation

- Have students create and use flash cards with scatterplots on one side and the names of the appropriate curves of best fit on the other.
- Use Web applets to manipulate curves of best fit on an interactive whiteboard.
- Provide students with a curve on graph paper, and ask them to identify some data points that would make it the curve of best fit for that data.

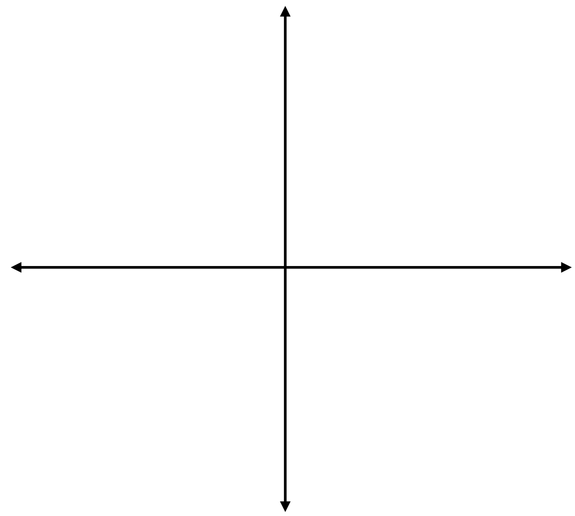
Curve of Best Fit Introductory Exercise

Using your knowledge and a graphing calculator, draw a sample graph of each of the following functions.

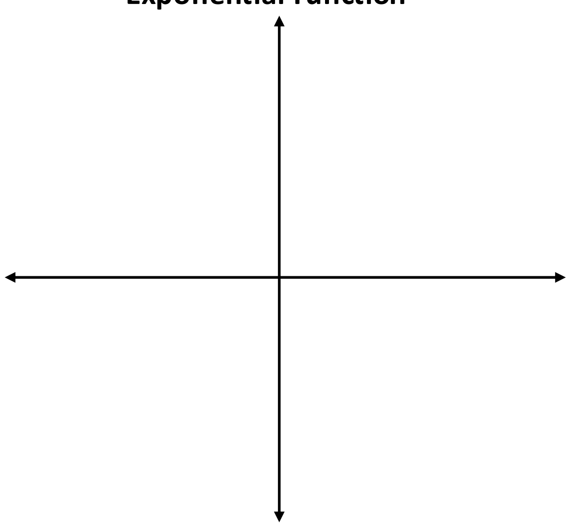
Quadratic Polynomial Function



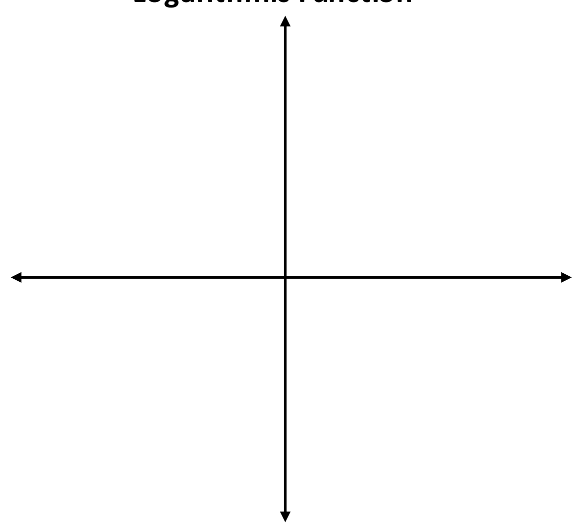
Cubic Polynomial Function



Exponential Function



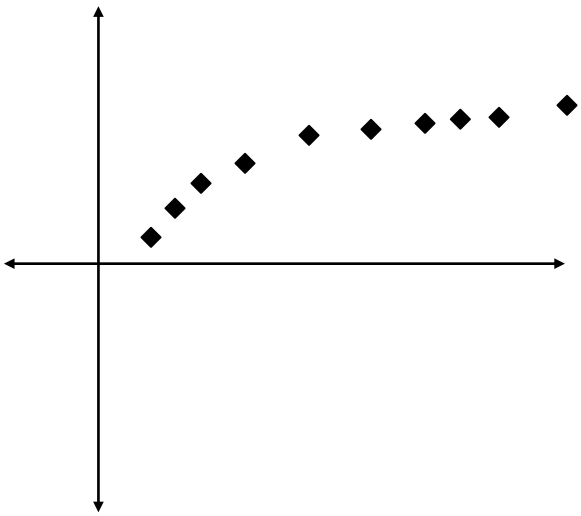
Logarithmic Function

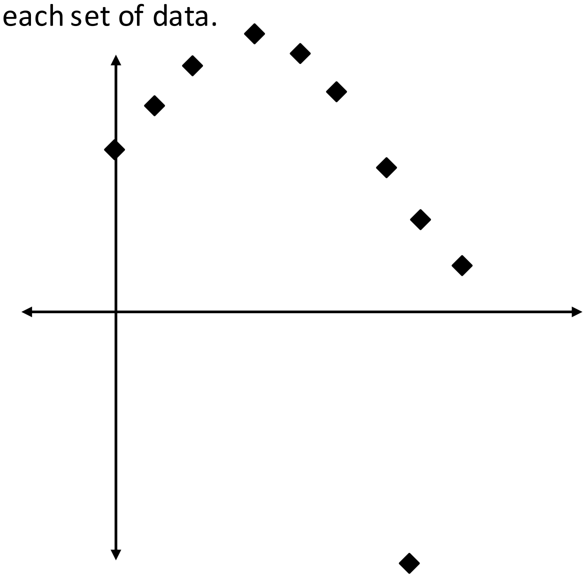


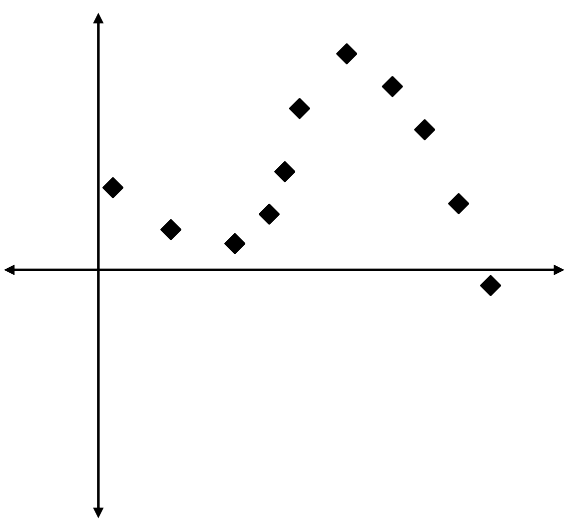
Describe each graph in your own words.

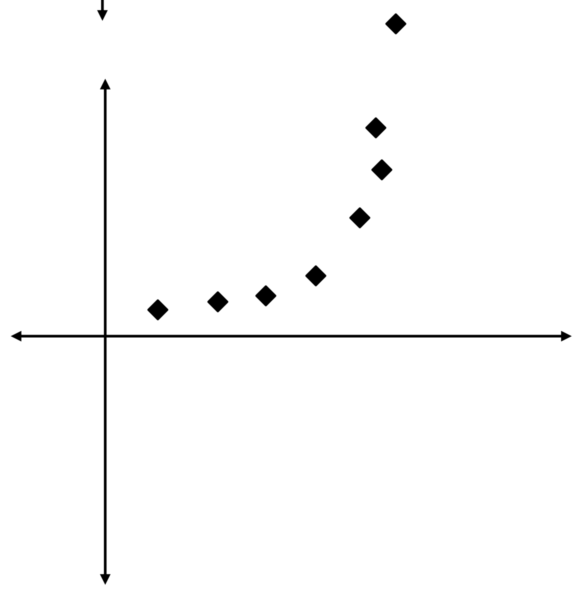
Curve of Best Fit Exploration

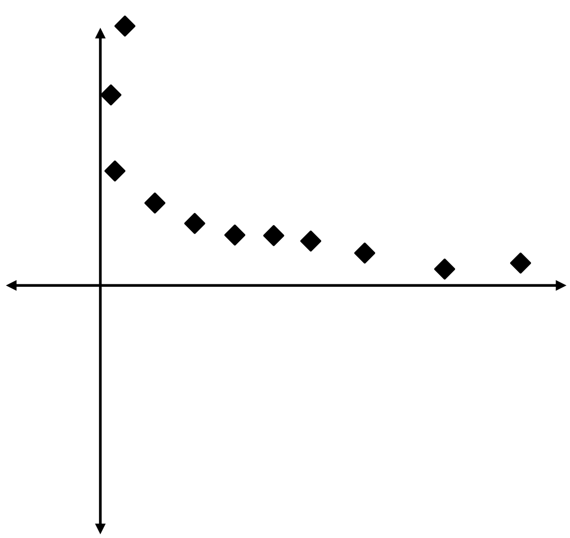
Draw and describe the graph of a model that best fits each set of data.

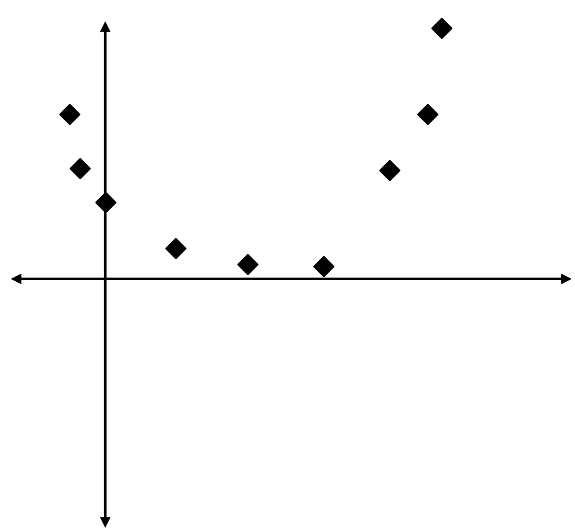












Activity 1: Data and Regressions

1. Place the provided circular regions on the centimeter grid paper, and trace them.
2. Count the number of square centimeters in the area of each circular region, and record the data in the table.

Radius of circular region	Area: number of square centimeters
1 cm	
2 cm	
3 cm	
4 cm	
5 cm	
6 cm	

3. Record the radius of each circular region in L1 in the calculator and the area of the circular region in L2.
4. Use the calculator to generate a scattergraph.
5. Examine the scattergraph carefully, and decide into which family of functions the graph might fit.
6. Using the regression equation capabilities of the calculator and your knowledge of function families, find the regression equation that you believe best fits your data, and graph the curve through the data points.
7. Using the equation of the curve, predict the area of a circular region with a radius of 12 cm.
8. What would be the radius of a circular region that covers 450 square cm? (Hint: Work backwards to find the answer.)

Activity 2: Data and Regressions

1. Using the data in the table at right, enter the year in L1 of your graphing calculator, the total number of marriages in L2, and the total number of divorces in L3.
2. Calculate the divorce rate for each year, and put that data in L4.
3. Which type of regression best fits the data from 1960 to 1976?
4. If the pattern continued in this way, what would be the predicted divorce rate in 1994?
5. Which type of regression best fits the data from 1978 to 1992?
6. Based on the data, what would be the predicted divorce rate for 1995?

Marriages and divorces in the U.S., 1960–1992		
Year	Marriages	Divorces
1960	1,523,000	393,000
1962	1,557,000	413,000
1964	1,725,000	450,000
1966	1,857,000	499,000
1968	2,069,258	584,000
1970	2,158,802	708,000
1972	2,282,154	845,000
1974	2,229,667	977,000
1976	2,154,807	1,083,000
1978	2,282,272	1,130,000
1980	2,406,708	1,182,000
1982	2,495,000	1,180,000
1984	2,487,000	1,155,000
1986	2,400,000	1,159,000
1988	2,389,000	1,183,000
1990	2,448,000	1,175,000
1992	2,362,000	1,215,000

Extension

7. What is a possible explanation for the different regressions in the marriage and divorce rates?
8. Do you find anything "odd" about this data? If so, explain.

Activity 3: M&Ms® Experiment

The following experiment is designed to simulate a natural occurrence.

Toss number	Number remaining
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	

Part I. Data Collection

1. Count the M&Ms® in the cup given to your group, and record this number in the chart to the right of toss number 0.
2. Shake the cup, and carefully dump the M&Ms on the paper towel. Remove and set aside those M&Ms that landed with the *M* showing. Count the pieces remaining, and record this number in the chart to the right of toss number 1. Pour these pieces back in the cup.
3. Again, shake the cup, dump the pieces, remove and set aside those with the *M* showing, count the remainder, record the number, and pour the remainder back in the cup.
4. Keep repeating the process until no *M*s appear.
5. Graph on graph paper the collected data from the chart.

Part II. Data Analysis

1. The data collected can be modeled with an exponential curve of the form $y = ab^x$. Using the graph from Part I, develop an equation to fit the data collected.
2. Give the coordinates of the beginning point in your data.
3. What information does this give about the equation being developed?
4. Locate a second point on the graph. Using this specified point and the original data point, find the value of *b* in the general equation.
5. Write the completed equation for the graph.
6. How well does this equation predict the remaining data on the graph? Check the generated value for *y* and the graphed value for *y* for several chosen values of *x*.
7. How “good a fit” is the equation that you developed?

Activity 4: Music and Mathematics

Stringed instruments, such as violins and guitars, produce the different pitches of a musical scale by producing different vibrating frequencies. The vibrating frequency of a string depends on the vibrating length of the string and the tension on it. For a string under a constant tension, the vibrating frequency varies inversely with its vibrating length—i.e., the shorter (smaller) the string, the higher (larger) its vibrating frequency; the longer (larger) the string, the lower (smaller) its vibrating frequency.

1. Complete the table to find the vibrating length of the string for each pitch of a C major scale. Round your answers to the nearest whole number. Assume that each string is under the same tension.

Pitch	**C**	**D**	**E**	**F**	**G**	**A**	**B**	**C**
Vibrating frequency (cycles/sec.)	**523**	**587**	**659**	**698**	**784**	**880**	**988**	**1046**
Vibrating length of string (mm)	**420**							

2. Write a function that models this variation.

3. Describe how the values of the vibrating frequency change in relation to the vibrating length of the string.

4. Make a scattergraph of your data. Describe the graph.

5. What is true about the product of the vibrating frequency and vibrating length for the first pitch, C?

6. What is true about the product of the vibrating frequency and vibrating length for the second pitch, D?

7. How can you determine the string length for the second pitch, D?

The Ewok-Jawa Problem

1. A band of 45 Ewoks crash-landed in the National Forest last night. This sounds like a small problem, but the population will grow at the rapid rate of 22% per year. Write an equation to describe the population in any given year. Also create a table that shows the Ewok population every five years from this year to the year 2050.
2. Coincidentally, a band of Jawas crash-landed near the Ewoks. The Jawas have a population growth modeled by the equation $A = 105(0.91)^t$, where A is the population at any time, t, given in years. Is this population increasing? Decreasing? Stagnant? Explain your reasoning.
3. In 2015, the two tribes begin fighting so that the casualties of war offset the Ewok birth rate. Write the compound equation to describe the Ewok population from 2015 to 2050.
4. The Jawas lose 15% of their members each year in battle. When will their tribe be extinct?
5. Using a graphing calculator, graph and label the equations for each of the four scenarios above. Include a brief explanation of each graph's implications.