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Purpose of Presentation:
Action required by state or federal law or regulation.

Previous Review or Action:
Previous review and action. Specify date and action taken below:
Date: March 17, 2016: First Review
Date: April 14, 2016: Public Hearing
Date: April 19, 2016: Public Hearing

Action Requested:
Final review: Action requested at this meeting.

Alignment with Board of Education Goals: Please indicate (X) all that apply:

| Goal 1: Accountability for Student Learning |
| Goal 2: Rigorous Standards to Promote College and Career Readiness |
| Goal 3: Expanded Opportunities to Learn |
| Goal 4: Nurturing Young Learners |
| Goal 5: Highly Qualified and Effective Educators |
| Goal 6: Sound Policies for Student Success |
| Goal 7: Safe and Secure Schools |
| Other Priority or Initiative. Specify: |

Background Information and Statutory Authority:
Goal 2: The Board of Education has made a commitment to maintain rigorous and relevant expectations for students that meet or exceed national and international benchmarks for college and career readiness.

*Code of Virginia*, Section 22.1-253.13:1-B… “The Board of Education shall establish a regular schedule, in a manner it deems appropriate, for the review, and revision as may be necessary of the Standards of Learning in all subject areas. Such review of each subject area shall occur at least once every seven years. Nothing in this section shall be construed to prohibit the Board from conducting such review and revision on a more frequent basis…”

New academic content Standards of Learning for mathematics were first developed in 1995. Pursuant to legislation from the 2000 Virginia General Assembly, the Board of Education established a seven-year cycle for review of the Standards of Learning. As a result, the 1995 *Mathematics Standards of Learning*
were reviewed in 2001 and 2009. The projected timeline for the review and revision of the 2009 *Mathematics Standards of Learning* and *Curriculum Framework* during the 2015-2016 school year was received by the Board of Education in March 2015.

In accordance with the timeline, the Virginia Department of Education (VDOE) produced a draft of the Proposed Revised 2016 *Mathematics Standards of Learning* and Proposed Revised 2016 *Mathematics Standards of Learning Curriculum Framework* documents found in Attachment A and Attachment B, respectively. The following list summarizes the actions involved in the review and revision process.

- Received public comments from stakeholders on the review and revision of the 2009 *Mathematics Standards of Learning and Curriculum Framework*
  - Received 74 sets of comments, including 71 from educators, mathematics education organizations, and school divisions, two from parents, and one from a university
  - Received comments March 27-April 27, 2015

- Convened a steering committee to review public comments and make recommendations for revisions to the standards
  - Comprised of nine school division mathematics supervisors that led grade-band and content subgroups of the review committee
  - VDOE staff from Instruction and Assessment served in an advisory capacity
  - Met June 1-3, 2015

- Convened a review committee to review public comments and make recommendations for revisions to the standards and Curriculum Framework
  - Comprised of the steering committee and 30 educators, including 19 classroom teachers, two school-based mathematics specialists, eight school division mathematics supervisors, and one assistant principal
  - Members represented all eight Superintendents’ Regions
  - VDOE staff from Instruction and Assessment served in an advisory capacity
  - Met June 22-26, 2015

- Developed a draft of the proposed standards and Curriculum Framework

- Received input, both electronically and in person, from external reviewers on the proposed draft of the standards and Curriculum Framework
  - Invited two-year and four-year colleges, the Virginia Mathematics and Science Coalition, the Virginia Council for Mathematics Supervision, the Virginia Council of Teachers of Mathematics, the Virginia Council for Mathematics Specialists, and the top ten employers in Virginia to participate in the external review process
  - Received 30 sets of comments representing two-year and four-year colleges, the Virginia Mathematics and Science Coalition, the Virginia Council for Mathematics Supervision, the Virginia Council of Teachers of Mathematics, the Virginia Council for Mathematics Specialists, Wells Fargo Bank, and Optima/Sentara Healthcare
  - Convened an external review meeting for discussion of comments
  - Met December 9, 2015

- Received support for the review process from the Assessment Development staff, including providing Instruction staff with insights from work with development of state assessments, working with content review committees, and providing technical reviews of proposed revisions.

- Received public comments from stakeholders on the Proposed 2016 *Mathematics Standards of Learning and Proposed 2016 Mathematics Standards of Learning Curriculum Framework*
  - Received 103 sets of comments from educators, mathematics education organizations, and school divisions
  - Received comments March 18-April 25, 2016
Held public hearings in Montgomery County and Henrico County

• Reviewed public comment and developed the Proposed Revised 2016 Mathematics Standards of Learning and Proposed Revised 2016 Mathematics Standards of Learning Curriculum Framework

The Mathematics Standards of Learning identify academic content for essential components of the mathematics curriculum at different grade levels for Virginia’s public schools. The Mathematics Standards of Learning Curriculum Framework, a companion document to the Mathematics Standards of Learning, amplifies the standards and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn.

The Proposed Revised 2016 Mathematics Standards of Learning and the Proposed Revised 2016 Mathematics Standards of Learning Curriculum Framework were compared to the expectations of the 2009 National Assessment for Educational Progress (NAEP) and the 2015 Trends in International Mathematics and Science Study (TIMSS). The revised standards were found to be highly correlated to the expectations for each by the end of grade 4 and grade 8 respectively.

In support of Governor McAuliffe’s focus on strengthening the 21st century work force, the proposed revisions to the standards and Curriculum Framework strengthen support for teachers and educational leaders through improvements to the standards and Curriculum Framework, strengthen pathways within K-12 mathematics education through a focus on improving the vertical progression of mathematics content, and will better prepare students for college and careers through a greater emphasis on critical thinking and problem solving.

Summary of Major Elements:
The attached drafts of the Proposed Revised 2016 Mathematics Standards of Learning and the Proposed Revised 2016 Mathematics Standards of Learning Curriculum Framework include revisions since first review in response to public comment, as listed.

Proposed Revised 2016 Mathematics Standards of Learning

• Edits to introductory statements
• Edits to provide consistency and parallelism in language
• Edits to improve the progression of mathematics content which led to changes in selected standards (e.g., revisions to the Patterns, Functions, and Algebra strand in grades 6-8, which encompass standards 6.12, 7.10, and 8.16, develop the concept of proportional relationships, slope of a line, and linear functions)

Proposed Revised 2016 Mathematics Standards of Learning Curriculum Framework

• Additions to provide greater support to teachers through definitions, connections, and examples
• Edits to clarify expectations through improved wording
• Edits to provide consistency and parallelism in language
• Edits to improve the progression of mathematics content
• Edits to word choice or content limiters to inform instruction and assessment

All edits found in the proposed revised drafts have been tracked using the following system:
• a single underline (sample) indicates content added to the 2009 standards or curriculum framework;
• a single strikethrough (sample) indicates content deleted from the 2009 standards or curriculum framework;
• a double underline (sample) indicates content added to the 2009 standards or curriculum framework subsequent to first review; and
• a double strikethrough (sample) indicates content deleted from the 2009 standards or curriculum framework subsequent to first review.

Comments have been included to provide school divisions assistance in revising local curricula and for schools and teachers to inform and revise instructional pacing and planning. A few examples follow:

<table>
<thead>
<tr>
<th>Tracking Statement (example)</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Moved to 3.4a]</td>
<td>Content moved from this location to standard 3.4 sub-bullet a</td>
</tr>
<tr>
<td>[Moved from 3.4a]</td>
<td>Content moved to this location from standard 3.4 sub-bullet a</td>
</tr>
<tr>
<td>[Moved to EKS]</td>
<td>Content moved from this location to the Essential Knowledge and Skills column</td>
</tr>
<tr>
<td>[ Moved to US ]</td>
<td>Content moved from this location to the Understanding the Standard column</td>
</tr>
<tr>
<td>[Reordered]</td>
<td>Content reordered within this standard or column</td>
</tr>
<tr>
<td>[Included in 8.6b]</td>
<td>Content deleted because it is already included in standard 8.6 sub-bullet b</td>
</tr>
<tr>
<td>[Combined with bullet above]</td>
<td>Content remained in this location, but was combined with another related bullet</td>
</tr>
<tr>
<td>[Incorporated into 7.2 and 7.3]</td>
<td>Content deleted from this location and incorporated into another related standard</td>
</tr>
</tbody>
</table>

**Impact on Fiscal and Human Resources:**
The revisions can be absorbed by the agency’s existing resources at this time. If the agency is required to absorb additional responsibilities related to this activity, other services may be impacted.

**Timetable for Further Review/Action:**
Upon approval of the Proposed Revised 2016 Mathematics Standards of Learning and Proposed Revised 2016 Mathematics Standards of Learning Curriculum Framework, the Department of Education will implement the standards and curriculum framework according to the proposed implementation timeline found in Table 1.

**Superintendent’s Recommendation:**
The Superintendent of Public Instruction recommends that the Board of Education approve the Proposed Revised 2016 Mathematics Standards of Learning and the Proposed Revised 2016 Mathematics Standards of Learning Curriculum Framework and authorize the Department of Education to make clarifying and/or technical edits.

**Rationale for Action:**
The Code of Virginia requires the review and revision of the Standards of Learning in each subject area every seven years. Action by the Virginia Board of Education allows for the Virginia Department of Education to provide school divisions with the new Mathematics Standards of Learning and Curriculum Framework that reflect over 200 public comments submitted during the review process.
Table 1

2016 Mathematics Standards of Learning Anticipated Implementation Timeline and Communication Plan

<table>
<thead>
<tr>
<th>School Year</th>
<th>Date</th>
<th>Action</th>
<th>Communication</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>2016-2017</td>
<td>September</td>
<td>Board of Education approves the 2016 Mathematics Standards of Learning (SOL) and 2016 Mathematics Standards of Learning Curriculum Framework (CF)</td>
<td>Announce approval of the SOL and CF, implementation timeline, and expectations for incorporating the new standards into local curricula</td>
<td>Superintendent’s Memo, VDOE social media, TeacherDirect, and other communication channels</td>
</tr>
<tr>
<td></td>
<td>December</td>
<td>Final versions of the 2016 Mathematics SOL, 2016 Mathematics CF, and summary of changes posted</td>
<td>Announce posting of final versions and expectations for incorporating the new standards into local curricula</td>
<td>Superintendent’s Memo, VDOE social media, TeacherDirect, and other communication channels</td>
</tr>
<tr>
<td></td>
<td>Spring/Summer</td>
<td>VDOE provides professional development on the changes to the SOL and CF</td>
<td>Announce professional development</td>
<td>Superintendent’s Memo, VDOE social media, TeacherDirect, and other communication channels</td>
</tr>
<tr>
<td></td>
<td>Spring/Summer</td>
<td>VDOE conducts the textbook review process in accordance with previously approved BOE textbook adoption timeline</td>
<td>Announce the process and call for nominees to serve</td>
<td>Superintendent’s Memo, VDOE social media, TeacherDirect, and other communication channels</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>School divisions have incorporated 2016 Mathematics SOL and CF into written curricula</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2017-2018</td>
<td>Full year</td>
<td>2009 Mathematics SOL and 2016 Mathematics SOL included in the written and taught curricula</td>
<td>Detail instruction and assessment timeline and expectations</td>
<td>Superintendent’s Memo, VDOE social media, TeacherDirect, and other communication channels</td>
</tr>
<tr>
<td></td>
<td>Fall/Winter</td>
<td>VDOE provides SOL Practice Items aligned with the 2016 Mathematics SOL</td>
<td>Announce posting of resource</td>
<td>Superintendent’s Memo, VDOE social media, TeacherDirect, and other communication channels</td>
</tr>
<tr>
<td></td>
<td>Spring/Summer</td>
<td>VDOE provides professional development on the changes to the SOL and CF</td>
<td>Announce professional development</td>
<td>Superintendent’s Memo, VDOE social media, TeacherDirect, and other communication channels</td>
</tr>
<tr>
<td></td>
<td>Spring</td>
<td>VDOE completes the textbook review process and seeks approval from the Board of Education</td>
<td>Announce completion</td>
<td>Superintendent’s Memo, VDOE social media, TeacherDirect, and other communication channels</td>
</tr>
<tr>
<td></td>
<td>Fall and Spring</td>
<td>SOL assessments measure the 2009 Mathematics SOL and include field test items measuring the 2016 Mathematics SOL</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2018-2019</td>
<td>Full year</td>
<td>Written and taught curricula reflect the 2016 Mathematics SOL</td>
<td>Detail instruction and assessment timeline and expectations</td>
<td>Superintendent’s Memo, VDOE social media, TeacherDirect, and other communication channels</td>
</tr>
<tr>
<td></td>
<td>Fall and Spring</td>
<td>SOL assessments measure the 2016 Mathematics SOL</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Mathematics Standards of Learning for Virginia Public Schools

Board of Education Commonwealth of Virginia

February 2009 September 2016
Preface

The Standards of Learning in this publication represent a significant development in public education in Virginia. These standards focus on the mathematical knowledge and skills all students need for the future, and they have been aligned with national expectations for postsecondary success.

In 1995, the Virginia Board of Education published Standards of Learning in English, mathematics, science, and history and social science for kindergarten through grade 12. Subsequently, Standards of Learning were developed for all academic content areas. The Standards of Learning provide a framework for instructional programs designed to raise the academic achievement of all students in Virginia and are an important part of Virginia’s efforts to provide challenging educational programs in the public schools prepare them for college and careers.

The Standards of Learning set reasonable targets and expectations for what teachers need to teach and students need to learn. The standards are not intended to encompass the entire curriculum for a given grade level or course or to prescribe how the content should be taught; the standards are to be incorporated into a broader, locally designed curriculum. Teachers are encouraged to go beyond the standards and select instructional strategies and assessment methods appropriate for their students.

The Standards of Learning are recognized as a model for other states. Pursuant to legislation from the 2000 Virginia General Assembly, the Board of Education established a seven-year cycle for review of the Standards of Learning. As a result, the 1995 Mathematics Standards of Learning were reviewed in 2001, 2009, and 2016, the results of which are contained in this document. The standards were revised through a series of public hearings and the efforts of parents, teachers, administrators, representatives from higher education, and the business and industry leaders community. The standards set clear, concise, and measurable academic expectations for young people students. Parents and guardians are encouraged to work with their children, their children’s teachers, and their children’s schools to help them achieve these academic standards.

A major objective of Virginia’s educational agenda is to give the citizens of Virginia a program of public education that is among the best in the nation and that meets the needs of all young people in Virginia. These Standards of Learning chart the course for achieving that objective.
Introduction

Preparing Virginia’s students today require more rigorous mathematical knowledge and skills to pursue higher education, to compete in a global workforce, and to be informed citizens requires rigorous mathematical knowledge and skills. Students must gain an understanding of fundamental ideas in number sense, computation, measurement, geometry, probability, data analysis and statistics, and algebra and functions, and they must develop proficiency in mathematical skills.

The 2016 Mathematics Standards of Learning for mathematics identify academic content for essential components of the mathematics curriculum at different grade levels for Virginia’s public schools. Recommendations and reports from Achieve, Information from the College Board, and ACT, as well as the National Assessment of Educational Progress (NAEP) Frameworks, the Curriculum Focal Points from the National Council of Teachers of Mathematics (NCTM), Principles and Standards for School Mathematics from NCTM, Focus in High School Mathematics: Reasoning and Sense Making from NCTM, the Singapore Curricula, the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report from the American Statistical Association, and the Report of the President’s National Mathematics Advisory Panel were considered in identifying mathematics content necessary for success for all students in postsecondary pursuits.

Standards are identified for kindergarten through grade eight and for a core set of high school courses. Throughout a student’s mathematics schooling from kindergarten through grade eight, specific content strands or topics are included. These content strands are Number and Number Sense; Computation and Estimation; Measurement and Geometry; Probability and Statistics; and Patterns, Functions, and Algebra. The Standards of Learning for within each strand progress in complexity at each grade level and throughout the high school courses content. While the standards are organized by strand and identified numerically, local curricula and pacing guides should determine the instructional sequence of the content.

The 2016 Mathematics Standards of Learning Curriculum Framework is a companion document to the 2016 Mathematics Standards of Learning that amplifies the Mathematics Standards of Learning standards and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students, provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn.

The Standards of Learning are not intended to encompass the entire curriculum for a given grade level or course or to prescribe how the content should be taught. Teachers are encouraged to go beyond the standards and select instructional strategies and assessment methods appropriate for their students.
Mathematical Process Goals for Students

Students today require more rigorous mathematical knowledge and skills to pursue higher education, to compete in a technologically sophisticated work force, and to be informed citizens. Students must gain an understanding of fundamental ideas in arithmetic, measurement, geometry, probability, data analysis and statistics, and algebra and functions, and they must develop proficiency in mathematical skills. In addition, students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving in science, engineering, business and industry, government, and practical affairs. Hence, the use of technology must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skills should be the shared responsibility of teachers of all disciplines.

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-life world data and situations within and outside mathematics and then apply appropriate strategies to find determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problem types. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become competent mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students to clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will learn to use number sense to apply proportional and spatial reasoning and to reason from a variety of representations such as graphs, tables, and charts.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics to one another and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections between different areas of mathematics and between mathematics and other disciplines, especially science and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.
Mathematical Representations
Students will represent and describe mathematical ideas, generalizations, and relationships with using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections more easily among different representations – physical, visual, symbolic, graphical, numerical, algebraic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation.

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
**Algebra Readiness**

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

**Equity**

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
Kindergarten

The kindergarten standards place emphasis on developing the concept of number by counting; combining, sorting, and comparing sets of objects; recognizing, and describing, and creating simple repeating patterns; and recognizing shapes and sizes of figures and objects. Students will investigate nonstandard measurement through direct comparisons, collect data, and create graphs. The concept of fractions will be introduced through sharing experiences.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized mathematical vocabulary and language patterns is crucial to a student’s understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six-content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be instruction in the process of problem solving will need to be integrated early and continuously into each student’s mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

Focus: Whole Number Concepts

K.1 The student, given two sets, each containing 10 or fewer concrete objects, will identify and describe one set as having more, fewer, or the same number of members as the other set, using the concept of one-to-one correspondence. [Moved to K.2]

K.12 The student, given a set containing 15 or fewer concrete objects, will:
   a) tell how many are in the given set of 20 or fewer objects by counting the number of objects orally; and
   b) read, write, and represent numbers from 0 through 20, the numeral to tell how many are in the set; and
   c) select the corresponding numeral from a given set of numerals. [Moved to EKS]

K.2 The student, given no more than three two sets, each set containing 10 or fewer concrete objects, will:
   a) identify, compare and describe one set as having more, fewer, or the same number of members as the other set(s), and using the concept of one-to-one correspondence,
   b) compare and order sets from least to greatest and greatest to least.

K.4 The student will, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth, and the ordered position of each object. [Ordinal positions moved to 1.3]
   a) recognize and describe with fluency part-whole relationships for numbers up to 5; and
   b) investigate and describe part-whole relationships for numbers up to 10.
K.43 The student will
a) count forward orally by ones from 0 to 100, starting at any number between 0 and 100; 
b) count and backward orally by ones when given any number between 1 and 100 from 10; and
c) identify one more than a number and one less than a number, and the number after, without counting, and the number before when given any number between 0 and 100 and identify the number before, without counting, when given any number between 1 and 10 count by fives and tens to 100. [Count by fives included in 1.2]; and 
d) count forward by tens to determine the total number of objects to 100.

K.5 The student will identify the parts of a set and/or region that represent fractions for halves and fourths. investigate fractions by representing and solving practical problems involving equal sharing with two or four sharers.

Computation and Estimation
Focus: Whole Number Operations
K.6 The student will model and solve single-step story and picture problems with sums to 10 and differences within 10 adding and subtracting whole numbers, using up to 10 concrete objects.

Measurement and Geometry
Focus: Instruments and Attributes
K.7 The student will recognize the attributes of a penny, nickel, dime, and quarter and identify the value of each coin, number of pennies equivalent to a nickel, a dime, and a quarter. [Value of a collection included in 1.7]

K.8 The student will identify the instruments used to measure length (ruler), weight (scale), time (clock: digital and analog; calendar: day, month, and season), and temperature (thermometer). [Moved each instrument to the standard where the content is first taught (ruler – SOL 2.8; scale – SOL 2.8; clock – SOL 1.9; thermometer – SOL 2.11]

K.89 The student will investigate the passage of time by reading and interpreting a calendar. [Moved from 1.11] tell time to the hour, using analog and digital clocks. [Time to hour moved to 1.9a]

K.910 The student will compare two objects or events, using direct comparisons or nonstandard units of measure, according to one or more of the following attributes: length (shorter, longer), height (taller, shorter), weight (heavier, lighter), temperature (hotter, colder), volume (more, less), and time (longer, shorter). Examples of nonstandard units include foot length, hand span, new pencil, paper clip, and block. [Nonstandard units included in 1.10]

Geometry
Focus: Plane Figures
K.1011 The student will
a) identify, describe, and trace plane geometric figures (circle, triangle, square, and rectangle); and
b) compare the size (smaller, larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle); and
c) describe the location of one object relative to another (above, below, next to) and identify representations of plane figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space. [Moved from K.12]
K.12  The student will describe the location of one object relative to another (above, below, next to) and identify representations of plane geometric figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space. [Moved to K.10c]

Probability and Statistics
Focus: Data Collection and Display
K.11  The student will gather data by counting and tallying. [Tallying included in 1.14a]
   a) collect, organize, and represent data; and
   b) read and interpret data in object graphs, pictures graphs, and tables. [Moved from K.14]

K.14  The student will display gathered data in object graphs, picture graphs, and tables, and will answer questions related to the data. [Moved to K.11b]

Patterns, Functions, and Algebra
Focus: Attributes and Patterning
K.12  The student will sort and classify objects according to one attributes.
K.13  The student will identify, describe, and extend, create, and transfer repeating patterns.
Grade One

The first-grade standards place emphasis on counting, sorting, and ordering sets of up to 1100 objects; recognizing and describing simple repeating and growing patterns; and tracing, describing, and sorting plane geometric figures. Students’ understanding of number will be expanded through recognizing and describing part-whole relationships for numbers up to 10, learning and applying the basic addition facts through the nines table, sums of 20, and the corresponding subtraction facts, solving story and picture problems using addition and subtraction within 20; using nonstandard units to measure; and organizing and interpreting data. Fractional concepts will be expanded through sharing scenarios involving halves and fourths.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized mathematical vocabulary and language patterns is crucial to a student’s understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be instruction in the process of problem solving will need to be integrated early and continuously into each student’s mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

Focus: Place Value and Fraction Concepts

1.1 The student will
   a) count forward orally by ones to 110 from 0 to 100, starting at any number between 0 and 110, and write the corresponding numerals; and
   b) write the numerals 0 to 110 in sequence and out-of-sequence, group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.
   c) count backward orally by ones when given any number between 1 and 30;
   d) count forward orally by ones, twos, fives, and tens to determine the total number of objects to 110.

1.2 The student, given up to 110 objects, will count forward by ones, twos, fives, and tens to 100 and backward by ones from 30:
   a) group a collection into tens and ones and write the corresponding numeral;
   b) compare two numbers between 0 and 110 represented pictorially or with concrete objects, using the words greater than, less than or equal to; and
   c) order three or fewer sets from least to greatest and greatest to least.
The student, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth, and the ordered position of each object. [Moved from K.3]

The student will identify the parts of a set and/or region that represent fractions for halves, thirds, and fourths; and write the fractions.

a) represent and solve practical problems involving investigating fractions by solving equal sharing problems with two, or four, or eight sharers; and

b) verbally identify the parts of a region that represent and name fractions for halves and fourths using models.

The student, given a familiar problem situation involving magnitude, will

a) select a reasonable order of magnitude from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, 500); and

b) explain the reasonableness of the choice.

[Moved from Computation and Estimation strand]

**Computation and Estimation**

**Focus: Whole Number Operations**

1.5 The student will recall basic addition facts with sums to 18 or less and the corresponding subtraction facts. [Moved to 1.7; addition/subtraction to 18 included in 2.6b]

The student will create and solve one single-step story and picture problems using basic addition and subtraction within 20 facts with sums to 18 or less and the corresponding subtraction facts.

1.7 The student will recall basic

a) recognize and describe with fluency part-whole relationships for numbers up to 10; and

b) demonstrate fluency with addition and subtraction within facts with sums to 10 or less and the corresponding subtraction facts.

**Measurement and Geometry**

**Focus: Time and Nonstandard Measurement**

1.8 The student will

a) identify the number of pennies equivalent to a nickel, a dime, and a quarter; and

b) determine the value of a collection of like coins (pennies, nickels, and dimes) whose total value is 100 cents or less.

1.9 The student will investigate the passage of time by

a) telling time to the hour and half-hour, using analog and digital clocks; and

b) reading and interpreting a calendar.

1.10 The student will use nonstandard units to measure and compare length, weight/mass, and volume.

1.11 The student will compare, using the concepts of more, less, and equivalent.

a) the volumes of two given containers; and

b) the weight/mass of two objects, using a balance scale.
1.11 The student will use calendar language appropriately (e.g., names of the months, today, yesterday, next week, last week). [Moved to K.98 and 1.9b]

**Geometry**

**Focus: Characteristics of Plane Figures**

1.112 The student will

a) identify, and trace, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, vertices, and right angles; and,

b) identify and describe representations of circles, squares, rectangles, and triangles in different environments, regardless of orientation, and explain reasoning. [Moved from 1.13]

1.13 The student will construct, model, and describe objects in the environment as geometric shapes (triangle, rectangle, square, and circle) and explain the reasonableness of each choice. [Moved to 1.11b]

**Probability and Statistics**

**Focus: Data Collection and Interpretation**

1.124 The student will

a) investigate, identify, and describe various forms of data collection (e.g., recording daily temperature, lunch count, attendance, favorite ice cream), using tables, picture graphs, and object graphs; and [Examples moved to US]

b) read and interpret data displayed in tables, picture graphs, and object graphs, using the vocabulary more, less, fewer, greater than, less than, and equal to. [Moved from 1.15]

1.15 The student will interpret information displayed in a picture or object graph, using the vocabulary more, less, fewer, greater than, less than, and equal to. [Moved to 1.12b]

**Patterns, Functions, and Algebra**

**Focus: Patterning and Equivalence**

1.136 The student will sort and classify concrete objects according to one or more attributes including color, size, shape, and thickness. [Included in EKS]

1.147 The student will identify, recognize, describe, extend, and create, and transfer a wide variety of growing and repeating patterns. [Transfer included to match EKS bullet]

1.158 The student will demonstrate an understanding of equality through the use of the equal symbol.
Grade Two

The second-grade standards extend the study of number and spatial sense to include three-digit whole numbers and solid geometric figures. Students will continue to learn, use, and gain proficiency in the basic addition facts through the tenses of 20 and the corresponding subtraction facts within 20. Students will begin to use U.S. Customary and metric units to measure length and weight; predict using simple probability; and create and interpret pictureographs and bar graphs. Students will work with a variety of patterns and will develop knowledge understanding of equality by identifying missing numbers in addition and subtraction facts.

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Number and Number Sense

Focus: Place Value, Number Patterns, and Fraction Concepts

2.1 The student will
   a) read, write, and identify the place and value of each digit in a three-digit numeral, using numeration with and without models;
   b) identify the number that is 10 more, 10 less, 100 more, and 100 less than a given number up to 999;
   c) round two-digit numbers to the nearest ten; and
d) compare and order two whole numbers between 0 and 999, using symbols (>, <, or =) and words (greater than, less than, or equal to). [Symbols and words included in EKS]

2.24 The student will
   a) count forward by twos, fives, and tens to 120, starting at various multiples of 2, 5, or 10;
   b) count backward by tens from 120; and [Increased to 120 to correlate to Grade 1 change]
   c) recognize use objects to determine whether a number is even or odd numbers.

2.32 The student will
   a) count and identify the ordinal positions first through twentieth, using an ordered set of objects; and
   b) write the ordinal numbers, 1st through 20th. [Edited to match EKS].

2.43 The student will
a) name and write fractions represented by a set, region, or length model; identify the parts of a set and/or region that represent fractions for halves, fourths, eighths, thirds, fourths, sixths, eighths, and tenths;  
b) write the representation parts with models and with symbols; and  
c) compare the unit fractions for halves, fourths, eighths, thirds, and fourths, sixths, eighths, and tenths, with models.

Computation and Estimation
Focus: Number Relationships and Operations
2.5 The student will recall  
a) recognize and use the relationships between addition and subtraction to solve single-step practical problems, with whole numbers to 20; and [Moved from 2.9]  
b) demonstrate fluency with addition and subtraction facts with for sums to within 20 or less and the corresponding subtraction facts.

2.6 The student, given two whole numbers whose sum is 99 or less, will  
a) estimate the sums and differences; and [Differences moved from 2.7a]  
b) find the determine sums and differences, using various methods of calculation; and [Differences moved from 2.7b]  
c) create and solve single-step and two-step practical problems involving addition and subtraction. [Moved from 2.8 and 2.21]

2.7 The student, given two whole numbers, each of which is 99 or less, will  
a) estimate the difference; and [Moved to 2.6a]  
b) find the difference, using various methods of calculation. [Moved to 2.6b]

2.8 The student will create and solve one- and two-step addition and subtraction problems, using data from simple tables, picture graphs, and bar graphs. [Combined with 2.6c]

2.9 The student will recognize and describe the related facts that represent and describe the inverse relationship between addition and subtraction. [Moved to 2.5b EKS]

Measurement and Geometry
Focus: Money, Linear Measurement, Weight/Mass, and Volume
2.7.10 The student will  
a) count and compare a collection of pennies, nickels, dimes, and quarters whose total value is $2.00 or less; and  
b) correctly use the cent symbol (¢), dollar symbol ($), and decimal point (€).

2.8.11 The student will estimate and measure  
a) length to the nearest centimeter and inch; and [Centimeters included in 3.8]  
b) weight/mass of objects in to the nearest pounds/ounces and kilograms/grams, using a scale; and [Ounces, kilograms/grams included in 4.8]  
e) liquid volume in cups, pints, quarts, gallons, and liters. [Included in 3.8]

2.9.12 The student will tell and write time to the nearest five minutes, using analog and digital clocks.

2.1043 The student will  
a) determine past and future days of the week; and  
b) identify specific days and dates on a given calendar.
2.1114 The student will read the temperature on a Celsius and/or Fahrenheit thermometer to the nearest 10 degrees. [Celsius/Fahrenheit included in EKS]

**Geometry**

**Focus: Symmetry and Plane and Solid Figures**

2.1215 The student will
a) draw a line of symmetry in a figure; and
b) identify and create figures with at least one line of symmetry.

2.1316 The student will identify, describe, compare, and contrast plane and solid geometric figures (circle/sphere, square/cube, and rectangle/rectangular prism).

**Probability and Statistics**

**Focus: Applications of Data**

2.151417 The student will use data from experiments to construct picture graphs, pictographs, and bar graphs.

a) collect, organize, and represent data in picture pictographs and bar graphs; and
b) read and interpret data represented in picture pictographs and bar graphs.

2.141518 The student will use data from probability experiments to predict outcomes when the experiment is repeated.

2.19 The student will analyze data displayed in picture graphs, pictographs, and bar graphs. [Moved to 2.1415b]

**Patterns, Functions, and Algebra**

**Focus: Patterning and Numerical Sentences**

2.1620 The student will identify, describe, create, and extend and transfer a wide variety of patterns found in objects, pictures, and numbers.

2.21 The student will solve problems by completing numerical sentences involving the basic facts for addition and subtraction. The student will create story problems, using the numerical sentences. [Moved to 2.5 and 2.6]

2.1722 The student will demonstrate an understanding of equality by recognizing that through the use of the equal symbol in an equation indicates equivalent quantities and the use of the not equal symbol indicates that quantities are not equivalent.
Grade Three

The third-grade standards place emphasis on developing an understanding of, and solving problems that involve, multiplication and division facts through the twelve times table, products of 100, 10 × 10. Students will be fluent in the basic addition facts through the tens table and the corresponding subtraction facts. Students will apply knowledge of place value and the properties of addition and multiplication as strategies for solving problems. Concrete materials and two-dimensional models and pictorial representations will be used to introduce addition and subtraction with fractions and the concept of probability as the measurement of chance. Students will use standard units (U.S. Customary and metric) to measure temperature, length, and liquid volume, and weight and identify relevant properties of shapes, points, line segments, rays, angles, vertices, and lines will be explored and investigated and describe the identity and commutative properties for addition and multiplication.

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Number and Number Sense

Focus: Place Value and Fractions

3.1 The student will
a) read and write six-digit numerals and identify the place value and value of each digit in a six-digit whole number, with and without models;

b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and

c) compare two and order whole numbers between 0 and 9,999 or less, using symbols (>, <, or =) and words (greater than, less than, or equal to). [Symbols and words included in EKS]

3.2 The student will recognize and use the inverse relationships between addition/subtraction and multiplication/division to complete basic fact sentences. The student will use these relationships to solve problems. [Addition/subtraction included in 2.5; multiplication/division moved to 3.4 EKS]

3.23 The student will
a) name and write fractions (including improper fractions and mixed numbers) represented by a model;

b) model represent fractions (including improper fractions and mixed numbers) and write the fractions’ names with models and symbols; and
c) use models to compare proper fractions having like and unlike denominators, using words and symbols (>, <, or =, or ≠) with models.

**Computation and Estimation**

**Focus: Computation and Fraction Operations**

3.34 The student will
a) estimate solutions to and determine the sum or difference of two whole numbers; and
b) create and solve single-step and multistep practical problems involving the sums or differences of two whole numbers, each 9,999 or less, with or without regrouping.

3.45 The student will recall multiplication facts through the twelves table, and the corresponding division facts. [Recall moved to 4.4a]

a) represent multiplication and division through 10 × 10, using a variety of approaches and models; [Moved from 3.6]
b) create and solve single-step practical problems that involve multiplication and division through 10 × 10; and [Create and solve multiplication moved from 3.6; create and solve with division is new expectation]
c) demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10; [Recall of facts to 12 × 12 included in 4.4a]
d) create and solve single-step practical problems that involve multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less. [Moved from 3.6]

3.6 The student will represent multiplication and division, using area, set, and number line models, and create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 5 or less. [Moved to 3.4a and 3.4ed]

3.5 The student will solve practical problems that involve addition and subtraction with proper fractions having like denominators of 12 or less.

**Measurement and Geometry**

**Focus: U.S. Customary and Metric Units, Area and Perimeter, and Time**

3.68 The student will
a) determine, by counting, the value of a collection of bills and coins whose total value is $5.00 or less;
b) compare the value of the two sets of coins or two sets of coins and bills and coins; and
c) make change from $5.00 or less. [Edited to match EKS]

3.79 The student will estimate and use U.S. Customary and metric units to measure
a) length to the nearest \( \frac{1}{2} \) inch, inch, foot, yard, centimeter, and meter; and
b) liquid volume in cups, pints, quarts, gallons, and liters;
e) weight/mass in ounces, pounds, grams, and kilograms; and [Included in 4.8]
d) area and perimeter. [Moved to 3.8a,b]

3.81 The student will estimate and [Moved from 3.9d EKS]
a) measure the distance around a polygon in order to determine its perimeter using U.S. Customary and metric units[Moved from 3.9d EKS]; and
b) count the number of square units needed to cover a given surface in order to determine its area.

3.94 The student will
a) tell time to the nearest minute, using analog and digital clocks; and
b) determine solve practical problems related to elapsed time in one-hour increments over within a 12-hour period; and

c) identify equivalent periods of time and solve practical problems related to equivalent periods of time. [Moved from 3.12]

3.12 The student will identify equivalent periods of time, including relationships among days, months, and years, as well as minutes and hours. [Moved to 3.9c]

3.104 The student will read temperature to the nearest degree from a Celsius thermometer and a Fahrenheit thermometer. Real thermometers and physical models of thermometers will be used. [Thermometer information moved to EKS]

3.12+ The student will

a) define polygon; [Moved from 4.12a]

b) identify and name polygons with 10 or fewer sides; and-[Moved from 4.12b]

c) combine and subdivide polygons with 3 or 4 sides and name the resulting polygons.

3.14 The student will identify, describe, compare, and contrast characteristics of plane and solid geometric figures (circle, square, rectangle, triangle, cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models. [Moved to 4.11]

3.11 The student will identify and draw representations of points, lines, line segments, rays, and angles, and lines.

3.13 The student will identify and describe congruent and noncongruent plane figures.

**Probability and Statistics**

**Focus: Applications of Data and Chance**

3.15 The student will collect, and organize, and represent data, using observations, measurements, surveys, or experiments in picture pictographs or bar graphs; and;

b) construct a line plot, a picture graph, or a bar graph to represent the data; and[Line plot moved to 5.16; construct picture pictographs and bar graphs moved to 3.14a]

e) read and interpret the data represented in picture graphs and bar graphs, and picture graphs and write a sentence analyzing the data.

3.14 The student will investigate and describe the concept of probability as a measurement of chance and list possible results outcomes for a given situation single event.

**Patterns, Functions, and Algebra**

**Focus: Patterns and Property Concepts**

3.16 The student will recognize and identify, describe, create, and extend and transfer patterns found in objects, pictures, numbers and tables, a variety of patterns formed using numbers, tables, and pictures, and extend the patterns, using the same or different forms.

3.17 The student will create equations to represent equivalent mathematical relationships. [Moved from 3.20 EKS]

a) investigate the identity and the commutative properties for addition and multiplication; and [Use of properties moved to 3.3 EKS and 3.4 EKS]
b) identify examples of the identity and commutative properties for addition and multiplication.
Grade Four

The fourth-grade standards place emphasis on multiplication and division with whole numbers and solving problems involving addition and subtraction of fractions and decimals by finding common multiples and factors. Students will develop fluency with the basic multiplication facts through the twelves table 12 x 12 and the corresponding division facts as they become proficient in multiplying larger numbers. Students will apply knowledge of place value and the properties of addition and multiplication as strategies for solving problems. Students also will refine their estimation skills for computations and measurements. Students will identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices. Students will describe and compare characteristics of plane and solid figures. Concrete materials and two-dimensional models and pictorial representations will be used to solve problems involving perimeter and area, patterns, probability, and equivalence of fractions and decimals. Students will recognize images of figures resulting from geometric transformations such as reflection, translation, and rotation. Students will investigate and describe the associative property for addition and multiplication.

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Number and Number Sense
Focus: Place Value, Fractions, and Decimals

4.1 The student will
   a) read, write, and identify orally and in writing the place and value for each digit in a whole number expressed through millions nine-digit whole number;
   b) compare and order two whole numbers expressed through millions, using symbols (> , <, or =); and [Symbols included in EKS]
   c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

4.2 The student will
   a) compare and order fractions and mixed numbers, with and without models;
   b) represent equivalent fractions; and
   c) identify the division statement that represents a fraction with models and in context.

4.3 The student will
   a) read, write, represent, and identify decimals expressed through thousandths;
   b) round decimals to the nearest whole number, tenth, and hundredth; [Round to tenth and hundredth included in 5.1]
c) compare and order decimals; and
d) given a model, write the decimal and fraction equivalents.

**Computation and Estimation**

*Focus: Factors and Multiples, and Fraction and Decimal Operations*

4.4 The student will
a) demonstrate fluency with multiplication facts through 12 x 12, and the corresponding division facts; [Moved from 3.5] estimate sums, differences, products, and quotients of whole numbers;
b) estimate and determine sums, differences, and products of add, subtract, and multiply whole numbers;
c) estimate and determine quotients of divide whole numbers, finding quotients with and without remainders; and
d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division problems with whole numbers.

4.5 The student will
a) determine common multiples and factors, including least common multiple and greatest common factor;
b) add and subtract fractions and mixed numbers having like and unlike denominators that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors; and
c) add and subtract with decimals; and [Moved to 4.6a]
d) solve single-step and multistep practical problems involving addition and subtraction with fractions and with decimals and mixed numbers. [Decimals limited to single-step only and moved to 4.6b; solve multistep problems with decimals fractions included in 5.5b, 5.6a]

4.6 The student will
a) add and subtract with decimals; and [Moved from 4.5c]
b) solve single-step and multistep practical problems involving addition and subtraction with decimals. [Moved from 4.5d; decimals changed to single-step only; multistep with decimals included in 5.5b]

**Measurement and Geometry**

*Focus: Equivalence within U.S. Customary and Metric Systems*

4.6 The student will
a) estimate and measure weight/mass and describe the results in U.S. Customary and metric units as appropriate; and [Moved to 4.8b]
b) identify equivalent measurements between units within the U.S. Customary system [Moved to 4.8c] (ounces, pounds, and tons) [Moved to 4.8 EKS] and between units within the metric system (grams and kilograms). [Included in 5.9a]

4.7 The student will
a) estimate and measure length, and describe the result in both metric and U.S. Customary units; and [Moved to 4.8a]
b) identify equivalent measurements between units within the U.S. Customary system (inches and feet; feet and yards; inches and yards; yards and miles) [Moved to 4.8c] and between units within the metric system (millimeters and centimeters; centimeters and meters; and millimeters and meters). [Included in 5.9a]
4.7 The student will solve practical problems that involve determining perimeter and area in U.S. Customary and metric units.

4.8 The student will
\(a\) estimate and measure length and describe the result in U.S. Customary and metric units; and
\(b\) estimate and measure liquid volume and describe the result in U.S. Customary units; and
\(c\) estimate and measure weight/mass and describe the result in U.S. Customary and metric units; and
\(d\) given the equivalent measure of one unit, identify equivalent measurements of length, weight/mass, and liquid volume between units within the U.S. Customary system (cups, pints, quarts, and gallons); and
\(e\) solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary.

4.9 The student will determine practical problems related to elapsed time in hours and minutes within a 12-hour period.

4.10 The student will
\(a\) identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices; and
\(b\) identify and describe representations of lines that illustrate intersecting, parallelism, and perpendicularity.

4.11 The student will
\(a\) investigate congruence of plane figures after geometric transformations, such as reflection, translation, and rotation, using mirrors, paper folding, and tracing; and
\(b\) recognize the images of figures resulting from geometric transformations, such as translation, reflection, and rotation.

4.12 The student will classify quadrilaterals as parallelograms, rectangles, squares, rhombi/parallelograms, and/or trapezoids.
\(a\) define polygon; and
\(b\) identify polygons with 10 or fewer sides.

4.13 The student will
\(a\) predict the likelihood of an outcome of a simple event; and
\(b\) represent probability as a number between 0 and 1, inclusive; and
\(c\) create a model or practical problem to represent a given probability.

4.14 The student will
a) collect, organize, and represent data in a bar graph and line graph display; and
b) make observations and inferences about interpret data represented in bar graphs and line graphs; interpret data from a variety of graphs; and
c) compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph; a chart and a line graph; a pictograph and a bar graph). [Added to align with EKS bullet]

Patterns, Functions, and Algebra
Focus: Geometric Patterns, Equality, and Properties
4.15 The student will recognize, identify, describe, create, and extend numerical and geometric patterns found in objects, pictures, numbers, and tables.

4.16 The student will
a) recognize and demonstrate the meaning of equality in an equation; and
b) investigate and describe the associative property for addition and multiplication. [Moved to 5.19]
Grade Five

The fifth-grade standards place emphasis on number sense with whole numbers, fractions, and decimals. This focus includes concepts of prime and composite numbers, identifying even and odd numbers, and solving problems using order of operations for positive whole numbers. Students will develop proficiency in the use of fractions and decimals to solve practical problems. Students will collect, display, and analyze data in a variety of ways and solve probability problems, using a sample space, or a tree diagram, or the Fundamental Counting Principle. Students also will solve problems involving volume, area, and perimeter. Students will be introduced to variable expressions with a variable and open sentences, and will model one-step linear equations in one variable, using addition and subtraction. Students will solve problems investigating and recognizing using strategies including place value and the properties of addition and multiplication—the distributive property. All of these skills assist in the development of the algebraic concepts needed for success in the middle grades.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized mathematical vocabulary and language patterns is crucial to a student’s understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be instruction in the process of problem solving will need to be integrated early and continuously into each student’s mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

Focus: Prime and Composite Numbers and Rounding Decimals

5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.

5.2 The student will
   a) recognize and name, represent and identify equivalencies among fractions and in their equivalent decimals, with and without models; form and vice versa; and
   b) compare and order fractions, mixed numbers, and/or decimals in a given set from least to greatest and greatest to least.

5.3 The student will
   a) identify and describe the characteristics of prime and composite numbers; and
   b) identify and describe the characteristics of even and odd numbers.

Computation and Estimation

Focus: Multistep Applications and Order of Operations

5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.
5.5 The student will
a) find the sum, difference, estimate and determine the product, and quotient of two numbers expressed as decimals through thousandths (divisors with only one nonzero digit), and [Addition and subtraction with decimals included in 4.6]
b) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals, and single-step practical problems involving division of decimals.

5.6 The student will
a) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers; and express answers in simplest form. [Included in EKS]
b) solve single-step practical problems involving multiplication that involve modeling and determining the product of a whole number, limited to 12 or less, and a unit proper fraction, with models.

5.7 The student will evaluate whole number numerical expressions, using the order of operations limited to parentheses, addition, subtraction, multiplication, and division. [Moved to EKS]

Measurement and Geometry
Focus: Perimeter, Area, Volume, and Equivalent Measures
5.8 The student will
a) find solve practical problems that involve perimeter, area, and volume in standard units of measure; and
b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation;
ed) identify equivalent measurements within the metric system; [Moved to 5.9a]
d) estimate and then measure to solve problems, using U.S. Customary and metric units; and
[U.S. Customary included in 4.8d; Metric moved to 5.9b]e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units. [Moved to 5.9 EKS]

5.9 The student will
a) given the equivalent measure of one unit, identify equivalent measurements within the metric system; and [Moved from 5.8c]b) solve practical problems involving length, mass, and liquid volume using metric units. [Moved from 5.8d]

5.10 The student will identify and describe the diameter, radius, chord, and circumference of a circle.

5.11 The student will determine solve practical problems related to elapsed time in hours and minutes within a 24-hour period.

5.12 The student will classify and measure right, acute, obtuse, and straight angles. [Classify moved from 5.12a]

Geometry
Focus: Classification and Subdividing
5.13 The student will classify
a) angles as right, acute, obtuse, or straight; and [Classification of angles moved to 5.12]
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ba) classify triangles as right, acute, or obtuse, and equilateral, scalene, or isosceles; and
b) investigate the sum of the interior angles in a triangle and determine an unknown angle measure.

5.14.3 The student, using plane figures (square, rectangle, triangle, parallelogram, rhombus, and trapezoid), will
a) develop definitions of these plane figures;[Included in 4.12] recognize and apply transformations, such as translation, reflection, and rotation;[Moved from 4.11b] and
b) investigate and describe the results of combining and subdividing plane figures/polygons.

Probability and Statistics
Focus: Outcomes and Measures of Center
5.15.4 The student will make predictions and determine the probability of an outcome by constructing a sample space or using the Fundamental (Basic) Counting Principle.

5.16.5 The student, given a practical problem situation, will collect, organize, and interpret data in a variety of forms, using
a) represent data in line plots, and a stem-and-leaf plots; and line graphs; and [Line plot moved from 3.17; Line graphs included in 4.14; Line graphs included in 4.14]
b) make observations and inferences about interpret data represented in a line plots, and a stem-and-leaf plots; and, and line graphs. [Line plot moved from 3.17; Line graphs included in 4.14]
c) compare data represented in a line plot with the same data represented in a stem-and-leaf plot.

5.17.6 The student will solve practical problems that involve
a) describing mean, median, and mode as measures of center;
b) describing mean as fair share;
c) describing the range of a set of data as a measure of spread; and [Reordered]
d) determine the mean, median, mode, and range of a set of data. ; and
d) describe the range of a set of data as a measure of variation. [Reordered and edited]

Patterns, Functions, and Algebra
Focus: Equations and Properties
5.18.7 The student will identify, describe, create, and express, and extend the relationship of found in a number patterns and express the relationship found in objects, pictures, numbers and tables.

5.19.8 The student will
a) investigate and describe the concept of variable;
b) write an open sentence/equation to represent a given mathematical relationship, using a variable;
c) use an variable expression with a variable to represent a given verbal quantitative expression involving one operation; and model one-step linear equations in one variable, using addition and subtraction; and [Moved to 6.14]
d) create a problem situation based on a given open sentence/equation, using a single variable.

5.20.9 The student will investigate and identify/recognize the distributive property of multiplication over addition:
a) the identity properties of addition and multiplication; [Moved from 3.20]
b) the commutative properties of addition and multiplication; [Moved from 3.20] and
e) the associative properties of addition and multiplication; and [Moved from 4.16b]
d) the distributive property of multiplication over addition. [Reordered][Application of properties moved to 5.4; naming of properties incorporated in 5.4 US]
Grade Six

The sixth-grade standards provide a transition from the emphasis placed on whole number arithmetic in the elementary grades to foundations of algebra. The standards emphasize include a focus on rational numbers and operations involving rational numbers. Students will use ratios to compare data sets; recognize decimals, fractions, and percents as ratios; solve single-step and multistep problems, using positive rational numbers; and gain a foundation in the understanding of and operations with integers. Students will solve problems involving area and perimeter, and begin to graph in a coordinate plane. In addition, students will build on the concept of graphical representation of data developed in the elementary grades and develop concepts regarding measures of center. Students will solve linear equations and inequalities in one variable, and use algebraic terminology. Students will represent proportional relationships using two variables as a precursor to the development of the concept of linear functions. Students will solve problems involving area, perimeter, and surface area, work with \( \pi \) (pi), and focus on the relationships among the properties of quadrilaterals. In addition, students will focus on applications of probability and statistics.

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Mathematics has its own language, and The acquisition of specialized mathematical vocabulary and language patterns is crucial to a student’s understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six-content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be Instruction in the process of problem solving will need to be integrated early and continuously into each student’s mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

Focus: Relationships among Fractions, Decimals, and Percents

6.1 The student will describe and compare data sets; represent relationships between quantities, using ratios, and will use appropriate notations, such as \( \frac{a}{b} \), \( a \) to \( b \), and \( a:b \).

6.2 The student will
a) investigate and describe fractions, decimals, and percents as ratios [Moved to EKS]
represent and determine equivalencies among fractions, mixed numbers, decimals, and percents; and
b) identify a given fraction, decimal, or percent from a representation; [Moved to EKS Included in 6.2a]
c) demonstrate equivalent relationships among fractions, decimals, and percents; and
4b) compare and order fractions, decimals, and percents positive rational numbers.

6.3 The student will
a) identify and represent integers;

b) compare and order and compare integers; and
c) identify and describe absolute value of integers.

6.4 The student will demonstrate multiple representations of multiplication and division of fractions. [Moved to 6.5 EKS]

6.45 The student will investigate and describe concepts of positive recognize and represent patterns with whole number exponents and perfect squares.

**Computation and Estimation**

Focus: Applications of Operations with Rational Numbers

6.56 The student will
a) multiply and divide fractions and mixed numbers; and
b) estimate solutions and then solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and

c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals. [Moved from 6.7]

6.7 The student will solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of decimals. [Moved to 6.5c]

6.6 The student will
a) add, subtract, multiply, and divide integers; and [Moved from 7.3]
b) solve practical problems involving operations with integers, and [Moved from 7.3]
c) simplify numerical expressions involving integers. [Moved and modified from 6.8] [Combined with 6.7, 2016 BOE first draft]

6.78 The student will evaluate whole numbers; simplify numerical expressions involving integers, using the order of operations. [Combined with 6.6, 2016]

**Measurement and Geometry**

Focus: Problem Solving with Area, Perimeter, Volume, and Surface Area

6.9 The student will make ballpark comparisons between measurements in the U.S. Customary System of measurement and measurements in the metric system. [Included in 7.3 EKS]

6.7810 The student will
a) define derive \( \pi \) (pi) as the ratio of the circumference of a circle to its diameter;
b) solve problems, including practical problems, involving circumference and area of a circle, given the diameter or radius; and
c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles; and.
d) describe and determine the volume and surface area of a rectangular prism. [Included in 7.4a]

**Geometry**

Focus: Properties and Relationships

6.8941 The student will
a) identify the coordinates of a point in components of the coordinate plane; and
b) identify the coordinates of a point and graph ordered pairs in a coordinate plane.

6.94102 The student will determine congruence of segments, angles, and polygons.
6.13 The student will describe and identify properties of quadrilaterals. [Moved to Included in 7.6]

Probability and Statistics
Focus: Practical Applications of Statistics
6.101114 The student, given a problem-practical situation, will
a) construct represent data in a circle graphs;
b) draw conclusions and make predictions, observations and inferences about data represented in a using circle graphs; and
c) compare and contrast circle graphs with the same data represented in bar graphs, picture graphs, and line plots that present information from the same data set.

6.111215 The student will
a) describe represent the mean of a data set graphically as the balance point; and
b) decide which measure of center is appropriate for a given purpose situation; and
be) determine the impact on measures of center when a single value of a data set is added, removed, or changed. [Moved from 5.16 EKS]

6.16 The student will
a) compare and contrast dependent and independent events; and [Moved to 8.11a]
b) determine probabilities for dependent and independent events. [Included in 8.11b]

Patterns, Functions, and Algebra
Focus: Variable Equations and Properties
6.17 The student will identify and extend geometric [Included in AFDA.1 EKS and AII.5] and arithmetic [Included in AI.7 EKS, AFDA.1 EKS and AII.5] sequences.

6.1213 The student will
a) write an equation in the form $y = kx$, where $k$ is the constant of proportionality, to represent a proportional relationship between two quantities, including those arising from practical problem situations; and
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
c) determine if a proportional relationship exists between two quantities; and
d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs; and equations.

6.131418 The student will solve one-step linear equations in one variable, involving whole number coefficients and positive rational solutions including practical problems that require the solution of a one-step linear equation in one variable.

6.19 The student will investigate and recognize
a) the identity properties for addition and multiplication;
b) the multiplicative property of zero; and
c) the inverse property for multiplication. [Included in the EKS for 6.67, 6.1314, 6.1415]

6.141520 The student will
a) represent a practical situation with a graph inequalities linear inequality in one variable; and
b) solve one-step linear inequalities in one variable and graph the solution on a number line.
Grade Seven

The seventh-grade standards continue to emphasize the foundations of algebra. Students who successfully complete the seventh-grade standards should be prepared to study Algebra I in grade eight. The standards address the concept of and operations with rational numbers by continuing their study from sixth-grade. Students will build on the concept of ratios to solve problems involving topics in grade seven include proportional reasoning, integer computation. Students will solve problems involving volume and surface area and focus on the relationships among the properties of quadrilaterals. Probability is investigated through comparing experimental results to theoretical expectations. Students continue to develop their understanding of solving two-step linear equations and inequalities in one variable, by applying the properties of real numbers and recognizing different representations for. Students discern between proportional and non-proportional relationships and begin to develop a concept of slope as rate of change. Students will apply the properties of real numbers in solving equations, solve inequalities, and use data analysis techniques to make inferences, conjectures, and predictions.

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Number and Number Sense
Focus: Proportional Reasoning

7.1 The student will
a) investigate and describe the concept of negative exponents for powers of ten;
b) determine compare and order scientific notation for numbers greater than zero written in scientific notation;
c) compare and order fractions, decimals, percents and numbers written in scientific notation rational numbers;
d) determine square roots of perfect squares; and
e) identify and describe absolute value for of rational numbers.

7.2 The student will describe and represent arithmetic sequences [Included in AI.7 EKS, AFDA.1 EKS and AII.5] and geometric sequences [Included in AFDA.1 EKS and AII.5] using variable expressions.
Computation and Estimation
Focus: Integer Operations and Proportional Reasoning
7.23 The student will
a) model addition, subtraction, multiplication and division of integers [Moved to 6.6a EKS] simplify numerical expressions involving rational numbers using the order of operations; [Moved from 8.1a] and
b) add, subtract, multiply, and divide integers [Moved to 6.6a] solve practical problems involving operations with rational numbers.

7.34 The student will solve single-step and multistep practical problems, using proportional reasoning.

Measurement and Geometry
Focus: Proportional Reasoning
7.45 The student will
a) describe and determine the volume and surface area of rectangular prisms and [Moved from 7.5 EKS] cylinders; and
b) solve problems, including practical problems, involving the volume and surface area of rectangular prisms and cylinders.; and
c) describe how changing one measured attribute of a rectangular prism affects its volume and surface area. [Included in 8.6b]

7.56 The student will determine whether plane figures—quadrilaterals and triangles—are similar and write proportions to express the relationships between solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar figures—quadrilaterals and triangles.

Geometry
Focus: Relationships between Figures
7.67 The student will
a) compare and contrast the following quadrilaterals based on their properties: parallelogram, rectangle, square, rhombus, and trapezoid.
b) solve problems to determine unknown side lengths or angle measures of quadrilaterals.

7.78 The student, given a polygon in the coordinate plane, will apply and contrast transformations (translations and reflections of right triangles or rectangles, dilations, rotations, and translations) by graphing in the coordinate plane. [Dilations included in 8.7 and G.3; Rotations included in G.3]

Probability and Statistics
Focus: Applications of Statistics and Probability
7.89 The student will
a) determine the theoretical and experimental probabilities of an event; and
b) investigate and describe the difference between the experimental probability and theoretical probability of an event.

7.10 The student will determine the probability of compound events, using the Fundamental (Basic) Counting Principle. [Moved to 5.15]
7.94  The student, given data for a practical situation, will
a) construct and analyze data in histograms; and
b) make observations and inferences about data represented in a histogram; and

c) compare and contrast histograms with other types of graphs presenting information from the same data set represented in stem-and-leaf plots, line plots, and circle graphs.

Patterns, Functions, and Algebra
Focus: Linear Equations

7.12  The student will represent relationships with tables, graphs, rules, and words. [Included in 7.10]

7.10  The student will represent relationships with tables, graphs, rules, and words.

a) determine if a proportional relationship exists between two quantities, given a table of values or a graph; [Moved to 6.12c, 2016]
b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations. [Moved to 6.12b, 2016]
c) determine the slope, \( m \), as rate of change in a proportional relationship between two quantities, given two ordered pairs, and write an equation in the form \( y = mx \) form where \( m \) represents the relationship;
d) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in \( y = mx \) form where \( m \) represents the slope as rate of change;

c) determine the \( y \)-intercept, \( b \), in an additive relationship between two quantities and write an equation in the form \( y = x + b \) to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the \( y \)-intercept and an ordered pair, or given the equation in the form \( y = x + b \), where \( b \) represents the \( y \)-intercept; and
e) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, tables, equations, and graphs.

7.11  The student will

a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and [Included in 7.12 EKS]
b) evaluate algebraic expressions for given replacement values of the variables.

7.12  The student will

a) solve one- and [Included in 6.14] two-step linear equations in one variable; and
b) solve practical problems requiring the solution of a one- and two-step linear equations in one variable.

7.13  The student will

a) solve one- and two-step [two-step moved from 8.15b] linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division; and
b) graph the solutions to inequalities on the number line.
7.16 The student will apply the following properties of operations with real numbers: a) the commutative and associative properties for addition and multiplication; b) the distributive property; c) the additive and multiplicative identity properties; d) the additive and multiplicative inverse properties; and e) the multiplicative property of zero. [Incorporated into 7.2, 7.11, 7.12, and 7.13]
Grade Eight

The eighth-grade standards continue to build on the concepts needed for success in high school level algebra, and geometry, and statistics are intended to serve two purposes. First, the standards contain content that reviews or extends concepts and skills learned in previous grades. Second, they contain new content that prepares students for more abstract concepts in algebra and geometry. The eighth-grade standards provide students additional instruction and time to acquire the concepts and skills necessary for success in Algebra I. Students will gain proficiency in computation with rational explore real numbers and the subsets of the real number system, and will use Proportional reasoning is expounded upon as students to solve a variety of problems. Students find the volume and surface area of more complex three-dimensional figures and apply transformations to geometric shapes in the coordinate plane. Students will verify and apply the Pythagorean Theorem creating a foundation for further study of triangular relationships in geometry. New Students will represent data, both univariate and bivariate data, and make predictions by observing data patterns. Students build upon the algebraic concepts developed in courses that the standards for Grades 6 and 7 Mathematics include simplifying algebraic expressions, solving multistep equations and inequalities, and graphing linear equations functions, visualizing three-dimensional shapes represented in two-dimensional drawings, and applying transformations to geometric shapes in the coordinate plane. Students will verify and apply the Pythagorean Theorem and represent relations and functions, using tables, graphs, and rules. The eighth-grade standards provide a more solid foundation in Algebra I for those students not ready for Algebra I in grade eight in middle school mathematics.

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Number and Number Sense

Focus: Relationships within the Real Number System

8.1 The student will
a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and [Combined with 7.28.14a]
b) compare and order real numbers, decimals, fractions, percents, and numbers written in scientific notation. [Moved to EKS]

8.2 The student will describe orally and in writing the relationships between the subsets of the real number system.
8.3 The student will
   a) estimate and determine the two consecutive integers between which a square root lies; and [Moved from Computation and Estimation strand 8.5b]
   b) determine both the positive and negative square roots of a given perfect square. [Moved from Computation and Estimation strand 8.5 EKS]

Computation and Estimation
Focus: Practical Applications of Operations with Real Numbers
8.43 The student will
   a) solve practical problems involving consumer applications, rational numbers, percents, ratios, and proportions; and
   b) determine the percent increase or decrease for a given situation.

8.4 The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables. [Moved to 8.14a]

8.5 The student will
   a) determine whether a given number is a perfect square; and [Included in 7.1]
   b) find the two consecutive whole numbers between which a square root lies. [Included in 8.3a]

Measurement and Geometry
Focus: Problem Solving
8.56 The student will use
   a) verify by measuring and describe the relationships among pairs of angles that are vertical angles, adjacent angles, supplementary angles, and complementary angles; and
   b) to determine the measure of unknown angles of less than 360°.

8.67 The student will
   a) investigate and solve problems, including practical problems, involving volume and surface area of rectangular prisms, cylinders, cones, and square-based pyramids; and
   b) describe how changing one measured attribute of a rectangular prism affects the volume and surface area.

Geometry
Focus: Problem Solving with 2- and 3-Dimensional Figures
8.78 The student will
   a) given a polygon, apply transformations, to include translations, rotations, reflections, and dilations, in the coordinate plane figures; and
   b) identify practical applications of transformations.

8.89 The student will construct a three-dimensional model, given the top or bottom, side, and front views.

8.910 The student will
   a) verify the Pythagorean Theorem; and
   b) apply the Pythagorean Theorem.

8.1044 The student will solve practical area and perimeter problems, including practical problems, involving composite plane figures.
Probability and Statistics
Focus: Statistical Analysis of Graphs and Problem Situations
8.1112 The student will
a) compare and contrast the probability of independent and dependent events; and [Moved from 6.16] with and without replacement.
b) determine probabilities for independent and dependent events.

8.12 The student will
a) represent numerical data in boxplots; [Moved from A.10]
b) make observations and inferences about data represented in boxplots; and [Moved from A.10]
c) compare and analyze two data sets using boxplots with the same data represented in histograms, stem and leaf plots, and line plots. [Moved from A.10]

8.13 The student will
a) represent data in scatterplots, make comparisons, predictions, and inferences, using information displayed in graphs; and
b) make observations and inferences about data represented in construct and analyze scatterplots; and
c) use a drawing to estimate the line of best fit for data represented in a scatterplot.

Patterns, Functions, and Algebra
Focus: Linear Relationships
8.144 The student will
a) apply the order of operations to evaluate an algebraic expressions for given replacement values of the variables [Moved from Computation and Estimation strand 8.4 and Number and Number Sense strand 8.1a]; and
b) simplify expressions in one variable.

8.154 The student will make connections between any two representations (tables, graphs, words, and rules) of a given relationship.
a) determine whether a given relation is a function; and
b) determine the domain and range of a function. [Moved from 8.17]

8.174 The student will
a) solve multistep linear equations in one variable with the variable on one and both two sides of the equation, including practical problems requiring the solution of a multistep linear equation in one variable.
b) solve two-step linear inequalities and graph the results on a number line; and [Moved to 8.18]
e) identify properties of operations used to solve an equation. [Incorporated into 8.14, 8.17, and 8.18 EKS]

8.16 The student will
a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;
b) identify the slope and y-intercept of a linear function given a table of values, the graph or given the an equation in $y = mx + b$ form;
c) determine the independent and dependent variable, given a practical situation modeled by a linear function;
d) graph a linear function given the equation in $y = mx + b$ form; and two variables.
c) distinguish between proportional and non-proportional situations modeled by linear functions; make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

8.17 The student will identify the domain, range, independent variable, or dependent variable in a given situation. [Moved to 8.15b]

8.18 The student will solve multistep linear inequalities in one variable with the variable on one and both sides of the inequality symbol, including practical problems, and graph the solution on a number line. [Moved from 8.15b]
Algebra I

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The standards below outline the content for a one-year course in Algebra I. All students are expected to achieve the Algebra I standards. The study of Algebra I assists students in generalizing patterns or modeling relevant, practical situations with algebraic models. In order to assist students in developing meaning and connecting algebraic concepts to geometry and statistics, when planning for instruction, consideration should be given to the sequential development of concepts and skills by using concrete materials to assist students in making the transition from the arithmetic numeric to the symbolic. Students should be helped to make connections and build relationships between algebra and arithmetic, geometry, and probability and statistics. Connections between Algebra I and also should be made to other subject areas through practical applications. This approach to teaching algebra should may assist in helping students attach meaning to the abstract concepts of algebra.

These standards require students to use algebra as a tool for representing and solving a variety of practical problems. Tables and graphs will be used to interpret algebraic expressions, equations, and inequalities and to analyze behaviors of functions. These standards include a transformational approach to graphing functions and writing equations when given the graph of the equation. Transformational graphing builds a strong connection between algebraic and graphic representations of functions.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Throughout the course, students should be encouraged to engage in discourse about mathematics with teachers and other students, use the language and symbols of mathematics in representations and communication, discuss problems and problem solving, and develop confidence in themselves as mathematics students.

Expressions and Operations

A.1 The student will
a) represent verbal quantitative situations algebraically; and
b) evaluate these algebraic expressions for given replacement values of the variables.

A.2 The student will perform operations on polynomials, including
a) applying the laws of exponents to perform operations on expressions;
b) adding, subtracting, multiplying, and dividing polynomials; and
c) factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations. [Moved to EKS]

A.3 The student will simplify express the
a) square roots of non-negative rational whole numbers and monomial algebraic expressions;
b) cube roots of integers whole rational numbers; and the square root of a monomial algebraic expression in simplest radical form
c) numerical expressions containing square or cube roots.

Equations and Inequalities
A.4 The student will solve multistep linear and quadratic equations in two variables, including
a) multistep linear equations in one variable algebraically; solving literal equations (formulas) for a given variable;
b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets; solving quadratic equations in one variable algebraically and graphically; solving literal equations for a specified variable; multistep linear equations algebraically and graphically;
ed) solving systems of two linear equations in two variables algebraically and graphically; and
ef) solving real-world practical problems involving equations and systems of equations.
Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

A.5 The student will solve multistep linear inequalities in two variables, including
a) solving multistep linear inequalities in one variable algebraically and represent the solution graphically;
b) representing the solution of linear inequalities in two variables algebraically and graphically; justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets;
c) solving real-world practical problems involving inequalities; and
d) representing the solution to solving systems of inequalities algebraically and graphically.

A.6 The student will graph linear equations and linear inequalities in two variables, including
a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined;
b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line; and
c) graph linear equations in two variables.

Functions
A.7 The student will investigate and analyze function (linear and quadratic) function families and their characteristics both algebraically and graphically, including
a) determining whether a relation is a function;
b) domain and range;
c) zeros of a function;
d) x- and y-intercepts;
e) finding the values of a function for elements in its domain; and
f) making connections between any two and among multiple representations of functions using verbal descriptions, tables, equations, and graphs, including concrete, verbal, numeric, graphic, and algebraic.

A.8 The student, given a data set or practical situation, in a real-world context, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.
Statistics

A.9 The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores. [Included in AFDA.7 and Algebra II.11]

A.10 The student will compare and contrast multiple univariate data sets, using box-and-whisker plots. [Moved to 8.12]

A.911 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve real-world practical problems, using mathematical models of linear and quadratic functions. Mathematical models will include linear and quadratic functions.
Geometry

This course is designed for students who have successfully completed the standards for Algebra I. All students are expected to achieve the Geometry standards. The course includes an emphasis on developing reasoning skills through the exploration of geometric relationships including, among other things, properties of geometric figures, trigonometric relationships, and mathematical proofs reasoning to justify conclusions. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods. Methods of justification will include paragraph proofs, two-column proofs, indirect proofs, coordinate proofs, algebraic methods, and verbal arguments. A gradual development of formal proof will be encouraged. Inductive and intuitive approaches to proof as well as deductive axiomatic methods should be used.

This set of standards includes emphasis on two- and three-dimensional reasoning skills, coordinate and transformational geometry, and the use of geometric models to solve problems. A variety of applications and some general problem-solving techniques, including algebraic skills, should be used to implement these standards. Graphing utilities (calculators, computers, and other technology tools) Calculators, computers, graphing utilities (graphing calculators or computer graphing simulators), and dynamic geometry software applications, and other appropriate technology tools will be used to assist in teaching and learning. Any technology that will enhance student learning should be used.

Reasoning, Lines, and Transformations

G.1 The student will use deductive reasoning to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
a) identifying the converse, inverse, and contrapositive of a conditional statement;
b) translating a short verbal argument into symbolic form; and
c) determining the validity of a logical argument. [Added from EKS] using Venn diagrams to represent set relationships [Moved to DM.12]; and
d) using deductive reasoning.

G.2 The student will use the relationships between angles formed by two lines cut intersected by a transversal to
a) determine whether prove two or more lines are parallel; and
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
e) solve real-world problems, including practical problems, involving angles formed when parallel lines are cut intersected by a transversal.

G.3 The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include
a) investigating and using formulas for determining distance, midpoint, and slope;
b) applying slope to verify and determine whether lines are parallel or perpendicular;
c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.
G.4 The student will construct and justify the constructions of
   a) a line segment congruent to a given line segment;
   b) the perpendicular bisector of a line segment;
   c) a perpendicular to a given line from a point not on the line;
   d) a perpendicular to a given line at a given point on the line;
   e) the bisector of a given angle,
   f) an angle congruent to a given angle; and
   g) a line parallel to a given line through a point not on the given line; and
   h) an equilateral triangle, a square, and a regular hexagon inscribed in a circle. [Moved from EKS]

Triangles
G.5 The student, given information concerning the lengths of sides and/or measures of angles in
   triangles, will solve problems, including practical problems. This will include
   a) ordering the sides by length, given the angle measures;
   b) ordering the angles by degree measure, given the side lengths;
   c) determining whether a triangle exists; and
   d) determining the range in which the length of the third side must lie.
   These concepts will be considered in the context of real-world situations.

G.6 The student, given information in the form of a figure or statement, will prove two triangles
   are congruent, using algebraic and coordinate methods as well as deductive proofs.

G.7 The student, given information in the form of a figure or statement, will prove two triangles
   are similar, using algebraic and coordinate methods as well as deductive proofs.

G.8 The student will solve real-world problems, including practical problems, involving
   right triangles. This will include applying by using the
   a) the Pythagorean Theorem and its converse,
   b) properties of special right triangles, and
   c) right triangle trigonometry, trigonometric ratios.

Polygons and Circles
G.9 The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to
   solve real-world problems, including practical problems.

G.10 The student will solve real-world problems, including practical problems, involving
    angles of convex polygons. This will include determining the
    a) sum of the interior and/or exterior angles;
    b) measure of an interior and/or exterior angle; and
    c) number of sides of a regular polygon.

G.11 The student will solve problems, including practical problems, by applying properties of
    circles, use angles, arcs, chords, tangents, and secants to. This will include determining
    a) investigate, verify, and apply properties of circles, angle measures formed
       by intersecting chords, secants, and/or tangents;
    b) solve real-world problems involving properties of circles, lengths of
       segments formed by intersecting chords, secants, and/or tangents; and
    c) find arc lengths; and
    d) and areas of a sectors in circles.

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G.12  The student will solve problems involving circles, given the coordinates of the center of a circle and a point on the circle, will write the equation of the circle.

Three-Dimensional Figures

G.13  The student will use formulas for surface area and volume of three-dimensional objects to solve real-world practical problems.

G.14  The student will use the concepts of similarity to apply the concepts of similarity to geometric objects into two- or three-dimensional geometric figures. This will include:
   a) comparing ratios between side-lengths, perimeters, areas, and volumes of similar figures;
   b) determining how changes in one or more dimensions of an object figure affect area and/or volume of the object figure;
   c) determining how changes in area and/or volume of an object figure affect one or more dimensions of the object figure; and
   d) solving real-world problems, including practical problems, about similar geometric objects.
Algebra, Functions, and Data Analysis

The following standards outline the content for a one-year course in Algebra, Functions, and Data Analysis. This course is designed for students who have successfully completed the standards for Algebra I and may benefit from additional support in their transition to Algebra II. Within the context of mathematical modeling and data analysis, students will study functions and their behaviors, systems of inequalities, probability, experimental design and implementation, and analysis of data. Data will be generated by practical applications arising from science, business, and finance. Students will solve problems that require the formulation of linear, quadratic, exponential, or logarithmic equations or a system of equations.

Through the investigation of mathematical models and interpretation/analysis of data from real life relevant, applied contexts and situations, students will strengthen conceptual understandings in mathematics and further develop connections between algebra and statistics. Students should use the language and symbols of mathematics in representations and communication, both orally and in writing, throughout the course.

These standards include a transformational approach to graphing functions and writing equations when given the graph of the equation. Transformational graphing builds a strong connection between algebraic and graphic representations of functions.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

The infusion of technology (graphing calculator and/or computer software) in this course will assist in modeling and investigating functions and data analysis.

Algebra and Functions

AFDA.1 The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) function families and their characteristics. Key concepts include
  a) domain and range continuity; [Reordered]
  b) intervals on which a function is increasing or decreasing local and absolute maxima and minima [Reordered];
  c) domain and range absolute maximum and minimum [Reordered];
  d) zeros;
  e) intercepts;
  f) values of a function for elements in its domain intervals in which the function is increasing/decreasing [Reordered];
  g) connections between and among multiple any two representations of functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic;
  h) end behaviors; and
  hi vertical and horizontal asymptotes.

AFDA.2 The student will use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic) function.

AFDA.3 The student will collect and analyze data, and generate and determine the equation for of the curve of best fit in order to make predictions, and solve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world practical problems or applications using models of linear, quadratic, and exponential functions. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.
AFDA.4 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate multiple representations of functions for analysis, interpretation, and prediction.

AFDA.5 The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

Data Analysis

AFDA.6 The student will calculate probabilities. Key concepts include
   a) conditional probability;
   b) dependent and independent events;
   c) addition and multiplication rules mutually exclusive events;
   d) counting techniques (permutations and combinations); and
   e) Law of Large Numbers.

AFDA.7 The student will analyze the normal distribution. Key concepts include:
   a) describe the distribution of a univariate data set using descriptive statistics characteristics of normally distributed data;
   b) identify and describe properties of a normal distribution percentiles;
   c) interpret and compare z-scores for normally distributed data normalizing data using z-scores; and
   d) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve area under the standard normal curve and probability.

AFDA.8 The student will design and conduct an experiment/survey. Key concepts include
   a) sample size;
   b) sampling technique;
   c) controlling sources of bias and experimental error;
   d) data collection; and
   e) data analysis and reporting.
Algebra II

The standards below outline the content for a one-year course in Algebra II. Students enrolled in Algebra II are assumed to have mastered those concepts outlined in the Algebra I standards. All students preparing for postsecondary and advanced technical studies are expected to achieve the Algebra II standards. A thorough treatment of advanced algebraic concepts will be provided through the study of functions, “families of functions,” equations, inequalities, systems of equations and inequalities, polynomials, rational and radical equations, complex numbers, and sequences and series. Emphasis will be placed on practical applications and modeling throughout the course of study. Oral and written communication concerning the language of algebra, logic of procedures, and interpretation of results should also permeate the course.

These standards include a transformational approach to graphing functions. Transformational graphing uses translation, reflection, dilation, and rotation to generate a “family of graphs” from a given graph “parent” function and builds a strong connection between algebraic and graphic representations of functions. Students will vary the coefficients and constants of an equation, observe the changes in the graph of the equation, and make generalizations that can be applied to many graphs.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Graphing utilities (graphing calculators or computer graphing simulators), computers, spreadsheets, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through mathematical modeling and aid in the investigation and study of functions. They also provide an effective tool for solving and verifying solutions to equations and inequalities. Any other available technology that will enhance student learning should be used.

Expressions and Operations

AII.1 The student, given rational, radical, or polynomial expressions, will
  a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
  b) add, subtract, multiply, divide, and simplify radical expressions containing rational
     numbers and variables, and expressions containing rational exponents;
  c) write radical expressions as expressions containing rational exponents and vice
     versa;[Moved to EKS]; and
  d) factor polynomials completely in one or two variables.

AII.2* The student will investigate and apply the properties of arithmetic and geometric
  sequences and series to solve real-world problems, including writing the first n terms, finding the n
  \textsuperscript{th} term, and evaluating summation formulas. Notation will include \(\Sigma\) and \(a_n\).
*Standard AII.2 will be assessed in the Functions and Statistics reporting category. (Revised March 2011)[Moved to AII.5 in the Functions strand]

AII.23 The student will perform operations on complex numbers, express the results in simplest
  form using patterns of the powers of \(i\), and identify field properties that are valid for the
  complex numbers.
Equations and Inequalities

**AII. 34** The student will solve, algebraically and graphically,
- a) absolute value linear equations and inequalities;
- b) quadratic equations over the set of complex numbers;
- c) equations containing rational algebraic expressions; and
- d) equations containing radical expressions.

Graphing calculators will be used for solving and for confirming the algebraic solutions. [Moved to EKS]

**AII. 45** The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic equations, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions. [Moved to EKS]

Functions

**AII. 52** The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world practical problems, including writing the first $n$ terms, finding determining the $n^{th}$ term, and evaluating summation formulas. Notation will include $\sum$ and $a_n$. [Moved from Expressions and Operations strand]

**AII. 6** For absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions, the student will
- a) recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families; and will
- b) use knowledge of transformations to convert between graphic and symbolic forms of equations and the corresponding graph of functions.

A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions. [Moved to EKS]

**AII. 7** The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions families algebraically and graphically. Key concepts include
- a) domain, and range, and continuity including limited and discontinuous domains and ranges [Moved to EKS];
- b) intervals in which a function is increasing or decreasing; [Reordered]
- c) maxima and minima extrema;
- d) zeros;
- e) $x$ - and $y$ -intercepts;
- f) intervals in which a function is increasing or decreasing values of a function for elements in its domain;
- g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic asymptotes;
- h) end behavior;
- i) vertical and horizontal asymptotes; [Reordered]
- j) inverse of a function; and
- k) composition of multiple functions algebraically and graphically.

Graphing calculators will be used as a tool to assist in investigation of functions. [Moved to EKS]

**AII. 8** The student will investigate and describe the relationships among solutions of an equation, zeros of a function, $x$-intercepts of a graph, and factors of a polynomial expression.
Statistics
AII.9 The student will collect and analyze data, determine the equation of the curve of best fit, in order to make predictions, and solve real-world practical problems, using mathematical models of linear, quadratic, and exponential functions. Mathematical models will include polynomial, exponential, and logarithmic functions.

AII.10 The student will identify, represent, create, and solve real-world problems, including practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.

AII.11 The student will
a) describe the distribution of a univariate data set using descriptive statistics;
b) identify and describe properties of a normal distribution;
c) interpret and compare z-scores for normally distributed data; and
d) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

AII.12 The student will compute and distinguish between permutations and combinations and use technology for applications.
**Trigonometry**

The standards below outline the content for a one-semester course in trigonometry. Students enrolled in trigonometry are assumed to have mastered those concepts outlined in the Algebra II standards. A thorough treatment of trigonometry will be provided through the study of trigonometric definitions, applications, graphing, and solving trigonometric equations and inequalities. Emphasis should also be placed on using connections between right triangle ratios, trigonometric functions, and circular functions. In addition, applications and modeling should be included throughout the course of study. Oral and written communication concerning the language of mathematics, logic of procedure, and interpretation of results should also permeate the course.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of trigonometric functions and their inverses. They also provide a powerful tool for solving and verifying solutions to trigonometric equations and inequalities.

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**T.1**

The student, given a point other than the origin on the terminal side of an angle in standard position, or the value of the trigonometric function of the angle, will determine the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.

**T.2**

The student, given the value of one trigonometric function, will find the values of the other trigonometric functions, using the definitions and properties of the trigonometric functions. [Combined into T.1]

The student will develop and apply the properties of the unit circle in degrees and radians. [Moved and rewritten from T.3]

**T.93**

The student will find, without the aid of a calculator, the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting angle measures from radians to degrees and vice versa. [Moved to T.2]

The student will solve problems, including practical problems, involving
a) arc length and area of sectors in circles using radians and degrees; and
b) linear and angular velocity.

**T.74**

The student will determine, with the aid of a calculator, the value of any trigonometric function and inverse trigonometric function.

**T.5**

The student will verify basic trigonometric identities and make substitutions, using the basic identities.

**T.36**

The student, given one of the six trigonometric functions in standard form, will
a) state the domain and the range of the function;
b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;
c) sketch the graph of the function by using transformations for at least a two-period interval; and
d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.

T.47 The student will graph the six inverse trigonometric functions, identify the domain and range of the inverse trigonometric functions, and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included. [Included in EKS]

T.68 The student will solve trigonometric equations and inequalities that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities.

T.89 The student will identify, create, and solve real-world practical problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines. [Included in EKS]
Algebra II and Trigonometry

The standards for this combined course in Algebra II and Trigonometry include all of the standards listed for Algebra II and Trigonometry. This course is designed for advanced students who are capable of a more rigorous course at an accelerated pace. The standards listed for this course provide the foundation for students to pursue a sequence of advanced mathematical studies from Mathematical Analysis to Advanced Placement Calculus.

Expressions and Operations

AII/T.1 The student, given rational, radical, or polynomial expressions, will
   a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
   b) add, subtract, multiply, divide, and simplify radical expressions containing rational
      numbers and variables, and expressions containing rational exponents;
   c) write radical expressions as expressions containing rational exponents and vice versa; and
   d) factor polynomials completely.

AII/T.2 The student will investigate and apply the properties of arithmetic and geometric sequences
   and series to solve real-world problems, including writing the first $n$ terms, finding the $n^{th}$
   term, and evaluating summation formulas. Notation will include $\sum$ and $a_n$.

AII/T.3 The student will perform operations on complex numbers, express the results in simplest
   form using patterns of the powers of $i$, and identify field properties that are valid for the
   complex numbers.

Equations and Inequalities

AII/T.4 The student will solve, algebraically and graphically,
   a) absolute value equations and inequalities;
   b) quadratic equations over the set of complex numbers;
   c) equations containing rational algebraic expressions; and
   d) equations containing radical expressions.
   Graphing calculators will be used for solving and for confirming the algebraic solutions.

AII/T.5 The student will solve nonlinear systems of equations, including linear-quadratic and
   quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool
   to visualize graphs and predict the number of solutions.

Functions

AII/T.6 The student will recognize the general shape of function (absolute value, square root, cube root,
   rational, polynomial, exponential, and logarithmic) families and will convert between graphic
   and symbolic forms of functions. A transformational approach to graphing will be employed.
   Graphing calculators will be used as a tool to investigate the shapes and behaviors of these
   functions.
AII/T.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include
a) domain and range, including limited and discontinuous domains and ranges;
b) zeros;
c) \text{-} and \text{y}-intercepts;
d) intervals in which a function is increasing or decreasing;
e) asymptotes;
f) end behavior;
g) inverse of a function; and
h) composition of multiple functions.
Graphing calculators will be used as a tool to assist in the investigation of functions.

AII/T.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, \text{-}intercepts of a graph, and factors of a polynomial expression.

Statistics

AII/T.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

AII/T.10 The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.

AII/T.11 The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.

AII/T.12 The student will compute and distinguish between permutations and combinations and use technology for applications.

Trigonometry

AII/T.13 The student, given a point other than the origin on the terminal side of an angle, will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.

AII/T.14 The student, given the value of one trigonometric function, will find the values of the other trigonometric functions, using the definitions and properties of the trigonometric functions.

AII/T.15 The student will find, without the aid of a calculator, the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting angle measures from radians to degrees and vice versa.

AII/T.16 The student will find, with the aid of a calculator, the value of any trigonometric function and inverse trigonometric function.

AII/T.17 The student will verify basic trigonometric identities and make substitutions, using the basic identities.

AII/T.18 The student, given one of the six trigonometric functions in standard form, will
a) state the domain and the range of the function;
b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;
c) sketch the graph of the function by using transformations for at least a two-period interval; and
d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.
AII/T.19—The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.

AII/T.20—The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities.

AII/T.21—The student will identify, create, and solve real-world problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.
Computer Mathematics

This course is intended to provide students with experiences in using computer programming techniques and skills to solve problems that can be set up as mathematical models. Students enrolled in Computer Mathematics are assumed to have studied the concepts and skills in Algebra I and beginning geometry. Students who successfully complete the standards for this course may earn credit toward meeting the mathematics graduation requirement. It is recognized that many students will gain computer skills in other mathematics courses or in a separate curriculum outside of mathematics and prior to high school. In such cases, the standards indicated by an asterisk (*) should be included in the student’s course of study and treated as a review.

Even though computer ideas should be introduced in the context of mathematical concepts, problem solving per se should be developed in the most general sense, making the techniques applicable by students in many other environments. Strategies include defining the problem; developing, refining, and implementing a plan; and testing and revising the solution. Programming, ranging from simple programs involving only a few lines to complex programs involving subprograms, should permeate the entire course and may include programming a graphing calculator or scripting a problem solution in a database or spreadsheet. Programming concepts, problem-solving strategies, and mathematical applications should be integrated throughout the course.

These standards identify fundamental principles and concepts in the field of computer science that will be used within the context of mathematical problem solving in a variety of applications. As students develop and refine skills in logic, organization, and precise expression, they will apply those skills to enhance learning in all disciplines.

**COM.1** The student will design and apply computer programming techniques and skills to solve practical real-world problems in mathematics arising from consumer, business, and other applications in mathematics. Problems will include opportunities for students to analyze data in charts, graphs, and tables and to use their knowledge of equations, formulas, and functions to solve these problems.

*COM.2* The student will design, write, document, test, and debug, and document a computer program. Programming documentation will include preconditions and postconditions of program segments, input/output specifications, the step-by-step plan, the test data, a sample run, and the program listing with appropriately placed comments. [Moved to US]

*COM.3* The student will write program specifications that define the constraints of a given problem. These specifications will include descriptions of preconditions, postconditions, the desired output, analysis of the available input, and an indication as to whether or not the problem is solvable under the given conditions. [Moved to US]

*COM.4* The student will design an step-by-step plan (algorithm) to solve a given problem. The plan will be in the form of a program flowchart, pseudo code, hierarchy chart, and/or data-flow diagram. [Moved to US]

*COM.5* The student will divide a given problem into manageable sections (modules) by task and implement the solution. The modules will include an appropriate user-defined functions, subroutines, and procedures. Enrichment topics might include user-defined libraries (units) and object-oriented programming.
The student will design and implement the input phase of a program, which will include designing screen layout, and getting information into the program by way of user interaction, data statements, and/or file input, and validating input. The input phase will also include methods of filtering out invalid data (error trapping).

The student will design and implement the output phase of a computer program, which will include designing output layout, accessing a variety of available output devices, using output statements, and labeling results.

The student will design and implement computer graphics to enhance output, which will include topics appropriate for the available programming environment as well as student background. Students will use graphics as an end in itself, as an enhancement to other output, and as a vehicle for reinforcing programming techniques.

The student will define simple and use appropriate variable data types that include integer, real (fixed and scientific notation), character, string, and Boolean, and object.

The student will use appropriate variable data types, including integer, real (fixed and scientific notation), character, string, and Boolean. This will also include variables representing structured data types. [Combined with COM.15]

The student will describe the way the computer stores, accesses, and processes variables, including the following topics: the use of variables versus constants, variables' addresses, pointers, parameter passing, scope of variables, and local versus global variables.

The student will translate a mathematical expressions into a computer programming statement expressions, which involves by declaring variables, writing assignment statements, and using the order of operations.

The student will select and implement call built-in (library) functions into processing data, as appropriate.

The student will implement conditional statements that include “if/then” statements, “if/then/else” statements, case statements, and Boolean logic.

The student will implement various mechanisms for performing iteration with an algorithm loops, including iterative loops. Other topics will include single entry point, single exit point, preconditions, and postconditions.

The student will select and implement appropriate data structures, including arrays (one- and/or two-dimensional and/or multidimensional), files, and record objects. Implementation will include creating the data structure, putting information into the structure, and retrieving information from the structure.

The student will implement pre-existing defined algorithms, including sort routines, search routines, and simple animation routines.

The student will test a program using an appropriate set of data. The set of test data should include boundary cases and test all branches of a program, be appropriate and complete for the type of program being tested.
COM.1819 The student will debug a program, using appropriate techniques (e.g., appropriately placed controlled breaks, the printing of intermediate results, other debugging tools available in the programming environment), and identify the difference between syntax errors, runtime errors, and logic errors.

COM.20 The student will design, write, test, debug, and document a complete structured program that requires the synthesis of many of the concepts contained in previous standards. [Included in COM.2]
Probability and Statistics

The following standards outline the content of a one-year course in Probability and Statistics. If a one-
semester course is desired, the standards with an asterisk dagger (†) would apply. Students enrolled in
this course are assumed to have mastered the concepts identified in the Standards of Learning for Algebra
II. The purpose of the course is to present basic concepts and techniques for collecting and analyzing data,
drawing conclusions, and making predictions.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching
and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and
statistical behaviors and provide a powerful tool for solving and verifying solutions.

A graphing calculator is essential for every student taking the Probability and Statistics course and is
required for the Advanced Placement Statistics Examination. The calculator may not fully substitute for a
computer, however. In the absence of a computer for student use, teachers may provide students with
eramples of computer output generated by a statistical software package.

*PS.1† The student will analyze graphical displays of univariate data, including dotplots, stemplots,
boxplots, cumulative frequency graphs, and histograms, to identify and describe patterns and
departures from patterns, using central tendency, spread, clusters, gaps, and
outliers. Appropriate technology will be used to create graphical displays. [Moved to EKS]

*PS.2† The student will analyze numerical characteristics of univariate data sets to describe patterns
and departures from patterns, using mean, median, mode, variance, standard deviation,
interquartile range, range, and outliers.

*PS.3† The student will compare distributions of two or more univariate data sets, numerically and
graphically, analyzing center and spread (within group and between group variations),
clusters and gaps, shapes, outliers, or other unusual features.

*PS.4† The student will analyze scatterplots to identify and describe the relationship between two
variables, using shape; strength of relationship; clusters; positive, negative, or no association;
outliers; and influential points.

PS.5 The student will find determine and interpret linear correlation, use the method of least
squares regression to model the linear relationship between two variables, and use the
residual plots to assess linearity.

PS.6 The student will make logarithmic and power transformations to achieve linearity.

PS.7† The student, using two-way tables and other graphical displays, will analyze categorical data
to describe patterns and departure from patterns and to find determine marginal frequency and
relative frequencies, including conditional frequencies.

*PS.8† The student will describe the methods of data collection in a census, sample survey,
experiment, and observational study and identify an appropriate method of solution for a
given problem setting.

*PS.9† The student will plan and conduct a survey. The plan will address sampling techniques (e.g.,
simple random and stratified) [Moved to EKS] and methods to reduce bias.

PS.10† The student will plan and conduct an well-designed experiment. The plan will address
control, randomization, replication, blinding, and measurement of experimental error.
*PS.11* The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive.

*PS.12* The student will find determine probabilities (relative frequency and theoretical), including conditional probabilities for events that are either dependent or independent, by applying the Law of Large Numbers concept, the addition rule, and the multiplication rule.

*PS.13* The student will develop, interpret, and apply the binomial and geometric probability distributions for discrete random variables, including computing the mean and standard deviation for the binomial and geometric variables.

PS.14 The student will simulate probability distributions, including binomial and geometric.

PS.15 The student will identify random variables as independent or dependent and find determine the mean and standard deviations for random variables and sums and differences of independent random variables.

*PS.16* The student will identify properties of a normal distribution and apply the normal distribution to determine probabilities, using a table or graphing calculator [Moved to EKS].

*PS.17* The student, given data from a large sample, will find determine and interpret appropriate point estimates and confidence intervals for parameters. The parameters will include proportion and mean, difference between two proportions, and difference between two means (independent and paired), and slope of a least-squares regression line.

PS.18 The student will apply and interpret the logic of an appropriate hypothesis-testing procedure. Tests will include large sample tests for proportion, mean, difference between two proportions, and difference between two means (independent and paired), and Chi-squared tests for goodness of fit, homogeneity of proportions, and independence; and slope of a least-squares regression line.

PS.19 The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means.

PS.20 The student will identify properties of a *t*-distribution and apply *t*-distributions to single-sample and two-sample (independent and matched pairs) *t*-procedures using tables or graphing calculators.
Discrete Mathematics

The following standards outline the content of a one-year course in Discrete Mathematics. If a one-semester course is desired, the standards with an asterisk/dagger (*) would apply. Students enrolled in Discrete Mathematics are assumed to have mastered the concepts outlined in the Standards of Learning for Algebra II.

Discrete mathematics may be described as the study of mathematical properties of sets and systems that have a countable (discrete) number of elements. With the advent of modern technology, discrete (discontinuous) models have become as important as continuous models. In this course, the main focus is problem solving in a discrete setting. Techniques that are not considered in the current traditional courses of algebra, geometry, and calculus will be utilized. As students solve problems, they will analyze and determine whether or not a solution exists (existence problems), investigate how many solutions exist (counting problems), and focus on finding the best solution (optimization problems). Connections will be made to other disciplines. The importance of discrete mathematics has been influenced by computers.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Modern technology (graphing calculators and/or computers) will be an integral component of this course.

*DM.1† The student will model problems, using vertex-edge graphs. The concepts of valence, connectedness, paths, planarity, and directed graphs will be investigated. Adjacency matrices and matrix operations will be used to solve problems (e.g., food chains, number of paths).[Included in US]

*DM.2† The student will solve problems through investigation and application of circuits, cycles, Euler Paths, Euler Circuits, Hamilton Paths, and Hamilton Circuits. Optimal solutions will be sought using existing algorithms and student-created algorithms.

*DM.3† The student will apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization. Graph coloring and chromatic number will be used.[Moved to EKS]

*DM.4 The student will apply algorithms, such as Kruskal’s, Prim’s, or Dijkstra’s, [Included in EKS] relating to trees, networks, and paths. Appropriate technology will be used to determine the number of possible solutions and generate solutions when a feasible number exists.

*DM.105 The student will use algorithms to schedule tasks in order to determine a minimum project time. The algorithms will include critical path analysis, the list-processing algorithm, and student-created algorithms.

*DM.116 The student will solve linear programming problems. Appropriate technology will be used to facilitate the use of matrices, graphing techniques, and the Simplex method of determining solutions.[Included in EKS]

DM.57 The student will analyze and describe the issue of fair division in discrete and continuous cases. (e.g., cake cutting, estate division[Included in EKS]). Algorithms for continuous and discrete cases will be applied.

DM.68 The student will investigate and describe weighted voting and the results of various election methods. These may include approval and preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners.
DM.79 The student will identify apportionment inconsistencies that apply to issues such as salary caps in sports and allocation of representatives to Congress. Historical and current methods will be compared.

DM.1210 The student will use the recursive process and difference equations with the aid of appropriate technology to generate
  a) compound interest;
  b) sequences and series;
  c) fractals;
  d) population growth models; and
  e) the Fibonacci sequence.

DM.811 The student will describe and apply sorting algorithms and coding algorithms used in sorting, processing, and communicating information. These will include
  a) bubble sort, merge sort, and network sort; and
  b) ISBN, UPC, Zip, and banking codes.[Included in EKS]

DM.912† The student will select, justify, and apply an appropriate technique to solve a logic problem. Techniques will include Venn diagrams, truth tables, and matrices.[Included in EKS]

DM.13 The student will apply the formulas of combinatorics in the areas of
  a) the Fundamental (Basic) Counting Principle;
  b) knapsack and bin-packing problems;
  c) permutations and combinations; and
  d) the pigeonhole principle.
Mathematical Analysis

The standards below outline the content for a one-year course in Mathematical Analysis. Students enrolled in Mathematical Analysis are assumed to have mastered Geometry and Algebra II concepts and have some exposure to trigonometry. Mathematical Analysis develops students’ understanding of algebraic and transcendental functions, parametric and polar equations, sequences and series, and vectors. The content of this course serves as appropriate preparation for a calculus course.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Graph calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of functions and their inverses. They also provide a powerful tool for solving and verifying solutions to equations and inequalities.

MA.1 The student will investigate and identify the characteristic properties of polynomial, rational, piecewise, and step and rational functions and use these to sketch their graphs of the functions of the functions. This will include determining zeros, upper and lower bounds, y-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics. [Included in EKS]

MA.32 The student will apply compositions of functions and inverses of functions to real-world practical situations. Analytical methods and graphing utilities will be used to investigate and verify the domain and range of resulting functions. [Included in EKS]

MA.53 The student will investigate and describe the continuity of functions, using graphs and algebraic methods. [Moved to EKS]

MA.124 The student will expand binomials having positive integral exponents through the use of the Binomial Theorem, the formula for combinations, and Pascal’s Triangle. [Included in EKS]

MA.135 The student will find the sum (sigma notation included) of finite and infinite convergent series, which will lead to an intuitive approach to a limit.

MA.146 The student will use mathematical induction to prove formulas and mathematical statements.

MA.47 The student will find the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution. [Included in EKS]

MA.68 The student will investigate, graph, and identify the characteristic properties of conic sections from equations in (h, k) vertex and standard forms. Transformations in the coordinate plane will be used to graph conic sections. [Included in EKS]

MA.29 The student will investigate and identify the characteristics of exponential and logarithmic functions to graph the function, in order to graph these functions and solve equations, and solve real-world practical problems. This will include the role of e, natural and common logarithms, laws of exponents and logarithms, and the solution of logarithmic and exponential equations. [Included in EKS]
MA.910 The student will investigate and identify the characteristics of the graphs of polar equations, using graphing utilities. This will include classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations. [Included in EKS]

MA.711 The student will perform operations with vectors in the coordinate plane and solve real-world practical problems using vectors. This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components. [Included in EKS]

MA.1012 The student will use parametric equations to model and solve application practical problems.

MA.813 The student will identify, create, and solve real-world practical problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines. [Included in the EKS.]

MA.1114 The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.
Mathematics Standards of Learning

Curriculum Framework 2009

Kindergarten

Board of Education
Commonwealth of Virginia
Introduction

The 2009-2016 Mathematics Standards of Learning, Curriculum Framework, is a companion document to the 2009-2016 Mathematics Standards of Learning, and amplifies the Mathematics Standards of Learning by further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. Provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

*Essential Understandings*
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. It contains background information for the teacher (K-8). It may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers’ knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

*Essential Knowledge and Skills*
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate in each standard is outlined. This is not meant to be an exhaustive list of student expectations or a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student’s problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
Students in grades K–3 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, fractions, number relationships (“more than,” “less than,” and “equal to”), and the effects of single-step and multistep computations. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.
### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- A set is a collection of distinct elements or items.
- A one-to-one correspondence exists when two sets have an equal number of items.
- Strategies for developing the concept of one-to-one matching involve set comparisons without counting. Hands-on experiences in matching items between two sets by moving, touching, and aligning objects, using one-to-one correspondence, enable visual as well as kinesthetic comparisons of the number of items in the two sets.
- Students can also count to make comparisons between two sets without matching the sets, using one-to-one correspondence.
- Students are generally familiar with the concept of more, but have had little experience with the term less. It is important to use the terms together to build an understanding of their relationship. For example, when asking which group has more, follow with which group has less and vice versa.

### ESSENTIAL UNDERSTANDINGS

- All students should
- Understand how quantities relate to each other, which leads to an understanding of how numbers are related to each other.

### ESSENTIAL KNOWLEDGE AND SKILLS

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
  - Match each member of one set with each member of another set, using the concept of one-to-one correspondence to compare the number of members between sets, where each set contains 10 or fewer objects.
  - Compare and describe two sets of 10 or fewer objects, using the terms more, fewer, and the same.
  - Given a set of objects, construct a second set which has more, fewer or the same number of objects.
### STANDARD K.21

**STRAND: NUMBER AND NUMBER SENSE**

**GRADE LEVEL K**

K.21 The student, given a set containing 15 or fewer concrete objects, will

- **a)** tell how many are in the given set of 20 or fewer objects by counting the number of objects orally; and
- **b)** read, write, and represent numbers from 0 through 20; the numeral to tell how many are in the set; and
- **c)** select the corresponding numeral from a given set of numerals. [Moved to EKS]

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- There are three developmental levels of counting:
  - rote sequence;
  - one-to-one correspondence; and
  - the cardinality of numbers.

- Counting involves two separate skills: verbalizing the list (rote sequence counting) of standard number words in order (“one, two, three, …”) and connecting this sequence with the objects in the set being counted, using one-to-one correspondence. Association of number words with collections of objects is achieved by moving, touching, or pointing to objects as the number words are spoken. Objects may be presented in random order or arranged for easy counting.

- When counting objects, students should:
  - Say the number names in standard order;
  - Count one item for each number word (one-to-one correspondence);
  - Understand that the number of objects is the same regardless of their arrangement or the order in which they were counted (conservation of number);
  - Understand that the last number names the total amount of objects counted (cardinality). By asking the question, “How many are there?” after the student has counted a collection of objects, teachers can determine if students have cardinality of number; and
  - Understand that each successive number name refers to a quantity that is one larger.

- Cardinality is knowing how many are in a set by recognizing that the last counting word tells the total number in a set.

- After having a student count a collection of objects, the teacher may be able to assess whether the student has cardinality of

### ESSENTIAL UNDERSTANDINGS

All students should

- Read and write numerals from 0 through 15. [Moved to EKS]

- Understand that the total number of objects can be found by counting.

- Understand that the last counted number describes the total amount in the set.

- Understand that if the set is empty, it has 0 elements. [Moved to US]

- Understand that changing the spatial arrangement of a set of objects does not change the total amount of the set.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Count orally to tell how many are in a given set containing 20 or fewer concrete objects, using one-to-one correspondence, and identify the corresponding numeral. (a)

- Read, write, and represent numbers from 0-20 to include:
  - Construct a set of objects that corresponds to a given numeral, including an empty set. (b) [Reordered]
  - Read and write the numerals from 0 through 20. [Reordered]
  - Identify written numerals from 0 through 20 represented in random order. (b)
  - Identify the numeral that corresponds to the total number of objects in a given set. Select the numeral from a given set of numerals that corresponds to a set of 20 or fewer concrete objects. (b)

- Write the numerals from 0 through 15. [Reordered]
  - Write a numeral that corresponds to a set of 20 or fewer concrete objects. (b)

- Construct a set of objects that corresponds to a given numeral, including an empty set. [Reordered]
The student, given a set containing 15 or fewer concrete objects, will

a) tell how many are in the given set by counting the number of objects orally; and

b) read, write, and represent numbers from 0 through 20, the numeral to tell how many are in the set; and
c) select the corresponding numeral from a given set of numerals. [Moved to EKS]

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</table>
| number by asking the question, “How many are there?”  
Students who do not yet have cardinality of number are often unable to tell you how many objects there were without recounting them.  
• Kinesthetic involvement (e.g., tracing the numbers, using tactile materials, such as sand, sandpaper, carpeting, or finger paint) facilitates the writing of numerals.  
• Articulating the characteristics of each numeral when writing numbers has been found to reduce the amount of time it takes to learn to write numerals.  
If a set is empty, it has zero (0) objects or elements. Zero (0) is both a number and a digit. As a number, it plays a central role in mathematics as the additive identity of the integers, real numbers, and many other algebraic structures. As a digit, zero is used as a placeholder in our number systems.  
• Symbolic reversals in numeral writing are common for this age and should not be mistaken for lack of understanding.  
• Describing a teen number as a ten and some more, will help students name how many are in a set of 13-19 objects. This also lays a foundation for place value. Conservation of number and cardinality principle are two important milestones in development to attaching meaning to counting.  
• The cardinality principle refers to the concept that the last counted number describes the total amount of the counted set. It is an extension of one-to-one correspondence.  
• Conservation of number is the understanding that the number of objects remains the same when they are rearranged spatially. |
The student, given no more than three two-sets, each set containing 10 or fewer concrete objects, will

a) identify, compare and describe one set as having more, fewer, or the same number of members objects as the other set(s), and using the concept of one-to-one correspondence.

b) compare and order sets from least to greatest and greatest to least.

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<tr>
<td>A set is a collection of objects distinct elements or items.</td>
<td>All students should understand how quantities relate to each other, which leads to an understanding of how numbers are related to each other.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
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<tr>
<td>A one-to-one correspondence exists when two sets have an equal number of items. [Included in K.1]</td>
<td></td>
<td>• Match each member of one set with each member of another set, using the concept of one-to-one correspondence to compare the number of members between sets, where each set contains 10 or fewer objects. [Included in US].</td>
</tr>
<tr>
<td>Sets can be compared by matching, lining up the objects, visually estimating magnitude, recognizing quantities without counting (subitizing), or counting the number of objects in each set.</td>
<td></td>
<td>• Compare and describe two no more than three sets of 10 or fewer objects, using the terms more, fewer, and the same. (a)</td>
</tr>
<tr>
<td>Strategies for developing the concept of one-to-one matching involve set comparisons without counting. Hands-on experiences in matching items between two sets by moving, touching, and aligning objects, using one-to-one correspondence, enable visual as well as kinesthetic comparisons of the number of items in the two sets.</td>
<td></td>
<td>• Given a set of objects, construct a second set which has more, fewer or the same number of objects. (a)</td>
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<tr>
<td>Students can also count to make comparisons between two sets without matching the sets, using one-to-one correspondence.</td>
<td></td>
<td>• Compare and order three or fewer sets, each set containing 10 or fewer concrete objects, from least to greatest and greatest to least. (b)</td>
</tr>
<tr>
<td>Comparing sets is an extension of conservation of number (e.g., 5 is 5 whether it is 5 marbles or 5 basketballs even though 5 basketballs take up more space). When comparing objects, the set can be arranged differently while still containing the same number (e.g., 5 marbles in a cup is the same as 5 marbles on the floor).</td>
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<tr>
<td>Comparing objects is an extension of cardinality. Cardinality is knowing how many are in a set by recognizing that the last counting word tells the total number in a set.</td>
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</table>
The student, given **no more than three two-sets**, each set containing 10 or fewer concrete objects, will

a) **identify, compare and describe** one set as having more, fewer, or the same number of members objects as the other set(s),

and using the concept of one-to-one correspondence.

b) **compare and order** sets from least to greatest and greatest to least.

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<tr>
<td>• After having a student count a collection of objects, the teacher may be able to assess whether the student has cardinality of number by asking the question, “How many are there?” Students who do not yet have cardinality of number are often unable to tell you how many objects there were without recounting them.</td>
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<tr>
<td>• Students are generally familiar with the concept of <em>more</em>, but may have had little experience with the term <em>less</em>. It is important to use the terms together to build an understanding of their relationship. For example, when asking which group has more, follow with which group has less <em>and vice versa</em>.</td>
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</table>
K.34 The student will, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth, and the ordered position of each object.[Ordinal positions moved to 1.3]  
   a) recognize and describe with fluency part-whole relationships for numbers up to 5; and  
   b) investigate and describe part-whole relationships for numbers up to 10.

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<td><strong>(Background Information for Instructor Use Only)</strong></td>
<td>All students should use ordinal numbers to describe the position of objects in a sequence.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
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<tr>
<td>• Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently.</td>
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<td>• Recognize and describe with fluency part-whole relationships for numbers up to 5 in a variety of configurations. (a)</td>
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<tr>
<td>• Flexibility requires knowledge of more than one approach to solving a particular kind of problem. Being flexible allows students to choose an appropriate strategy for the numbers involved.</td>
<td></td>
<td>• Investigate and describe part-whole relationships for numbers up to 10 using a variety of configurations. (b)</td>
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<tr>
<td>• Composing and decomposing numbers flexibly forms a basis for understanding properties of the operations and later formal algebraic concepts and procedures.</td>
<td></td>
<td>• Identify the ordinal positions first through tenth using ordered sets of ten concrete objects and/or pictures of such sets presented from left-to-right; right-to-left; top-to-bottom; and/or bottom-to-top.[Ordinal positions moved to 1.3]</td>
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<tr>
<td>• Parts of 5 and 10 should be represented in a variety of ways, such as five frames, ten frames, strings of beads, arrangements of tiles or toothpicks, dot cards, beaded strings or number frames.</td>
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<tr>
<td>• Dot patterns should be presented in both regular and irregular arrangements. This will help students to understand that numbers are made up of parts, and will later assist them in combining parts as well as counting on.</td>
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<tr>
<td>• Numbers can be composed and decomposed using part-part-whole relationships (e.g., 4 can be decomposed as 3 and 1, 2 and 2, 4 and 0).</td>
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The student will, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth, and the ordered position of each object. [Ordinal positions moved to 1.3]

a) recognize and describe with fluency part-whole relationships for numbers up to 5; and

b) investigate and describe part-whole relationships for numbers up to 10.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Quickly recognizing and naming the number of objects in a small group without counting is called subitizing. The size of the group a student can subitize is dependent upon the arrangement of the dots or objects. At this age, students should subitize regular arrangements up to 5.

- When students are able to combine or separate groups to create a number, they are building a foundation for addition and subtraction.

- Decomposition of 5 and 10 develops benchmarks of 5 and 10 that are essential in building place value understanding through the understanding of decomposition of the numbers of 5 and 10.

- Flexibility requires knowledge of more than one approach to solving a particular kind of problem. Being flexible allows students to choose an appropriate strategy for the numbers involved.

- Composing and decomposing numbers flexibly forms a basis for understanding properties of the operations and later formal algebraic concepts and procedures.

- Accuracy is the ability to determine a correct answer using knowledge of number facts and other important number relationships.

- Efficiency is the ability to carry out a strategy easily when solving a problem without getting bogged down in too many steps or lose track of the logic of the strategy being used.

- Mathematically fluent students are not only able to provide correct answers quickly but also to use facts and computation strategies they know to efficiently determine answers that they...
K.34 The student will, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth, and the ordered position of each object. [Ordinal positions moved to 1.3]

a) recognize and describe with fluency part-whole relationships for numbers up to 5; and
b) investigate and describe part-whole relationships for numbers up to 10.

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<td>do not know.</td>
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<tr>
<td>• Understanding the cardinal and ordinal meanings of numbers are necessary to quantify, measure, and identify the order of objects.</td>
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<tr>
<td>• An ordinal number is a number that names the place or position of an object in a sequence or set (e.g., first, third). Ordered position, ordinal position, and ordinality are terms that refer to the place or position of an object in a sequence or set.</td>
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<tr>
<td>• The ordinal position is determined by where one starts in an ordered set of objects or sequence of objects.</td>
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<tr>
<td>• The ordinal meaning of numbers is developed by identifying and verbalizing the place or position of objects in a set or sequence (e.g., the student’s position in line when students are lined up alphabetically by first name). [Moved to Grade 1]</td>
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</table>
The student will:

a) count forward orally by ones from 0 to 100, starting at any number between 0 and 100;

b) count and backward orally by ones when given any number between 1 and 10 from 10; and

c) identify one more than a number and one less than a number; and the number after, without counting, and the number before when given any number between 0 and 100 and identify the number before, without counting, when given any number between 1 and 10; count by fives and tens to 100. [Count by fives included in 1.2]; and

d) count forward by tens to determine the total number of objects to 100.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Counting skills are essential components of the development of number ideas; however, they are only one of the indicators of the understanding of numbers.

- Counting forward by rote, supported by visuals such as the hundred chart or number path, advances the child’s development of sequencing.

- The natural numbers are 1, 2, 3, 4…. The whole numbers are 0, 1, 2, 3, 4…. Students should count the whole numbers 0, 1, 2, 3, 4….

- A number path is a counting model where each number is represented within a square and the squares can be clearly counted.

  Example of a Number Path

  1 2 3 4 5 6 7 8 9 10

- A number line is a length model where each number represents its length from zero. When young children use a number line as a counting tool in solving problems, they often confuse what should be counted—the numbers or the spaces between the numbers. A number path is more appropriate for students at this age.

- Counting backward by rote lays the foundation for subtraction. Students should count backward beginning with 10, 9, 8,… through …3, 2, 1, 0.

- Counting forward and backward leads to the development of counting on and counting back.

ESSENTIAL UNDERSTANDINGS

All students should

- Use the correct oral counting sequence in both forward and backward counting situations.

- Understand that skip counting can be used to count a collection of objects.

- Describe patterns in skip counting and use those patterns to predict the next number or numbers in the skip counting sequence.

- Understand that numeric relationships include one more than, one less than, two more than, two less than, etc.

- Understand benchmarks of five and ten.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Count forward orally by ones from 0 to 100, starting at any number between 0 and 100. (a)

- Count backward orally from 10 to 0 when given any number between 0 and 10. (b)

- Identify the number after, without counting, and the number before when given any number between 0 and 100. (c)

- Identify the number before, without counting, when given any number between 1 and 10. (c)

- Count forward orally by tens, starting at 0, to determine the total number of objects to 100. (d)

- Recognize the relationship of one more than and one less than a number using objects (i.e., five and one more is six; and one less than ten is nine).

- Group 100 or fewer objects together into sets of fives or tens and then count them by fives or by tens.

- Investigate and recognize the pattern of counting by fives to 100, using a variety of tools. [Included in 1.2]

- Investigate and recognize the pattern of counting by tens to 100, using a variety of tools.
**STANDARD K.43**

**STRAND: NUMBER AND NUMBER SENSE**

**GRADE LEVEL K**

**K.43** The student will

a) count forward orally by ones from 0 to 100 starting at any number between 0 and 100;

b) count and backward orally by ones when given any number between 1 and 10 from 10; and

c) identify one more than a number and one less than a number; and the number after, without counting, and the number before when given any number between 0 and 100 and identify the number before, without counting, when given any number between 1 and 10 count by fives and tens to 100. (Count by fives included in 1.2); and

d) count forward by tens to determine the total number of objects to 100.

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<td>• Connecting rote counting to the counting of collections is necessary for students to understand the meaning of a number.</td>
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<tr>
<td>• Identifying the number after and/or the number before any given numbers demonstrates an understanding of number relationships as opposed to a memorized sequence of numbers.</td>
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<tr>
<td>• Providing experiences in counting beyond 100 will help students who often struggle with going over the century mark. The patterns developed as a result of skip counting are precursors for recognizing numeric patterns, functional relationships, and concepts underlying money, time telling, and multiplication. Powerful models for developing these concepts include, but are not limited to, counters, hundred chart, and calculators. Skip counting by fives lays the foundation for reading a clock effectively and telling time to the nearest five minutes, counting money, and developing the multiplication facts for five. Skip counting by tens is a precursor for use of place value, addition, counting money, and multiplying by multiples of 10. Calculators can be used to display the numeric patterns that result from skip counting.</td>
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STANDARD K.5  
STRAIGHT: NUMBER AND NUMBER SENSE  
GRADE LEVEL K

K.5 The student will identify the parts of a set and/or region that represent fractions for halves and fourths, investigate fractions by representing and solving practical problems involving equal sharing with two or four sharers.

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<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
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<tr>
<td>• Practical situations with fractions should involve real life problems in which students themselves are determining how to subdivide a whole into equal parts, test those parts to be sure they are equal, and use those parts to recreate the whole.</td>
<td>• Understand that fractional parts are equal shares of a whole region or a whole set. [Moved to US]</td>
<td>• Given a practical situation, share a whole equally with two or four sharers.</td>
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<tr>
<td>• Fractions can have different meanings: part-whole, division, measurement, ratio, and operator. The focus of this grade level is to develop the idea of equal sharing (division) and part-whole relationships. Fraction notation will be introduced in second grade.</td>
<td>• Understand that the fraction name (half, fourth) tells the number of equal parts in the whole. [Moved to US]</td>
<td>• Given a practical situation, represent fair shares concretely or pictorially.</td>
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<tr>
<td>• An equal sharing problem is an idea that young children understand equal sharing problems intuitively because of their experiences sharing objects with siblings, friends, etc. Consider the following examples:</td>
<td>• Understand that the fraction name (half, fourth) of the set model is a subset of the whole set with equal numbers.</td>
<td>• Given a practical situation, describe shares as equal pieces or parts of the whole (e.g., halves).</td>
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<tr>
<td>– Two children sharing six sandwiches</td>
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<td>• Recognize fractions as representing parts of equal size of a whole.</td>
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<tr>
<td>– Two children sharing one sandwich</td>
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<td>• Given a region, identify half and/or a fourth of the region.</td>
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<tr>
<td>Four children sharing one piece of paper</td>
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<td>• Given a set, identify half and/or a fourth of the set.</td>
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<td>Four children sharing two sticks of clay</td>
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<td>– Two children sharing three brownies</td>
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<td>Four children sharing five brownies</td>
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<tr>
<td>For two children sharing one sandwich, a child might say that each will get half of the sandwich. For two children sharing four brownies, a child might say they each get half of a brownie, while another child might say they will get one of the two pieces.</td>
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<tr>
<td>• Teachers should use vocabulary such as halves and fourths. Students may name the parts as halves but may also use language such as “one piece out of the two pieces” to describe half. But students at this level should not be expected to use fraction vocabulary or notation at this age. Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions furthers this</td>
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Mathematics Standards of Learning Curriculum Framework 2009-2016: Kindergarten 13
K.5 The student will identify the parts of a set and/or region that represent fractions for halves and fourths. Investigate fractions by representing and solving practical problems involving equal sharing with two or four shares.

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<td>Students should be encouraged to create drawings, use concrete objects, or utilize other representations to solve problems.</td>
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<tr>
<td>Fraction models at this level should be able to be continuously divided (e.g., cookies, brownies). It is important to use models that can be continuously divided when there are remainders so those remainders can be cut into as many equal parts as needed.</td>
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<tr>
<td>A fraction is a way of representing part of a whole (as in a region/area model) or part of a group (as in a set model).</td>
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<td>In each fraction model, the fractional parts must have the same area (e.g., equal shares of a whole).</td>
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<td>Equal parts may be different shapes but maintain the same value (e.g., a sandwich could be cut in two equal pieces vertically, horizontally, or diagonally to represent halves).</td>
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<tr>
<td>The fraction name (half or fourth) tells the number of equal parts in the whole.</td>
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<tr>
<td>The fractional parts of a set model are subsets of an equal number. For example, in a set of ten cubes, each half would be a subset of five cubes.</td>
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<tr>
<td>Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions furthers this development (e.g., fourths means “four equal parts of a whole” or ( \frac{1}{4} ) represents one of four parts of equal size when a pizza is shared among four students.</td>
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A variety of contexts and problem types are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from real-life experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as “How many books do we have altogether?” or “How many cookies are left if I eat two?” or “I have three more candies than you do.” Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling, discussing, and recording a variety of problem situations. This approach helps students transition from the concrete to the representation to the symbolic in order to develop meaning for the operations and how they relate to each other.

In grades K–3, computation and estimation instruction focuses on:

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition and subtraction, multiplication, division and related facts;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as skip counting and solving problems that involve equal groupings of objects as well as problems that involve sharing equally, and initial work on performing initial operations with fractions.
The student will model and solve single-step story and picture problems with sums to 10 and differences within 10 adding and subtracting whole numbers, using up to 10 concrete objects.

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<td>* Whole numbers are 0, 1, 2, 3, 4, 5, 6, and so on.</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>* Students should experience a variety of problem types related to addition and subtraction.</td>
<td>* Understand that addition means putting things together and that subtraction is the inverse of addition and means to separate things out.</td>
<td>* Model and solve various types of story and picture problems using 10 or fewer concrete objects. (Types of problems should include joining, separating, and part-part-whole scenarios.)</td>
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<td>* The problem types most appropriate for students at this level include:</td>
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<td>* Combine two sets with known quantities in each set, and count the combined set using up to 10 concrete objects, to determine the sum, where the sum is not greater than 10.</td>
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<tr>
<td>Join (result unknown) – Example: Sue had 4 pennies. Josh gave her 2 more. How many pennies does Sue have altogether?</td>
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<td>* Given a set of 10 or fewer concrete objects, remove, take away, or separate part of the set and determine the result.</td>
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<td>Separate (result unknown) – Example: Sue had 8 pennies. She gave 5 pennies to Josh. How many pennies does Sue have now?</td>
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<tr>
<td>Part-part-whole (whole unknown) – Example: Josh has 4 red balloons and 3 blue balloons. How many balloons does he have?</td>
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<tr>
<td>Part-part-whole (both parts unknown) – Example: Josh has 5 balloons. Some of them are red and some of them are blue. How many balloons can be blue and how many can be red?</td>
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<tr>
<td>* Addition is join problems involve the process of combining or joining sets or quantities. Separate problems</td>
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<tr>
<td>Subtraction can be viewed as a taking away or separating process or as comparing to find the difference between two sets. Part-part-whole problems involve two quantities that are combined into one whole but no physical action is required. Comparison problems that ask how many more or how many fewer should be reserved for first and second grade.</td>
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<td>* Operation symbols (+, -) are introduced in first grade.</td>
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<td>* Single-step refers to the least number of steps necessary to solve a problem.</td>
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K.6 The student will model and solve single-step story and picture problems with sums to 10 and differences within 10 adding and subtracting whole numbers, using up to 10 concrete objects.

- Number relationships help students develop strategies for addition and subtraction. These strategies include:
  - Instant recognition of the amount in a set of objects (subitize) that are arranged in a familiar pattern such as the dots on number cubes; and
  - One more than, one less than, two more than, two less than. [Reordered]

- Counting on from the larger set to determine the sum of the combined sets is one strategy for finding a sum.

- Counting backward from the larger set to determine the difference between two sets is one strategy for subtraction.

- Number relationships, including the following, help students develop strategies for adding and subtracting:
  - Instant recognition of the amount in a set of objects that are arranged in a familiar pattern such as the dots on number cubes
  - One more than, one less than, two more than, two less than. [Reordered]
The exploration of measurement and geometry in the primary grades allows students to learn more about the world around them. Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K–3 should encounter measurement in many normal situations, from their daily lives, from their use of the calendar and from science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, area, perimeter, volume, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both several techniques, and direct comparison, nonstandard units, and standard tools to find determine measurements and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on:

- observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.
• **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

• **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

• **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
**STANDARD K.7**  
**STRAND: MEASUREMENT AND GEOMETRY**  
**GRADE LEVEL K**

<table>
<thead>
<tr>
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| • Involvement in varied activities such as physically manipulating coins and making comparisons about their sizes, colors, and values are prerequisites to the skills of coin recognition and valuation.  
• Counting money helps students gain an awareness of consumer skills and the use of money in everyday life.  
• A variety of classroom experiences in which students manipulate physical models of money and count forward to determine the value of a collection of coins are important activities to ensure competence with using money.  
• Students need experiences counting collections of pennies. This can promote one-to-one correspondence, as a penny is worth one cent.  
• Students need experiences to develop the concept that a nickel has a value of five cents even though it is one object (which is the same as five pennies), that a dime has a value of ten cents (which is the same as ten pennies), and a quarter has a value of twenty-five cents (which is the same as twenty-five pennies), even though each coin (nickel, dime, quarter) is only one object. | All students should  
- Develop common referents for identifying pennies, nickels, dimes, and quarters.  
- Understand the value of a collection of coins whose value is 10 cents or less. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to  
- Describe the properties/characteristics/attributes (e.g., color, relative size) of a penny, nickel, dime, and quarter.  
- Identify a penny, nickel, dime, and quarter.  
- Identify that the number of pennies equivalent to a nickel, a dime, and a quarter (i.e., a nickel has the same value as five pennies), value of each coin (a penny is worth one cent, a nickel is worth five cents, a dime is worth ten cents, and a quarter is worth twenty-five cents) a nickel is the same value as five pennies.  
- Count a randomly placed collection of pennies and/or nickels (or models of pennies and/or nickels) whose value is 10 cents or less, and determine the value of the collection. |

K.7 The student will recognize the attributes of a penny, nickel, dime, and quarter and identify the value of each coin. Number of pennies equivalent to a nickel, a dime, and a quarter, and will determine the value of a collection of pennies and/or nickels whose total value is 10 cents or less. [Value of a collection included in 1.7]
K.8 The student will identify the instruments used to measure length (ruler), weight (scale), time (clock: digital and analog; calendar: day, month, and season), and temperature (thermometer). [Moved each instrument to the standard where the content is first taught (ruler – SOL 2.8; scale – SOL 2.8; clock – SOL 1.9; thermometer – SOL 2.11)]

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<tr>
<td>▪ Many experiences in measuring physical objects, using nonstandard and standard units of measure, help to develop an intuitive understanding of measurement and will help students connect a tool with its purpose in measuring.</td>
<td>All students should identify an appropriate measuring tool for a given unit of measure.</td>
<td>The student will use problem-solving, mathematical communication, mathematical reasoning, connections, and representations to:</td>
</tr>
<tr>
<td>▪ Selecting from among various measuring instruments and determining which can be used to solve various real-life problems are introduced at this level.</td>
<td></td>
<td>▪ Identify a ruler as an instrument to measure length. [Moved to 2.8]</td>
</tr>
<tr>
<td>▪ A precursor to connecting tools to a type of measurement is an introduction to the concepts of length, weight, time, and temperature.</td>
<td></td>
<td>▪ Identify different types of scales as instruments to measure weight. [Moved to 2.8]</td>
</tr>
</tbody>
</table>

Mathematics Standards of Learning Curriculum Framework 2009-2016: Kindergarten 21
K.89 The student will investigate the passage of time by reading and interpreting a calendar. [Moved from 1.11] Tell time to the hour, using analog and digital clocks. [Time to hour moved to 1.9a]

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<tr>
<td>• Practical situations are appropriate to develop a sense of the interval of time between events (e.g., club or team meetings occur every week on Monday: there is a week between meetings).</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• The calendar is a way to represent units of time (e.g., days, weeks, months, and a year).</td>
<td>• Apply an appropriate technique, depending on the type of clock, to determine time to the nearest hour.</td>
<td>• Identify the months of the year. [Moved from 1.11]</td>
</tr>
<tr>
<td>• Using a calendar develops the concept of a day as a 24-hour period rather than a period of time from sunrise to sunset. [Moved from 1.11]</td>
<td></td>
<td>• Identify the seven days in a week. [Moved from 1.11]</td>
</tr>
<tr>
<td>• Many experiences in relating time on the hour to daily routines and school schedules (e.g., catching the bus, lunch time, recess time, and resource time) help students develop personal referents for time.</td>
<td></td>
<td>• Determine the days/dates before and after a given day/date (e.g., yesterday, today, tomorrow). [Moved from 1.11]</td>
</tr>
<tr>
<td>• Making sense of telling time to the nearest hour is reinforced when students recognize the positions of the hands on an analog clock and identify the corresponding time to the hour.</td>
<td></td>
<td>• Tell time on an analog clock to the hour.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Tell time on a digital clock to the hour.</td>
</tr>
</tbody>
</table>
The student will compare two objects or events, using direct comparisons or nonstandard units of measure, according to one or more of the following attributes: length (shorter, longer), height (taller, shorter), weight (heavier, lighter), temperature (hotter, colder), volume (more, less), and time (longer, shorter). Examples of nonstandard units include foot length, hand span, new pencil, paper clip, and block. [Nonstandard units included in 1.10]

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<td>All students should</td>
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</tr>
<tr>
<td>• Students need to identify the attribute that they are measuring (e.g., length, height, weight, temperature, volume) before they begin to measure. [Reordered]</td>
<td>• Compare and order objects according to their attributes.</td>
<td>• Compare and describe lengths of two objects (as shorter or longer), using direct comparison (e.g., the bus is longer than the car) or nonstandard units of measure (e.g., foot length, hand span, new pencil, paper clip, block).</td>
</tr>
<tr>
<td>• Multiple hands-on experiences are needed to gain the ability to compare the attributes of objects. [Reordered]</td>
<td>• Develop an understanding of measuring with nonstandard and standard units of measure.</td>
<td>• Compare and describe heights of two objects (as taller or shorter), using direct comparison or nonstandard units of measure (e.g., book, hand span, new pencil, paper clip, block).</td>
</tr>
<tr>
<td>• Students develop conservation of measurement when they understand that the attributes do not change when the object is manipulated (e.g., a piece of string that is coiled maintains its length as it is straightened; the volume of water does not change when poured from a pitcher into a fish tank.)</td>
<td>• Recognize attributes (length, height, weight, temperature) that can be measured.</td>
<td>• Compare and describe weights of two objects (as heavier or lighter), using direct comparison or nonstandard units of measure (e.g., book, cubes, new pencil, paper clip, block).</td>
</tr>
<tr>
<td>Length is the distance between two points.</td>
<td>• Compare and describe temperatures of two objects or environment (as hotter or colder), using direct comparison.</td>
<td>• Compare and describe volumes of two containers (as more or less), using direct comparison.</td>
</tr>
<tr>
<td>Height is the vertical length of a perpendicular to its base from the bottom or base of something to the top.</td>
<td>• Compare and describe the amount of time spent on two events (as longer or shorter), using direct comparison.</td>
<td>• Compare and describe the amount of time spent on two events (as longer or shorter), using direct comparison.</td>
</tr>
<tr>
<td>Weight is a measure of the heaviness of an object.</td>
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<td></td>
</tr>
<tr>
<td>Temperature is the degree of hotness or coldness of an object (e.g., a body) or environment.</td>
<td></td>
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<tr>
<td>Volume is the measure of the capacity of a container.</td>
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</tr>
<tr>
<td>Time is the measure of an event from its beginning to end. Students could compare the difference between the time spent sliding down the slide versus the time spent walking around the school building.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students need to identify the attribute that they are measuring (e.g., length, height, weight, temperature) before they begin to measure. [Reordered]</td>
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<td>Multiple hands-on experiences are needed to gain the ability to compare the attributes of objects. [Reordered]</td>
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Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on:

- observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
The student will:

a) identify, and describe, and trace plane geometric figures (circle, triangle, square, and rectangle); and
b) compare the size (smaller, larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle); and
c) describe the location of one object relative to another (above, below, next to) and identify representations of plane figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.

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<tr>
<td>• An important part of the geometry strand in grades K through 2 is the naming and describing of figures. Children move from their own vocabulary and begin to incorporate conventional terminology as the teacher uses geometric terms.</td>
</tr>
<tr>
<td>• Early experiences with comparing, sorting, combining, and subdividing figures assist students in analyzing the characteristics of plane figures.</td>
</tr>
<tr>
<td>• Attribute blocks, relational attribute blocks, and tangrams, are among the manipulatives that are particularly appropriate for sorting and comparing size and shape.</td>
</tr>
<tr>
<td>• Students should be given opportunities to construct plane figures using multiple tools (e.g., clay, straws, and paper and scissors).</td>
</tr>
<tr>
<td>• Presentation of triangles, rectangles, and squares should be made in a variety of spatial orientations so that students are less likely to develop common misconception that triangles, rectangles, and squares must have one side parallel to the bottom of the page on which they are printed. [Moved from K.12 EKS]</td>
</tr>
<tr>
<td>• A common misconception students have when a figure such as a square is rotated is they will frequently refer to the rotated square as a diamond. Clarification needs to be ongoing — e.g., a square is a square regardless of its location in space; there is no plane figure called a diamond.</td>
</tr>
<tr>
<td>• A plane geometric figure is any closed, two-dimensional shape that can be drawn in a plane, closed figure. Circles and polygons are examples of plane geometric figures.</td>
</tr>
<tr>
<td>• A vertex is the point at which two or more lines, line segments, or rays meet to form an angle. The term “vertices” is the plural form of “vertex.”</td>
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<td>All students should</td>
</tr>
<tr>
<td>• Use their knowledge of plane figures to help them systematically represent and describe their world.</td>
</tr>
<tr>
<td>• Identify the characteristics of plane geometric figures (circle, triangle, square, and rectangle).</td>
</tr>
<tr>
<td>• Compare the size and shape of plane geometric figures by using strategies to sort and/or group and begin to refine the vocabulary used to explain their strategies.</td>
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<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
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<tr>
<td>• Identify a circle, triangle, square, and rectangle. (a)</td>
</tr>
<tr>
<td>• Describe the characteristics of triangles, squares, and rectangles, including number of sides and number of vertices. (a)</td>
</tr>
<tr>
<td>• Describe a circle using terms such as round and curved. (a)</td>
</tr>
<tr>
<td>• Trace a circle, triangle, square, and rectangle. [Included in 1.11]</td>
</tr>
<tr>
<td>• Compare and group plane geometric figures (circle, triangle, square, and rectangle) according to their relative sizes (larger, smaller). (b)</td>
</tr>
<tr>
<td>• Compare and group plane geometric figures (circle, triangle, square, and rectangle) according to their shapes. (b)</td>
</tr>
<tr>
<td>• Distinguish between examples and nonexamples of identified geometric plane figures (circle, triangle, square, and rectangle). (b)</td>
</tr>
<tr>
<td>• Identify pictorial representations of a circle, triangle, square, and rectangle, regardless of their position and orientation in space. (c) [Moved from K.12]</td>
</tr>
<tr>
<td>• Describe the location of one object relative to another, using the terms above, below, and next to. (c) [Moved from K.12]</td>
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K.1041 The student will
a) identify, and describe, and trace plane geometric figures (circle, triangle, square, and rectangle); and
b) compare the size (smaller, larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle); and
c) describe the location of one object relative to another (above, below, next to) and identify representations of plane figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.[Moved from K.12]

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<tr>
<td>- Presentation of triangles, rectangles, and squares should be made in a variety of spatial orientations so that students do not develop the common misconception that triangles, rectangles, and squares must have one side parallel to the bottom of the page on which they are printed.</td>
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<td>- The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.</td>
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<td>- <strong>Level 0: Pre-recognition.</strong> Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.</td>
<td></td>
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<tr>
<td>- <strong>Level 1: Visualization.</strong> Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., “I know it’s a rectangle because it looks like a door, and I know that a door is a rectangle.”)</td>
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<tr>
<td>- <strong>Level 2: Analysis.</strong> Properties are perceived, but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., “I know it’s a rectangle because it is closed; it has four sides and four right angles.”).</td>
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<tr>
<td>- A polygon is a closed plane figure composed of at least three line segments that do not cross geometric figure that</td>
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<tr>
<td>- has sides that are line segments;</td>
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<tr>
<td>- is simple (its sides do not cross);</td>
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<tr>
<td>- is closed; and</td>
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<tr>
<td>- is two-dimensional (lies in a plane).</td>
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The student will

a) identify, and describe, and trace plane geometric figures (circle, triangle, square, and rectangle); and

b) compare the size (smaller, larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle); and

c) describe the location of one object relative to another (above, below, next to) and identify representations of plane figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.

### UNDERSTANDING THE STANDARD

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<tr>
<td>• A triangle is a polygon with three angles and three sides.</td>
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</tr>
<tr>
<td>• Children should be shown to have experiences with different types of triangles (e.g., such as equilateral, isosceles, scalene, right, acute, and obtuse); however, at this level, they are not expected to name the various types.</td>
<td></td>
</tr>
<tr>
<td>• A quadrilateral is a polygon with four sides.</td>
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</tr>
<tr>
<td>• A rectangle is a quadrilateral with four right angles.</td>
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</tr>
<tr>
<td>• A square is a quadrilateral with all four congruent (equal length) sides and four right angles of equal length. At this level, students might describe a square as a special rectangle with four sides of equal length.</td>
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</tr>
<tr>
<td>• Students, at this level, do not need to use the terms polygon, quadrilateral, or congruent.</td>
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</tr>
<tr>
<td>• A circle is the set of closed curve with all points in a plane that are the same distance from a fixed point called the center. A circle is not a polygon because it does not have straight sides.</td>
<td></td>
</tr>
<tr>
<td>• Representations of circles, squares, rectangles, and triangles can be found in the students’ environment at school and at home. Students should have opportunities to identify/classify things in their environment by the type of figures those things represent. Early experiences with comparing and sorting figures assist students in analyzing the characteristics of plane geometric figures. Attribute blocks, relational attribute blocks, and tangrams are among the manipulatives that are particularly appropriate for sorting and comparing size.</td>
<td></td>
</tr>
</tbody>
</table>
The student will
a) identify, **and** describe, and trace plane geometric figures (circle, triangle, square, and rectangle); and
b) compare the size (**smaller**, **larger**, **smaller**) and shape of plane geometric figures (circle, triangle, square, and rectangle); and
c) describe the location of one object relative to another (above, below, next to) and identify representations of plane figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.

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<tr>
<td>Clay, straws, and paper and scissors are several manipulatives that are appropriate for constructing geometric figures.</td>
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The student will describe the location of one object relative to another (above, below, next to) and identify representations of plane geometric figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space. [Moved to K.10c]

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<tr>
<td>• Representations of circles, squares, rectangles, and triangles can be found in the students’ environment at school and at home. Students should have opportunities to identify/classify things in their environment by the type of figures those things represent.</td>
<td>• Understand that objects can have different orientations in space.</td>
<td>• Identify pictorial representations of a circle, triangle, square, and rectangle, regardless of their position and orientation in space. [Moved to K.10c]</td>
</tr>
<tr>
<td>• Children are often confused when a figure such as a square is rotated; they frequently refer to the rotated square as a diamond. Clarification needs to be ongoing — i.e., a square is a square regardless of its location in space; there is no such geometric figure as a diamond.</td>
<td></td>
<td>• Describe the location of one object relative to another, using the terms above, below, and next to. [Moved to K.10c]</td>
</tr>
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</table>
Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible*, *unlikely*, *equally likely*, *more likely*, and *certain*.

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.
K.1113 The student will gather data by counting and tallying. [Tallying included in 1.14a] 
a) collect, organize, and represent data; and  
b) read and interpret data in object graphs, picture graphs, and tables. [Moved from K.14]

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<td>• Data are pieces of information collected about people or things. The primary purpose of collecting data is to answer questions. The primary purpose of interpreting data is to inform decisions (e.g., which type of clothing to pack for a vacation based on a weather graph or which type of lunch to serve based upon class favorites).</td>
</tr>
<tr>
<td>• Methods for organizing data could include five or ten frames, surveys, checklists, or various methods of grouping concrete materials.</td>
</tr>
<tr>
<td>• At this level, data gathered and displayed by students should be limited to 16 or fewer data points for no more than 4 categories.</td>
</tr>
<tr>
<td>• Students should have opportunities to interpret graphs, created with the assistance of the teacher, that contain data points where their entire class is represented (e.g., tables that show who brought their lunch and who will buy their lunch for any given day, picture graph showing how students traveled to school – bus, car, walk).</td>
</tr>
<tr>
<td>• A key should be included where appropriate.</td>
</tr>
<tr>
<td>• Tallying is a method for gathering information. Tally marks are used to show how often something happens or occurs. Each tally mark represents one occurrence. Tally marks are clustered into groups of five, with four vertical marks representing the first four occurrences and the fifth mark crossing the first four on a diagonal to represent the fifth occurrence.</td>
</tr>
<tr>
<td>• When data are presented in an organized manner, students can interpret and discuss the results and implications of their investigation (e.g., identifying parts of the data that have special characteristics, including categories with the greatest, the least, or the same number of responses).</td>
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</tbody>
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<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
</tr>
</thead>
<tbody>
<tr>
<td>All students should</td>
</tr>
<tr>
<td>• Pose questions and gather data.</td>
</tr>
<tr>
<td>• Understand how data are collected and presented in an organized manner by counting and tallying.</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Use counting and tallying to gather data on categories identified by the teacher and/or student (e.g., favorites, number of siblings, days of various types of weather during a given month, types/numbers of pets, types of shoes, flowers in the garden). Data points, collected by students, should be limited to 16 or fewer for no more than four categories. (a)</td>
</tr>
<tr>
<td>• Represent data by arranging concrete objects into organized groups to form a simple object graph. (a)</td>
</tr>
<tr>
<td>• Represent gathered data, using pictures to form a simple picture graph (e.g., a picture graph of the weather for a month). (a)</td>
</tr>
<tr>
<td>• Represent gathered data in tables (vertically or horizontally). (a)</td>
</tr>
<tr>
<td>• Answer questions related to the gathered data displayed in object graphs, picture graphs, and tables:</td>
</tr>
<tr>
<td>– Read the graph to determine the categories of data and the data as a whole (e.g., the total number of responses) and its parts (e.g., five people are wearing sneakers); and</td>
</tr>
<tr>
<td>– Interpret the data that represents numerical relationships, including categories with the greatest, the least, or the same. (b) [Moved from K.14]</td>
</tr>
</tbody>
</table>
The student will gather data by counting and tallying. [Tallying included in 1.14a]

a) collect, organize, and represent data; and
b) read and interpret data in object graphs, picture graphs, and tables. [Moved from K.14]

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- In the process of collecting gathering data, students make decisions about what is relevant to their investigation (e.g., when collecting data on their classmates’ favorite pets, deciding to limit the categories to common pets).

- When students begin to collect data, they recognize the need to categorize, which helps develop the understanding of “things that go together.” Categorical data are used when constructing picture graphs and bar graphs.

- Different types of representations emphasize different things about the same data. [Moved from K.14 US]

- Object graphs are graphs that use concrete materials to represent the categorical data that are collected (e.g., cubes stacked by the month, with one cube representing the birthday month of each student). [Moved from K.14 US]

- Picture graphs are graphs that use pictures to represent and compare information. [Moved from K.14 US] At this level, each picture should represent 1 data point.

- Tables are an orderly arrangement of data in which the data are arranged in columns and rows in an essentially rectangular format. Tables may be used to display numerical relationships or to organize lists. [Moved from K.14 US]

- Students represent data to convey results of their investigations at a glance, using concrete objects, pictures, and numbers to give a “picture” of the organized data. [Moved from K.14 Us]

- Graphs can be used to make connections between mathematics and science or social studies (e.g., types of plants found in the school yard, how students get to school). [Moved from K.14 US]
The student will gather data by counting and tallying.\(^{[}\text{Tallying included in 1.14a]}\)

a) collect, organize, and represent data; and
b) read and interpret data in object graphs, picture graphs, and tables.\(^{[}\text{Moved from K.14]}\)

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<tbody>
<tr>
<td>Students should have experiences answering questions related to the analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses).(^{[}\text{Moved from K.14 US]})</td>
<td></td>
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</table>
K.14 The student will display gathered data in object graphs, picture graphs, and tables, and will answer questions related to the data. [Moved to K.11b]

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<tr>
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<tbody>
<tr>
<td>All students should</td>
<td>The student will use problem-solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
<td></td>
</tr>
<tr>
<td>• Understand that data can be represented using concrete objects, pictures, and graphs.</td>
<td>• Display data by arranging concrete objects into organized groups to form a simple object graph.</td>
<td></td>
</tr>
<tr>
<td>• Understand that different types of representations emphasize different things about the same data.</td>
<td>• Display gathered data, using pictures to form a simple picture graph (e.g., a picture graph of the types of shoes worn by students on a given day).</td>
<td></td>
</tr>
<tr>
<td>• Understand that picture graphs use pictures to show and compare information; object graphs use concrete materials to represent categorical data; and tables can be used to show an orderly arrangement of data in columns and rows.</td>
<td>• Display gathered data in tables, either in rows or columns.</td>
<td></td>
</tr>
<tr>
<td>• Answer questions related to the gathered data displayed in object graphs, picture graphs, and tables by:</td>
<td>• Answer questions related to the gathered data displayed in object graphs, picture graphs, and tables by:</td>
<td></td>
</tr>
<tr>
<td>– Describing the categories of data and the data as a whole (e.g., the total number of responses) and its parts.</td>
<td>– Describing the categories of data and the data as a whole (e.g., the total number of responses) and its parts.</td>
<td></td>
</tr>
<tr>
<td>– Identifying parts of the data that represent numerical relationships, including categories with the greatest, the least, or the same.</td>
<td>– Identifying parts of the data that represent numerical relationships, including categories with the greatest, the least, or the same.</td>
<td></td>
</tr>
</tbody>
</table>

Object graphs are graphs that use concrete materials to represent the categorical data that are collected (e.g., cubes stacked by the month, with one cube representing the birthday month of each student).

Picture graphs are graphs that use pictures to show and compare information.

Tables are an orderly arrangement of data in which the data are arranged in columns and rows in an essentially rectangular format. Tables may be used to display some type of numerical relationship or organized lists (e.g., input/output functions, tables showing one candy costs five cents and two candies cost ten cents).

Students represent data to convey results of their investigations at a glance, using concrete objects, pictures, and numbers to give a “picture” of the organized data.

When data are displayed in an organized manner, children can describe the results of their investigations.

Graphs can be used to make connections between mathematics and social studies and/or science (e.g., job areas and the different people that work in these areas: health—doctors and nurses; education—teachers and principals).

Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be asked. [Moved to K.11b]
Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children’s mathematical development.

Students in kindergarten through third grades K-2 develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns. Students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to algebra.
K.1215  The student will sort and classify objects according to one attribute.

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<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>- Objects can be sorted and classified in different ways. [Moved from middle column].</td>
<td>- Understand that the same set of objects can be sorted and classified in different ways. [Moved to US]</td>
<td>- Identify the attributes of an object such as color, size, shape, thickness, etc. [Moved from US]</td>
</tr>
<tr>
<td>- To classify is to arrange or organize a set of materials according to a category or attribute (a quality or characteristic). An object has many attributes such as color, size, shape, thickness, etc. [Moved to EKS]</td>
<td></td>
<td>- Sort objects into appropriate groups (categories) based on one attribute (e.g., large bears and small bears).</td>
</tr>
<tr>
<td>- General similarities and differences among objects are easily observed by children entering kindergarten, who are able to focus on any one attribute. The teacher’s task is to move students toward a more sophisticated understanding of classification in which two or more attributes connect or differentiate sets, such as those found in nature (e.g., leaves having both different colors and different figures).</td>
<td></td>
<td>- Classify sets of objects into groups (categories) of one attribute.</td>
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<tr>
<td></td>
<td></td>
<td>- Label attributes of a set of objects that has been sorted.</td>
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<tr>
<td></td>
<td></td>
<td>- Name multiple ways to sort a set of objects.</td>
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</table>
### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Patterning recognition is a fundamental cornerstone of mathematics, particularly algebra. The process of generalization leads to the foundation of algebraic reasoning.
- Opportunities to create, identify, describe, extend, and transfer repeating patterns are essential to the primary school experience and lay the foundation for thinking algebraically.
- Patterning should include:
  - creating a given pattern, using objects, sounds, movements, and pictures;
  - recording a pattern with pictures or symbols;
  - transferring a pattern into a different representation (e.g., the pattern snap, snap, clap changed to a blue, blue, red pattern, or changed to an AAB repeating pattern); and
  - analyzing patterns in practical situations (e.g., calendar, seasons, days of the week).
- The part of the pattern that repeats is called the core.
- At this level students should have experiences extending patterns when given a complete repetition of a core (e.g., ABCABCABC) as well as when the final repetition of the core is incomplete (e.g., ABCABCA... or Red, Blue, Green, Red, Blue, Green, Red, Blue...).
- In a repeating pattern, the core of the pattern is the string of elements that repeats. By identifying the core, students demonstrate their understanding of the pattern.
- Students should recognize that the sound pattern ‘snap, clap, snap, clap’ is the same in form as the color pattern ‘red, blue, red, blue’.
- Pattern recognition and the extension of the pattern allow students to make predictions.
- The simplest types of patterns are repeating patterns. The patterns can be oral, such as the refrain in “Old MacDonald’s Farm” (“E-i-e-i-o”), or physical with clapping and snapping.

### ESSENTIAL UNDERSTANDINGS

All students should

- Understand that patterns are a way to recognize order and organize their world and to predict what comes next in an arrangement.
- Understand that the sound pattern ‘snap, clap, snap, clap’ is the same in form as the color pattern ‘red, blue, red, blue’.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Observe and identify and describe the basic repeating pattern (core) (the part of the sequence that repeats) found in repeating patterns of common objects, sounds, and movements, and pictures that occur in practical situations.
- Identify the core in a repeating pattern.
- Extend a repeating pattern by adding at least two complete repetitions of the core to the pattern.
- Create a repeating pattern.
- Compare similarities and differences between patterns.
- Transfer a repeating pattern from one representation to another.
K.1316 The student will identify, describe, and extend, create, and transfer repeating patterns.

<table>
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<tr>
<td>patterns, or combinations of both, such as is found in songs like the “Hokey Pokey.” In each case, students need to identify the basic unit of the pattern and repeat it. Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience.</td>
<td></td>
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<tr>
<td><strong>Examples of repeating patterns:</strong> (repeating the core) are</td>
<td></td>
<td></td>
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<tr>
<td>– ABABABAB;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– ABCABC;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– ABBAABBA;</td>
<td></td>
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<tr>
<td>– AABBAABBAABB; and</td>
<td></td>
<td></td>
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<tr>
<td>– AABAABAB;</td>
<td></td>
<td></td>
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<tr>
<td>– AABCAABC; and</td>
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<td></td>
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<tr>
<td>– ABACABAC.</td>
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<tr>
<td><strong>Examples of growing patterns include:</strong></td>
<td></td>
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<td>–</td>
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<tr>
<td>1 2 3 4</td>
<td></td>
<td></td>
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<tr>
<td>– 10, 20, 30, 40, 50...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– ★△★★ ★★★★△</td>
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Introduction

The 2009-2016 Mathematics Standards of Learning Curriculum Framework, is a companion document to the 2009-2016 Mathematics Standards of Learning, and amplifies the Mathematics Standards of Learning by further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. It provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

**Essential Understandings**

This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

**Understanding the Standard**

This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. Background information for the teacher (K-8). It contains content. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers’ knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

**Essential Knowledge and Skills**

This section provides a detailed expansion of the mathematics knowledge and skills that What each student should know and be able to demonstrate in each standard is outlined. This is not meant to be an exhaustive list of student expectations or a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise. [Reordered and rewritten]
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student’s problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
Students in grades K–3 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, fractions, number relationships (“more than,” “less than,” and “equal to”), and the effects of single-step and multistep computations. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.
1.1 The student will

- **a)** count forward orally, by ones, to 110 from 0 to 100, starting at any number between 0 and 110, and write the corresponding numerals; [Reordered]

- **b)** write the numerals 0 to 110 in sequence and out-of-sequence group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value. [Moved to 1.2]

- **c)** count backward orally by ones when given any number between 1 and 30; [Moved from 1.2]

- **d)** count forward orally by ones, twos, fives, and tens to determine the total number of objects to 110. [Moved from 1.2]

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<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• There are three developmental levels of counting:</td>
<td>- Associate oral number names with the correct numeral and set of objects.</td>
<td>• Count by rote forward orally, by ones, from 0 to 110 starting at any number between 0 and 110, 100, using the correct name for each numeral: (a)</td>
</tr>
<tr>
<td>- rote sequence;</td>
<td>- Understand that 1 and 10 are special units of numbers (e.g., 10 is 10 ones, but it is also 1 ten).</td>
<td>- Use the correct oral counting sequence to tell how many objects are in a set. (a, c)</td>
</tr>
<tr>
<td>- one-to-one correspondence; and</td>
<td>- Understand the ten-to-one relationship of ones and tens (10 ones equals 1 ten).</td>
<td>- Write numerals 0-110 correctly in sequence and out of sequence. (a, b)</td>
</tr>
<tr>
<td>- the cardinality of numbers.</td>
<td>- Understand that numbers are written to show how many tens and how many ones are in the number.</td>
<td>- Write each numeral from 0 to 100.</td>
</tr>
<tr>
<td>• Counting involves two separate skills: verbalizing the list (rote sequence counting) of standard number words in order (“one, two, three, …”) and connecting this sequence with the items in the set being counted, using one-to-one correspondence. Association of number words with collections of objects is achieved by moving, touching, or pointing to objects as the number words are spoken.</td>
<td>- Understand that groups of tens and ones can be used to tell how many.</td>
<td>- Read two-digit numbers when shown a numeral, a Base 10 model of the number, or a pictorial representation of the number.</td>
</tr>
<tr>
<td>• The last number stated represents the number of objects in the set. This is the total number of objects in the set is known as the cardinality of the set. After having a student count a collection of objects, the teacher may be able to assess whether the student has cardinality of number by asking the question, “How many are there?” Students who do not yet have cardinality of number are often unable to tell you how many objects there were without recounting them.</td>
<td></td>
<td>- Represent a given two-digit number with a model or picture. (b)</td>
</tr>
<tr>
<td>• Rote counting is a prerequisite skill for the understanding of addition (1 more), subtraction (1 less), and the ten-to-one concept of place value.</td>
<td></td>
<td>- Given a model or a pictorial representation, write the corresponding numeral. (a, b, d)</td>
</tr>
<tr>
<td>• Conservation of number is applied when students understand that a group of ten objects is still ten objects regardless of whether they are arranged in a cup, group, stack, etc.</td>
<td></td>
<td>- Count backward orally, by ones, when given any number between 1 and 30. (b) [Moved from 1.2]</td>
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<tr>
<td></td>
<td></td>
<td>- Count forward orally by ones, twos, fives, and tens to determine the total number of objects to 110. (c, d) [Moved from 1.2]</td>
</tr>
</tbody>
</table>
1.1 The student will
a) count forward orally, by ones, to 110 from 0 to 100, starting at any number between 0 and 110, and write the corresponding numerals; and
b) write the numerals 0 to 110 in sequence and out-of-sequence a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.

c) count backward orally by ones when given any number between 1 and 30;

d) count forward orally by ones, twos, fives, and tens to determine the total number of objects to 110.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Unitizing is the concept that a group of objects can be counted as one unit (e.g., 10 ones can be counted as 1 ten.)
- Articulating the characteristics of each numeral when writing numbers has been found to reduce the amount of time it takes to learn to write numerals.
- Using objects and asking the questions, “How many are in each group?”, “How many groups are there?” and “What is the total number you have?” as students learn to skip count, supports students as they solidify their understanding of cardinality and develop multiplicative reasoning.
- The patterns developed as a result of skip counting are precursors for recognizing numeric patterns, functional relationships, and concepts underlying money, time, telling, and multiplication. Powerful models for developing these concepts include counters, number paths, and hundred charts.

Example of a Number Path

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

- A number path is a counting model where each number is represented within a square and the squares can be clearly counted.
- A number line is a length model with each number being represented by its length from zero. When young children use a number line as a counting tool, they often confuse what should be counted, the numbers or the spaces.

ESSENTIAL UNDERSTANDINGS

- Identify the place value (ones, tens) of each digit in a two-digit numeral (e.g., The place value of the 2 in the number 23 is tens. The value of the 2 in the number 23 is 20).
- Group a collection of objects into sets of tens and ones. Write the numeral that corresponds to the total number of objects in a given collection of objects that have been grouped into sets of tens and ones.

ESSENTIAL KNOWLEDGE AND SKILLS
1.1 The student will

a) **count forward orally by ones, to 110 from 0 to 100, starting at any number between 0 and 110, and write the corresponding numerals.** [Reordered] and

b) **write the numerals 0 to 110 in sequence and out-of-sequence, group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.** [Moved to 1.2a] represent a number from 0-110 using concrete objects;

c) **count backward orally by ones when given any number between 1 and 30;** [Moved from 1.2] and

d) **count forward orally by ones, twos, fives, and tens to determine the total number of objects to 110.** [Moved from 1.2]

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
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<tbody>
<tr>
<td>between the numbers. A number path is more appropriate for students at this age.</td>
</tr>
</tbody>
</table>

- Counting backward by rote lays the foundation for subtraction.
- Skip counting by twos supports the development of the concept of even numbers and the development of multiplication facts for two.
- Skip counting by fives lays the foundation for telling time to the nearest five minutes, counting money, and developing the multiplication facts for five.
- Skip counting by tens is a precursor for place value, addition (10 more), subtraction (10 less), counting money, and multiplying by multiples of 10.
- Manipulatives that can be physically connected and separated into groups of tens and leftover ones, such as connecting or snap cubes, beans on craft sticks, pennies in cups, bundle of sticks, or beads on pipe cleaners should be used.
- Ten-to-one trading activities with manipulatives on place value mats and base ten blocks are more appropriate in grade 2.
- The number system is based on a pattern of tens where each place has ten times the value of the place to its right. This is known as the ten-to-one concept of place value.
- Opportunities to experience the relationships among tens and ones through hands-on experiences with manipulatives are essential to developing the ten-to-one place value concept of our number system and to understanding the value of each digit in a
1.1 The student will
   a) count **forward orally, by ones, to 110 from 0 to 100**, starting at any number between 0 and 110, and write the corresponding numerals; and
   b) write the numerals 0 to 110 in sequence and out-of-sequence a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value. [Moved to 1.2a] represent a number from 0-110 using concrete objects;
   c) count backward orally by ones when given any number between 1 and 30; [Moved from 1.2] and
   d) count forward orally by ones, twos, fives, and tens to determine the total number of objects to 110. [Moved from 1.2]

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>two-digit number. Ten-to-one trading activities with manipulatives on place value mats provide excellent experiences for developing the understanding of the places in the Base-10 system. Models that clearly illustrate the relationships among tens and ones are physically proportional (e.g., the tens piece is ten times larger than the ones piece). Providing students with opportunities to model two-digit numbers expressed with groups of ones and tens will help students understand the ideas of trading, regrouping, and equality. Recording the numeral when using physical and pictorial models leads to an understanding that the position of each digit in a numeral determines the size of the group it represents. [Moved to 1.2]</td>
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</tbody>
</table>

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 1

5
1.2 The student, given up to 110 objects, will count forward by ones, twos, fives, and tens to 100 and backward by ones from 30. a) group a collection into tens and ones and write the corresponding numeral; b) compare two numbers between 0 and 110 represented pictorially or with concrete objects, using the words greater than, less than or equal to; and c) order three or fewer sets from least to greatest and greatest to least.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- The patterns developed as a result of skip counting are precursors for recognizing numeric patterns, functional relationships, and concepts underlying money, time telling, and multiplication. Powerful models for developing these concepts include counters, number line, hundred chart, and calculators.
- Skip counting by twos supports the development of the concept of even numbers.
- Skip counting by fives lays the foundation for reading a clock effectively and telling time to the nearest five minutes, counting money, and developing the multiplication facts for five.
- Skip counting by tens is a precursor for use of place value, addition, counting money, and multiplying by multiples of 10.
- Counting backward by rote lays the foundation for subtraction. Students should count backward beginning with 30, 29, 28,... through ..., 3, 2, 1, 0. Conservation of number is applied when students understand that a group of ten objects is still ten objects regardless of whether they are arranged in a cup, group, stack, etc.
- Unitizing is the concept that a group of objects can be counted as one unit (e.g., 10 ones can be counted as 1 ten.)
- The number system is based on a pattern of tens where each place has ten times the value of the place to its right. This is known as the ten-to-one concept of place value.

ESSENTIAL UNDERSTANDINGS

All students should
- Understand that collections of objects can be grouped and skip counting can be used to count the collection.
- Describe patterns in counting by ones (both forward and backward) andskip counting and use those patterns to predict the next number in the counting sequence.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
- Count by ones, twos, fives, and tens to 100, using concrete objects, such as counters, connecting cubes, pennies, nickels, and dimes.
- Demonstrate a one to one correspondence when counting by ones with concrete objects or representations.
- Skip count orally by twos, fives and tens to 100 starting at various multiples of 2, 5, or 10.
- Count backward by ones from 30.
- Group a collection of objects into sets of tens and ones. (a)
- Write the numeral that corresponds to the total number of objects in a given collection of objects that have been grouped into sets of tens and ones. (a)
- Identify the place and value of each digit in a two-digit numeral (e.g., in the number 23, the 2 is in the tens place and the value of the 2 is 20). (a)
- Knows Identifies the number of tens and ones that can be made from any number up to 100 (e.g., 47 is 47 ones or can also be grouped into 4 tens with 7 ones left over). (a)
- Compare two numbers between 0 and 110 represented pictorially or with concrete objects, using the words greater than, less than or equal to. (b)
- Order three or fewer sets, each set containing up to 110 objects, from least to greatest and greatest to least. (c)
1.2 The student, given up to 110 objects, will count forward by ones, twos, fives, and tens to 100 and backward by ones from 30.\[Moved to 1.1\]

a) group a collection into tens and ones and write the corresponding numeral;\[Moved from 1.1b\]

b) compare two numbers between 0 and 110 represented pictorially or with concrete objects, using the words greater than, less than or equal to; and

c) order three or fewer sets from least to greatest and greatest to least.

UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Opportunities to experience the relationships among tens and ones through hands-on experiences with manipulatives are essential to developing the ten-to-one place value concept of our number system and to understanding the value of each digit in a two-digit number.

- Models that clearly illustrate the relationships among tens and ones as physically proportional are most appropriate for this grade (e.g., the tens piece is ten times larger than the ones piece).

- Knowledge of place value is essential when comparing numbers.

- Students are generally familiar with the concept of more, and have limited experience with the term less. It is important to use the terms together to build understanding of their relationship. For example, when asking which group has more, follow by asking which group has less.

- Recording the numeral when using physical and pictorial models leads to an understanding that the position of each digit in a numeral determines the size of the group it represents.

- Manipulatives that can be physically connected and separated into groups of tens and leftover ones, such as connecting or snap cubes, beans on craft sticks, pennies in cups, bundle of sticks, or beads on pipe cleaners should be used.\[Reordered\]

- Ten-to-one trading activities with manipulatives on place value mats and Base-10 blocks are more appropriate in grade 2.

- Calculators can be used to reinforce skip counting.
STANDARD 1.3 STRAND: NUMBER AND NUMBER SENSE GRADE LEVEL 1

1.3 The student, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth, and the ordered position of each object. [Moved from K.3]

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- An understanding of the cardinal and ordinal meanings of numbers is necessary to quantify, measure, and identify the order of objects. [Moved from K.3]

- An ordinal number is a number that names the place or position of an object in a sequence or set (e.g., first, third). Ordered position, ordinal position, and ordinality are terms that refer to the place or position of an object in a sequence or set. [Moved from K.3]

- The ordinal position is determined by where one starts in an ordered set of objects or sequence of objects. [Moved from K.3]

- The ordinal meaning of numbers is developed by identifying and verbalizing the place or position of objects in a set or sequence (e.g., the student’s position in line when students are lined up alphabetically by first name). [Moved from K.3]

- Students at this level do not need to recognize or read the written words for ordinal numbers (i.e., first, second, third, etc.). [Moved from K.3]

- Practical applications of ordinal numbers can be experienced through calendar and patterning activities.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify the ordinal positions first through tenth using ordered sets of ten concrete objects and/or pictures of such sets presented from
  - left-to-right;
  - right-to-left;
  - top-to-bottom; and/or
  - bottom-to-top. [Moved from K.3]
The student will identify the parts of a set and/or region that represent fractions for halves, thirds, and fourths and write the fractions.

a) represent and solve practical problems involving investigate fractions by solving equal sharing problems with two, or four, or eight sharers; and

b) verbally identify the parts of a region that represent and name fractions for halves, and fourths, and eighths, using models.

### UNDERSTANDING THE STANDARD

**Background Information for Instructor Use Only**

- Practical situations with fractions should involve real life problems in which students themselves are determining how to subdivide a whole into equal parts, test those parts to be sure they are equal, and use those parts to recreate the whole.

- When working with fractions, the whole must be defined.

- Fractions can have different meanings: part-whole, division, measurement, ratio, and operator. The focus of this grade level is to develop the idea of equal sharing (division) and part-whole relationships. Fraction notation will be introduced in second grade.

- An equal sharing problem is an idea that young children understand intuitively because of their experiences sharing objects with siblings, friends, etc. Consider the following examples:
  - Two children sharing six sandwiches
  - Two children sharing one sandwich.
  - Four children sharing one piece of paper.
  - Four children sharing two sticks of clay.
  - Four children sharing five brownies.
  - Eight children sharing one chocolate bar.
  - Eight children sharing five sandwiches.

- Fraction models that can be continuously divided should be used at this grade (e.g., cookies, brownies). It is important to use models that can be continuously divided when there are remainders so those remainders can be cut into as many equal parts as needed.

- Students should be encouraged to observe and state what

### ESSENTIAL UNDERSTANDINGS

All students should

- Understand that a fraction represents a part of a whole.

- Understand that fractional parts are equal shares of a whole.

- Understand that the fraction name (half, third, fourth) tells the number of equal parts in the whole.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Given a practical situation, share a whole equally with two or four or eight sharers. (a)

- Given a practical situation, represent fair shares pictorially. (a)

- Given a practical situation, describe shares as equal pieces or parts of the whole (e.g., halves, fourths). (a)

- Represent a whole to show it having two equal parts and identify one-half (\(\frac{1}{2}\)), and two halves (\(\frac{2}{2}\)).

- Represent a whole to show it having three equal parts and identify one-third (\(\frac{1}{3}\)), two-thirds (\(\frac{2}{3}\)) and three-thirds (\(\frac{3}{3}\)).

- Represent a whole to show it having four equal parts and identify one-fourth (\(\frac{1}{4}\)), two-fourths (\(\frac{2}{4}\)), three-fourths (\(\frac{3}{4}\)) and four-fourths (\(\frac{4}{4}\)).

- Identify and model halves, thirds, and fourths. Represent halves and fourths, and eighths of a whole, using the set model (e.g., connecting cubes and counters), and a region/area models (e.g., pie pieces, pattern blocks, geoboards, paper folding, and drawings). (a)
The student will identify the parts of a set and/or region that represent fractions for halves, thirds, and fourths and write the fractions.

a) represent and solve practical problems involving fractions by solving equal sharing problems with two, or four, or eight sharers; and

b) verbally identify the parts of a region that represent and name fractions for halves, and fourths, and eighths, using models.
1.43 The student will identify the parts of a set and/or region that represent fractions for halves, thirds, and fourths and write the fractions.

a) represent and solve practical problems involving fractions by solving equal sharing problems with two, or four, or eight sharers; and

b) verbally identify the parts of a region that represent fractions for halves, and fourths, and eighths, using models.

**UNDERSTANDING THE STANDARD**

(Background Information for Instructor Use Only)

<table>
<thead>
<tr>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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<tbody>
<tr>
<td>understanding of a fraction</td>
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<tr>
<td>• At this level, students are expected to first understand the part-whole relationship (e.g., three out of four equal parts) before being expected to recognize or use symbolic representations for fractions (e.g., $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$).</td>
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<tr>
<td>• Students should have opportunities to make connections and comparisons among fraction representations by connecting concrete or pictorial representations with spoken representations (e.g., “one-half,” “one out of two equal parts,” “or “two-thirds,” “two out of three equal parts,” and “one-half is more than one-fourth of the same whole”).</td>
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<tr>
<td>• Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions (e.g., thirds means “three equal parts of a whole”) furthers this development.</td>
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</table>
A variety of contexts and problem types are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from practical experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as “How many books do we have altogether?” or “How many cookies are left if I eat two?” or “I have three more candies than you do.” Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling, discussing, and recording a variety of problem situations. This approach helps students transition from the concrete to the representation to the symbolic in order to develop meaning for the operations and how they relate to each other.

In grades K–3, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition, subtraction, multiplication, division and related facts;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as skip counting and solving problems that involve equal groupings of objects and as well as problems that involve sharing equally, the initial work; and performing initial operations with fractions.
1.54 The student, given a familiar problem situation involving magnitude, will
a) select a reasonable order of magnitude from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, 500); and
b) explain the reasonableness of the choice.

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<tbody>
<tr>
<td>Magnitude refers to the size of a set.</td>
<td>All students should develop an understanding of the order of magnitude (size) of whole numbers and use this knowledge to estimate quantities.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
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<tr>
<td>Exploring ways to estimate the number of objects in a set, based on appearance (e.g., clustering, grouping, comparing), enhances the development of number sense.</td>
<td></td>
<td>select a reasonable order of magnitude for a given set from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, and 500 jelly beans in jars) in a familiar problem situation. (a)</td>
</tr>
<tr>
<td>To estimate means to find determine a number that is close to the exact amount. When asking for an estimate, teachers might ask, “about how much?” or “about how many?” or “Is this about 10 or about 50?”</td>
<td></td>
<td>Given a familiar problem situation involving magnitude, explain why a particular estimate was chosen as the most reasonable from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral. (b)</td>
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<tr>
<td>Students should be provided opportunities to estimate a quantity, given a benchmark of 10 and/or 100 objects.</td>
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</table>
### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Associate the terms *addition*, *adding*, and *sum* with the concept of joining or combining.
- Associate the terms *subtraction*, *subtracting*, *minus*, and *difference* with the process of taking away or separating (i.e., removing a set of objects from the given set of objects, finding the difference between two numbers, or comparing two numbers).
- Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the basic addition and subtraction facts include counting back; “one more than,” “two more than” facts; “one less than,” “two less than” facts; “doubles” to recall addition facts (e.g., $2 + 2 = \_\_$; $3 + 3 = \_\_$); “near doubles” (e.g., $3 + 4 = (3 + 3) + 1 = \_\_$); “make ten” facts (e.g., at least one addend of 8 or 9); “think addition for subtraction” (e.g., for $9 - 5 = \_\_$, think “5 and what number makes 9?”); use of the commutative property, without naming the property (e.g., $4 + 3$ is the same as $3 + 4$); use of related facts (e.g., $4 + 3 = 7$, $3 + 4 = 7$, $7 - 4 = 3$, and $7 - 3 = 4$); use of the additive identity property (e.g., $4 + 0 = 4$), without naming the property but saying, “When you add zero to a number, you always get the original number.”; and use patterns to make sums (e.g., $0 + 5 = 5$, $1 + 4 = 5$, $2 + 3 = 5$, etc.).
- Manipulatives should be used to develop an understanding of addition and subtraction facts. Automaticity of facts can be achieved through constant practice which may include games, hands-on activities, flash cards, and paper and pencil.
- Students should first master facts to 10 and then master facts to 18.

### ESSENTIAL UNDERSTANDINGS

- All students should
  - Develop an understanding of the addition and subtraction relationship.
  - Develop addition and subtraction strategies for fact recall.
  - Develop fluency with basic number combinations for addition and subtraction.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify $+$ as a symbol for addition, $-$ as a symbol for subtraction, and $=$ as a symbol for equality.
- Recall and state orally the basic addition facts for sums with two addends to 18 or less and the corresponding subtraction facts.
- Recall and write the basic addition facts for sums to 18 or less and the corresponding subtraction facts, when addition or subtraction problems are presented in either horizontal or vertical written format.
**STANDARD 1.6**  
**STRAND: COMPUTATION AND ESTIMATION**  
**GRADE LEVEL 1**

1.6 The student will create and solve **single step** story and picture problems using **basic addition and subtraction within 20 facts** with sums to 18 20 or less and the corresponding subtraction facts.

| UNDERSTANDING THE STANDARD  
(Background Information for Instructor Use Only) | ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
<table>
<thead>
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</thead>
<tbody>
<tr>
<td><strong>• Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship.</strong></td>
<td>All students should understand various meanings of addition and subtraction in a variety of situations. Understand that creating and solving problems involves the use of addition and/or subtraction.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to <strong>• Interpret</strong> create and solve single-step oral or written story and picture problems involving one-step solutions, using basic addition and subtraction within 20 facts (sums to 18 20 or less and the corresponding subtraction facts). <strong>• Identify</strong> a correct number sentence to solve an oral or written story and picture problem, selecting from among basic addition and/or subtraction equations (e.g., number sentences) facts. <strong>• Combine</strong> parts contained in larger numbers up to 20 by using related combinations (e.g., 9 + 7 can be thought of as 9 broken up into 2 and 7; using doubles 7 + 7 = 14; 14 + 2 = 16 or 7 broken up into 1 and 6; making a ten 1 + 9 = 10; 10 + 6 = 16). <strong>• Explain</strong> strategies used to solve addition and subtraction problems within 20 using spoken words, objects, pictorial models, and number sentences.</td>
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<tr>
<td><strong>• The problem solving process is enhanced when students:</strong></td>
<td></td>
<td><strong>• Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the basic addition and subtraction facts include</strong></td>
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<tr>
<td>– create their own story problems; and</td>
<td>All students should understand various meanings of addition and subtraction in a variety of situations. Understand that creating and solving problems involves the use of addition and/or subtraction.</td>
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<td>– visualize the action in the story problem and draw a picture to show their thinking; and</td>
<td><strong>• Interpret</strong> create and solve single-step oral or written story and picture problems involving one-step solutions, using basic addition and subtraction within 20 facts (sums to 18 20 or less and the corresponding subtraction facts).</td>
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<tr>
<td>– model the word problems, using manipulatives, representations, or number sentences/equations.</td>
<td><strong>• Identify</strong> a correct number sentence to solve an oral or written story and picture problem, selecting from among basic addition and/or subtraction equations (e.g., number sentences) facts.</td>
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<tr>
<td><strong>• The least number of steps necessary to solve a single-step problem is one.</strong></td>
<td><strong>• Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the basic addition and subtraction facts include</strong></td>
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<td><strong>• In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as in all, altogether, difference, etc. encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions as well as challenges in upcoming grades and courses.</strong></td>
<td>– counting on;</td>
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</tr>
<tr>
<td><strong>• Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the basic addition and subtraction facts include</strong></td>
<td>– counting back;</td>
<td></td>
</tr>
<tr>
<td>– “one more than,” “two more than”;</td>
<td>– “one less than,” “two less than”;</td>
<td></td>
</tr>
<tr>
<td>– “doubles” (e.g., 6 + 6 = __);</td>
<td>– “near doubles” (e.g., 7 + 8 = (7 + 7) + 1 = or (8 + 8) - 1);</td>
<td></td>
</tr>
<tr>
<td>– “make ten” (e.g., 7 + 4 can be thought of as 7 + 3 + 1 in order to make a 10);</td>
<td>– “think addition for subtraction” (e.g., for 9 – 5 = __, think “5 and what number makes 9?”);</td>
<td></td>
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<tr>
<td>– “use of the commutative property” (e.g., 14 + 3 is the same as 3 + 14);</td>
<td>– use of related facts (e.g.,14 + 3 = 17, 3 + 14 = 17, 17 – 4 =</td>
<td></td>
</tr>
<tr>
<td>– use of related facts (e.g.,14 + 3 = 17, 3 + 14 = 17, 17 – 4 =</td>
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</tbody>
</table>
1.6 The student will create and solve one single-step story and picture problems using basic addition and subtraction within 20 facts with sums to 18 or less and the corresponding subtraction facts.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
</table>
| 13, and $17 - 13 = 4$;  
- use of the additive identity property (e.g., $14 + 0 = 14$); and  
- use patterns to make sums (e.g., $0 + 15 = 15$, $1 + 14 = 15$, $2 + 13 = 15$, etc.). | | |
| Students at this level are not expected to use the parentheses or to name the properties. | | |
| Students should develop fluency with facts to 10 and then utilize strategies and known facts to 10 to determine facts to 20. | | |
| Flexibility with facts to 10 should be applied to facts to 20 (e.g., when adding $4 + 7$, it is appropriate to think of $4$ as $3 + 1$ in order to combine $3$ and $7$ to make a 10 whereas adding $4 + 8$, it is appropriate to think of $4$ as $2 + 2$ in order to combine $8$ and $2$ to make a 10). Students should experience a variety of problem types related to addition and subtraction, including  
- join and separate problems (action involved): join (for example: Sam had 8 pennies. Tom gave him 3 more. How many pennies does Sam have now?) separate (for example: Sam had 11 pennies. He gave 3 to Tom. How many pennies does Sam have now?)  
- part-part-whole problems (no action involved): missing part (for example: There are 8 marbles. Five are shown. How many are missing?)  
- classification problems (for example: Jane had 12 hats. Only 3 of the hats are blue. How many are not blue?)  
- comparison problems (for example: Bill is 7 years old. Alice is 4 years old. How much younger is Alice than Bill?). | | |
| Extensive research has been undertaken over the last several decades regarding different problem types. Many of these studies have been published in professional mathematics education publications using different labels and terminology to describe the varied problem types. | | |
| Students should have exposure to a variety of problem types related to addition and subtraction. Examples are represented in | | |
1.6  The student will create and solve one single-step story and picture problems using basic addition and subtraction within 20 facts with sums to 18 or less and the corresponding subtraction facts.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

It is important to note that Join Problems (with start unknown), Separate Problems (with start unknown), Compare Problems (with larger unknown – using ‘fewer’) and Compare problems (with smaller unknown – using ‘more’) are the most difficult and should be mastered in grade 2.

#### LESSON: COMMON ADDITION AND SUBTRACTION PROBLEM TYPES

<table>
<thead>
<tr>
<th>Join</th>
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</thead>
<tbody>
<tr>
<td>(Result Unknown)</td>
<td>(Change Unknown)</td>
<td>(Start Unknown)</td>
</tr>
<tr>
<td>Sue had 9 pencils. Alex gave her 5 more pencils. How many pencils does Sue have all together?</td>
<td>Sue had 9 pencils. Alex gave her some more pencils. Now Sue has 14 pencils. How many did Alex give her?</td>
<td>Sue had some pencils. Alex gave her 5 more. Now Sue has 14 pencils. How many pencils did Sue have to start with?</td>
</tr>
<tr>
<td>Separate</td>
<td>Separate</td>
<td>Separate</td>
</tr>
<tr>
<td>(Result Unknown)</td>
<td>(Change Unknown)</td>
<td>(Start Unknown)</td>
</tr>
<tr>
<td>Brooke had 10 cookies. She gave 6 cookies to Joe. How many cookies does Brooke have now?</td>
<td>Brooke had 10 cookies. She gave some to Joe. She has 4 cookies left. How many cookies did Brooke give to Joe?</td>
<td>Brooke had some cookies. She gave 6 to Joe. Now she has 4 cookies left. How many cookies did Brooke start with?</td>
</tr>
<tr>
<td>Part-Part-Whole</td>
<td>Part-Part-Whole</td>
<td>Part-Part-Whole</td>
</tr>
<tr>
<td>(Whole Unknown)</td>
<td>(One Part Unknown)</td>
<td>(Both Parts Unknown)</td>
</tr>
<tr>
<td>Lisa has 4 red markers and 8 blue markers. How many markers does she have?</td>
<td>Lisa has 12 markers. Four of the markers are red and the rest are blue. How many blue markers does Lisa have?</td>
<td>Lisa has a pack of red and blue markers. She has 12 markers in all. How many markers could be red? How many could be blue?</td>
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<thead>
<tr>
<th>Compare</th>
<th>Compare</th>
<th>Compare</th>
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<tbody>
<tr>
<td>(Difference Unknown)</td>
<td>(Bigger Unknown)</td>
<td>(Smaller Unknown)</td>
</tr>
<tr>
<td>Ryan has 7 books and Chris has 2 books. How many more books does Ryan have than Chris?</td>
<td>Chris has 2 books. Ryan has 5 more books than Chris. How many books does Ryan have?</td>
<td>Ryan has 2 more books than Chris. Ryan has 7 books. How many books does Chris have?</td>
</tr>
<tr>
<td>Ryan has 7 books. Chris has 2 books. How many fewer books does Chris have than Ryan?</td>
<td>Chris has 5 fewer books than Ryan. Chris has 2 books. How many books does Ryan have?</td>
<td>Chris has 5 fewer books than Ryan. Ryan has 7 books. How many books does Chris have?</td>
</tr>
</tbody>
</table>
1.75 The student will recall basic
a) recognize and describe with fluency part-whole relationships for numbers up to 10; and
b) demonstrate fluency with addition and subtraction within facts with for sums to 10 or less and the corresponding subtraction facts.

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<tr>
<td>• Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently.</td>
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<tr>
<td>• Flexibility requires knowledge of more than one approach to solving a particular kind of problem. Being flexible allows students to choose an appropriate strategy for the numbers involved.</td>
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<tr>
<td>• Mathematically fluent students are not only able to provide correct answers quickly but also to use facts and computation strategies they know to efficiently determine answers that they do not know.</td>
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<tr>
<td>• Composing and decomposing numbers flexibly forms a basis for understanding properties of the operations and later formal algebraic concepts and procedures.</td>
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<tr>
<td>• Parts of numbers to 10 should be represented in a variety of different ways, such as five frames, ten frames, strings of beads, arrangements of tiles or toothpicks, dot cards, rekenreks or beaded number frames.</td>
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<tr>
<td>• Dot patterns should be presented in both regular and irregular arrangements. This will help students to understand that numbers are made up of parts, and will later assist them in combining parts as well as counting on.</td>
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<tr>
<td>All students should</td>
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<tr>
<td>• Develop an understanding of the addition and subtraction relationship.</td>
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<tr>
<td>• Develop addition and subtraction strategies for fact recall.</td>
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<tr>
<td>• Develop fluency with basic number combinations for addition and subtraction.</td>
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<tr>
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<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
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<tr>
<td>• Recognize and describe with fluency part-whole relationships for numbers up to 10 in a variety of configurations. (a)</td>
</tr>
<tr>
<td>• Identify + as a symbol for addition, – as a symbol for subtraction, and = as a symbol for equality. (b)</td>
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<tr>
<td>• Demonstrate fluency with recall and state orally the basic addition facts for sums with two addends to 18 or less and the corresponding subtraction facts, when addition or subtraction problems are presented in either horizontal or vertical written format.</td>
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</tbody>
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The student will recall basic mathematics and describe with fluency part-whole relationships for numbers up to 10; and demonstrate fluency with addition and subtraction within facts with for sums to 10 or less and the corresponding subtraction facts.

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<tr>
<td>• Accuracy is the ability to determine a correct answer using knowledge of number facts and other important number relationships.</td>
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<tr>
<td>• Efficiency is the ability to carry out a strategy easily when solving a problem without getting bogged down in too many steps or lose track of the logic of the strategy being used.</td>
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<tr>
<td>• Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship.</td>
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<tr>
<td>• Manipulatives should be used to develop an understanding of addition and subtraction facts.[Reordered]</td>
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<tr>
<td>• Automaticity of facts can be achieved through meaningful constant practice which may include games, hands-on activities, flash cards, and paper and pencil ten frames.[Reordered]</td>
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<tr>
<td>• Subtraction is the inverse of addition. Subtraction can be viewed as a process of taking away or separating, or as a process of comparing two sets to determine the difference between them.</td>
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<tr>
<td>• Associate the terms addition, adding, and sum with the concept of joining or combining.</td>
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<tr>
<td>• Associate the terms subtraction, subtracting, minus, and difference with the process of taking away or separating (i.e., removing a set of objects from the given set of objects, finding determining the difference between two numbers, or comparing two numbers to determine the difference).</td>
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<tr>
<td>• Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the basic addition and subtraction facts include – counting on;</td>
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1.75 The student will recall basic
a) recognize and describe with fluency part-whole relationships for numbers up to 10; and
b) demonstrate fluency with addition and subtraction within facts with sums to 10 or less and the corresponding subtraction facts.

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<td>- counting back;</td>
<td>- “one more than,” “two more than” facts;</td>
<td>- Students at this level are not expected to name the properties.</td>
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<td>- “one less than,” “two less than” facts;</td>
<td>- “doubles” to recall addition facts (e.g., 2 + 2 = __; 3 + 3 = __);</td>
<td>- Manipulatives should be used to develop an understanding of addition and subtraction facts. [Reordered]</td>
</tr>
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<td>- “near doubles” [e.g., 3 + 4 = (3 + 3) + 1 = __];</td>
<td>- “make ten” facts (7 + 4 can be thought of as 7 + 3 + 1 in order to make a ten e.g., at least one addend of 8 or 9);</td>
<td>- Automaticity of facts can be achieved through constant practice which may include games, hands-on activities, flash cards, and paper and pencil. [Reordered]</td>
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<td>- “think addition for subtraction” (e.g., for 9 – 5 = __, think “5 and what number makes 9?”);</td>
<td>- use of the commutative property, without naming the property (e.g., 4 + 3 is the same as 3 + 4);</td>
<td>- Students should first master facts to 10 and then master facts to 18.</td>
</tr>
<tr>
<td>- use of related facts (e.g., 4 + 3 = 7, 3 + 4 = 7, 7 – 4 = 3, and 7 – 3 = 4);</td>
<td>- use of the additive identity property (e.g., 4 + 0 = 4); without naming the property but saying, “When you add zero to a number, you always get the original number.” and</td>
<td></td>
</tr>
<tr>
<td>- use of patterns to make sums (e.g., 0 + 5 = 5, 1 + 4 = 5, 2 + 3 = 5, etc.).</td>
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The exploration of measurement and geometry in the primary grades allows students to learn more about the world around them. Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K–3 should encounter measurement in many normal situations, from their daily lives, from their use of the calendar and from science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, area, perimeter, volume, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both several techniques and direct comparison, nonstandard units, and standard tools to find determine measurements and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

[Moved from Geometry strand introduction]
Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language. The focus of instruction at this level is on

- observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below). The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Many experiences with coins help students develop an understanding of money, such as:
  - drawing pennies to show the value of a given coin (e.g., a nickel, a dime, or a quarter);
  - playing store and purchasing classroom objects, using play money (pennies);
  - using skip counting to count a collection of like coins;
  - representing the value of coins using a variety of organizers, such as five/ten frames or hundreds charts, pictures; and
  - representing the value of a nickel, a dime, and a quarter, using pennies; and
  - trading the equivalent value of pennies for a nickel, a dime, and a quarter, using play money.

- Counting coins is an application of unitizing.

- Unitizing is the concept that a group of objects can be counted as one unit (e.g., 10 pennies can be counted as 1 dime.)

- Counting money helps students gain an awareness of consumer skills and the use of money in everyday life.

- A variety of classroom experiences in which students manipulate physical models of money and count forward to determine the value of a collection of coins are important activities to ensure develop competence with counting money.

- Establishing a one-to-one correspondence between the number names and the items in a set of coins (pennies, nickels, or dimes) is essential for an accurate count.

- The last number stated represents the value of a collection of coins being counted. This is known as the cardinality of the set.

### ESSENTIAL UNDERSTANDINGS

- All students should
  - Develop an understanding of exchanging the appropriate number of pennies for a nickel, a dime, or a quarter.
  - Develop an understanding of place value by skip counting a collection of coins by ones, fives, and tens.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify the value of a nickel, a dime, and a quarter in terms of pennies. (a) [Included in K.7]

- Recognize the characteristics of pennies, nickels, and dimes (e.g., color, size). [Included in K.7]

- Count by ones to determine the value of a collection of pennies whose total value is 100 cents or less. (b)

- Group a collection of pennies by fives and tens as a way to determine the value. The total value of the collection is 100 cents or less. (b)

- Count by fives to determine the value of a collection of nickels whose total value is 100 cents or less. (b)

- Count by tens to determine the value of a collection of dimes whose total value is 100 cents or less. (b)

- Count by ones, fives, and tens to determine the value of a collection of pennies and nickels, pennies and dimes, and nickels and dimes whose total value is 100 cents or less.

- Count by ones, fives, and tens to determine the value of a collection of pennies, nickels, and dimes whose total value is 100 cents or less. [Moved to 2.7]
STANDARD 1.98 STRAND: MEASUREMENT AND GEOMETRY GRADE LEVEL 1

1.98 The student will investigate the passage of time by
   a) telling time to the hour and half-hour, using analog and digital clocks; and
   b) reading and interpreting a calendar. [Moved from 1.11]

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<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Many experiences using clocks help students develop an understanding of the telling of time to the hour and half-hour, including</td>
<td>• Understand how to tell time to the half-hour, using an analog and digital clock.</td>
<td>• Identify different types of clocks (analog and digital) as instruments to measure time. [Moved from K.8] (a)</td>
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<tr>
<td>– identifying the parts of an analog clock (minute and hour hands);</td>
<td>• Understand the concepts of a.m., p.m., minutes, and hours.</td>
<td>• Tell time shown on an analog clock to the hour and half-hour. (a)</td>
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<tr>
<td>– demonstrating a given time to the hour and half-hour, using a model clock;</td>
<td>• Understand that there are sixty minutes in an hour.</td>
<td>• Tell time shown on a digital clock to the hour and half-hour. (a)</td>
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<tr>
<td>– writing correct digital time to the hour and half-hour; and</td>
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<td>• Match a written time (e.g., 1:00, 3:30, 11:00) to the time shown on a digital and analog clock to the hour and half-hour. (a)</td>
</tr>
<tr>
<td>– relating time on the hour and half-hour to daily routines and school schedules (e.g., the times of TV programs, bedtime, resource time, lunch time, recess time); and</td>
<td></td>
<td>• Read a calendar to locate a given day or date. (b)</td>
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<tr>
<td>– connecting the hour and half hour to fraction concepts.</td>
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<td>• Determine the days/dates before and after a given day/date (e.g., today is the 30th, so yesterday must have been the ?). (b)</td>
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<td>• Determine the date that is a specific number of days or weeks in the past or in the future from a given date, using a calendar. (b) [Moved from 1.11]</td>
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<td>Read a calendar to locate a given day or date (e.g., What day of the week is the 10th? Saturday is what date?). (b)</td>
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<td>Identify specific dates (e.g., the third Monday in a given month). (b) [Moved from 1.11] Given a calendar, determine the number of any day of the week (e.g., How many Fridays are in the month of October?). (b)</td>
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<tr>
<td>• Practical situations are appropriate to develop a sense of the interval of time between events (e.g., Boy Scout club meetings occur every week on Monday: there is a week between meetings).</td>
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<tr>
<td>• The calendar is a way to represent units of time (e.g., days, weeks, and months).</td>
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<tr>
<td>• Using a calendar develops the concept of day as a 24-hour period rather than a period of time from sunrise to sunset. [Moved from 1.11]</td>
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</table>
1.109 The student will use nonstandard units to measure and compare length, weight/mass, and volume.

**UNDERSTANDING THE STANDARD**

(Background Information for Instructor Use Only)

- The process of measurement involves selecting a unit of measure, comparing the unit to the object to be measured, counting the number of times the unit is used to measure the object, and arriving at an approximate total number of units.

- Measurement involves comparing an attribute of an object to the same attribute of the unit of measurement (e.g., the length of a cube measures the length of a book; the weight/mass of a cube measures the weight/mass of the book; the volume of the cube measures the volume of a book).

- Premature use of instruments or formulas leaves children without the understanding necessary for solving measurement problems.

- When children’s initial explorations of length, weight/mass, and volume involve the use of nonstandard units, they develop some understanding about the need for standard measurement units for length, weight/mass, and volume, especially when they communicate about these measures.

- The level of difficulty in measuring volume can be increased by varying and mixing the sizes and/or shapes of the containers (e.g., using short, wide containers as well as tall, narrow containers).

- Students develop conservation of measurement when they understand that the attributes do not change when the object is manipulated (e.g., a piece of string that is coiled maintains its length as it is straightened; the volume of water does not change when poured from a pitcher into a fish tank.)

- Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object’s mass, although they use the term weight (e.g., “How much does it weigh?” versus “What is its mass?”).

**ESSENTIAL UNDERSTANDINGS**

**All students should**

- Understand that measurement involves comparing an attribute of an object to the same attribute of the unit of measurement (e.g., the length of a cube measures the length of a book. The weight/mass of the cube measures the weight/mass of the book. The volume of the cube measures the volume of a book).

- Understand how to measure length, weight/mass, and volume using various nonstandard units of measurement.

**ESSENTIAL KNOWLEDGE AND SKILLS**

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Measure the length of objects, using various nonstandard units (e.g., connecting cubes, paper clips, erasers).

- Compare the length of two objects, using the terms longer/shorter, taller/shorter, same as.

- Measure the weight/mass of objects, using a balance or pan scale with various nonstandard units (e.g., paper clips, bean bags, cubes).

- Identify a balance scale or a pan scale as a tool for measuring weight.

- Compare the weight/mass of two objects, using the terms lighter, heavier, or the same, using a balance scale.

- Measure the volume of objects, using various nonstandard units (e.g., connecting cubes, blocks, rice, water).

- Compare the volumes of two containers to determine if the volume of one is more, less, or equivalent to the other, using nonstandard units of measure (e.g., a spoonful or scoopful of rice, sand, jellybeans).

- Compare the volumes of two containers to determine if the volume of one is more, less, or equivalent to the other by pouring the contents of one container into the other.

[Moved from 1.10]
The student will use nonstandard units to measure and compare length, weight/mass, and volume.[Compare moved from 1.10]

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<td>• Physically measuring the weights of objects, using a balance scale, helps students develop an intuitive idea of what it means to say something is “lighter,” “heavier,” or “the same.”</td>
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<tr>
<td>• Balance scales are instruments used for comparing weight/mass. A balance scale usually has a beam that is supported in the center. On each side of the beam are two identical trays. When the trays hold equal weights, the beam is level, and the scale is “balanced.”—The tray containing less weight/mass will rise and the tray containing more weight/mass will fall.</td>
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</tr>
<tr>
<td>• Experience estimating the weights of two objects (one in each hand) using the terms “lighter,” “heavier”, or “the same” promotes an understanding of the concept of balance.[Moved from 1.10]</td>
<td></td>
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</tr>
</tbody>
</table>
1.10 The student will compare, using the concepts of more, less, and equivalent, [Compare moved to 1.10]
   a) the volumes of two given containers; and [Moved to 1.10 EKS]
   b) the weight/mass of two objects, using a balance scale. [Moved to 1.10 EKS]

<table>
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<tbody>
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<td>(Background Information for Instructor Use Only)</td>
<td>All students should</td>
<td>The student will use problem-solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>- Estimation is a commonly used strategy to compare the volumes of two containers.</td>
<td>- Understand how to fill containers with objects to determine their volume and compare the volumes of two containers.</td>
<td>- Compare the volumes of two containers to determine if the volume of one is more, less, or equivalent to the other, using nonstandard units of measure (e.g., a spoonful or scoopful). [Moved to 1.10]</td>
</tr>
<tr>
<td>- Determining the volume of a container by counting the number of nonstandard units (e.g., a spoonful, or scoopful of concrete material, such as jelly beans, sand, water, or rice) that can be held by the container is a precursor to comparing volumes.</td>
<td>- Understand that a balance scale can be used to compare the weights of two objects using the terms more, less, or equivalent.</td>
<td>- Compare the volumes of two containers to determine if the volume of one is more, less, or equivalent to the other by pouring the contents of one container into the other. [Moved to 1.10]</td>
</tr>
<tr>
<td>- A variety of activities that focus on directly comparing the volume of objects leads to an understanding of volume.</td>
<td></td>
<td>- Compare the weight/mass of two objects, using the terms lighter, heavier, or the same, using a balance scale. The pan containing less weight/mass will rise and the pan containing more weight/mass will fall. If the objects are of equivalent weight/mass, the two pans will balance. [Moved to 1.10]</td>
</tr>
<tr>
<td>- The level of difficulty in measuring volume can be increased by varying and mixing the sizes of the containers (e.g., using short, wide containers as well as tall, narrow containers).</td>
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</tr>
<tr>
<td>- Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object’s mass, although they use the term weight (e.g., “How much does it weigh?” versus “What is its mass?”).</td>
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<tr>
<td>- Balance scales are instruments used for comparing weight/mass. A balance scale usually has a beam that is supported in the center. On each side of the beam are two identical trays. When the trays hold equal weights, the beam is level, and the scale is “balanced.”</td>
<td></td>
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</tr>
<tr>
<td>- Physically measuring the weights of objects, using a balance scale, helps students develop an intuitive idea of what it means to say something is “lighter,” “heavier,” or “the same.”</td>
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</tr>
<tr>
<td>- Experience estimating the weights of two objects (one in each hand) using the terms lighter, heavier, or the same promotes an understanding of the concept of balance.</td>
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</tbody>
</table>
STANDARD 1.11  STRAND: MEASUREMENT  GRADE LEVEL 1

1.11 The student will use calendar language appropriately (e.g., names of the months, today, yesterday, next week, last week). [Moved to K.9 and 1.9]

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<tr>
<td>Practical situations are appropriate to develop a sense of the interval of time between events (e.g., Boy Scout meetings occur every week on Monday; there is a week between meetings).</td>
<td>All students should understand how to use a calendar as a way to measure time.</td>
<td>The student will use problem-solving, mathematical communication, mathematical reasoning, connections, and representations to:</td>
</tr>
<tr>
<td>The calendar is a way to represent units of time (e.g., days, weeks, and months).</td>
<td>Read a calendar to locate a given day or date. [Moved to 1.9]</td>
<td></td>
</tr>
<tr>
<td>Using a calendar develops the concept of day as a 24-hour period rather than a period of time from sunrise to sunset.</td>
<td>Identify the months of the year. [Moved to K.9]</td>
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<tr>
<td></td>
<td>Identify the seven days in a week. [Moved to K.9]</td>
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<tr>
<td></td>
<td>Determine the days/dates before and after a given day/date (e.g., yesterday, today, tomorrow). [Moved to 1.9 and K.9]</td>
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</tr>
<tr>
<td></td>
<td>Determine the date that is a specific number of days or weeks in the past or in the future from a given date, using a calendar. [Moved to 1.9]</td>
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<tr>
<td></td>
<td>Identify specific dates (e.g., the third Monday in a given month). [Moved to 1.9]</td>
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</table>
Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on
• observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;
• sorting objects and ordering them directly by comparing them one to the other;
• describing, comparing, contrasting, sorting, and classifying figures; and
• exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to these figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

• **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

• **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

• **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
1.112 The student will
   a) Identify, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, vertices, and right angles; and
   b) identify and describe representations of circles, squares, rectangles, and triangles in different environments, regardless of orientation, and explain reasoning. [Moved from 1.13]

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<td>(Background Information for Instructor Use Only)</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Early experiences with comparing, sorting, and subdividing figures assist students in analyzing the characteristics of plane geometric figures.</td>
<td>• Develop strategies to sort and/or group plane geometric figures and refine the vocabulary used to explain their strategies.</td>
<td>• Identify the name of the plane figure when given information about the number of sides, vertices, and angles. (a) [Reordered]</td>
</tr>
<tr>
<td>• A plane geometric figure is any closed, two-dimensional shape that can be drawn in a plane closed figure. Circles and polygons are examples of plane geometric figures.</td>
<td></td>
<td>• Trace triangles, squares, rectangles, and circles. (a) [Reordered]</td>
</tr>
<tr>
<td>• The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.</td>
<td>• Level 0: Pre-recognition. Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.</td>
<td>• Describe a circle using terms such as round and curved. (a) [Reordered]</td>
</tr>
<tr>
<td>• Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., “I know it’s a rectangle because it looks like a door, and I know that a door is a rectangle”).</td>
<td>• Level 2: Analysis. Properties are perceived, but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., “I know it’s a rectangle because it is closed; it has four sides and four right angles, and opposite sides are parallel.”).</td>
<td>• Describe triangles, squares, and rectangles by the number of sides, vertices, and right angles. Recognize that rectangles and squares have special types of angles called right angles. (a)</td>
</tr>
<tr>
<td>• Students should have experiences with various plane geometric polygons.</td>
<td>• Students should have experiences with various plane geometric polygons.</td>
<td>• Identify the name of the geometric figure when given information about the number of sides, vertices, and right angles. [Reordered]</td>
</tr>
<tr>
<td>• Triangles could be equilateral, right, obtuse, acute, etc.</td>
<td></td>
<td>• Sort plane geometric figures into appropriate subsets (categories) based on their characteristics (number of sides, vertices, angles, curved, etc.). (a) [Reordered]</td>
</tr>
<tr>
<td>• Quadrilaterals could be rectangles, squares, trapezoids, rhombi,</td>
<td></td>
<td>• Identify and describe representations of circles, squares, rectangles, and triangles, regardless of orientation, in different environments and explain reasoning. (b) [Moved from 1.13]</td>
</tr>
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</table>
1.1112 The student will
  a) Identify, and trace, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, vertices, and right angles; and
  b) identify and describe representations of circles, squares, rectangles, and triangles in different environments, regardless of orientation, and explain reasoning. [Moved from 1.13]

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<td>(Background Information for Instructor Use Only)</td>
<td>etc.</td>
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<tr>
<td>• A vertex is the point at which two or more lines, line segments, lines, or rays meet to form an angle. The term “vertices” is the plural form of vertex.</td>
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<tr>
<td>• An angle is formed by two rays that share a common endpoint called the vertex. Angles are found wherever lines or line segments intersect.</td>
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<tr>
<td>• A polygon is a closed plane figure composed of at least three line segments that do not cross.</td>
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<tr>
<td>• A polygon is a plane geometric figure which has sides that are line segments; is simple (its sides do not cross); is closed; and is two-dimensional (lies in a plane).</td>
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<tr>
<td>• A triangle is a polygon with three angles and three sides.</td>
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<tr>
<td>• A quadrilateral is a polygon with four sides.</td>
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<tr>
<td>• A rectangle is a quadrilateral with four right angles.</td>
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<tr>
<td>• A square is a rectangle-quadrilateral with four congruent (equal length) sides of equal length, and four right angles. At this level, students might describe a square as a special rectangle with four sides of equal length.</td>
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<tr>
<td>• Students at this level do not need to use the terms polygon, quadrilateral, or congruent.</td>
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<td>• A right angle measures exactly 90°.</td>
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1.1112 The student will:

a) Identify, and trace, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, vertices, and right angles; and

b) Identify and describe representations of circles, squares, rectangles, and triangles in different environments, regardless of orientation, and explain reasoning. [Moved from 1.13]

**UNDERSTANDING THE STANDARD**

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<td>• A circle is the set of a closed curve with all points in one plane and that are the same distance from a fixed point (called the center). A circle is not a polygon, because it does not have straight sides.</td>
</tr>
<tr>
<td>• Transformations (translations, reflections, and rotation) can be used to change the location of objects.</td>
</tr>
<tr>
<td>• Presentation of triangles, rectangles, and squares should be made in a variety of spatial orientations so that students do not develop the common misconception that triangles, rectangles, and squares must have one side parallel to the bottom of the page on which they are printed.</td>
</tr>
<tr>
<td>• Representations of circles, squares, rectangles, and triangles can be found in the students’ environment at school and at home. Students should have opportunities to identify/classify things in the environment by the type of figure those things represent. [Moved from 1.13]</td>
</tr>
<tr>
<td>• A common misconception students have when a figure such as a square is rotated is they will frequently refer to the rotated square as a diamond. Clarification needs to be ongoing — e.g., a square is a square regardless of its location in space; there is no plane figure called a diamond. [Moved from 1.13]</td>
</tr>
<tr>
<td>• Building geometric and spatial capabilities fosters enthusiasm for mathematics while providing a context to develop spatial sense. [Moved from 1.13]</td>
</tr>
<tr>
<td>• Polygons can be constructed using other polygons (e.g., six equilateral triangles can be used to construct a hexagon; a triangle can be added to a rectangle to create a pentagon, etc.). [Moved from 1.13]</td>
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</tbody>
</table>
1.1112 The student will
   a) identify, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, vertices, and right angles; and,
   b) identify and describe representations of circles, squares, rectangles, and triangles in different environments, regardless of orientation, and explain reasoning.[Moved from 1.13]

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Early experiences with comparing, sorting, and subdividing figures or manipulatives (e.g. patterns blocks) assist students in analyzing the characteristics of plane geometric figures.

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</table>
1.13 The student will construct, model, and describe objects in the environment as geometric shapes (triangle, rectangle, square, and circle) and explain the reasonableness of each choice. [Moved to 1.11b]

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<tr>
<td>• Representations of circles, squares, rectangles, and triangles can be found in the students' environment at school and at home. Students should have opportunities to identify/classify things in their environment by the type of figure those things represent.</td>
<td>All students should</td>
<td>The student will use problem-solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• A common misconception students have when a figure such as a square is rotated is they will frequently refer to the rotated square as a diamond. Clarification needs to be ongoing — i.e., a square is a square regardless of its location in space; there is no such geometric figure as a diamond.</td>
<td>• Understand that geometric figures are integral parts of the environment.</td>
<td>• Construct plane geometric figures.</td>
</tr>
<tr>
<td>• Building geometric and spatial capabilities fosters enthusiasm for mathematics while providing a context to develop spatial sense.</td>
<td>• Use familiarity with the figure, structure, and location to develop spatial reasoning.</td>
<td>• Identify models of representations of circles, squares, rectangles, and triangles in the environment at school and home and tell why they represent those figures.</td>
</tr>
<tr>
<td>• Polygons can be constructed using other polygons (e.g., six equilateral triangles can be used to construct a hexagon, a triangle can be added to a rectangle to create a pentagon, etc.).</td>
<td></td>
<td>• Describe representations of circles, squares, rectangles, and triangles in the environment and explain the reasonableness of the choice.</td>
</tr>
<tr>
<td>[Moved to 1.11 EKS]</td>
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<td>[Moved to 1.11 EKS]</td>
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</tbody>
</table>
Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible, unlikely, equally likely, more likely,* and *certain.*

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.
1.1214 The student will

a) investigate, identify, and describe collect, organize, and represent various forms of data collection (e.g., recording daily temperature, lunch count, attendance, favorite ice cream), using tables, picture graphs, and object graphs; and [Examples moved to US]

b) read and interpret data displayed in tables, picture graphs, and object graphs, using the vocabulary more, less, fewer, greater than, less than, and equal to. [Moved from 1.15]

**UNDERSTANDING THE STANDARD**
(Background Information for Instructor Use Only)

- Data are pieces of information collected about people or things. The primary purpose of collecting data is to answer questions. The primary purpose of interpreting data is inform decisions (e.g., which type of clothing to pack for a vacation based on a weather graph or which type of lunch to serve based upon class favorites).

- Students’ questions about the physical world can often be answered by collecting data and observing the results.

- Data are information collected about people or things. The primary purpose of collecting data is to answer questions.

- After generating questions, students decide what information is needed and how it can be collected.

- The collection of the data often leads to new questions to be investigated.

- The entire process broadens children’s views of mathematics and its usefulness.

- Data collection could involve voting, informal surveys, tallying, and charts (e.g., recording daily temperature, lunch count, attendance, favorite ice cream).

- Surveys, which are data-collecting tools that list choices, should have a limited number of questions at the primary grades.

- Tallying is a method for gathering information. Tally marks are used to show how often something happens or occurs. Each tally mark represents one occurrence. Tally marks are clustered into groups of five, with four vertical marks representing the first four occurrences and the fifth mark crossing the first four on a diagonal to represent the fifth occurrence.

**ESSENTIAL UNDERSTANDINGS**

All students should

- Understand how data can be collected and presented in an organized manner.
- Understand that data gathered and analyzed from observations and surveys can have an impact on our everyday lives.

**ESSENTIAL KNOWLEDGE AND SKILLS**

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Investigate Collect and organize data using various forms of data collection, including (e.g., counting and tallying, informal surveys, observations, and voting). Data points, collected by students, should be limited to 16 or fewer for no more than four categories. (a)

- Represent data in tables, picture graphs, and object graphs. (a)

- Identify and describe various forms of data collection in practical situations (e.g., recording daily temperature, lunch count, attendance, and favorite ice cream.)

- Analyze information displayed in tables, picture graphs, and object graphs (horizontally or vertically represented):
  - Read the graph to determine the categories of data and the data as a whole (e.g., the total number of responses) and its parts (e.g., 15 people are wearing sneakers); and
  - Interpret the data that represents numerical relationships, to include using the words more, less, fewer, greater than, less than, and equal to. (b) [Moved from 1.15 EKS]
1.1214 The student will

a) investigate, identify, and describe various forms of data collection (e.g., recording daily temperature, lunch count, attendance, favorite ice cream), using tables, picture graphs, and object graphs; and

b) read and interpret data displayed in tables, picture graphs, and object graphs, using the vocabulary more, less, fewer, greater than, less than, and equal to.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)

- Picture graphs are graphs that use pictures to represent and compare information. At this level, each picture should represent 1 data point.

- Object graphs are graphs that use concrete materials to represent and compare the categorical data that are collected (e.g., cubes stacked by the month, with one cube representing the birthday month of each student).

- Tables are an orderly arrangement of data in which the data are arranged in columns and rows in an essentially rectangular format. Tables may be used to display some type of numerical relationship or organized lists.

- At this level, data gathered and displayed by students should be limited to 16 or fewer data points for no more than 4 categories.

- Students should have opportunities to interpret graphs, created with the assistance of the teacher, that contain data points where their entire class is represented (e.g., tables that show who brought their lunch and who will buy their lunch for any given day, picture graph showing how students traveled to school – bus, car, walk).

ESSENTIAL UNDERSTANDINGS

ESSENTIAL KNOWLEDGE AND SKILLS
1.15 The student will interpret information displayed in a picture or object graph, using the vocabulary more, less, fewer, greater than, less than, and equal to. [Moved to 1.1.2b]

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<td>All students should</td>
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</tr>
<tr>
<td>• Statistics is used to describe and interpret with numbers the world around us; it is a tool for problem solving.</td>
<td>• Understand that picture graphs use pictures to represent and compare data while object graphs use concrete objects to represent and compare data.</td>
<td>• Compare one category to another in a graph, indicating which has more or which has less, or which is equal to.</td>
</tr>
<tr>
<td>• Students’ questions about everyday life can often be answered by collecting and interpreting data.</td>
<td>• Understand that data can be analyzed and interpreted, using the terms more, less, fewer, greater than, less than, and equal to.</td>
<td>• Interpret information displayed in object graphs and picture graphs, using the words more, less, fewer, greater than, less than, and equal to.</td>
</tr>
<tr>
<td>• Organized data provides a clearer picture for interpretation. Data may be described in object graphs and picture graphs.</td>
<td>• Interpretation of the data could lead to additional questions to be investigated.</td>
<td>• Find answers to questions, using graphs (e.g., “Which category has more?”, “How many more?”, and “How many in all?”).</td>
</tr>
<tr>
<td>• Picture graphs are graphs that use pictures to show and compare information.</td>
<td></td>
<td>[Moved to 1.1.2b EKS]</td>
</tr>
<tr>
<td>• Object graphs are graphs that use concrete materials to represent the categorical data that are collected (e.g., cubes stacked by the month, with one cube representing the birthday month of each student).</td>
<td></td>
<td></td>
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<tr>
<td>• Interpretation of the data could lead to additional questions to be investigated.</td>
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Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children’s mathematical development.

Students in kindergarten through third grade develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns. Students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to algebra.
The student will sort and classify concrete objects according to one or more attributes, including color, size, shape, and thickness. [Included in EKS]

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>Sorting, classifying, and ordering objects facilitate work with patterns, geometric shapes, and data.</td>
<td>• Understand the same set of objects can be sorted and classified in different ways. [Moved to US]</td>
<td></td>
</tr>
<tr>
<td>The same set of objects can be sorted and classified in different ways. [Moved from middle column]</td>
<td></td>
<td>• Sort and classify objects into appropriate subsets (categories) based on one or two attributes, such as size, shape, color, and/or thickness (e.g., sort a set of objects that are both red and thick).</td>
</tr>
<tr>
<td>To classify is to arrange or organize a set of materials according to a category or attribute (a quality or characteristic).</td>
<td></td>
<td>• Label attributes of a set of objects that has been sorted.</td>
</tr>
<tr>
<td>A venn diagram can be a helpful tool when sorting by more than one attribute.</td>
<td></td>
<td>• Name multiple ways to sort a set of objects.</td>
</tr>
<tr>
<td>One way to explore attributes is to investigate non-examples (e.g., a triangle could be a non-example in a sort of rectangles and circles).</td>
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<tr>
<td>General similarities and differences among items are easily observed by primary students, who can begin to focus on more than one attribute at a time. During the primary grades, the teacher’s task is to move students toward a more sophisticated understanding of classification in which two or more attributes connect or differentiate sets, such as those found in nature (e.g., leaves with different colors and different shapes).</td>
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</tbody>
</table>
1.1417 The student will identify, recognize, describe, extend, and create, and transfer a wide variety of growing and repeating patterns. [Transfer included to match EKS bullet.]

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
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<tbody>
<tr>
<td>Opportunities to create, identify, describe, extend, and transfer patterns are essential to the primary school experience and lay the foundation for thinking algebraically.</td>
</tr>
<tr>
<td>Patterns allow children to recognize order, generalize, and predict.</td>
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<tr>
<td>Patterning should include:</td>
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<tr>
<td>– reproducing a given pattern, using objects, sounds, movements and pictures/manipulatives;</td>
</tr>
<tr>
<td>– recording a pattern with pictures or symbols;</td>
</tr>
<tr>
<td>– transferring a pattern into a different form or different representation (e.g., blue–blue–red–green to an AABC repeating pattern); and</td>
</tr>
<tr>
<td>– analyzing patterns in practical situations (e.g., calendar, seasons, days of the week).</td>
</tr>
<tr>
<td>The simplest types of patterns are in a repeating pattern. The part of the pattern that repeats is the core. The patterns can be oral, such as the refrain in “Old MacDonald’s Farm” (“e-i-e-i-o”), or physical with clapping and snapping patterns, or combinations of both, such as is found in songs like the “Hokey Pokey.” In each case, students need to identify the basic unit of the pattern and repeat it.</td>
</tr>
<tr>
<td>Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience. [Reordered]</td>
</tr>
<tr>
<td>At this level students should have experiences extending patterns when given a complete repetition of a core (e.g., ABACABACABAC) as well as when the final repetition of the core is incomplete (e.g., AABBAABBA…; Red, Blue, Green, Red, Blue, Green, Red, Blue…).</td>
</tr>
<tr>
<td>Examples of repeating patterns include:</td>
</tr>
<tr>
<td>– AABCAABC;</td>
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<tr>
<td>– ABACABAC;</td>
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<tr>
<td>All students should</td>
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<tr>
<td>– Understand that patterns are a way to recognize order, to organize their world, and to predict what comes next in an arrangement.</td>
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<tr>
<td>– Recognize and state the core of a pattern.</td>
</tr>
<tr>
<td>– Analyze how both repeating and growing patterns are generated.</td>
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<tr>
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<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>– Recognize Identify the pattern in a given rhythmic, color, geometric figure, or numerical sequence.</td>
</tr>
<tr>
<td>– Describe the pattern in a given rhythmic, color, geometric figure, or numerical sequence in terms of the core (the part of the sequence that repeats).</td>
</tr>
<tr>
<td>– Extend a repeating or growing pattern, using manipulatives, geometric figures, numbers, or calculators.</td>
</tr>
<tr>
<td>– Create a repeating or growing pattern, using manipulatives, geometric figures, numbers, or calculators (e.g., the growing patterns 2, 3, 2, 4, 2, 5, 2, 6, …, 2, …). [Reordered]</td>
</tr>
<tr>
<td>– Transfer a pattern from one form to another.</td>
</tr>
<tr>
<td>– Create a repeating or growing pattern, using manipulatives, geometric figures, numbers, or calculators (e.g., the growing patterns 2, 3, 2, 4, 2, 5, 2, 6, 2, …). [Reordered]</td>
</tr>
</tbody>
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1.1417 The student will identify, recognize, describe, extend, and create, and transfer a wide variety of growing and repeating patterns. [Transfer included to match EKS bullet.]

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</tr>
<tr>
<td>– ABBCCABBC;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– AABCAABC; and</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– ABACDABACD.</td>
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<tr>
<td>• Growing patterns involve a progression from step to step which make them more difficult for students than repeating patterns. Students must determine what comes next and also begin the process of generalization, which leads to the foundation of algebraic reasoning. Students need experiences identifying what changes and what stays the same in a growing pattern. Growing patterns may be represented in various ways, including dot patterns, staircases, pictures, etc.</td>
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<tr>
<td>• Examples of growing patterns include:</td>
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<tr>
<td>• Transferring a pattern is creating the pattern in a different form or representation.</td>
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<tr>
<td>• Examples of pattern transfers include:</td>
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<tr>
<td>– 1, 2, 3, 4... has the same structure as 10, 11, 12, 13...</td>
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<tr>
<td>– ABABAB... has the same structure as red, blue, red, blue, red blue</td>
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<tr>
<td>– Snap, clap, jump, clap, snap, clap, jump, clap has the same structure as ABCBABC</td>
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<tr>
<td>• Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns in both arithmetic and geometric formats.</td>
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</table>
1.1417 The student will **identify, recognize, describe, extend, and create, and transfer** a wide variety of growing and repeating patterns. [Transfer included to match EKS bullet.]

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<tr>
<td>• Create an arithmetic number pattern. Sample numeric patterns include 6, 9, 12, 15, 18, ... (growing pattern); 20, 18, 16, 14, ... (growing pattern); 1, 2, 4, 7, 11, 16, ... (growing pattern); and 1, 3, 5, 1, 3, 5, 1, 3, 5... (repeating pattern).</td>
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<tr>
<td>• In geometric patterns, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation is the result of turning a figure around a point or a vertex, and reflection is the result of flipping a figure over a line.</td>
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</tbody>
</table>
The student will demonstrate an understanding of equality through the use of the equal symbol.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Equality can be shown by measuring with a balance scale or a number balance. [Reordered]
- At this level, students should represent equality using objects, words, and symbolically through the use of the equal symbol while inequality should be communicated primarily through words such as “not equal,” “not equivalent,” etc.
- An equation (number sentence) is a mathematical statement representing two expressions that are equivalent. It consists of two expressions, one on each side of an 'equal' symbol (e.g., $5 + 3 = 8$, $8 = 5 + 3$ and $4 + 3 = 9 - 2$).
- An equation, such as $3 + 5 = 6 + 2$, can be represented using a balance scale, with equal amounts on each side.
- Students at this level are not expected to use the term expression.
- Equations can be written with an equal symbol. The equal symbol means “is the same as” or “another name for” or “equal in value”. [Moved from middle column]
- A common misunderstanding is that the equal symbol always means “the answer comes next.” The equal symbol can represent a balance between expressions. [Reordered]
- Equations have expressions of equal value on both sides (e.g., $5 + 3 = 8$, $8 = 5 + 3$ and $4 + 3 = 5 + 2$). [Reordered]
- Equality can be shown by measuring with a balance scale or a number balance. [Reordered]
- Manipulatives such as rods, connecting cubes, and counters can be used to model equations.
- An expression is a representation of a quantity. It contains made up of numbers, variables, and/or computational operation symbols. It does not have an equal symbol (e.g., $5$, $4 + 3$, $8 - 2$).
- An equation is a mathematical statement that two expressions are the same. Equations are written with an equal sign.

### ESSENTIAL UNDERSTANDINGS

**All students should**

- Understand that the equal sign means “is the same as” or “another name for” or “equal in value”.
- Understand that equality represents a balance concept. Both sides of the equation balance because they are equal (they have the same value).

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify the concepts of equality,
- Identify equivalent values (reordered) and represent equalities through the use of objects, words, and the equal (=) symbol.
- Identify and describe expressions that are not equal (e.g., $4 + 3$ is not equal to $3 + 5$).
- Recognize that the equations $4 + 2 = 2 + 4$ and $6 + 1 = 4 + 3$ can be used to represent the relationship between two expressions of equal value (e.g., $4 + 2 = 2 + 4$ and $6 + 1 = 4 + 3$).
- Model an equation that represents the relationship of two expressions of equal value.
- Identify equivalent values (e.g., $3 = 3$, $4 + 3 = 9 - 2 - 1 + 3$, $4 + 7 - 2 = 5$, etc.).
1.1518 The student will demonstrate an understanding of equality through the use of the equal sign symbol.

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<tr>
<td>• Equations have expressions of equal value on both sides (e.g., (5 + 3 = 8), (8 = 5 + 3) and (4 + 3 = 9 - 2)).</td>
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<tr>
<td>• An equation can be represented using balance scales. There must be the same amount on each side of an equal sign (e.g., (5 + 3 = 3 + 5)).</td>
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<tr>
<td>• A common misunderstanding is that the equal sign always means the answer. The equal sign can represent an equality.</td>
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</tr>
<tr>
<td>• Equations should be shown in many different forms (e.g., (6 = 6), (4 + 2 = 6), (5 + 1 = 4 + 2), (6 = 4 + 2), (4 = 6 - 2)).</td>
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<tr>
<td>• Inequalities such as (5 &lt; 4 + 3) are not equations. Equations must have the equal sign symbol (e.g., (5 + 6 = 11)).</td>
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<tr>
<td>• It is important for children to understand that the expression (3 + 5) is another representation of eight.</td>
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<tr>
<td>• The equal sign is used when two representations name the same number, (5 + 3 = 10 - 2). These two expressions in the equation represent the same number, eight.</td>
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<tr>
<td>• Equations should be routinely modeled in conjunction with story problems.</td>
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<tr>
<td>• Solving missing addend problems and stories helps with the understanding of equality and use of the equal sign symbol (e.g., There are 4 red birds in the tree. Some black birds fly to the tree. Now there are six birds in the tree. How many black birds flew to the tree? (4 + _ = 6)).</td>
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</table>
Mathematics Standards of Learning

Curriculum Framework 2009

Grade 2

Board of Education
Commonwealth of Virginia
The 2009-2016 Mathematics Standards of Learning Curriculum Framework, is a companion document to the 2009-2016 Mathematics Standards of Learning, and amplifies the Mathematics Standards of Learning by further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. It provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

**Essential Understandings**
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

**Understanding the Standard**
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instructionbackground information for the teacher. It contains context- The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers’ knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

**Essential Knowledge and Skills**
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that What each student should know and be able to demonstratedo in each standard is outlined. This is not meant to be an exhaustive list of student expectations nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student’s problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students’ mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
Students in grades K–2 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, fractions, number relationships (“more than,” “less than,” and “equal to”), and the effects of single-step and multistep computations. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.
2.1 The student will
a) read, write, and identify the place and value of each digit in a three-digit numeral, using numeration with and without models;
b) identify the number that is 10 more, 10 less, 100 more, and 100 less than a given number, up to 999;
c) round two-digit numbers to the nearest ten; and [Reordered]
d) compare and order two whole numbers between 0 and 999, using symbols (>, <, or =) and words (greater than, less than, or equal to); and [Symbols and words included in EKS]
d) round two-digit numbers to the nearest ten. [Reordered]

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| The number system is based on a simple pattern of tens where each place has ten times the value of the place to its right. Numbers are written to show how many hundreds, tens, and ones are in the number. [Moved from middle column] Opportunities to experience the relationships among hundreds, tens, and ones through hands-on experiences with manipulatives are essential to developing the ten-to-one place value concept of our number system and to understanding the value of each digit in a three-digit number. This structure is helpful when comparing and ordering numbers. Manipulatives that can be physically connected and separated into groups of tens and leftover ones (e.g., snap cubes, beans on craft sticks, pennies in cups, bundle of sticks, beads on pipe cleaners, etc.) should be used. Ten-to-one trading activities with manipulatives on place value mats provide excellent experiences for developing the understanding of the places in the base-10 system. [Reordered] Models that clearly illustrate the relationships among hundreds, tens, and ones, tens, and hundreds, are physically proportional (e.g., the tens piece is ten times larger than the ones piece). Students need to understand that 10 and 100 are special units of numbers (e.g., 10 is 10 ones, but it is also 1 ten). Flexibility in thinking about numbers is critical. For example, 123 is 123 ones; or 1 hundred, 2 tens, and 3 ones; or 12 tens and 3 ones. (e.g., 84 is 8 tens and 4 ones, or 7 tens and 14 ones, or 15 ones.) | All students should
- Understand the ten-to-one relationship of ones, tens, and hundreds (10 ones equals 1 ten; 10 tens equals 1 hundred).
- Understand that numbers are written to show how many hundreds, tens, and ones are in the number. [Moved to US]
- Understand that rounding gives a close, easy-to-use number to use when an exact number is not needed for the situation at hand.
- Understand that a knowledge of place value is essential when comparing numbers.
- Understand the relative magnitude of numbers by comparing numbers. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
- Demonstrate the understanding of the ten-to-one relationships among ones, tens, and hundreds, using manipulatives (e.g., Base-10 blocks, bundles of 10 sticks). (a)
- Determine the place value of each digit in a three-digit numeral presented as a pictorial representation (e.g., a picture of Base-10 blocks) or as a physical representation (e.g., actual Base-10 blocks).
- Write numerals, using a Base-10 model or picture, pictorial representation (i.e., a picture of base-10 blocks). (a)
- Read three-digit numbers when shown a numeral, a Base-10 model of the number, or a pictorial representation of the number. (a)
- Identify and write the place value (ones, tens, hundreds) of each digit in a three-digit numeral. (a)
- Determine the value of each digit in a three-digit numeral (e.g., in 352, the 5 represents 5 tens and its value is 50). (a)
- Use models to represent numbers in multiple ways, according to place value (e.g., 256 can be 2 hundreds, 14 tens, and 16 ones, but also 25 tens and 6 ones). (a)
- Use place value understanding to identify the number that is 10 more, 10 less, 100 more, or 100 less than a given number. |
2.1 The student will
a) read, write, and identify the place and value of each digit in a three-digit numeral, using numeration with and without models;
b) identify the number that is 10 more, 10 less, 100 more, and 100 less than a given number, up to 999;
c) round two-digit numbers to the nearest ten; and [Reordered]
d) compare and order two whole numbers between 0 and 999, using symbols (>, <, or =) and words (greater than, less than, or equal to); and [Symbols and words included in EKS]
d) round two-digit numbers to the nearest ten. [Reordered]
2.1 The student will
a) read, write, and identify the place and value of each digit in a three-digit numeral, using numeration with and without models;
b) identify the number that is 10 more, 10 less, 100 more, and 100 less than a given number, up to 999;
c) round two-digit numbers to the nearest ten; and [Reordered]
dc) compare and order two whole numbers between 0 and 999, using symbols (> , <, or = ) and words (greater than, less than, or equal to); and [Symbols and words included in EKS]
d) round two-digit numbers to the nearest ten. [Reordered]

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

<table>
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<tr>
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<tbody>
<tr>
<td>to round to.</td>
</tr>
<tr>
<td>– If the digit is less than 5, leave the digit in the rounding place as it is, and change the digit to the right of the rounding place to zero.</td>
</tr>
<tr>
<td>– If the digit is 5 or greater, add 1 to the digit in the rounding place, and change the digit to the right of the rounding place to zero.</td>
</tr>
</tbody>
</table>

A procedure for comparing two numbers by examining place value may include the following:
– Line up the numbers by place value lining up the ones.
– Beginning at the left, find the first place value where the digits are different.
– Compare the digits in this place value to determine which number is greater (or which is less).
– Use the appropriate symbol > or < or words greater than or less than to compare the numbers in the order in which they are presented.
– If both numbers are the same, use the symbol = or the words equal to.

• Mathematical symbols (>, <) used to compare two unequal numbers are called inequality symbols.
The student will
a) count forward by twos, fives, and tens to 120, starting at various multiples of 2, 5, or 10;
b) count backward by tens from 120; and[c] recognize use objects to determine whether a number is even or odd numbers.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
</table>
| • Collections of objects can be grouped and skip counting can be used to count the collection.[Moved from middle column.] | All students should  
- Understand that collections of objects can be grouped and skip counting can be used to count the collection.[Moved to US]  
- Describe patterns in skip counting and use those patterns to predict the next number in the counting sequence. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to  
- Determine patterns created by counting by twos, fives, and tens to 120 on a hundred number charts. (a)  
- Describe patterns in skip counting and use those patterns to predict the next number in the counting sequence. (a)[Moved from middle column]  
- Skip count by twos, fives, and tens to 120 from various multiples of 2, 5, and 10. (a)  
- Skip count by twos to 120 starting from any multiple of 2, fives, and tens to 100. (a)  
- Skip count by five to 120 starting at any multiple of 5. (a)  
- Skip count by ten to 120 starting at any multiple of 10. (a)  
- Count backward by tens from 120. (b)  
- Use objects to determine whether a number is odd or even (e.g., dividing collections of objects into two equal groups or pairing objects).[Moved example from US] (c)  

| Skip counting by twos supports the development of the concept of even numbers. |  
|---|---|---|
### STANDARD 2.24  
**STAND: NUMBER AND NUMBER SENSE**

#### GRADE LEVEL 2

**2.24** The student will  
  a) count forward by twos, fives, and tens to 120, starting at various multiples of 2, 5, or 10;  
  b) count backward by tens from 120; and  
  c) recognize use objects to determine whether a number is even or odd numbers.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>when each object has a pair or partner. When an object is left over, or does not have a pair, then the number is odd.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.32 The student will
   a) count and identify the ordinal positions first through twentieth, using an ordered set of objects; and
   b) write the ordinal numbers, 1\textsuperscript{st} through 20\textsuperscript{th}. [Edited to match EKS]

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>The cardinal and ordinal meanings of numbers are necessary to quantify, measure, and identify the order of objects.</td>
<td>Use ordinal numbers to describe the position of an object in a sequence or set.</td>
<td>Count an ordered set of objects, using the ordinal number words first through twentieth. (a)</td>
</tr>
<tr>
<td>The ordinal meaning of numbers is developed by identifying and verbalizing the place or position of objects in a set or sequence (e.g., a student’s position in line when students are lined up alphabetically by first name).</td>
<td></td>
<td>Identify the ordinal positions first through twentieth, using an ordered set of objects.</td>
</tr>
<tr>
<td>An ordinal number is a number that names the place or position of an object in a sequence or set (e.g., first, third). Ordered position, ordinal position, and ordinality are terms that refer to the place or position of an object in a sequence or set.</td>
<td></td>
<td>Identify the ordinal positions first through twentieth, using an ordered set of objects presented in lines or rows from</td>
</tr>
<tr>
<td>The ordinal position is determined by where one starts in an ordered set of objects or sequence of objects (e.g., from the left, right, top, bottom).</td>
<td></td>
<td>– left to right;</td>
</tr>
<tr>
<td>The ordinal meaning of numbers is developed by identifying and verbalizing the place or position of objects in a set or sequence (e.g., a student’s position in line when students are lined up alphabetically by first name).</td>
<td></td>
<td>– right to left;</td>
</tr>
<tr>
<td>Ordinal position can also be emphasized through sequencing events (e.g., months in a year, days in a month, or sequencing events in a story).</td>
<td></td>
<td>– top to bottom; and</td>
</tr>
<tr>
<td>Practical applications of ordinal numbers can be experienced through calendar and patterning activities.</td>
<td></td>
<td>– bottom to top. (a)</td>
</tr>
<tr>
<td>Cardinality can be compared with ordinality when comparing the results of counting. There is obvious similarity between the ordinal number words third through twentieth and the cardinal number words three through twenty.</td>
<td></td>
<td>Write 1\textsuperscript{st}, 2\textsuperscript{nd}, 3\textsuperscript{rd}, through 20\textsuperscript{th} in numerals. (b)</td>
</tr>
</tbody>
</table>
STANDARD 2.43 STRAND: NUMBER AND NUMBER SENSE GRADE LEVEL 2

2.43 The student will
a) name and write fractions represented by a set, region, or length model identify the parts of a set and/or region that represent fractions for halves, fourths, eighths, thirds, and sixths, tenths, and sixths, tenths; and
b) write the represent fractional parts with models and with symbols; and

c) compare the unit fractions for halves, fourths, eighths, thirds, and sixths, with models, eighths, and tenths.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)

- Students need opportunities to solve practical problems involving fractions in which students themselves are determining how to subdivide a whole into equal parts, test those parts to be sure they are equal, and use those parts to count the fractional parts and recreate the whole.

- Counting unit fractional parts as they build the whole (e.g., one-fourth, two-fourths, three-fourths, and four-fourths), will support students understanding that four-fourths makes one whole and prepares them for the study of multiplying unit fractions (e.g., \( \frac{4}{4} = 1 \)) in later grades.

- When working with fractions, the whole should be defined.

- A fraction is a numerical way of representing part of a whole (as in a region/area model), or part of a group (as in a set model), or part of a length (as in a measurement model).

- In each fractional region/area model, the parts must be equal (i.e., each piece must have the same area); the size of each chip in a set must be equal). In problems with fractions, a whole is broken into equal-size parts and reassembled into one whole. The pieces could have a different shape as shown in the examples below.

- In a set model, the set represents the whole and each item represents an equivalent part of the set. For example, in a set of six counters, one counter represents one-sixth of the set.

ESSENTIAL UNDERSTANDINGS

All students should

- Understand that fractional parts are equal shares of a whole or a whole set.

- Understand that the fraction name (half, fourth) tells the number of equal parts in the whole.

- Understand that when working with unit fractions, the larger the denominator, the smaller the part and therefore the smaller the fraction.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Recognize fractions as representing equal-size parts of a whole.

- Name and write fractions represented by a set model showing halves, fourths, eighths, thirds, and sixths. (a, b)

- Name and write fractions represented by a region/area model showing halves, fourths, eighths, thirds, and sixths. (a, b)

- Name and write fractions represented by a length model showing halves, fourths, eighths, thirds, and sixths. (a, b)

- Identify the fractional parts of a whole or a set for \( \frac{2}{2}, \frac{3}{3}, \frac{2}{4}, \frac{7}{7}, \frac{3}{10}, \text{ etc.} \)

- Identify the fraction names (halves, thirds, fourths, sixths, eighths, tenths) for the fraction notations \( \frac{2}{2}, \frac{3}{3}, \frac{2}{4}, \frac{7}{7}, \frac{3}{10}, \text{ etc.} \)

- Represent, with models and with symbols, fractional parts of a whole for halves, fourths, eighths, thirds, and sixths, using:
  - region/area models (e.g., pie pieces, pattern blocks, geoboards);
  - sets (e.g., chips, counters, cubes); and
  - length/measurement models (e.g., fraction strips or bars).
The student will:

a) name and write fractions represented by a set, region, or length model and identify the parts of a set and/or region that represent fractions for halves, fourths, eighths, thirds, and fourths, sixths, eighths, and tenths;

b) write the represent fractional parts with models and with symbols; and

c) compare the unit fractions for halves, fourths, eighths, thirds, and fourths, sixths, with models, eighths, and tenths.

**UNDERSTANDING THE STANDARD**

(Background Information for Instructor Use Only)

- In the set model, the set can be subdivided into subsets with the same number of items in each subset. For example, a set of six counters can be subdivided into two subsets of three counters each and each subset represents one-half of the whole set.

- In the primary grades, students may benefit from experiences with sets that are comprised of congruent figures (e.g., 12 eggs in a carton) before working with sets that have noncongruent parts.

- In a length model, each length represents an equal part of the whole. For example, given a strip of paper, students could fold the strip into four equal parts, each part representing one-fourth. Students will notice that there are four one-fourths in the entire length of the strip of paper that has been divided into fourths in a cube train with a length of 8 cubes, each cube represents one-eighth. Students would be able to state that there are eight one-eighths and that is the same as one whole. If they have 9 cubes, they could make one whole train with one-eighth left over.

- Students need opportunities to use models (region/area or length/measurement) to count fractional parts that go beyond one whole. For instance, if students are counting five pie pieces and building the pie as they count, where each piece is equivalent to one-fourth of a pie, they might say “one-fourth, two-fourths, three-fourths, four-fourths, five-fourths.” As a result of building the whole while they are counting, they begin to realize that four-fourths make one whole and the fifth-fourth starts another whole. They will begin to generalize that when the numerator and the denominator are the same, they have one whole. They also will compare the unit fractions for halves, fourths, eighths, thirds, and fourths, sixths, with models, eighths, and tenths.

**ESSENTIAL UNDERSTANDINGS**

- Compare unit fractions with models or pictures (\(\frac{1}{2}\), \(\frac{1}{3}\), \(\frac{1}{4}\), \(\frac{1}{6}\), \(\frac{1}{8}\), and \(\frac{1}{10}\)) for halves, fourths, eighths, thirds, and sixths) using the words (greater than, less than or equal to) and the symbols (>, <, =), with models.

**ESSENTIAL KNOWLEDGE AND SKILLS**

- Using same-size fraction pieces, from region/area models or length/measurement models, count the pieces (e.g., one-fourth, two-fourths, three-fourths, etc.) and compare those pieces to one whole (e.g., four-fourths will make one whole, one-fourth is less than a whole, five-fourths is more than one whole or is equal to one and one-fourth). (b) [Reordered]
The student will:

a) name and write fractions represented by a set, region, or length model; identify the parts of a set and/or region that represent fractions for halves, fourths, eighths, thirds, and fourths, sixths, eighths, and tenths;

b) write the represent fractional parts with models and with symbols; and

c) compare the unit fractions for halves, fourths, eighths, thirds, and fourths, sixths, with models, eighths, and tenths.

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<td>begin to see a fraction as the sum of unit fractions (e.g., three-fourths contains three one-fourths or four-fourths contains four one-fourths which is equal to one whole). $\frac{3}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ This provides students with a visual for when one whole is reached and develops a greater understanding of numerator and denominator.</td>
<td>Students will learn to write names for fractions greater than one and for mixed numbers in grade 3.</td>
<td>Students should have experiences dividing a whole into additional parts. As the whole is divided into more parts, students understand that each part becomes smaller (e.g., folding a paper in half one time creates 2 halves; folding it in half again, creates 4 fourths, which are smaller; folding it in half again, creates 8 eighths, which are even smaller). The same concept can be applied to thirds and sixths.</td>
</tr>
<tr>
<td>Students will learn to write names for fractions greater than one and for mixed numbers in grade 3.</td>
<td>Creating models that have a fractional value greater than one whole and describing those models as having a whole and leftover equal-sized pieces are the foundation for understanding mixed numbers in grade 3.</td>
<td>The value of a fraction is dependent on both the number of</td>
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</table>
STANDARD 2.43  STRAND: NUMBER AND NUMBER SENSE  GRADE LEVEL 2

2.43 The student will
   a) name and write fractions represented by a set, region, or length model identify the parts of a set and/or region that represent fractions for halves, fourths, eighths, thirds, and fourths, sixths, eighths, and tenths;
   b) write the represent fractional parts with models and with symbols; and
   c) compare the unit fractions for halves, fourths, eighths, thirds, and fourths, sixths, with models, eighths, and tenths.

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<td>equal equivalent parts in a whole (denominator) and the number of those parts being considered (numerator).</td>
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<tr>
<td>The denominator tells how many equal parts are in the whole or set. The numerator tells how many of those parts are being described.</td>
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<td>Students should have opportunities to make connections among fraction representations by connecting concrete or pictorial representations with spoken or symbolic representations.</td>
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<tr>
<td>Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions will further this development (e.g., thirds means “three equal parts of a whole” or $\frac{1}{3}$ represents one of three equal-size parts when a pizza is shared among three students) will further this development.</td>
<td></td>
<td></td>
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<tr>
<td>A unit fraction is when there is a one as in which the numerator is one.</td>
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<td></td>
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<tr>
<td>Using models when comparing unit fractions builds a mental image of fractions and the understanding that as the number of pieces of a whole increases, the size of one single piece decreases will assist in developing the concept that (i.e., the larger the denominator the smaller the piece; therefore, $\frac{1}{3} &gt; \frac{1}{4}$).</td>
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</table>
A variety of contexts and problem types are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from real-life experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as “How many books do we have altogether if Jackie gives us five more?” or “How many cookies are left if I eat two? About how many students are at two tables?” or “I have three more candies than you do.” Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling, discussing, and recording a variety of problem situations. This approach helps students transition from the concrete to the representation to the symbolic in order to develop meaning for the operations and how they relate to each other.

In grades K–23, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition and subtraction within 20, multiplication, division and related facts;
- gaining facility in manipulating whole numbers to add and subtract, and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as skip counting and solving problems that involve equal groupings of objects and as well as problems that involve sharing equally, the initial work of performing initial operations with fractions.
The student will recall:

a) recognize and use the relationships between addition and subtraction to solve single-step practical problems, with whole numbers to 20; and [Moved from 2.9]

b) demonstrate fluency with addition and subtraction facts with sums to within 20 or less and the corresponding subtraction facts.

### UNDERSTANDING THE STANDARD

**Background Information for Instructor Use Only**

- Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently.
- Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship.
- Associate the terms addition, adding, plus, and sum with the concept of joining or combining.
- Associate the terms subtraction, subtracting, minus, and difference with the concept of process of “taking away” or separating (i.e., removing a set of objects from the given set of objects, finding determining the difference between two numbers, or comparing two numbers).
- Concrete models should be used initially to develop an understanding of addition and subtraction facts.
- Recognizing and using patterns and learning to represent situations mathematically are important aspects of primary mathematics. [Moved from EKS 2.21]
- An equation (number sentence) is a mathematical statement that two expressions are equivalent. It consists of two expressions, one on each side of an ‘equal’ symbol (e.g., \( 5 + 3 = 8 \), \( 8 = 5 + 3 \) and \( 4 + 3 = 9 - 2 \)).
- An equation can be represented using balance scales, with equal amounts on each side (e.g., \( 3 + 5 = 6 + 2 \)).
- An expression is a representation of a quantity. It contains numbers, variables, and/or computational operation symbols. It does not have an equal sign (e.g., \( 5, 4 + 3, 8 - 2 \)). It is not necessary for students at this level to use the term ‘expression.’

### ESSENTIAL UNDERSTANDINGS

- All students should:
  - Understand that addition involves combining and subtraction involves separating.
  - Develop fluency in recalling facts for addition and subtraction.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

- Recognize and use the relationship between addition and subtraction to solve single-step practical problems, with whole numbers to 20. (ba)
- Determine the missing number in an equation (number sentence) (e.g., \( 3 + \square = 5 \) or \( \square + 2 = 5 \); \( 5 - \square = 3 \) or \( 5 - 2 = \square \)). (ba) [Moved from 2.9 EKS]
- Write the related facts for a given addition or subtraction fact (e.g., given \( 3 + 4 = 7 \), write \( 7 - 4 = 3 \) and \( 7 - 3 = 4 \)). (ba) [Moved from 2.9 EKS]
- Recall and write the basic Demonstrate fluency with addition facts for sums to 20 or less and the corresponding subtraction within 20 facts, when addition or subtraction problems are presented in either horizontal or vertical written format. (a) [Moved to US] [Reordered]
STANDARD 2.5  STRAND: COMPUTATION AND ESTIMATION  GRADE LEVEL 2

2.5  The student will recall
   a) recognize and use the relationships between addition and subtraction to solve single-step practical problems, with whole numbers to 20; and [moved from 2.9]
   b) demonstrate fluency with addition and subtraction facts with sums to within 20 or less and the corresponding subtraction facts.

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<tbody>
<tr>
<td>• The patterns formed by related facts facilitate the solution of problems involving a missing addend in an addition sentence or a missing part in a subtraction sentence. [moved from EKS 2.21]</td>
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<tr>
<td>• Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the basic addition and subtraction facts include</td>
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<td>- counting on;</td>
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<td>- counting back;</td>
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<tr>
<td>- “one more than,” “two more than” facts;</td>
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<tr>
<td>- “one less than,” “two less than” facts;</td>
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<tr>
<td>- “doubles” to recall addition facts (e.g., 2 + 2 = □; 3 + 3 = □);</td>
</tr>
<tr>
<td>- “near doubles” [e.g., 3 + 4 = (3 + 3) + 1 = □];</td>
</tr>
<tr>
<td>- “make ten” facts (7 + 4 can be thought of as 3 + 3 + 1 in order to make a ten) [e.g., at least one addend of 8 or 9];</td>
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<tr>
<td>- “think addition for subtraction,” (e.g., for 9 – 5 = □, think “5 and what number makes 9?”);</td>
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<tr>
<td>- use of the commutative property without naming the property (e.g., 4 + 3 is the same as 3 + 4);</td>
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<tr>
<td>- use of related facts (e.g., 4 + 3 = 7, 3 + 4 = 7, 7 – 4 = 3, and 7 – 3 = 4); and</td>
</tr>
<tr>
<td>- use of the additive identity property (e.g., 4 + 0 = 4) without naming the property but saying, “When you add zero to a number, you always get the original number”; and</td>
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<tr>
<td>- use patterns to make sums (e.g., 0 + 5 = 5, 1 + 4 = 5, 2 + 3 = 5, etc.)</td>
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<tr>
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<tbody>
<tr>
<td>• Grade 2 students should begin to explore the properties of addition as strategies for solving addition and subtraction problems using a variety of representations.</td>
</tr>
</tbody>
</table>
The student will recall

a) recognize and use the relationships between addition and subtraction to solve single-step practical problems, with whole numbers to 20; and [Moved from 2.9]

b) demonstrate fluency with addition and subtraction facts with sums to within 20 or less and the corresponding subtraction facts.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- The properties of the operations are “rules” about how numbers work and how they relate to one another. Students at this level do not need to use the formal terms for these properties but should utilize these properties to further develop flexibility and fluency in solving problems. The following properties are most appropriate for exploration at this level:
  - The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4).
  - The identity property of addition states that if zero is added to a given number, the sum is the same as the given number (e.g., 0 + 2 = 2).
  - The associative property of addition states that the sum stays the same when the grouping of addends is changed [e.g., 4 + (6 + 7) = (4 + 6) + 7].

- Addition and subtraction problems should be presented in both horizontal and vertical written format.

- Equations may be written with sums and differences at the beginning of the equation (e.g., 8 = 5 + 3). [Moved from EKS]

- Models such as 10 or 20 frames and part-part-whole diagrams help develop an understanding of relationships between equations and operations.

- Manipulatives should be used initially to develop an understanding of addition and subtraction facts and to engage students in meaningful memorization. Rote recall of the facts is often achieved through constant practice and may come from a

ESSENTIAL UNDERSTANDINGS

ESSENTIAL KNOWLEDGE AND SKILLS
2.5 The student will recall

a) recognize and use the relationships between addition and subtraction to solve single-step practical problems, with whole numbers to 20; and [Moved from 2.9]

b) demonstrate fluency with addition and subtraction facts with sums to within 20 or less and the corresponding subtraction facts.

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<td>variety of formats, including presentation through counting on, related facts, flash cards, practice sheets, and/or games.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The student, given two whole numbers whose sum is 99 or less, will

a) estimate the sums and differences; and [Differences moved from 2.7a]
b) find the determine sums and differences, using various methods of calculation; and [Differences moved from 2.7b]
c) create and solve single-step and two-step practical problems involving addition and subtraction. [Moved from 2.8 and 2.21]

### UNDERSTANDING THE STANDARD
**Background Information for Instructor Use Only**
- An equation (number sentence) is a mathematical statement representing two expressions that are equivalent. It consists of two expressions, one on each side of an 'equal' symbol (e.g., $5 + 3 = 8, 8 = 5 + 3$ and $4 + 3 = 9 - 2$). An equation can be represented using a balance scale, with equal amounts on each side (e.g., $3 + 5 = 6 + 2$).
- An expression represents a quantity. It contains numbers, variables, and/or computational operation symbols. It does not have an equal symbol (e.g., $5, 4 + 3, 8, 2$). At this level, students are not expected to understand the terms expression or variable.
- Estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed.
- Estimation is a number sense skill used instead of finding an exact answer. When an actual computation is not necessary, an estimate will suffice.
- Estimation also can be used before solving a problem to check the reasonableness of the sum or difference when an exact answer is required.
- Rounding is one strategy used to estimate.
- Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship.
- Grade 2 students should begin to explore the properties of addition as strategies for solving addition and subtraction problems using a variety of representations, including manipulatives and diagrams.
- The properties of the operations are “rules” about how numbers

### ESSENTIAL UNDERSTANDINGS
All students should
- Understand that estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed.
- Understand that estimation skills are also valuable in determining the reasonableness of the sum when solving for the exact answer is needed.
- Understand that addition is used to join groups in practical situations when exact answers are needed.
- Develop flexible methods of adding whole numbers by combining numbers in a variety of ways to find the sum, most depending on place values.

### ESSENTIAL KNOWLEDGE AND SKILLS
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
- Regroup 10 ones for 1 ten, using Base 10 models, when finding the sum of two whole numbers whose sum is 99 or less.
- Estimate the sum of two whole numbers whose sum is 99 or less and recognize whether the estimation is reasonable (e.g., $27 + 41$ is about 70, because 27 is about 30 and 41 is about 40, and $30 + 40 = 70$). (a)
- Estimate the difference between two whole numbers each 99 or less and recognize whether the estimate is reasonable. (a)
- Find-Determine the sum of two whole numbers whose sum is 99 or less, using various methods, using Base 10 models, such as Base 10 blocks and bundles of tens. (b)
- Determine the difference of two whole numbers each 99 or less, using various methods. (b) [Moved to 2.6b]
- Create and solve single-step practical problems involving addition and subtraction. (c)
- Create and solve two-step practical problems involving addition, subtraction, or both addition and subtraction. (c)
- Solve single-step and two-step practical problems presented vertically or horizontally that require finding determining the sum of two whole numbers whose sum is 99 or less, using paper and pencil. (c)
- Solve problems, using mental computation strategies, involving addition of two whole numbers whose sum is 99 or
2.6 The student, given two whole numbers whose sum is 99 or less, will
   a) estimate the sums and differences; and
   b) find the sums and differences, using various methods of calculation;
   c) create and solve single-step and two-step practical problems involving addition and subtraction.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>work and how they relate to one another. Students at this level do not need to use the formal terms for these properties but should utilize these properties to further develop flexibility and fluency in solving problems. The following properties are most appropriate for exploration at this level:</td>
<td>• The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., $4 + 3 = 3 + 4$).</td>
<td>• Solve single-step and two-step practical problems that require determining the difference between whole numbers each 99 or less. (c)</td>
</tr>
<tr>
<td>• The identity property of addition states that if zero is added to a given number, the sum is the same as the given number.</td>
<td>• The associative property of addition states that the sum stays the same when the grouping of addends is changed (e.g., $4 + (6 + 7) = (4 + 6) + 7$).</td>
<td></td>
</tr>
<tr>
<td>• Extensive research has been undertaken over the last several decades regarding different problem types. Many of these studies have been published in professional mathematics education publications using different labels and terminology to describe the varied problem types.</td>
<td>• Students should experience a variety of problem types related to addition and subtraction. Problem type examples are included in the following chart:</td>
<td></td>
</tr>
</tbody>
</table>
2.6 The student, given two whole numbers whose sum is 99 or less, will
a) estimate the sums and differences; and [Differences moved from 2.7a]
b) find the determine sums and differences, using various methods of calculation; and [Differences moved from 2.7b]
c) create and solve single-step and two-step practical problems involving addition and subtraction.[Moved from 2.8 and 2.21]

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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GRADE 2: COMMON ADDITION AND SUBTRACTION PROBLEM TYPES</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Join (Result Unknown)</td>
<td>Join (Change Unknown)</td>
<td>Join (Start Unknown)</td>
</tr>
<tr>
<td>Sue had 28 pencils. Alex</td>
<td>Sue had 28 pencils. Alex</td>
<td>Sue had some pencils. Alex</td>
</tr>
<tr>
<td>gave her 14 more pencils. How many pencils does Sue have all together?</td>
<td>gave her some more pencils. Now Sue has 42 pencils. How many did Alex give her?</td>
<td>gave her 14 more. Now Sue has 42 pencils. How many pencils did Sue have to start with?</td>
</tr>
<tr>
<td>Separate (Result Unknown)</td>
<td>Separate (Change Unknown)</td>
<td>Separate (Start Unknown)</td>
</tr>
<tr>
<td>Brooke had 35 marbles.</td>
<td>Brooke had 35 marbles.</td>
<td>Brooke had some marbles.</td>
</tr>
<tr>
<td>She gave 19 marbles to Joe. How many marbles does Brooke have now?</td>
<td>She gave some to Joe. She has 16 marbles left. How many marbles did Brooke give to Joe?</td>
<td>She gave 19 to Joe. Now she has 16 marbles left. How many marbles did Brooke start with?</td>
</tr>
<tr>
<td>Part-Part-Whole (Whole</td>
<td>Part-Part-Whole (One Part Unknown)</td>
<td>Part-Part-Whole (Both Parts Unknown)</td>
</tr>
<tr>
<td>Unknown)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The teacher has 20 red</td>
<td>The teacher has 45</td>
<td>The teacher has a tub of red</td>
</tr>
<tr>
<td>markers and 25 blue</td>
<td>markers. Twenty of the</td>
<td>and blue markers. She has</td>
</tr>
<tr>
<td>markers. How many</td>
<td>markers are red, and the</td>
<td>45 markers in all. How many</td>
</tr>
<tr>
<td>markers does he have?</td>
<td>rest are blue. How</td>
<td>many markers could be red?</td>
</tr>
<tr>
<td>Reference (Difference</td>
<td>many blue markers does</td>
<td>How many could be blue?</td>
</tr>
<tr>
<td>Unknown)</td>
<td>he have?</td>
<td></td>
</tr>
<tr>
<td>Ryan has 20 books and</td>
<td>Ryan has 9 books. Ryan</td>
<td>Ryan has 11 more books than</td>
</tr>
<tr>
<td>Chris has 9 books. How</td>
<td>has 11 more books than</td>
<td>Chris. Ryan has 20 books. How</td>
</tr>
<tr>
<td>many more books does</td>
<td>Chris. How many books</td>
<td>many books does Chris have?</td>
</tr>
<tr>
<td>Ryan have than Chris?</td>
<td>does Ryan have?</td>
<td></td>
</tr>
<tr>
<td>Ryan has 20 books. Chris</td>
<td>Ryan has 11 fewer books</td>
<td>Ryan has 11 fewer books than</td>
</tr>
<tr>
<td>has 9 books. How many</td>
<td>than Ryan. Chris has 9</td>
<td>Ryan. Ryan has 20 books. How</td>
</tr>
<tr>
<td>fewer books does Chris</td>
<td>books. How many books</td>
<td>many books does Chris have?</td>
</tr>
<tr>
<td>have than Ryan?</td>
<td>does Ryan have?</td>
<td></td>
</tr>
</tbody>
</table>

- By estimating the result of an addition problem, a place value orientation for the answer is established.
STANDARD 2.6 STRAND: COMPUTATION AND ESTIMATION GRADE LEVEL 2

2.6 The student, given two whole numbers whose sum is 99 or less, will
a) estimate the sums and differences; and
b) find the determine sums and differences, using various methods of calculation; and

2.6c) create and solve single-step and two-step practical problems involving addition and subtraction.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as in all, altogether, difference, etc. encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions as well as challenges in upcoming grades and courses.

- Strategies for adding and subtracting two-digit numbers can include, but are not limited to, using concrete objects, a hundred chart, number line, and invented strategies.

- Mental computation helps build number sense in students. Strategies for mentally adding or subtracting two-digit numbers should be included. Some of these strategies may include: making ten, partial sums, and counting on, among others.

- Partial sums:
  
  \[
  56 + 41 = \_
  \]
  
  \[
  50 + 40 = 90 \]
  
  \[
  6 + 1 = 7 \]
  
  \[
  90 + 7 = 97 \]

- Counting on:
  
  \[
  36 + 62 = \_
  \]
  
  \[
  36 + 60 = 96 \]
  
  \[
  96 + 2 = 98 \]

- Partial Sums

<table>
<thead>
<tr>
<th>Partial Sums</th>
<th>Counting On</th>
</tr>
</thead>
<tbody>
<tr>
<td>56 + 41 =</td>
<td>36 + 62 =</td>
</tr>
<tr>
<td>30 + 40 = 90</td>
<td>36 + 60 = 96</td>
</tr>
<tr>
<td>6 + 1 = 7</td>
<td>96 + 2 = 98</td>
</tr>
<tr>
<td>90 + 7 = 97</td>
<td></td>
</tr>
</tbody>
</table>

- Addition means to combine or join quantities.
- The terms used in addition are...
2.6 The student, given two whole numbers whose sum is 99 or less, will
a) estimate the sums and differences; and [Differences moved from 2.7a]
b) find the determine sums and differences, using various methods of calculation; and [Differences moved from 2.7b]
c) create and solve single-step and two-step practical problems involving addition and subtraction. [Moved from 2.8 and 2.21]

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Strategies for adding two-digit numbers can include, but are not limited to, using a hundreds chart, number line, and invented strategies.
- Building an understanding of the algorithm by first using concrete materials and then a do-and-write approach connects it to the written form of the algorithm.
- The traditional algorithm for two-digit numbers is contrary to the natural inclination to begin with the left-hand number.
- Regrouping is used in addition when a sum in a particular place value is 10 or greater.
- Mental computational strategies for subtracting two-digit numbers should be student-invented strategies. Some of these strategies may include:

<table>
<thead>
<tr>
<th>Operation</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>87 - 25 =</td>
<td>62</td>
</tr>
<tr>
<td>20 + 60 =</td>
<td>80</td>
</tr>
<tr>
<td>5 + 2 =</td>
<td>7</td>
</tr>
<tr>
<td>60 + 2 =</td>
<td>62</td>
</tr>
<tr>
<td>98 - 41 =</td>
<td>57</td>
</tr>
</tbody>
</table>

- The terms used in addition are

<table>
<thead>
<tr>
<th>Equation</th>
<th>Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>23 + 46</td>
<td>addend</td>
</tr>
<tr>
<td>69</td>
<td>sum</td>
</tr>
</tbody>
</table>

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 2
STANDARD 2.6  STRAND: COMPUTATION AND ESTIMATION  GRADE LEVEL 2

2.6  The student, given two whole numbers whose sum is 99 or less, will
a)  estimate the sums and differences; and
b)  find the determine sums and differences, using various methods of calculation;
and

c)  create and solve single-step and two-step practical problems involving addition and subtraction.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- The terms often used in subtraction are
  
  - minuend  →  98
  - subtrahend  →  -41
  - difference  →  57

- At this level, students do not need to use the terms for addition and subtraction shown above.

- Problem solving means engaging in a task for which a solution or a method of solution is not known in advance. Solving problems using data and graphs offers one way to connect mathematics to practical situations.

- The problem solving process is enhanced when students:
  - create their own story problems; and
  - model word problems, using manipulatives, drawings, or acting out the problem.

- The least number of steps necessary to solve a single-step problem is one.

- Using concrete materials (e.g., base-10 blocks, connecting cubes, beans and cups, etc.) to explore, model and stimulate discussion about a variety of problem situations helps students understand regrouping and enables them to move from the concrete to the abstract. Regrouping is used in addition and subtraction algorithms.

- Conceptual understanding begins with concrete and contextual experiences. Next, students must make connections that serve as a bridge to the symbolic. Student-created representations, such as drawings, diagrams, tally marks, graphs, or written comments are strategies that help students make these connections.
### STANDARD 2.7  
**STRAND: COMPUTATION AND ESTIMATION**

#### GRADE LEVEL 2

**Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 2**

2.7 The student, given two whole numbers, each of which is 99 or less, will
   a) estimate the difference; and [Moved to 2.6a]
   b) find the difference, using various methods of calculation. [Moved to 2.6b]

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
</tr>
<tr>
<td>• Estimation is a number sense skill used instead of finding an exact answer. When an estimate is needed, the actual computation is not necessary.</td>
</tr>
<tr>
<td>• Rounding is one strategy used to estimate.</td>
</tr>
<tr>
<td>• Estimation is also used before solving a problem to check the reasonableness of the sum when an exact answer is required.</td>
</tr>
<tr>
<td>• By estimating the result of a subtraction problem, a place value orientation for the answer is established.</td>
</tr>
<tr>
<td>• Subtraction is the inverse operation of addition and is used for different reasons:</td>
</tr>
<tr>
<td>- to remove one amount from another;</td>
</tr>
<tr>
<td>- to compare one amount to another; and</td>
</tr>
<tr>
<td>- to find the missing quantity when the whole quantity and part of the quantity are known.</td>
</tr>
<tr>
<td>• Three terms often used in subtraction are</td>
</tr>
<tr>
<td><strong>minuend</strong> → 98</td>
</tr>
<tr>
<td><strong>subtrahend</strong> → 41</td>
</tr>
<tr>
<td><strong>difference</strong> → 57</td>
</tr>
<tr>
<td>• Regrouping is a process of renaming a number to make subtraction easier.</td>
</tr>
<tr>
<td>• An understanding of the subtraction algorithm should be built by first using concrete materials and then employing a do-and-write approach (i.e., use the manipulatives, then record what you have done). This connects the activity to the written form of the algorithm.</td>
</tr>
<tr>
<td>• Mental computational strategies for subtracting two-digit numbers might include</td>
</tr>
<tr>
<td>- lead-digit or front-end strategy:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
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<tbody>
<tr>
<td>All students should</td>
</tr>
<tr>
<td>• Understand that estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed.</td>
</tr>
<tr>
<td>• Understand that estimation skills are also valuable in determining the reasonableness of the difference when solving for the exact answer is needed.</td>
</tr>
<tr>
<td>• Understand that subtraction is used in practical situations when exact answers are needed.</td>
</tr>
<tr>
<td>• Develop flexible methods of subtracting whole numbers to find the difference, by combining numbers in a variety of ways, most depending on place values.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Regroup 1 ten for 10 ones, using Base-10 models, such as Base-10 blocks and bundles of tens.</td>
</tr>
<tr>
<td>• Estimate the difference of two whole numbers each 99 or less and recognize whether the estimation is reasonable. [Moved to 2.6a]</td>
</tr>
<tr>
<td>• Find the difference of two whole numbers each 99 or less, using Base-10 models, such as Base-10 blocks and bundles of tens. [Moved to 2.6b]</td>
</tr>
<tr>
<td>• Solve problems presented vertically or horizontally that require finding the difference between two whole numbers each 99 or less, using paper and pencil. [Moved to 2.6b]</td>
</tr>
<tr>
<td>• Solve problems, using mental computation strategies, involving subtraction of two whole numbers each 99 or less. [Moved to 2.6 US]</td>
</tr>
</tbody>
</table>
2.7 The student, given two whole numbers, each of which is 99 or less, will
a) estimate the difference; and [Moved to 2.6a]
b) find the difference, using various methods of calculation. [Moved to 2.6b]

<table>
<thead>
<tr>
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</tr>
</thead>
</table>
| 56 – 21 = __  
50 – 20 = 30  
6 – 1 = 5  
30 + 5 = 35  
counting up:  
87 – 25 = __  
20 + 60 = 80  
5 + 2 = 7  
60 + 2 = 62  
or  
87 – 25 = __  
25 + 60 = 85  
85 + 2 = 87  
60 + 2 = 62  
or  
87 – 25 = __  
25 + 2 = 27  
27 + 60 = 87  
2 + 60 = 62  
partial differences:  
98 – 41 = __  
90 – 40 = 50  
8 – 1 = 7  
50 + 2 = 57. |

Strategies for subtracting two-digit numbers may include using a hundreds chart, number line, and invented strategies.
### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Problem solving means engaging in a task for which a solution or a method of solution is not known in advance. Solving problems using data and graphs offers a natural way to connect mathematics to practical situations.
- The ability to retrieve information from simple charts and picture graphs is a necessary prerequisite to solving problems.
- An example of an approach to solving problems is Polya’s four-step plan:
  - Understand: Retell the problem.
  - Plan: Decide what the operation is.
  - Solve: Write a number sentence.
  - Look back: Does the answer make sense?
- The problem solving process is enhanced when students create their own story problems; and model word problems, using manipulatives or drawings.

### ESSENTIAL UNDERSTANDINGS

- All students should
- Develop strategies for solving practical problems.
- Enhance problem solving skills by creating their own problems.

### ESSENTIAL KNOWLEDGE AND SKILLS

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:
  - Identify the appropriate data and the operation needed to solve an addition or subtraction problem where the data are presented in a simple table, picture graph, or bar graph. [Combined with 2.6]
  - Solve addition and subtraction problems requiring a one- or two-step solution, using data from simple tables, picture graphs, bar graphs, and everyday life situations. [Combined with 2.6]
  - Create a one- or two-step addition or subtraction problem using data from simple tables, picture graphs, and bar graphs whose sum is 99 or less. [Combined with 2.6]
2.9 The student will recognize and describe the related facts that represent and describe the inverse relationship between addition and subtraction. [Moved to 2.5b]

<table>
<thead>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Addition and subtraction are inverse operations, that is, one undoes the other:</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>3 + 4 = 7 7 – 3 = 4</td>
<td>• Understand how addition and subtraction relate to one another.</td>
<td>• Determine the missing number in a number sentence (e.g., 3 + __ = 5 or __ + 2 = 5; 5 – __ = 3 or 5 – 2 = __). [Moved to 2.5 EKS]</td>
</tr>
<tr>
<td>7 – 4 = 3 4 + 3 = 7</td>
<td></td>
<td>• Write the related facts for a given addition or subtraction fact (e.g., given 3 + 4 = 7, write 7 – 4 = 3 and 7 – 3 = 4). [Moved to 2.5 EKS]</td>
</tr>
<tr>
<td>• For each addition fact, there is a related subtraction fact.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Developing strategies for solving missing addends problems and the missing part of subtraction facts builds an understanding of the link between addition and subtraction. To solve</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 – 5 = __, think 5 + __ = 9.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Demonstrate joining and separating sets to investigate the relationship between addition and subtraction.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 2
The exploration of measurement and geometry in the primary grades allows students to learn more about the world around them. Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K–3 should encounter measurement in many normal situations, from their daily lives, from their use of the calendar and from science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, area, perimeter, volume, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both several techniques and nonstandard and standard tools to find determinations and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

[Moved from Geometry strand introduction]

Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on

- observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

Level 2: Analysis. Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
The student will
a) count and compare a collection of pennies, nickels, dimes, and quarters whose total value is $2.00 or less; and
b) correctly use the cent symbol (¢), dollar symbol ($), and decimal point (·).

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The money system used in the United States consists of coins and bills based on relationships involving ones, fives, and tens, making it easy to count money.</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• The dollar is the basic unit.</td>
<td>• Understand how to count and compare a collection of coins and one-dollar bills whose total value is $2.00 or less.</td>
<td>• Determine the value of a collection of coins and one-dollar bills whose total value is $2.00 or less. (a)</td>
</tr>
<tr>
<td>• Emphasis is placed on the verbal expression of the symbols for cents and dollars (e.g., $0.35 and 35¢ are both read as “thirty-five cents”; $3.00 is read as “three dollars”).</td>
<td>• Understand the proper use of the cent symbol (¢), dollar sign ($), and decimal point (·).</td>
<td>• Count by ones, fives, tens, and twenty-fives to determine the value of a collection of coins whose total value is $2.00 or less. (a) [Moved from 1.7 EKS]</td>
</tr>
<tr>
<td>• Money can be counted by grouping coins and bills to determine the value of each group and then adding to determine the total value.</td>
<td></td>
<td>• Compare the values of two sets of coins and one-dollar bills (each set having a total value of $2.00 or less), using the terms greater than, less than, or equal to.</td>
</tr>
<tr>
<td>• The value of a collection of coins and bills can be determined by counting on, beginning with the highest value, and/or by grouping the coins and bills into groups that are easier to count.</td>
<td></td>
<td>• Simulate everyday opportunities to count and compare a collection of coins and one-dollar bills whose total value is $2.00 or less. [Moved to US]</td>
</tr>
<tr>
<td>• Simulate everyday opportunities to count and compare a collection of coins and one-dollar bills whose total value is $2.00 or less.</td>
<td></td>
<td>• Use the cent (¢) and dollar ($) symbols and decimal point (·) to write a value of money which is $2.00 or less. (b)</td>
</tr>
<tr>
<td>• Emphasis is placed on the verbal expression of the symbols for cents and dollars (e.g., $0.35 and 35¢ are both read as “thirty-five cents”; $2.00 is read as “two dollars”).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The most common way to add amounts of money is to “count on” the amount to be added.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### UNDERSTANDING THE STANDARD

**Background Information for Instructor Use Only**

- The process of measurement involves selecting a unit of measure, comparing the unit to the object to be measured, counting the number of times the unit is used to measure the object, and arriving at an approximate total number of units. [Copied from 1.10]

- Measurement involves comparing an attribute of an object to the same attribute of the unit of measurement (e.g., the length of a cube measures the length of a book; the weight of the cube measures the weight of the book).

- A clear concept of the size of one unit is necessary before one can measure to the nearest unit.

- Knowledge of the exact relationships within the metric or U.S. Customary system of measurement for measuring liquid volume, such as 4 cups to a quart, is not required at this grade level. [Included in 4.6 and 5.8]

- Practical experiences measuring liquid volume, using a variety of actual measuring devices (e.g., containers for a cup, pint, quart, gallon, and liter), will help students build a foundation for estimating liquid volume with these measures.

- The experience of making a ruler out of individual units of length can lead to greater understanding of using one. A ruler takes those units of length and numbers them. Measurement of length is counting the number of units. A “broken ruler” is a useful tool for students to use in order to develop an understanding of counting the number of units.

- Students benefit from experiences that allow them to explore the relationship between the size of the unit of measurement and the number of units needed to measure the length of an object.

### ESSENTIAL UNDERSTANDINGS

- All students should
  - Understand that centimeters/inches are units used to measure length.
  - Understand how to estimate and measure to determine a linear measure to the nearest centimeter and inch.
  - Understand that pounds/ounces and kilograms/grams are units used to measure weight/mass.
  - Understand how to use a scale to determine the weight/mass of an object and use the appropriate unit for measuring weight/mass.
  - Understand that cups, pints, quarts, gallons, and liters are units used to measure liquid volume.
  - Understand how to use measuring devices to determine liquid volume in both metric and customary units.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify a ruler as an instrument to measure length. (a) [Moved from K.8 EKS]
- Estimate and then measure the length of various line segments and objects to the nearest inch and centimeter using a ruler. (a)
- Identify different types of scales as instruments to measure weight/mass. (Moved from K.8 EKS) (b)
- Estimate and then measure the weight/mass of objects to the nearest pound, ounces, and kilograms, using a scale. (b)
- Estimate and measure liquid volume in cups, pints, quarts, gallons, and liters. [Included in 3.8 EKS]
2.811 The student will estimate and measure
   a) length to the nearest centimeter and inch; [Centimeters included in 3.8]
   b) weight/mass of objects into the nearest pounds/ounces and kilograms/grams, using a scale; and, [Ounces, kilograms/grams included in 4.8]
   c) liquid volume in cups, pints, quarts, gallons, and liters. [Included in 3.8]

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>• Proper placement of a ruler when measuring length (i.e., placing the end of the ruler at one end of the item to be measured) should be demonstrated.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object’s mass, although they use the term weight (e.g., “How much does it weigh?” versus “What is its mass?”).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• A balance is a scale for measuring mass. To determine the mass of an object by using a two-pan balance, first level both sides of the balance by putting standard units of mass on one side to counterbalance the object on the other, then find the sum of the standard units of mass required to level the balance.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Benchmarks of common objects need to be established for one pound and one kilogram. Practical experience measuring the mass/weight of familiar objects helps to establish benchmarks.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Pounds and kilograms are not compared at this level.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The terms cups, pints, quarts, gallons, and liters are introduced as terms used to describe the liquid volume of everyday containers.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The exact relationship between a quart and a liter is not expected at this level.</td>
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<td></td>
</tr>
</tbody>
</table>
2.912 The student will tell and write time to the nearest five minutes, using analog and digital clocks.

<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>• Telling time requires reading a clock. The position of the two hands on an analog clock is read to tell the time. A digital clock shows the time by displaying the time in numbers which are read as the hour and minutes.</td>
<td>All students should  ■ Apply an appropriate technique to determine time to the nearest five minutes, using analog and digital clocks.  ■ Demonstrate an understanding of counting by fives to predict five minute intervals when telling time to the nearest five minutes.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to  ■ Show, tell, and write time to the nearest five minutes, using an analog and digital clock.  ■ Match a written time (e.g., 4:20, 10:05, 1:50) to a time shown on a clock face to the nearest five minutes.  ■ Match the time (to the nearest five minutes) shown on a clock face to a written time.</td>
</tr>
<tr>
<td>• Counting by fives is beneficial when telling time to the nearest five minutes.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Students should develop an understanding that there are sixty minutes in an hour.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The use of a demonstration clock with gears ensures that the positions of the hour hand and the minute hand are precise at all times.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The face of an analog clock can be divided into 4 equal parts, called quarter hours, of 15 minutes each.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.1013 The student will
a) determine past and future days of the week; and
b) identify specific days and dates on a given calendar.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>All students should understand how to use a calendar as a way to measure time.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:</td>
</tr>
</tbody>
</table>
| • The calendar is a way to represent units of time (e.g., days, weeks, and months, and years). | | • Determine the days/dates before and after a given day/date.  
  (a)[Included in K.8 and 1.9] |
| • Using a calendar develops the concept of day as a 24-hour period rather than a period of time from sunrise to sunset. | | • Determine the day that is a specific number of days or weeks in the past or in the future from a given date, using a calendar.  (a) |
| • Practical situations are appropriate to develop a sense of the interval of time between events (e.g., Boy Scout club meetings occur every week on Monday: there is a week between meetings). | | • Identify specific days and dates (e.g., What is the third Monday in a given month? On or what day of the week does May 11th fall?  (b) |
The student will read the temperature on a Celsius and/or Fahrenheit thermometer to the nearest 10 degrees. [Celsius/Fahrenheit included in EKS]

**UNDERSTANDING THE STANDARD**

(Background Information for Instructor Use Only)

- The symbols for degrees in Celsius (°C) and degrees in Fahrenheit (°F) should be used to write temperatures.
- Celsius and Fahrenheit temperatures should be related to everyday occurrences by measuring the temperature of the classroom, the outside, liquids, body temperature, and other things found in the environment (e.g., classroom; temperature on the playground; temperature of warm and cold liquids; body temperature).
- Estimating and measuring temperatures in the environment in Fahrenheit and Celsius require the use of real thermometers.
- A variety of physical models (e.g., circular, linear) should be used to represent the temperature determined by a real thermometer.
- Reading temperature in degrees Celsius will begin in grade 3.

**ESSENTIAL UNDERSTANDINGS**

All students should
- Understand how to measure temperature in Celsius and Fahrenheit with a thermometer.

**ESSENTIAL KNOWLEDGE AND SKILLS**

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify different types of thermometers as instruments used to measure temperature.[Moved from K.8 EKS]
- Read temperature in Celsius and Fahrenheit to the nearest ten degrees on thermometers (real world, physical model, and pictorial representation). Read temperature to the nearest 10 degrees from real Celsius and Fahrenheit thermometers and from physical models (including pictorial representations) of such thermometers.
Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on
- observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to these figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
2.1215 The student will
a) draw a line of symmetry in a figure; and
b) identify and create figures with at least one line of symmetry.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- A line of symmetry divides a symmetrical figure, object, or arrangement of objects into two congruent parts, that are congruent if one part is reflected over the line of symmetry. Each of which is the mirror image of the other. An example is shown below:

```
/   /
  | |
  | |
  v
```

Lines of symmetry are not limited to horizontal and vertical lines.

- Children learn about symmetry through hands-on experiences with geometric figures and the creation of geometric pictures and patterns.

- Guided explorations of the study of symmetry by using mirrors, miras, paper folding, and pattern blocks will enhance students’ understanding of the attributes of symmetrical figures.

- Congruent figures have exactly the same size and shape.

- Noncongruent figures do not have exactly the same size and shape. Congruent figures remain congruent even if they are in different spatial orientations.

- While investigating symmetry, children move figures, such as pattern blocks, intuitively, thereby exploring transformations of those figures. A transformation is the movement of a figure — either a translation, rotation, or reflection. A translation is the result of sliding a figure in any direction; rotation is the result of turning a figure around a point or a vertex; and reflection is the result of flipping a figure over a line. Children at this level do not need to know the terms related to transformations of figures.

### ESSENTIAL UNDERSTANDINGS

- All students should
  - Develop strategies to determine whether or not a figure has at least one line of symmetry.
  - Develop strategies to create figures with at least one line of symmetry.
  - Understand that some figures may have more than one line of symmetry.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Draw a line of symmetry in a figure. (a) [Reordered]
- Identify figures with at least one line of symmetry, using various concrete materials (e.g., mirrors, paper folding, pattern blocks). (b)
- Draw a line of symmetry — horizontal, vertical, and diagonal — in a figure. [Moved to US]
- Determine a line of symmetry that results in two congruent parts — figures that have the same size and shape and explain reasoning. (a, b)
- Create figures with at least one line of symmetry using various concrete materials. (b)
2.1316 The student will identify, describe, compare, and contrast plane and solid geometric figures (circle/sphere, square/cube, and rectangle/rectangular prism).

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- A plane figure is any closed, two-dimensional shape that can be drawn in a plane.
- The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.
  - **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
  - **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., “I know it’s a rectangle because it looks like a door, and I know that a door is a rectangle.”).
  - **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., “I know it’s a rectangle because it is closed; it has four sides and four right angles, and opposite sides are parallel.”).

- An important part of geometry is naming and describing figures in two dimensions (plane figures) and three dimensions (solid figures).
  - A vertex is a point at which two or more lines, line segments, lines, or rays meet to form an angle. In solid figures a vertex is the point at which three or more edges meet.
  - An angle is formed by two rays that share a common endpoint called the vertex. Angles are found wherever lines or line segments intersect.
  - A **plane figure** is any two-dimensional shape that can be drawn in a plane.

### ESSENTIAL UNDERSTANDINGS

- All students should
  - Understand the differences between plane and solid figures while recognizing the inter-relatedness of the two.
  - Understand that a solid figure is made up of a set of plane figures.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Determine similarities and differences between related plane and solid figures (e.g., circle/sphere, square/cube, rectangle/rectangular prism), using models and cutouts.

- Trace faces of solid figures (e.g., cube and rectangular prism) to create the set of plane figures related to the solid figure.

- Identify and describe plane and solid figures (e.g., circle/sphere, square/cube, rectangle/rectangular prism), according to their characteristics (the number of sides, vertices, and angles) and shape of their faces, edges, and vertices using models. Squares and rectangles have four right angles.

- Identify and describe solid figures (e.g., sphere, cube, and rectangular prism), according to the shape of their faces, number of edges, and number of vertices, using models.

- Compare and contrast plane and solid geometric figures (e.g., circle/sphere, square/cube, and rectangle/rectangular prism) according to their characteristics (number and shape of their faces, edges, bases, edges, vertices, and angles).
2.1316 The student will identify, describe, compare, and contrast plane and solid geometric figures (circle/sphere, square/cube, and rectangle/rectangular prism).

<table>
<thead>
<tr>
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<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>drawn in a plane</strong> figures formed by lines that are curved, straight, or a combination of both. They have angles and sides.**</td>
<td><strong>The identification of plane and solid figures is accomplished by working with and handling objects.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>A solid figure is a three-dimensional figure, having length, width, and height.</strong></td>
<td><strong>A solid figure is a three-dimensional figure, having length, width, and height.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Tracing faces of solid figures is valuable to understanding the set of plane figures related to the solid figure (e.g., cube and rectangular prism).</strong> [Reordered]</td>
<td><strong>Tracing faces of solid figures is valuable to understanding the set of plane figures related to the solid figure (e.g., cube and rectangular prism).</strong> [Reordered]</td>
<td></td>
</tr>
<tr>
<td><strong>A circle is a closed curve in a plane with all its points in a plane that are the same distance from a point called the center.</strong></td>
<td><strong>A circle is a closed curve in a plane with all its points in a plane that are the same distance from a point called the center.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>A sphere is a solid figure with all of its points the same distance from its center.</strong></td>
<td><strong>A sphere is a solid figure with all of its points the same distance from its center.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>A rectangle is a quadrilateral plane figure with four right angles. A square is a special type of rectangle.</strong></td>
<td><strong>A rectangle is a quadrilateral plane figure with four right angles. A square is a special type of rectangle.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>The edge is the line segment where two faces of a solid figure intersect.</strong></td>
<td><strong>The edge is the line segment where two faces of a solid figure intersect.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>A face is any flat side of a polygon that serves as one side of a solid figure (e.g., a square is a face of a cube).</strong></td>
<td><strong>A face is any flat side of a polygon that serves as one side of a solid figure (e.g., a square is a face of a cube).</strong></td>
<td></td>
</tr>
<tr>
<td><strong>A square is a rectangle with four congruent (equal length) sides of equal length and four right angles.</strong></td>
<td><strong>A square is a rectangle with four congruent (equal length) sides of equal length and four right angles.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>A right angle measures exactly 90°.</strong></td>
<td><strong>A right angle measures exactly 90°.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>A rectangular prism is a solid figure in which all six faces are rectangles. A rectangular prism has 8 vertices and 12 edges.</strong></td>
<td><strong>A rectangular prism is a solid figure in which all six faces are rectangles. A rectangular prism has 8 vertices and 12 edges.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>A cube is a solid figure with six congruent, square faces. All edges are the same length. A cube has 8 vertices and 12 edges. It is a type of rectangular prism.</strong></td>
<td><strong>A cube is a solid figure with six congruent, square faces. All edges are the same length. A cube has 8 vertices and 12 edges. It is a type of rectangular prism.</strong></td>
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</tr>
</tbody>
</table>
The student will identify, describe, compare, and contrast plane and solid geometric figures (circle/sphere, square/cube, and rectangle/rectangular prism).

- A base is a special face of a solid figure.
- Tracing faces of solid figures is valuable to cubes and rectangular prisms and decomposing cubes and rectangular prisms along their edges helps students understand the set of plane figures related to the solid figure (e.g., cube and rectangular prism).
- The relationship between plane and solid geometric figures, such as the square and the cube or the rectangle and the rectangular prism helps build the foundation for future geometric study of faces, edges, angles, and vertices.

The following chart defines the characteristics of solid geometric figures included at this grade level:

<table>
<thead>
<tr>
<th>Solid Geometric Figure</th>
<th># of Faces</th>
<th>Shape of Faces</th>
<th># of Edges</th>
<th># of Vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube</td>
<td>6</td>
<td>Squares</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>Rectangular Prism</td>
<td>6</td>
<td>Rectangles</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>Square Pyramid</td>
<td>5</td>
<td>Square/Triangles</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>Sphere</td>
<td>0</td>
<td>N/A</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cone</td>
<td>1</td>
<td>Circles</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Cylinder</td>
<td>2</td>
<td>Circles</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

* The most commonly referenced cones and cylinders have circular bases.
Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of which events are more or less likely to occur, using the terms impossible, unlikely, equally likely, more likely, and certain.

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.
The student will use data from experiments to construct picture graphs, pictographs, and bar graphs.

**a)** collect, organize, and represent data in picture graphs and bar graphs; and

**b)** read and interpret data represented in picture graphs and bar graphs. [Moved from 2.19]

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Data can be collected and organized in picture graphs and bar graphs.[Moved from center column.]

- The purpose of a graph is to represent data gathered to answer a question.

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*Our Favorite Pets*

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cat</td>
<td>Dog</td>
<td>Horse</td>
<td>Fish</td>
</tr>
<tr>
<td><img src="image" alt="Cat" /></td>
<td><img src="image" alt="Dog" /></td>
<td><img src="image" alt="Horse" /></td>
<td><img src="image" alt="Fish" /></td>
</tr>
</tbody>
</table>

- Picture graphs (also called pictographs) are graphs that use pictures to show and compare information. An example of a picture graph is:

- At this level, the number of categories on a pictograph should be limited to four when a student is creating a graph, and six when a student is interpreting and analyzing a graph.

- A picture pictograph uses pictures or symbols to represent one or more objects (data points). [Moved to US]

### ESSENTIAL UNDERSTANDINGS

All students should

- Understand that data may be generated from experiments.

- Understand how data can be collected and organized in picture graphs, pictographs, and bar graphs. [Moved to US]

- Understand that picture graphs use pictures to show and compare data.

- Understand that pictographs use a symbol of an object, person, etc.

- Understand that bar graphs can be used to compare categorical data.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Collect and Organize data from experiments, using various forms of data collection (e.g., lists, tables, objects, pictures, symbols, tally marks, and charts) in order to construct a graph. Data points, collected by students, should be limited to 16 or fewer for no more than four categories. (a)

- Read the information presented horizontally and vertically on picture graphs, pictographs, and bar graphs.

- Collect no more than 16 pieces of data to answer a given question.

- Represent data from experiments by constructing picture graphs, pictographs, and bar graphs (limited to 16 or fewer data points for no more than four categories). (a)

- Label the axes on a bar graph, limiting the number of categories (categorical data) to four and the increments to multiples of whole numbers (e.g., multiples of 1, 2, or 5).[Moved to EKSUS]

- On a pictograph, limit the number of categories to four and include a key where appropriate. [Moved to EKSUS]

- Analyze information displayed in picture graphs and interpret data represented in pictographs and bar graphs with up to 25 data points for no more than 6 categories (represented horizontally or vertically). State orally and in writing (at least one statement) that includes one or more of the following:
2.15417 The student will use data from experiments to construct picture graphs, pictographs, and bar graphs.  
a) collect, organize, and represent data in picture pictographs and bar graphs; and  
b) read and interpret data represented in picture pictographs and bar graphs.  

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
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<tbody>
<tr>
<td><strong>ESSENTIAL UNDERSTANDINGS</strong></td>
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<tr>
<td><strong>ESSENTIAL KNOWLEDGE AND SKILLS</strong></td>
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<tr>
<td>(Background Information for Instructor Use Only)</td>
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</table>

- A key is provided in a graph to assist in the analysis of the displayed data. [Moved from 2.19 US]
- The key should be provided for the symbol in a picture pictograph graph when the symbol represents more than one piece of data (e.g., \(\square\) represents five people in a graph). At this level, each symbol should represent 1, 2, 5, or 10 pieces of data.
- Pictographs are graphs that use symbols to show and compare information. A student can be represented as a stick figure in a pictograph. A key should be used to indicate what the symbol represents (e.g., one picture of a sneaker represents five sneakers in a graph of shoe types). An example of a pictograph picture pictograph is:

<table>
<thead>
<tr>
<th>Cat</th>
<th>Dog</th>
<th>Horse</th>
<th>Fish</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Cat" /></td>
<td><img src="image" alt="Dog" /></td>
<td><img src="image" alt="Horse" /></td>
<td><img src="image" alt="Fish" /></td>
</tr>
</tbody>
</table>

\(\square = 4\) students

- In prior grades, students worked with simple pictographs with a scale of one (i.e., each picture represented only one item) making a key unnecessary.
- Students’ prior knowledge and work with skip counting helps them to identify the number of pictures or symbols to be used in a

- Describes the categories of data and the data as a whole (e.g., adding together all data points will equal the total number of responses);
- Identifies parts of the data that have special characteristics; including categories with the greatest, the least, or the same;
- Uses the data to make comparisons; and
- Makes predictions and generalizations. (b) [Moved from 2.19 EKS]
2.15417 The student will use data from experiments to construct picture graphs, pictographs, and bar graphs.  
  a) collect, organize, and represent data in picture pictographs and bar graphs; and  
  b) read and interpret data represented in picture pictographs and bar graphs. [Moved from 2.19]

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<tbody>
<tr>
<td>pictograph.</td>
<td></td>
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</tr>
<tr>
<td>• Definitions for the terms picture graph and pictographs vary. Pictographs are most often defined as a pictorial representation of numerical data. The focus of instruction should be placed on reading and using the key in analyzing the graph. There is no need for students to distinguish between a picture graph and a pictograph.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| • Bar graphs are used to compare counts of different categories (categorical data). Using grid paper may ensure more accurate graphs.  
  – A bar graph uses parallel, horizontal or vertical parallel bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category.  
  – There is space before, between, and after each of the bars.  
  – The axis displaying the scale that represents the count for the categories should begin at zero and extend one increment above the greatest recorded piece of data. **Second In grade 2 students should be collecting data that are recorded in increments of whole numbers limited to, usually – multiples of 1, 2, or 5.**  
  – At this level, the number of categories on a picture bar graph should be limited to four. A key should be included where appropriate. [Moved from EKS]  
  – Each axis should be labeled, and the graph should be given a title. | | |
| • Statements that represent an analysis and interpretation of the data in the graph should be discussed with students and written (e.g., similarities and differences, least and greatest, the categories, total number of responses, etc.). [Moved from 2.19 US] | | |
| • Data gathered and displayed by students should be limited to 16 or fewer data points for no more than 4 categories. However, | | |
2.15417 The student will use data from experiments to construct picture graphs, pictographs, and bar graphs. 
a) collect, organize, and represent data in picture pictographs and bar graphs; and 
b) read and interpret data represented in picture pictographs and bar graphs. [Moved from 2.19]
The student will use data from probability experiments to predict outcomes when the experiment is repeated.

### UNDERSTANDING THE STANDARD

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<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
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</thead>
<tbody>
<tr>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>Understand that data may be generated from experiments.</td>
<td>Conduct probability experiments, using multicolored spinners, colored tiles, or number cubes and use the data from the experiments to predict outcomes if the experiment is repeated.</td>
</tr>
<tr>
<td>Understand that the likelihood of an event occurring is to predict the probability of it happening.</td>
<td>Record the results of probability experiments, using tables, charts, and tally marks.</td>
</tr>
<tr>
<td>Probability is the chance of an event occurring. (e.g., the probability of landing on a particular color when flipping a two-colored chip is ( \frac{1}{2} ), representing one of two possible outcomes).</td>
<td>Interpret the results of probability experiments (e.g., the two-colored spinner landed on red 5 out of 10 times).</td>
</tr>
<tr>
<td>An event is a possible outcome in probability. Simple events include the possible outcomes when tossing a coin (heads or tails), when rolling a random number cube generator (a number cube or a die where there are six equally likely outcomes and the probability of one outcome is ( \frac{1}{6} )), or when spinning a spinner.</td>
<td>Predict which of two events is more or less likely to occur if an experiment is repeated.</td>
</tr>
<tr>
<td>If all the outcomes of an event are equally likely to occur, the probability of an event is equal to the number of favorable outcomes divided by the total number of possible outcomes: the probability of the event = ( \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}} ).</td>
<td></td>
</tr>
<tr>
<td>At this level, students need to understand only this fractional representation of probability (e.g., the probability of getting heads when flipping a coin is ( \frac{1}{2} )).</td>
<td></td>
</tr>
<tr>
<td>Students should have opportunities to describe in informal terms...</td>
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</table>

A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in investigations and have opportunities to use manipulatives.

Investigation of experimental probability is continued through informal activities, such as dropping a two-colored counter (usually a chip that has a different color on each side), using a multicolored spinner (a circular spinner that is divided equally into two, three, four, six or eight or more equal "pie" parts where each part is filled with a different color), using spinners with numbers, or rolling random number cube generators (dice).

Probability is the chance of an event occurring (e.g., the probability of landing on a particular color when flipping a two-colored chip is \( \frac{1}{2} \), representing one of two possible outcomes).

An event is a possible outcome in probability. Simple events include the possible outcomes when tossing a coin (heads or tails), when rolling a random number cube generator (a number cube or a die where there are six equally likely outcomes and the probability of one outcome is \( \frac{1}{6} \)), or when spinning a spinner.
The student will use data from probability experiments to predict outcomes when the experiment is repeated.

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<tbody>
<tr>
<td>(i.e., impossible, unlikely, as likely as, equally likely, likely, and certain) the degree of likelihood of an event occurring. Activities should include practical examples.</td>
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</table>
2.19 The student will analyze data displayed in picture graphs, pictographs, and bar graphs. [Moved to 2.1415b]

<table>
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<tbody>
<tr>
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<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>● Statements that represent an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be discussed with students and written.</td>
<td>● Understand how to read the key used in a graph to assist in the analysis of the displayed data.</td>
<td>● Analyze information from simple picture graphs, pictographs, and bar graphs by writing at least one statement that covers one or both of the following:</td>
</tr>
<tr>
<td>● When data are displayed in an organized manner, the results of investigations can be described, and the questions posed can be answered.</td>
<td>● Understand how to interpret data in order to analyze it.</td>
<td>- Describe the categories of data and the data as a whole (e.g., the total number of responses).</td>
</tr>
<tr>
<td></td>
<td>● Understand how to analyze data in order to answer the questions posed, make predictions, and generalizations.</td>
<td>- Identify parts of the data that have special characteristics, including categories with the greatest, the least, or the same. [Moved to 2.1415 EKS]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>● Select the best analysis of a graph from a set of possible analyses of the graph. [Moved to 2.1415 US]</td>
</tr>
</tbody>
</table>
Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children’s mathematical development.

Students in kindergarten through third grade develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns. Students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to algebra.
The student will identify, describe, create, and extend, and transfer a wide variety of patterns found in objects, pictures, and numbers.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Patterning is a fundamental cornerstone of mathematics, particularly algebra. The process of generalization leads to the foundation of algebraic reasoning.
- Opportunities to create, identify, describe, extend, and transfer patterns are essential to the primary school experience and lay the foundation for thinking algebraically. [Reordered and edited]
- The part of the pattern that repeats is called the core.
- Identifying and extending patterns is an important process in mathematical thinking.
- Analysis of patterns in the real world (e.g., patterns on a butterfly’s wings, patterns on a ladybug’s shell) leads to the analysis of mathematical patterns such as number patterns and geometric patterns.
- Reproduction of a given pattern in a different manifestation, using symbols and objects, lays the foundation for writing numbers symbolically or algebraically.
- The simplest types of patterns are repeating patterns. Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience. [Reordered]
- Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns in both arithmetic and geometric formats, involve a progression from step to step which make them more difficult for students than repeating patterns. Students must determine what comes next and also begin the process of generalization, which leads to the foundation of algebraic reasoning. Students need experiences identifying what changes and what stays the same in a growing pattern. Growing patterns may be represented in various ways.

### ESSENTIAL UNDERSTANDINGS

- All students should
  - Understand patterns are a way to recognize order and to predict what comes next in an arrangement.
  - Analyze how both repeating and growing patterns are generated.

### ESSENTIAL KNOWLEDGE AND SKILLS

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
  - Identify a **pattern as growing** or **repeating** pattern, from a given geometric or numeric sequence.
  - Predict the next number, geometric figure, symbol, picture, or object in a given pattern.
  - Describe the core (the part of the sequence that repeats) of a given growing or repeating pattern.
  - Describe how a given growing pattern is changing.
  - Create a growing or repeating pattern, using numbers, pictures, or objects. [Reordered]
  - Extend a given pattern, using numbers, geometric figures, symbols, pictures, or objects.
  - Create a new pattern, using numbers, geometric figures, pictures, symbols, or objects. [Reordered]
  - Recognize the same pattern in different manifestations. Transfer a given growing or repeating pattern from one form to another using numbers, pictures, or objects.
The student will identify, describe, create, and extend, and transfer a wide variety of patterns found in objects, pictures, and numbers.

<table>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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<tbody>
<tr>
<td><strong>Including dot patterns, staircases, pictures, etc.</strong></td>
<td><strong>ESSENTIAL UNDERSTANDINGS</strong></td>
<td><strong>ESSENTIAL KNOWLEDGE AND SKILLS</strong></td>
</tr>
<tr>
<td>• In numeric patterns, students must determine the difference, called the <em>common difference</em>, between each succeeding number in order to determine what is added to each previous number to obtain the next number. Students do not need to use the term <em>common difference</em> at this level. Create an arithmetic number pattern.</td>
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<tr>
<td>• Sample numeric patterns include:</td>
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<tr>
<td>– 6, 9, 12, 15, 18, … (growing pattern);</td>
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<td></td>
</tr>
<tr>
<td>– 20, 18, 16, 14, … (growing pattern);</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– 1, 2, 4, 7, 11, 16, 22, 33, 44, 42, 4, 6, 8, 10, … (growing pattern); and</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– 1, 3, 5, 1, 3, 5, 1, 3, 5… (repeating pattern).</td>
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<tr>
<td>In grade 2, growing numeric patterns will only include increasing values.</td>
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<tr>
<td>• In geometric patterns using objects or figures, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation is the result of turning a figure around a point or a vertex, and reflection is the result of flipping a figure over a line.</td>
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<td>• Examples of patterns using objects or figures include:</td>
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<tr>
<td><img src="image1" alt="Pattern 1" /></td>
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<td></td>
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<tr>
<td><img src="image2" alt="Pattern 2" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image3" alt="Pattern 3" /></td>
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<td></td>
</tr>
<tr>
<td>• Transferring a pattern is creating the pattern in a different form or representation.</td>
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</table>
2.1620 The student will identify, describe, create, and extend, and transfer a wide variety of patterns found in objects, pictures, and numbers.

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<tbody>
<tr>
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<tr>
<td>• Examples of pattern transfers include:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>‒ 10, 20, 30, 40 has the same structure as 14, 24, 34, 44;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>‒ □ ○ △ □ ○ △ ○ has the same structure as □ ○ △ □ ○ △ ○ ; and</td>
<td></td>
<td></td>
</tr>
<tr>
<td>‒ 1, 3, 5, 1, 3, 5, 1, 3, 5 has the same structure as ABCABC₄</td>
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</table>
2.21 The student will solve problems by completing numerical sentences involving the basic facts for addition and subtraction. The student will create story problems, using the numerical sentences. [Moved to 2.5 and 2.6]

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</thead>
<tbody>
<tr>
<td>• Recognizing and using patterns and learning to represent situations mathematically are important aspects of primary mathematics.</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Discussing what a word problem is saying and writing a number sentence are precursors to solving word problems.</td>
<td>• Use mathematical models to represent and understand quantitative relationships.</td>
<td>• Solve problems by completing a numerical sentence involving the basic facts for addition and subtraction (e.g., (3 + _ = 7), or (9 - _ = 2)). [Moved to 2.5]</td>
</tr>
<tr>
<td>• The patterns formed by related basic facts facilitate the solution of problems involving a missing addend in an addition sentence or a missing part (subtrahend) in a subtraction sentence.</td>
<td>• Understand various meanings of addition and subtraction and the relationship between the two operations.</td>
<td>• Create a story problem for a given numerical sentence. [Moved to 2.6]</td>
</tr>
<tr>
<td>• Making mathematical models to represent simple addition and subtraction problems facilitates their solution.</td>
<td>• Understand how to write missing addend and missing subtrahend sentences.</td>
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<tr>
<td>• By using story problems and numerical sentences, students begin to explore forming equations and representing quantities using variables.</td>
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<td></td>
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<tr>
<td>• Students can begin to understand the use of a symbol (e.g., (_), (?), or (\square)) to represent an unknown quantity.</td>
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</table>

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 2
2.1722 The student will demonstrate an understanding of equality by recognizing that through the use of the equal symbol (=) in an equation indicates equivalent quantities and the use of the not equal symbol (≠) indicates that quantities are not equivalent.

<table>
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<tbody>
<tr>
<td>• The equal symbol (=) means that the values on either side are the same (equivalent) (balanced).</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• The not equal (≠) symbol means that the values on either side are not the same (equivalent) (not balanced).</td>
<td>• Understand that the equal symbol means equivalent (same as) quantities.</td>
<td>• Identify the equality symbol (=) as the symbol used to indicate that the values on either side are equal.</td>
</tr>
<tr>
<td>• In order for students to develop the concept of equality, students need to see the = symbol used in various appropriate locations (e.g., 3 + 4 = 7 and 5 = 2 + 3).</td>
<td>• The inequality symbol (≠) means not equivalent.</td>
<td>• Identify the not equal symbol (≠) and inequality (≠) as the symbol used to indicate that two values on either side are not equal.</td>
</tr>
<tr>
<td>• An equation (number sentence) is a mathematical statement representing two expressions that are equivalent. It consists of two expressions, one on each side of an 'equal' symbol (e.g., 5 + 3 = 8, 8 = 5 + 3 and 4 + 3 = 9 - 2). An equation can be represented using a number balance scale, with equal amounts on each side (e.g., 3 + 5 = 6 + 2).</td>
<td>• Identify equivalent values and equation expressions that are equal (e.g., 8 = 8 and 8 = 4 + 4).</td>
<td>• Identify equivalent values and equation expressions that are not equal (e.g., 8 ≠ 9 and 4 + 3 ≠ 8).</td>
</tr>
<tr>
<td>• An expression represents a quantity. It contains numbers, variables, and/or computational operation symbols. It does not have an equal symbol (e.g., 5, 4 + 3, 8-2). Students at this level are not expected to use the terms expression or variable.</td>
<td>• Identify and use the appropriate symbol to distinguish between equal and not equal quantities (e.g., 8 = 2 - 7 = 315 - 5 and 1 + 4 ≠ 6 + 29 + 24 = 10 + 23; 45 - 9 = 46 - 10; 15 + 16 ≠ 31 + 15).</td>
<td>• Use a model to represent the relationship of two expressions of equal value and two expressions that are not equivalent.</td>
</tr>
<tr>
<td>• Manipulatives such as connecting cubes, counters, and number scales can be used to model equations.</td>
<td>• A number sentence is an equation with numbers (e.g., 6 + 3 = 9; or 6 + 3 = 4 + 5).</td>
<td></td>
</tr>
</tbody>
</table>
Grade 3

Board of Education
Commonwealth of Virginia
Acknowledgements
The Virginia Department of Education wishes to express sincere thanks to Deborah Kiger Bliss, Lois A. Williams, Ed.D., and Felicia Dyke, Ph.D., who assisted in the development of the 2009 Mathematics Standards of Learning Curriculum Framework.

Acknowledgements
The Virginia Department of Education wishes to express sincere thanks to Michael Bolling, who assisted in the development of the 2016 Mathematics Standards of Learning and 2016 Mathematics Standards of Learning Curriculum Framework.

NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

Introduction

The 2009-2016 Mathematics Standards of Learning Curriculum Framework is a companion document to the 2009-2016 Mathematics Standards of Learning, and amplifies the Mathematics Standards of Learning by further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

**Essential Understandings**
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

**Understanding the Standard**
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction; background information for the teacher (K-8). It contains content—The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers’ knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

**Essential Knowledge and Skills**
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that What each student should know and be able to demonstrated in each standard is outlined. This is not meant to be an exhaustive list of student expectations, nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student’s problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
Students in grades K–3 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, fractions, number relationships (“more than,” “less than,” and “equal to”), and the effects of single-step and multistep computations. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

[Mathematics instruction in grades 3 through 5 should continue to foster the development of number sense, with greater emphasis on decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction in grades 3-5 allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names), and making connections among mathematics concepts as well as to other content areas. Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g., $\frac{1}{2}$ or 0.5). Students should apply their knowledge of number and number sense to investigate and solve a variety of problem types.
### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- The structure of the Base-10 number system is based upon a simple pattern of tens, where each place is ten times the value of the place to its right. This structure is known as a ten-to-one place value relationship, which is helpful in comparing and ordering numbers.
- Models that clearly illustrate the relationships among hundreds, tens, and ones are physically proportional (e.g., the tens piece is ten times larger than the ones piece).
- Place value refers to the value of each digit and depends upon the position of the digit in the number. In the number 7,864, the eight is in the hundreds place, and the value of the 8 is eight hundred.
- Flexibility in thinking about numbers — or “decomposition” of numbers (e.g., 42,345 is 42 hundreds, 3 tens, and 4 ones, or 2 thousands, 34 tens, and 5 ones, or 22 hundreds, 13 tens, and 15 ones, etc.) — is critical and supports understandings essential to addition/subtraction and multiplication/division. This flexibility also builds background understanding for the ideas that students use when regrouping (e.g., when subtracting 18 from 174, a student may choose to regroup and think of 174 as 1 hundred, 6 tens, and 14 ones, while another child might regroup 174 as 1 hundred, 5 tens, and 24 ones. Then subtract 18 from 24).
- Whole numbers may be written in a variety of formats:  
  - Standard: 123,456; 
  - Written: one hundred twenty-three thousand, four hundred fifty-six; and 
  - Expanded: 100,000 + 20,000 + 3,000 + 400 + 50 + 6 = (4 × 100,000) + (2 × 10,000) + (3 × 1,000) + (4 × 100) + (5 × 10) + (6 × 1) or 100,000 + 20,000 + 3,000 + 400 + 50 + 6 = (4 × 100,000) + (2 × 10,000) + (3 × 1,000) + (4 × 100) + (5 × 10) + (6 × 1).

### ESSENTIAL UNDERSTANDINGS

All students should

- Understand that knowledge of place value is essential when comparing numbers.
- Understand the relationships in the place value system, where each place is ten times the value of the place to its right.
- Understand that rounding gives an estimate to use when exact numbers are not needed for the situation.
- Understand the relative magnitude of numbers by comparing numbers.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Represent numbers up to 9,999 in multiple ways, according to place value (e.g., 256 can be 1 hundred, 14 tens, and 16 ones, but also 25 tens and 6 ones), with and without models. (a)
- Investigate and identify the place and value for each digit in a six-digit numeral, using Base 10 manipulatives (e.g., Base 10 blocks).
- Use the patterns in the place value system to read and write numbers.
- Identify the place (ones, tens, hundreds, thousands, etc.) of each digit in a six-digit numeral, with and without models. (a)
- Determine the value of each digit in a six-digit numeral and whole number (e.g., in 165,724, the 7 represents 7 hundreds and its value is 700). (a)
- Read six-digit numerals orally. (a)
- Write six-digit numerals in standard form that are stated verbally or written in words. (a)
- Round a given numeral, 9,999 or less, to the nearest ten, hundred, and thousand. (b)
- Solve problems, using rounding of numbers, 9,999 or less, to the nearest ten, hundred, and thousand. (b)
- Determine which of two whole numbers between 0 and 9,999 is greater. (c)
### Standard 3.1

#### Strand: Number and Number Sense

**Grade Level 3**

3.1 The student will

a) read, and write six-digit numerals and identify the place value and value of each digit in a six-digit whole number, with and without models;

b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and

c) compare and order whole numbers between 0 and each 9,999 or less, using symbols (>, <, or =) and words (greater than, less than, or equal to). [Symbols and words included in EKS]

<table>
<thead>
<tr>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>• Determine which of two whole numbers between 0 and 9,999 is less. (c)</td>
<td>• Use the terms greater than, less than, and equal to, and not equal to when comparing two whole numbers. (c)</td>
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<tr>
<td>+ (6 x 1),</td>
<td>• Compare two whole numbers between 0 and each 9,999 or less, using the symbols &gt;, &lt;, or =. (c)</td>
<td>• Order up to three numbers, each 9,999 or less, represented with concrete objects, pictorially, or symbolically from least to greatest and greatest to least. (c)</td>
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<tr>
<td>• Numbers are arranged into groups of three places called periods (ones, thousands, millions, and so on). Places within the periods repeat (hundreds, tens, ones). Commas are used to separate the periods. Knowing the place value and period of a number helps students find and determine the value of a digit in any number as well as read and write numbers.</td>
<td>• Reading and writing large numbers should be related to numbers that have meanings (e.g., numbers found in the students’ environment). Concrete materials, such as Base-10 blocks may be used to represent whole numbers through thousands. Larger numbers may be represented on place value charts.</td>
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<td>• To read a whole number through the hundred thousands place,</td>
<td>• Rounding is one of the estimation strategies that is often used to assess the reasonableness of a solution or to give an estimate of an amount.</td>
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<td>– read the digits to the first comma;</td>
<td>• Students should explore reasons for estimation, using practical experiences, and use rounding to solve practical situations problems.</td>
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<td>– say the name of the period (e.g., “thousands”); then</td>
<td>• The concept of rounding may be introduced through the use of a number line. When given a number to round, locate it on the number line. Next, determine the closest multiple of ten, hundred, or thousand it is between. Then identify to which it is closer.</td>
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</tbody>
</table>
STANDARD 3.1  STRAND: NUMBER AND NUMBER SENSE  GRADE LEVEL 3

3.1 The student will
a) read, and write, six-digit numerals and identify the place value and value of each digit in a six-digit whole number, with and without models;
b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and
c) compare two and order whole numbers between 0 and each 9,999 or less, using symbols (>, <, or =) and words (greater than, less than, or equal to). [Symbols and words included in EKS]

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</thead>
</table>
| • A procedure for rounding numbers to the nearest ten, hundred, or thousand is as follows:  
  — Look one place to the right of the digit to which you wish to round.  
  — If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero.  
  — If the digit is 5 or greater, add 1 to the digit in the rounding place, and change the digits to the right of the rounding place to zero. | | |
| • A procedure for comparing two numbers by examining may include the following:  
  — Line up the numbers by place value by lining up the ones.  
  — Beginning at the left, find the first place value where the digits are different.  
  — Compare the digits in this place value to determine which number is greater (or which is less).  
  — Use the appropriate symbol > or < or the words greater than or less than to compare the numbers in the order in which they are presented.  
  — If both numbers are the same, use the symbol = or the words equal to. | | |
3.2 The student will recognize and use the inverse relationships between addition/subtraction and multiplication/division to complete basic fact sentences. The student will use these relationships to solve problems. [Addition/subtraction included in 2.5; multiplication/division moved to 3.4 EKS]

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<tbody>
<tr>
<td>• Addition and subtraction are inverse operations, as are multiplication and division.</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
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<tr>
<td>• In building thinking strategies for subtraction, an emphasis is placed on connecting the subtraction fact to the related addition fact. The same is true for division, where the division fact is tied to the related multiplication fact. Building fact sentences helps strengthen this relationship.</td>
<td>• Understand how addition and subtraction are related.</td>
<td>• Use the inverse relationships between addition/subtraction and multiplication/division to solve related basic fact sentences. For example, ( 5 + 3 = 8 ) and ( 8 - 3 = ) __; ( 4 \times 3 = 12 ) and ( 12 \div 4 = ) __.</td>
</tr>
<tr>
<td>• Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship.</td>
<td>• Understand how multiplication and division are related.</td>
<td>• Write three related basic fact sentences when given one basic fact sentence for addition/subtraction and for multiplication/division. For example, given ( 3 \times 2 = 6 ), solve the related facts ( _ \times 3 = 6 ), ( 6 \div 3 = _ ) and ( 6 \div _ = 3 ).</td>
</tr>
<tr>
<td>• Multiplication and division should be taught concurrently in order to develop understanding of the inverse relationship.</td>
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</tbody>
</table>
3.23  The student will

a) name and write fractions (including improper fractions) and mixed numbers represented by a model;

b) model represent fractions (including improper fractions and mixed numbers) and write the fractions’ names with models and symbols; and

c) use models to compare proper fractions having like and unlike denominators, using words and symbols (> , <, or ≠, or ≠ ), with models.

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**UNDERSTANDING THE STANDARD**

(Background Information for Instructor Use Only)

- When working with fractions, the whole must be defined.
- A fraction is a numerical way of representing part of a whole (as in region/area or as in a measurement model), or part of a group (as in a set model), or part of a length (as in a length/measurement model). Fractions are used to name a part of one thing or a part of a collection of things. Models can include pattern blocks, fraction bars, rulers, number line, etc.
- Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction can be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., 3 \(\frac{5}{8}\)).
- The value of a fraction is dependent on both the number of equivalent parts in a whole (denominator) and the number of those parts being considered (numerator).
- Model A fractional part of a whole can be modeled using
  - region/area models (e.g., pie pieces, pattern blocks, geoboards, drawings);
  - set models (e.g., chips, counters, cubes, drawings); and
  - length/measurement models (e.g., rods, connecting cubes, number lines, rulers, and drawings). (b) [Moved from 3.7]
- In each area/region and length/measurement model, the whole is divided or partitioned into equal-sized (congruent) parts with area of equivalent value. The fractional parts are not always congruent.

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**ESSENTIAL UNDERSTANDINGS**

All students should

- Understand that the whole must be defined.
- Understand that the numerator tells the number of equal parts that represent a whole.
- Understand that the denominator tells the number of equal-sized parts that represent a whole.
- Understand that the value of a fraction is dependent on both the number of parts in a whole (denominator) and the number of those parts being considered (numerator). [Moved to US]
- Understand that a proper fraction is a fraction whose numerator is smaller than its denominator. [Moved to US]
- That an improper fraction is a fraction whose numerator is equal to or greater than the denominator and is one or greater than one. [Moved to US]

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**ESSENTIAL KNOWLEDGE AND SKILLS**

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Name and write fractions (including proper and improper fractions) and mixed numbers with denominators of 12 or less in symbols represented by both concrete and/or pictorial models (drawing set, region/area, and length/measurement models or concrete materials) to include halves, thirds, fourths, eighths, tenths, and twelfths. (a)
- Use concrete materials and pictures to model at least halves, thirds, fourths, eighths, tenths, and twelfths.
- Represent a given fraction (including proper or improper and mixed numbers), using concrete materials, pictures, or pictorial models (set, area/region, length/measurement models, and symbols. (b) [Moved from 3.7]
- Identify a fraction represented by a model as the sum of unit fractions (e.g., \(\frac{1}{5}\) + \(\frac{2}{5}\) = \(\frac{3}{5}\)).
- Using a model of a fraction greater than one, count the fractional parts to name and write it as an improper fraction and as a mixed number (e.g., \(\frac{1}{4} + \frac{3}{4} = \frac{1}{3}\) or \(\frac{2}{3} = \frac{7}{3}\)). (b)
- Compare proper fractions using the terms greater than, less than, or equal to, or not equal to and the symbols (<, >, and =). Comparisons are made between fractions with both like and unlike denominators, using with concrete or pictorial models (length/measurement, region/area, and set). concrete materials and pictures. (c)
The student will
a) name and write fractions (including improper fractions and mixed numbers) represented by a model;
b) model represent fractions (including improper fractions and mixed numbers) and write the fractions’ names with models and symbols; and
c) use models to compare proper fractions having like and unlike denominators, using words and symbols (> , <, or = , or ≠), with models.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

and could have a different shape as shown in the examples below:

- In the set model, each member of the set is an equivalent part of the set. The members of the set do not have to be equal in size. In set models, the whole needs to be defined, but members of the set may have different sizes and shapes. For instance, if a whole is defined as a set of 10 animals, the animals within the set may be different. Further, for example, students should be able to identify/write a fraction that represents the number of monkeys as representing half of the animals in the following set of animals.

- In the primary grades, students may benefit from experiences with sets that are comprised of congruent figures (e.g., 12 eggs in a carton) before working with sets that have noncongruent parts.
- The denominator tells how many equal parts are in the whole or set. The numerator tells how many of those parts are being considered.
- Students need opportunities to use models to count fractional

ESSENTIAL UNDERSTANDINGS

US]
- Understand that an improper fraction can be expressed as a whole number or a mixed number.
- Understand that a mixed number is written as a whole number and a proper fraction.

ESSENTIAL KNOWLEDGE AND SKILLS

- Use models to compare fractions to the benchmarks of 0, 1/2, 1, and 1. (c)
3.23 The student will
a) name and write fractions (including improper fractions) and mixed numbers represented by a model;

b) model represent fractions (including improper fractions and mixed numbers), and write the fractions’ names with models and symbols; and

c) use models to compare proper fractions having like and unlike denominators, using words and symbols (> , <, or =, or ≠), with models.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

**parts that go beyond a whole.** For instance, if students are counting five slices of cake and building the cake as they count, where each slice is equivalent to one-fourth, they might say "one-fourth, two-fourths, three-fourths, four-fourths, five-fourths." As a result of building the whole while they are counting, they begin to realize that four-fourths make one whole and the fifth-fourth starts another whole and begin to develop flexibility in naming this amount in different ways (e.g., five-fourths or one and one-fourth). They will begin to generalize that when the numerator and the denominator are the same, there is one whole and when the numerator is larger than the denominator, there is more than one whole. They also will begin to see a fraction as the sum of unit fractions (e.g., three-fourths contains three one-fourths or four-fourths contains four one-fourths which is equal to one whole). This provides students with a visual, as in the example below, for when one whole is reached and develops a greater understanding of numerator and denominator.

<table>
<thead>
<tr>
<th>1/4</th>
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<td>5/4</td>
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\[
\frac{5}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \quad \text{and} \quad \frac{5}{4} = 1 \frac{1}{4}
\]

- A proper fraction is a fraction whose numerator is less than the denominator. A proper fraction is a fraction that is always less than one.

- An improper fraction is a fraction whose numerator is greater than or equal to the denominator. An improper fraction is a fraction that is greater than or equal to one.

- An improper fraction greater than one can also be expressed as a mixed number. A mixed number is written as a whole number
### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Models, benchmarks, and equivalent forms are helpful in judging the size of fractions. [Copied from 4.2]
- Experiences at this level should include exploring and reasoning about comparing and ordering fractions with common numerators, common denominators, or comparing to the benchmark of $\frac{1}{2}$ and 1.
- Students should have a variety of experiences focusing on comparing:
  - fractions with like denominators;
  - fractions with like numerators;
  - fractions that are more than one whole and less than one whole; and
  - fractions close to zero, close to one-half, and close to one whole.
- Comparing unit fractions (a fraction in which the numerator is one) builds a mental image of fractions and the understanding that as the number of pieces of a whole increases, the size of one single piece decreases (e.g., $\frac{1}{5}$ of a bar is smaller than $\frac{1}{4}$ of the same bar). [Reordered]
- Comparing fractions to a benchmark on a number line (e.g., close to 0, less than $\frac{1}{2}$, exactly $\frac{1}{2}$, greater than $\frac{1}{2}$ (or close to 1)) facilitates the comparison of fractions when using concrete materials or pictorial models. [Reordered]
- Provide opportunities to make connections among fraction...
The student will

a) name and write fractions (including improper fractions) and mixed numbers represented by a model;

b) model represent fractions (including improper fractions and mixed numbers) and write the fractions' names with models and symbols; and

c) use models to compare proper fractions having like and unlike denominators, using words and symbols (> , <, =, or ≠), with models.

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<td>representations by connecting concrete or pictorial representations with oral language and symbolic representations.</td>
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<tr>
<td>• Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions (e.g., thirds means “three equal parts of a whole,” ( \frac{1}{3} ) represents one of three equal-size parts when a pizza is shared among three students, or three-fourths means “three of four equal parts of a whole”) furthers this development.</td>
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<tr>
<td>• Comparing unit fractions (a fraction in which the numerator is one) builds a mental image of fractions and the understanding that as the number of pieces of a whole increases, the size of one single piece decreases (e.g., ( \frac{1}{2} ) of a bar is smaller than ( \frac{1}{4} ) of a bar). [Reordered]</td>
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<tr>
<td>• Comparing fractions to a benchmark on a number line (e.g., close to 0, less than ( \frac{1}{2} ), exactly ( \frac{1}{2} ), greater than ( \frac{1}{2} ), or close to 1) facilitates the comparison of fractions when using concrete materials or pictorial models. [Reordered]</td>
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A variety of contexts are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from real-life experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as “How many books do we have altogether?” or “How many cookies are left if I eat two?” or “I have three more candies than you do.” Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling, discussing, and recording a variety of problem situations. This approach helps students transition from the concrete to the representation to the symbolic in order to develop meaning for the operations and how they relate to each other.

In grades K–3, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition, subtraction, multiplication, division and related facts;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as equal groupings of objects and sharing equally; and
- performing initial operations with fractions.

Computation and estimation in grades 3-5 should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students’ understanding of these operations to work with decimals. Instruction should focus on computation activities that enable students to model, explain, and develop proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades. Multiplication and division with decimals will be explored in grade 5.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability not only to identify but to use relationships between operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use properties of operations to solve problems [e.g., \(7 \times 28\) is equivalent to \((7 \times 20) + (7 \times 8)\)].

Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals, using benchmarks (e.g., \(\frac{2}{2} + \frac{1}{2}\) must be less than 1 because both fractions are less than \(\frac{1}{2}\)). Using estimation, students should develop strategies to recognize the reasonableness of their solutions.
Additionally, students should enhance their ability to select an appropriate problem solving method from among estimation, mental mathematics, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.
The student will:
a) estimate solutions to and determine the sum or difference of two whole numbers; and
b) create and solve single-step and multistep practical problems involving the sums or differences of two whole numbers, each 9,999 or less, with or without regrouping.

**UNDERSTANDING THE STANDARD**
(Background Information for Instructor Use Only)

- Extensive research has been undertaken over the last several decades regarding different problem types. Many of these studies have been published in professional mathematics education publications using different labels and terminology to describe the varied problem types.
- Students should experience a variety of problem types related to addition and subtraction. Examples are included in the following chart:

<table>
<thead>
<tr>
<th>UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>- Understand that estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed. [Moved to US]</td>
<td>- Determine whether an estimate or an exact answer is an appropriate solution for practical addition and subtraction problems situations involving single-step and multistep problems. (a, b)</td>
</tr>
<tr>
<td>- Understand that estimation skills are also valuable in determining the reasonableness of the sum or difference when solving for the exact answer is needed. [Moved to US]</td>
<td>- Determine whether to add or subtract in practical problem situations.</td>
</tr>
<tr>
<td>- Develop and use strategies to estimate whole number sums and differences to determine the reasonableness of an exact answer.</td>
<td>- Estimate the sum or difference of two whole numbers with sums to 9,999 or less when an exact answer is not required. (a)</td>
</tr>
<tr>
<td>- Develop flexible methods of adding whole numbers by combining numbers in a variety of ways, most depending on place values. [Moved to US]</td>
<td>- Estimate the difference of two whole numbers, each 9,999 or less. (a)</td>
</tr>
</tbody>
</table>

- Apply strategies, including place value and the properties of addition, to add or subtract two whole numbers with sums to 9,999 or less. (a, b) [Moved from 3.20]
- Apply strategies, including place value and the properties of addition, to subtract two whole numbers, each 9,999 or less. (a)
- Recognize and use related Use inverse relationships between addition and subtraction facts to solve practical problems. (b) [Moved from 3.2]
- Solve practical problems involving the sum of two whole numbers, each 9,999 or less, with or without regrouping, using calculators, paper and pencil, or mental computation in practical problem situations.
- Solve practical problems involving the difference of two whole numbers, each 9,999 or less, with or without regrouping, using calculators, paper and pencil, or mental computation in practical problem situations.
3.34 The student will
a) estimate solutions to and determine the sum or difference of two whole numbers; and
b) create and solve single-step and multistep practical problems involving the sum or differences of two whole numbers, each 9,999 or less, with or without regrouping.

### UNDERSTANDING THE STANDARD

#### ESSENTIAL UNDERSTANDINGS

<table>
<thead>
<tr>
<th>GRADE 3: COMMON ADDITION AND SUBTRACTION PROBLEM TYPES</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Join (Result Unknown)</strong></td>
<td>whole numbers, each 9,999 or less, with or without regrouping, using calculators, paper and pencil, or mental computation in practical problem situations.</td>
</tr>
<tr>
<td><strong>Join (Change Unknown)</strong></td>
<td>- Use strategies when adding and subtracting, including application of the properties of addition. (a, b) [Moved from 3.20] [Reordered]</td>
</tr>
<tr>
<td><strong>Join (Start Unknown)</strong></td>
<td>- Create and solve single-step and multistep practical problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping. (b)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Separate (Result Unknown)</strong></th>
<th><strong>Separate (Change Unknown)</strong></th>
<th><strong>Separate (Start Unknown)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sue had 214 sheets of paper. Alex gave her 95 more sheets of paper. How many sheets of paper does Sue have all together?</td>
<td>Sue had 214 sheets of paper. Alex gave her some more sheets of paper. How many sheets of paper does Sue have all together?</td>
<td>Sue had some sheets of paper. Alex gave her 95 more sheets of paper. How many sheets of paper does Sue have all together?</td>
</tr>
<tr>
<td>Brooke had 145 marbles. She gave 26 marbles to Joe. How many marbles does Brooke have now?</td>
<td>Brooke had 145 marbles. She gave some to Joe. How many marbles did Brooke have before giving some to Joe?</td>
<td>Brooke had some marbles. She gave 26 to Joe. How many marbles did Brooke have before giving 26 to Joe?</td>
</tr>
<tr>
<td>Part-Part-Whole (Whole Unknown)</td>
<td>Part-Part-Whole (One Part Unknown)</td>
<td>Part-Part-Whole (Both Parts Unknown)</td>
</tr>
<tr>
<td>There are 29 boys and 36 girls in the third grade at a school. How many boys and girls are in third grade all?</td>
<td>There are 65 students in third grade. Twenty-nine of the students are boys, and the rest are girls. How many girls are in third grade?</td>
<td>There are 65 students in third grade. Some of the students are girls and some of the students are boys. How many students could be girls and how many students could be boys?</td>
</tr>
<tr>
<td>Compare (Difference Unknown)</td>
<td>Compare (Bigger Unknown)</td>
<td>Compare (Smaller Unknown)</td>
</tr>
<tr>
<td>Mr. Ross has 325 books. Mrs. King has 196 books. How many more books does Mr. Ross have than Mrs. King?</td>
<td>Mrs. King has 196 books. Mr. Ross has 129 more books than Mrs. King. How many books does Mr. Ross have?</td>
<td>Mr. Ross has 129 more books than Mrs. King. Mr. Ross has 325 books. How many books does Mrs. King have?</td>
</tr>
<tr>
<td>Mr. Ross has 325 books. Mrs. King has 196 books. How many fewer books does Mr. Ross have than Mrs. King?</td>
<td>Mrs. King has 196 fewer books than Mr. Ross. How many books does Mrs. Ross have?</td>
<td>Mrs. King has 129 fewer books than Mr. Ross. Mr. Ross has 325 books. How many books does Mrs. King have?</td>
</tr>
</tbody>
</table>
The student will:

a) estimate solutions to and determine the sum or difference of two whole numbers; and

b) create and solve single-step and multistep practical problems involving the sum or differences of two whole numbers, each 9,999 or less, with or without regrouping.

**UNDERSTANDING THE STANDARD**

(Background Information for Instructor Use Only)

- **Rate and a** Combination problem types are introduced in grade 5.
- **In problem solving, emphasis should be placed on thinking and reasoning rather than on key words.** Focusing on key words such as *in all, altogether, difference, etc.* encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions as well as challenges in upcoming grades and courses.
- **Addition** is the combining of quantities; it uses the following terms:
  
  \[
  \begin{align*}
  \text{addend} & \rightarrow 423 \\
  \text{addend} & \rightarrow +246 \\
  \text{sum} & \rightarrow 669
  \end{align*}
  \]

- **Subtraction** is the inverse of addition; it yields the difference between two numbers and uses the following terms:
  
  \[
  \begin{align*}
  \text{minuend} & \rightarrow 7,698 \\
  \text{subtrahend} & \rightarrow -5,341 \\
  \text{difference} & \rightarrow 2,357
  \end{align*}
  \]

- **An algorithm** is a step-by-step method for computing.

- **Flexible methods of adding whole numbers by combining numbers in a variety of ways, most depending on place values, are useful.** [Moved from middle column]

- **The least number of steps necessary to solve a single-step problem is one.**

- **An example of an approach to solving problems is Polya’s four-step plan:**

**ESSENTIAL UNDERSTANDINGS**

**ESSENTIAL KNOWLEDGE AND SKILLS**
3.34 The student will
a) estimate solutions to and determine the sum or difference of two whole numbers; and
b) create and solve single-step and multistep practical problems involving the sum or differences of two whole numbers, each 9,999 or less, with or without regrouping.

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<td></td>
<td></td>
</tr>
<tr>
<td>- Understand: Retell the problem; read it twice; take notes; study the charts or diagrams; look up words and symbols that are new.</td>
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<tr>
<td>- Plan: Decide what operation(s) and sequence of steps to use to solve the problem.</td>
<td></td>
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<tr>
<td>- Solve: Follow the plan and work accurately. If the first attempt does not work, try another plan.</td>
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<td></td>
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<tr>
<td>- Look back: Does the answer make sense?</td>
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</tr>
<tr>
<td>- Estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed. [Moved from middle column]</td>
<td></td>
<td></td>
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<tr>
<td>- Estimation skills are also valuable in determining the reasonableness of the sum or difference when solving for the exact answer is needed. [Moved from middle column]</td>
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</tr>
<tr>
<td>- Knowing whether to find an exact answer or to make an estimate is learned through practical experiences in recognizing which is appropriate.</td>
<td></td>
<td></td>
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<tr>
<td>- When an exact answer is required, opportunities to explore whether the answer can be determined mentally or must involve paper and pencil or calculators help students select the correct most efficient approach.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Determining whether an estimate is appropriate and using a variety of strategies to estimate requires experiences with problem situations involving estimation.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- There are a variety of mental mathematics strategies for each basic operation, and opportunities to practice these strategies give students the tools to use them at appropriate times. For example, with addition, mental mathematics strategies include:</td>
<td></td>
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<tr>
<td>- adding doubles;</td>
<td></td>
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The student will

a) estimate solutions to and determine the sum or difference of two whole numbers; and

b) create and solve single-step and multistep practical problems involving the sum or differences of two whole numbers, each 9,999 or less, with or without regrouping.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- using addition by counting up for solving subtraction problems;
- adding 9: add 10 and subtract 1; and
- Making 10: for column addition, look for numbers that group together to make 10.

- At this level, it is not necessary for students to use the formal terms for these properties.

- Grade 3 students should explore and apply the properties of addition as strategies for solving addition and subtraction problems using a variety of representations (e.g., manipulatives, diagrams, symbols, etc.).

- The properties of the operations are “rules” about how numbers work and how they relate to one another. Students at this level do not need to use the formal terms for these properties but should utilize these properties to further develop flexibility and fluency in solving problems. The following properties of addition are most appropriate for exploration at this level:
  - The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., $4 + 3 = 3 + 4$). [Moved from 3.20]
  - The identity property of addition states that if zero is added to a given number, the sum is the same as the given number. [Moved from 3.20]
  - The associative property of addition states that the sum stays the same when the grouping of addends is changed [e.g., $15 + (35 + 16) = (15 + 35) + 16$].

- Using concrete materials (e.g., base-10 materials to blocks, connecting cubes, beans and cups, etc.) to explore, model and stimulate discussion about a variety of problem situations helps students understand regrouping and enables them to move from
The student will

a) estimate solutions to and determine the sum or difference of two whole numbers; and

b) create and solve single-step and multistep practical problems involving the sum or differences of two whole numbers, each 9,999 or less, with or without regrouping.

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<tr>
<td>the concrete to the abstract. Regrouping is used in addition and subtraction algorithms.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Exploring concepts through concrete experiences develops conceptual understanding begins with concrete experiences. Next, the children must make connections that serve as a bridge to the symbolic. One strategy used to make connections is Student-created representations, such as drawings, diagrams, tally marks, graphs, or written comments are strategies that help students make these connections.</td>
<td></td>
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</tbody>
</table>
The student will recall multiplication facts through the twelves table, and the corresponding division facts.

a) represent multiplication and division through \(10 \times 10\), using a variety of approaches and models; [Moved from 3.6]

b) create and solve single-step practical problems that involve multiplication and division through \(10 \times 10\); and [Create and solve multiplication moved from 3.6; create and solve with division is new expectation]

c) demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10; and [Recall of facts to 12 x 12 included in 4.4a]

d) create and solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less. [Moved from 3.6]

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and practical problems involving equal-sized groups, arrays, and length models.

- To extend the understanding of multiplication, three models may be used:
  - The equal-sets or equal-groups model lends itself to sorting a variety of concrete objects into equal groups and reinforces the concept of multiplication as a way to find the total number of items in a collection of groups, with the same amount in each group, and the total number of items can be found by repeated addition or skip counting.
  
  1 2 3 4 5 6 7 8 9 10

  - The array model, consisting of rows and columns (e.g., 3 rows of 4 columns for a 3-by-4 array), helps build an understanding of the commutative property.

  1 2 3 4
  5 6 7 8
  9 10 11 12

  - The length model (e.g., a number line) also reinforces repeated addition or skip counting. [Reordered]

  0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

- There is an inverse relationship between multiplication and

### ESSENTIAL UNDERSTANDINGS

- All students should
  - Develop fluency with number combinations for multiplication and division.
  - Understand that multiplication is repeated addition.
  - Understand that division is the inverse of multiplication. [Moved to US]
  - Understand that patterns and relationships exist in the facts. [Moved to US]
  - Understand that number relationships can be used to learn and retain the facts. [Moved to US]

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Recall and state the multiplication and division facts through the twelves table. [Moved to 4.4 EKS]
  
- Recall and write the multiplication and division facts through the twelves table.
  
- Represent multiplication using a variety of approaches and models (e.g., repeated addition, equal-sized groups, arrays, equal jumps on a number line, skip counting). (a)
  
- Represent division using a variety of approaches and models (e.g., repeated subtraction, equal sharing, forming equal groups). (a)
  
- Create practical problems to represent a multiplication or division fact. (b)
  
- Use multiplication and division basic facts to represent a given situation, using a number sentence. (b)

- Recognize and use the inverse relationship between multiplication and division to solve practical problems. (b) [Moved from 3.2]

- Write three related equations (fact sentences) when given one basic-equation (fact sentence) for addition/subtraction and for multiplication and division (e.g., given \(6 \times 7 = 42\), write \(7 \times 6 = 42\), \(42 \div 7 = 6\), and \(42 \div 6 = 7\). (b) [Moved from 3.2 EKS]
The student will recall multiplication facts through the twelves table, and the corresponding division facts. [Recall moved to 4.4a]

a) represent multiplication and division through $10 \times 10$, using a variety of approaches and models;[Moved from 3.6]

b) create and solve single-step practical problems that involve multiplication and division through $10 \times 10$; and[Create and solve multiplication moved from 3.6; create and solve with division is new expectation]

c) demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10; and [Recall of facts to 12 x 12 included in 4.4a]

d) create and solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less.[Moved from 3.6]

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<tr>
<td>The terms associated with multiplication are listed below:</td>
<td>Solve single-step practical problems that involve multiplication and division of whole numbers through $10 \times 10$. (b)</td>
<td>Solve single-step practical problems that involve multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less. (c,d) [Moved from 3.5]</td>
</tr>
<tr>
<td>factor $\rightarrow$ 54</td>
<td>Demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10. (c)</td>
<td>Apply strategies, including place value and the properties of multiplication when multiplying and dividing whole numbers. (a, b, c, d) [Moved from 3.20]</td>
</tr>
<tr>
<td>factor $\rightarrow$ $\times 3$</td>
<td>Create and solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less. (c,d) [Moved from 3.5]</td>
<td></td>
</tr>
<tr>
<td>product $\rightarrow$ 162</td>
<td>Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently.</td>
<td></td>
</tr>
</tbody>
</table>

The number line model above shows two jumps of three between 6 and 0, answering the question of how many jumps of three go from 6 to 0; therefore, $6 \div 3 = 2$. |
The student will recall multiplication facts through the twelves table, and the corresponding division facts. [Recall moved to 4.4a]

a) represent multiplication and division through $10 \times 10$, using a variety of approaches and models; [Moved from 3.6]

b) create and solve single-step practical problems that involve multiplication and division through $10 \times 10$; and [Create and solve multiplication moved from 3.6; create and solve with division is new expectation]

c) demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10; and [Recall of facts to 12 x 12 included in 4.4a]

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<td></td>
</tr>
<tr>
<td>The development of computational fluency relies on quick access to number facts. There are patterns and relationships that exist in the facts. These relationships can be used to learn and retain the facts. By studying patterns and relationships, students build a foundation for fluency with multiplication and division facts.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A certain amount of practice is necessary to develop fluency with computational strategies; however, the practice must be motivating and systematic if students are to develop fluency in computation, whether mental, with manipulative materials, or with paper and pencil. [Moved to 4.4a US]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beginning with learning the foundational multiplication facts for 0, 1, 2, 5, and 10 allows students to utilize prior skip counting skills and the use of doubles to solve problems. Understanding and using the foundational facts can be helpful in deriving and learning all multiplication facts. For example, decomposing one of the factors in $7 \times 6$, allows for the use of the foundational facts of 5s and 2s. This knowledge can be combined to learn the facts for 7 (e.g., $7 \times 6$ can be thought of as $5 \times 6 + 5 \times 2$).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>As students work to solve multiplication and division problems, they naturally tend to utilize strategies that involve place value understanding and properties of the operations. Applying the commutative property of multiplication (e.g., $5 \times 8 = 8 \times 5$) reduces in half the number of multiplication facts that students learn. The distributive property of multiplication allows students to find the answer to a problem such as $6 \times 7$ by decomposing 7 into 3 and 4 (e.g., $6 \times 7 = 6 \times (3 + 4)$) allowing them to think about $(6 \times 3) + (6 \times 4) = 18 + 24 = 36$.</td>
<td></td>
<td></td>
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3.45 The student will recall multiplication facts through the twelves table, and the corresponding division facts. [Recall moved to 4.4a]

a) represent multiplication and division through $10 \times 10$, using a variety of approaches and models; [Moved from 3.6]

b) create and solve single-step practical problems that involve multiplication and division through $10 \times 10$; and [Create and solve multiplication moved from 3.6; create and solve with division is new expectation]

c) demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10; and [Recall of facts to $12 \times 12$ included in 4.4a]

d) create and solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less. [Moved from 3.6]

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Strategies that allow students to derive unknown facts from facts they do know include: doubles (2s facts), doubling twice (4s facts), five facts (half of ten), decomposing into known facts (e.g., $7 \times 8$ can be thought of as $5 \times 8 + 2 \times 8$).

- Strategies to learn the multiplication facts through the twelves table include an understanding of multiples/skip counting, properties of zero and one as factors, pattern of nines, commutative property, and related facts. [Moved to 4.4a US]

- Extensive research has been undertaken over the last several decades regarding different problem types. Many of these studies have been published in professional mathematics education publications using different labels and terminology to describe the varied problem types.

- Students should experience a variety of problem types related to multiplication and division. Some examples are included in the following chart:
The student will recall multiplication facts through the twelves table, and the corresponding division facts. [Recall moved to 4.4a] 

a) represent multiplication and division through $10 \times 10$, using a variety of approaches and models; [Moved from 3.6] 

b) create and solve single-step practical problems that involve multiplication and division through $10 \times 10$; and [Create and solve multiplication moved from 3.6; create and solve with division is new expectation] 

c) demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10; and [Recall of facts to 12 x 12 included in 4.4a] 

d) create and solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less. [Moved from 3.6]

### UNDERSTANDING THE STANDARD

**Grade 3: Common Multiplication and Division Problem Types**

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<tr>
<th>Equal Group Problems</th>
<th>Multiplicative Comparison Problems</th>
<th>Array Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Unknown (Multiplication)</td>
<td>Result Unknown</td>
<td>Whole Unknown</td>
</tr>
<tr>
<td>Size of Groups Unknown (Partitive Division)</td>
<td>Start Unknown</td>
<td>There were 5 baseball teams competing at the field. Each team had 9 baseball players. How many baseball players were there all together?</td>
</tr>
<tr>
<td>Number of Groups Unknown (Measurement Division)</td>
<td>Comparison Factor Unknown</td>
<td>There are 27 children playing on teams at the field. The children are divided equally among 3 teams. How many children are on each team?</td>
</tr>
<tr>
<td>There are 5 boxes of markers. Each box contains 6 markers. How many markers are there in all?</td>
<td>Tyrone ran 3 miles. Jasmine ran 4 times as many miles as Tyrone. How many miles did Jasmine run?</td>
<td>There are 27 children playing on teams at the field. There are 9 children on each team. How many teams are there?</td>
</tr>
<tr>
<td>If 50 markers are shared equally among 5 friends, how many markers will each friend get?</td>
<td>Jasmine ran 12 miles. Tyrone ran 3 miles. How many times more miles did Jasmine run than Tyrone?</td>
<td>Note: Area problems will be included in Grades 4 and 5.</td>
</tr>
<tr>
<td>If 50 markers are placed into school boxes with each box containing 6 markers. How many school boxes can be filled?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### ESSENTIAL UNDERSTANDINGS

- The least number of steps necessary to solve a single-step problem is one.

### ESSENTIAL KNOWLEDGE AND SKILLS

- Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic.
The student will recall multiplication facts through the twelves table, and the corresponding division facts. [Recall moved to 4.4a]
a) represent multiplication and division through $10 \times 10$, using a variety of approaches and models; [Moved from 3.6]
b) create and solve single-step practical problems that involve multiplication and division through $10 \times 10$; and [Create and solve multiplication moved from 3.6; create and solve with division is new expectation]
c) demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10; and [Recall of facts to 12 x 12 included in 4.4a]
d) create and solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less. [Moved from 3.6]

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<tr>
<td>relationships. These relationships remain true regardless of the numbers. [Moved from 3.20 US]</td>
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<tr>
<td>• Grade 3 students should explore and apply the properties of multiplication and addition as strategies for solving multiplication and division problems using a variety of representations (e.g., manipulatives, diagrams, and symbols).</td>
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<tr>
<td>• The properties of the operations are “rules” about how numbers work and how they relate to one another. Students at this level do not need to use the formal terms for these properties but should utilize these properties to further develop flexibility and fluency in solving problems. The following properties are most appropriate for exploration at this level:</td>
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<tr>
<td>– The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., $2 \times 3 = 3 \times 2$). [Moved from 3.20]</td>
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<tr>
<td>– The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number. [Moved from 3.20]</td>
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<tr>
<td>– The associative property of addition states that the sum stays the same when the grouping of addends is changed (e.g., $15 + (35 + 16) = (15 + 35) + 16$).</td>
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<tr>
<td>– The associative property of multiplication states that the product stays the same when the grouping of factors is changed (e.g., $6 \times (3 \times 5) = (6 \times 3) \times 5$). [Copied from 4.16]</td>
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<tr>
<td>– The distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products. [Copied from</td>
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</table>
The student will recall multiplication facts through the twelves table, and the corresponding division facts. [Recall moved to 4.4a]  
3.45  
a) represent multiplication and division through $10 \times 10$, using a variety of approaches and models; [Moved from 3.6]  
b) create and solve single-step practical problems that involve multiplication and division through $10 \times 10$; and [Create and solve multiplication moved from 3.6; create and solve with division is new expectation]  
c) demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10; and [Recall of facts to 12 x 12 included in 4.4a]  
d) create and solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less. [Moved from 3.6]

### UNDERSTANDING THE STANDARD  
(Background Information for Instructor Use Only)

| 5.19 |  
|---|---|---|
| $8 \times 7 = (5 + 2)$ | $40 + 16$ |
| $(8 \times 5) + (8 \times 2)$ | 56 |
| $5 \times 23 = (20 + 3)$ | $100 + 15$ |
| $(5 \times 20) + (5 \times 3)$ | 115 |

- At this level, it is not necessary for students to use the formal terms for the properties.
- Strategies for solving problems that involve multiplication or division may include mental strategies, partial products, the standard algorithm, and the commutative, associative, and distributive properties. [Moved from 3.20]
- In order to develop and use strategies to learn the multiplication facts through the twelves table, students should use concrete materials, hundred chart, and mental mathematics.
- To extend the understanding of multiplication, three models may be used:
  - The equal sets or equal groups model lends itself to sorting a variety of concrete objects into equal groups and reinforces repeated addition or skip counting.
  - The array model, consisting of rows and columns (e.g., 3 rows of 4 columns for a 3 by 4 array) helps build the commutative property.
3.45 The student will recall multiplication facts through the twelves table, and the corresponding division facts. [Recall moved to 4.4a]

a) represent multiplication and division through 10 \times 10, using a variety of approaches and models; [Moved from 3.6]

b) create and solve single-step practical problems that involve multiplication and division through 10 \times 10; and [Create and solve multiplication moved from 3.6; create and solve with division is new expectation]

c) demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10; and [Recall of facts to 12 x 12 included in 4.4a]

d) create and solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less. [Moved from 3.6]

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<tr>
<td>• The length model (e.g., a number line) also reinforces repeated addition or skip counting. [Reordered]</td>
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### STANDARD 3.6

**STRAND: COMPUTATION AND ESTIMATION**  
**GRADE LEVEL: 3**

#### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- The multiplication and division facts through the twelves tables should be modeled.
- Multiplication is a shortcut for repeated addition. The terms associated with multiplication are listed below:
  - factor → 54
  - factor → × 3
  - product → 162
- Creating real-life problems and solving them facilitates the connection between mathematics and everyday experiences (e.g., area problems).
- The use of Base-10 blocks and repeated addition can serve as a model. For example, $4 \times 12$ is read as four sets consisting of one rod and two units. The sum is renamed as four rods and eight units or 48. This can be thought of as $42 + 12 + 12 + 12$ = (SET)
- The use of Base-10 blocks and the array model can be used to solve the same problem. A rectangle array that is one rod and two units long by four units wide is formed. The area of this array is represented by 4 rods and 8 units.
- The number line model can be used to solve a multiplication problem such as $3 \times 4$. This is represented on the number line by three jumps of four.
- The number line model can be used to solve a division problem such as $6 \div 3$ and is represented on the number line by noting how many jumps of three go from 6 to 0.

### ESSENTIAL UNDERSTANDINGS

- All students should
  - Understand the meanings of multiplication and division.
  - Understand the models used to represent multiplying and dividing whole numbers.

### ESSENTIAL KNOWLEDGE AND SKILLS

- The student will use problem-solving, mathematical communication, mathematical reasoning, connections, and representations to
  - Model multiplication, using area, set, and number line models.
  - Model division, using area, set, and number line models.
  - Solve multiplication problems, using the multiplication algorithm, where one factor is 99 or less and the second factor is 5 or less.
  - Create and solve word problems involving multiplication, where one factor is 99 or less and the second factor is 5 or less.

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Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 3 27
The student will **solve practical problems that involve addition** and subtraction with proper fractions having like denominators of 12 or less.

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<tr>
<td>A proper fraction is a fraction whose numerator is less than the denominator. A proper fraction is a fraction that is always less than one.</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
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<tr>
<td>An improper fraction is a fraction whose numerator is greater than or equal to the denominator. An improper fraction is a fraction that is equal to or greater than one.</td>
<td>• Understand that a proper fraction is a fraction whose numerator is smaller than its denominator.</td>
<td>• Demonstrate a fractional part of a whole, using</td>
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<tr>
<td>An improper fraction can be expressed as a mixed number. A mixed number is written as a whole number and a proper fraction.</td>
<td>• Understand that an improper fraction is a fraction whose numerator is greater than or equal to the denominator and is one or greater than one.</td>
<td>region/area models (e.g., pie pieces, pattern blocks, geoboards, drawings);</td>
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<tr>
<td>Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction can be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{1}{5}$).</td>
<td>• Understand that an improper fraction can be expressed as a whole number or a mixed number.</td>
<td>set models (e.g., chips, counters, cubes, drawings); and</td>
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<tr>
<td>When adding or subtracting fractions, an answer greater than one can be expressed as an improper fraction or the equivalent mixed number. For example:</td>
<td>• Understand that a mixed number is written as a whole number and a proper fraction. A mixed number is the sum of a whole number and the proper fraction.</td>
<td>length/measurement models (e.g., nonstandard units such as rods, connecting cubes, and drawings). [Moved to 3.2 EKS]</td>
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<td>$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$.</td>
<td>• Understand that computation with fractions uses the same strategies as whole number computation.</td>
<td>• Name and write fractions and mixed numbers represented by drawings or concrete materials. [Moved to 3.2 EKS]</td>
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<tr>
<td>The strategies of addition and subtraction applied to fractions are the same as the strategies applied to whole numbers.</td>
<td>• When adding and subtracting fractions the fractions must represent like size units (e.g. one-fifth added to three-fifths is four-fifths). This understanding builds the foundation for why common denominators are necessary in future work with adding unlike fractions and for work in algebra when adding polynomial expressions.</td>
<td>• Represent a given fraction or mixed number, using concrete materials, pictures, and symbols. For example, write the symbol for one-fourth and represent it with concrete materials and/or pictures. [Moved to 3.2 EKS]</td>
</tr>
<tr>
<td>Reasonable answers to problems involving addition and subtraction of fractions can be established by using benchmarks</td>
<td>• Solve practical problems that involve addition and subtraction with proper fractions having like denominators of 12 or less, using concrete materials and pictorial models representing area/regions (e.g., circles, squares, and rectangles), length/measurements (e.g., fraction bars and strips), and sets (e.g., counters).</td>
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3.57 The student will **solve practical problems that involve addition** and **subtraction with** proper fractions having like denominators of 12 or less.

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<td>such as 0, ( \frac{1}{2} ), and 1. For example, ( \frac{3}{5} ) and ( \frac{4}{5} ) are each greater than ( \frac{1}{2} ), so their sum is greater than 1.</td>
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<tr>
<td>Concrete materials and pictorial models representing area/regions (e.g., circles, squares, and rectangles), length/measurements (fraction bars and strips), and sets (counters) can be used to add and subtract fractions having like denominators of 12 or less.</td>
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</table>
Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K–3 should encounter measurement in many normal situations, from their daily use of the calendar and from science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, area, perimeter, volume, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both techniques and nonstandard and standard tools to find measurements and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

[Moved from grade 4 Measurement strand introduction]

Students in grades 3-5 should be actively involved in measurement activities that require a dynamic interaction among students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects.

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, perimeter, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary), to measure length, weight/mass, liquid volume, area, perimeter, temperature, and time. Student understanding of measurement continues to be enhanced by using appropriate tools such as rulers, balances, clocks, and thermometers.

[Moved from grade 4 Geometry stand introduction]

The study of geometry helps students represent and make sense of the world. In grades 3 - 5 reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.
Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles.

[Moved from geometry strand introduction] The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)
The student will:

a) determine, by counting, the value of a collection of bills and coins whose total value is $5.00 or less;

b) compare the value of the two sets of coins or two sets of coins and bills and coins; and

c) make change from $5.00 or less. [Edited to match EKS]

**UNDERSTANDING THE STANDARD**

(Background Information for Instructor Use Only)

- Simulate everyday opportunities to count and compare a collection of coins and one-dollar bills whose total value is $5.00 or less.

- The value of a collection of coins and bills can be determined by counting on, beginning with the highest value, and/or by grouping the coins and bills.

- A variety of skills can be used to determine the change after a purchase, including:

  - counting on, using coins and bills, i.e., starting with the amount to be paid (purchase price),

  - counting forward to the next dollar, and then counting forward by dollar bills to reach the amount from which to make change; and

  - mentally calculating the difference.

**ESSENTIAL UNDERSTANDINGS**

All students should:

- Understand that a collection of coins and bills has a value that can be counted.

- Understand how to make change from $5.00 or less.

**ESSENTIAL KNOWLEDGE AND SKILLS**

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

- Count and determine the value of collections of coins and bills up to whose total value is $5.00 or less. (a)

- Compare the values of two sets of coins or two sets of coins and bills, up to $5.00, using the terms greater than, less than, and equal to. (b)

- Make change from $5.00 or less. (c)
3.79 The student will estimate and use U.S. Customary and metric units to measure
a) length to the nearest \( \frac{1}{2} \) inch, inch, foot, yard, centimeter, and meter; and
b) liquid volume in cups, pints, quarts, gallons, and liters;
c) weight/mass in ounces, pounds, grams, and kilograms; and [Included in 4.8]
d) area and perimeter. [Moved to 3.8a, b]

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| Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object’s mass, although they use the term weight (e.g., “How much does it weigh?” versus “What is its mass?”). | All students should
- Understand how to estimate measures of length, liquid volume, weight/mass, area and perimeter.
- Understand how to determine the actual measure of length, liquid volume, weight/mass, area and perimeter.
- Understand that perimeter is a measure of the distance around a polygon.
- Understand that area is a measure of square units needed to cover a surface. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
- Estimate and use U.S. Customary and metric units to measure lengths of objects to the nearest \( \frac{1}{2} \) of an inch, inch, foot, yard, centimeter, and meter. (a)
- Determine the actual measure of length using U.S. Customary and metric units to measure objects to the nearest \( \frac{1}{2} \) of an inch, foot, yard, centimeter, and meter. (a)
- Estimate and use U.S. Customary and metric units to measure liquid volume to the nearest cup, pint, quart, gallon, and liter. (b)
- Determine the actual measure of liquid volume using U.S. Customary and metric units to measure to the nearest cup, pint, quart, gallon, and liter. (b)
- Estimate and use U.S. Customary and metric units to measure the weight/mass of objects to the nearest ounce, pound, gram, and kilogram.
- Determine the actual measure of weight/mass using U.S. Customary and metric units to measure the weight/mass of objects to the nearest ounce, pound, gram, and kilogram. [Included in 4.6]
- Estimate and use U.S. Customary and metric units to measure area and perimeter. [Moved to 3.8 EKS]
- Determine the actual measure of area or perimeter using U.S. Customary and metric units. [Moved to 3.8 EKS] |
| The concept of a standard measurement unit is one of the major ideas in understanding measurement. Familiarity with standard units is developed through hands-on experiences of comparing, estimating, measuring, and constructing. | | |
| Benchmarks of common objects need to be established for each of the specified units of measure (e.g., the liquid volume of a small glass of orange juice is about one cup; mass of a mathematics book is about one kilogram). Practical experiences measuring the mass/volume of familiar objects help to establish benchmarks and facilitates the student’s ability to estimate measures. | | |
| One unit of measure may be more appropriate than another to use when measuring an object, depending on the size of the object and the degree of accuracy desired. | | |
| Correct use of measurement tools is essential to understanding the concepts of measurement. | | |
| Perimeter is the distance around any two-dimensional figure and is found by adding the measures of the sides. | | |
| Area is a two-dimensional measure and is therefore measured in square units. | | |
| Area is the number of square units needed to cover a figure, or | | |

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 3
3.79 The student will estimate and use U.S. Customary and metric units to measure
   a) length to the nearest \( \frac{1}{2} \) inch, inch, foot, yard, centimeter, and meter; and
   b) liquid volume in cups, pints, quarts, gallons, and liters;
   c) weight/mass in ounces, pounds, grams, and kilograms; and [Included in 4.8]
   d) area and perimeter. [Moved to 3.8a,b]

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<td>more precisely, it is the measure in square units of the interior region of a two-dimensional figure.</td>
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### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- A polygon is a closed plane figure composed of at least three line segments that do not cross. None of the sides are curved.
- A plane figure is any closed, two-dimensional shape.
- Perimeter is a measure of the path or distance around a polygon and is found by adding the measures of the sides of a plane figure.
- Area is the number of iterations of a two-dimensional unit needed to cover a surface. The two-dimensional unit is usually a square, but it could also be another shape such as a rectangle or an equilateral triangle.
- The unit of measure used to find the perimeter or area is stated along with the numerical value when expressing the perimeter or area of a figure (e.g., the perimeter of the book cover is 38 inches and the area of the book cover is 90 square inches).
- Opportunities to explore the concepts of perimeter and area should involve hands-on experiences (e.g., placing tiles/toothpicks around a polygon and counting the number of tiles/toothpicks to determine its perimeter and filling or covering a polygon with cubes/tiles (square units) and counting the cubes/tiles to determine its area).

### ESSENTIAL UNDERSTANDINGS

- All students should
  - Understand the meaning of a polygon as a closed figure with at least three sides.
  - Understand that perimeter is a measure of the distance around a polygon.
  - Understand how to determine the perimeter by counting the number of units around a polygon.
  - Understand that area is a measure of square units needed to cover a surface.
  - Understand how to determine the area by counting the number of square units.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Estimate and use U.S. Customary and metric units to measure the distance around a polygon with no more than 6 sides each side of a variety of polygons and add the measures of the sides to determine the perimeter of each polygon. (a) [Estimate and use moved from 3.9d]

- Determine the area of a given surface by estimating and then counting the number of square units needed to cover the surface. (b)
The student will:

a) tell time to the nearest minute, using analog and digital clocks; and

b) determine solve practical problems related to elapsed time in one-hour increments over within a 12-hour period; and

c) identify equivalent periods of time and solve practical problems related to equivalent periods of time. [Moved from 3.12]

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<td>(Background Information for Instructor Use Only)</td>
<td>All students should &lt;br&gt; - Apply appropriate techniques to determine time to the nearest minute, using analog and digital clocks. &lt;br&gt; - Understand how to determine elapsed time in one-hour increments over a 12-hour period.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to &lt;br&gt; - Tell time to the nearest minute, using analog and digital clocks. (a) &lt;br&gt; - Match a written time (e.g., 4:38, 7:09, 12:51) to the times shown on analog and digital clocks to written times and to each other in the nearest minute. (a) &lt;br&gt; - When given the beginning time and ending time, determine the elapsed time in one-hour increments within a 12-hour period (times do not cross between a.m. and p.m.). &lt;br&gt; - Solve practical problems in relation to time that has related to elapsed time in one-hour increments, within a 12-hour period (within a.m. or within p.m.): &lt;br&gt;   - when given the beginning time and the ending time, determine the time that has elapsed; (b) &lt;br&gt;   - when given the beginning time and amount of elapsed time in one-hour increments, determine the ending time; or (b) &lt;br&gt;   - when given the ending time and the elapsed time in one-hour increments, determine the beginning time. (b) &lt;br&gt; - Identify the number of minutes in an hour and the number of hours in a day. (c) [Moved from 3.12] &lt;br&gt; - Identify equivalent relationships observed in a calendar, including the approximate number of days in a given month (about 30), the number of days in a week, the number of days in a year (about 365 1/4), and the number of months in a year. (c) [Moved from 3.12]</td>
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- While digital clocks make reading time easy, it is necessary to ensure that students understand that there are sixty minutes in an hour.
- Use of a demonstration clock with gears ensures that the positions of the hour hand and the minute hand are precise when time is read.
- Students need to understand that time has passed or will pass in equal increments (i.e., seconds, minutes, or hours).
- Elapsed time is the amount of time that has passed between two given times.
- Elapsed time should be modeled and demonstrated using geared analog clocks and timelines.
- It is necessary to ensure that students need to understand that there are sixty minutes in an hour when using analog and digital clocks.
- Elapsed time can be found by counting on from the beginning time or counting back from the ending time, to the finishing time.
- Count the number of whole hours between the beginning time and the finishing time.
- For example, to find the elapsed time between 7 a.m. and 10 a.m., students can count on to find the difference between the times (7 and 10), so the total elapsed time is 3 hours.
- The knowledge that a year has about 365 and 1/4 days will help students understand the necessity of adding a full day every fourth year, called a leap year. [Moved from 3.12]
3.911 The student will
a) tell time to the nearest minute, using analog and digital clocks; and
b) determine practical problems related to elapsed time in one-hour increments over within a 12-hour period; and
c) identify equivalent periods of time and solve practical problems related to equivalent periods of time. [Moved from 3.12]

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- The use of a calendar facilitates the understanding of time relationships between days and months, days and weeks, days and years, and months and years. Students need to know the relationship between periods of time (e.g., if there are 24 hours in one day, how many hours are in three days?). [Moved from 3.12]
- The use of an analog clock facilitates the understanding of time relationships between minutes and hours and hours and days. [Moved from 3.12]

ESSENTIAL UNDERSTANDINGS

ESSENTIAL KNOWLEDGE AND SKILLS

- Solve practical problems in relation to equivalent periods of time to include:
  - approximate days in 5 or fewer months;
  - days in 5 or fewer weeks;
  - months in 5 or fewer years;
  - minutes in 5 or fewer hours; and
  - hours in 5 or fewer days. (c)
3.12 The student will identify equivalent periods of time, including relationships among days, months, and years, as well as minutes and hours. [Moved to 3.9c]

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<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The knowledge that a year has (365) and (\frac{1}{4}) days will help students understand the necessity of adding a full day every fourth year, called a leap year. [Moved to 3.9 US]</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• The use of a calendar facilitates the understanding of time relationships between days and months, days and weeks, and months and years. [Moved to 3.9 US]</td>
<td>• Understand the relationship that exists among periods of time, using calendars, and clocks.</td>
<td>• Identify equivalent relationships observed in a calendar, including the number of days in a given month, the number of days in a week, the number of days in a year, and the number of months in a year. [Moved to 3.9 EKS]</td>
</tr>
<tr>
<td>• Recognize that students need to know the relationships, such as if there are 24 hours in one day, how many hours are in three days? [Moved to 3.9] If the date is January 6, what date would it be in two weeks? How many weeks are in March, April, and May?</td>
<td></td>
<td>• Identify the number of minutes in an hour and the number of hours in a day. [Moved to 3.9 EKS]</td>
</tr>
<tr>
<td>• The use of an analog clock facilitates the understanding of time relationships between minutes and hours and days. [Moved to 3.9 US]</td>
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</tr>
</tbody>
</table>
3.1013 The student will read temperature to the nearest degree from a Celsius thermometer and a Fahrenheit thermometer. Real thermometers and physical models of thermometers will be used. [Thermometer information moved to EKS]

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<tbody>
<tr>
<td>• Estimating and measuring temperatures in the environment in Fahrenheit and Celsius require the use of real thermometers.</td>
<td>All students should understand how to measure temperature in Celsius and Fahrenheit with a thermometer.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• A variety of physical models (e.g., circular, linear) should can be used to represent the temperature determined by a real thermometer.</td>
<td></td>
<td>• Read Celsius and Fahrenheit temperatures to the nearest degree from using real Celsius and Fahrenheit thermometers, physical models, or (including using pictorial representations) of such thermometers.</td>
</tr>
<tr>
<td>• The symbols for degrees in Celsius (°C) and degrees in Fahrenheit (°F) should be used to write temperatures.</td>
<td></td>
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</tr>
<tr>
<td>• Celsius and Fahrenheit temperatures should be related to everyday occurrences by measuring the temperature of the classroom, the outside, liquids, body temperature, and other things found in the environment, things found in the student’s environment (e.g., the temperature of the classroom: temperature on the playground; temperature of warm and cold liquids; body temperature).</td>
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<tr>
<td>• At this level, scale increments should be limited to 1 or 2.</td>
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</tbody>
</table>
Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on

- observing, identifying, describing, comparing, contrasting and investigating solid objects and their faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to these figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
3.1211 The student will

a) define polygon;[Moved from 4.12a]
b) identify and name polygons with 10 or fewer sides; and[Moved from 4.12b]
c) combine and subdivide polygons with 3 or 4 sides and name the resulting polygons.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- A polygon is a closed plane figure composed of at least 3 line segments that do not cross. None of the sides are curved.
- Polygons may be described using line segments and angles by their attributes (e.g. sides and vertices). Line segments form the sides of a polygon and angles are formed by two line segments coming together at a vertex of a polygon.
- A rectangle, square, trapezoid, parallelogram, and rhombus are all classified as quadrilaterals.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to

- Define a polygon. (a)[Moved from 4.12 EKS]
- Classify figures as polygons or not polygons. (a)[Moved from 4.12 EKS]
- Identify and name polygons with 10 or fewer sides in various orientations:
  - triangle is a 3-sided polygon;
  - quadrilateral is a 4-sided polygon;
  - pentagon is a 5-sided polygon;
  - hexagon is a 6-sided polygon;
  - heptagon is a 7-sided polygon;
  - octagon is a 8-sided polygon;
  - nonagon is a 9-sided polygon; and
  - decagon is a 10-sided polygon. (b)[Moved from 4.12 EKS]
- Combine no more than three polygons, where each has 3 or 4 sides, and name the resulting polygon. (c)
- Subdivide a 3- or 4-sided polygon into no more than three parts and name the resulting polygons. (c)
### UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)
- The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.
  - **Level 0:** Pre-recognition. Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
  - **Level 1:** Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., “I know it’s a rectangle because it looks like a door, and I know that the door is a rectangle.”).
  - **Level 2:** Analysis. Properties are perceived, but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., “I know it’s a rectangle because it’s closed, it has four sides and four right angles, and opposite sides are parallel.”).
- A plane geometric figure is any two-dimensional closed figure. Circles and polygons are examples of plane geometric figures.
- Three-dimensional figures are called solid figures or simply solids. Solids enclose a region of space. The interior of both plane and solid figures are not part of the figure. Solids are classified by the types of surfaces they have. These surfaces may be flat, curved, or both.
- The study of geometric figures must be active, using visual images and concrete materials.
- Access to a variety of concrete tools such as graph paper, pattern blocks, geoboards, and geometric solids is greatly enhanced by computer software tools that support exploration.

### ESSENTIAL UNDERSTANDINGS

<table>
<thead>
<tr>
<th>All students should</th>
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<tbody>
<tr>
<td>- Understand how to identify and describe plane and solid geometric figures by using relevant characteristics.</td>
</tr>
<tr>
<td>- Understand the similarities and differences between plane and solid figures.</td>
</tr>
</tbody>
</table>

### ESSENTIAL KNOWLEDGE AND SKILLS

<table>
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<tr>
<th>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</th>
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<tbody>
<tr>
<td>- Identify models and pictures of plane geometric figures (circle, square, rectangle, and triangle) and solid geometric figures (cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by name.</td>
</tr>
<tr>
<td>- Identify and describe plane geometric figures by counting the number of sides and angles.</td>
</tr>
<tr>
<td>- Identify and describe solid geometric figures by counting the number of angles, vertices, edges, and by the number and shape of faces.</td>
</tr>
<tr>
<td>- Compare and contrast characteristics of plane and solid geometric figures (e.g., circle/sphere, square/cube, triangle/square pyramid, and rectangle/rectangular prism), by counting the number of sides, angles, vertices, edges, and the number and shape of faces.</td>
</tr>
<tr>
<td>- Compare and contrast characteristics of solid geometric figures (e.g., cube, rectangular pyramid, square pyramid, sphere, cylinder, and cone) to similar objects in everyday life (e.g., a party hat is like a cone).</td>
</tr>
<tr>
<td>- Identify characteristics of solid geometric figures (cylinder, cone, cube, square pyramid, and rectangular prism).</td>
</tr>
</tbody>
</table>

[Moved to 4.11 EKS]
The student will identify, describe, compare, and contrast characteristics of plane and solid geometric figures (circle, square, rectangle, triangle, cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models.

### UNDERSTANDING THE STANDARD

**Opportunity must be provided for building and using geometric vocabulary to describe plane and solid figures.**

- A cube is a solid figure with six congruent square faces and with every edge the same length. A cube has 8 vertices and 12 edges.
- A cylinder is a solid figure formed by two congruent parallel circles joined by a curved surface.
- A cone is a solid, pointed figure that has a flat, round face (usually a circle) that is joined to a vertex by a curved surface.
- A rectangular prism is a solid figure in which all six faces are rectangles with three pair of parallel congruent opposite faces.
- A sphere is a solid figure with all of its points the same distance from its center.
- A square pyramid is a solid figure with one square face and four triangular faces that share a common vertex.
- A face is a polygon that serves as one side of a solid figure (e.g., a square is a face of a cube).
- An angle is formed by two rays with a common endpoint. This endpoint is called the vertex. Angles are found wherever lines intersect. An angle can be named in three different ways by using – three letters to name, in this order, a point on one ray, the vertex, and a point on the other ray; – one letter at the vertex; or – a number written inside the rays of the angle.
- An edge is the line segment where two faces of a solid figure intersect.
- A vertex is the point at which two lines, line segments, or rays meet to form an angle. It is also the point on a three dimensional figure where three or more faces intersect.
3.14 The student will identify, describe, compare, and contrast characteristics of plane and solid geometric figures (circle, square, rectangle, triangle, cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models.

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<tr>
<td>• Students should be reminded that a solid geometric object is hollow rather than solid. The “solid” indicates a three-dimensional figure.</td>
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</tbody>
</table>
### UNDERSTANDING THE STANDARD

**Background Information for Instructor Use Only**

- A point is an exact location in space. It has no length, width, or height. Usually, a point is named with a capital letter. **[Naming is included in grade 4]**

- A line is a collection of points extending indefinitely in both directions. It has no endpoints. When a line is drawn, at least two points on it can be marked and given capital letter names. The line can also be named with a single, lower case letter. Arrows must be drawn to show that the line goes on in both directions infinitely. **[Naming is included in grade 4]**

- A line segment is part of a line. It has two endpoints and includes all the points between and including those endpoints. The endpoints are used to name a line segment. **[Naming is included in grade 4]**

- A ray is part of a line. It has one endpoint and continues on and extends indefinitely in one direction.

- An angle is formed by two rays having a common endpoint. This endpoint is called the vertex. Angles are found wherever lines or line segments intersect.

- Geometric notation to name lines, line segments and rays is included in grade 4.

- Angles are found wherever lines and line segments intersect. An angle can be named in three different ways by using:
  - three letters to name, in this order, a point on one ray, the vertex, and a point on the other ray;
  - one letter at the vertex; or
  - a number written inside the rays of the angle.

   Angle rulers may be particularly useful in developing the concept of an angle.

### ESSENTIAL UNDERSTANDINGS

**All students should**

- Understand that line segments and angles are fundamental components of plane polygons.

- Understand that a line segment is a part of a line, has two endpoints, and contains all the points between those two endpoints.

- Understand that points make up a line.

- Understand that a line continues indefinitely in two opposite directions.

- Understand that a ray is part of a line, has one endpoint, and continues indefinitely in only one direction.

- Understand that an angle is formed by two rays having a common endpoint.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify examples of points, lines, line segments, rays, and angles, and lines.

- Describe endpoints and vertices as they relate to lines, line segments, rays, and angles.

- Draw representations of points, line segments, rays, angles, and lines, using a ruler or straightedge.
### UNDERSTANDING THE STANDARD

#### Background Information for Instructor Use Only

- **Essential Understandings**
  - **Essential Knowledge and Skills**
  - **The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**
    - Identify examples of congruent and noncongruent figures. Verify their congruence by laying one on top of the other using drawings or models.
    - Determine and explain why plane figures are congruent or noncongruent, using tracing procedures.

#### ESSENTIAL UNDERSTANDINGS

- **All students should**
  - Understand that congruent plane figures match exactly.
  - Understand that noncongruent plane figures do not match exactly.
  - Understand that congruent plane figures remain congruent even if they are in different spatial orientations.
  - Understand that noncongruent plane figures remain noncongruent even if they are in different spatial orientations.

#### ESSENTIAL KNOWLEDGE AND SKILLS

- **The student will identify and describe congruent and noncongruent plane figures.**
- **Congruent plane figures** are figures having exactly the same size and shape. Noncongruent plane figures are figures that do not have exactly the same size and shape. Opportunities for exploring figures that are congruent and/or noncongruent can best be accomplished by using physical models.
- **Congruent plane figures** remain congruent even if they are in different spatial orientations.
- **Have students identify** figures that are congruent or noncongruent **may be identified** by using direct comparisons and/or tracing procedures.
Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms impossible, unlikely, equally likely, more likely, and certain.

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

[Moved from grade 4 strand introduction]
Students entering grades 3-5 have begun to explore the concepts of the measurement of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students have had initial experiences in predicting whether a game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction at grades 3-5 is to deepen their understanding of the concepts of probability by:

- offering opportunities to set up models simulating practical events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities that instill a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data collection strategies. Students should continue to focus on:

- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to practical experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions (“How many?” “How much?”) from the data displays;
- identifying and justifying comparisons (“Which is the most? Which is the least?” “Which is the same? Which is different?”) about the information;
- comparing their initial predictions to the actual results; and
- communicating to others their interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data analysis methods as powerful means for decision making.
The student will

a) collect, and organize, and represent data, using observations, measurements, surveys, or experiments in picture pictographs or bar graphs; and [Process included in EKS]

b) construct a line plot, a picture graph, or a bar graph to represent the data; and [Line plot moved to 5.16; construct picture pictographs and bar graphs moved to 3.1415a]

c) read and interpret the data represented in pictographs and bar graphs and write a sentence analyzing the data.

### UNDERSTANDING THE STANDARD

**Background Information for Instructor Use Only**

- Investigations involving data should occur frequently and relate to students’ experiences, interests, and environment.
- Formulating questions for investigations is student-generated at this level. For example: What is the cafeteria lunch preferred by students in the class when four lunch menus are offered? If a new student enters our class tomorrow what color eyes will they likely have?
- The purpose of a graph is to represent data gathered to answer a question.
- A picture pictograph uses pictures or symbols to represent one or more objects.
- A key should be provided for the symbol in a picture pictograph when the symbol represents more than one piece of data (e.g., represents five people in a graph). The key is used in a graph to assist in the analysis of the displayed data. One-half of a symbol represents one-half of the value of the symbol being used, as indicated in the key.
- Students’ prior knowledge and work with skip counting helps them to identify the number of pictures or symbols to be used in a pictograph.
- Definitions for the terms picture graph and pictographs vary. Pictographs are most often defined as a pictorial representation of numerical data. The focus of instruction should be placed on reading and using the key in analyzing the graph. There is no need for students to distinguish between a picture graph and a pictograph.

### ESSENTIAL UNDERSTANDINGS

- **All students should**
  - Understand how data can be collected and organized.
  - Understand that data can be displayed in different types of graphs depending on the data. [Moved to US]
  - Understand how to construct a line plot, picture graph, or bar graph.
  - Understand that data sets can be interpreted and analyzed to draw conclusions.

### ESSENTIAL KNOWLEDGE AND SKILLS

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
  - Formulate questions to investigate. (a)
  - Design data investigations to answer formulated questions, limiting the number of categories for data collection to four. (a)
  - Collect and organize data, using various forms of data collections (e.g., surveys, polls, questionnaires, scientific experiments, and observations). (a)
  - Organize data and construct a line plot with no more than 30 data points. [Moved to 5.16 EKS]
  - Read, interpret, and analyze information from line plots by writing at least one statement. [Moved to 5.16 EKS]
    - Label each axis on a bar graph and give the bar graph a title. Limit increments on the numerical axis to whole numbers representing multiples of 1, 2, 5, or 10. (a)
  - Represent data in a picture pictograph (limited to 16 or fewer data points for no more than four categories). (a)
  - Analyze the data represented in picture pictographs and bar graphs, orally and in writing. (b)
    - Read the information presented on a simple bar or
### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Bar graphs are used to compare counts of different categories (categorical data). Using grid paper ensures more accurate graphs.
- A bar graph uses **parallel** horizontal or vertical **parallel** bars to represent counts for categories. One bar is used for each category, with the length of the bar representing the count for that category.
- There is space before, between, and after **each** of the bars.
- The axis displaying the scale representing the count for the categories should begin at zero and extend one increment above the greatest recorded piece of data. Third Grade 3 students should collect data that are recorded in increments of whole numbers, usually limited to multiples of 1, 2, 5, or 10.
- Each axis should be labeled, and the graph should be given a title.
- Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be written (e.g., similarities and differences, least and greatest, the categories, and total number of responses).
- Statements should also express a prediction based on the analysis and interpretation of the characteristics of the data in the graph (e.g., The lunch room should serve pizza more often since that was the lunch students have liked the most.)
- A line plot shows the frequency of data on a number line. Line plots are used to show the spread of the data and quickly identify the range, mode, and any outliers.

### ESSENTIAL UNDERSTANDINGS

- **picture** pictograph (e.g., the title, the categories, the description of the two axes). (b)

### ESSENTIAL KNOWLEDGE AND SKILLS

- Analyze and interpret information from **picture** pictograph and bar graphs, with up to 30 data points and up to 8 categories, describe interpretation orally and by writing at least one sentence. (b)
  - Describe the categories of data and the data as a whole (e.g., data were collected on four ways to cook or prepare eggs — scrambled, fried, hard boiled, and egg salad — eaten by students). (b)
  - Identify parts of the data that have special characteristics, including categories with the greatest, the least, or the same (e.g., most students prefer scrambled eggs). (b)
  - Select a correct interpretation of a graph from a set of interpretations of the graph, where one is correct and the remaining are incorrect. For example, a bar graph containing data on four ways to cook or prepare eggs—eaten by students—show that more students prefer scrambled eggs. A correct answer response, if given, would be that more students prefer scrambled eggs than any other way to cook or prepare eggs. (b)
The student will
a) collect, and organize, and represent data, using observations, measurements, surveys, or experiments in picture pictographs or bar graphs; and\[Process included in EKS\]
b) construct a line plot, a picture graph, or a bar graph to represent the data; and\[Line plot moved to 5.16; construct picture pictographs and bar graphs moved to 3.1415a\]
c) read and interpret the data represented in pictographs and bar graphs and picture graphs and write a sentence analyzing the data.

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<tr>
<td>Number of Books Read</td>
<td><img src="image" alt="Graph" /></td>
<td></td>
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<tr>
<td>10 11 12 13 14 15 16 17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Each x represents one student</td>
<td></td>
<td></td>
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<tr>
<td>• When data are displayed in an organized manner, the results of the investigations can be described and the posed question answered.</td>
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<tr>
<td>• Recognition of appropriate and inappropriate statements begins at this level with graph interpretations.</td>
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</table>
3.14518  The student will investigate and describe the concept of probability as a **measurement of** chance and list possible results **outcomes** for a given situation **single event**.

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</tr>
<tr>
<td>• A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives.</td>
<td>• Investigate, understand, and apply basic concepts of probability.</td>
<td>• Define probability as the <strong>measurement</strong> of chance that an event will happen.</td>
</tr>
<tr>
<td>• Investigation of experimental probability is continued at this level through informal activities using materials such as two-colored counters, spinners, and random number <strong>generators</strong> (number cubes).</td>
<td>• Understand that probability is the chance of an event happening.</td>
<td>• List all possible outcomes for a <strong>given situation single event</strong> (e.g., heads and tails are the two possible outcomes of flipping a coin). <strong>Limit</strong> The number of outcomes <strong>may be limited</strong> to 12 or fewer.</td>
</tr>
<tr>
<td>• Probability is the <strong>measurement</strong> of chance of an event occurring.</td>
<td></td>
<td>• Identify-Describe the degree of likelihood of an outcome occurring using terms such as <strong>impossible</strong>, <strong>unlikely</strong>, <strong>as likely as</strong>, <strong>equally likely</strong>, <strong>equally likely</strong>, <strong>likely</strong>, and <strong>certain</strong>.</td>
</tr>
<tr>
<td>• The probability of an event occurring is the ratio of desired outcomes to the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event $= \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$.</td>
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<tr>
<td>• The probability of an event occurring is represented by a ratio between 0 and 1. An event is “impossible” if it has a probability of 0 (e.g., the probability that the month of April will have 31 days). An event is “certain” if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow morning).</td>
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<tr>
<td>• When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time).</td>
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<tr>
<td>• Students should have opportunities to describe in informal terms (e.g., <strong>impossible</strong>, <strong>unlikely</strong>, <strong>as likely as</strong>, <strong>equally likely</strong>, <strong>equally likely</strong>, <strong>likely</strong>, and <strong>certain</strong>) the degree of likelihood of an event occurring. Activities should include real-life examples.</td>
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<tr>
<td>• For any event, such as flipping a coin, spinning a spinner, or rolling a number cube, the <strong>equally likely</strong> things that can happen are called <strong>outcomes</strong>. For example, there are two <strong>equally likely</strong> possible outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up; when flipping a coin, each of the outcomes is <strong>equally likely</strong>.</td>
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3.14518 The student will investigate and describe the concept of probability as a measurement of chance and list possible results for a given situation.

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<td>A sample space represents all possible outcomes of an experiment. The sample space may be organized in a list, table, or chart.</td>
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</tr>
<tr>
<td>Students in grade 5 will have experiences with probability that involve combinations (i.e., how many different outfits can be made given 3 shirts and two pants).</td>
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</tr>
</tbody>
</table>
Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children’s mathematical development.

Students in kindergarten through third grade develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns. These students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to algebra.

Students entering grades 3-5 have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships within a pattern.

The focus of instruction is to help students develop a solid use of patterning as a problem solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They interpret the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By interpreting mathematical situations and models, students begin to represent these, using symbols and variables to write “rules” for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.
### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Developing fluency and flexibility in identifying, describing, transforming, and extending patterns is fundamental to mathematics, particularly algebraic reasoning.
- Exploring patterns requires active physical and mental involvement.
- The use of materials to extend patterns permits experimentation or trial-and-error problem solving approaches that are almost impossible without them.
- Reproduction of a given pattern in a different representation, using symbols and objects, lays the foundation for writing numbers symbolically or algebraically.
- The simplest types of patterns are repeating patterns. In each case, students need to identify the basic unit or core of the pattern and repeat it. Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience.
- Growing patterns are more difficult for students to understand than repeating patterns because not only must they identify the core, they must also look for a generalization or relationship that will tell them how the pattern is changing from step to step. Determining what comes next, they must also begin the process of generalization. Growing patterns are patterns in which the change can be described as an increase or decrease by a constant value. Students need experiences with growing patterns in both arithmetic and geometric formats using objects, pictures, numbers, and tables.
- In numeric patterns, students must determine the difference, called the common difference, between each succeeding number in order to determine what is added to each previous number to obtain the next number. Students do not need to use the term common difference at this level.

### ESSENTIAL UNDERSTANDINGS

- All students should:
  - Understand that numeric and geometric patterns can be expressed in words or symbols. [Moved to US]
  - Understand the structure of a pattern and how it grows or changes.
  - Understand that mathematical relationships exist in patterns.
  - Understand that patterns can be translated from one representation to another. [Moved to US]

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

- Recognize, identify, and describe repeating and growing numeric and geometric patterns using words, objects, pictures, numbers, and tables (e.g., skip counting, addition tables, and multiplication tables).
- Identify a missing term in a pattern (e.g., 4, 6, 8, 10, 12).
- Describe Create repeating and growing numeric and geometric patterns formed using objects, pictures, numbers, and tables, and or pictures, using the same or different forms.
- Extend or identify missing parts in repeating and growing patterns of numbers or figures using concrete objects, pictures, numbers, and tables, and or pictures.
- Transfer repeating and growing patterns using numbers, tables, objects, and pictures. [Moved from US]
- Apply input and output rules involving addition and subtraction to solve problems in various contexts. Solve problems that involve the application of input and output rules limited to addition and subtraction of whole numbers.
- When given the rule, determine the missing values in a list or table. (Rules will be limited to addition and subtraction of whole numbers.)
The student will recognize and identify, describe, create, and extend and transfer patterns found in objects, pictures, numbers and tables, a variety of patterns formed using numbers, tables, and pictures, and extend the patterns, using the same or different forms.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Sample numeric patterns include:</td>
<td></td>
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<tr>
<td>- 6, 9, 12, 15, 18, …(growing pattern);</td>
<td></td>
<td></td>
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<tr>
<td>- 1, 2, 4, 7, 11, 16, …(growing pattern);</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- 20, 18, 16, 14,…(growing pattern); and</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- 1, 3, 5, 1, 3, 5, 1, 3, 5 …(repeating pattern).</td>
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<tr>
<td>● Students should have experiences with numeric patterns that are additive in nature Numeric patterns, at this level, will be limited to addition and subtraction of whole numbers.</td>
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<tr>
<td>● In geometric figure patterns, students must often recognize transformations or changes of position in the plane of a figure, particularly rotation or reflection. Rotation is the result of turning a figure around a point or a vertex, and reflection is the result of flipping a figure over a line. Students at this level do not need to know the terms related to transformations of figures.</td>
<td></td>
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<tr>
<td>● Sample geometric figure patterns include:</td>
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<tr>
<td>- O Δ O O Δ Δ O O O Δ Δ Δ; and</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- □□ ★★ □□ ★★ □□ ★★ □□ ★★ . . . (repeating pattern).</td>
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<tr>
<td>● A table of values can be analyzed to determine the pattern that has been used, and that pattern can then be used to find the next value.</td>
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<tr>
<td>● Sample pattern transfers include:</td>
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</tr>
<tr>
<td>- 2, 5, 8, 11, 14, 17 is the same pattern has the same structure as 4, 7, 10, 13, 16, 19</td>
<td></td>
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</tr>
</tbody>
</table>
The student will recognize and identify, describe, create, and extend patterns found in objects, pictures, numbers and tables. A variety of patterns formed using numbers, tables, and pictures, and extend the patterns, using the same or different forms.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- The same pattern has the same structure as [ ]

- Input/output tables with a given rule provide direction for what to do to the input to get the output and can then be used to determine an unknown value. Applying rules to the input to find the output builds the foundation for functional thinking. Sample input/output tables that require applying the rule to the input to get the output:

<table>
<thead>
<tr>
<th>Rule: Add 4</th>
<th>Rule: Subtract 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>Output</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>?</td>
</tr>
<tr>
<td>8</td>
<td>?</td>
</tr>
</tbody>
</table>

### ESSENTIAL UNDERSTANDINGS

### ESSENTIAL KNOWLEDGE AND SKILLS
STANDARD 3.1720  STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA  GRADE LEVEL 3

3.1720 The student will create equations to represent equivalent mathematical relationships.\[Moved from 3.20 EKS\]

a) investigate the identity and the commutative properties for addition and multiplication; and \[Use of properties moved to 3.3 EKS and 3.4 EKS\]

b) identify examples of the identity and commutative properties for addition and multiplication.

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</thead>
<tbody>
<tr>
<td>Mathematical relationships can be expressed using equations (number sentences). [Moved from middle column]</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>A number sentence is an equation with numbers (e.g., 6 + 3 = 9; or 6 + 3 = 4 + 5). [Reordered]</td>
<td>• Understand that mathematical relationships can be expressed using number sentences. [Moved to US]</td>
<td>• Investigate the identity property for addition and determine that when the number zero is added to another number or another number is added to the number zero, that number remains unchanged. Examples of the identity property for addition are 0 + 2 = 2; 5 + 0 = 5. [Moved to 3.3 US]</td>
</tr>
<tr>
<td>The equal symbol (=) means that the values on either side are equivalent (balanced).</td>
<td>• Understand the identity property for addition.</td>
<td>• Investigate the identity property for multiplication and determine that when the number one is multiplied by another number or another number is multiplied by the number one, that number remains unchanged. Examples of the identity property for multiplication are 1 x 3 = 3; 6 x 1 = 6. [Moved to 3.4 US]</td>
</tr>
<tr>
<td>The not equal (≠) symbol means that the values on either side are not equivalent (not balanced).</td>
<td>• Understand the identity property for multiplication.</td>
<td>• Recognize that the commutative property for addition is an order property. Changing the order of the addends does not change the sum (5 + 4 = 9 and 4 + 5 = 9). [Moved to 3.3 US]</td>
</tr>
<tr>
<td>Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers. [Moved to 3.4 US]</td>
<td>• Understand the commutative property of addition.</td>
<td>• Recognize that the commutative property for multiplication is an order property. Changing the order of the factors does not change the product (2 x 3 = 3 x 2). [Moved to 3.4 US]</td>
</tr>
<tr>
<td>An expression is a representation of a quantity. It contains numbers, variables, and/or computational operation symbols. It does not have an equal symbol (e.g., 5, 4 + 3, 8 - 2, 2 x 7).</td>
<td>• Understand the commutative property of multiplication.</td>
<td>• Identify equivalent values and expressions (e.g., 5 + 3 = 4 + 2 and 8 = 4 + 4).</td>
</tr>
<tr>
<td>An equation is a mathematical sentence that two expressions are equivalent. It consists of two expressions, one on each side of an equal symbol (e.g., 5 + 3 = 8, 8 = 5 + 3 and 4 + 3 = 9 - 2).</td>
<td>• Understand that quantities on both sides of an equals sign must be equal. [Moved to US]</td>
<td>• Identify and use the appropriate symbol to distinguish between expressions that are equal and expressions that are not equal (e.g., 256 - 13 = 220 + 23; 143 + 17 = 140 + 20; 457 + 100 ≠ 557 + 100).</td>
</tr>
<tr>
<td>An equation can be represented using balance scales, with equal amounts on each side (e.g., 3 + 5 = 6 + 2).</td>
<td>• Understand that quantities on both sides of the not equal sign are not equal.</td>
<td>• Create equations. Write number sentences to represent equivalent mathematical relationships (e.g., 4 x 3 = 14 - 2).</td>
</tr>
<tr>
<td>The commutative property for addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4). Similarly, the commutative property for multiplication states that changing the order of the factors does not affect the product (e.g., 2 x 3 = 3 x 2).</td>
<td>• The identity property for addition states that if zero is added to a given number, the sum is the same as the given number. The identity property of multiplication states that if a given number is</td>
<td></td>
</tr>
<tr>
<td>The identity property for addition states that if zero is added to a given number, the sum is the same as the given number. The identity property of multiplication states that if a given number is</td>
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The student will create equations to represent equivalent mathematical relationships. [Moved from 3.20 EKS]

a) investigate the identity and the commutative properties for addition and multiplication; and [Use of properties moved to 3.3 EKS and 3.4 EKS]

b) identify examples of the identity and commutative properties for addition and multiplication.

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<tbody>
<tr>
<td>multiplied by one, the product is the same as the given number.</td>
<td></td>
<td>properties for addition and multiplication.</td>
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<td>A number sentence is an equation with numbers (e.g., 6 + 3 = 9; or 6 + 3 = 4 + 5). [Reordered]</td>
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</table>
The 2009-2016 Mathematics Standards of Learning Curriculum Framework is a companion document to the 2009-2016 Mathematics Standards of Learning, and amplifies the Mathematics Standards of Learning by and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. Provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instructionbackground information for the teacher (K-8). It contains content — The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers’ knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that What each student should know and be able to demonstrate in each standard is outlined. This is not meant to be an exhaustive list of student expectations nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise. [Reordered and rewritten]
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student’s problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
FOCUS 34–5

STAND: NUMBER AND NUMBER SENSE

GRADE LEVEL 4

Mathematics instruction in grades 4 and 3-5 should continue to foster the development of number sense, especially with greater emphasis on decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction in grades 4 and 3-5 allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names), and making connections among mathematics concepts as well as to other content areas. Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g., $\frac{1}{2}$ or 0.5). Students should apply their knowledge of number and number sense to investigate and solve a variety of problem types.
STANDARD 4.1  STRAND: NUMBER AND NUMBER SENSE  GRADE LEVEL 4

4.1 The student will
a) read, write, and identify orally and in writing the place and value of each digit in a whole number expressed through millions; nine-digit whole number;
b) compare and order two whole numbers expressed through millions, using symbols (> , <, or =); and [Symbols included in EKS]
c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- The structure of the base-10 number system is based upon a simple pattern of tens, in which the value of each place is ten times the value of the place to its right.
- Place value refers to the value of each digit and depends upon the position of the digit in the number. For example, in the number 7,864,352, the eight is in the hundred thousand place, and the value of the 8 is eight hundred thousand or 800,000.
- Whole numbers may be written in a variety of forms:
  - Standard: 1,234,567
  - Written: one million, two hundred thirty-four thousand, five hundred sixty-seven
  - Expanded: \((4 \times 1,000,000) + (2 \times 100,000) + (3 \times 10,000) + (4 \times 1,000) + (5 \times 100) + (6 \times 10) + (7 \times 1) = 1,234,567\)
- Numbers are arranged into groups of three places called periods (ones, thousands, millions, ...). The value of the places within the periods repeat (hundreds, tens, ones). Commas are used to separate the periods. Knowing the value of the place and period of a number helps students determine values of digits in any number as well as read and write numbers. Students at this level will work with numbers through the millions period (nine-digit numbers).
- Reading and writing large numbers should be meaningful for students. Experiences can be provided that relate practical situations (e.g., numbers found in the students’ environment including population, number of school lunches sold statewide in a day, etc.).
- Concrete materials such as base-10 blocks or and bundles of sticks may be used to represent whole numbers through

<table>
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<td>All students should</td>
</tr>
<tr>
<td>- Understand the relationships in the place value system in which the value of each place is ten times the value of the place to its right.</td>
</tr>
<tr>
<td>- Use the patterns in the place value system to read and write numbers.</td>
</tr>
<tr>
<td>- Understand that reading place value correctly is essential when comparing numbers.</td>
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<tr>
<td>- Understand that rounding gives a close number to use when exact numbers are not needed for the situation at hand.</td>
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<tr>
<td>- Develop strategies for rounding.</td>
</tr>
</tbody>
</table>

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify and communicate, both orally and in written form, the place and value for each digit in whole numbers expressed through the one millions place of a nine-digit whole number. (a)
- Read nine-digit whole numbers, presented in standard form expressed through the one millions place that are presented in standard format, and select the matching represent the same number in written format. (a)
- Write nine-digit whole numbers through the one millions place in standard format when the numbers are presented orally or in written format. (a)
- Identify and use the symbols for greater than, less than, equal to, and not equal to. [Added to next bullet]
- Compare two whole numbers expressed through the one millions, using the words greater than, less than, equal to, and not equal to or using the symbols >, <, or =. (b)
- Order up to four whole numbers expressed through millions. (b)
- Round whole numbers expressed through the one millions place to the nearest thousand, ten thousand, and hundred-thousand place. (c)
- Identify the greatest whole number and least whole range of numbers that round to a given thousand, ten thousand, and hundred thousand. (c)
4.1 The student will

a) read, write, and identify orally and in writing the place and value of each digit in a whole number expressed through millions; nine-digit whole number;
b) compare and order two whole numbers expressed through millions, using symbols (> , < , or = ); and [Symbols included in EKS]
c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

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<td>thousands. Larger numbers may be represented by digit cards and place value charts or on number lines.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number lines are useful tools when developing a conceptual understanding of rounding with whole numbers. When given a number to round, locate it on the number line. Next, determine the closest multiples of thousand, ten-thousand, or hundred-thousand it is between. Then identify to which it is closer.</td>
<td></td>
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<tr>
<td>Mathematical symbols (&gt; , &lt; ) used to compare two unequal numbers are called inequality symbols.</td>
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<td>A procedure for comparing two numbers by examining place value may include the following:</td>
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<tr>
<td>— Compare the digits in the numbers to determine which number is greater (or which is less).</td>
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<tr>
<td>— Use a number line to identify the appropriate placement of the numbers based on the place value of the digits.</td>
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<tr>
<td>— Use the appropriate symbol &gt; or &lt; or words greater than or less than to compare the numbers in the order in which they are presented.</td>
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<tr>
<td>— If both numbers have the same value, use the symbol = or words equal to.</td>
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<tr>
<td>A strategy for rounding numbers to the nearest thousand, ten thousand, and hundred thousand is as follows:</td>
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<tr>
<td>— Use a number line to determine the rounded number (e.g., when rounding 4,367,925 to the nearest thousand, identify the ‘thousands’ the number would fall between on the number line, then determine the thousand that the number is closest to).</td>
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<td></td>
<td>4,367,000</td>
<td>? 4,368,000</td>
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</tbody>
</table>
4.1 The student will
   a) read, write, and identify orally and in writing the place and value of each digit in a whole number expressed through millions; nine-digit whole number;
   b) compare and order two whole numbers expressed through millions, using symbols (>, <, or =); and [Symbols included in EKS]
   c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

**UNDERSTANDING THE STANDARD**
(Background Information for Instructor Use Only)

---

- Look one place to the right of the digit to which you wish to round.
- If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero.
- If the digit is 5 or greater, add 1 to the digit in the rounding place and change the digits to the right of the rounding place to zero.

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<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
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</tbody>
</table>
STANDARD 4.2

STRAND: NUMBER AND NUMBER SENSE 
GRADE LEVEL 4

4.2 The student will
   a) compare and order fractions and mixed numbers, with and without models; *
   b) represent equivalent fractions;* and
   c) identify the division statement that represents a fraction, with models and in context.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- A fraction is a way of representing part of a whole region (as in an area model) or a part of a group (as in a set model) or as a location on a number line part of a length (as in a measurement model). A fraction is used to name a part of one thing or a part of a collection of things.

- In the area model and length/measurement fraction models, the parts must be equal. In the set model, the elements of the set do not have to be equal (e.g., “What fraction of the class is wearing the color red?”).

- In a set model, each member of the set is an equivalent part of the set. In set models, the whole needs to be defined, but members of the set may have different sizes and shapes. For instance if a whole is defined as a set of 10 animals, the animals within the set may be different. For example, students should be able to identify monkeys as representing \( \frac{1}{2} \) of the animals in the following set.

- Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction can be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., \( 3 \frac{2}{5} \)).

ESSENTIAL UNDERSTANDINGS

All students should

- Develop an understanding of fractions as parts of unit wholes, as parts of a collection, and as locations on a number line.

- Understand that a mixed number is a fraction that has two parts: a whole number and a proper fraction. The mixed number is the sum of these two parts. [Moved to US]

- Use models, benchmarks, and equivalent forms to judge the size of fractions.

- Recognize that a whole divided into nine equal parts has smaller parts than if the whole had been divided into five equal parts.

- Recognize and generate equivalent forms of commonly used fractions and decimals.

- Understand the division statement that represents a fraction.

- Understand that the more

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Compare and order no more than four fractions having like and unlike denominators of 12 or less, using manipulative concrete and pictorial models and drawings, such as region/area models. (a)

- Use benchmarks (e.g., 0, \( \frac{1}{2} \) or 1) to compare and order no more than four fractions having unlike denominators of 12 or less. (a) [Reordered]

- Compare and order no more than four fractions with like denominators of 12 or less by comparing number of parts (numerators) (e.g., \( \frac{1}{5} < \frac{3}{5} \)). (a)

- Compare and order no more than four fractions with like numerators and unlike denominators of 12 or less by comparing the size of the parts (e.g., \( \frac{3}{5} < \frac{3}{9} \)). (a) Compare and order fractions having unlike denominators of 12 or less by comparing the fractions to benchmarks (e.g., \( 0, \frac{1}{2} \) or 1) to determine their relationships to the benchmarks or by finding a common denominator.

- Compare and order no more than four proper fractions (proper or improper), and/or mixed numbers, and improper fractions having denominators of 12 or less. (a)

- Use the symbols >, <, and =, and \( \neq \) to compare the numerical value of proper fractions (proper or improper) fractions and/or mixed numbers having denominators of 12
4.2 The student will
   a) compare and order fractions and mixed numbers, with and without models;*
   b) represent equivalent fractions;* and
   c) identify the division statement that represents a fraction, with models and in context.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- The value of a fraction is dependent on both the number of equal parts in a whole (denominator) and the number of those parts being considered (numerator).
- The denominator tells how many equal parts are in the whole or set. The numerator tells how many of those parts are being counted or described.
- The more parts the whole is divided into, the smaller the parts (e.g., \(\frac{1}{5} < \frac{1}{3}\)).
- When fractions have the same denominator, they are said to have “common denominators” or “like denominators.” Comparing fractions with like denominators involves comparing only the numerators.
- Strategies for comparing fractions having unlike denominators may include
  - comparing fractions to familiar benchmarks (e.g., 0, \(\frac{1}{2}\), 1);
  - finding equivalent fractions, using manipulative models such as fraction strips, number lines, fraction circles, rods, pattern blocks, cubes, base-10 blocks, tangrams, graph paper, or patterns in a multiplication chart and patterns; and
  - finding a common denominator by finding the least common multiple (LCM) of both denominators and then rewriting each fraction as an equivalent fraction, using the LCM as the denominator.
- A variety of fraction models should be used to expand students’ understanding of fractions and mixed numbers:
  - Region/area models: a surface or area is subdivided into smaller equal parts, and each part is compared with the whole (e.g., fraction circles, pattern blocks, geoboards, grid paper).

### ESSENTIAL UNDERSTANDINGS
- parts the whole is divided into, the smaller the parts (e.g., \(\frac{1}{5} < \frac{1}{3}\)).

### ESSENTIAL KNOWLEDGE AND SKILLS

- Represent equivalent fractions through twelfths, using region/area models, set models, and measurement/length models. (b)
- Identify the division statement that represents a fraction with models and in context (e.g., \(\frac{3}{5}\) means the same as 3 divided by 5 or \(\frac{3}{5}\) represents the amount of muffin each of five children will receive when sharing 3 muffins equally). (c)
4.2 The student will
a) compare and order fractions and mixed numbers, with and without models;*
b) represent equivalent fractions;* and
c) identify the division statement that represents a fraction, with models and in context.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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| Set models: the whole is understood to be a set of objects, and subsets of the whole make up fractional parts (e.g., counters, chips).
| Measurement models: similar to area models but lengths instead of areas are compared (e.g., fraction strips, rods, cubes, number lines, rulers). |
| A proper fraction is a fraction whose numerator is less than the denominator. A proper fraction is a fraction that is always less than one. |
| An improper fraction is a fraction whose numerator is greater than or equal to the denominator. An improper fraction is a fraction that is equal to or greater than one. |
| An improper fraction can be expressed as a mixed number. A mixed number is written as a whole number and a proper fraction. |
| A mixed number has two parts: a whole number and a fraction. |
| Equivalent fractions name the same amount. Students should use a variety of representations and models to identify different names for equivalent fractions. |
| Students should focus on finding equivalent fractions of familiar fractions such as halves, thirds, fourths, sixths, eighths, tenths, and twelfths. |
| Decimals and fractions represent the same relationships; however, they are presented in two different formats. The decimal 0.25 is written as $\frac{1}{4}$. When presented with the fraction $\frac{3}{5}$ representing division, the division expression representing the fraction is written as $\div \frac{3}{5}$. |

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 4
4.2 The student will
a) compare and order fractions and mixed numbers, with and without models;*
b) represent equivalent fractions;* and
c) identify the division statement that represents a fraction, with models and in context.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<tr>
<td>• The fraction ( \frac{3}{4} ) may be interpreted as the amount of cake each person will receive when 3 cakes are divided equally among 4 people.</td>
<td>ESSENTIAL UNDERSTANDINGS</td>
<td>ESSENTIAL KNOWLEDGE AND SKILLS</td>
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</tbody>
</table>
### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

- Investigate the ten-to-one place value relationship for decimals through thousandths, using base-10 manipulatives (e.g., place value mats/charts, decimal squares, and base-10 blocks, money). (a)
- Represent and identify decimals expressed through thousandths, using base-10 manipulatives, pictorial representations, and numerical symbols (e.g., relate the appropriate drawing to 0.05). (a)
- Identify and communicate, both orally and in written form, the position and value of a decimal through thousandths. For example, in 0.385, the 8 is in the hundredths place and has a value of 0.08. (a)
- Read and write decimals expressed through thousandths, using base-10 manipulatives, drawings, and numerical symbols. (a)
- Round decimals expressed through thousandths to the nearest whole number, tenth, and hundredth. (b)
- Compare two decimals expressed through thousandths, using the symbols >, <, =, and ≠ and the words greater than, less than, equal to, and not equal to. (c)
- Order a set of up to four decimals, expressed through thousandths, from least to greatest or greatest to least. (c)

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Decimal numbers are another way of writing fractions. Expand the set of whole numbers and like fractions they are a way of representing part of a whole.

- The structure of the base-10 number system is based upon a simple pattern of tens, where each place is ten times the value of the place to its right. This is known as a ten-to-one place value relationship (e.g., in 2.35, 3 is in the tenths place since it takes ten one-tenths to make one whole). Use base-10 proportional manipulatives, such as place value mats/charts, decimal squares, base-10 blocks, meter sticks, as well as the ten-to-one non-proportional model, money, to investigate this relationship.

- Base-10 models concretely relate fractions to decimals (e.g., 10- by-10 grids, meter sticks, number lines, decimal squares, decimal circles, money). Reordered]

- Understanding the system of tens means that ten tenths represents one whole, ten hundredths represents one tenth, ten thousandths represents one hundredth.

- A decimal point separates the whole number places from the places that are less than one. Place values extend infinitely in two directions from a decimal point. A number containing a decimal point is called a decimal number or simply a decimal.

- To read decimals,
  - read the whole number to the left of the decimal point, if there is one;
  - read the decimal point as “and”;
  - read the digits to the right of the decimal point just as you would read a whole number; and
  - say the name of the place value of the digit in the smallest
4.3 The student will
a) read, write, represent, and identify decimals expressed through thousandths;
b) round decimals to the nearest whole number, tenth, and hundredth;[Round to tenth and hundredth included in 5.1]
c) compare and order decimals; and

On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<td>• Any decimal less than 1 will include a leading zero, for example 0.125 which can be read as “zero and one hundred twenty-five thousandths” or as “one hundred twenty-five thousandths.”</td>
<td>• Represent fractions for halves, fourths, fifths, and tenths as decimals through hundredths, using concrete objects (e.g., demonstrate the relationship between the fraction $\frac{1}{4}$ and its decimal equivalent 0.25). (d)</td>
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<tr>
<td>• Decimals may be written in a variety of forms:</td>
<td>• Relate fractions to decimals, using concrete objects (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, decimal circles, money). (d)</td>
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<td>Standard: 26.537</td>
<td>• Write the decimal and fraction equivalent for a given model (e.g., $\frac{1}{4} = 0.25$ or $0.25 = \frac{1}{4}$, $1.25 = \frac{5}{4}$ or $1\frac{1}{4}$). (d)</td>
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<tr>
<td>Written: twenty-six and five hundred thirty-seven thousandths</td>
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<tr>
<td>Expanded: $(2 \times 10) + (6 \times 1) + (5 \times 0.1) + (3 \times 0.01) + (7 \times 0.001)$</td>
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<tr>
<td>$20 + 6 + 0.5 + 0.03 + 0.007$</td>
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<tr>
<td>• Decimals and fractions represent the same relationships; however, they are presented in two different formats. The decimal 0.25 is written as $\frac{1}{4}$. Decimal numbers are another way of writing fractions. When presented with the fraction $\frac{3}{5}$, the division expression representing a fraction is written as 3 divided by 5. The Base-10 models concretely relate fractions to decimals (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, money).</td>
<td>• The procedure for Strategies for rounding whole numbers can be applied to rounding decimals; numbers is similar to the procedure for rounding whole numbers.</td>
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<tr>
<td>• Number lines are useful tools when developing a conceptual understanding of rounding with decimals. When given a decimal to round to the nearest whole or ones place, locate it on the number line. Next, determine the two whole numbers it is between. Then identify to which it is closer.</td>
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Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 4
4.3 The student will
   a) read, write, represent, and identify decimals expressed through thousandths;
   b) round decimals to the nearest whole number, tenth, and hundredth; [Round to tenth and hundredth included in 5.1]
   c) compare and order decimals; and
   d) given a model, write the decimal and fraction equivalents.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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</table>
| A strategy for rounding decimal numbers to the nearest tenth and hundredth is as follows:  
  - Look one place to the right of the digit you want to round to.  
  - If the digit is 5 or greater, add 1 to the digit in the rounding place, and drop the digits to the right of the rounding place.  
  - If the digit is less than 5, leave the digit in the rounding place as it is, and drop the digits to the right of the rounding place.  
| Different strategies for rounding decimals include:  
  - Use a number line to locate a decimal between two numbers. For example, 18.83 is closer to 18.8 than to 18.9.  
  - Compare the digits in the numbers to determine which number is greater (or which is less).  
  - Compare the value of decimals, using the symbols >, <, = (e.g., 0.83 > 0.8 or 0.19 < 0.2).  
  - Order the value of decimals, from least to greatest and greatest to least (e.g., 0.83, 0.82, 0.84).  
| Decimal numbers are another way of writing fractions (halves, fourths, fifths, and tenths). The Base-10 models concretely relate fractions to decimals (e.g., 10 by 10 grids, meter sticks, number lines, decimal squares, decimal circles, money). Provide a fraction model (halves, fourths, fifths, and tenths) and ask students for its decimal equivalent.  
| Provide a decimal model and ask students for its fraction equivalent (halves, fourths, fifths, and tenths). |
Computation and estimation in grades 4 and 5 should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students’ understanding of these operations to work with decimals. Instruction should focus on computation activities that enable students to model, explain, and develop proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract with common fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades. Multiplication and division with decimals will be explored in grade 5.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability to identify and use relationships between operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use (not identify) properties of operations to solve problems [e.g., \( 7 \times 28 \) is equivalent to \( (7 \times 20) + (7 \times 8) \)].

Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals, using benchmarks (e.g., \( \frac{2}{5} + \frac{1}{3} \) must be less than 1 because both fractions are less than \( \frac{1}{2} \)). Using estimation, students should develop strategies to recognize the reasonableness of their computations.

Additionally, students should enhance their ability to select an appropriate problem solving method from among estimation, mental mathematics, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.
The student will

a) demonstrate fluency with multiplication facts through $12 \times 12$ and the corresponding division facts; *[Moved from 3.5]*

b) estimate and determine sums, differences, products, and quotients of whole numbers; *[Moved from 3.5]*

c) estimate and determine quotients of divide-whole numbers, finding quotients with and without remainders; *[Moved from 3.5]*

d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division problems with whole numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.*

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**UNDERSTANDING THE STANDARD**

(Background Information for Instructor Use Only)

- Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently.
- The development of computational fluency relies on quick access to number facts. There are patterns and relationships that exist in the facts. These relationships can be used to learn and retain the facts.
- A certain amount of practice is necessary to develop fluency with computational strategies; however, the practice must be motivating and systematic if students are to develop fluency in computation, whether mental, with manipulative materials, or with paper and pencil. [Reordered]
- In grade 3, students developed an understanding of the meanings of multiplication and division of whole numbers through activities and practical problems involving equal-sized groups, arrays, and length models. In addition, grade 3 students have worked on fluency of facts for 0, 1, 2, 5, 10.
- Three models used to develop an understanding of multiplication include:
  - The equal-sets or equal-groups model lends itself to sorting a variety of concrete objects into equal groups and reinforces the concept of multiplication as a way to find the total number of items in a collection of groups, with the same amount in each group, and the total number of items can be found by repeated addition or skip counting.

**ESSENTIAL UNDERSTANDINGS**

- All students should
  - Develop and use strategies to estimate whole number sums and differences and to judge the reasonableness of such results.
  - Understand that addition and subtraction are inverse operations.
  - Understand that division is the operation of making equal groups or equal shares. When the original amount and the number of shares are known, divide to find the size of each share. When the original amount and the size of each share are known, divide to find the number of shares.
  - Understand that multiplication and division are inverse operations.
  - Understand various representations of division and the terms used in

**ESSENTIAL KNOWLEDGE AND SKILLS**

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Demonstrate fluency with multiplication through $12 \times 12$ and the corresponding division facts. (a) [Moved from 3.5]

- Use strategies when adding and subtracting, including application of the properties of addition. (b, d) [Use of properties moved from 3.20 and 5.19] [Reordered and reworded]

- Use strategies when multiplying and dividing, including application of the properties of multiplication. (b, c, d) [Use of properties moved from 3.20 and 5.19] [Reordered]

- Estimate whole number sums, differences, products, and quotients, with and without context. (b, c)

- Refine estimates by adjusting the final amount, using terms such as closer to, between, and a little more than. [Reordered]

- Apply strategies, including place value and the properties of addition to determine the sum or difference of two whole numbers, each 999,999 or less, in vertical and horizontal form with or without regrouping, using paper and pencil, and using a calculator. (b) [Use of properties moved from 3.20 and 4.16]

- Estimate and find.
STANDARD 4.4  STRAND: COMPUTATION AND ESTIMATION  GRADE LEVEL 4

4.4 The student will

a) demonstrate fluency with multiplication facts through $12 \times 12$ and the corresponding division facts;*[Moved from 3.5]*
estimate sums, differences, products, and quotients of whole numbers;

b) estimate and determine sums, differences, and products of add, subtract, and multiply whole numbers;*

c) estimate and determine quotients of divide whole numbers, finding quotients with and without remainders;* and
d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division problems with whole numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<td>- The array model, consisting of rows and columns (e.g., 3 rows of 4 columns for a 3-by-4 array), helps build an understanding of the commutative property.</td>
<td>division are dividend, divisor, and quotient. dividend ÷ divisor = quotient(divisor) ÷ (dividend)</td>
<td>determine the products of two whole numbers when one factor has two digits or fewer, and the other factor has three digits or fewer, using paper and pencil and calculators. (b) [Use of properties moved from 3.20, 4.16, and 5.19]</td>
</tr>
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<td>- The array model, consisting of rows and columns (e.g., 3 rows of 4 columns for a 3-by-4 array), helps build an understanding of the commutative property.</td>
<td>Understand how to solve single-step and multistep problems involving whole number operations.</td>
<td>Estimate and find Apply strategies, including place value and the properties of multiplication and/or addition, to determine the quotient of two whole numbers, given a one-digit divisor and a two- or three-digit dividend, with and without remainders. (c)</td>
</tr>
<tr>
<td>- The length model (e.g., a number line) also reinforces repeated addition or skip counting. [Reordered]</td>
<td>- There is an inverse relationship between multiplication and division.</td>
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<tr>
<td>- The length model (e.g., a number line) also reinforces repeated addition or skip counting. [Reordered]</td>
<td>- The terms associated with multiplication are listed below: factor $\rightarrow$ 54 factor $\rightarrow$ $\times$ 3 product $\rightarrow$ 162 [Moved from 3.6]</td>
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<tr>
<td>- There is an inverse relationship between multiplication and division.</td>
<td>- The number line model can be used to solve a multiplication problem such as $3 \times 4$. This is represented on the number line by three jumps of four or four jumps of three, depending on the context of the problem. [Moved from 3.6]</td>
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<td>- The array model, consisting of rows and columns (e.g., 3 rows of 4 columns for a 3-by-4 array), helps build an understanding of the commutative property.</td>
<td>- Understand how to solve single-step and multistep problems involving whole number operations.</td>
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[Reordered]
STANDARD 4.4  STRAND: COMPUTATION AND ESTIMATION  GRADE LEVEL 4

4.4  The student will
a)  demonstrate fluency with multiplication facts through $12 \times 12$ and the corresponding division facts;*[Moved from 3.5] estimate sums, differences, products, and quotients of whole numbers;
b)  estimate and determine sums, differences, and products of add, subtract, and multiply whole numbers;*
c)  estimate and determine quotients of divide-whole numbers, finding quotients with and without remainders;* and
d)  create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division problems with whole numbers.

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<tbody>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>• The number line model can be used to solve a division problem such as $6 \div 3$ and is represented on the number line by noting how many jumps of three go from 6 to 0.[Moved from 3.6]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The number line model above shows two jumps of three between 6 and 0, answering the question of how many jumps of three go from 6 to 0; therefore, $6 \div 3 = 2$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• In order to develop and use strategies to learn the multiplication facts through the twelves table, students should use concrete materials, hundred chart, and mental mathematics. Strategies to learn the multiplication facts include an understanding of multiples, skip counting, properties of zero and one as factors, pattern of nines, commutative property, and related facts.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Investigating arithmetic operations with whole numbers helps students learn about the different properties of arithmetic relationships. These relationships remain true regardless of the whole numbers.[Moved from 4.16]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Grade 4 students should explore and apply the properties of addition and multiplication as strategies for solving addition, subtraction, multiplication, and division problems using a variety of representations (e.g., manipulatives, diagrams, and symbols),</td>
<td></td>
<td></td>
</tr>
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</table>
4.4 The student will

a) demonstrate fluency with multiplication facts through $12 \times 12$ and the corresponding division facts; *[Moved from 3.5]*
estimate sums, differences, products, and quotients of whole numbers;

b) estimate and determine sums, differences, and products of add, subtract, and multiply whole numbers; *[Moved from 3.5]*
c) estimate and determine quotients of divide whole numbers, finding quotients with and without remainders; *[Moved from 3.5]* and
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<tr>
<td>• The properties of the operations are “rules” about how numbers work and how they relate to one another. Students at this level do not need to use the formal terms for these properties but should utilize these properties to further develop flexibility and fluency in solving problems. The following properties are most appropriate for exploration at this level:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– The identity property of addition states that if zero is added to a given number, the sum is the same as the given number. The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., $24 + 136 = 136 + 24$). Similarly, the commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., $12 \times 43 = 43 \times 12$).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– The associative property of addition states that the sum stays the same when the grouping of addends is changed (e.g., $15 + (35 + 16) = (15 + 35) + 16$). The associative property of multiplication states that the product stays the same when the grouping of factors is changed (e.g., $16 \times (40 \times 5) = (16 \times 40) \times 5$).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Using the distributive property with whole numbers helps with understanding mathematical relationships. <em>[Moved from middle column]</em> The distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products.</td>
<td></td>
<td></td>
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4.4 The student will

a) demonstrate fluency with multiplication facts through $12 \times 12$ and the corresponding division facts; *[Moved from 3.5] estimate sums, differences, products, and quotients of whole numbers;

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<td></td>
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<tr>
<td>products. Several examples are shown below:</td>
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<td></td>
</tr>
<tr>
<td>- $3(9) = 3(5 + 4)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3(5 + 4) = 3 \times 5 + 3 \times 4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- $5 \times (3 + 7) = (5 \times 3) + (5 \times 7)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- $(2 \times 3) + (2 \times 5) = 2 \times (3 + 5)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- $9 \times 23$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$9(20 + 3)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$180 + 27$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$207$</td>
<td></td>
<td></td>
</tr>
</tbody>
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- Students will begin to use the formal terms for the properties in grade 5.
- A sum is the result of adding two or more numbers.
4.4 The student will
  a) demonstrate fluency with multiplication facts through 12 × 12 and the corresponding division facts;* [Moved from 3.5]
estimate sums, differences, products, and quotients of whole numbers;
  b) estimate and determine sums, differences, and products of add, subtract, and multiply whole numbers;*
  c) estimate and determine quotients of divide whole numbers, finding quotients with and without remainders;* and
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<tr>
<td>A difference is the amount that remains after one quantity is subtracted from another.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The problem solving process is enhanced when students create and solve their own practical problems and model problems using manipulatives and drawings.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The least number of steps necessary to solve a single-step problem is one.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as in all, altogether, difference, etc. encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions as well as challenges in upcoming grades and courses.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extensive research has been undertaken over the last several decades regarding different problem types. Many of these studies have been published in professional mathematics education publications using different labels and terminology to describe the varied problem types.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students should experience a variety of problem types related to multiplication and division. Some examples are included in the following chart:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
STANDARD 4.4  STRAND: COMPUTATION AND ESTIMATION  GRADE LEVEL 4

4.4 The student will
   a) demonstrate fluency with multiplication facts through $12 \times 12$ and the corresponding division facts;*[Moved from 3.5] estimate sums, differences, products, and quotients of whole numbers;
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<th>GRADE 4: COMMON MULTIPLICATION AND DIVISION PROBLEM TYPES</th>
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<tbody>
<tr>
<td>Equal Group Problems</td>
<td></td>
</tr>
<tr>
<td>Whole Unknown (Multiplication)</td>
<td></td>
</tr>
<tr>
<td>There are 6 boxes of crayons. Each box contains 24 crayons. How many crayons are there in all?</td>
<td></td>
</tr>
<tr>
<td>Number of Groups Unknown (Partition Division)</td>
<td></td>
</tr>
<tr>
<td>If 144 crayons are placed into school boxes with each box containing 24 crayons, How many school boxes can be filled?</td>
<td></td>
</tr>
<tr>
<td>Size of Groups Unknown (Measurement Division)</td>
<td></td>
</tr>
<tr>
<td>If 144 crayons are shared equally among 6 friends, how many crayons will each friend get?</td>
<td></td>
</tr>
</tbody>
</table>

Multiplicative Comparison Problems

<table>
<thead>
<tr>
<th>Result Unknown</th>
<th>Start Unknown</th>
<th>Comparison Factor Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tyrone ran 30 miles last month. Jasmine ran 4 times as many miles as Tyrone during the same month. How many miles did Jasmine run?</td>
<td>Jasmine ran 130 miles. She ran 4 times as many miles as Tyrone. How many miles did Tyrone run?</td>
<td>Jasmine ran 130 miles. Tyrone ran 30 miles. How many times more miles did Jasmine run than Tyrone?</td>
</tr>
</tbody>
</table>

Array or Area Problems

<table>
<thead>
<tr>
<th>Whole Unknown</th>
<th>One Dimension Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>There are 12 baseball teams competing in the tournament. Each team has 9 baseball players. How many baseball players are there all together?</td>
<td>There are 108 baseball players competing in the tournament. The players are divided equally among 12 teams. How many players are on each team?</td>
</tr>
<tr>
<td>Mr. Myers’s dog pen measures 15 feet by 22 feet. How many square feet are in the dog pen?</td>
<td>There are 108 baseball players competing in the tournament. There are exactly 9 players on each team. How many teams are there?</td>
</tr>
<tr>
<td>The dog pen covers 60 square feet. The length of the dog pen is 15 feet. What is the width of the dog pen?</td>
<td>The dog pen covers 60 square feet. The length of the dog pen is 15 feet. What is the width of the dog pen?</td>
</tr>
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ESSENTIAL KNOWLEDGE AND SKILLS
4.4 The student will
a) demonstrate fluency with multiplication facts through $12 \times 12$ and the corresponding division facts;*[Moved from 3.5] estimate sums, differences, products, and quotients of whole numbers;
b) estimate and determine sums, differences, and products of add, subtract, and multiply whole numbers;*
c) estimate and determine quotients of divide whole numbers, finding quotients with and without remainders;* and
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<tr>
<td>• Estimation can be used to determine the approximation for and then to verify the reasonableness of sums, differences, products, and quotients of whole numbers.[Moved from middle column] An estimate is a number close to an exact solution that lies within a range of the exact solution and the estimation strategy used in a particular problem determines how close the number is to the exact solution. An estimate tells about how much or about how many.</td>
<td>• Strategies such as rounding up or down, front-end, and compatible numbers may be used to estimate sums, differences, products, and quotients of whole numbers.</td>
<td></td>
</tr>
<tr>
<td>• Different strategies for estimating include using compatible numbers to estimate sums and differences and using front-end estimation for sums and differences.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Compatible numbers are numbers that are easy to work with mentally. Number pairs that are easy to add or subtract are compatible. When estimating a sum, replace actual numbers with compatible numbers (e.g., $52 + 74$ can be estimated by using the compatible numbers $50 + 75$). When estimating a difference, use numbers that are close to the original numbers. Tens and hundreds are easy to subtract (e.g., $83 - 38$ is close to $80 - 40$).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The front-end strategy for estimating is computing with the front digits. Front-end estimation for addition can be used even when the addends have a different number of digits. The procedure requires the addition of the values of the digits in the greatest of the smallest number. For example:</td>
<td></td>
<td></td>
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### UNDERSTANDING THE STANDARD

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<tr>
<th>Action</th>
<th>Number 1</th>
<th>Number 2</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>2367</td>
<td></td>
<td></td>
<td>2300</td>
</tr>
<tr>
<td>243</td>
<td></td>
<td></td>
<td>200</td>
</tr>
<tr>
<td>+ 1186</td>
<td></td>
<td></td>
<td>1100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3600</td>
</tr>
</tbody>
</table>

- Front-end or leading-digit estimation always gives a sum less than the actual sum; however, the estimate can be adjusted or refined so that it is closer to the actual sum.

- Addition is the combining of quantities; it uses the following terms:
  - **addend** → 45,623
  - **addend** → ± 37,846
  - **sum** → 83,469

- Subtraction is the inverse of addition; it yields the difference between two numbers and uses the following terms:
  - **minuend** → 45,698
  - **subtrahend** → −32,741
  - **difference** → 12,957

- Before adding or subtracting with paper and pencil, addition and subtraction problems in horizontal form should be rewritten in vertical form by lining up the places vertically.

- Using Base-10 materials to model and stimulate discussion about a variety of problem situations helps students understand regrouping and enables them to move from the concrete to the pictorial, to the abstract. Regrouping is used in addition and subtraction algorithms. In addition, when the sum in a place is 10
The student will

a) demonstrate fluency with multiplication facts through \(12 \times 12\) and the corresponding division facts; *[Moved from 3.5]*

b) estimate and determine sums, differences, and products of add, subtract, and multiply whole numbers; *

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<td>or more, is used to regroup the sums so that there is only one digit in each place. In subtraction, when the number (minuend) in a place is not enough from which to subtract, regrouping is required. A certain amount of practice is necessary to develop fluency with computational strategies for multidigit numbers; however, the practice must be meaningful, motivating, and systematic if students are to develop fluency in computation, whether mentally, with manipulative materials, or with paper and pencil. Calculators are an appropriate tool for computing sums and differences of large numbers, particularly when mastery of the algorithm has been demonstrated.</td>
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</table>

- In multiplication, one factor represents the number of equal groups and the other factor represents the number in or size of each group. The product is the total of all the parts. Multiplication can also refer to a multiplicative comparison, such as: “Gwen has six times as many stickers as Phillip”. Both situations should be modeled with manipulatives.

- The terms associated with multiplication are

  \[ \begin{align*}
  \text{factor} & \rightarrow 76 \\
  \text{factor} & \rightarrow \times 23 \\
  \text{product} & \rightarrow 81,748 
  \end{align*} \]

- Models of multiplication may include repeated addition and collections of like sets of objects; partial products; and area or array models.

- One model of multiplication is repeated addition.
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### UNDERSTANDING THE STANDARD

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- Another model of multiplication is the “Partial Product” model.
  
  $24 \times 3$
  
  $24 \times 3 = 72$

- Another model of multiplication is the “Area Model” (which also represents partial products) and should be modeled first with Base 10 blocks. (e.g., $23 \times 68$)

- Students should continue to develop fluency with single digit multiplication facts and their related division facts.

- Calculators should be used to solve problems that require tedious calculations.

- Estimation should be used to check the reasonableness of the product. Examples of estimation strategies include the following:

  - The front-end method: multiply the front digits and then complete the product by recording the number of zeros found in the factors. It is important to develop understanding of this process before using the step-by-step procedure.
    
    $523 \times 31$ → $500 \times 30$
    
    $15,000$
    
    - This is $3 \times 5 = 15$ with 3 zeros.
    
    - Compatible numbers: replace factors with compatible numbers.

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4.4 The student will

a) **demonstrate fluency with multiplication facts through 12 × 12 and the corresponding division facts:**

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<td>and then multiply. Opportunities for students to discover patterns with 10 and powers of 10 should be provided.</td>
<td>Division is the operation of making equal groups or equal shares. When the original amount and the number of shares are known, divide to find the size of each share. When the original amount and the size of each share are known, divide to find the number of shares. Both situations may be modeled with Base-10 manipulatives.</td>
<td>Multiplication and division are inverse operations.</td>
</tr>
<tr>
<td>× 11 → 10</td>
<td>Division is the operation of making equal groups or shares. When the original amount and the number of shares are known, divide to determine the size of each share. When the original amount and the size of each share are known, divide to determine the number of shares. Both situations may be modeled with base-10 manipulatives.</td>
<td>Division is the inverse of multiplication. Terms used in division are dividend, divisor, and quotient.</td>
</tr>
<tr>
<td>64 → 64</td>
<td>- dividend ÷ divisor = quotient</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- quotient = divisor ÷ dividend</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Opportunities to invent division algorithms help students make sense of the algorithm. Teachers should teach division by Students benefit from experiences with various methods of division, such as repeated multiplication, and repeated subtraction and partial</td>
<td></td>
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The student will:

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<tr>
<td>Quotients (Before teaching the traditional long division algorithm)</td>
<td></td>
<td></td>
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<tr>
<td>Students need exposure to various types of practical problems in which they must interpret the quotient and remainder based on the context. The chart below includes one example of each type of problem.</td>
<td></td>
<td></td>
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#### MAKING SENSE OF THE REMAINDER IN DIVISION

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<thead>
<tr>
<th>TYPE OF PROBLEM</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remainder is not needed and can be left over (or discarded).</td>
<td>Bill has 29 pencils to share fairly with 6 friends. How many pencils will each friend receive? 4 pencils with 5 pencils left over</td>
</tr>
<tr>
<td>Remainder is partitioned and represented as a fraction or decimal.</td>
<td>Six friends will share 29 ounces of juice. How many ounces will each person get if all of the juice is shared equally? $4 \frac{5}{6}$ ounces</td>
</tr>
<tr>
<td>Remainder forces the answer to be increased to the next whole number.</td>
<td>There are 29 people going to the party by car. How many cars will be needed if each car holds 6 people? 5 cars</td>
</tr>
<tr>
<td>Remainder forces the answer to be rounded (giving an approximate answer).</td>
<td>Six children will share a bag of candy containing 29 pieces. About how many pieces of candy will each child get? About 5 pieces of candy</td>
</tr>
</tbody>
</table>

- Students will solve problems involving the division of decimals in grades 5 and 6.
## UNDERSTANDING THE STANDARD

### (Background Information for Instructor Use Only)
- A factor of a whole number is an integer whole number that divides evenly into that number with no remainder of zero.
- A factor of a number is a divisor of the number.
- A common factor of two or more numbers is a divisor that all of the numbers share. [Reordered]
- The greatest common factor of two or more numbers is the largest of the common factors that all of the numbers share. [Reordered]
- A multiple of a number is the product of the number and any natural number is a multiple of the number.
- A common factor of two or more numbers is a divisor that all of the numbers share.
- Common multiples and common factors can be useful when simplifying fractions. [Moved from middle column]
- The least common multiple of two or more numbers is the smallest common lowest number that is a multiple of all of the given numbers.
- The greatest common factor of two or more numbers is the largest of the common factors that all of the numbers share.
- Estimation keeps the focus on the meaning of the numbers and operations, encourages reflective thinking, and helps build informal number sense with fractions. Students can reason with

### ESSENTIAL UNDERSTANDINGS
- All students should
  - Understand and use common multiples and common factors for simplifying fractions.
  - Develop and use strategies to estimate addition and subtraction involving fractions and decimals.
  - Use visual models to add and subtract with fractions and decimals.

### ESSENTIAL KNOWLEDGE AND SKILLS
- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
  - Find common multiples and common factors of numbers. (a)
  - Determine the least common multiple and greatest common factor of no more than 3 numbers. (a)
  - Use least common multiple and/or greatest common factor to find a common denominator for fractions using common multiples and factors. Common denominators should not exceed 60. (b)
  - Estimate the sum or difference of two fractions. (b, c)
  - Add and subtract with fractions (proper or improper) and/or mixed numbers, having like and unlike denominators whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction using common multiples and factors. Subtraction with fractions will be limited to problems that do not require regrouping. (b)
  - Add and subtract with fractions having unlike denominators whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction using common multiples and factors. [Included in bullet above]

---

4.5 The student will

a) determine common multiples and factors, including least common multiple and greatest common factor;

b) add and subtract fractions and mixed numbers having like and unlike denominators that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors,* and

c) add and subtract with decimals; and [Moved to 4.6a]

d) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers and with decimals. [Decimals limited to single-step only and moved to 4.6b; solve multistep with decimals included in 5.5b; solve multistep problems with fractions included in 5.6a]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.
4.5 The student will

a) determine common multiples and factors, including least common multiple and greatest common factor;

b) add and subtract fractions and mixed numbers having like and unlike denominators, that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors,* and

c) add and subtract with decimals; and [Moved to 4.6a]

cd) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers and

with decimals. [Decimals limited to single-step only and moved to 4.6b; solve multistep with decimals included in 5.5b solve

multistep problems with fractions included in 5.6a]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<tr>
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<tbody>
<tr>
<td>benchmarks to get an estimate without using an algorithm.</td>
<td>Solve single-step and multistep practical problems that involve adding and subtracting fractions and mixed numbers (proper or improper) and/or mixed numbers, having like and unlike denominators whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction using common multiples and factors. Subtraction with fractions will be limited to problems that do not require regrouping. (c)</td>
<td>Students should investigate addition and subtraction with fractions, using a variety of models (e.g., fraction circles, fraction strips, linking cubes, number lines, pattern blocks).</td>
</tr>
<tr>
<td>Students should investigate addition and subtraction with fractions, using a variety of models (e.g., fraction circles, fraction strips, linking cubes, number lines, pattern blocks).</td>
<td>Add and subtract with decimals through thousandths, using concrete materials, pictorial representations, and paper and pencil. [Moved to 4.6 EKS]</td>
<td>While this standard requires instruction in solving problems with denominators of 2, 3, 4, 5, 6, 8, 10, and 12, students would benefit from experiences with other denominators.</td>
</tr>
<tr>
<td>When students use the least common multiple to determine common denominators to add or subtract fractions with unlike denominators, the least common multiple may be greater than 12, but will not exceed 60.</td>
<td>Solve single-step and multistep problems that involve adding and subtracting fractions and mixed numbers through thousandths. [Moved to 4.6 EKS]</td>
<td>Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction can be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3 \frac{5}{2}$).</td>
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<td>Instruction involving addition and subtraction of fractions should include experiences with proper fractions, improper fractions, and mixed numbers as addends, minuends, subtrahends, sums, and differences.</td>
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<tr>
<td>At this level, subtraction with fractions will be limited to.</td>
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The student will:

a) determine common multiples and factors, including least common multiple and greatest common factor;

b) add and subtract fractions and mixed numbers having like and unlike denominators, that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions; using common multiples and factors;* and

c) add and subtract with decimals; and [Moved to 4.6a]

d) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers and with decimals. [Decimals limited to single-step only and moved to 4.6b; solve multistep with decimals included in 5.5b solve multistep problems with fractions included in 5.6a]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<tr>
<td>problems that do not require regrouping.</td>
<td>A fraction is in simplest form when its numerator and denominator have no common factors other than 1. The numerator can be greater than the denominator. [Copied from SOL5.6]</td>
<td>The least number of steps necessary to solve a single-step problem is one.</td>
</tr>
<tr>
<td></td>
<td>The least number of steps necessary to solve a single-step problem is one.</td>
<td>Addition and subtraction of decimals may be explored, using a variety of models (e.g., 10-by-10 grids, number lines, money). [Moved from 4.5 U.S.]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as in all, altogether, difference, etc. encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The least number of steps necessary to solve a single-step problem is one.</td>
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</table>
The student will

a) determine common multiples and factors, including least common multiple and greatest common factor;
b) add and subtract fractions and mixed numbers having like and unlike denominators; that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors;* and
c) add and subtract with decimals; and[Moved to 4.6a]
cd) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers and with decimals.[Decimals limited to single-step only and moved to 4.6b; solve multistep with decimals included in 5.5b solve multistep problems with fractions included in 5.6a]

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<td>When adding or subtracting with fractions having like denominators, add or subtract the numerators and use the same denominator. Write the answer in simplest form using common multiples and factors.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>When adding or subtracting with fractions having unlike denominators, rewrite them as fractions with a common denominator. The least common multiple (LCM) of the unlike denominators is a common denominator (LCD). Write the answer in simplest form using common multiples and factors.</td>
<td></td>
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</tr>
<tr>
<td>Addition and subtraction of decimals may be explored, using a variety of models (e.g., 10-by-10 grids, number lines, money). [Moved to 4.6]</td>
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</tr>
<tr>
<td>For decimal computation, the same ideas developed for whole number computation may be used, and these ideas may be applied to decimals, giving careful attention to the placement of the decimal point in the solution. Lining up tenths to tenths, hundredths to hundredths, etc., helps to establish the correct placement of the decimal.</td>
<td></td>
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<tr>
<td>Fractions may be related to decimals by using models (e.g., 10-by-10 grids, decimal squares, money).</td>
<td></td>
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</tr>
</tbody>
</table>
4.6  The student will
   a) add and subtract with decimals;* and[Moved from 4.5c]
   b) solve single-step and multistep practical problems involving addition and subtraction with decimals.[Moved from 4.5d; decimals changed to single-step only; multistep with decimals included in 5.5b]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Addition and subtraction of decimals may be explored, using a variety of models (e.g., 10-by-10 grids, number lines, money). [Moved from 4.5 US]
- The problem solving process is enhanced when students create and solve their own practical problems and model problems using manipulatives and drawings.
- In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as in all, altogether, difference, etc. encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions.
- The least number of steps necessary to solve a single-step problem is one.

### ESSENTIAL UNDERSTANDINGS

- Estimate sums and differences of decimals. (a)[Moved from 4.5 EKS]
- Add and subtract with decimals through thousandths, using concrete materials, pictorial representations, and paper and pencil. (a)[Moved from 4.5 EKS]
- Solve single-step and multistep practical problems that involve adding and subtracting with decimals through thousandths. (b) [Moved from 4.5 EKS; multistep included in 5.5b]
Students in grades 4 and 5 should be actively involved in measurement activities that require a dynamic interaction between students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects.

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, perimeter, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary), to measure length, weight/mass, liquid volume/capacity, area, perimeter, temperature, and time. Students' understanding of measurement by continues to be enhanced through experiences using appropriate tools such as rulers, balances, clocks, and thermometers. The process of measuring is identical for any attribute (i.e., length, weight/mass, liquid volume/capacity, area): choose a unit, compare that unit to the object, and report the number of units.

[Moved from geometry stand introduction]

The study of geometry helps students represent and make sense of the world. In grades 3 through 5, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)
4.6 The student will
   a) estimate and measure weight/mass and describe the results in U.S. Customary and metric units as appropriate; and
   b) identify equivalent measurements between units within the U.S. Customary system and between units within the metric system (grams and kilograms).

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

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<tbody>
<tr>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>Use benchmarks to estimate and measure weight/mass.</td>
<td>Determine an appropriate unit of measure (e.g., ounce, pound, ton, gram, kilogram) to use when measuring everyday objects in both metric and U.S. Customary units. [Moved to 4.8 EKS]</td>
</tr>
<tr>
<td>Identify equivalent measures between units within the U.S. Customary and between units within the metric measurements.</td>
<td>Measure objects in both metric and U.S. Customary units (e.g., ounce, pound, ton, gram, or kilogram) to the nearest appropriate measure, using a variety of measuring instruments. [Moved to 4.8 EKS]</td>
</tr>
<tr>
<td>Students should estimate the mass/weight of everyday objects (e.g., foods, pencils, book bags, shoes), using appropriate metric or U.S. Customary units.</td>
<td>Record the mass of an object including the appropriate unit of measure (e.g., 24 grams). [Moved to 4.8 EKS]</td>
</tr>
</tbody>
</table>
4.7 The student will
a) estimate and measure length, and describe the result in both metric and U.S. Customary units; and [Moved to 4.8a]
b) identify equivalent measurements between units within the U.S. Customary system (inches and feet; feet and yards; inches and yards; yards and miles) [Moved to 4.8c] and between units within the metric system (millimeters and centimeters; centimeters and meters; and millimeters and meters). [Included in 5.9a].

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<td>All students should</td>
<td>The student will use problem-solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Length is the distance along a line or figure from one point to another. [Moved to 4.8 US]</td>
<td>Use benchmarks to estimate and measure length.</td>
<td>• Determine an appropriate unit of measure (e.g., inch, foot, yard, mile, millimeter, centimeter, and meter) to use when measuring everyday objects in both metric and U.S. Customary units. [Moved to 4.8 EKS]</td>
</tr>
<tr>
<td>• U.S. Customary units for measurement of length include inches, feet, yards, and miles. Appropriate measuring devices include rulers, yardsticks, and tape measures. Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter ruler, meter stick, and tape measure. [Moved to 4.8 US]</td>
<td>Understand how to convert units of length between the U.S. Customary and metric systems, using ballpark comparisons.</td>
<td>• Estimate the length of everyday objects (e.g., books, windows, tables) in both metric and U.S. Customary units of measure. [Moved to 4.8 EKS]</td>
</tr>
<tr>
<td>• Practical experience measuring the length of familiar objects helps to establish benchmarks and facilitates the student’s ability to estimate length.</td>
<td>Understand the relationship between U.S. Customary units and the relationship between metric units.</td>
<td>• Measure the length of objects in both metric and U.S. Customary units, measuring to the nearest inch (1/2, 1/4, 1/8), foot, yard, mile, millimeter, centimeter, and meter, and record the length including the appropriate unit of measure (e.g., 24 inches). [Moved to 4.8 EKS]</td>
</tr>
<tr>
<td>• Students should estimate the length of everyday objects (e.g., books, windows, tables) in both metric and U.S. Customary units of measure.</td>
<td></td>
<td>• Compare estimates of the length of objects with the actual measurement of the length of objects. [Moved to 4.8 EKS]</td>
</tr>
<tr>
<td>• When measuring with U.S. Customary units, students should be able to measure to the nearest part of an inch (1/2, 1/4, 1/8), inch, foot, or yard. [Moved to 4.8 US]</td>
<td></td>
<td>• Identify equivalent measures of length between units within the U.S. Customary measurements [Moved to 4.8 EKS] and between units within the metric measurements. [Included in 5.9a]</td>
</tr>
</tbody>
</table>
The student will solve practical problems that involve determining perimeter and area in standard U.S. Customary and metric units of measure.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Perimeter is the path or distance around any plane figure.
- To determine the perimeter of any polygon, determine the sum of the lengths of the sides.
- Area is the surface included within a plane figure. Area is measured by the number of square units needed to cover a surface or plane figure.
- Students should have opportunities to investigate and discover, using manipulatives, the formulas for the area of a square and the area of a rectangle.
  - Area of a square = side length × side length
  - Area of rectangle = length × width
- Perimeter and area should always be labeled with the appropriate unit of measure.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Determine the perimeter of a polygon with no more than 10 sides when the lengths of the sides are given, with diagrams.
- Determine the perimeter and area of a rectangle when given the measure of two adjacent sides, with and without diagrams. [Moved from 5.8 EKS]
- Determine the perimeter and area of a square when the measure of one side is given, with and without diagrams. [Moved from 5.8 EKS]
- Label the perimeter and area with the appropriate unit of measure. [Moved to EKS]
- Solve practical problems that involve determining perimeter and area in standard U.S. Customary and metric units of measure.
The student will:

a) estimate and measure length and describe the result in U.S. Customary and metric units; [Moved from 4.7a]

b) estimate and measure liquid volume and describe the results in U.S. Customary units; [Included in 3.7b]

c) estimate and measure weight/mass and describe the results in U.S. Customary and metric units; [Moved from 4.6a]; and

d) given the equivalent measure of one unit, identify equivalent measurements of length, weight/mass and liquid volume
   between units within the U.S. Customary system; (cups, pints, quarts, and gallons); [Moved to EKS]; and

d) solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units.

### UNDERSTANDING THE STANDARD

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| - U.S. Customary units for measurement of liquid volume include cups, pints, quarts, and gallons.
| - The measurement of the object must include the unit of measure along with the number of iterations.
| - Students should measure the liquid volume of everyday objects in U.S. Customary units, including cups, pints, quarts, gallons, and record the volume including the appropriate unit of measure (e.g., 24 gallons). [Reordered]
| - Practical experience measuring liquid volume of familiar objects helps to establish benchmarks and facilitates the student’s ability to estimate liquid volume.
| - Students should estimate the liquid volume of containers in U.S. Customary units to the nearest cup, pint, quart, and gallon.
| - Length is the distance between two points along a line. [Moved from 4.7 US]
| - U.S. Customary units for measurement of length include inches, feet, yards, and miles. Appropriate measuring devices include rulers, yardsticks, and tape measures.
| - Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter ruler, meter stick, and tape measure. [Moved from 4.7 US]
| - When measuring with U.S. Customary units, students should be able to measure to the nearest part of an inch \( \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \), foot, or

### ESSENTIAL UNDERSTANDINGS

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<tr>
<td>- Use benchmarks to estimate and measure volume.</td>
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<tr>
<td>- Identify equivalent measurements between units within the U.S. Customary system.</td>
</tr>
</tbody>
</table>

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

- Determine an appropriate unit of measure (inch, foot, yard, mile, millimeter, centimeter, and meter) to use when measuring length in both U.S. Customary and metric units. (a) [Moved from 4.7 EKS]

- Estimate and measure length in both U.S. Customary and metric units, measuring to the nearest part of an inch \( \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \), and to the nearest foot, yard, millimeter, centimeter, or meter, and record the length including the unit of measure (e.g., 24 inches). (a) [Moved from 4.7 EKS]

- Compare estimates of the length with the actual measurement of the length. (a) [Moved from 4.7 EKS]

- Determine an appropriate unit of measure (ounce, pound, gram, kilogram) to use when measuring everyday objects in both U.S. Customary and metric units. (b) [Moved from 4.6 EKS]

- Estimate and measure the weight/mass of objects in both U.S. Customary and metric units (ounce, pound, gram, or kilogram) to the nearest appropriate measure, using a variety of measuring instruments. (b) [Moved from 4.6 EKS]

- Record the weight/mass of an object with the unit of measure (e.g., 24 grams). (b) [Moved from 4.6 EKS]
4.8 The student will
a) estimate and measure length and describe the result in U.S. Customary and metric units; [Moved from 4.7a]
ab) estimate and measure liquid volume and describe the results in U.S. Customary units [Included in 3.7b]
estimate and measure weight/mass and describe the results in U.S. Customary and metric units [Moved from 4.6a]; and
bc) given the equivalent measure of one unit, identify equivalent measurements of length, weight/mass and liquid volume between units within the U.S. Customary system; (cups, pints, quarts, and gallons) [Moved to EKS]; and
d) solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

ESSENTIAL UNDERSTANDINGS

ESSENTIAL KNOWLEDGE AND SKILLS

• Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object’s mass, although they use the term weight (e.g., “How much does it weigh?” versus “What is its mass?”). [Moved from 4.6 US]

• Balances are appropriate measuring devices to measure weight in U.S. Customary units (ounces, pounds) and mass in metric units (grams, kilograms). [Moved from 4.6 US]

• Practical experience measuring the weight/mass of familiar objects (e.g., foods, pencils, book bags, shoes) helps to establish benchmarks and facilitates the student’s ability to estimate weight/mass. [Moved from 4.6 US]

• Students should measure the liquid volume of everyday objects in U.S. Customary units, including cups, pints, quarts, and gallons, and record the volume including the appropriate unit of measure (e.g., 24 gallons). [Reordered]

• Students at this level will be given the equivalent measure of one unit when asked to determine equivalencies between units in the U.S. Customary system.

  – For example, students will be told 1 gallon is equivalent to 4 quarts and then will need to apply that relationship to determine problems such as:

    - the number of quarts in 5 gallons:

• Determine an appropriate unit of measure (cups, pints, quarts, gallons) to use when measuring liquid volume in U.S. Customary units.

• Estimate the liquid volume of containers in U.S. Customary units of measure to the nearest cup, pint, quart, and gallon. [Include in 3.7 EKS]

• Measure the liquid volume of everyday objects in U.S. Customary units, including cups, pints, quarts, and gallons, and record the volume including the appropriate unit of measure (e.g., 24 gallons). [Included in 3.7 EKS]

• Given the equivalent measure of one unit, identify equivalent measures of volume between units within the U.S. Customary system for:

  – length: inches and feet, feet and yards, inches and yards, yards and miles; [Moved from 4.7b],
  – weight/mass: ounces, pounds; [Moved from 4.6b] and
  – liquid volume: cups, pints, quarts, and gallons [Moved from 4.8b]. (c)

• Solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary. (d)
4.8 The student will
   a) estimate and measure length and describe the result in U.S. Customary and metric units; [Moved from 4.7a]
   b) estimate and measure liquid volume and describe the results in U.S. Customary units [Included in 3.7b]
   c) estimate and measure weight/mass and describe the results in U.S. Customary and metric units [Moved from 4.6a];
   d) given the equivalent measure of one unit, identify equivalent measurements of length, weight/mass and liquid volume
   between units within the U.S. Customary system; (cups, pints, quarts, and gallons) [Moved to EKS], and
   e) solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units.

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<td>- the number of gallons equal to 20 quarts;</td>
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<tr>
<td>- When empty Tim’s 10 gallon container, can hold how many quarts?; or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Maria has 20 quarts of lemonade. How many empty gallon containers will she be able to fill?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
STANDARD 4.9 STRAND: MEASUREMENT AND GEOMETRY GRADE LEVEL 4

4.9 The student will determine **solve practical problems related to** elapsed time in hours and minutes within a 12-hour period.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Elapsed time is the amount of time that has passed between two given times.</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Elapsed time should be modeled and demonstrated using analog clocks and timelines.</td>
<td>• Understanding the “counting on” strategy for determining elapsed time in hour and minute increments over a 12-hour period from a.m. to a.m. or p.m. to p.m.</td>
<td>• Determine the elapsed time in hours and minutes within a 12-hour period (times can cross between a.m. and p.m.):</td>
</tr>
<tr>
<td>• Elapsed time can be found by counting on from the beginning time or counting back from the ending time.</td>
<td></td>
<td>• Solve practical problems in relation to elapsed time that has elapsed in hours and minutes, within a 12-hour period (within a.m., within p.m., and across a.m. and p.m.):</td>
</tr>
<tr>
<td></td>
<td>— Count the number of whole hours between the beginning time and the finishing time.</td>
<td>— when given the beginning time and the ending time, determine the time that has elapsed;</td>
</tr>
<tr>
<td></td>
<td>— Count the remaining minutes.</td>
<td>— when given the beginning time and amount of elapsed time in hours and minutes, determine the ending time;</td>
</tr>
<tr>
<td></td>
<td>— Add the hours and minutes.</td>
<td>or</td>
</tr>
<tr>
<td></td>
<td>For example, to find the elapsed time between 10:15 a.m. and 1:25 p.m., count 10 minutes; and then, add 3 hours to 10 minutes to find the total elapsed time of 3 hours and 10 minutes.</td>
<td>— when given the ending time and the elapsed time in hours and minutes, determine the beginning time.</td>
</tr>
</tbody>
</table>
The study of geometry helps students represent and make sense of the world. At the fourth- and fifth-grade levels, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)
### STANDARD 4.10
**STRAND: MEASUREMENT AND GEOMETRY**
**GRADE LEVEL 4**

#### 4.10 The student will
- **a)** identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices; and
- **b)** identify and describe representations of lines that illustrate intersecting, parallelism, and perpendicularity lines.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(Background Information for Instructor Use Only)</strong></td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>- Points, lines, line segments, rays, and angles, including endpoints and vertices are fundamental components of noncircular geometric figures.</td>
<td>- Understand that points, lines, line segments, rays, and angles, including endpoints and vertices are fundamental components of noncircular geometric figures.</td>
<td>- Identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices. (a)</td>
</tr>
<tr>
<td>- A point is a location in space. It has no length, width, or height. A point is usually named with a capital letter.</td>
<td>- Understand that the shortest distance between two points on a flat surface is a line segment.</td>
<td>- Understand that lines in a plane can intersect or are parallel. Perpendicularity is a special case of intersection. [Moved to US]</td>
</tr>
<tr>
<td>- The shortest distance between two points in a plane on a flat surface, is a line segment.</td>
<td>- Understand that lines in a plane either intersect or are parallel. Perpendicularity is a special case of intersection.</td>
<td>- Use symbolic notation to name points, lines, line segments, rays, and angles. (a)</td>
</tr>
<tr>
<td>- A point is a location in space. It has no length, width, or height. A point is usually named with a capital letter.</td>
<td>- Identify practical situations that illustrate parallel, intersecting, and perpendicular lines.</td>
<td>- Identify practical situations that illustrate parallel, intersecting, and perpendicular lines. (b)</td>
</tr>
<tr>
<td>- A line is a collection of points extending going on and on infinitely in both directions. It has no endpoints. When a line is drawn, at least two points on it can be marked and given capital letter names. Arrows must be drawn to show that the line goes on in both directions infinitely (e.g., ( \overline{AB} ) read as “line AB”).</td>
<td>- Identify practical situations that illustrate parallel, intersecting, and perpendicular lines. (b)</td>
<td>- Use symbolic notation to describe parallel lines and perpendicular lines. (b)</td>
</tr>
<tr>
<td>- A line segment is part of a line. It has two endpoints and includes all the points between and including those endpoints. To name a line segment, name the endpoints (e.g., ( \overline{AB} ) read as “line segment AB”).</td>
<td>- Identify parallel, perpendicular, and intersecting line segments in plane and solid figures. (b)</td>
<td></td>
</tr>
<tr>
<td>- A ray is part of a line. It has one endpoint and extends continues infinitely in one direction. To name a ray, say the name of its endpoint first and then say the name of one other point on the ray (e.g., ( \overline{AB} ) read as “ray AB”).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- An angle is formed by two rays that share a common have the same endpoint. These rays form an angle. This endpoint is called the vertex. Angles are found wherever lines or line segments intersect.</td>
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</tr>
<tr>
<td>- An angle can be named in three different ways by using:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- three letters to name, in this order: a point on one ray, the vertex, and a point on the other ray;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- one letter at the vertex; or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- a number written inside the rays of the angle.</td>
<td></td>
<td></td>
</tr>
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</table>
4.10 The student will
   a) identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices; and
   b) identify and describe representations of lines that illustrate intersecting, parallelism, and perpendicularity lines.

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<td>(Background Information for Instructor Use Only)</td>
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<td></td>
</tr>
<tr>
<td>• A vertex is the point at which two lines, line segments, or rays meet to form an angle. In solid three-dimensional figures, a vertex is the point at which three or more edges meet.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Lines in a plane either intersect or are parallel. Perpendicularity is a special case of intersection.</td>
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<tr>
<td>• Intersecting lines have one point in common.</td>
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<td></td>
</tr>
<tr>
<td>• Perpendicular lines are special intersecting lines that form right angles where they intersect. The symbol ( \perp ) is used to indicate that two lines are perpendicular. For example, the notation ( \overline{AB} \perp \overline{CD} ) is read as “line ( AB ) is perpendicular to line ( CD )”</td>
<td></td>
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</tbody>
</table>

[Diagram: \( \overline{AB} \perp \overline{CD} \)]

• Students need experiences using geometric markings in figures to indicate congruence of sides and angles and to indicate parallel sides.

• Parallel lines are lines that lie in the same plane and never do not intersect. Parallel lines are always the same distance apart and do not share any points. The symbol \( \parallel \) indicates that two or more lines are parallel. For example, the notation \( \overline{BC} \parallel \overline{FG} \) is read as “line \( BC \) is parallel to line \( FG \)”.

[Diagram: \( \overline{BC} \parallel \overline{FG} \)]

• Students should explore intersection, parallelism, and perpendicularity in both two and three dimensions. For example, students should analyze the relationships between the edges of a cube. Which edges are parallel? Which are perpendicular? What
4.10 The student will
   a) identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices; and
   b) identify and describe representations of lines that illustrate intersecting, parallelism, and perpendicularity lines.

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<tr>
<td>plane contains the upper left edge and the lower right edge of the cube? Students can visualize this by using the classroom itself to notice the lines formed by the intersection of the ceiling and walls, of the floor and wall, and of two walls.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.11 The student will
   a) investigate congruence of plane figures after geometric transformations, such as reflection, translation, and rotation, using mirrors, paper folding, and tracing; and
   b) recognize the images of figures resulting from geometric transformations, such as translation, reflection, and rotation.

[Moved to 5.14]

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<td>All students should</td>
<td>The student will use problem-solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td></td>
<td>- Understand the meaning of the term congruent.</td>
<td>Recognize the congruence of plane figures resulting from geometric transformations such as translation, reflection, and rotation, using mirrors, paper folding, and tracing.</td>
</tr>
<tr>
<td></td>
<td>- Understand how to identify congruent figures.</td>
<td>[Moved to 5.14]</td>
</tr>
<tr>
<td></td>
<td>- Understand that the orientation of figures does not affect congruency or noncongruency.</td>
<td></td>
</tr>
<tr>
<td>• The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Level 0: Pre-recognition. Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.</td>
<td></td>
<td></td>
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<tr>
<td>- Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)</td>
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<tr>
<td>- Level 2: Analysis. Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)</td>
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<tr>
<td>- Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusion are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)</td>
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• Congruent figures are figures having exactly the same size and shape. [Moved to 5.14 US]

• Opportunities for exploring figures that are congruent and/or noncongruent can best be accomplished by using physical models.
4.11 The student will
a) investigate congruence of plane figures after geometric transformations, such as reflection, translation, and rotation, using mirrors, paper folding, and tracing; and
b) recognize the images of figures resulting from geometric transformations, such as translation, reflection, and rotation.

[Moved to 5.14]

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<tbody>
<tr>
<td>• A translation is a transformation in which an image is formed by moving every point on a figure the same distance in the same direction. [Moved to 5.14 US]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• A reflection is a transformation in which a figure is flipped over a line called the line of reflection. All corresponding points in the image and preimage are equidistant from the line of reflection. [Moved to 5.14 US]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• A rotation is a transformation in which an image is formed by turning its preimage about a point. [Moved to 5.14]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The resulting figure of a translation, reflection, or rotation is congruent to the original figure. [Moved to 5.14 US]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.11 The student will identify, describe, compare, and contrast characteristics of plane and solid figures by identifying relevant according to their characteristics, including the (number of angles, vertices, and edges, and the number and shape of faces), using concrete models and pictorial representations.[Moved from 3.14]

<table>
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<tbody>
<tr>
<td><strong>(Background Information for Instructor Use Only)</strong></td>
<td><strong>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</strong></td>
<td><strong>Identify concrete models and pictorial representations of solid figures (cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by name.[Moved from 3.14 EKS]</strong></td>
</tr>
<tr>
<td>• The study of geometric figures must be active, using visual images and concrete materials (tools such as graph paper, pattern blocks, geoboards, and geometric solids, and computer software tools).</td>
<td>• Identify and describe characteristics of solid figures (cube, rectangular prism, square pyramid, sphere, cone, cylinder) by counting according to their characteristics (the number of angles, vertices, edges, and by the number and shape of faces).[Moved from 3.14 EKS]</td>
<td></td>
</tr>
<tr>
<td>• Opportunity must be provided for building and using geometric vocabulary to describe plane and solid figures.</td>
<td>• Compare and contrast characteristics of plane and solid figures (circle/sphere, square/cube, triangle/square pyramid, rectangle/rectangular prism), by counting according to their characteristics (number of sides, angles, vertices, edges, and the number and shape of faces).[Moved from 3.14 EKS]</td>
<td></td>
</tr>
<tr>
<td>• A plane figure is any closed, two-dimensional shape that can be drawn in a plane. A plane is a flat surface with no thickness.</td>
<td><strong>ESSENTIAL UNDERSTANDINGS</strong></td>
<td><strong>ESSENTIAL KNOWLEDGE AND SKILLS</strong></td>
</tr>
<tr>
<td>• A solid figure is a three-dimensional figure, having length, width, and height.</td>
<td><strong>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</strong></td>
<td><strong>Identify concrete models and pictorial representations of solid figures (cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by name.[Moved from 3.14 EKS]</strong></td>
</tr>
<tr>
<td>• A face is any flat surface of a solid figure.</td>
<td><strong>Identify and describe characteristics of solid figures (cube, rectangular prism, square pyramid, sphere, cone, cylinder) by counting according to their characteristics (the number of angles, vertices, edges, and by the number and shape of faces).[Moved from 3.14 EKS]</strong></td>
<td></td>
</tr>
<tr>
<td>• An angle is formed by two rays with a common endpoint called the vertex. Angles are found wherever lines and/or line segments intersect.</td>
<td><strong>Compare and contrast characteristics of plane and solid figures (circle/sphere, square/cube, triangle/square pyramid, rectangle/rectangular prism), by counting according to their characteristics (number of sides, angles, vertices, edges, and the number and shape of faces).[Moved from 3.14 EKS]</strong></td>
<td></td>
</tr>
<tr>
<td>• An edge is the line segment where two faces of a solid figure intersect.</td>
<td><strong>ESSENTIAL UNDERSTANDINGS</strong></td>
<td><strong>ESSENTIAL KNOWLEDGE AND SKILLS</strong></td>
</tr>
<tr>
<td>• A vertex is the point at which two or more lines, line segments, or rays meet to form an angle. A vertex in a solid figure is the point at which three or more faces meet.</td>
<td><strong>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</strong></td>
<td><strong>Identify concrete models and pictorial representations of solid figures (cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by name.[Moved from 3.14 EKS]</strong></td>
</tr>
<tr>
<td>• A cube is a solid figure with six congruent, square faces. All edges are the same length. A cube has 8 vertices and 12 edges.</td>
<td><strong>Identify and describe characteristics of solid figures (cube, rectangular prism, square pyramid, sphere, cone, cylinder) by counting according to their characteristics (the number of angles, vertices, edges, and by the number and shape of faces).[Moved from 3.14 EKS]</strong></td>
<td></td>
</tr>
<tr>
<td>• A cylinder is a solid figure formed by two congruent parallel faces called bases, joined by a curved surface. The most commonly referenced cylinders have circular bases.</td>
<td><strong>Compare and contrast characteristics of plane and solid figures (circle/sphere, square/cube, triangle/square pyramid, rectangle/rectangular prism), by counting according to their characteristics (number of sides, angles, vertices, edges, and the number and shape of faces).[Moved from 3.14 EKS]</strong></td>
<td></td>
</tr>
<tr>
<td>• A base is a special face of a solid figure.</td>
<td><strong>ESSENTIAL UNDERSTANDINGS</strong></td>
<td><strong>ESSENTIAL KNOWLEDGE AND SKILLS</strong></td>
</tr>
<tr>
<td>• A cone is a solid figure formed by a face called a base that is joined to a vertex by a curved surface. The most commonly referenced cones have a circular base.</td>
<td><strong>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</strong></td>
<td><strong>Identify concrete models and pictorial representations of solid figures (cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by name.[Moved from 3.14 EKS]</strong></td>
</tr>
</tbody>
</table>
The student will identify, describe, compare, and contrast characteristics of plane and solid figures by identifying relevant according to their characteristics, including the (number of angles, vertices, and edges, and the number and shape of faces), using concrete models and pictorial representations. [Moved from 3.14]

<table>
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<tr>
<th>Underlying the Standard</th>
<th>Essential Understandings</th>
<th>Essential Knowledge and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shape of Figures</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Solid Geometric Figure</strong></td>
<td><strong># of Faces</strong></td>
<td><strong>Shape of Faces</strong></td>
</tr>
<tr>
<td>Cube</td>
<td>6</td>
<td>Squares</td>
</tr>
<tr>
<td>Rectangular Prism</td>
<td>6</td>
<td>Rectangles</td>
</tr>
<tr>
<td>Square Pyramid</td>
<td>5</td>
<td>Square/Triangles</td>
</tr>
<tr>
<td>Sphere</td>
<td>0</td>
<td>N/A</td>
</tr>
<tr>
<td>Cone</td>
<td>2</td>
<td>Circle</td>
</tr>
<tr>
<td>Cylinder</td>
<td>2</td>
<td>Circle</td>
</tr>
</tbody>
</table>

* The most commonly referenced cones and cylinders have circular faces.
4.12 The student will classify quadrilaterals as parallelograms, rectangles, squares, rhombi, and/or trapezoids.
   a) define polygon; and
   b) identify polygons with 10 or fewer sides. [Polygons other than quadrilaterals moved to 3.11]

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<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to</td>
</tr>
<tr>
<td>• A polygon is a closed plane geometric figure composed of at least three line segments that do not cross. None of the sides are curved.</td>
<td>• Identify polygons with 10 or fewer sides in everyday situations.</td>
<td>• Define and identify properties of polygons with 10 or fewer sides. [Polygons other than quadrilaterals moved to 3.13]</td>
</tr>
<tr>
<td>• A triangle is a polygon with three angles and three sides.</td>
<td>• Identify polygons with 10 or fewer sides in multiple orientations (rotations, reflections, and translations of the polygons). [Polygons other than quadrilaterals moved to 3.13]</td>
<td></td>
</tr>
<tr>
<td>• A quadrilateral is a polygon with four sides.</td>
<td></td>
<td>• Develop definitions for parallelograms, rectangles, squares, rhombi, and trapezoids. [Reworded]</td>
</tr>
<tr>
<td>• A parallelogram is a quadrilateral with both pairs of opposite sides parallel and congruent. [Reordered]</td>
<td></td>
<td>• Identify properties of quadrilaterals including parallel, perpendicular, and congruent sides. [Reworded]</td>
</tr>
<tr>
<td>• Congruent figures have the same size and shape. Congruent sides are the same length.</td>
<td></td>
<td>• Classify quadrilaterals as parallelograms, rectangles, squares, rhombi, and/or trapezoids. [Reworded]</td>
</tr>
<tr>
<td>• A rectangle is a quadrilateral with four right angles and therefore opposite sides that are parallel and congruent.</td>
<td></td>
<td>• Compare and contrast the properties of quadrilaterals.</td>
</tr>
<tr>
<td>• The geometric markings shown on the rectangle below indicate parallel sides with an equal number of arrows and congruent sides indicated with an equal number of hatch (hash) marks.</td>
<td></td>
<td>• Identify parallel sides, congruent sides, and right angles using geometric markings to denote properties of quadrilaterals.</td>
</tr>
<tr>
<td>• A square is a rectangle with four congruent sides of equal length and four right angles.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• A trapezoid is a quadrilateral with exactly one pair of parallel sides.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• A parallelogram is a quadrilateral with both pairs of opposite sides parallel. [Reordered]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• A rhombus is a quadrilateral with 4 four congruent sides.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Properties of a rhombus include the following:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– opposite sides are congruent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– opposite sides are parallel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– opposite angles are congruent</td>
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The student will classify quadrilaterals as parallelograms, rectangles, squares, rhombi, and/or trapezoids.

a) define polygon; and
b) identify polygons with 10 or fewer sides. [Polygons other than quadrilaterals moved to 3.11]

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<tr>
<td>• A pentagon is a 5-sided polygon.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• A hexagon is a 6-sided polygon.</td>
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<td></td>
</tr>
<tr>
<td>• A heptagon is a 7-sided polygon.</td>
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<td></td>
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<tr>
<td>• An octagon is an 8-sided polygon.</td>
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<td></td>
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<tr>
<td>• A nonagon is a 9-sided polygon.</td>
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<td></td>
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<tr>
<td>• A decagon is a 10-sided polygon. [Polygon information moved to 3.11]</td>
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</tbody>
</table>
Students entering grades 4 and 5 have begun to explore the concepts of the measurement of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students have had initial experiences in predicting whether a game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction in grades 4 and 5 is to deepen their understanding of the concepts of probability by

- offering opportunities to set up models simulating practical events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities that instill a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data collection strategies. Students should continue to focus on

- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to practical experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions (“How many?” “How much?”) from the data displays;
- identifying and justifying comparisons (“Which is the most? Which is the least?” “Which is the same?” “Which is different?”) about the information;
- comparing their initial predictions to the actual results; and
- writing a few sentences to communicating to others their interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data analysis methods as powerful means for decision making.
STANDARD 4.13 STRAND: PROBABILITY AND STATISTICS GRADE LEVEL 4

4.13 The student will
   a) predict determine the likelihood of an outcome of a simple event;
   b) represent probability as a number between 0 and 1, inclusive; and
   c) create a model or practical problem to represent a given probability.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives.
- Probability is the chance measure of likelihood that an event occurring will occur. An event is a collection of outcomes from an investigation or experiment.
- The terms certain, likely, equally likely, unlikely, and impossible can be used to describe the likelihood of an event. [Moved from middle column]
- If all outcomes of an event are equally likely, the probability of an event occurring is the ratio of desired outcomes to the denominator represents the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event equals number of favorable outcomes total number of possible outcomes.
- Probability is quantified as a number The probability of an event occurring is represented by a ratio between 0 and 1. An event is “impossible” if it has a probability of 0 (e.g., if 8 balls in a bag, 4 yellow, 4 blue, that a red ball could be selected the probability that the month of April will have 21 days). An event is “certain” if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow, morning if 10 coins, all pennies, are in a bag that a penny could be selected).
- When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time).

ESSENTIAL UNDERSTANDINGS

All students should
- Understand and apply basic concepts of probability.
- Describe events as likely or unlikely and discuss the degree of likelihood, using the terms certain, likely, equally likely, unlikely, and impossible. [Moved to US]
- Predict the likelihood of an outcome of a simple event and test the prediction.
- Understand that the measure of the probability of an event can be represented by a number between 0 and 1, inclusive.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Model and determine all possible outcomes of a given simple event where there are no more than 24 possible outcomes, using a variety of manipulatives (e.g., such as coins, number cubes, and spinners). (a)
- Determine the outcome of an event that is least likely to occur (less than half) or most likely to occur (greater than half) where there are no more than 24 possible outcomes. (a) [Reordered]
- Write the probability of a given simple event as a fraction, where there are no more than 24 possible outcomes the total number of possible outcomes is 24 or fewer. (b)
- Identify Determine the likelihood of an event occurring and relate it to its whole number or fractional representation (e.g., impossible or 0; equally likely or 1). (b)
- Determine the outcome of an event that is least likely to occur (less than half) or most likely to occur (greater than half) when the number of possible outcomes is 24 or less.
- Create a model or practical problem to represent a given probability. (c)

Represent probability as a point between 0 and 1, inclusively, on a number line.

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 4 51
4.13 The student will
   a) predict-determine the likelihood of an outcome of a simple event;
   b) represent probability as a number between 0 and 1, inclusive; and
   c) create a model or practical problem to represent a given probability.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Conduct experiments to determine the probability of an event occurring for a given number of trials (no more than 25 trials), using manipulatives (e.g., the number of times “heads” occurs when flipping a coin 10 times; the chance that when the names of 12 classmates are put in a shoebox, a name that begins with D will be drawn). Equally likely events can be represented with fractions of equivalent value. For example, on a spinner with 8 sections of equal size, where three of the sections are labeled G (green) and three are labeled B (blue), the chances of landing on green or on blue are equally likely; the probability of each of these events is the same, or \( \frac{3}{8} \).

- Students should have opportunities to describe in informal terms (i.e., impossible, unlikely, as likely as unlikely, equally likely, likely, and certain) the degree of likelihood of an event occurring.

- Activities should include practical examples.

- For an event such as flipping a coin, the equally likely things that can happen are called outcomes. For example, there are two equally likely possible outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up. The two possible outcomes, heads up or tails up, are equally likely.

- For another event such as spinning a spinner that is one-third red and two-thirds blue, the two outcomes, red and blue, are not equally likely. This is an unfair spinner (since it is not divided equally), therefore, the outcomes are not equally likely.
The student will
a) **predict** the likelihood of an outcome of a simple event;
   b) **represent** probability as a number between 0 and 1, inclusive; and
   c) **create** a model or practical problem to represent a given probability.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Students need opportunities to create a model or practical problem that represents a given probability. For example, if asked to create a box of marbles where the probability of selecting a black marble is \( \frac{4}{9} \), sample responses might include:

<p>| | | |</p>
<table>
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<tbody>
<tr>
<td>black</td>
<td>black</td>
<td>red</td>
</tr>
<tr>
<td>black</td>
<td>yellow</td>
<td>yellow</td>
</tr>
<tr>
<td>black</td>
<td>black</td>
<td>black</td>
</tr>
</tbody>
</table>
The student will

a) collect, organize, and represent data in a bar graph and line graph display; and
b) make observations and inferences about interpret data represented in bar graphs and line graphs, interpret data from a variety of graphs; and
c) compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph; a chart and a line graph; a pictograph and a bar graph).[Added to align with EKS bullet]

<table>
<thead>
<tr>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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</thead>
</table>
| Data displayed in bar and line graphs can be interpreted so that informed decisions can be made.[Moved from middle column] The title and labels of the graph provide the foundation for interpreting the data.[Moved from middle column] There are two types of data: categorical (e.g. qualitative) and numerical (e.g. quantitative). Categorical data are observations about characteristics that can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. While students need to be aware of the differences, they do not have to know the terms for each type of data. Bar graphs display grouped data such as categories using rectangular bars whose length represents the quantity the bar represents. Bar graphs should be used to compare counts of different categories (categorical or qualitative data). Using grid paper can assist students in creating more accurate graphs with greater accuracy.[Reordered] | All students should
- Understand the difference between representing categorical data and representing numerical data.
- Understand that line graphs show change over time (numerical data).
- Understand that bar graphs should be used to compare counts of different categories (categorical data).
- Understand how data displayed in bar and line graphs can be interpreted so that informed decisions can be made.[Moved to US] | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
- Collect data, using, for example, observations, measurement, surveys, scientific experiments, polls, or questionnaires. (a)
- Organize data into a chart or table. (a)
- Construct and display. Represent data in bar graphs, labeling one axis with equal whole number increments of 1 or more (numerical data) (e.g., 2, 5, 10, or 100) and the other axis with categories related to the title of the graph (categorical data) (e.g., swimming, fishing, boating, and water skiing as the categories of “Favorite Summer Sports”). (a)
- Construct and display. Represent data in line graphs, labeling the vertical axis with equal whole number increments of 1 or more and the horizontal axis with continuous data commonly related to time (e.g., hours, days, months, years, and age). Line graphs will have no more than 10 identified points along a continuum for continuous data. For example, growth charts showing age versus the height place across the horizontal axis (e.g., 1 month, 2 months, 3 months, and 4 months).[Example moved to US1] (a)
- Title the graph or identify an appropriate title; the title in a given graph and label or identify the axes. Label the axis or identify the appropriate labels. (a)
STANDARD 4.14 STRAND: PROBABILITY AND STATISTICS GRADE LEVEL 4

4.14 The student will
a) collect, organize, and represent data in a bar graph and line graph display; and
b) make observations and inferences about interpret data represented in bar graphs and line graphs, interpret data from a variety of graphs; and
c) compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph; a chart and a line graph; a pictograph and a bar graph).[Added to align with EKS bullet]

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- A bar graph uses horizontal or vertical bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category.
- There is space before, between, and after the bars.
- The axis that displays the scale representing the count for the categories should begin at zero and extend one increment above the greatest recorded piece of data. Fourth Grade 4 students should collect and represent data that are recorded in increments of whole numbers, usually multiples of 1, 2, 5, 10, or 100.
- Each axis should be labeled, and the graph should be given a title.[Reordered]

- Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be written.[Reordered]

- Line graphs are used to show how two continuous variables (data sets, numerical or quantitative data) are related. Line graphs may be used to show how one variable changes over time (numerical or quantitative data), that is the relationship between some attribute that can be quantified at certain points in time. If this one variable is not continuous, then a broken line is used. By looking at a line graph, it can be determined whether the variable change in the data set is increasing, decreasing, or staying the same over time.
  - The values along the horizontal axis represent continuous data on a given variable, usually some measure of time (e.g., time in years, months, or days). The data presented on a line

ESSENTIAL UNDERSTANDINGS

- Interpret data by making observations from simple line and bar graphs by describing the characteristics of the data and the data as a whole (e.g., the time period when the temperature increased the most, the category with the greatest/least, categories with the same number of responses, similarities and differences, the total number). Data points will be limited to 30 and categories to 8. One set of data will be represented on a graph. (b)

- Interpret data by making inferences from line and bar graphs. (b)

- Interpret the data to answer the question posed, and compare the answer to the prediction (e.g., “The summer sport preferred by most is swimming, which is what I predicted before collecting the data.”). (b)

- Write at least one sentence to describe the analysis and interpretation of the data, identifying parts of the data that have special characteristics, including categories with the greatest, the least, or the same. (b)

- Compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph; a chart and a line graph; a pictograph and a bar graph). (c)
4.14 The student will
a) collect, organize, and represent data in a bar graph and line graph display; and
b) make observations and inferences about data represented in bar graphs and line graphs; and
c) compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph; a chart and a line graph; a pictograph and a bar graph).

**UNDERSTANDING THE STANDARD**  
(Background Information for Instructor Use Only)

- A line graph is referred to as “continuous data,” as it represents data collected over a continuous period of time.
- The values along the vertical axis are the scale and represent the frequency with which those values occur in the data set; the frequency with which those values occur in the data set represent the range of values in the collected data set at the given time interval on the horizontal axis. The scale values on the vertical axis should represent equal increments of multiples of whole numbers, fractions, or decimals, depending upon the data being collected. The scale should extend one increment above the greatest recorded piece of data.
- Plot a point to represent the data collected for each time increment. Use line segments to connect the points in order moving left to right.
- Each axis should be labeled, and the graph should be given a title.
- A line graph tells whether something has increased, decreased, or stayed the same with the passage of time. Statements representing an analysis and interpretation of the characteristics of the data in the graph should be included (e.g., trends of increase and/or decrease, and least and greatest).

- For example, a line graph documenting data gathered during a planting cycle might show length of time and the height of a plant at any given interval.
- Different situations call for different types of graphs. The way data is displayed is often dependent upon what someone is trying to communicate.
- Comparing different types of representations (charts and graphs) provide students an opportunity to learn how different graphs can

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Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 4
4.14 The student will
a) collect, organize, and represent data in a bar graph and line graph display; and
b) make observations and inferences about interpret data represented in bar graphs and line graphs; and
c) compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph; a chart and a line graph; a pictograph and a bar graph).

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<tr>
<td>show different things about the same data. Following construction of graphs, students benefit from discussions around what information each graph provides.</td>
<td>- Tables or charts organize the exact data and display numerical information. They do not show visual comparisons which generally means it takes longer to understand or to examine trends.</td>
<td>- Line graphs display data that changes continuously over time. This allows overall increases or decreases to be seen more readily.</td>
</tr>
<tr>
<td>- Bar graphs can be easily used to compare data and see relationships. They provide a visual display comparing the numerical values of different categories. The scale of a bar graph may affect how one perceives the data.</td>
<td>- Examples of some questions that could be explored in comparing a chart to a line graph include: In which representation do you readily see the increase or decrease of temperature over time? In which representation is it easiest to determine when the greatest rise in temperature occurred?</td>
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</table>

### Temperature Over Time

<table>
<thead>
<tr>
<th>Time</th>
<th>Temperature</th>
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<tbody>
<tr>
<td>9 a.m.</td>
<td>12</td>
</tr>
<tr>
<td>10 a.m.</td>
<td>26</td>
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<tr>
<td>11 a.m.</td>
<td>33</td>
</tr>
<tr>
<td>12 a.m.</td>
<td>39</td>
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</tbody>
</table>
4.14 The student will

(Added to align with EKS bullet)

a) collect, organize, and represent data in a bar graph and line graph display; and

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| • Bar graphs should be used to compare counts of different categories (categorical data). Using grid paper ensures more accurate graphs.  
  --- A bar graph uses parallel, horizontal or vertical bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category.  
  --- There is space before, between, and after the bars.  
  --- The axis that displays the scale representing the count for the categories should extend one increment above the greatest recorded piece of data. Fourth-grade students should collect data that are recorded in increments of whole numbers, usually multiples of 1, 2, 5, 10, or 100.  
  --- Each axis should be labeled, and the graph should be given a title.  
  --- Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be written. [Bar graph information reordered] | | |
Students entering grades 4 and 5 have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships in open number sentences within a pattern.

The focus of instruction is to help students develop a solid use of patterning as a problem solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They interpret the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By interpreting mathematical situations and models, students begin to represent these, using symbols and variables to write “rules” for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.
### UNDERSTANDING THE STANDARD

**Background Information for Instructor Use Only**

- Patterns and functions can be represented in many ways and described using words, tables, graphs, and symbols. [Moved from middle column]

- Most patterning activities should involve making connections between some form of concrete materials and numerical representations to make up a pattern (e.g., number sequence, table, description).

- Students will identify and extend a wide variety of patterns, including rhythmic, geometric, graphic, numerical, and algebraic. Numeric patterns, at this level, will include both growing (addition and subtraction only) and repeating patterns (limited to addition, subtraction, and multiplication of whole numbers and addition and subtraction of fractions with like denominators of 12 or less).

- Students need experiences with growing patterns using concrete materials and calculators. Growing patterns can increase or decrease by a constant value.

- Reproduction of a given pattern in a different representation, using symbols and objects, lays the foundation for writing the relationship symbolically or algebraically.

- Sample growing patterns that are, or can be represented as numerical (arithmetic) growing patterns include:
  - 20, 18, 16, 14, 4, 8, 16, …;
  - 8, 10, 13, 17, …, 0.35, 0.55, 0.75;
  - \( \frac{1}{4}, \frac{3}{4}, \frac{1}{4}, \frac{3}{4}, \ldots \); and
  - \[
  \begin{array}{c}
  \text{1st} \\
  \text{2nd} \\
  \text{3rd} \\
  \text{4th}
  \end{array}
  \]

### ESSENTIAL UNDERSTANDINGS

**All students should**

- Understand that patterns and functions can be represented in many ways and described using words, tables, graphs, and symbols. [Moved to US]

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify and describe geometric and numerical patterns, using words, objects, pictures, numbers, and tables, symbols, or words.

- Create geometric and numerical patterns, using concrete materials, objects, pictures, numbers, and tables, and words.

- Extend geometric and numerical patterns, using objects, pictures, numbers, and tables, concrete materials, number lines, tables, and words.

- Identify, describe, create, and extend single-operation input and output rules involving addition and subtraction to solve problems in various contexts. Solve practical problems that involve identifying, describing, and extending single-operation input and output rules, limited to addition, subtraction, and multiplication of whole numbers and addition and subtraction of fractions with like denominators of 12 or less.

- Identify the rule in a single-operation, addition or subtraction, numerical pattern found in a list or table, limited to addition, subtraction, and multiplication of whole numbers.
4.15 The student will recognize, identify, describe, create, and extend numerical and geometric patterns found in objects, pictures, numbers, and tables.

**UNDERSTANDING THE STANDARD**
*(Background Information for Instructor Use Only)*

- Students in grade 3 had experiences working with input/output tables. At this level, tables of values input/output tables should be analyzed for a pattern to determine an unknown value or describe the rule that explains how to find the output when given the input-the next value. Determining and applying rules builds the foundation for functional thinking. Sample input/output tables that require determination of the rule or missing terms can be found below: Examples include:

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Input</td>
<td>Output</td>
<td>Input</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>442</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
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<td>75</td>
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<tr>
<td>10</td>
<td>17</td>
<td>50</td>
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<td>1.0</td>
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4.16 The student will
a) recognize and demonstrate the meaning of equality in an equation; and
b) investigate and describe the associative property for addition and multiplication. [Moved to 5.19]

<table>
<thead>
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<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Mathematical relationships can be expressed using equations. [Moved from middle column]</td>
<td>• Understand that mathematical relationships can be expressed using equations. [Moved to US]</td>
<td>• Recognize and demonstrate that the equals sign (=) relates equivalent quantities in an equation.</td>
</tr>
<tr>
<td>• Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers. [Moved to 4.4]</td>
<td>• Understand that quantities on both sides of an equation must be equal.</td>
<td>• Write an equation to represent the relationship between equivalent mathematical expressions relationships (e.g., $4 \times 3 = 2 \times 6; 10 + 8 = 36 \div 2; 12 \times 4 = 60 \div 12$).</td>
</tr>
<tr>
<td>• The commutative property for addition states that changing the order of the addends does not affect the sum (e.g., $4 + 3 = 3 + 4$). Similarly, the commutative property for multiplication states that changing the order of the factors does not affect the product (e.g., $2 \times 3 = 3 \times 2$). [Moved to 4.4]</td>
<td>• Understand that the associative property for addition means you can change the groupings of three or more addends without changing the sum.</td>
<td>• Recognize and demonstrate appropriate use of the equals sign (=) and the not equal sign (≠) in an equation.</td>
</tr>
<tr>
<td>• The associative property for addition states that the sum stays the same when the grouping of addends is changed [e.g., $(15 + (35 + 16)) = (15 + 35) + 16$]. The associative property for multiplication states that the product stays the same when the grouping of factors is changed [e.g., $(6 \times (3 \times 5)) = (6 \times 3) \times 5$]. [Moved to 5.19]</td>
<td>• Understand that the associative property for multiplication means you can change the groupings of three or more factors without changing the product.</td>
<td>• Identify and use the appropriate symbol to distinguish between expressions that are equal and expressions that are not equal, using addition, subtraction, multiplication, and division (e.g., $4 \times 12 = 6 \times 8$ and $64 \div 8 \neq 8 \times 8$).</td>
</tr>
<tr>
<td>• An expression is a representation of a quantity. It is made up of numbers, variables, and/or computational symbols. It does not have an equal symbol (e.g., $8, 15 \times 12$).</td>
<td>• Investigate and describe the associative property for addition as $(6 + 2) + 3 = 6 + (2 + 3)$. [Moved to 5.19]</td>
<td>• Investigate and describe the associative property for multiplication as $(3 \times 2) \times 4 = 3 \times (2 \times 4)$. [Moved to 5.19]</td>
</tr>
<tr>
<td>• An equation represents the relationship between two expressions of equal value. (e.g., $12 \times 3 = 72 + 2$)</td>
<td>• The equal symbol (=) means that the values on either side are equivalent (balanced).</td>
<td></td>
</tr>
<tr>
<td>• The equal symbol (=) means that the values on either side are equivalent (balanced).</td>
<td>• The not equal (≠) symbol means that the values on either side are not equivalent (not balanced).</td>
<td></td>
</tr>
</tbody>
</table>
Mathematics Standards of Learning

Curriculum Framework 2009-2016

Grade 5

Board of Education
Commonwealth of Virginia
The 2009-2016 Mathematics Standards of Learning Curriculum Framework, is a companion document to the 2009-2016 Mathematics Standards of Learning, and amplifies the Mathematics Standards of Learning by and further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. It provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

**Essential Understandings**

This section delineates the key concepts, ideas, and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

**Understanding the Standard**

This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. Background information for the teacher (K-8). It contains content that may extend the teachers’ knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

**Essential Knowledge and Skills**

Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate in each standard is outlined. This is not meant to be an exhaustive list of student expectations or a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance students’ problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
Mathematics instruction in grades 4 and 3-5 should continue to foster the development of number sense, especially with greater emphasis on decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction in grades 4 and 3-5 allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names), and making connections among mathematical concepts as well as to other content areas. Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g., $\frac{1}{2}$ or 0.5). Students should apply their knowledge of number and number sense to investigate and solve a variety of problem types.
# STANDARD 5.1  
## STRAND: NUMBER AND NUMBER SENSE  
### GRADE LEVEL 5

**5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.**

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The structure of the base-10 number system is based upon a simple pattern of tens in which each place is ten times the value of the place to its right. This is known as a ten-to-one place value relationship. To investigate this relationship, use base-10 proportional manipulatives, such as place value mats/charts, decimal squares, base-10 blocks, meter sticks, as well as the ten-to-one non-proportional model and money, to investigate this relationship.</td>
<td>• All students should understand that decimals are rounded in a way that is similar to the way whole numbers are rounded.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to round decimal numbers to the nearest whole number, tenth, or hundredth.</td>
</tr>
<tr>
<td>• A decimal point separates the whole number places from the places less than one. Place values extend infinitely in two directions from a decimal point. A number containing a decimal point is called a <em>decimal number</em> or simply a <em>decimal</em>.</td>
<td>• Understand that decimal numbers can be rounded to estimate when exact numbers are not needed for the situation at hand.</td>
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<tr>
<td>• To read decimals,</td>
<td></td>
<td></td>
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<tr>
<td>– read the whole number to the left of the decimal point, if there is one;</td>
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<td></td>
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<tr>
<td>– read the decimal point as “and”;</td>
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<tr>
<td>– read the digits to the right of the decimal point just as you would read a whole number; and</td>
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<tr>
<td>– say the name of the place value of the digit in the smallest place.</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>– say the name of the place value of the digit in the smallest place.</td>
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<tr>
<td>• Any decimal less than 1 will include a leading zero (e.g., 0.125). This number may be read as “zero and one hundred twenty-five thousandths” or as “one hundred twenty-five thousandths.”</td>
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<tr>
<td>• Decimals may be written in a variety of forms:</td>
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<tr>
<td>– Standard: 23.456</td>
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<tr>
<td>– Written: Twenty-three and four hundred fifty-six thousandths</td>
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<td></td>
</tr>
<tr>
<td>– Expanded: ((2 \times 10) + (3 \times 1) + (4 \times 0.1))</td>
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</tbody>
</table>
5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.

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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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</thead>
<tbody>
<tr>
<td>(5 \times 0.01) \div (6 \times 0.001)</td>
<td></td>
<td></td>
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<tr>
<td>• To help students identify the ten-to-one place value relationship for decimals through thousandths, use Base-10 manipulatives, such as place value mats/charts, decimal squares, Base-10 blocks, and money. Decimals can be rounded to the nearest whole number, tenth, or hundredth, in situations when exact numbers are not needed. Strategies for rounding whole numbers can be applied to rounding decimals.</td>
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<tr>
<td>• Number lines are tools that can be used in developing a conceptual understanding of rounding decimals. One strategy includes creating a number line that shows the decimal that is to be rounded. Locate it on the number line. Next, determine the closest multiples of whole numbers, tenths, or hundredths, it is between. Then identify to which it is closer.</td>
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<tr>
<td>• Strategies for rounding decimal numbers to the nearest whole number, tenth and hundredth are as follows:</td>
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<tr>
<td>- Look one place to the right of the digit to which you wish to round.</td>
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<tr>
<td>- If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero.</td>
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<tr>
<td>- If the digit is 5 or greater, add 1 to the digit in the rounding place and change the digits to the right of the rounding place to zero.</td>
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<td></td>
</tr>
<tr>
<td>• Create a number line that shows the decimal that is to be rounded.</td>
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<tr>
<td>• The position of the decimal will help children conceptualize the number's placement relative for rounding. An example is to round 5.747 to the nearest hundredth:</td>
<td></td>
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</tr>
</tbody>
</table>

![Number Line Example](image-url)
5.2 The student will

a) recognize and name, represent and identify equivalencies among fractions and in their equivalent decimals, with and without models; form and vice versa* and

b) compare and order fractions, mixed numbers, and/or decimals, in a given set, from least to greatest and greatest to least.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

<table>
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</table>

- Students should recognize, name, and focus on finding determinative equivalent decimals of familiar fractions with denominators that are factors of 100 making connections to tenths and hundredths. (e.g., \( \frac{2}{5} = 4/10 \) or 0.4) and (e.g., \( \frac{7}{20} = 35/100 \) or 0.35), such as halves, fourths, fifths, eighths, and tenths.
- Students should be able to determine equivalent relationships between decimals and fractions with denominators up to 12.
- Students should have experience with fractions such as \( \frac{1}{8} \) whose decimal representation is a terminating decimal (e.g., \( \frac{1}{8} = 0.125 \)) and with fractions such as \( \frac{3}{9} \), whose decimal representation does not end but continues to repeat (e.g., \( \frac{2}{3} = 0.222666… \)). The repeating decimal can be written with ellipses (three dots) as in 0.222666… or denoted with a bar above the digits that repeat as in \( 0.\overline{2} \).
- To help students compare the value of two decimals through thousandths, use manipulatives, such as place value mats/charts, 10-by-10 grids, decimal squares, base-10 blocks, meter sticks, number lines, and money.
- Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction can be expressed as a mixed number. A mixed number is written with

<table>
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<tbody>
<tr>
<td>All students should</td>
</tr>
<tr>
<td>- Understand the relationship between fractions and their decimal form and vice versa.</td>
</tr>
<tr>
<td>- Understand that fractions and decimals can be compared and ordered from least to greatest and greatest to least.</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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</thead>
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<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>- Represent fractions with denominators that are thirds, eighths, and factors of 100 in their equivalent decimal form with concrete or pictorial models. (a)</td>
</tr>
<tr>
<td>- (halves, fourths, fifths, eighths, tenths, and twelfths) in their equivalent decimal form and vice versa.</td>
</tr>
<tr>
<td>- Represent decimals in their equivalent fraction form (thirds, eighths, and factors of 100) with concrete or pictorial models. (a)</td>
</tr>
<tr>
<td>- Recognize and name, identify equivalent relationships between decimals and fractions with denominators up to 12 that are thirds, eighths, and factors of 100 in their equivalent decimal form without models. (a)</td>
</tr>
<tr>
<td>- Compare and order from least to greatest and greatest to least a given set of no more than four numbers written as decimals, fractions (proper or improper), and/or mixed numbers with denominators of 12 or less. (b)</td>
</tr>
<tr>
<td>- Use the symbols ( &gt;, &lt;, =, ) and ( \neq ) to compare decimals through thousandths, fractions (proper or improper fractions), improper fractions, and/or mixed numbers, having denominators of 12 or less. (b)</td>
</tr>
</tbody>
</table>
5.2 The student will

a) recognize and name, represent and identify equivalencies among fractions and in their equivalent decimals, with and without models; form and vice versa* and

b) compare and order fractions, mixed numbers, and/or decimals, in a given set, from least to greatest and greatest to least.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<tr>
<td>two parts: a whole number and a proper fraction (e.g., $3 \frac{5}{8}$).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A proper fraction is a fraction whose numerator is less than the denominator. A proper fraction is a fraction that is always less than one.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>An improper fraction is a fraction whose numerator is greater than or equal to the denominator. An improper fraction is a fraction that is equal to or greater than one.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>An improper fraction can be expressed as a mixed number. A mixed number is written as a whole number and a proper fraction.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| A procedure for comparing two decimals by examining may include the following:  
  - Line up the decimal numbers at their decimal points.  
  - Beginning at the left, find the first place value where the digits are different.  
  - Compare the digits in this place value to determine which number is greater (or which is less).  
  - Use the appropriate symbol > or < or the words greater than or less than to compare the numbers in the order in which they are presented.  
  - If both numbers are the same, use the symbol = or words equal to.  
  Two numbers can be compared by examining place value and/or using a number line. | | |
| Decimals and fractions represent the same relationships; however, they are presented in two different formats. Decimal numbers are another way of writing fractions. An amount less than one whole can be represented by a fraction or by an equivalent decimal. | | |
5.2 The student will
a) recognize and name, represent and identify equivalencies among fractions and in their equivalent decimals, with and without models; form and vice versa;* and
b) compare and order fractions, mixed numbers, and/or decimals, in a given set, from least to greatest and greatest to least.*

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<tbody>
<tr>
<td>• Base-10 models (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, money) concretely relate fractions to decimals and vice versa; demonstrate the relationship between fractions and decimals.</td>
<td></td>
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</tbody>
</table>
STANDARD 5.3 STRAND: NUMBER AND NUMBER SENSE GRADE LEVEL 5

5.3 The student will
a) identify and describe the characteristics of prime and composite numbers; and
b) identify and describe the characteristics of even and odd numbers.

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>Natural numbers are the counting numbers starting at 1.</td>
<td>All students should understand and use the unique characteristics of certain sets of numbers, including prime, composite, even, and odd numbers.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>A prime number is a natural number, other than 1, that has exactly two different factors, 1 and the number itself.</td>
<td></td>
<td>identify prime numbers less than or equal to 100. (a)</td>
</tr>
<tr>
<td>A composite number is a natural number that has more than two different factors other than one and itself.</td>
<td></td>
<td>Identify composite numbers less than or equal to 100. (a)</td>
</tr>
<tr>
<td>The number 1 is neither prime nor composite because it has only one set of factors and both factors are 1 itself.</td>
<td></td>
<td>Demonstrate with manipulatives concrete or pictorial representations and explain orally or in writing why a number is prime or composite. (a)</td>
</tr>
<tr>
<td>The prime factorization of a number is a representation of the number as the product of its prime factors. For example, the prime factorization of 18 is $2 \times 3 \times 3$.</td>
<td></td>
<td>Identify which numbers are even or odd. (b)</td>
</tr>
<tr>
<td>Prime factorization concepts can be developed by using factor trees.</td>
<td></td>
<td>Explain and demonstrate with manipulatives concrete or pictorial representations and explain orally or in writing why a number is even or odd. (b)</td>
</tr>
<tr>
<td>Prime or composite numbers can be represented by rectangular models or rectangular arrays on grid paper. A prime number can be represented by only one rectangular array (e.g., 7 can be represented by a $7 \times 1$ and a $1 \times 7$). A composite number can always be represented by two or more rectangular arrays (e.g., 9 can be represented by a $9 \times 1$, a $1 \times 9$, or a $3 \times 3$).</td>
<td></td>
<td>Demonstrate with concrete manipulatives or pictorial representations and explain orally or in writing why the sum or difference of two numbers is even or odd. (b)</td>
</tr>
<tr>
<td>Divisibility rules are useful tools in identifying prime and composite numbers.</td>
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</tr>
<tr>
<td>Odd and even numbers can be explored in different ways (e.g., dividing collections of objects into two equal groups or pairing objects). When pairing objects, the number of objects is even when each object has a pair or partner. When an object is left over, or does not have a pair, then the number is odd.</td>
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<tr>
<td>Students should use manipulatives (e.g., base-10 blocks, cubes, tiles, hundreds board, etc.) to explore and categorize numbers into groups of odd or even.</td>
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<tr>
<td>Examples of ways to use manipulatives to show even and odd...</td>
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</table>
STANDARD 5.3  

STRAND: NUMBER AND NUMBER SENSE  
GRADE LEVEL 5

5.3  
The student will
a) identify and describe the characteristics of prime and composite numbers; and
b) identify and describe the characteristics of even and odd numbers.

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</table>

- Numbers may include (but are not limited to):
  - For an even number (e.g., 12), six pairs of counters can be formed with no remainder, or two groups of six counters can be formed with no remainder; and
  - For an odd number, such as 13: (a) six pairs of counters can be formed with one counter remaining, or (b) two groups of six counters can be formed with one counter remaining.

- Students should use rules to categorize numbers into groups of odd or even. Rules can include:
  - An odd number does not have 2 as a factor and is not divisible by 2.
  - The sum of two even numbers is even.
  - The sum of two odd numbers is even.
  - The sum of an even number and an odd number is odd.
  - Even numbers have an even number or zero in the ones place.
  - Odd numbers have an odd number in the ones place.
  - An even number has 2 as a factor and is divisible by 2.
  - The product of two even numbers is even.
  - The product of two odd numbers is odd.
  - The product of an even number and an odd number is even.
Computation and estimation in grades 4 and 5 should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students’ understanding of these operations to work with decimals. Instruction should focus on computation activities that enable students to model, explain, and develop proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract with common fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades. Multiplication and division with decimals will be explored in grade 5.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability not only to identify but to use relationships between operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use (not identify) properties of operations to solve problems [e.g., \(7 \times 28\) is equivalent to \((7 \times 20) + (7 \times 8)\)].

Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals, using benchmarks (e.g., \(\frac{2}{5} + \frac{1}{3}\) must be less than 1 because both fractions are less than \(\frac{1}{2}\)). Using estimation, students should develop strategies to recognize the reasonableness of their computational solutions.

Additionally, students should enhance their ability to select an appropriate problem solving method from among estimation, mental mathematics, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.
5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>- An example of an approach to solving problems is Polya’s four-step-plan:</td>
<td>Understand the meaning of mathematical operations and how these operations relate to one another when creating and solving single-step and multistep word problems.</td>
<td>- Select appropriate methods and tools from among paper and pencil, estimation, mental computation, and calculators according to the context and nature of the computation in order to compute with whole numbers.</td>
</tr>
<tr>
<td>- Understand: Retell the problem; read it twice; take notes; study the charts or diagrams; look up words and symbols that are new.</td>
<td>- The problem-solving process is enhanced when students create and solve their own practical problems and model problems using manipulatives and drawings.</td>
<td>- Create single-step and multistep problems involving the operations of addition, subtraction, multiplication, and division with and without remainders of whole numbers, using practical situations.</td>
</tr>
<tr>
<td>- Plan: Decide what operation(s) to use and what sequence of steps to use to solve the problem.</td>
<td>- In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as <em>in all, altogether, difference, etc.</em> encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions as well as challenges in upcoming grades and courses.</td>
<td>- Estimate the sum, difference, product, and quotient of whole number computations.</td>
</tr>
<tr>
<td>- Solve: Follow the plan and work accurately. If the first attempt doesn’t work, try another plan.</td>
<td>- Estimation gives a rough idea of an amount. Strategies such as front-end, rounding, and mental computation may be used to estimate sums, differences, products, and quotients. Addition, subtraction, multiplication, and division of whole numbers.</td>
<td>- Apply strategies, including place value and application of the properties of addition and multiplication to solve single-step and multistep problems involving addition, subtraction, multiplication, and division, with and without remainders, of whole numbers, using paper and pencil, mental computation, and calculators, in which:</td>
</tr>
<tr>
<td>- Look back: Does the answer make sense?</td>
<td>- Examples of problems to be solved by using estimation strategies are encountered in shopping for groceries, buying school supplies, budgeting allowance, and sharing the cost of a pizza or the prize money from a contest.</td>
<td>- sums, differences, and products will do not exceed five digits;</td>
</tr>
<tr>
<td>- The problem-solving process is enhanced when students create and solve their own practical problems and model problems using manipulatives and drawings.</td>
<td>- Estimation can be used to determine a reasonable range for the answer to computation and to verify the reasonableness of sums, differences, products, and quotients of whole numbers.</td>
<td>- multipliers will factors do not exceed two digits by three digits [Moved from 4.4]</td>
</tr>
<tr>
<td>- In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as <em>in all, altogether, difference, etc.</em> encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions as well as challenges in upcoming grades and courses.</td>
<td>- Examples of problems to be solved by using estimation strategies are encountered in shopping for groceries, buying school supplies, budgeting allowance, and sharing the cost of a pizza or the prize money from a contest.</td>
<td>- divisors will do not exceed two digits; or</td>
</tr>
<tr>
<td>- Estimation gives a rough idea of an amount. Strategies such as front-end, rounding, and mental computation may be used to estimate sums, differences, products, and quotients. Addition, subtraction, multiplication, and division of whole numbers.</td>
<td>- Estimation can be used to determine a reasonable range for the answer to computation and to verify the reasonableness of sums, differences, products, and quotients of whole numbers.</td>
<td>- dividends will do not exceed four digits.</td>
</tr>
<tr>
<td>- Examples of problems to be solved by using estimation strategies are encountered in shopping for groceries, buying school supplies, budgeting allowance, and sharing the cost of a pizza or the prize money from a contest.</td>
<td>- Use the context of a practical problem to interpret the quotient and remainder.</td>
<td>- Use two or more operational steps to solve a multistep problem. Operations can be the same or different. [Moved to US]</td>
</tr>
</tbody>
</table>
5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.

<table>
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<tbody>
<tr>
<td>• The least number of steps necessary to solve a single-step problem is one.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• A multistep problem incorporates two or more operational steps (operations can be the same or different). [Moved from EKS]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Extensive research has been undertaken over the last several decades regarding different problem types. Many of these studies have been published in professional mathematics education publications using different labels and terminology to describe the varied problem types.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Students should experience a variety of problem types related to multiplication and division. Some examples are included in the following chart:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.

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<tbody>
<tr>
<td><strong>GRADE 5: COMMON MULTIPLICATION AND DIVISION PROBLEM TYPES</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal Group Problems</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whole Unknown (Multiplication)</td>
<td>Size of Groups Unknown (Partitive Division)</td>
<td>Number of Groups Unknown (Measurement Division)</td>
</tr>
<tr>
<td><strong>Multiplicative Comparison Problems</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Result Unknown</td>
<td>Start Unknown</td>
<td>Comparison Factor Unknown</td>
</tr>
<tr>
<td>Tyrone traveled 125 miles last month. Jasmine traveled 15 times as many miles as Tyrone during the same month. How many miles did Jasmine travel?</td>
<td>Jasmine traveled 1,995 miles last summer. She traveled 12 times as many miles as Tyrone during the same summer. How many miles did Tyrone travel?</td>
<td>Jasmine traveled 1,275 miles in December. Tyrone traveled 85 miles in December. How many times more miles did Jasmine travel than Tyrone?</td>
</tr>
<tr>
<td><strong>Array or Area Problems</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whole Unknown</td>
<td>One Dimension Unknown</td>
<td></td>
</tr>
<tr>
<td>There are 28 sections of parking lot at the stadium. There are 115 cars parked in each section of the parking lot at the stadium. How many cars are parked at the stadium all together?</td>
<td>There are 3,220 cars parked at the stadium. The cars are divided evenly among each of the 28 sections of parking lot. How many cars are parked in each section?</td>
<td></td>
</tr>
<tr>
<td>Mr. Myers’ barn measures 35 feet by 110 feet. How many square feet are in the barn?</td>
<td>There are exactly 115 cars parked in each section. How many sections are filled with cars?</td>
<td></td>
</tr>
<tr>
<td>Mr. Myers’ barn covers 3,850 square feet. The width of the barn is 35 feet. What is the length of the barn?</td>
<td>Mr. Myers’ barn covers 3,850 square feet. The width of the barn is 35 feet. What is the length of the barn?</td>
<td></td>
</tr>
<tr>
<td><strong>COMBINATION PROBLEMS</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outcomes Unknown</td>
<td>Factor Unknown</td>
<td></td>
</tr>
<tr>
<td>An experiment involves tossing a coin and rolling a die. How many different outcomes are possible?</td>
<td>Mike bought some new shorts and shirts that can all be worn together. He has a total of 12 different outfits. If he bought 3 pairs of pants, how many shirts did he buy?</td>
<td></td>
</tr>
<tr>
<td>Kelly has 2 pairs of pants and 3 shirts that can all be worn together. How many different outfits consisting of a pair of pants and a shirt does she have?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

### ESSENTIAL UNDERSTANDINGS

- Students also need exposure to various types of practical problems in which they must interpret the quotient and remainder based on the context. The chart below includes one example of each type of problem.

<table>
<thead>
<tr>
<th>MAKING SENSE OF THE REMAINDER IN DIVISION</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYPE OF PROBLEM</td>
</tr>
<tr>
<td>Remainder is not needed and can be left over (or discarded).</td>
</tr>
<tr>
<td>Remainder is partitioned and represented as a fraction or decimal.</td>
</tr>
<tr>
<td>Remainder forces the answer to be increased to the next whole number.</td>
</tr>
<tr>
<td>Remainder forces the answer to be rounded (giving an approximate answer).</td>
</tr>
</tbody>
</table>

- Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers. [Moved from 3.20]

- Grade 5 students should explore and apply the properties of addition and multiplication as strategies for solving addition, subtraction, multiplication, and division problems using a variety of representations (e.g., manipulatives, diagrams, and symbols).

- The properties of the operations are “rules” about how numbers work and how they relate to one another. Students at this level do not need to use the formal terms for these properties but should utilize these properties to further develop flexibility and fluency.
5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.

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<tbody>
<tr>
<td>in solving problems. The following properties are most appropriate for exploration at this level:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., (4 + 3 = 3 + 4)). Similarly, the commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., (2 \times 3 = 3 \times 2)). [Moved from 3.20]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– The identity property of addition states that if zero is added to a given number, the sum is the same as the given number. The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number. [Moved from 3.20]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– The associative property of addition states that the sum stays the same when the grouping of addends is changed (e.g., (15 + (35 + 16) = (15 + 35) + 16)).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– The associative property of multiplication states that the product stays the same when the grouping of factors is changed (e.g., (6 \times (3 \times 5) = (6 \times 3) \times 5)). [Moved from 4.16]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– The distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products. [Moved from 5.20]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– Examples of the distributive property include:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(- 3(9) = 3(5 + 4))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(- 3(54 + 4) = 3 \times 54 + 3 \times 4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(- 5 \times (3 + 7) = (5 \times 3) + (5 \times 7))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(- (2 \times 3) + (2 \times 5) = 2 \times (3 + 5))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(- 9 \times 23 = 9(20+3))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(- 180 + 27 = 207)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.

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<td></td>
<td></td>
</tr>
</tbody>
</table>

- **34 × 8**
  - 30
  - 4
  - $8 \times 30 = \frac{240}{32}$

- **23 × 12**
  - $(20 + 3) \times (10 + 2)$
  - $(20 \times 10) + (20 \times 2) + (3 \times 10) + (3 \times 2)$
  - $200 + 40 + 30 + 6$
  - $276$
5.5  

The student will  
b) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals, and single-step practical problems involving division of decimals.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

<table>
<thead>
<tr>
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</tr>
</thead>
</table>
| • Addition and subtraction of decimals may be investigated using a variety of models (e.g., 10-by-10 grids, number lines, money).  
• Decimal computation uses similar procedures as those developed for whole number computation, applying and applying them to decimal place value computation, giving careful attention to the placement of the decimal point in the solution.  
• Multiplication of decimals follows the same procedure as multiplication of whole numbers. The only difference is that a decimal point must be correctly placed in the product giving careful attention to the placement of the decimal point in the solution.  
• The product of decimals is dependent upon the two factors being multiplied.  
• In cases where an exact product is not required, the product of decimals can be estimated using strategies for multiplying whole numbers, such as front-end and compatible numbers, or rounding. In each case, the student needs to determine where to place the decimal point to ensure that the product is reasonable.  
• Estimation keeps the focus on the meaning of the numbers and operations, encourages reflective thinking, and helps build informal number sense with decimals. Students can reason with benchmarks to get an estimate without using an algorithm.  
• Estimation can be used to determine a reasonable range for the answer to computation and to verify the reasonableness of sums, differences, products, and quotients of decimals. | All students should:  
• Use similar procedures as those developed for whole number computation and apply them to decimal place values, giving careful attention to the placement of the decimal point in the solution.  
• Select appropriate methods and tools from among paper and pencil, estimation, mental computation, and calculators according to the context and nature of the computation in order to compute with decimal numbers.  
• Understand the various meanings of division and its effect on whole numbers.  
• Understand various representations of division, i.e.,  
  - dividend ÷ divisor = quotient  
  - dividend ÷ divisor = quotient [Moved to]  
  - quotient = dividend ÷ divisor  
  - quotient = dividend ÷ ÷ divisor  
• Use estimation to check the reasonableness of a product and quotient.  
• Use multiple representations to model multiplication and division of decimals and whole numbers. (a) | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to  
• Estimate and determine the product of two decimal numbers in which:  
  - the multipliers do not exceed two digits by two digits (e.g., 2.3 × 4.5, 0.08 × 0.9, 0.85 × 2.3, 1.8 ÷ 5); and  
  - the products do not exceed the thousandths place. (Leading zeroes will not be considered when counting digits.)  
    (a)  
• Estimate and determine the quotient of two numbers in which:  
  - quotients do not exceed 4 digits with or without a decimal point;  
  - quotients may include whole numbers, tenths, hundredths, or thousandths;  
  - divisors are limited to a single digit whole number or a decimal expressed as tenths; and  
  - no more than one additional zero will need to be annexed. (a)  
• Use multiple representations to model multiplication and division of decimals and whole numbers. (a) |
5.5 The student will

a) find the sum, difference, estimate and determine the product, and quotient of two numbers expressed as involving decimals through thousandths (divisors with only one nonzero digit);* and [Addition and subtraction with decimals included in 4.6)

b) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals, and single-step practical problems involving division of decimals.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>US</td>
<td></td>
</tr>
<tr>
<td>• Division is the operation of making equal groups or shares. When the original amount and the number of shares are known, divide to find determine the size of each share. When the original amount and the size of each share are known, divide to find determining the number of shares. Both situations may be modeled with base-10 manipulatives.</td>
<td></td>
<td>• Create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals, and single-step practical problems involving division of decimals. (b)</td>
</tr>
<tr>
<td>• The fair-share concept of decimal division can be modeled, using manipulatives (e.g., base-10 blocks). Multiplication and division of decimals can be represented with arrays, paper folding, repeated addition, repeated subtraction, base-10 models, and area models.</td>
<td></td>
<td>- Determine an appropriate method of calculation to find the sum, difference, product, and quotient of two numbers expressed as decimals through thousandths, selecting from among paper and pencil, estimation, mental computation, and calculators.</td>
</tr>
<tr>
<td>• Students in grade 4 studied decimals through thousandths and solved practical problems that involved addition and subtraction of decimals. Consideration should be given to creating division problems with decimals that do not exceed quotients in the thousandths. Teachers may desire to work backwards in creating appropriate decimal division problems meeting the parameters for grade 5 students.</td>
<td></td>
<td>- Estimate to find the number that is closest to the sum, difference, and product of two numbers expressed as decimals through thousandths.</td>
</tr>
<tr>
<td>• Examples of appropriate decimal division problems for grade 5 students include, but are not limited to:</td>
<td></td>
<td>- Find the sum, difference, and product of two numbers expressed as decimals through thousandths, using paper and pencil, estimation, mental computation, and calculators.</td>
</tr>
<tr>
<td>- 2.38 ÷ 4; 6 ÷ 0.2; 1.78 ÷ 0.5; etc.</td>
<td></td>
<td>- Determine the quotient, given a dividend expressed as a decimal through thousandths and a single-digit divisor. For example, 5.4 divided by 2 and 2.4 divided by 5.</td>
</tr>
<tr>
<td>- A scientist collected 3 water samples from local streams. Each sample was the same size, and she collected 1.35 liters of water in all. What was the volume of each water sample?</td>
<td></td>
<td>- Use estimation to check the reasonableness of a sum, difference, product, and quotient. [Reordered]</td>
</tr>
<tr>
<td>- There is exactly 12 liters of sports drink available to the tennis team. If each tennis player will be served 0.5 liters, how many players can be served?</td>
<td></td>
<td>- Create and solve single-step and multistep problems.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- A multistep problem needs to incorporate two or more operational steps (operations can be the same or different).</td>
</tr>
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</table>
The student will

a) find the sum, difference, estimate and determine the product, and quotient of two numbers expressed as involving decimals through thousandths (divisors with only one nonzero digit);* and Addition and subtraction with decimals included in 4.6

b) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals, and single-step practical problems involving division of decimals.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

**UNDERSTANDING THE STANDARD**

**ESSENTIAL UNDERSTANDINGS**

- The relay team race is exactly 4.8 miles long. Each person on the team is expected to run 0.8 miles. How many team members will be needed to cover the total distance?

- Division with decimals is performed the same way as division of whole numbers. The only difference is the placement of the decimal point in the quotient.

- When solving division problems, numbers may need to be expressed as equivalent decimals by annexing zeros. Annexing zeros is sometimes necessary when dividing decimal numbers. This occurs when a zero must be added in the dividend as a place holder.

- The quotient can be estimated, given a dividend expressed as a decimal through thousandths (and no adding of zeros to the dividend during the division process) and a single-digit divisor.

- Estimation can be used to check the reasonableness of a quotient.

- Division is the inverse of multiplication; therefore, multiplication and division are inverse operations.

- Terms used in division are dividend, divisor, and quotient.

  \[
  \text{dividend} \div \text{divisor} = \text{quotient}
  \]

  \[
  \underbrace{\text{divisor}}_{\text{dividend}} \quad \overline{\text{quotient}}
  \]

  \[
  \begin{align*}
  \text{dividend} \\
  \text{divisor} = \text{quotient} \end{align*}
  
  \[
  \text{Moved from US}
  \]

- There are a variety of algorithms for division such as repeated
5.5 The student will
a) find the sum, difference, estimate and determine the product, and quotient of two numbers expressed as involving decimals; decimals through thousandths (divisors with only one nonzero digit);* and [Addition and subtraction with decimals included in 4.6]

b) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals, and single-step practical problems involving division of decimals.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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| multiplication and subtraction. Experience with these algorithms may enhance understanding of the traditional long division algorithm.  
- A multistep problem needs to incorporate no more than two operational steps (operations can be the same or different).  
- The least number of steps necessary to solve a single-step problem is one. |  |  |
STANDARD 5.6

STRAND: COMPUTATION AND ESTIMATION

GRADE LEVEL 5

5.6 The student will
   a) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers; and express answers in simplest form. [Included in EKS]
   a) solve single-step practical problems involving multiplication that involve modeling and determining the product of a whole number, limited to 12 or less, and a unit proper fraction, with models.*

   *On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<tbody>
<tr>
<td>A fraction can be expressed in simplest form (simplest equivalent fraction) by dividing the numerator and denominator by their greatest common factor.</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>When the numerator and denominator have no common factors other than 1, then the fraction is in simplest form.</td>
<td>Develop and use strategies to estimate and compute addition and subtraction of fractions.</td>
<td>Solve single-step and multistep practical problems involving addition and subtraction with proper or improper fractions having like and unlike denominators and or mixed numbers. Denominators in the problems should be limited to 12 or less, and answers should be expressed in simplest form. (a)</td>
</tr>
<tr>
<td>Fractions having like denominators means the same as fractions having common denominators.</td>
<td>Understand the concept of least common multiple and least common denominator as they are important when adding and subtracting fractions.</td>
<td>Solve single-step and multistep practical problems involving addition and subtraction with mixed numbers having like and unlike denominators, with and without regrouping. Denominators in the problems should be limited to 12 or less, and answers should be expressed in simplest form. (a) [Mixed numbers included in EKS above]</td>
</tr>
<tr>
<td>Equivalent fractions name the same amount. To find equivalent fractions, multiply or divide the numerator and denominator by the same nonzero number.</td>
<td>Understand that a fraction is in simplest form when its numerator and denominator have no common factors other than 1. The numerator can be greater than the denominator.</td>
<td>Use estimation to check the reasonableness of a sum or difference. (a) [Included in EKS]</td>
</tr>
<tr>
<td>Addition and subtraction with fractions and mixed numbers can be modeled using a variety of concrete material and pictorial representations as well as paper and pencil.</td>
<td></td>
<td>Solve single-step and multistep practical problems that include modeling and determining the product involving multiplication of a whole number and a unit fraction (e.g., 9 ( \times \frac{1}{2}, 12 \div \frac{1}{4} )). The denominator will be a factor of the whole number, the whole number will not exceed 60, limited to 12 or less, and a proper fraction (e.g., 6 ( \times \frac{1}{2}, \frac{5}{8}, \frac{9}{2} )), with models. The denominator will be a factor of the whole number, and answers should be expressed in simplest form. (b)</td>
</tr>
<tr>
<td>Estimation keeps the focus on the meaning of the numbers and operations, encourages reflective thinking, and helps build informal number sense with fractions. Students can reason with benchmarks to get an estimate without using an algorithm. Estimation can be used to check the reasonableness of an answer.</td>
<td></td>
<td>Apply the inverse property of multiplication in models.</td>
</tr>
<tr>
<td>To add, subtract, and compare fractions and mixed numbers, it often helps to find the least common denominator. The least common denominator (LCD) of two or more fractions is the least common multiple (LCM) of the denominators.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>To add or subtract with fractions having the same or like denominators, add or subtract the numerators and write in simplest form.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>To add or subtract with fractions that do not have the same denominator.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.6  The student will

a) **solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers;**
and express answers in simplest form. [Included in EKS]

b) **solve single-step practical problems involving multiplication that involve modeling and determining the product of a whole number, limited to 12 or less, and a unit proper fraction, with models.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
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<tbody>
<tr>
<td>denominator, first find equivalent fractions with the least common denominator. Then add or subtract and write the answer in simplest form.</td>
<td>(For example, use a visual fraction model to represent ( \frac{4}{4} ) as the product of ( 4 \times \frac{1}{4} ). (b)</td>
</tr>
<tr>
<td>A mixed number has two parts: a whole number and a fraction. The value of a mixed number is the sum of its two parts.</td>
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</tr>
<tr>
<td>To add or subtract with mixed numbers, students may use a number line, draw a picture, rewrite fractions with like denominators, or rewrite mixed numbers as fractions.</td>
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</tr>
<tr>
<td>A unit fraction is a fraction in which the numerator is one.</td>
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</tr>
<tr>
<td>Models for representing multiplication of fractions may include arrays, paper folding, repeated addition, fraction strips or rods, pattern blocks, or area models.</td>
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</tr>
<tr>
<td>Students should begin with exploring multiplication with fractions by solving problems that involve a whole number and a unit fraction.</td>
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</tr>
<tr>
<td>When multiplying a whole number by a fraction such as ( 6 \cdot \frac{1}{2} ), the meaning is the same as with multiplication of whole numbers: ( 6 ) groups the size of ( \frac{1}{2} ) of the whole.</td>
<td></td>
</tr>
</tbody>
</table>
| \[
\begin{array}{cccccc}
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\end{array}
\]
\( 6 \times \frac{1}{2} = \frac{6}{2} = 3 \) |
| When multiplying a fraction by a whole number such as \( \frac{1}{2} \cdot 6 \), we are trying to determine a part of the whole (e.g., one-half of six). | |
5.6 The student will
   a) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers; and express answers in simplest form. [Included in EKS]
   a) b) solve single-step practical problems involving multiplication that involve modeling and determining the product of a whole number, limited to 12 or less, and a unit proper fraction, with models.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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- The inverse property of multiplication states that the product of a number and its multiplicative inverse always equals one. The inverse property of multiplication states that every number has a multiplicative inverse and the product of multiplicative inverses is 1 (e.g., 5 and $\frac{1}{5}$ are multiplicative inverses because $5 \times \frac{1}{5} = 1$). The multiplicative inverse of a given number can be called the reciprocal of the number. Students at this level do not need to use the term for the properties of the operations.

- Multiplying a whole number by a unit fraction can be related to dividing the whole number by the denominator of the fraction. For example, $\frac{1}{2}$ of 6 is equivalent to 2. This understanding forms a foundation for learning how to multiply a whole number by a proper fraction.

- At this level, students will use models to solve problems that involve multiplication of a whole number, limited to 12 or less, and a unit proper fraction where the denominator is a factor of the
## STANDARD 5.6

**STANDAD 5.6 STRAND: COMPUTATION AND ESTIMATION GRADE LEVEL 5**

5.6 The student will

a) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers; and express answers in simplest form. [Included in EKS]

a)b) solve single-step practical problems involving multiplication that involve modeling and determining the product of a whole number, limited to 12 or less, and a unit proper fraction, with models.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- **Examples of problems students in grade 5 should be able to solve include, but are not limited to, the following:** For example, a model for \( \frac{3}{4} \times 8 \) or \( 8 \times \frac{3}{4} \) shows that the answer is three groups of \( \frac{1}{4} \times 8 \).

\[
\frac{128}{8} \quad \text{or} \quad \frac{1}{8} \times 8
\]

- **Examples of problems grade 5 students should be able to solve include, but are not limited to the following:**
  - If nine children each bring \( \frac{1}{3} \) of a cup of candy for the party, how many thirds will there be? What will be the total number of cups of candy?
  - If it takes \( \frac{3}{4} \) cup of ice cream to fill an ice cream cone, how much ice cream will be needed to fill eight cones?

- **Answers** Resulting fractions should be expressed in simplest form.

- Problems where the denominator is not a factor of the whole number (e.g., \( \frac{1}{8} \times 6 \) or \( 6 \times \frac{1}{8} \)) will be a focus in grade 6.

- The fraction \( \frac{2}{3} \) may be interpreted as the amount of cake each person gets when 3 cakes are divided equally among 4 people.

- The least number of steps necessary to solve a single-step problem is one.

### ESSENTIAL UNDERSTANDINGS

### ESSENTIAL KNOWLEDGE AND SKILLS
STANDARD 5.7  STRAND: COMPUTATION AND ESTIMATION  GRADE LEVEL 5

5.7 The student will simplify evaluate whole number numerical expressions, using the order of operations limited to parentheses, addition, subtraction, multiplication, and division.* [Moved to EKS]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<tr>
<td>• An expression is a representation of a quantity. It is made up of numbers, variables, and/or computational symbols, and grouping symbols. It does not have an equal symbol (e.g., $15 \times 12$).</td>
<td>All students should understand that the order of operations describes the order to use to simplify expressions containing more than one operation.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to use the order of operations to simplify whole number numerical expressions, by using the order of operations in a demonstrated step-by-step approach limited to addition, subtraction, multiplication, and division. Expressions may contain parentheses.</td>
</tr>
<tr>
<td>• An expression, like a phrase, has no equal sign.</td>
<td></td>
<td>• Find the value of numerical expressions, using the order of operations.</td>
</tr>
<tr>
<td>• Expressions containing more than one operation are simplified by using the order of operations.</td>
<td></td>
<td>• Given a whole number numerical expression involving more than one operation, describe which operation is completed first, which is second, etc.</td>
</tr>
<tr>
<td>• The order of operations is a convention that defines the computation order to follow in simplifying an expression. It ensures that there is only one correct value.</td>
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<tr>
<td>• The order of operations is as follows:</td>
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<tr>
<td>— First, complete all operations within grouping symbols. If there are grouping symbols within other grouping symbols, do the innermost operation first. (Students in grade 5 are not expected to simplify expressions having parentheses within other grouping symbols.)</td>
<td></td>
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<tr>
<td>— If there are multiple operations within the parentheses, apply the order of operations.</td>
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<tr>
<td>— Second, evaluate all exponential expressions. (Students in grade 5 are not expected to simplify expressions with exponents.)</td>
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<tr>
<td>— Third, multiply and/or divide in order from left to right.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>— Fourth, add and/or subtract in order from left to right.</td>
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</table>
Students in grades 4 and 5 should be actively involved in measurement activities that require a dynamic interaction among students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects.

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, perimeter, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary), to measure length, weight/mass, liquid volume/capacity, area, perimeter, temperature, and time. Students continue to enhance their understanding of measurement by continuing to be enhanced through experiences using appropriate tools such as rulers, balances, clocks, and thermometers. The process of measuring is identical for any attribute (i.e., length, weight/mass, liquid volume/capacity, area): choose a unit, compare that unit to the object, and report the number of units.

[ Moved from Geometry Strand introduction ]

The study of geometry helps students represent and make sense of the world. In grades 3 through 5, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
• **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

• **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)
The student will

a) solve practical problems that involve finding perimeter, area, and volume in standard units of measure; and
b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation;

c) identify equivalent measurements within the metric system;[Moved to 5.9a]
d) estimate and then measure to solve problems, using U.S. Customary and metric units; and[U.S. Customary included in 4.8d; Metric moved to 5.9b]
e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units.[Moved to 5.9EKS]

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

• A plane figure is any closed, two-dimensional shape.
• Perimeter is the path or distance around any plane figure. It is a measure of length.
• Area is the surface included within a plane figure. Area is measured by the number of square units needed to cover a surface or plane figure.
• Volume of a three-dimensional figure is a measure of capacity and is measured in cubic units.
• A polygon is a closed plane figure composed of at least three line segments that do not cross.
• To find the perimeter of any polygon, add the lengths of the sides.
• Students should label the perimeter, area, and volume with the appropriate unit of linear, square, or cubic measure.
• Area is the number of square units needed to cover a surface or figure.
• A right triangle has one right angle.
• Students should explore area investigation, using manipulatives, to discover the formulas for the area of a square, rectangle, and right triangle, and volume of a rectangular solid. Area of a rectangle = Length × Width
• Area of a square = Side × Side

ESSENTIAL UNDERSTANDINGS

All students should
• Understand the concepts of perimeter, area, and volume.
• Understand and use appropriate units of measure for perimeter, area, and volume.
• Understand the difference between using perimeter, area, and volume in a given situation.
• Understand how to select a measuring device and unit of measure to solve problems involving measurement.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

• Solve practical problems that involve determining the perimeter, area, and volume in standard units of measure (a)
• Determine the perimeter of a polygon, with or without diagrams, when
  – the lengths of all sides of a polygon that is not a rectangle or a square are given;
  – the length and width of a rectangle are given; or
  – the length of a side of a square is given. (a)
• Estimate and determine the perimeter of a polygon, and area of a square and rectangle using only whole number measurements, given in metric or U.S. Customary units, and record the solution with the appropriate unit of measure (e.g., 24 square inches).
• Develop a procedure for determining the area of a right triangle following the parameters listed above, using only whole number measurements given in metric or U.S. Customary units, and record the solution with the appropriate unit of measure (e.g., 24.12 square inches). (a)
• Estimate and determine the area of a square, with or without diagrams, when the length of a side is given.[Moved to 4.7 EKS]
5.8 The student will
   a) solve practical problems that involve finding perimeter, area, and volume in standard units of measure; and
   b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation;
   c) identify equivalent measurements within the metric system; [Moved to 5.9a]
   d) estimate and then measure to solve problems, using U.S. Customary and metric units; and [U.S. Customary included in 4.8d; Metric moved to 5.9b]
   e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units. [Moved to 5.9EKS]

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Area of a right triangle = \( \frac{1}{2} \) base \( \times \) height
- Volume of a rectangular solid = length \( \times \) width \( \times \) height
  - Students would benefit from opportunities that include the use of benchmark fractions (e.g., \( \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \)) in determining perimeter.
  - The area of a rectangle can be determined by multiplying the length of the base by the length of the height.
  - The diagonal of the rectangle below has been divided, creating two right triangles. Adjoining sides of the rectangle form the legs of the right triangles and therefore are congruent to the side lengths of the rectangle. The representation illustrates that the area of each right triangle is half the area of the rectangle. Exploring decomposition of shapes helps students develop algorithms for determining area of various shapes (e.g., area of a triangle is \( \frac{1}{2} \) base \( \times \) height).

ESSENTIAL UNDERSTANDINGS

- Estimate and determine the area of a rectangle, with or without diagrams, when the length and width are given. [Moved to 4.7EKS]
- Estimate and determine the area of a right triangle, with or without diagrams, when the base and the height are given. (a)
- Differentiate among the concepts of area, perimeter, and volume.
- Develop a procedure for finding determining volume using manipulatives (e.g., cubes). (a)
- Estimate and determine the volume of a rectangular prism with diagrams, when the length, width, and height are given, using whole number measurements. Record the solution with the appropriate unit of measure (e.g., 12 cubic inches). (a)
- Describe practical situations where area, perimeter, area, and volume are appropriate measures to use, and justify their choices orally or in writing. (b)
- Identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation. (b)
- Identify equivalent measurements within the metric system for the following:
5.8 The student will
a) solve practical problems that involve finding perimeter, area, and volume in standard units of measure; and
b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation;
c) identify equivalent measurements within the metric system; [Moved to 5.9a]
d) estimate and then measure to solve problems, using U.S. Customary and metric units; and [U.S. Customary included in 4.8d; Metric moved to 5.9b]
e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units. [Moved to 5.9 EKS]

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<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>-- length: millimeters, centimeters, meters, and kilometers. [Moved to 5.9 EKS]</td>
<td>-- length: part of an inch ((\frac{1}{8}, \frac{1}{4}, \frac{1}{2})), inches, feet, yards, millimeters, centimeters, meters, and kilometers. [Moved to 5.9b]</td>
</tr>
<tr>
<td>* Two congruent right triangles can always be arranged to form a square or a rectangle.</td>
<td>-- mass: grams and kilograms. [Moved to 5.9 EKS]</td>
<td>-- mass: ounces, pounds, and tons. [Moved to 5.9b]</td>
</tr>
<tr>
<td>* To develop the formula for determining the volume of a rectangular prism, volume = length (\times) width (\times) height, students will benefit from experiences filling rectangular prisms (e.g., shoe boxes, cereal boxes) with cubes by first covering the bottom of the box and then building up the layers to fill the entire box.</td>
<td>-- liquid volume: milliliters, and liters. [Moved to 5.9 EKS]</td>
<td>-- liquid volume: cups, pints, quarts, gallons, milliliters, and liters. [Moved to 5.9b]</td>
</tr>
<tr>
<td>* Length is the distance along a line or figure from one point to another. [Moved to 5.9 EKS]</td>
<td></td>
<td>-- area: square units; and [Moved to 5.8a]</td>
</tr>
<tr>
<td>* U.S. Customary units for measurement of length include inches, feet, yards, and miles. Appropriate measuring devices include rulers, yardsticks, and tape measures. Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter ruler, meter stick, and tape measure. [Moved to 5.9 EKS]</td>
<td></td>
<td>-- temperature: Celsius and Fahrenheit units.</td>
</tr>
<tr>
<td>* When measuring with U.S. Customary units, students should be able to measure to the nearest part of an inch ((\frac{1}{2}, \frac{1}{4}, \frac{1}{8})), foot, or yard. [Moved to 5.9 EKS]</td>
<td></td>
<td>-- Water freezes at 0°C and 32°F.</td>
</tr>
<tr>
<td>* Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight that an object changes is dependent on</td>
<td></td>
<td>-- Water boils at 100°C and 212°F.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-- Normal body temperature is about 37°C and 98.6°F.</td>
</tr>
</tbody>
</table>
The student will:

a) solve practical problems that involve finding perimeter, area, and volume in standard units of measure; and

b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation.

c) identify equivalent measurements within the metric system; [Moved to 5.9a]

b) estimate and then measure to solve problems, using U.S. Customary and metric units; and [U.S. Customary included in 4.8d; Metric moved to 5.9b]

e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units. [Moved to 5.9 EKS]

### UNDERSTANDING THE STANDARD

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<td>the gravitational pull at its location. In everyday life, most people are actually interested in determining an object’s mass, although they use the term ‘weight’ (e.g., “How much does it weigh?” versus “What is its mass?”). [Moved to 5.9 EKS]</td>
<td>[Moved to 5.9 EKS]</td>
<td>[Moved to 5.9 EKS]</td>
</tr>
<tr>
<td>Appropriate measuring devices to measure mass in U.S. Customary units (ounces, pounds) and metric units (grams, kilograms) are balances. [Moved to 5.9 EKS]</td>
<td>[Moved to 5.9 EKS]</td>
<td>[Moved to 5.9 EKS]</td>
</tr>
<tr>
<td>U.S. Customary units to measure liquid volume (capacity) include cups, pints, quarts, and gallons. Metric units to measure liquid volume (capacity) include milliliters and liters. [Moved to 5.9 EKS]</td>
<td>[Moved to 5.9 EKS]</td>
<td>[Moved to 5.9 EKS]</td>
</tr>
<tr>
<td>Temperature is measured using a thermometer. The U.S. Customary unit of measure is degrees Fahrenheit; the metric unit of measure is degrees Celsius.</td>
<td>[Moved to 5.9 EKS]</td>
<td>[Moved to 5.9 EKS]</td>
</tr>
<tr>
<td>Practical experience measuring familiar objects helps students establish benchmarks and facilitates students’ ability to use the units of measure to make estimates. [Moved to 5.9 EKS]</td>
<td>[Moved to 5.9 EKS]</td>
<td>[Moved to 5.9 EKS]</td>
</tr>
</tbody>
</table>
5.9 The student will

a) given the equivalent measure of one unit, identify equivalent measurements within the metric system; and [Moved from 5.8c]  
b) solve practical problems involving length, mass, and liquid volume using metric units. [Moved from 5.8d]  

<table>
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</tr>
</thead>
</table>
| • Length is the distance between two points along a line. [Moved from 4.7 US]  
• Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter ruler, meter stick, and tape measure. [Moved from 4.7 US]  
• Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object’s mass, although they use the term weight (e.g., “How much does it weigh?” versus “What is its mass?”). [Moved from 5.8 US]  
• Balances are appropriate measuring devices to measure mass in U.S. Customary units (ounces, pounds) and metric units (grams, kilograms). [Moved from 5.8 US]  
• Metric units to measure liquid volume (capacity) include milliliters and liters. [Moved from 5.8 US]  
• Practical experience measuring familiar objects helps students establish benchmarks and facilitates students’ ability to use the appropriate units of measure to make estimates. [Moved from 5.8 US]  
• Students at this level will be given the equivalent measure of one unit when asked to determine equivalencies between units in the metric system. An example can be found below. Students will be told 1 kilometer is equivalent to 1,000 meters. Then asked to apply that relationship to determine:  
  – the number of meters in 3.5 kilometers;  
  – the number of kilometers equal to 2,100 meters; or  
| The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to  
• Given the equivalent measure of one unit, identify equivalent measurements within the metric system for the following:  
  – length: millimeters, centimeters, meters, and kilometers;  
  – mass: grams and kilograms;  
  – liquid volume: milliliters, and liters. (a) [Moved from 5.8 EKS]  
• Estimate and measure to solve practical problems that involve metric units:  
  – length: millimeters, centimeters, meters, and kilometers;  
  – mass: grams and kilograms; and  
  – liquid volume: milliliters, and liters. (b) [Moved from 5.8 EKS] |
The student will

a) given the equivalent measure of one unit, identify equivalent measurements within the metric system; and [Moved from 5.8c]
b) solve practical problems involving length, mass, and liquid volume using metric units. [Moved from 5.8d]

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<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appropriate examples of linear metric measurement equivalents at this grade level may include, but are not limited to:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.35 meters = 435 centimeters</td>
<td></td>
<td></td>
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<tr>
<td>148 millimeters = 14.8 centimeters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seth ran 2.78 kilometers on Saturday. How many meters are equivalent to 2.78 kilometers?</td>
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</tbody>
</table>
The student will identify and describe the diameter, radius, chord, and circumference of a circle.

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<tbody>
<tr>
<td>• A circle is a set of points on a flat surface (in a plane) with every point equidistant that are the same distance from a given point called the center.</td>
<td><strong>All students should</strong></td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• A chord is a line segment connecting any two points on a circle. A chord may or may not go through the center of a circle. The diameter is the longest chord of a circle.</td>
<td></td>
<td>• Identify and describe the diameter, radius, chord, and circumference of a circle.</td>
</tr>
<tr>
<td>• Students will benefit from understanding that a chord goes from one side of the circle to the other, but does not need to pass through the center.</td>
<td></td>
<td>• Investigate and describe the relationship between</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- diameter and radius;</td>
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<tr>
<td></td>
<td></td>
<td>- diameter and chord;</td>
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<tr>
<td></td>
<td></td>
<td>- radius and circumference; and</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- diameter and circumference.</td>
</tr>
<tr>
<td>• A diameter is a chord that goes through the center of a circle. The diameter is two times the radius. The length of the diameter of a circle is twice the length of the radius. [Moved from EKS]</td>
<td></td>
<td>The length of the diameter of a circle is twice the length of the radius. [Moved to US]</td>
</tr>
<tr>
<td>• A radius is a line segment joining from the center of a circle to any point on the circle. Two radii end-to-end form a diameter of a circle.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Circumference is the distance around or “perimeter” of a circle. <strong>An approximation for circumference is about three times larger than the diameter of a circle.</strong> The <strong>An approximation for circumference is about six times the radius of a circle.</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.1110 The student will determine an amount of solve practical problems related to elapsed time in hours and minutes within a 24-hour period.

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<tbody>
<tr>
<td>• Elapsed time is the amount of time that has passed between two given times.</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Elapsed time can be found by counting on from the beginning time or counting back from the finishing time.</td>
<td>Understand that elapsed time can be found by counting on from the beginning time to the finishing time.</td>
<td>Determine elapsed time in hours and minutes within a 24-hour period.</td>
</tr>
<tr>
<td>— Count the number of whole hours between the beginning time and the finishing time.</td>
<td></td>
<td>• Solve practical problems related to elapsed time in hours and minutes within a 24-hour period:</td>
</tr>
<tr>
<td>— Count the remaining minutes.</td>
<td>Add the hours and minutes.</td>
<td>— when given the beginning time and the ending time, determine the time that has elapsed;</td>
</tr>
<tr>
<td>— Add the hours and minutes.</td>
<td>For example, to find the elapsed time between 10:15 a.m. and 1:25 p.m., count on as follows:</td>
<td>— when given the beginning time and amount of elapsed time in hours and minutes, determine the ending time;</td>
</tr>
<tr>
<td>For example, to find the elapsed time between 10:15 a.m. and 1:25 p.m., count on as follows:</td>
<td>from 10:15 a.m. to 1:15 p.m., count 3 hours; from 1:15 p.m. to 1:25 p.m., count 10 minutes; and then add 3 hours to 10 minutes to find the total elapsed time of 3 hours and 10 minutes.</td>
<td>or</td>
</tr>
<tr>
<td></td>
<td></td>
<td>— when given the ending time and the elapsed time in hours and minutes, determine the beginning time.</td>
</tr>
</tbody>
</table>
## STANDARD 5.1211

**STRAND: MEASUREMENT AND GEOMETRY**

**GRADE LEVEL 5**

### 5.1211

The student will classify and measure right, acute, obtuse, and straight angles. [Classify moved from 5.12a]

#### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Angles can be classified as right, acute, obtuse, or straight according to their measures. [Moved from 5.12 US]
- Angles are measured in degrees. There are up to 360 degrees in an angle. A degree is \(\frac{1}{360}\) of a complete rotation of a full circle. There are 360 degrees in a circle.
- To measure the number of degrees in an angle, use a protractor or an angle ruler.
- A right angle measures exactly 90°.
- An acute angle measures greater than 0° but less than 90°.
- An obtuse angle measures greater than 90° but less than 180°.
- A straight angle forms an angle measures exactly 180°.
- Before measuring an angle, students should first compare it to a right angle to determine whether the measure of the angle is less than or greater than 90°.
- Students should recognize angle measure as additive. When an angle is decomposed into nonoverlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. [Moved from EKS]
- Students should understand how to work with a protractor or angle ruler as well as available computer software to measure and draw angles and triangles.

#### ESSENTIAL UNDERSTANDINGS

- All students should
  - Understand how to measure acute, right, obtuse, and straight angles.

#### ESSENTIAL KNOWLEDGE AND SKILLS

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
  - Classify angles as right, acute, obtuse, or straight. [Moved from 5.12a]
  - Identify the appropriate tools (e.g., protractor and straightedge or angle ruler as well as available software) used to measure and draw angles and triangles.
  - Measure right, acute, straight, and obtuse angles, using appropriate tools, and identify their measures in degrees.
  - Recognize angle measure as additive. When an angle is decomposed into nonoverlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. [Moved to US]
  - Solve addition and subtraction problems to find unknown angle measures on a diagram in practical and mathematical problems. (e.g., by using an equation with a symbol for the unknown angle measure).‡

‡Revised March 2011
The study of geometry helps students represent and make sense of the world. At the fourth- and fifth-grade levels, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)
5.1312 The student will classify
a) angles as right, acute, obtuse, or straight; and
b) classify triangles as right, acute, or obtuse, and equilateral, scalene, or isosceles; and
b) investigate the sum of the interior angles in a triangle and determine an unknown angle measure.

**UNDERSTANDING THE STANDARD**
(Background Information for Instructor Use Only)

- Angles can be classified as right, acute, obtuse, or straight according to their measures.
- A triangle can be classified as either right, acute, or obtuse according to the measure of its largest angle.
- Triangles may also be classified according to the measure of their sides, e.g., scalene (no sides congruent), isosceles (at least two sides congruent) and equilateral (all sides congruent).
- An equilateral triangle (with three congruent sides) is a special case of an isosceles triangle (which has at least two congruent sides).
- Triangles can be classified by the measure of their largest angle and by the measure of their sides (i.e., an isosceles right triangle).

- Congruent sides are denoted with the same number of hatch (or hash) marks on each congruent side.
- A right angle measures exactly 90°.
- An acute angle measures greater than 0° but less than 90°.
- An obtuse angle measures greater than 90° but less than 180°.
- A straight angle forms an angle that measures exactly 180°.
- A right triangle has one right angle.
- The hypotenuse of a right triangle is the side opposite the right angle. The hypotenuse of a right triangle is always the longest side.

**ESSENTIAL UNDERSTANDINGS**

- All students should
  - Understand that angles can be classified as right, acute, obtuse, or straight according to their measures.
  - Understand that a triangle can be classified as either right, acute, or obtuse according to the measure of its largest angle.
  - Understand that a triangle can be classified as equilateral, scalene, or isosceles according to the number of sides with equal length.

**ESSENTIAL KNOWLEDGE AND SKILLS**

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Classify angles as right, acute, straight, or obtuse.
- Classify triangles as right, acute, or obtuse.
- Classify triangles as equilateral, scalene, or isosceles.
- Compare and contrast the properties of triangles.
- Identify congruent sides and right angles using geometric markings to denote properties of triangles.
- Identify the hypotenuse and legs of a right triangle.
- Use models to prove that the sum of the interior angles of a triangle is 180° and use that relationship to determine an unknown angle measure in a triangle.
The student will classify:

a) angles as right, acute, obtuse, or straight; and

b) classify triangles as right, acute, or obtuse, and equilateral, scalene, or isosceles; and

b) investigate the sum of the interior angles in a triangle and determine an unknown angle measure.

### UNDERSTANDING THE STANDARD

#### (Background Information for Instructor Use Only)

- **ESSENTIAL UNDERSTANDINGS**
  - Side of the right triangle. The legs of a right triangle form the right angle.
  - An obtuse triangle has one obtuse angle.
  - An acute triangle has three acute angles (or no angle measuring 90° or greater).
  - A scalene triangle has no congruent sides.
  - An isosceles triangle has at least two congruent sides. The angles opposite of the congruent sides are congruent.
  - An equilateral triangle has three congruent sides. All angles of an equilateral triangle are congruent and measure 60°.
  - To facilitate the exploration of relationships, ask students whether a right triangle can have an obtuse angle. Why or why not? Can an obtuse triangle have more than one obtuse angle? Why or why not? What type of angles are the two angles other than the right angle in a right triangle? What type of angles are the two angles other than the obtuse angle in an obtuse triangle?
The student, using plane figures (square, rectangle, triangle, parallelogram, rhombus, and trapezoid), will
a) develop definitions of these plane figures [Included in 4.12] recognize and apply transformations, such as translation, reflection, and rotation [Moved from 4.11b] and
b) investigate and describe the results of combining and subdividing plane figures polygons.

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<tr>
<td>A transformation of a figure (preimage) changes the size, shape, or position of the figure to a new figure (image). Transformations can be explored using mirrors, paper folding and tracing.</td>
<td>All students should understand that simple plane figures can be combined to make more complicated figures that complicated figures can be subdivided into simple plane figures.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to develop definitions for squares, rectangles, triangles, parallelograms, rhombi, and trapezoids. [Included in 4.12]</td>
</tr>
<tr>
<td>Congruent figures have the same size and shape [Moved from 4.11 US]</td>
<td></td>
<td>Apply transformations to polygons in order to determine congruence [Moved from 4.11a and edited]</td>
</tr>
<tr>
<td>A translation is a transformation in which an image is formed by moving every point on the preimage the same distance in the same direction. [Moved from 4.11 US]</td>
<td></td>
<td>Recognize that translations, reflections, and rotations preserve congruency. [Moved from 4.11 EKS]</td>
</tr>
<tr>
<td>A reflection is a transformation in which an image is formed by reflecting the preimage over a line called the line of reflection. All corresponding points in the image and preimage are equidistant from the line of reflection [Moved from 4.11 US]</td>
<td></td>
<td>Identify the image of a polygon resulting from a single transformation (translation, reflection, or rotation). [Moved from 4.11b]</td>
</tr>
<tr>
<td>A rotation is a transformation in which an image is formed by rotating the preimage about a point called the center of rotation. The center of rotation may or may not be on the preimage. [Moved from 4.11 US]</td>
<td></td>
<td>Investigate and describe the results of combining and subdividing polygons plane figures.</td>
</tr>
<tr>
<td>The resulting figure of a translation, reflection, or rotation is congruent to the original figure. [Moved from 4.11 US]</td>
<td></td>
<td>Compare and contrast the characteristics of a given polygon that has been subdivided, with the characteristics of the resulting parts.</td>
</tr>
<tr>
<td>The orientation of figures does not affect congruency or noncongruency.</td>
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</tr>
<tr>
<td>A triangle is a polygon with three sides. Triangles may be classified according to the measure of their angles, i.e., right, acute, or obtuse. Triangles may also be classified according to the measure of their sides, i.e., scalene (no sides congruent), isosceles (at least two sides congruent) and equilateral (all sides congruent). [Moved to 5.13]</td>
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<tr>
<td>A quadrilateral is a polygon with four sides.</td>
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</tbody>
</table>
The student, using plane figures (square, rectangle, triangle, parallelogram, rhombus, and trapezoid), will
a) develop definitions of these plane figures [Included in 4.12] recognize and apply transformations, such as translation, reflection, and rotation; [Moved from 4.11b] and
b) investigate and describe the results of combining and subdividing plane figures polygons.
5.1413 The student, using plane figures (square, rectangle, triangle, parallelogram, rhombus, and trapezoid), will
a) develop definitions of these plane figures [Included in 4.12] recognize and apply transformations, such as translation, reflection, and rotation; [Moved from 4.11b] and
b) investigate and describe the results of combining and subdividing plane figures polygons.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- A polygon is a closed plane figure composed of at least three line segments that do not cross.
- Two or more figures polygons can be combined to form a new figure polygon. Students should be able to identify the figures that have been combined.
- The region of a polygon may be subdivided into two or more regions that represent figures. A figure polygon that can be divided into more than one of the basic figures is said to be a composite figure (or shape). Students should understand how to divide the region of a polygon into familiar figures using concrete materials (e.g., pattern blocks, tangrams, geoboards, grid paper, paper (folding), etc.)

### ESSENTIAL UNDERSTANDINGS

- This diagonal of the rectangle has been divided in half diagonally creating above subdivides the rectangle in half and creates two right triangles. The figure can also be formed by combining two right triangles that are congruent. The resulting figure shows that adjoining sides of the rectangle form the legs of the right triangle and are therefore congruent to the side lengths of the rectangle. The representation illustrates that the area of each right triangle is half the area of the rectangle. Exploring decomposition of shapes helps students develop algorithms for determining area of various shapes (e.g., area of a triangle is \( \frac{1}{2} \times \text{base} \times \text{height} \)).
- Congruent sides are denoted with the same number of hatch (or hash) marks on each congruent side. For example, a side on a polygon with 2 hatch marks is congruent to the side with 2 hatch marks on a congruent polygon or within the same polygon.
Students entering grades 4 and 5 have begun to explore the concepts of the measurement of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students have had initial experiences in predicting whether a game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction is to deepen their understanding of the concepts of probability by:

- offering opportunities to set up models simulating practical events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities that instill a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data collection strategies. Students should continue to focus on:

- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to practical experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions (“How many?” “How much?”) from the data displays;
- identifying and justifying comparisons (“Which is the most? Which is the least?” “Which is the same? Which is different?”) about the information;
- comparing their initial predictions to the actual results; and
- writing a few sentences to communicating to others their interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data analysis methods as powerful means for decision making.
5.1514 The student will make predictions and determine the probability of an outcome by constructing a sample space or using the Fundamental (Basic) Counting Principle.

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<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>All students should</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td></td>
<td>- Understand that the basic concepts of probability can be applied to make predictions of outcomes of simple experiments.</td>
<td>- Construct a sample space, using a tree diagram to identify all possible outcomes of a single event.</td>
</tr>
<tr>
<td></td>
<td>- Understand that a sample space represents all possible outcomes of an experiment.</td>
<td>- Construct a sample space, using a list or chart to represent all possible outcomes of a single event.</td>
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<tr>
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<td></td>
<td>- Predict and determine the probability of an outcome by constructing a sample space. The sample space will have a total of 24 or less equally likely possible outcomes.</td>
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<td></td>
<td>- Compute the number of possible outcomes by using the Fundamental (Basic) Counting Principle. [Moved from 7.10]</td>
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</tbody>
</table>

- A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives, tables, tree diagrams, and lists. [Copied from 4.13]
- Probability is the chance of an event occurring measure of likelihood that an event will occur. An event is a collection of outcomes from an investigation or experiment.
- The probability of an event occurring is the ratio of desired outcomes to all possible outcomes. Probability can be expressed as a fraction, where the numerator represents the number of favorable outcomes and the denominator represents the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event = number of favorable outcomes total number of possible outcomes.
- Probability is quantified as a number. The probability of an event occurring is represented by a ratio between 0 and 1. An event is “impossible” if it has a probability of 0 (e.g., the probability that the month of April will have 31 days). An event is “certain” if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow morning if today is Thursday then tomorrow will be Friday).
- When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time).
- Students should have opportunities to describe in informal terms (i.e., impossible, unlikely, as likely as unlikely, as likely as equally likely, likely, and certain) the degree of likelihood of an event occurring. Activities should include practical examples.
- For any event such as flipping a coin, the equally likely things that can happen are called outcomes. For example, there are two
5.1514 The student will make predictions and determine the probability of an outcome by constructing a sample space or using the Fundamental (Basic) Counting Principle.

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<td>equally likely outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up.</td>
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<td></td>
</tr>
<tr>
<td>A sample space represents all possible outcomes of an experiment. The sample space may be organized in a list, chart, or tree diagram.</td>
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</tr>
<tr>
<td>Tree diagrams can be used to illustrate all possible outcomes in a sample space. The Fundamental Counting Principle describes how to find the number of outcomes when there are multiple choices. For example, how many different outfit combinations can you make from two shirts (red and blue) and three pants (black, white, khaki)? The sample space displayed in a tree diagram would show that there are $2 \times 3 = 6$ (Fundamental Counting Principle) the outfit combinations: red-black; red-white; red-khaki; blue-black; blue-white; blue-khaki. Exploring the use of tree diagrams for modeling combinations helps students develop the Fundamental Counting Principle. For this problem applying the Fundamental Counting Principle shows there are $2 \times 3 = 6$ outcomes.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Fundamental (Basic) Counting Principle is a computational procedure to determine the total number of possible outcomes when there are multiple choices or several events. It is the product of the number of outcomes for each choice or event that can be chosen individually. [Moved from 7.10]. For example, the possible final outcomes or outfits of four shirts (green, yellow, blue, red), two shorts (tan or black), and three shoes (sneakers, sandals, flip flops) is $4 \times 2 \times 3 = 24$ outfits. A spinner with eight equal-sized sections is equally likely to land on any one of the sections, three of which are red, three green, and two yellow. Have students write a problem statement involving probability, such as, “What is the probability that the spinner will land on green?”</td>
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</table>
STANDARD 5.1615  
STRAND: PROBABILITY AND STATISTICS  
GRADE LEVEL 5

The student, given a practical problem situation, will collect, organize, and interpret data in a variety of forms, using:

a) represent data in line plots, and stem-and-leaf plots, and line graphs; and [Line plot moved from 3.17; Line graphs included in 4.14]

b) make observations and inferences about interpret data represented in a line plots, and stem-and-leaf plots, and line graph. [Line plot moved from 3.17; Line graphs included in 4.14]; and

c) compare data represented in a line plot with the same data represented in a stem-and-leaf plot.

### UNDERSTANDING THE STANDARD  
(Background Information for Instructor Use Only)

- The emphasis in all work with statistics should be on the analysis of the data and the communication of the analysis, rather than on a single correct answer. Data analysis should include opportunities to describe the data, recognize patterns or trends, and make predictions.
- Statistical investigations should be active, with students formulating questions about something in their environment and finding determining quantitative ways to answer the questions.
- Investigations that support collecting data can be brief class surveys or more extended projects taking many days.
- Through experiences displaying data in a variety of graphical representations, students learn to select an appropriate representation (i.e., a representation that is more helpful in analyzing and interpreting the data to answer questions and make predictions).
- There are two types of data: categorical and numerical. Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. While students need to be aware of the differences, they do not have to know the terms for each type of data.
- A line plot shows the frequency of data on a number line. Line plots are used to show the spread of the data and quickly identify the range, median, modes, and any outliers.

### ESSENTIAL UNDERSTANDINGS

All students should

- Understand how to interpret collected and organized data.
- Understand that stem and leaf plots list data in a meaningful arrangement. It helps in finding median, modes, minimum and maximum values, and ranges.
- Understand that line graphs show changes over time.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

- Formulate the question that will guide the data collection. Collect data, using observations (e.g., weather), measurement (e.g., shoe sizes), surveys (e.g., hours watching television), or experiments (e.g., plant growth). (a)
- Organize the data into a chart, table, line plot, and stem-and-leaf plots, and line graphs. (a)[Line plot moved from 3.17]
- Represent data in a line plot. Line plots will have no more than 30 data points. [Moved from 3.17 (a)]
- Display data in line graphs and stem-and-leaf plots.
- Construct Represent data in a line graph, labeling the vertical axis with equal whole numbers, decimal, or fractional increments and the horizontal axis with continuous data commonly related to time (e.g., hours, days, months, years, and age). Line graphs will have no more than six 10 identified points along a continuum for continuous data (e.g., the decades: 1950s, 1960s, 1970s, 1980s, 1990s, and 2000s). (a)
- Construct Represent data in a stem-and-leaf plot to organize and display data, where the stem is listed in ascending order and the leaves are in ascending order, with or without commas between leaves. Stem-and-leaf plots will be limited to no more than 30 data points. (a)
STANDARD 5.1615 STRAND: PROBABILITY AND STATISTICS GRADE LEVEL 5

5.1615 The student, given a practical problem situation, will collect, organize, and interpret data in a variety of forms, using:

a) represent data in line plots, and stem-and-leaf plots, and line graphs; and

b) make observations and inferences about data represented in a line plots, and stem-and-leaf plots, and line graph.

c) compare data represented in a line plot with the same data represented in a stem-and-leaf plot.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

<table>
<thead>
<tr>
<th>Number of Books Read</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
</tr>
<tr>
<td>x</td>
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<tr>
<td>x</td>
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<td>x</td>
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<tr>
<td>x</td>
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<tr>
<td>x</td>
</tr>
<tr>
<td>x</td>
</tr>
<tr>
<td>10 11 12 13 14 15 16 17</td>
</tr>
</tbody>
</table>

Each x represents one student.

Each x represents one student.

ESSENTIAL UNDERSTANDINGS

- Title the given graph or identify an appropriate title. Interpret the data in a variety of forms (e.g., orally or in written form). (a)

- Interpret data by making observations from line plots, and stem-and-leaf plots, and line graphs, by describing the characteristics of the data and the data as a whole. One set of data will be represented on a graph. (b)

- Interpret data by making inferences from line plots, and stem-and-leaf plots, and line graphs. (b)

- Compare two different representations of the same data (e.g., a set of data displayed on a chart and line graph, line plot and chart, data represented in a line plot and with the same data represented in a stem-and-leaf plot. (b)
The student, given a **practical** problem-situation, will collect, organize, and interpret data in a variety of forms, using:

- **a)** represent data in **line plots**, **stem-and-leaf plots**, and **line graphs**; and
- **b)** make observations and inferences about **interpret data represented in a line plots**, **stem-and-leaf plots**, and **line graph**; and
- **c)** compare data represented in a line plot with the same data represented in a stem-and-leaf plot.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- The information displayed in different graphs may be examined to determine how data are or are not related, differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inferences).

- A stem and leaf plot uses columns to display a summary of discrete numerical data while maintaining the individual data points. A stem-and-leaf plot displays data to show its shape and distribution.

- A stem-and-leaf plot is useful when data have a range greater than 25 and allows the exact values of data to be listed in a meaningful array. Data covering a range of 25 numbers are best displayed in a stem-and-leaf plot and are utilized to organize numerical data from least to greatest, using the digits of the greatest to group data.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3, 6, 9</td>
</tr>
<tr>
<td>1</td>
<td>2, 5, 7, 8</td>
</tr>
<tr>
<td>2</td>
<td>4, 6</td>
</tr>
<tr>
<td>3</td>
<td>1, 3, 7, 7, 7</td>
</tr>
<tr>
<td>4</td>
<td>0, 4, 8</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1, 2, 2, 3, 8</td>
</tr>
</tbody>
</table>

$3|5 = 35$

- The data is organized from least to greatest.
5.1615 The student, given a practical problem situation, will collect, organize, and interpret data in a variety of forms, using
a) represent data in line plots, and stem-and-leaf plots, and line graphs; and [Line plot moved from 3.17; Line graphs included
in 4.14]
b) make observations and inferences about interpret data represented in a line plots, and stem-and-leaf plots, and line
graph.[Line plot moved from 3.17; Line graphs included in 4.14]; and

c) compare data represented in a line plot with the same data represented in a stem-and-leaf plot.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Each value should be separated into a stem and a leaf [e.g., two-digit numbers are separated into stems (tens) and leaves (ones)].
- The stems are listed vertically from least to greatest with a line to their right. The leaves are listed horizontally, also from least to greatest, and can be separated by spaces or commas. Every value is recorded regardless of the number of repeats. No stem can be skipped. For example, in the stem and leaf plot above, there are no data for the stem 5, 5 should be listed showing no leaves.
- A key is often included to explain how to read the plot.
- Different situations call for different types of graphs. The way data is displayed is often dependent upon what someone is trying to communicate.
- Comparing different types of representations (e.g., charts graphs, line plots, etc.) provides students an opportunity to learn how different graphs can show different things about the same data. Following construction of representations, students benefit from discussions around what information each representation provides.
- Tables or charts organize the exact data and display numerical information. They do not show visual comparisons which generally means it takes longer to understand or to examine trends.
- Bar graphs can be easily used to compare data and see relationships. They provide a visual display comparing the numerical values of different categories. The scale of a bar graph may affect how one perceives the data.
5.1615 The student, given a practical problem situation, will collect, organize, and interpret data in a variety of forms, using:

a) represent data in line plots, stem-and-leaf plots, and line graphs; and

b) make observations and inferences about data represented in a line plots, stem-and-leaf plots, and line graph.

c) compare data represented in a line plot with the same data represented in a stem-and-leaf plot.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Comparisons, predictions and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions.

- Sample questions that could be explored in comparing different representations such as a chart to a line plot to a stem-and-leaf plot could include: In which representation can you quickly identify the mode? The range? What predictions can you make?
The student will solve practical problems that involve
a) describing mean, median, and mode as measures of center;
b) describing mean as fair share;
c) describing the range of a set of data as a measure of spread; and [Reordered] 
d) finding-determining the mean, median, mode, and range of a set of data. 
(Reordered and edited)

-- Revised March 2011
5.17.16 The student will solve practical problems that involve
   a) describing mean, median, and mode as measures of center;
   b) describing mean as fair share;
   c) describing the range of a set of data as a measure of spread; and [Reordered]
   d) finding determining the mean, median, mode, and range of a set of data, and
   d) describing the range of a set of data as a measure of variation. [Reordered and edited]

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The range is the spread of a set of data. The range of a set of data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>is the difference between the greatest and least values in the data set. It is determined by subtracting the least number in the data set from the greatest number in the data set. An example is ordering test scores from least to greatest: 73, 77, 84, 87, 89, 91, 94. The greatest score in the data set is 94 and the least score is 73, so the least score is subtracted from the greatest score or 94 - 73 = 21. The range of these test scores is 21.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Students need to learn more than how to identify the mean, median, mode, and range of a set of data. They need to build an understanding of what the number tells them about the data, and they need to see those values in the context of other characteristics of the data in order to best describe the results.</td>
<td></td>
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</tbody>
</table>
Students entering grades 4 and 5 have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships in open number sentences within a pattern.

The focus of instruction is to help students develop a solid use of patterning as a problem solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They interpret the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By interpreting mathematical situations and models, students begin to represent these, using symbols and variables to write “rules” for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.
### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Mathematical relationships exist in patterns. [Moved from middle column] There are an infinite number of patterns.
- Patterns and functions can be represented in many ways and described using words, tables, and symbols. [Moved from middle column] The simplest types of patterns are repeating patterns. In such patterns, students need to identify the basic unit of the pattern and repeat it.
- Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Can increase or decrease by a constant value.
- Students need experiences with exploring growing patterns using concrete materials and calculators. [Moved from EKS] Calculators are valuable tools for generating and analyzing patterns. The emphasis is not on computation but on identifying and describing patterns.
- Patterns at this level may include addition, subtraction, or multiplication of whole numbers, addition or subtraction of fractions (with denominators 12 or less), and decimals expressed in tenths or hundredths. Several sample numerical patterns are included below:
  - $6, 9, 12, 15, 18, \ldots$
  - $5, 7, 9, 11, 13, \ldots$
  - $1, 2, 4, 7, 11, 16, \ldots$
  - $2, 4, 8, 16, 32, \ldots$
  - $32, 30, 28, 26, 24, \ldots$ and
  - $1, 5, 25, 125, 625, \ldots$
  - $0.15, 0.35, 0.55, 0.75, \ldots$
  - $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{3}{4}, \frac{1}{4}, \ldots$ and
  - $128, 64, 32, 16, 8$

### ESSENTIAL UNDERSTANDINGS

All students should
- Understand that patterns and functions can be represented in many ways and described using words, tables, and symbols.
- Understand the structure of a pattern and how it grows or changes using concrete materials and calculators.
- Understand that mathematical relationships exist in patterns. [Moved to US]
- Understand that expressions use symbols to define a relationship and shows how each number in the list, after the first number, is related to the preceding number.
- Understand that expressions can be numerical or variable or a combination of numbers and variables.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
- Describe numerical and geometric patterns formed by using concrete materials and calculators. [Moved to US]
- Identify, create, describe, and extend patterns using concrete materials, number lines, tables, or pictures.
- Describe and express the relationship found in patterns, using words, tables, and symbols to express the relationship. [Reordered]
- Identify, describe, create, and extend single-operation input and output rules involving addition, subtraction, multiplication, or division to solve problems in various contexts. Solve practical problems that involve identifying, describing, and extending single-operation input and output rules (limited to addition, subtraction and multiplication of whole numbers; addition and subtraction of fractions, with denominators of 12 or less; and addition and subtraction of decimals expressed in tenths or hundredths).
- Identify the rule in a single-operation (addition, subtraction, multiplication or division) numerical pattern found in a list or table (limited to addition, subtraction and multiplication of whole numbers; addition and subtraction of fractions, with denominators of 12 or less; and addition and subtraction of decimals expressed in tenths or hundredths).
The student will identify, describe, create, and express, and extend the relationship found in a number patterns and express the relationship found in objects, pictures, numbers and tables.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Students in grades 3 and 4 had experiences working with input/output tables to determine the rule or a missing value. Generalizing patterns to identify rules and applying rules builds the foundation for functional thinking. Sample input/output tables that require determination of the rule or missing terms can be found below:</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Rule: ?</td>
<td>Rule: ?</td>
</tr>
<tr>
<td></td>
<td>Input</td>
<td>Output</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>8.9</td>
</tr>
<tr>
<td>5</td>
<td>?</td>
<td>6.6</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>20</td>
<td>0.5</td>
</tr>
<tr>
<td>• A numerical expression is a representation of a quantity. It is made up of numbers, variables, and/or computational symbols. It does not have an equal sign (e.g., $15 \times 12$).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• An expression, like a phrase, has no equal sign.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• A verbal expression involving one operation can be represented by a variable expression that describes the relationship. Numbers are used when they are known; variables are used when the numbers are unknown. The example in the table below defines the relationship between the input number and output number as $x + 3$. [Reordered] Students at this level are not expected to write a variable expression to describe patterns. They might describe the pattern below as $+3$ or given any number, add three.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• When the pattern data are expressed in a T-table, an expression can represent that data. An example is:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The student will identify, describe, create, and express, and extend the relationship found in a number patterns and express the relationship found in objects, pictures, numbers, and tables.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>15</td>
<td>18</td>
</tr>
</tbody>
</table>

- A verbal quantitative expression involving one operation can be represented by a variable expression that describes what is going on. Numbers are used when they are known; variables are used when the numbers are unknown. For example, “a full box of cookies and four extra” can be represented by $b + 4$; “three full boxes of cookies” by $3b$; “a full box of cookies shared among four” by $\frac{b}{4}$ [Reordered].

- An variable mathematical algebraic expression contain is a variable or a combination of variables, numbers, and/or operation symbols and represents a mathematical relationship. An expression cannot be solved.
STANDARD 5.1918  

STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA  
GRADE LEVEL 5  

5.1918 The student will  

a) investigate and describe the concept of variable;  
b) write an open sentence/equation to represent a given mathematical relationship, using a variable;  
c) use an variable expression with a variable to represent a given verbal quantitative expression involving one operation; and  
d) create a problem situation based on a given open sentence/equation, using a single variable.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
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</table>

- A variable is a symbol that can stand for an unknown number or object (e.g., $a + 4 = 6$) or for a quantity that changes (e.g., the rule or generalization for the pattern for an input/output table such as $x + 2 = y$).
- An variable algebraic expression, an expression with a variable, is like a phrase: as a phrase does not have a verb, so an expression does not have an equals sign symbol ($=$).
- A verbal expression describing a relationship involving one operation can be represented by an variable expression with a variable that mathematically describes the relationship. (e.g., "three full boxes of cookies" = $24$)

<table>
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<th>ESSENTIAL UNDERSTANDINGS</th>
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</table>

- All students should understand that a variable is a symbol that can stand for an unknown number or object.  
- Understand that a variable expression is a variable or combination of variables, numbers, and symbols that represents a mathematical relationship.
- Understand that an open sentence has a variable and an equals sign ($=$).
- Understand that problem situations can be expressed as open sentences.

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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</table>

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to  

- Describe the concept of a variable (presented as boxes, letters, or other symbols) as a representation of an unknown quantity. (a)  
- Write an open sentence/equation with addition, subtraction, multiplication, or division, using a variable to represent an unknown quantity or number. (b)  
- Model one-step linear equations using a variety of concrete materials such as colored chips on an equation mat or weights on a balance scale. [Moved to 6.14]  
- Use a variable expression with a variable to represent a given verbal expression involving one operation (e.g., "5 more than a number") can be represented by $y + 5$. (c)  
- Create and write a word problem to match a given open sentence/equation with a single variable and one operation. (d)
5.1918 The student will

a) investigate and describe the concept of variable;
b) write an **open sentence equation** to represent a given mathematical relationship, using a variable;
c) **use a variable expression** with a variable to represent a given verbal quantitative expression involving one operation; and 
d) create a problem situation based on a given **open sentence equation**, using a single variable.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Another example of an</strong> open sentence equation is b + 3 = 23 and represents the answer to the word problem, “How many cookies are in a box if the box plus three more equals 23 cookies, where b stands for the number of cookies in the box?”</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teachers should consider varying the letters used (in addition to x) to represent variables. At this level, discuss how the symbol × is often used to represent multiplication and can often be confused with the variable x. Students can minimize this confusion by using parentheses [e.g., 4(x) = 20 or 4x = 20] or a small dot raised off the line to represent multiplication [4 • x = 20].</td>
<td></td>
<td></td>
</tr>
<tr>
<td>By using story problems and numerical sentences, students begin to explore forming equations and representing quantities using variables.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>An open sentence equation containing a variable is neither true nor false until the variable is replaced with a number and the value of the expressions on both sides are compared.</td>
<td></td>
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</tbody>
</table>
5.2019 The student will investigate and identify the distributive property of multiplication over addition.
   a) the identity properties of addition and multiplication; [Moved from 3.20]
   b) the commutative properties of addition and multiplication; [Moved from 3.20] and
   c) the associative properties of addition and multiplication; and [Moved from 4.16b]
   d) the distributive property of multiplication over addition. [Reordered] [Application of properties moved to 5.4; naming of properties incorporated in 5.4 US]

<table>
<thead>
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</thead>
</table>
| Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers. [Moved from 3.20] | All students should
   - Understand that the distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
   - Investigate and identify the identity properties of addition and multiplication of whole numbers, using diagrams and manipulatives. [a] [Moved from 3.20] |
| The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4). Similarly, the commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 2 × 3 = 3 × 2). [Moved from 3.20] | | - Investigate and identify equations that represent the identity properties of addition and multiplication of whole numbers. [a] [Moved from 3.20] |
| The identity property of addition states that if zero is added to a given number, the sum is the same as the given number. The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number. [Moved from 3.20] | | - Investigate and identify the commutative properties of addition and multiplication of whole numbers, using diagrams and manipulatives. [b] [Moved from 3.20] |
| The associative property of addition states that the sum stays the same when the grouping of addends is changed (e.g., 15 + (25 + 16) = (15 + 25) + 16). The associative property of multiplication states that the product stays the same when the grouping of factors is changed (e.g., 6 × (2 × 5) = (6 × 3) × 5). [Moved from 4.16] | | - Investigate and identify equations that represent the commutative properties of addition and multiplication of whole numbers. [b] [Moved from 3.20] |
| The distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products. [Moved to 5.4 US] | Examples of the distributive property include: (e.g.,
   - 3(0) = 3(5 + 4)
   - 2(54 + 4) = 2 × 54 + 2 × 4
   - 5 × (3 + 7) = (5 × 3) + (5 × 7) | - Investigate and identify the associative properties of addition and multiplication of whole numbers. [c] [Moved from 4.16] |
   | | - Investigate and recognize identify the distributive property of whole numbers, limited to multiplication over addition, using diagrams and manipulatives. [d] [Moved use of property to SOL 5.4 EKS] |
   | | - Investigate and recognize identify an equations that |
5.2019 The student will investigate and identify recognize the distributive property of multiplication over addition.

a) the identity properties of addition and multiplication;<sup>[Moved from 3.20]</sup>

b) the commutative properties of addition and multiplication;<sup>[Moved from 3.20] and</sup>

c) the associative properties of addition and multiplication; and<sup>[Moved from 4.16b]</sup>

d) the distributive property of multiplication over addition.<sup>[Reordered][Application of properties moved to 5.4; naming of properties incorporated in 5.4 US] </sup>

<table>
<thead>
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</thead>
<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td></td>
<td>represents the distributive property, when given several whole number equations, limited to multiplication over addition. (d)</td>
</tr>
</tbody>
</table>

- The distributive property can be used to simplify expressions (e.g., $9 \times 23 = 9(20 + 3) = 180 + 27 = 207$; or $5 \times 19 = 5(10 + 9) = 50 + 45 = 95$).<sup>[Reorganized and reordered]</sup>
The 2009-2016 Mathematics Standards of Learning Curriculum Framework is a companion document to the 2009-2016 Mathematics Standards of Learning, and amplifies the Mathematics Standards of Learning by further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instructionbackground information for the teacher (K-8). The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers’ knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that What each student should know and be able to demonstratedo in each standard is outlined. This is not meant to be an exhaustive list of student expectations nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
**Algebra Readiness**

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

**Equity**

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
In the middle grades, the focus of mathematics learning is to
• build on students’ concrete reasoning experiences developed in the elementary grades;
• construct a more advanced understanding of mathematics through active learning experiences;
• develop deep mathematical understandings required for success in abstract learning experiences; and
• apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

• Students in the middle grades focus on mastering rational numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to most middle school mathematics topics.

• Students develop an understanding of integers and rational numbers by using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.

• Students develop an understanding of the properties of operations on real numbers through experiences with rational numbers and by applying the order of operations.

• Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

Mathematics instruction in grades 6 – 8 continues to focus on the development of number sense, with emphasis on rational and real numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to many middle school mathematics topics.

Students develop an understanding of integers and rational numbers using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.

Students develop an understanding of real numbers and the properties of operations on real numbers through experiences with rational and irrational numbers and apply the order of operations.

Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.
The student will describe and compare data sets, represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, $a$ to $b$, and $a:b$.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between two quantities. Ratios are used in practical situations when there is a need to compare quantities. [Moved from middle column]

- In the elementary grades, students are taught that fractions represent a part-to-whole relationship. However, fractions may also express a measurement, an operator (multiplication), a quotient, or a ratio. Examples of fraction interpretations include:
  - Fractions as parts of wholes: $\frac{3}{4}$ represents three parts of a whole, where the whole is separated into four equal parts.
  - Fractions as measurement: the notation $\frac{3}{4}$ can be interpreted as three one-fourths of a unit.
  - Fractions as an operator: $\frac{3}{4}$ represents a multiplier of three-fourths of the original magnitude.
  - Fractions as a quotient: $\frac{3}{4}$ represents the result obtained when three is divided by four.
  - Fractions as a ratio: $\frac{3}{4}$ is a comparison of 3 of a quantity to the whole quantity of 4.

- A ratio may be written using a colon ($a:b$), or the word to ($a$ to $b$). It may also be written using a fraction or fraction notation ($\frac{a}{b}$), if the ratio expresses a part-to-whole comparison. [Moved from middle column]

- The order of the values in a ratio is directly related to the order in which the quantities are compared. For example, if asked for the ratio of the number of cats to dogs in a park, the ratio must be expressed as the number of cats to the number of dogs, in that order. [Reordered]

- Example: In a certain class, there is a ratio of 3 girls to 4 boys.

### ESSENTIAL UNDERSTANDINGS

#### What is a ratio?
A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a set and between two sets. A ratio can be written using fraction form ($\frac{a}{b}$), a colon (2:3), or the word to (2 to 3). [Moved to US]

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Describe/Represent a relationship within a set by comparing part of the set to the entire set.

- Describe/Represent a relationship between two sets by comparing part of one set to a corresponding part of the other set. [Included in bullet above]

- Describe/Represent a relationship between two sets by comparing all of one set to all of the other set. [Included in bullet above]

- Describe/Represent a relationship within a set by comparing one part of the set to another part of the same set. [Included in bullet above]

- Represent a relationship in words that makes a comparison by using the notations $\frac{a}{b}$, $a:b$, and $a$ to $b$.

- Create a relationship in words for a given ratio expressed symbolically.
The student will describe and compare datasets, represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, $a$ to $b$, and $a:b$.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Another comparison that could represent the relationship between these quantities is the ratio of 4 boys to 3 girls (4:3). Both ratios give the same information about the number of girls and boys in the class, but they are distinct ratios. When you switch the order of comparison (girls to boys vs. boys to girls), there are different ratios being expressed.

- Fractions may be used when determining equivalent ratios.
  
  - Example: The ratio of girls to boys in a class is 3:4, this can be interpreted as:
    
    \[
    \text{number of girls} = \frac{3}{4} \cdot \text{number of boys}.
    \]
    
    In a class with 16 boys, number of girls = $\frac{3}{4} \cdot 16 = 12$.

  - Example: A similar comparison could compare the ratio of boys to girls in the class as being 4:3, which can be interpreted as:
    
    \[
    \text{number of boys} = \frac{4}{3} \cdot \text{number of girls}.
    \]
    
    In a class with 12 girls, number of boys = $\frac{4}{3} \cdot 12 = 16$.

- A ratio can represent different comparisons within the same quantity or between different quantities.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>part-to-whole (within the same quantity)</td>
<td>compare part of a whole to the entire whole</td>
</tr>
<tr>
<td>part-to-part (within the same quantity)</td>
<td>compare part of a whole to another part of the same whole</td>
</tr>
<tr>
<td>whole-to-whole (different quantities)</td>
<td>compare all of one whole to all of another whole</td>
</tr>
<tr>
<td>part-to-part (different)</td>
<td>compare part of one whole to part of another whole</td>
</tr>
</tbody>
</table>
6.1 The student will describe and compare data sets, represent relationships between quantities using ratios, and will use appropriate notations, such as \( \frac{a}{b} \), \( a \) to \( b \), and \( a:b \).

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Examples: Given Set Quantity A and Set Quantity B, the following comparisons could be expressed.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Example</th>
<th>Ratio Notation(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>part-to-whole</td>
<td>compare the number of unfilled stars to the total number of stars in Set Quantity A</td>
<td>3:8; 3 to 8; ( \frac{3}{8} ) or ( 0.375; 37.5% )</td>
</tr>
<tr>
<td>part-to-part (^1)</td>
<td>compare the number of unfilled stars to the number of filled stars in Set Quantity A</td>
<td>3:5 or 3 to 5</td>
</tr>
<tr>
<td>whole-to-whole</td>
<td>compare the number of stars in Set Quantity A to the number of stars in Set Quantity B</td>
<td>8:5 or 8 to 5</td>
</tr>
<tr>
<td>part-to-part (^1)</td>
<td>compare the number of unfilled stars in Set Quantity A to the number of unfilled stars in Set Quantity B</td>
<td>3:2 or 3 to ( \frac{3}{2} ) or 1.5 to 1</td>
</tr>
</tbody>
</table>

\(^1\)Part-to-part comparisons and whole-to-whole comparisons are ratios that are not typically represented in fraction notation except in certain contexts, such as determining if two different ratios are equivalent.
STANDARD 6.1  STRAND: NUMBER AND NUMBER SENSE  GRADE LEVEL 6

The student will describe and compare data sets, represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, $a$ to $b$, and $a:b$.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>•  A ratio can compare part of a set to the entire set (part-whole comparison). [Included in table above]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>•  A ratio can compare part of a set to another part of the same set (part-part comparison). [Included in table above]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>•  A ratio can compare part of a set to a corresponding part of another set (part-part comparison). [Included in table above]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>•  A ratio can compare all of a set to all of another set; two real-world quantities. (whole-whole comparison). For Examples: miles per gallon, unit rate, and circumference to diameter of a circle.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>•  The order of the quantities in a ratio is directly related to the order of the quantities expressed in the relationship. For example, if asked for the ratio of the number of cats to dogs in a park, the ratio must be expressed as the number of cats to the number of dogs, in that order. [Reordered]</td>
<td></td>
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</tr>
<tr>
<td>•  A ratio is a multiplicative comparison of two numbers, measures, or quantities.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>•  All fractions are ratios and vice versa.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>•  Ratios may or may not be written in simplest form.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>•  Ratios can compare two parts of a whole.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>•  Rates can be expressed as ratios. A rate is a ratio that compares two quantities measured in different units. A unit rate is a rate with a denominator of 1. Examples of rates include miles/hour and revolutions/second. Other examples: rate: 142 heartbeats per 2 minutes; unit rate: 71 beats/min. rate: $4.90 for 10 stamps; $0.49/stamp.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6.2 The student will
a) investigate and describe fractions, decimals and percents as ratios [Moved to EKS] represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;*
and
b) identify a given fraction, decimal or percent from a representation; [Included in 6.2a]
c) demonstrate equivalent relationships among fractions, decimals, and percents; and
b) compare and order fractions, decimals, and percents positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Fractions, decimals and percents can be used to represent part-to-whole ratios.
  - Example: The ratio of dogs to the total number of pets at a grooming salon is 5:8. This implies that 5 out of every 8 pets being groomed is a dog. This part-to-whole ratio could be represented as the fraction \( \frac{5}{8} \) (of all pets are dogs), the decimal 0.625 (0.625 of the number of pets are dogs), or as the percent 62.5% (62.5% of the pets are dogs).

- Fractions, decimals, and percents are three different ways to express the same number. Any number that can be written as a fraction can be expressed as a terminating or repeating decimal or a percent. [Moved from middle column.]

- Equivalent relationships among fractions, decimals, and percents may be determined by using concrete materials and pictorial representations (e.g., fraction bars, base-10 blocks, fraction circles, number lines, colored counters, cubes, decimal squares, shaded figures, shaded grids, or calculators). [Reordered and reworded]

- Percent means “per 100” or how many “out of 100”; percent is another name for hundredths.

- A number followed by a percent symbol (%) is equivalent to a fraction with that number as the numerator and with 100 as the denominator (e.g., 30% = \( \frac{30}{100} = \frac{3}{10} = 0.3 \)).

ESSENTIAL UNDERSTANDINGS

What is the relationship among fractions, decimals and percents?
Fractions, decimals, and percents are three different ways to express the same number. A ratio can be written using fraction form \( \left( \frac{2}{3} \right) \), a colon (2:3), or the word to (2 to 3). Any number that can be written as a fraction can be expressed as a terminating or repeating decimal or a percent. [Moved to US]

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Investigate and describe Represent ratios as fractions (proper or improper), mixed numbers, decimals, and/or percents. (a) [Moved from 6.2a]

- Identify Determine the decimal and percent equivalents for numbers written in fraction form (proper or improper) or as a mixed number, including repeating decimals. (a)

- Represent fractions, decimals, and percents on a number line. [Combined Included in bullet above]

- Describe orally and in writing the equivalent relationships of equivalencies among decimals, percents, and fractions (proper or improper), and mixed numbers that have denominators that are 12 or less or factors of 100. (a)

- Represent, by shading a grid, a fraction, decimal, and percent. [Combined Included in bullet above]

- Represent in fraction, decimal, and percent form a given shaded region of a grid. [Combined Included in bullet above]

- Compare two decimals through thousandths using
### STANDARD 6.2  
**STRAND: NUMBER AND NUMBER SENSE**  
**GRADE LEVEL 6**

**6.2** The student will

- **a)** investigate and describe fractions, decimals and percents as ratios [Moved to EKS] represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;* and
- **b)** identify a given fraction, decimal or percent from a representation; [Included in 6.2a]
- **c)** demonstrate equivalent relationships among fractions, decimals, and percents; and
- **b)** compare and order fractions, decimals, and percents* positive rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<tr>
<td>expressed as fractions with a denominator of 100 (e.g., ( \frac{139}{100} = 139% = \frac{2}{\frac{139}{100}} )).</td>
<td>Percents can be expressed as decimals (e.g., ( \frac{38}{100} = 0.38; \frac{139}{100} = 1.39 )).</td>
<td>manipulatives, pictorial representations, number lines, and symbols (( &lt;,\leq,\geq,= )). [Included in 4.3 and 5.2]</td>
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<tr>
<td>Percents can be expressed as decimals (e.g., ( \frac{38}{100} = 0.38; \frac{139}{100} = 1.39 )).</td>
<td>Some fractions can be rewritten as equivalent fractions with denominators of powers of 10, and can be represented as decimals or percents (e.g., ( \frac{3}{5} = \frac{6}{10} = \frac{60}{100} = 0.60 = 60% )).</td>
<td>Compare two fractions (proper, improper, and mixed numbers) with denominators of 12 or less or factors of 100 using manipulatives, pictorial representations, number line, and symbols (( &lt;,\leq,\geq,= )). [Included in 4.2 and 5.2]</td>
</tr>
<tr>
<td>Decimals, fractions, and percents can be represented using concrete materials (e.g., Base 10 blocks, number lines, decimal squares, or grid paper). [Reordered]</td>
<td>Decimals, fractions, and percents can be represented using concrete materials (e.g., Base 10 blocks, number lines, decimal squares, or grid paper).</td>
<td>Compare two percents using pictorial representations and symbols (( &lt;,\leq,\geq,= )). [Included in 4.2 and 5.2]</td>
</tr>
<tr>
<td>Fractions, decimals, and percents can be represented by drawing shaded regions on grids using an area model, a set model, or by finding a location on number line. A measurement model. For example, the fraction ( \frac{1}{3} ) is shown using each of the three models.</td>
<td></td>
<td>Order no more than 3-four positive rational numbers expressed as fractions (proper or improper), mixed numbers, decimals, and percents (decimals through thousandths, fractions with denominators of 12 or less or factors of 100). Ordering may be in ascending or descending order. Fractions may be expressed as proper, improper, or mixed numbers. [Included in 4.2 and 5.2]</td>
</tr>
<tr>
<td>Percents are used in real life to solve practical problems involving for taxes, including sales, data description, and data comparison.</td>
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Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 6
6.2 The student will
   a) investigate and describe fractions, decimals and percents as ratios [Moved to EKS] represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;* and
   b) identify a given fraction, decimal or percent from a representation; [Included in 6.2a]
   c) demonstrate equivalent relationships among fractions, decimals, and percents; and
   b) compare and order fractions, decimals, and percents positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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**UNDERSTANDING THE STANDARD**
(Background Information for Instructor Use Only)

- The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where $a$ and $b$ are integers and $b$ does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of positive rational numbers are: $\sqrt{25}$, $0.275$, $0.75$, $\frac{22}{7}$, $4.59$.
- Students are not expected to know the names of the subsets of the real numbers until grade 8.
- Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction can be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$).
- A proper fraction is a fraction whose numerator is less than the denominator. A proper fraction is a fraction that is always less than one.
- An improper fraction is a fraction whose numerator is greater than or equal to the denominator. An improper fraction is a fraction that is equal to or greater than one.
- Fractions, decimals and percents are equivalent forms representing a given number. Any number that can be written as a fraction can be expressed as a terminating or repeating decimal.
6.2 The student will

a) investigate and describe fractions, decimals and percents as ratios [Moved to EKS] represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;* and

b) identify a given fraction, decimal or percent from a representation; [Included in 6.2a]

c) demonstrate equivalent relationships among fractions, decimals, and percents; and

d) compare and order fractions, decimals, and percents positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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### UNDERSTANDING THE STANDARD

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6.2 The student will

a) investigate and describe fractions, decimals, and percents as ratios [Moved to EKS] represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;* and

b) identify a given fraction, decimal, or percent from a representation; [Included in 6.2a]

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</tr>
</thead>
<tbody>
<tr>
<td>greater distance away from 1 whole than $\frac{8}{9}$: Therefore, $\frac{6}{7} &lt; \frac{8}{9}$</td>
<td>Students should have experience with some fractions such as $\frac{1}{8}$ whose have a decimal representation that is a terminating decimal (e.g., $\frac{1}{8} = 0.125$) and with some fractions such as $\frac{2}{9}$ whose have a decimal representation that does not end terminate but continues to repeat (e.g., $\frac{2}{9} = 0.222\ldots$). The repeating decimal can be written with ellipses (three dots) as in $0.222\ldots$ or denoted with a bar above the digits that repeat as in $0.\overline{2}$.</td>
<td></td>
</tr>
</tbody>
</table>
## UNDERSTANDING THE STANDARD

**ESSENTIAL UNDERSTANDINGS**

- The set of integers includes the set of whole numbers, and their opposites \{…,-2,-1,0,1,2,…\}, and zero. Zero has no opposite and is an integer that is neither positive nor negative. [Reordered]
- Integers are used in practical situations, such as temperature (above/below zero), deposits/withdrawals in a checking account, golf (above/below par), time lines, football yardage, positive and negative electrical charges, and altitude (above/below sea level). [Moved from middle column]
- Integers should be explored by modeling on a number line and using manipulatives, such as two-color counters, drawings, or algebra tiles.
- The opposite of a positive number is negative and the opposite of a negative number is positive.
- Positive integers are greater than zero.
- Negative integers are less than zero.
- Zero is an integer that is neither positive nor negative. [Reordered]
- A negative integer is always less than a positive integer.
- When comparing two negative integers, the negative integer that is closer to zero is greater.
- An integer and its opposite are the same distance from zero on a number line. [Note]
  - Example: the opposite of 3 is -3 and the opposite of -10 is 10.

**ESSENTIAL KNOWLEDGE AND SKILLS**

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Model integers, including models derived from practical situations. (a)
- Identify an integer represented by a point on a number line. (a)
- Represent integers on a number line. (a) [Included in bullet above]
- Compare and order integers using a number line. (b)
- Compare integers, using mathematical symbols (<, >, =). (b)
- Identify and describe the absolute value of an integer. (c)
6.3 The student will
   a) identify and represent integers;
   b) compare and order integers; and
   c) identify and describe absolute value of integers.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• On a conventional number line, a smaller number is always located to the left of a larger number (e.g., (-7) lies to the left of (-3), thus (-7 &lt; -3); (5) lies to the left of (8) thus (5 &lt; 8))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The absolute value of a number is the distance of a number from zero on the number line regardless of direction. Absolute value is represented using the symbol (</td>
<td>\</td>
<td>) as (\text{e.g., }</td>
</tr>
<tr>
<td>• On a conventional number line, a smaller number is always located to the left of a larger number (e.g., (-7) lies to the left of (-3), thus (-7 &lt; -3); (5) lies to the left of (8) thus (5 &lt; 8)).</td>
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<tr>
<td>• The absolute value of zero is zero.</td>
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</tbody>
</table>
6.4 The student will demonstrate multiple representations of multiplication and division of fractions. [Moved to 6.5 EKS]

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td></td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
</tbody>
</table>
| • Using manipulatives to build conceptual understanding and using pictures and sketches to link concrete examples to the symbolic enhance students’ understanding of operations with fractions and help students connect the meaning of whole number computation to fraction computation. | • When multiplying fractions, what is the meaning of the operation?
  When multiplying a whole by a fraction such as $3 \times \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. | • Demonstrate multiplication and division of fractions using multiple representations. |
| • Multiplication and division of fractions can be represented with arrays, paper folding, repeated addition, repeated subtraction, fraction strips, pattern blocks and area models. | • When multiplying a fraction by a fraction such as $\frac{2}{3} \times \frac{3}{4}$, we are asking for part of a part. 
  When multiplying a fraction by a whole number such as $\frac{1}{2} \times 6$, we are trying to find a part of the whole. | • Model algorithms for multiplying and dividing with fractions using appropriate representations. |
| • When multiplying a whole by a fraction such as $3 \times \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. | • What does it mean to divide with fractions?
  For measurement division, the divisor is the number of groups and the quotient will be the number of groups in the dividend. Division of fractions can be explained as how many of a given divisor are needed to equal the given dividend. In other words, for $\frac{1}{4} \div \frac{2}{3}$, the question is, “How many $\frac{2}{3}$ make $\frac{1}{4}$?” 
  For partition division the divisor is the size of the group, so the quotient answers the question, “How much is the whole?” or “How much for one?” | |
| • When multiplying a fraction by a fraction such as $\frac{2}{3} \times \frac{3}{4}$, we are asking for part of a part. | | |
| • When multiplying a fraction by a whole number such as $\frac{1}{2} \times 6$, we are trying to find a part of the whole. | | |
| • For measurement division, the divisor is the number of groups. You want to know how many are in each of those groups. Division of fractions can be explained as how many of a given divisor are needed to equal the given dividend. In other words, for $\frac{1}{4} \div \frac{2}{3}$, the question is, “How many $\frac{2}{3}$ make $\frac{1}{4}$?” | | |

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 6
6.4 The student will demonstrate multiple representations of multiplication and division of fractions. [Moved to 6.5 EKS]

<table>
<thead>
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<tbody>
<tr>
<td>For partition division the divisor is the size of the group, so the quotient answers the question, “How much is the whole?” or “How much for one?”</td>
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</table>
STANDARD 6.45

STRAND: NUMBER AND NUMBER SENSE

GRADE LEVEL 6

6.45 The student will investigate and describe concepts of positive representation patterns with whole number exponents and perfect squares.

<table>
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<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>What does exponential form represent? Exponential form is a short way to write repeated multiplication of a common factor such as $5 \times 5 \times 5 = 5^3$.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:</td>
</tr>
</tbody>
</table>
| | What is the relationship between perfect squares and a geometric square? A perfect square is the area of a geometric square whose side length is a whole number. | - Recognize and describe patterns with bases and exponents that are natural whole numbers, by using a calculator.
| | | - Recognize and describe patterns of perfect squares not to exceed $20^2$, by using grid paper, square tiles, tables, and calculators.
| | | - Recognize powers of ten with whole number exponents by examining patterns in a place value chart: $10^0 = 1, 10^1 = 10, 10^2 = 100, 10^3 = 1000, 10^4 = 10,000$.
| - The symbol in can be used in Grade 6 in place of “x” to indicate multiplication. [Moved from 6.2 US] | - Any real number other than zero raised to the zero power is 1. Zero to the zero power ($0^0$) is undefined. |
| - In exponential notation, the base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In $8^3$, 8 is the base and 3 is the exponent (e.g., $8^3 = 8 \cdot 8 \cdot 8$). [Same as middle column] | - A perfect square is the area of a geometric square whose side length is a whole number. |
| - A power of a number represents repeated multiplication of the number by itself (e.g., $2^3 = 8 \cdot 8 \cdot 8$ and is read “8 to the third power”). [Combined with bullet above] | - Perfect squares can be represented geometrically as the areas of squares whose sides are whole numbers (e.g., $1 \cdot 1, 2 \cdot 2$, or $3 \cdot 3$). This can be modeled with grid paper, tiles, geoboards, and virtual manipulatives. [Same as middle column] |
| - Any real number other than zero raised to the zero power is 1. Zero to the zero power ($0^0$) is undefined. | - The examination of patterns in place value of the powers of ten in grade 6 leads to the development of scientific notation in grade 7. | - Recognize powers of ten with whole number exponents by examining patterns in a place value chart: $10^0 = 1, 10^1 = 10, 10^2 = 100, 10^3 = 1000, 10^4 = 10,000$. |
In the middle grades, the focus of mathematics learning is to
• build on students’ concrete reasoning experiences developed in the elementary grades;
• construct a more advanced understanding of mathematics through active learning experiences;
• develop deep mathematical understandings required for success in abstract learning experiences; and
• apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

• Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense.

• Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate.

• Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonableness of answers.

• Students reinforce skills with operations with whole numbers, fractions, and decimals through problem solving and application activities.

Computation and estimation in grades 6-8 focus on developing conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring an understanding as to why procedures work and make sense.

Students develop and refine estimation strategies based on an understanding of number concepts, properties and relationships. The development of problem solving, using operations with integers and rational numbers, builds upon the strategies developed in the elementary grades. Students will reinforce these skills and build on the development of proportional reasoning and more advanced mathematical skills.

Students learn to make sense of the mathematical tools available by making valid judgments of the reasonableness of answers. Students will balance the ability to make precise calculations through the application of the order of operations with knowing when calculations may require estimation to obtain appropriate solutions to practical problems.
The student will

a) multiply and divide fractions and mixed numbers;* and

b) estimate solutions and then solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and

c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals. [Moved from 6.7]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- A fraction can be expressed in simplest form (simplest equivalent fraction) by dividing the numerator and denominator by their greatest common factor.
- When the numerator and denominator have no common factors other than 1, then the fraction is in simplest form.
- Simplifying fractions to simplest form assists with uniformity of answers.
- Addition and subtraction are inverse operations as are multiplication and division.
- Models for representing multiplication and division of fractions can include arrays, paper folding, repeated addition, repeated subtraction, fraction strips, fraction rods, pattern blocks, and area models. [Moved from 6.4]
- When a number is multiplied by a number greater than 1, the product is greater than the original number. But when a number is multiplied by a number between 0 and 1, the product is less than the original number. For example, \( \frac{1}{2} \cdot 4 = 2 \), 2 is greater than 1. However, \( \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} \), \( \frac{1}{8} \) is less than \( \frac{1}{2} \). [Moved from 6.4 middle column]
- It is helpful to use estimation to develop computational strategies. For example, \( \frac{7}{8} \cdot \frac{3}{4} \) is about \( \frac{3}{4} \) of 3, so the answer is between 2 and 3.

### ESSENTIAL UNDERSTANDINGS

- How are multiplication and division of fractions and multiplication and division of whole numbers alike? Fraction computation can be approached in the same way as whole number computation, applying those concepts to fractional parts.
- What is the role of estimation in solving problems? Estimation helps determine the reasonableness of answers.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Demonstrate/model multiplication and division of fractions (proper or improper) and mixed numbers using multiple representations. (a) [Moved from 6.4]
- Multiply and divide with fractions (proper or improper) and mixed numbers. Answers are expressed in simplest form. (a)
- Solve single-step and multistep practical problems that involve addition and subtraction with fractions (proper or improper) and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less. Answers are expressed in simplest form. (b)
- Solve single-step and multistep practical problems that involve multiplication and division with fractions (proper or improper) and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form. (b)
- Solve single-step and multistep practical problems involving addition, subtraction, multiplication and division with decimals. Divisors are limited to a three digit number, with decimal divisors limited to hundredths. (c) [Moved from 6.7]
6.56 The student will
a) multiply and divide fractions and mixed numbers;* and
b) estimate solutions and then solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and
c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals. [Moved from 6.7]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- When multiplying a whole number by a fraction such as $3 \cdot \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole.
- When multiplying a fraction by a fraction such as $\frac{2}{3} \cdot \frac{3}{4}$, we are asking for part of a part.
- When multiplying a fraction by a whole number such as $\frac{1}{2} \cdot 6$, we are trying to find a part of the whole.

- Partitioning division can be thought of as repeated subtraction or equal groups. For partition division, the divisor is the number of groups. You want to know how many are in each of those groups. For example, Diana is painting statues. She has $\frac{7}{8}$ gallon of paint remaining. Each statue requires $\frac{1}{16}$ of a gallon of paint. How many statues can she paint? Determine how many $\frac{1}{16}$ are in $\frac{7}{8}$. [Moved from SOL 6.4 middle column]

- In measurement division, the divisor is the size of the group, so the quotient answers the question, “How much is the whole?” or “How much for one?” For example, Jane bought $2 \frac{1}{2}$ pounds of apples for $4.50. How much did she pay per pound? In $2 \frac{1}{2}$ there are 10 thirds. If $4.50 is the cost of 10 thirds, then one third is $0.45. There are 3 thirds in one pound so 1 pound is 3($0.45) or $1.35. [Moved from SOL 6.4 middle column]
The student will
a) multiply and divide fractions and mixed numbers;* and
b) estimate solutions and then solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and
c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals. [Moved from 6.7]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<tr>
<td>Students do not need to use the terms partition division or measurement division.</td>
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<tr>
<td>A multistep problem is a problem that requires two or more steps to solve.</td>
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<tr>
<td>Different strategies can be used to estimate the result of computations and judge the reasonableness of the result. Example: What is an approximate answer for 2.19 ÷ 0.8? The answer is around 2 because 2.19 ÷ 0.8 is about 2 ÷ 1 = 2. [Moved from 6.7]</td>
<td></td>
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<tr>
<td>Understanding the placement of the decimal point is very important when determining quotients of decimals. Examining patterns with successive decimals provides meaning, such as dividing the dividend by 6, by 0.6, and by 0.06. [Moved from 6.7]</td>
<td></td>
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</tr>
<tr>
<td>Solving multistep problems in the context of practical situations enhances interconnectedness and proficiency with estimation strategies. [Moved from 6.7]</td>
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<tr>
<td>Examples of practical situations solved by using estimation strategies include shopping for groceries, buying school supplies, budgeting an allowance, and sharing the cost of a pizza or the prize money from a contest. [Moved from 6.7]</td>
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</tbody>
</table>
STANDARD 6.7 STRAND: COMPUTATION AND ESTIMATION GRADE LEVEL 6

6.7 The student will solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of decimals. [Moved to 6.5c]

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>• Different strategies can be used to estimate the result of computations and judge the reasonableness of the result. For example: What is an approximate answer for 2.19 ÷ 0.8? The answer is around 2 because $2 ÷ 1 = 2$.</td>
<td>• What is the role of estimation in solving problems? Estimation gives a reasonable solution to a problem when an exact answer is not required. If an exact answer is required, estimation allows you to know if the calculated answer is reasonable.</td>
<td>The student will use problem-solving, mathematical communication, mathematical reasoning, connections, and representations to:</td>
</tr>
<tr>
<td>• Understanding the placement of the decimal point is very important when finding quotients of decimals. Examining patterns with successive decimals provides meaning, such as dividing the dividend by 6, by 0.6, by 0.06, and by 0.006.</td>
<td></td>
<td>• Solve single-step and multistep practical problems involving addition, subtraction, multiplication and division with decimals expressed to thousandths with no more than two operations.</td>
</tr>
<tr>
<td>• Solving multistep problems in the context of real-life situations enhances interconnectedness and proficiency with estimation strategies.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Examples of practical situations solved by using estimation strategies include shopping for groceries, buying school supplies, budgeting an allowance, deciding what time to leave for school or the movies, and sharing a pizza or the prize money from a contest.</td>
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</tr>
</tbody>
</table>
6.6 The student will

a) add, subtract, multiply, and divide integers;* and [Moved from 7.3]
b) solve practical problems involving operations with integers; and [Moved from 7.3]
c) simplify numerical expressions involving integers.* [Moved and modified from 6.8] [Combined with 6.7, 2016 BOE first draft]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>• The set of integers is the set of whole numbers and their opposites (e.g., ..., -3, -2, -1, 0, 1, 2, 3...). Zero has no opposite and is neither positive nor negative. [Moved from 7.3]</td>
<td>• The sums, differences, products and quotients of integers are either positive, zero, or negative. How can this be demonstrated? This can be demonstrated through the use of patterns and models. [Moved to US]</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Integers are used in practical situations, such as temperature changes (above/below zero), balance in a checking account (deposits/withdrawals), golf, time lines, football yardage, and changes in altitude (above/below sea level). [Moved from 7.3]</td>
<td></td>
<td>• Model addition, subtraction, multiplication and division of integers using pictorial representations or concrete manipulatives. (a) [Moved from 7.3]</td>
</tr>
<tr>
<td>• Concrete experiences in formulating rules for adding, subtracting, multiplying, and dividing integers should be explored by examining patterns using calculators, using a number line, and using manipulatives, such as two-color counters, drawings, or by using algebra tiles. [Moved from 7.3]</td>
<td>• Add, subtract, multiply, and divide two integers. (a) [Moved from 7.3]</td>
<td>• Solve practical problems involving addition, subtraction, multiplication, and division with integers. (b) [Moved from 7.3]</td>
</tr>
<tr>
<td>• Sums, differences, products and quotients of integers are either positive, negative, or zero. This can be demonstrated through the use of patterns and models.</td>
<td>• Use the order of operations and apply the properties of real numbers to simplify numerical expressions involving more than two integers. Expressions should not include braces {} or brackets [], but may contain absolute value bars</td>
<td>(c) [Moved and modified from 6.8]</td>
</tr>
<tr>
<td>• The order of operations is a convention that defines the computation order to follow in simplifying an expression. [Moved from 6.8] Having an established convention ensures that there is only one correct result when simplifying an expression.</td>
<td></td>
<td>Simplification will be limited to three operations, which may include simplifying a whole number raised to an exponent of 1, 2 or 3. (c) [Moved and modified from 6.8]</td>
</tr>
<tr>
<td>• The order of operations is as follows:</td>
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<tr>
<td>− First, complete all operations within grouping symbols. If there are grouping symbols within other grouping symbols, do the innermost operation first.</td>
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<tr>
<td>− Second, evaluate all exponential expressions.</td>
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<tr>
<td>− Third, multiply and/or divide in order from left to right.</td>
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<td></td>
</tr>
<tr>
<td>− Fourth, add and/or subtract in order from left to right.</td>
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</table>
STANDARD 6.6  
**STRAND: COMPUTATION AND ESTIMATION**  
GRADE LEVEL 6

6.6 The student will

a) add, subtract, multiply, and divide integers;* and [Moved from 7.3]
b) solve practical problems involving operations with integers; and [Moved from 7.3]
c) simplify numerical expressions involving integers.* [Moved and modified from 6.8] [Combined with 6.7, 2016 BOE first draft]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<tr>
<td>Parentheses ( ), absolute value</td>
<td>[Moved from 6.8]</td>
<td>[Moved from 6.8]</td>
</tr>
<tr>
<td>value } (e.g., }</td>
<td>(e.g., }</td>
<td>(e.g., }</td>
</tr>
<tr>
<td>the division bar (e.g., }</td>
<td>should be treated as grouping symbols.</td>
<td></td>
</tr>
<tr>
<td>• Expressions are simplified using the order of operations and applying the properties of real numbers. Students should utilize the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of } in this standard):</td>
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<tr>
<td>- Commutative property of addition; } + } = } +</td>
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<tr>
<td>- Commutative property of multiplication; } \times</td>
<td></td>
<td></td>
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<tr>
<td>- Associative property of addition; ( + ) + } =</td>
<td></td>
<td></td>
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<tr>
<td>( + )</td>
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<td></td>
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<tr>
<td>- Associative property of multiplication; ( } )</td>
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<tr>
<td>- Subtraction and division are neither commutative nor associative.</td>
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<tr>
<td>- Distributive property (over addition/subtraction);</td>
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<tr>
<td>- Identity property of addition (additive identity property);</td>
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<tr>
<td>- Identity property of multiplication (multiplicative identity property);</td>
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<tr>
<td>- The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6.6 The student will
   a) add, subtract, multiply, and divide integers,* and [Moved from 7.3]
   b) solve practical problems involving operations with integers; and [Moved from 7.3]
   c) simplify numerical expressions involving integers.* [Moved and modified from 6.8] [Combined with 6.7, 2016 BOE first draft]

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<tr>
<td>are no identity elements for subtraction and division.</td>
<td>- Inverse property of addition (additive inverse property): $a + (-a) = 0$ and $(-a) + a = 0$</td>
<td></td>
</tr>
<tr>
<td>- Inverse property of addition (additive inverse property): $a + (-a) = 0$ and $(-a) + a = 0$</td>
<td>- Multiplicative property of zero: $a \cdot 0 = 0$ and $0 \cdot a = 0$</td>
<td></td>
</tr>
<tr>
<td>- Substitution property: if $a = b$ then $b$ can be substituted for $a$ in any expression, equation or inequality.</td>
<td>- The power of a number represents repeated multiplication of the number (e.g., $8^3 = 8 \cdot 8 \cdot 8$). The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In the example, 8 is the base, and 3 is the exponent. [Moved from 6.8]</td>
<td></td>
</tr>
<tr>
<td>- Any number, except 0, raised to the zero power is 1. Zero to the zero power ($0^0$) is undefined. [Moved from 6.8]</td>
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</tbody>
</table>
6.78 The student will evaluate whole numbers, simplify numerical expressions involving integers, using the order of operations. [*]

*On the state assessment, items measuring this objective are assessed without the use of a calculator. [Combined with 6.6, 2016]

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<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>What is the significance of the order of operations? The order of operations prescribes the order to use to simplify expressions containing more than one operation. It ensures that there is only one correct answer. [Moved to US]</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>The order of operations is a convention that defines the computation order to follow in simplifying an expression. Having an established convention ensures that there is only one correct result when simplifying an expression. [Moved from middle column]</td>
<td></td>
<td>Use the order of operations and apply the properties of real numbers to simplify numerical expressions involving integers by using the order of operations in a demonstrated step by step approach. The expressions should be limited to positive values and not include braces { }, but may use brackets [ ] or absolute value</td>
</tr>
<tr>
<td>The order of operations is as follows:</td>
<td></td>
<td>Find the value of numerical expressions, using order of operations, mental mathematics, and appropriate tools. Exponents are limited to positive values. [Combined with previous bullet]</td>
</tr>
<tr>
<td>First, complete all operations within grouping symbols.* If there are grouping symbols within other grouping symbols, do the innermost operation first.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second, evaluate all exponential expressions.</td>
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<tr>
<td>Third, multiply and/or divide in order from left to right.</td>
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<tr>
<td>Fourth, add and/or subtract in order from left to right.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parentheses ( ), brackets [ ], braces { }, absolute value</td>
<td>, (e.g.,</td>
<td>3(−5 + 2)</td>
</tr>
<tr>
<td>should be treated as grouping symbols.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expressions are simplified using the order of operations and applying the properties of real numbers.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The commutative property of addition states that changing the order of the addends does not affect the sum (e.g.,</td>
<td>5.6 + 4 = 4 + 5.6).</td>
<td></td>
</tr>
<tr>
<td>The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g.,</td>
<td>5.6 · 4 = 4 · 5.6).</td>
<td></td>
</tr>
<tr>
<td>The associative property of addition states that regrouping the addends does not change the sum (e.g.,</td>
<td>5.6 + (4 + 2) = (5.6 + 4) + 2).</td>
<td></td>
</tr>
<tr>
<td>The associative property of multiplication states that regrouping the factors does not change the product (e.g.,</td>
<td>5.6 · (4 · 2) = (5.6 · 4) · 2).</td>
<td></td>
</tr>
</tbody>
</table>

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 6
### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

<table>
<thead>
<tr>
<th>UNDERSTANDINGS</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Subtraction and division are neither commutative nor associative.</td>
<td></td>
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</tr>
<tr>
<td>- The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true. Distributive property examples:</td>
<td></td>
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</tr>
<tr>
<td>- $-5.2 \cdot (2 - 7) = (-5.2 \cdot 2) + (-5.2 \cdot 7)$, or</td>
<td></td>
<td></td>
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<tr>
<td>- $-5.2 \cdot (2 - 7) = (-5.2 - 2) \cdot (-5.2 - 7)$</td>
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<tr>
<td>- Converse of the distributive property example:</td>
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<tr>
<td>- $(-5.2 - 2) + (-5.2 - 7) = -5.2 \cdot (2 - 7)$, or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- $(-5.2 - 3) - (-5.2 - 7) = -5.2 - (2 - 7)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real number (e.g., $\frac{5}{2} - 6 + 0 = \frac{5}{2} - 6$).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number (e.g., $\sqrt{8} \cdot 1 = \sqrt{8}$.</td>
<td></td>
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</tr>
<tr>
<td>- Inverses are numbers that combine with other numbers and result in identity elements (e.g., $5 + (-5) = 0$; $\sqrt{8} \cdot 1 = \sqrt{8}$).</td>
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<tr>
<td>- The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., $5.6 + (-5.6) = 0$).</td>
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</tbody>
</table>
6.78 The student will evaluate whole numbers, simplify numerical expressions involving integers, using the order of operations.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator. [Combined with 6.6, 2016]

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<tr>
<td>The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., ( \frac{4}{4} = 1 )).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zero has no multiplicative inverse.</td>
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<tr>
<td>The multiplicative property of zero states that the product of any real number and zero is zero (e.g., ( 7 \cdot 0 = 0 ) and ( 0 \cdot 7 = 0 )).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Division by zero is not a possible mathematical operation. It is undefined.</td>
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<tr>
<td>The substitution property of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality. (e.g., if ( x = 5 + 8 ) then ( x = 3 ). Thus, 3 was substituted for ( 5 + 8 ) since they are equivalent).</td>
<td></td>
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</tr>
<tr>
<td>The power of a number represents repeated multiplication of the number (e.g., ( 8^3 = 8 \cdot 8 \cdot 8 )). The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In the example, 8 is the base, and 3 is the exponent.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Any number, except 0, raised to the zero power is 1. Zero to the zero power (0^0) is undefined.</td>
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</table>
Measurement and geometry in the middle grades, the focus of mathematics learning is to:
- build on students’ concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop the measurement skills that provide a natural context and connection among many mathematics concepts. Estimation skills are developed in determining length, weight/mass, liquid volume/capacity, and angle measure. Measurement is an essential part of mathematical explorations throughout the school year.
- Students continue to focus on experiences in which they measure objects physically and develop a deep understanding of the concepts and processes of measurement. Physical experiences in measuring various objects and quantities promote the long-term retention and understanding of measurement. Actual measurement activities are used to determine length, weight/mass, and liquid volume/capacity.
- Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, square-based pyramids, and cones.

Measurement and geometry in the middle grades provide a natural context and connection among many mathematical concepts. Students expand informal experiences with geometry and measurement in the elementary grades and develop a solid foundation for further exploration of these concepts in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.

Students develop measurement skills through exploration and estimation. Physical exploration to determine length, weight/mass, liquid volume/capacity, and angle measure are essential to develop a conceptual understanding of measurement. Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, square-based pyramids, and cones.

Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provide experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.

Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, square-based pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.
Students explore and develop an understanding of the Pythagorean Theorem. Understanding how the Pythagorean Theorem can be applied in practical situations has a far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

**Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

**Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

**Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

**Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)

**Level 4: Deduction.** Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)
6.9 The student will make ballpark comparisons between measurements in the U.S. Customary System of measurement and measurements in the metric system. [Included in 7.3 EKS]

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<tbody>
<tr>
<td>Making sense of various units of measure is an essential life skill, requiring reasonable estimates of what measurements mean, particularly in relation to other units of measure.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- 1 inch is about 2.5 centimeters.</td>
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</tr>
<tr>
<td>- 1 foot is about 30 centimeters.</td>
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<td></td>
</tr>
<tr>
<td>- 1 meter is a little longer than a yard, or about 40 inches.</td>
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<td></td>
</tr>
<tr>
<td>- 1 mile is slightly farther than 1.5 kilometers.</td>
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<tr>
<td>- 1 kilometer is slightly farther than half a mile.</td>
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<td></td>
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<tr>
<td>- 1 ounce is about 28 grams.</td>
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<td></td>
</tr>
<tr>
<td>- 1 nickel has the mass of about 5 grams.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- 1 kilogram is a little more than 2 pounds.</td>
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<td></td>
</tr>
<tr>
<td>- 1 quart is a little less than 1 liter.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- 1 liter is a little more than 1 quart.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Water freezes at 0°C and 32°F.</td>
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<td></td>
</tr>
<tr>
<td>- Water boils at 100°C and 212°F.</td>
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<tr>
<td>- Normal body temperature is about 37°C and 98°F.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Room temperature is about 20°C and 70°F.</td>
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</tr>
</tbody>
</table>

- Mass is the amount of matter in an object. Weight is the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location.

- What is the difference between weight and mass? Weight and mass are different. Mass is the amount of matter in an object. Weight is the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location.

- How do you determine which units to use at different times? Units of measure are determined by the attributes of the object being measured. Measures of length are expressed in linear units, measures of area are expressed in square units, and measures of volume are expressed in cubic units.

- Why are there two different measurement systems? Measurement systems are conventions invented by different cultures to meet their needs. The U.S. Customary System is the preferred method in the United States. The metric system is the preferred system worldwide.

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

  - Estimate the conversion of units of length, weight/mass, volume, and temperature between the U.S. Customary system and the metric system by using ballpark comparisons.

  - Ex: 1 L = 1qt.  Ex: 4L = 4 qts.

- Estimate measurements by comparing the object to be measured against a benchmark.
6.9 The student will make ballpark comparisons between measurements in the U.S. Customary System of measurement and measurements in the metric system. [Included in 7.3 EKS]

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<td>is determined by the situation.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Physically measuring objects along with using visual and symbolic representations improves student understanding of both the concepts and processes of measurement.</td>
<td></td>
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</tr>
</tbody>
</table>
6.7810 The student will
a) define \( \pi \) (pi) as the ratio of the circumference of a circle to its diameter;
b) solve problems, including practical problems, involving circumference and area of a circle, given the diameter or radius; and
c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles; and,
d) describe and determine the volume and surface area of a rectangular prism. [Included in 7.4a]

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<tr>
<td>• The value of pi (( \pi )) is the ratio of the circumference of a circle to its diameter. Thus, the circumference of a circle is proportional to its diameter.</td>
<td>• What is the relationship between the circumference and diameter of a circle? The circumference of a circle is about 3 times the measure of the diameter. [Moved to US]</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• The calculation of determining area and circumference may vary depending upon the approximation for pi. Common approximations for ( \pi ) include 3.14, ( \frac{22}{7} ), or the pi button on a calculator.</td>
<td>• What is the difference between area and perimeter? Perimeter is the distance around the outside of a figure while area is the measure of the amount of space enclosed by the perimeter. [Moved to US]</td>
<td>• Derive an approximation for ( \pi ) ((3.14 \text{ or } \frac{22}{7})) by gathering data and comparing the circumference to the diameter of various circles, using concrete materials or computer models. (a)</td>
</tr>
<tr>
<td>• Experiences in deriving the formulas for area, perimeter, and volume using manipulatives such as tiles, one-inch cubes, adding machine tape, graph paper, geoboards, or tracing paper, promote an understanding of the formulas and facility in their use.</td>
<td>• What is the relationship between area and surface area? Surface area is calculated for a three dimensional figure. It is the sum of the areas of the two-dimensional surfaces that make up the three-dimensional figure. [Moved to US]</td>
<td>• Find the circumference of a circle by substituting a value for the diameter or the radius into the formula ( C = \pi d \text{ or } C = 2\pi r ). [Included in bullet below]</td>
</tr>
<tr>
<td>• The perimeter of a polygon is the measure of the distance around the polygon.</td>
<td>• The area of a closed curve is the number of nonoverlapping squares units required to fill the region enclosed by the curve.</td>
<td>• Find the area of a circle by using the formula ( A = \pi r^2 ). [Included in bullet below]</td>
</tr>
<tr>
<td>• Perimeter is the path or distance around any plane figure. The perimeter of a circle is called the circumference. Circumference is the distance around or “perimeter” of a circle.</td>
<td>• The area of a closed curve is about three times the measure of its diameter. [Moved from middle column]</td>
<td>• Solve problems, including practical problems involving circumference and area of a circle when given the length of the diameter or radius. (b)</td>
</tr>
<tr>
<td>• The circumference of a circle is about three times the measure of its diameter. [Moved from middle column]</td>
<td>• The circumference of a circle is computed using ( C = \pi d \text{ or } C = 2\pi r ), where (d) is the diameter and (r) is the radius of the circle. [Reordered]</td>
<td>• Apply formulas to solve problems, including practical problems involving area and perimeter of triangles and rectangles. (c)</td>
</tr>
<tr>
<td>• The circumference of a circle is computed using ( C = \pi d \text{ or } C = 2\pi r ), where (d) is the diameter and (r) is the radius of the circle. [Reordered]</td>
<td>• The area of a closed curve is the number of nonoverlapping squares units required to fill the region enclosed by the curve.</td>
<td>• Solve practical problems involving circumference and area of a circle when given the diameter or radius. [Reordered]</td>
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<tr>
<td>• The area of a circle is computed using the formula ( A = \pi r^2 ), where (r) is the radius of the circle. [Reordered]</td>
<td>• The area of a circle is computed using the formula ( A = \pi r^2 ), where (r) is the radius of the circle. [Reordered]</td>
<td>• Create and solve problems that involve finding the circumference and area of a circle when given the diameter or radius. [Reordered]</td>
</tr>
<tr>
<td>• The area of a circle is computed using the formula ( A = \pi r^2 ), where (r) is the radius of the circle. [Reordered]</td>
<td></td>
<td>• Solve problems that require finding the surface area of</td>
</tr>
</tbody>
</table>

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 6
6.7810 The student will
a) define derive $\pi$ (pi) as the ratio of the circumference of a circle to its diameter;
b) solve problems, including practical problems, involving circumference and area of a circle, given the diameter or radius; and
c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles; and
d) describe and determine the volume and surface area of a rectangular prism. [Included in 7.4a]

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- The perimeter of a square whose side measures $s$ can be determined by multiplying 4 times by $s$ ($P = 4s$), and its area can be determined by squaring the length of one side times side ($A = s^2$).
- The perimeter of a rectangle is can be determined by computing the sum of twice the length and twice the width ($P = 2l + 2w$, or $P = 2(l + w)$), and its area is can be determined by computing the product of the length and the width ($A = lw$).
- The perimeter of a triangle can be determined by computing the sum of the side lengths ($P = a + b + c$), and its area can be determined by computing $\frac{1}{2}$ the product of base and the height ($A = \frac{1}{2}bh$).
- The value of $\pi$ is the ratio of the circumference of a circle to its diameter. [Reordered]
- The ratio of the circumference to the diameter of a circle is a constant value, $\pi$, which can be approximated by measuring various sizes of circles. [Reordered and rewritten]
- The fractional approximation of $\pi$ generally used is $\frac{22}{7}$. [Incorporated in bullet above]
- The decimal approximation of $\pi$ generally used is 3.14. [Incorporated in bullet above]
- The circumference of a circle is computed using $C = \pi d$ or $C = 2\pi r$, where $d$ is the diameter and $r$ is the radius of the circle. [Reordered]
The student will

a) **define derive** \( \pi \) (pi) as the ratio of the circumference of a circle to its diameter;

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<td></td>
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</tr>
<tr>
<td>• The surface area of a rectangular prism is the sum of the areas of all six faces ( (SA = 2lw + 2lh + 2wh) ). Moved to 7.4a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The volume of a rectangular prism is computed by multiplying the area of the base, ( B ), (length x width) by the height of the prism ( (V = lwh = Bh) ). [Moved to 7.4a]</td>
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</table>

\(^2\) Revised March 2011
In the middle grades, the focus of mathematics learning is to
• build on students’ concrete reasoning experiences developed in the elementary grades;
• construct a more advanced understanding of mathematics through active learning experiences;
• develop deep mathematical understandings required for success in abstract learning experiences; and
• apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

• Students expand the informal experiences they have had with geometry in the elementary grades and develop a solid foundation for the exploration of geometry in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.

• Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, mirrors, and geometry software provides experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.

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• Students explore and develop an understanding of the Pythagorean Theorem. Mastery of the use of the Pythagorean Theorem has far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

• **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

• **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

• **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)

Level 4: Deduction. Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)

[Included in Measurement and Geometry Strand Introduction]
STANDARD 6.8911 STRAND: MEASUREMENT AND GEOMETRY GRADE LEVEL 6

6.8911 The student will

a) identify the coordinates of a point in a components of the coordinate plane; and

b) identify the coordinates of a point and graph ordered pairs in a coordinate plane.

**UNDERSTANDING THE STANDARD**  
(Background Information for Instructor Use Only)

- In a coordinate plane, the coordinates of a point are typically represented by the ordered pair \((x, y)\), where \(x\) is the first coordinate and \(y\) is the second coordinate. However, any letters may be used to label the axes and the corresponding ordered pairs.
- Any given point is defined by only one ordered pair in the coordinate plane. [Moved from middle column]
- The grid lines on a coordinate plane are proportional and perpendicular.
- The axes of the coordinate plane are the two intersecting perpendicular lines that divide it into its four quadrants. The \(x\)-axis is the horizontal axis and the \(y\)-axis is the vertical axis.
- The quadrants of a coordinate plane are the four regions created by the two intersecting perpendicular number lines (\(x\) and \(y\)-axes). Quadrants are named in counterclockwise order. The signs on the ordered pairs for quadrant I are (+, +); for quadrant II, (−, +); for quadrant III, (−, −); and for quadrant IV, (+, −).
- In a coordinate plane, the origin is the point at the intersection of the \(x\)-axis and \(y\)-axis; the coordinates of this point are \((0, 0)\).
- For all points on the \(x\)-axis, the \(y\)-coordinate is 0. For all points on the \(y\)-axis, the \(x\)-coordinate is 0.
- The coordinates may be used to name the point. (e.g., the point \((2, 7)\)). It is not necessary to say “the point whose coordinates are \((2, 7)\)”. The first coordinate tells the location or distance of the point to the left or right of the \(y\)-axis and the second coordinate tells the location or distance of the point above or below the \(x\)-axis. For example, \((2, 7)\) is 2 units to the right of the \(y\)-axis and 7 units above the \(x\)-axis. [Moved from middle column]

**ESSENTIAL UNDERSTANDINGS**

- Can any given point be represented by more than one ordered pair? The coordinates of a point define its unique location in a coordinate plane. Any given point is defined by only one ordered pair. [Moved to US]
- In naming a point in the plane, does the order of the two coordinates matter? Yes. The first coordinate tells the location of the point to the left or right of the \(y\)-axis and the second point tells the location of the point above or below the \(x\)-axis. Point \((0, 0)\) is at the origin. [Moved to US]

**ESSENTIAL KNOWLEDGE AND SKILLS**

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify and label the axes, origin, and quadrants of a coordinate plane. (a)
- Identify and label the quadrants of a coordinate plane. [Included in bullet above]
- Identify the quadrant or the axis on which a point is positioned by examining the coordinates (ordered pair) of the point. Ordered pairs will be limited to coordinates expressed as integers. (a)
- Ordered pairs will be limited to coordinates expressed as integers. (b) [Included in bullets above and below]
- Graph ordered pairs in the four quadrants and on the axes of a coordinate plane. Ordered pairs will be limited to coordinates expressed as integers. (b)
- Identify ordered pairs represented by points in the four quadrants and on the axes of the coordinate plane. Ordered pairs will be limited to coordinates expressed as integers. (b)
- Relate the coordinates of a point to the distance from each axis and relate the coordinates of a single point to another point on the same horizontal or vertical line. Ordered pairs will be limited to coordinates expressed as integers. * (b)
- Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to determine the length of a side joining points with the same first coordinate or the same second coordinate.
The student will

a) identify the coordinates of a point in a components of the coordinate plane; and

b) identify the coordinates of a point and graph ordered pairs in a coordinate plane.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Coordinates of points having the same \( x \)-coordinate are located on the same vertical line. For example, \((2, 4)\) and \((2, -3)\) are both 2 units to the right of the \( y \)-axis and are vertically 7 units from each other.

- Coordinates of points having the same \( y \)-coordinate are located on the same horizontal line. For example, \((-4, -2)\) and \((2, -2)\) are both 2 units below the \( x \)-axis and are horizontally 6 units from each other.

### ESSENTIAL UNDERSTANDINGS

- Ordered pairs will be limited to coordinates expressed as integers. Apply these techniques in the context of solving practical and mathematical problems. (b) [Moved from 6.12]

### ESSENTIAL KNOWLEDGE AND SKILLS

\[ ^\dagger \text{Revised March 2011} \]
The student will determine congruence of segments, angles, and polygons.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>- The symbol for congruency is ( \cong ). [Reordered]</td>
<td>- Given two congruent figures, what inferences can be drawn about how the figures are related? The congruent figures will have exactly the same size and shape. [Included in US]</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:</td>
</tr>
<tr>
<td>- Congruent figures have exactly the same size and the same shape. Line segments are congruent if they have the same length. Angles are congruent if they have the same measure. Congruent polygons have an equal number of sides, and all the corresponding sides and angles are congruent. [Same as middle column] For example:</td>
<td>- Corresponding angles of congruent polygons will have the same measure.</td>
<td>- Identify regular polygons.</td>
</tr>
<tr>
<td>- A polygon is a closed plane figure composed of at least three line segments that do not cross.</td>
<td>- Corresponding sides of congruent polygons will have the same measure. [Included in US]</td>
<td>- Draw the lines of symmetry for to divide regular polygons into two congruent parts.</td>
</tr>
<tr>
<td>- A regular polygon has congruent sides and congruent interior angles.</td>
<td>- The number of lines of symmetry of a regular polygon is equal to the number of sides of the polygon.</td>
<td>- Determine the congruence of segments, angles, and polygons given their attributes. [Reordered]</td>
</tr>
<tr>
<td>- The number of lines of symmetry of a regular polygon is equal to the number of sides of the polygon.</td>
<td>- A line of symmetry divides a figure into two congruent parts, each of which is the mirror image of the other. Lines of symmetry are not limited to horizontal and vertical lines.</td>
<td>- Characterize Determine if polygons are congruent and/or noncongruent according to the measures of their sides and angles.</td>
</tr>
<tr>
<td>- Noncongruent figures may have the same shape but not the same size.</td>
<td>- Noncongruent figures may have the same shape but not the same size.</td>
<td></td>
</tr>
</tbody>
</table>

*Revised March 2014*
The student will determine congruence of segments, angles, and polygons.

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<thead>
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<tbody>
<tr>
<td>• The symbol for congruency is ( \equiv ). [Reordered]</td>
<td></td>
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</tr>
<tr>
<td>• The corresponding angles of congruent polygons have the same measure, and the corresponding sides of congruent polygons have the same measure. [Included in bullet above]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Students should be familiar with geometric markings in figures to indicate congruence of sides and angles and to indicate parallel sides. An equal number of hatch (hash) marks indicate that those sides are equal in length. An equal number of arrows indicate that those sides are parallel. An equal number of angle curves indicate that those angles have the same measure. See included diagram.</td>
<td></td>
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</tr>
<tr>
<td>[Diagram of a parallelogram with hatch marks, parallel arrows, and angle curves]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The determination of the congruence or noncongruence of two figures can be accomplished by placing one figure on top of the other or by comparing the measurements of all corresponding sides and angles. [Same as middle column]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Construction of congruent line segments, angles, and polygons helps students understand congruency.</td>
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</tbody>
</table>
## STANDARD 6.13 STRAND: GEOMETRY GRADE LEVEL 6

### 6.13 The student will describe and identify properties of quadrilaterals. [Included in 7.6]

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
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</tr>
</thead>
<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>- Can a figure belong to more than one subset of quadrilaterals? Any figure that has the attributes of more than one subset of quadrilaterals can belong to more than one subset. For example, rectangles have opposite sides of equal length. Squares have all 4 sides of equal length thereby meeting the attributes of both subsets.</td>
<td>The student will use problem-solving, mathematical communication, mathematical reasoning, connections, and representations to:</td>
</tr>
<tr>
<td>- A quadrilateral is a closed planar (two-dimensional) figure with four sides that are line segments.</td>
<td>- Sort and classify polygons as quadrilaterals, parallelograms, rectangles, trapezoids, kites, rhombi, and squares based on their properties. Properties include number of parallel sides, angle measures and number of congruent sides.</td>
<td>- Identify the sum of the measures of the angles of a quadrilateral as 360°.</td>
</tr>
<tr>
<td>- A parallelogram is a quadrilateral whose opposite sides are parallel and opposite angles are congruent.</td>
<td>- Quadrilaterals can be sorted according to common attributes, using a variety of materials.</td>
<td></td>
</tr>
<tr>
<td>- A rectangle is a parallelogram with four right angles.</td>
<td>- Quadrilaterals can be classified by the number of parallel sides: a parallelogram, rectangle, rhombus, and square each have two pairs of parallel sides; a trapezoid has only one pair of parallel sides; other quadrilaterals have no parallel sides.</td>
<td></td>
</tr>
<tr>
<td>- Rectangles have special characteristics (such as diagonals are bisectors) that are true for any rectangle.</td>
<td>- Quadrilaterals can be classified by the measures of their angles: a rectangle has four 90° angles; a trapezoid may have zero or two 90° angles.</td>
<td></td>
</tr>
<tr>
<td>- To bisect means to divide into two equal parts.</td>
<td>- Quadrilaterals can be classified by the number of congruent sides: a rhombus has four congruent sides; a square, which is a rhombus with four right angles, also has four congruent sides; a kite is a quadrilateral with two pairs of adjacent congruent sides. One pair of opposite angles is congruent.</td>
<td></td>
</tr>
<tr>
<td>- A square is a rectangle with four congruent sides or a rhombus with four right angles.</td>
<td>- Quadrilaterals can be classified by the measures of their angles: a rectangle has four 90° angles; a rhombus has four 90° angles.</td>
<td></td>
</tr>
<tr>
<td>- A rhombus is a parallelogram with four congruent sides.</td>
<td>- Quadrilaterals can be classified by the number of parallel sides: a parallelogram, rectangle, rhombus, and square each have two pairs of parallel sides; a trapezoid has only one pair of parallel sides; other quadrilaterals have no parallel sides.</td>
<td></td>
</tr>
<tr>
<td>- A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called bases, and the nonparallel sides are called legs. If the legs have the same length, then the trapezoid is an isosceles trapezoid.</td>
<td>- Quadrilaterals can be classified by the number of congruent sides: a rhombus has four congruent sides; a square, which is a rhombus with four right angles, also has four congruent sides; a kite is a quadrilateral with two pairs of adjacent congruent sides. One pair of opposite angles is congruent.</td>
<td></td>
</tr>
<tr>
<td>- A kite is a quadrilateral with two pairs of adjacent congruent sides. One pair of opposite angles is congruent.</td>
<td>- Quadrilaterals can be classified by the measures of their angles: a rectangle has four 90° angles; a trapezoid may have zero or two 90° angles.</td>
<td></td>
</tr>
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<td>- Quadrilaterals can be sorted according to common attributes, using a variety of materials.</td>
<td>- Quadrilaterals can be classified by the number of parallel sides: a parallelogram, rectangle, rhombus, and square each have two pairs of parallel sides; a trapezoid has only one pair of parallel sides; other quadrilaterals have no parallel sides.</td>
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<td>- Quadrilaterals can be classified by the measures of their angles: a rectangle has four 90° angles; a rhombus has four 90° angles.</td>
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Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 6
6.13 The student will describe and identify properties of quadrilaterals. [Included in 7.6]

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>parallelogram and a rectangle each have two pairs of congruent sides.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- A square is a special type of both a rectangle and a rhombus, which are special types of parallelograms, which are special types of quadrilaterals.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- The sum of the measures of the angles of a quadrilateral is 360°.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- A chart, graphic organizer, or Venn Diagram can be made to organize quadrilaterals according to attributes such as sides and/or angles.</td>
<td></td>
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</tr>
</tbody>
</table>
In the middle grades, the focus of mathematics learning is to
- build on students’ concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop an awareness of the power of data analysis and probability by building on their natural curiosity about data and making predictions.
- Students explore methods of data collection and use technology to represent data with various types of graphs. They learn that different types of graphs represent different types of data effectively. They use measures of center and dispersion to analyze and interpret data.
- Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability.
- Students explore experimental and theoretical probability through experiments and simulations by using concrete, active learning activities.

In the middle grades, students develop an awareness of the power of data analysis and the application of probability through fostering their natural curiosity about data and making predictions.

The exploration of various methods of data collection and representation allows students to become effective at using different types of graphs to represent different types of data. Students use measures of center and dispersion to analyze and interpret data.

Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability. Through experiments and simulations, students build on their understanding of the Fundamental Counting Principle from elementary mathematics to learn more about probability in the middle grades.
6.104114 The student, given a problem practical situation, will
a) construct represent data in a circle graphs;
b) draw conclusions and make predictions, observations and inferences about data represented in a using circle graphs; and
c) compare and contrast circle graphs with the same data represented in bar graphs, picture graphs, and line plots, that present information from the same data set.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Circle graphs are used for data showing a relationship of the parts to the whole. [Moved from middle column]
  - Example: the favorite fruit of 20 students in Mrs. Jones class was recorded in the table. Compare the same data displayed in both a circle graph and a bar graph.

<table>
<thead>
<tr>
<th>Fruit Preference</th>
<th># of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>banana</td>
<td>6</td>
</tr>
<tr>
<td>apple</td>
<td>7</td>
</tr>
<tr>
<td>pear</td>
<td>3</td>
</tr>
<tr>
<td>strawberry</td>
<td>4</td>
</tr>
</tbody>
</table>

Fruit Preferences in Mrs. Jones Class

ESSENTIAL UNDERSTANDINGS

- What types of data are best presented in a circle graph?
  Circle graphs are best used for data showing a relationship of the parts to the whole. [Moved to US]

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Collect, organize and display represent data in a circle graph by depicting information as fractional. Number of data values should be limited to allow for comparisons that have denominators of 12 or less or those that are factors of 100 (e.g., in a class of 20 students, 7 choose apples as a favorite fruit, so the comparison is 7 out of 20, \(\frac{7}{20}\), or 35%). (a)

- Draw conclusions and make predictions, observations and inferences about data represented in a circle graph. (b)

- Compare and contrast data represented in a circle graph with the same data represented in other graphical forms: bar graphs, picture graphs, and line plots. (c)
STANDARD 6.10.1114 STRAND: PROBABILITY AND STATISTICS GRADE LEVEL 6

6.10.1114 The student, given a problem-practical situation, will
a) construct represent data in a circle graphs;
b) draw conclusions and make predictions, observations and inferences about data represented in a using circle graphs; and
c) compare and contrast circle graphs with the same data represented in bar graphs, picture graphs, and line plots, that present information from the same data set.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Circle graphs can represent percent or frequency.</td>
<td>• Teachers should be reasonable about the selection of data values. The number of data values can affect how a circle graph is constructed (e.g., 10 out of 25 would be 40%, but 7 out of 9 would be 77.7%, making the construction of a circle graph more complex). Students should have experience constructing circle graphs, but a focus should be placed on the analysis of circle graphs.</td>
</tr>
<tr>
<td>• Circle graphs are not useful for data with large numbers of categories.</td>
<td>• Students are not expected to construct circle graphs by multiplying the percentage of data in a category by 360° in order to determine the central angle measure. Limiting comparisons to fraction parameters noted will assist students in estimating the central angle measures needed to constructing</td>
</tr>
</tbody>
</table>

Circle Preferences In Mrs. Jones Class

Fruit Preference

- bananas
- apple
- pear
- strawberry

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 6
6.1014 The student, given a problem situation, will
a) construct a circle graph;

b) draw conclusions and make predictions, observations and inferences about data represented in a circle graph; and

c) compare and contrast circle graphs with the same data represented in bar graphs, pictograph, and line plots, that present information from the same data set.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>circle graphs.</td>
<td>- To collect data for any problem situation, an experiment can be designed, a survey can be conducted, or other data-gathering strategies can be used. The data can be organized, displayed, analyzed, and interpreted to answer the problem.</td>
<td></td>
</tr>
<tr>
<td>- Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Different types of graphs can be used to display different types of categorical data. The way data is displayed is often dependent on what someone is trying to communicate.</td>
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</tr>
<tr>
<td>- A line plot is used for categorical and discrete numerical data and is used to show frequency of data on a number line. It is a simple way to organize data. Example:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Candy Bars</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[Diagram of a line plot with data points]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- A bar graph is used for categorical (and discrete) numerical data (e.g., number of months or number of people with a particular eye color) and is used to show comparisons.</td>
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</tr>
</tbody>
</table>
6.10.11  The student, given a problem-practical situation, will
a) construct represent data in a circle graphs;
  b) draw conclusions and make predictions, observations and inferences about data represented in a circle graphs; and
  c) compare and contrast circle graphs with the same data represented in bar graphs, picture graphs, and line plots, that present information from the same data set.

### UNDERSTANDING THE STANDARD

**(Background Information for Instructor Use Only)**

<table>
<thead>
<tr>
<th>Essential Understandings</th>
<th>Essential Knowledge and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>A picture graph is mainly used to show categorical data use continuous data (e.g., temperature and time). Picture graphs are used to show frequency and compare items. However, the use of partial pictures can give misleading information. Example: The Types of Pets We Have</td>
<td>All graphs must include a title, percent or number labels, and data categories should have labels for data categories, and a key. A key is essential to explain how to read the graph. A title is essential to explain what the graph represents.</td>
</tr>
<tr>
<td>Cat</td>
<td>Dog</td>
</tr>
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<td>😸</td>
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<tr>
<td>= 1 student</td>
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</tbody>
</table>

A circle graph is used for categorical and discrete numerical data. Circle graphs are used to show a relationship of the parts to a whole. A show a relationship of the parts to a whole.

- A scale should be chosen that is appropriate for the data values being represented.
6.1011 The student, given a problem-practical situation, will
a) construct represent data in a circle graphs;
   b) draw conclusions and make predictions, observations and inferences about data represented in a using circle graphs; and
   c) compare and contrast circle graphs with the same data represented in bar graphs, picture graphs, and line plots, that present information from the same data set.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
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<tbody>
<tr>
<td>• A key is essential to explain how to read the graph. [Included in bullet above]</td>
</tr>
<tr>
<td>• A title is essential to explain what the graph represents. [Included in bullet above]</td>
</tr>
<tr>
<td>• Data are analyzed by describing the various features and elements of a graph.</td>
</tr>
<tr>
<td>• Comparisons, predictions, and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions.</td>
</tr>
<tr>
<td>• The information displayed in different graphs may be examined to determine how data are or are not related, differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or “what could happen if” (inferences).</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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</tbody>
</table>
STANDARD 6.11215 STRAND: PROBABILITY AND STATISTICS GRADE LEVEL 6

6.11215 The student will

a) describe represent the mean of a data set graphically as the balance point; and

b) decide which determine an appropriate measure of center is appropriate for a given purpose situation; and

be) determine the impact on measures of center when a single value of a data set is added, removed, or changed. [Moved from 5.16 EKS]

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data.

- Measures of center are types of averages for a data set. They represent numbers that describe a data set. Mean, median, and mode are measures of center that are useful for describing the average for different situations.
  - Mean works well may be appropriate for sets of data with no very high or low numbers where there are no values much higher or lower than those in the rest of the data set.
  - Median is a good choice when data sets have a couple of values much higher or lower than most of the others.
  - Mode is a good descriptor to use when the set of data has some identical values, or when data is non-numeric (categorical) or when asked to choose the most popular item are not conducive to computation of other measures of central tendency, as when working with data in a yes or no survey.

- Mean can be defined as the point on a number line where the data distribution is balanced. This requires that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances from the mean of all the data points below the mean. This is the concept of mean as the balance point. [Reordered]

  - **Example:** Given the data set: 2, 3, 4, 7
    - The mean value of 4 can be represented on a number line as the balance point.

- ESSENTIAL UNDERSTANDINGS

  - What does the phrase “measure of center” mean? This is a collective term for the 3 types of averages for a set of data—mean, median, and mode. [Included in US]

  - What is meant by mean as balance point? Mean can be defined as the point on a number line where the data distribution is balanced. This means that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances below the mean. This is the concept of mean as the balance point. [Included in US]

- ESSENTIAL KNOWLEDGE AND SKILLS

  The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

  - Find Represent the mean for of a set of data graphically as the balance point represented in a line plot. (a)

  - Describe Determine three appropriate measures of center for a given situation in which each would best represent a set of data. (b)

  - Identify and draw a number line that demonstrates the concept of mean as balance point for a set of data.

  - Determine the impact on measures of center when a single value of a data set is added, removed, or changed. (eb) [Moved from 5.16 EKS]
**STANDARD 6.11215**  
**STRAND: PROBABILITY AND STATISTICS**  
**GRADE LEVEL 6**

6.11215 The student will:

- a) **describe** represent the **mean** of a data set graphically as the balance point; and
- b) **decide** which measure of center is appropriate for a given purpose/situation; and
- (c) **determine** the impact on measures of center when a single value of a data set is added, removed, or changed. [Moved from 5.16 EKS]

### UNDERSTANDING THE STANDARD  
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<table>
<thead>
<tr>
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<tbody>
<tr>
<td><img src="image" alt="Mean as Balance Point" /></td>
</tr>
</tbody>
</table>

- The mean can also be found by calculating it as the numerical average of the data set and is found by adding the numbers in the data set together and dividing the sum by the number of data pieces in the set.

- In grade 5 mathematics, mean is defined as fair-share.

- Mean can be defined as the point on a number line where the data distribution is balanced. This means that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances of all the data points below the mean. This is the concept of mean as the balance point. [Reordered]

- Defining mean as the balance point is a prerequisite for understanding standard deviation.

- The median is the middle value of a data set in ranked order. If there are an odd number of pieces of data, the median is the middle value in ranked order. If there is an even number of pieces of data, the median is the numerical average of the two middle values.

- The mode is the piece of data that occurs most frequently. If no value occurs more often than any other, there is no mode. If there is more than one value that occurs most often, all these most-frequently-occurring values are modes. When there are exactly two modes, the data set is bimodal.
6.16 The student will
a) compare and contrast dependent and independent events; and [Moved to 8.11a]
b) determine probabilities for dependent and independent events. [Included in 8.11b]

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- The probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes (sample space).
- The probability of an event occurring can be represented as a ratio or the equivalent fraction, decimal, or percent.
- The probability of an event occurring is a ratio between 0 and 1.
  - A probability of 0 means the event will never occur.
  - A probability of 1 means the event will always occur.
- A simple event is one event (e.g., pulling one sock out of a drawer and examining the probability of getting one color).
- Events are independent when the outcome of one has no effect on the outcome of the other. For example, rolling a number cube and flipping a coin are independent events.
- The probability of two independent events is found by using the following formula:
  \[ P(A \text{ and } B) = P(A) \cdot P(B) \].
- Ex: When rolling two number cubes simultaneously, what is the probability of rolling a 3 on one cube and a 4 on the other?
  \[ P(3 \text{ and } 4) = P(3) \cdot P(4) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \]

ESSENTIAL UNDERSTANDINGS

- How can you determine if a situation involves dependent or independent events?
- Events are independent when the outcome of one has no effect on the outcome of the other. Events are dependent when the outcome of one event is influenced by the outcome of the other.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Determine whether two events are dependent or independent.
- Compare and contrast dependent and independent events.
- Determine the probability of two dependent events.
- Determine the probability of two independent events.
6.16 The student will
   a) compare and contrast dependent and independent events; and [Moved to 8.11a]
   b) determine probabilities for dependent and independent events. [Included in 8.11b]

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Events are dependent when the outcome of one event is influenced by the outcome of the other. For example, when drawing two marbles from a bag, not replacing the first after it is drawn affects the outcome of the second draw.</td>
<td>The probability of two dependent events is found by using the following formula:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ P(\text{A and B}) = P(\text{A}) \times P(\text{B after A}) ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ex: You have a bag holding a blue ball, a red ball, and a yellow ball. What is the probability of picking a blue ball out of the bag on the first pick and then without replacing the blue ball in the bag, picking a red ball on the second pick?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ P(\text{blue and red}) = P(\text{blue}) \times P(\text{red after blue}) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} ]</td>
<td></td>
</tr>
</tbody>
</table>
In the middle grades, the focus of mathematics learning is to
- build on students’ concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students extend their knowledge of patterns developed in the elementary grades and through life experiences by investigating and describing functional relationships.
- Students learn to use algebraic concepts and terms appropriately. These concepts and terms include variable, term, coefficient, exponent, expression, equation, inequality, domain, and range. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.
- Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.
- Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.

Patterns, functions and algebra become a larger mathematical focus in the middle grades as students extend their knowledge of patterns developed in the elementary grades.

Students make connections between the numeric concepts of ratio and proportion and the algebraic relationships that exist within a set of equivalent ratios. Students use variable expressions to represent proportional relationships between two quantities and begin to connect the concept of a constant of proportionality to rate of change and slope. Representation of relationships between two quantities using tables, graphs, equations, or verbal descriptions allow students to connect their knowledge of patterns to the concept of functional relationships. Graphing linear equations in two variables in the coordinate plane is a focus of the study of functions which continues in high school mathematics.

Students learn to use algebraic concepts and terms appropriately. These concepts and terms include variable, term, coefficient, exponent, expression, equation, inequality, domain, and range. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades. Students learn to solve equations by using concrete materials. They expand their skills from one-step to multistep equations and inequalities through their application in practical situations.
6.17 The student will identify and extend geometric [Included in AFDA.1 EKS and AII.5] and arithmetic [Included in AI.7 EKS, AFDA.1 EKS and AII.5] sequences.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical patterns may include linear and exponential growth, perfect squares, triangular and other polygonal numbers, or Fibonacci numbers.</td>
<td>What is the difference between an arithmetic and a geometric sequence? While both are numerical patterns, arithmetic sequences are additive and geometric sequences are multiplicative.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>Arithmetic and geometric sequences are types of numerical patterns.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>In the numerical pattern of an arithmetic sequence, students must determine the difference, called the common difference, between each succeeding number in order to determine what is added to each previous number to obtain the next number. Sample numerical patterns are 6, 9, 12, 15, 18, ...; and 5, 7, 9, 11, 13, ....</td>
<td></td>
<td></td>
</tr>
<tr>
<td>In geometric number patterns, students must determine what each number is multiplied by to obtain the next number in the geometric sequence. This multiplier is called the common ratio. Sample geometric number patterns include 2, 4, 8, 16, 32, ...; 1, 5, 25, 125, 625, ...; and 80, 20, 5, 1.25, ....</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strategies to recognize and describe the differences between terms in numerical patterns include, but are not limited to, examining the change between consecutive terms, and finding common factors. An example is the pattern 1, 2, 4, 7, 11, 16, ....</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investigate and apply strategies to recognize and describe the change between terms in arithmetic patterns.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investigate and apply strategies to recognize and describe geometric patterns.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Describe verbally and in writing the relationships between consecutive terms in an arithmetic or geometric sequence.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extend and apply arithmetic and geometric sequences to similar situations.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extend arithmetic and geometric sequences in a table by using a given rule or mathematical relationship.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compare and contrast arithmetic and geometric sequences.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Identify the common difference for a given arithmetic sequence.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Identify the common ratio for a given geometric sequence.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6.1213 The student will
a) write an equation in the form \( y = kx \), where \( k \) is the constant of proportionality, to represent a proportional relationship between two quantities, including those arising from practical problems situations;
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
c) determine if a proportional relationship exists between two quantities; and [Moved from 7.10 (2016)]
d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs, and equations.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between quantities.
- Equivalent ratios arise by multiplying each value in a ratio by the same constant value. For example, the ratio of 4:2 would be equivalent to the ratio 8:4, since each value in the first ratio could be multiplied by 2 to obtain the second ratio.
- A proportional relationship consists of two quantities where there exists a constant number (constant of proportionality) such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.
- Proportional thinking requires students to thinking multiplicatively, versus additively. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e., one quantity is the result of multiplying the other quantity by a value). Therefore, it is important to use practical situations to model proportional relationships, since context can help students to see the relationship. Students will explore algebraic representations of additive relationships in Grade 7.

Example:

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Write an equation in the form \( y = kx \) representing a proportional relationship between quantities. The constant of proportionality, \( k \), will be limited to positive integers. (a)
- Given a ratio, make a table of equivalent ratios to represent a proportional relationship between two quantities. (a)
- Given a practical situation, make a table of equivalent ratios to represent a proportional relationship between two quantities. (a)
- Identify the unit rate of a proportional relationship represented by a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (b)
- Determine a missing value in a ratio table that represents a proportional relationship between two quantities using a unit rate. Unit rates are limited to positive values. (b)
- Determine if a proportional relationship exists between two quantities given a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive...
The student will:

a) write an equation in the form \( y = kx \), where \( k \) is the constant of proportionality, to represent a proportional relationship between two quantities, including those arising from practical problem situations;

b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine if a proportional relationship exists between two quantities; and [Moved from 7.10 (2016)]

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs, and equations.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

<table>
<thead>
<tr>
<th>Additive relationship:</th>
<th>Multiplicative relationship:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( y )</td>
</tr>
<tr>
<td>2</td>
<td>+8</td>
</tr>
<tr>
<td>3</td>
<td>+8</td>
</tr>
<tr>
<td>4</td>
<td>+8</td>
</tr>
<tr>
<td>5</td>
<td>+8</td>
</tr>
</tbody>
</table>

In the additive relationship, \( y \) is the result of adding 8 to \( x \).

In the multiplicative relationship, \( y \) is the result of multiplying 5 times \( x \).

The ordered pair (2, 10) is a quantity in both relationships, however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative.

- Students have had experiences with tables of values (input/output tables that are additive and multiplicative) in elementary grades.
- A ratio table is a table of values representing a proportional relationship that includes pairs of values that represent equivalent rates or ratios. A constant exists that can be multiplied by the measure of one quantity to get the measure of the other quantity for every ratio pair. The same proportional relationship exists between each pair of quantities in a ratio table.

Example: Given that the ratio of \( y \) to \( x \) in a proportional relationship is 8:4, create a ratio table that includes three additional equivalent ratios.

### ESSENTIAL KNOWLEDGE AND SKILLS

- Determine if a proportional relationship exists between two quantities given a graph of ordered pairs. Unit rates are limited to positive values. (c)
- Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs and equations. Unit rates are limited to positive values. (d)
The student will:

a) write an equation in the form $y = kx$, where $k$ is the constant of proportionality, to represent a proportional relationship between two quantities, including those arising from practical problems; 

b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table; 

c) determine if a proportional relationship exists between two quantities; and [Moved from 7.10 (2016)]

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs, and equations.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

Students have had experience with tables of values (input/output tables) in elementary grades and the concept of a ratio table should be connected to their prior knowledge of representing number patterns in tables.

- A rate is a ratio that involves two different units and how they relate to each other. Relationships between two units of measure are also rates (e.g., inches per foot).

- A unit rate describes how many units of the first quantity of a ratio correspond to one unit of the second quantity.
  - Example: If it costs $10 for 5 items at a store (a ratio of 10:5 comparing cost to the number of items), then the unit rate would be $2.00 per item (a ratio of 2:1 comparing cost to number of items).

<table>
<thead>
<tr>
<th># of items ($x$)</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost in $ ($y$)</td>
<td>$2.00$</td>
<td>$4.00$</td>
<td>$10.00$</td>
<td>$20.00$</td>
</tr>
</tbody>
</table>

- Any ratio can be converted into a unit rate by writing the ratio as a fraction and then dividing the numerator and denominator each by the value of the
The student will:

a) write an equation in the form \( y = kx \), where \( k \) is the constant of proportionality, to represent a proportional relationship between two quantities, including those arising from practical problems; 

b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table; 

c) determine if a proportional relationship exists between two quantities; and [Moved from 7.10 (2016)]

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs, and equations.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

**Example:** It costs $8 for 16 gourmet cookies at a bake sale. What is the price per cookie (unit rate) represented by this situation?

\[
\frac{8}{16} = \frac{8+16}{16+16} = \frac{0.5}{1}
\]

So, it would cost $0.50 per cookie, which would be the unit rate.

- Example: \( \frac{8}{16} \) and 40 to 10 are ratios, but are not unit rates. However, \( \frac{0.5}{1} \) and 4 to 1 are unit rates.

- Students in grade 6 should build a conceptual understanding of proportional relationships and unit rates before moving to more abstract representations and complex computations in higher grade levels. Students are not expected to use formal calculations for slope and unit rates (e.g., slope formula) in grade 6.

- A table of values representing a proportional relationship includes pairs of values that represent equivalent rates or ratios. A constant exists that can be multiplied by the measure of one quantity to get the measure of the other quantity for every ratio pair.

- Example of a proportional relationship:

  Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges $8 for each medium pizza. This ratio table represents the cost \((y)\) per number of pizzas ordered \((x)\).
6.12.13 The student will
a) write an equation in the form \( y = kx \), where \( k \) is the constant of proportionality, to represent a proportional relationship between two quantities, including those arising from practical problem situations;
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
c) determine if a proportional relationship exists between two quantities; and [Moved from 7.10 (2016)]
d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs, and equations.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x ) number of pizzas</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4 \times 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y ) total cost</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
</tr>
</tbody>
</table>

In this relationship, the ratio of \( y \) (cost in $) to \( x \) (number of pizzas) in each ordered pair is the same:

\[
\frac{8}{1} = \frac{16}{2} = \frac{24}{3} = \frac{32}{4}
\]

Example of a non-proportional relationship:

Uptown Pizza sells medium pizzas for $7 each but charges a $3 delivery fee per order. This table represents the cost per number of pizzas ordered.

<table>
<thead>
<tr>
<th>( x ) number of pizzas</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y ) total cost</td>
<td>10</td>
<td>17</td>
<td>24</td>
<td>31</td>
</tr>
</tbody>
</table>

The ratios represented in the table above are not equivalent.

In this relationship, the ratio of \( y \) to \( x \) in each ordered pair is not the same:

\[
\frac{10}{1} \neq \frac{17}{2} \neq \frac{24}{3} \neq \frac{31}{4}
\]

Other non-proportional relationships that are not linear will be studied in later courses.
STANDARD 6.1213 STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA GRADE LEVEL 6

6.1213 The student will

a) write an equation in the form \( y = kx \), where \( k \) is the constant of proportionality, to represent a proportional relationship between two quantities, including those arising from practical problems; situations;
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
c) determine if a proportional relationship exists between two quantities; and [Moved from 7.10 (2016)]
d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs, and equations.

UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Proportions represent equivalent ratios. Equivalent ratios arise by multiplying each value in a ratio pair by the same positive number. For example, the ratio of 3:2 would be equivalent to the ratio 6:4.
- A proportion is an equation that states that two ratios are equal. For example: \( \frac{2}{4} = \frac{1}{2} \) represents a proportion:
- A proportional relationship can be represented using verbal descriptions, ratio tables, graphs, and equations. Proportional relationships can be described verbally using the phrases “for each,” “for every,” and “per.”
- Proportional relationships involve collections of pairs of equivalent ratios that may be graphed in the coordinate plane. The graph of a proportional relationship includes ordered pairs \((x, y)\) that represent pairs of values that may be represented in a ratio table.
- Proportional relationships can be represented by the equation \( y = kx \), where \( k \) represents the constant of proportionality. An example of a proportional relationship: Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges $8 for each medium pizza. This table represents the cost per number of pizza ordered.

<table>
<thead>
<tr>
<th>x number of pizzas</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y total cost</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
</tr>
</tbody>
</table>

This relationship can be represented by \( y = 8x \) where \( y \) is the cost and \( x \) is the number of medium pizzas ordered. The constant of proportionality is evident with the table as the multiplicative relationship that exists within each row.
STANDARD 6.12.13 STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA GRADE LEVEL 6

6.12.13 The student will

a) write an equation in the form \( y = kx \), where \( k \) is the constant of proportionality, to represent a proportional relationship between two quantities, including those arising from practical problems situations;

b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine if a proportional relationship exists between two quantities; and [Moved from 7.10 (2016)]

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs, and equations.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

ESSENTIAL KNOWLEDGE AND SKILLS

ratio. In this example, it is 3.
An example of a non-proportional relationship: Uptown Pizza sells medium pizzas for $7 each but charges a $3 delivery fee per order. This table represents the cost per number of pizzas ordered.

<table>
<thead>
<tr>
<th>x number of pizzas</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y total cost</td>
<td>10</td>
<td>17</td>
<td>24</td>
<td>31</td>
</tr>
</tbody>
</table>

The ratios represented in the table are not equivalent. Other non-proportional relationships that are not linear will be studied in later courses.

- Proportional relationships can be expressed using verbal descriptions, tables, and graphs, and equations.
  - Example: (verbal description) To make a drink, mix 1 liter of syrup with 3 liters of water. If \( x \) represents how many liters of syrup are in the mixture and \( y \) represents how many liters of water are in the mixture, this proportional relationship can be represented using a ratio table:

<table>
<thead>
<tr>
<th>Syrup (liters) ( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water (liters) ( y )</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
</tr>
</tbody>
</table>

This proportional relationship can be represented using the equation \( y = 3x \), since for every one liter of syrup in the drink mixture there are 3 liters of water. The constant of proportionality is 3. The ratio of the amount of water \( y \) to the amount of syrup \( x \) is 3:1. Additionally, the proportional relationship may be graphed using the ordered pairs in the table.
6.1213 The student will

a) write an equation in the form \( y = kx \), where \( k \) is the constant of proportionality, to represent a proportional relationship between two quantities, including those arising from practical problems situations;

b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine if a proportional relationship exists between two quantities; and

[Moved from 7.10 (2016)]

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs, and equations.

**UNDERSTANDING THE STANDARD**

(Background Information for Instructor Use Only)

- A graph representing a proportional relationship is a straight line that passes through the origin \((0,0)\);

- The representation of the ratio between two quantities may depend upon the order in which the quantities are being compared.
  - Example: In the mixture example above, we could also compare the ratio of the liters of syrup per liters of water, as shown:

<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Graph of a proportional relationship" /></td>
<td></td>
</tr>
</tbody>
</table>
6.1213 The student will

a) write an equation in the form \( y = kx \), where \( k \) is the constant of proportionality, to represent a proportional relationship between two quantities, including those arising from practical problems; 

b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table; 

c) determine if a proportional relationship exists between two quantities; and [Moved from 7.10 (2016)]

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs, and equations.

### UNDERSTANDING THE STANDARD

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<thead>
<tr>
<th>Water (liters)</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Syrup (liters)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

In this comparison, the ratio of the amount of syrup (y) to the amount of water (x) would be 1:3.

The graph of this relationship could be represented by:

While students in grade 6 are not expected to graph comparisons with a ratio of 1:3, students should be aware of how the order in which quantities are compared affects the way in which the relationship is represented as a table of equivalent ratios, or as a graph.

- Double number lines diagrams can also be used to represent proportional relationships and create collections of pairs of equivalent ratios.
  - Example:
6.12.13 The student will
a) write an equation in the form \( y = kx \), where \( k \) is the constant of proportionality, to represent a proportional relationship between two quantities, including those arising from practical problems or situations;
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
c) determine if a proportional relationship exists between two quantities; and [Moved from 7.10 (2016)]
d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs, and equations.

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<tr>
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<tbody>
<tr>
<td><strong>(Background Information for Instructor Use Only)</strong></td>
<td></td>
</tr>
<tr>
<td>Water (liters)</td>
<td></td>
</tr>
<tr>
<td>[Diagram of water liters]</td>
<td></td>
</tr>
<tr>
<td>Syrup (liters)</td>
<td></td>
</tr>
<tr>
<td>[Diagram of syrup liters]</td>
<td></td>
</tr>
</tbody>
</table>

In this proportional relationship, there are three liters of water for each liter of syrup represented on the number lines.

- A graph representing a proportional relationship includes ordered pairs that lie in a straight line that, if extended, would pass through \((0, 0)\), creating a pattern of horizontal and vertical increases. The context of the problem and the type of data being represented by the graph must be considered when determining if the points are to be connected by a straight line on the graph.
  - Example of the graph of a non-proportional relationship:
The student will

a) write an equation in the form \( y = kx \), where \( k \) is the constant of proportionality, to represent a proportional relationship between two quantities, including those arising from practical problems situations;

b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine if a proportional relationship exists between two quantities; and [Moved from 7.10 (2016)]

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs, and equations.

**UNDERSTANDING THE STANDARD**

(Background Information for Instructor Use Only)

The relationship of distance \( (y) \) to time \( (x) \) is non-proportional. The ratio of \( y \) to \( x \) for each ordered pair is not equivalent. That is,

- \( \frac{11}{8} \neq \frac{9}{6} \neq \frac{5}{4} \neq \frac{3}{2} \neq \frac{1}{0} \)

The points of the graph do not lie in a straight line. Additionally, the line does not pass through the point \((0, 0)\), thus the relationship of \( y \) to \( x \) cannot be considered proportional.

- Practical situations that model proportional relationships can typically be represented by graphs in the first quadrant, since in most cases the values for \( x \) and \( y \) are not negative.

- If the relationship between \( x \) and \( y \) is proportional, it means that as \( x \) changes,
The student will

a) write an equation in the form $y = kx$, where $k$ is the constant of proportionality, to represent a proportional relationship between two quantities, including those arising from practical problem situations;

b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine if a proportional relationship exists between two quantities; and [Moved from 7.10 (2016)]

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs, and equations.

**UNDERSTANDING THE STANDARD**

(Background Information for Instructor Use Only)

- A unit rate could be used to find missing values in a ratio table.
  - Example: A store advertises a price of $25 for 5 DVDs. What would be the cost to purchase 2 DVDs? 3 DVDs? 4 DVDs?

<table>
<thead>
<tr>
<th># DVDs</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$5</td>
<td></td>
<td></td>
<td></td>
<td>$25</td>
</tr>
</tbody>
</table>

The ratio of $25 per 5 DVDs is also equivalent to a ratio of $5 per 1 DVD, which would be the unit rate for this relationship. This unit rate could be used to find the missing value of the ratio table above. If we multiply the number of DVDs by a constant multiplier of 5 (the unit rate) we obtain the total cost. Thus, 2 DVDs would cost $10, 3 DVDs would cost $15, and 4 DVDs would cost $20.
6.131418 The student will solve one-step linear equations in one variable, involving whole number coefficients and positive rational solutions including practical problems that require the solution of a one-step linear equation in one variable.

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<tr>
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<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>When solving an equation, why is it necessary to perform the same operation on both sides of an equal sign?</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to</td>
</tr>
<tr>
<td>• A one-step linear equation is an equation that requires one operation to solve may include, but not be limited to equations such as the following: (2x = 5); (y - 3 = -6); (\frac{1}{2}x = -3); (a - (-4) = 11).</td>
<td>To maintain equality, an operation performed on one side of an equation must be performed on the other side. [Included in US]</td>
<td>• Identify examples of the following algebraic vocabulary: equation, variable, expression, term, and coefficient. [Reordered]</td>
</tr>
<tr>
<td>• A variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable.</td>
<td></td>
<td>• Represent and solve a one-step linear equation in one variable, using a variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale.</td>
</tr>
<tr>
<td>• An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an &quot;equal sign (=)&quot; (e.g., (\frac{3}{7}, 5x, 140 - 38.2, 18 \cdot 21, 5 + x)).</td>
<td></td>
<td>• Apply properties of real numbers and properties of equality to solve a one-step equation in one variable by demonstrating the steps algebraically. Coefficients are limited to integers and unit fractions. Numeric terms are limited to integers.</td>
</tr>
<tr>
<td>• An expression that contains a variable is a variable expression. A variable expression is like a phrase: as a phrase does not have a verb, so an expression does not have an &quot;equal sign (=)&quot;. An expression cannot be solved.</td>
<td></td>
<td>• Identify and use the following algebraic terms appropriately: equation, variable, expression, term, and coefficient. [Reordered]</td>
</tr>
<tr>
<td>• A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used when the numbers are unknown. Example, the verbal expression &quot;a number multiplied by 5&quot; could be represented by the verbal expression &quot;n \cdot 5&quot; or &quot;5n&quot;.</td>
<td></td>
<td>• Confirm solutions to one-step linear equations in one variable.</td>
</tr>
<tr>
<td>• An algebraic expression is a variable expression that contains at least one variable (e.g., (x - 3)).</td>
<td></td>
<td>• Write verbal expressions and sentences as algebraic expressions and equations.</td>
</tr>
<tr>
<td>• A verbal sentence is a complete word statement (e.g., &quot;The sum of a number and two is five.&quot; could be represented by &quot;n + 2 = 5&quot;).</td>
<td></td>
<td>• Write algebraic expressions and equations as verbal expressions and sentences.</td>
</tr>
<tr>
<td>• An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., (2x = 7)).</td>
<td></td>
<td>• Represent and solve a practical problem with a one-step linear equation in one variable.</td>
</tr>
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</table>
6.131418 The student will solve one-step linear equations in one variable, involving whole number coefficients and positive rational solutions including practical problems that require the solution of a one-step linear equation in one variable.

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| A term is a number, variable, product, or quotient in an expression of sums and/or differences. In $7x^2 + 5x - 3$, there are three terms, $7x^2$, $5x$, and $3$.

- **Term:** a number, variable, product, or quotient in an expression of sums and/or differences.
- **Coefficient:** the numerical factor in a term. In the term $3xy^2$, $3$ is the coefficient; in the term $z$, $1$ is the coefficient.
- **Positive rational solutions:** are limited to whole numbers and positive fractions and decimals.
- **Equation:** a mathematical sentence stating that two expressions are equal.
- **Variable:** a symbol (placeholder) used to represent an unspecified member of a set or an unknown quantity.
- **Solution:** a value that makes it a true statement. Many equations have one solution and are represented as a point on a number line. Solving an equation or inequality involves a process of determining which value(s) from a specified set, if any, make the equation or inequality a true statement. Substitution can be used to determine whether a given value(s) makes an equation or inequality true.
- **Properties of real numbers and properties of equality:** can be used to solve equations, justify equation solutions, and express simplification. Students should utilize the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of $a$, $b$, or $c$ in this standard).
  - **Commutative property of addition:** states that changing the order of the addends does not affect the sum. For example, $(5.6 + 4 = 4 + 5.6) a + b = b + a$.
  - **Commutative property of multiplication:** states that
6.13.14.18 The student will solve one-step linear equations in one variable, involving whole number coefficients and positive rational solutions including practical problems that require the solution of a one-step linear equation in one variable.

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<td><strong>Changing the order of the factors does not affect the product</strong> <em>(e.g., 5.6 · 4 = 4 · 5.6; a · b = b · a)</em>.</td>
<td><strong>The associative property of addition states that regrouping the addends does not change the sum</strong> <em>(e.g., 5.6 + (4 + 3) = (5.6 + 4) + 3)</em>.</td>
<td><strong>Subtraction and division are neither commutative nor associative.</strong></td>
</tr>
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| **The associative property of multiplication states that regrouping the factors does not change the product** *(e.g., 5.6 · (4 · 3) = (5.6 · 4) · 3)*. | **The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true.**  
Distributive property examples:  
*5.2 · (3 + 7) = (5.2 · 3) + (5.2 · 7), or*  
*5.2 · (3 − 7) = (5.2 · 3) − (5.2 · 7)*.  
**Converse of the distributive property examples:**  
*(5.2 · 3) + (5.2 · 7) = 5.2 · (3 + 7), or*  
*(5.2 · 3) − (5.2 · 7) = 5.2 · (3 − 7)*. | **The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division.**  
**The identity property of addition (additive identity property): states that the sum of any real number and zero is equal to the given real number (e.g., 5/2 + 0 = 5/2; a + 0 = a and 0 + a = a).**  
**The identity property of multiplication (multiplicative identity property): states that the product of any real number and one is equal to the given real number.** |
STANDARD 6.131418 STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA GRADE LEVEL 6

6.131418 The student will solve one-step linear equations in one variable, involving whole number coefficients and positive rational solutions including practical problems that require the solution of a one-step linear equation in one variable.

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<td>- The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division.</td>
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<td>- Inverses are numbers that combine with other numbers and result in identity elements [e.g., ( 5 + (-5) = 0 ); ( \frac{1}{2} \cdot 5 = 1 )].</td>
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<td>- The Inverse property of addition (additive inverse property): states that the sum of a number and its additive inverse always equals zero (e.g., ( 5.6 + (-5.6) = 0 )) ( a + (-a) = 0 ).</td>
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</tr>
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<td>- The Inverse property of multiplication (multiplicative inverse property): states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., ( \frac{1}{a} \cdot a = 1 )).</td>
<td></td>
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<td>- Zero has no multiplicative inverse.</td>
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<td>- The Multiplicative property of zero: states that the product of any real number and zero is zero (e.g., ( 7 \cdot 0 = 0 )) ( 0 \cdot 7 = 0 ).</td>
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<td>- Division by zero is not a possible mathematical operation. It is undefined.</td>
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<td>- The Addition property of equality: states that any number must be added to both sides of an equation to maintain equality if ( a = b ), then ( a + c = b + c ).</td>
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<td>- The Subtraction property of equality: states that any number must be subtracted from both sides of an equation to maintain equality if ( a = b ), then ( a - c = b - c ).</td>
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<td>- The Multiplication property of equality: states that an equation must be multiplied on both sides by the same</td>
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6.131418 The student will solve one-step linear equations in one variable, involving whole number coefficients and positive rational solutions including practical problems that require the solution of a one-step linear equation in one variable.

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<td>number to maintain equality: if ( a = b ), then ( a \cdot c = b \cdot c ).</td>
<td></td>
<td></td>
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<tr>
<td>The ( \text{d} )ivision property of equality: states that an equation must be divided on both sides by the same number to maintain equality if ( a = b ) and ( c \neq 0 ), then ( \frac{a}{c} = \frac{b}{c} ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The ( \text{s} )ubstitution property: of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality. (e.g., if ( x = -5 + 8 ) then ( x = 3 ). Thus, 3 was substituted for (-5 + 8 ) since they are equivalent) if ( a = b ) then ( b ) can be substituted for ( a ) in any expression, equation or inequality.</td>
<td></td>
<td></td>
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### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- **Identity elements** are numbers that combine with other numbers without changing the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). There are no identity elements for subtraction and division.

- The **additive identity property** states that the sum of any real number and zero is equal to the given real number (e.g., $5 + 0 = 5$).

- The **multiplicative identity property** states that the product of any real number and one is equal to the given real number (e.g., $8 \cdot 1 = 8$).

- **Inverses** are numbers that combine with other numbers and result in identity elements.

- The **multiplicative inverse property** states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., $4 \cdot \frac{1}{4} = 1$).

- **Division by zero** is not a possible arithmetic operation. Division by zero is undefined.

### ESSENTIAL UNDERSTANDINGS

How are the identity properties for multiplication and addition the same? Different?

- **For each operation** the identity elements are numbers that combine with other numbers without changing the value of the other numbers. The additive identity is zero (0). The multiplicative identity is one (1).

- What is the result of multiplying any real number by zero?
  - The product is always zero.

- Do all real numbers have a multiplicative inverse?
  - No. Zero has no multiplicative inverse because there is no real number that can be multiplied by zero resulting in a product of one.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem-solving, mathematical communication, mathematical reasoning, connections, and representations to:

- Identify a real number equation that represents each property of operations with real numbers, when given several real number equations.

- Test the validity of properties by using examples of the properties of operations on real numbers.

- Identify the property of operations with real numbers that is illustrated by a real number equation.

**NOTE:** The commutative, associative and distributive properties are taught in previous grades.
STANDARD 6.141520  STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA  GRADE LEVEL 6

6.141520  The student will

a) represent a practical situation with a graph inequalities linear inequality in one variable; and
b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.

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<tbody>
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<td>• The solution set to an inequality is the set of all numbers that make the inequality true. [Reordered]</td>
<td>• In an inequality, does the order of the elements matter? Yes, the order does matter. For example, $x &gt; 5$ is not the same relationship as $5 &gt; x$. However, $x &lt; 5$ is the same relationship as $5 &lt; x$. [Moved to US]</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to</td>
</tr>
<tr>
<td>• Inequalities can represent practical situations. For example: Jaxon works at least 4 hours per week mowing lawns. Write an inequality representing this situation and graph the solution. $x \geq 4$ or $4 \leq x$.</td>
<td>• Given a verbal description, represent a practical situation with a one-variable linear inequality. (a)</td>
<td></td>
</tr>
<tr>
<td>Would Jaxon ever work 3 hours in a week? 6 hours?</td>
<td>• Apply properties of real numbers and the addition or subtraction properties of inequality to solve given a simple-one-step linear inequality in one variable with integers, and graph the relationship solution on a number line. Numeric terms being added or subtracted from the variable are limited to integers. (b)</td>
<td></td>
</tr>
<tr>
<td>• The variable in an inequality may represent values that are limited by the context of the problem or situation. For example: if the variable represents all children in a classroom who are taller than 36 inches, the variable will be limited to have a minimum and maximum value based on the heights of the children. Students are not expected to represent these situations with a compound inequality (e.g., $36 &lt; x &lt; 70$) but only recognize that the values satisfying the single inequality ($x &gt; 36$) will be limited by the context of the situation.</td>
<td>• Given the graph of an simple linear inequality with integers, represent the inequality two different ways (e.g., $x &lt; -5$ or $-5 &gt; x$) using symbols ($&lt;$, $&gt;$, $\leq$, $\geq$). (b)</td>
<td></td>
</tr>
<tr>
<td>• Inequalities using the $&lt;$ or $&gt;$ symbols are represented on a number line with an open circle on the number and a shaded line over the solution set. For example: When graphing $x &lt; 4$, use an open circle above the 4 to indicate that the 4 is not included.</td>
<td>• Identify a numerical value(s) that satisfies an is part of the solution set of a given inequality. (a, b)</td>
<td></td>
</tr>
<tr>
<td>• When graphing $x \leq 4$ fill in the circle above the 4 to indicate that the 4 is included. [Included in bullet below]</td>
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</tbody>
</table>

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 6
**STANDARD 6.141520**  
**STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA**  
**GRADE LEVEL 6**

6.141520 The student will
  
a) represent a practical situation with a graph inequalities linear inequality in one variable; and
  
b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.

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| (Background Information for Instructor Use Only) | • Inequalities using the ≤ or ≥ symbols are represented on a number line with a closed circle on the number and shaded line in the direction of the solution set.  
  
  **Example:** When graphing \( x \geq 4 \) fill in the circle above the 4 to indicate that the 4 is included.  
  
  ![Graph of Inequality](image)
  
  • The solution set to an inequality is the set of all numbers that make the inequality true. [Reordered]  
  
  • It is important for students to see inequalities written with the variable before the inequality symbol and after. **Example** \( x \geq -6 \) and \( 7 > y \). \( x > 5 \) is not the same relationship as \( 5 > x \). However, \( x > 5 \) is the same relationship as \( 5 < x \). [Moved from middle column]
  
  • A one-step linear inequality may include, but not be limited to inequalities such as the following: \( 2x > 5; y - 3 \leq -6; \frac{1}{3}x < -3; ax - (-4) \geq 11 \).
  
  • Solving an equation or inequality involves a process of determining which value(s) from a specified set, if any, make the equation or inequality a true statement. Substitution can be used to determine whether a given value(s) makes an equation or inequality true.
  
  • Properties of real numbers and properties of inequality can be used to solve inequalities, justify solutions, and express simplification. Students should utilize the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of \( a, b, \) or \( c \) in this standard).
  
  ---
  
  **Commutative property of addition:** states that changing the order of the addends does not affect the sum.
6.141520 The student will
a) represent a practical situation with a graph inequalities linear inequality in one variable; and
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<td><strong>(e.g., 5.6 + 4 = 4 + 5.6) a + b = b + a.</strong></td>
<td><strong>Commutative property of multiplication:</strong> states that changing the order of the factors does not affect the product (e.g., 5.6 · 1 = 1 · 5.6) a · b = b · a.</td>
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<td><strong>The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division.</strong></td>
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| **The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true.** | **Distributive property examples:**
- 5.2 · (3 + 7) = (5.2 · 3) + (5.2 · 7), or
- 5.2 · (3 – 7) = (5.2 · 3) – (5.2 · 7)
**Converse of the distributive property examples:**
- (5.2 · 3) + (5.2 · 7) = 5.2 · (3 + 7), or
- (5.2 · 3) – (5.2 · 7) = 5.2 · (3 – 7)** | |
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<td>is equal to the given real number (e.g., ( \frac{5}{2} + 0 = \frac{5}{2} ), ( a + 0 = a ) and ( 0 + a = a )).</td>
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<td>- The identity property of multiplication (multiplicative identity property): states that the product of any real number and one is equal to the given real number (e.g., ( \sqrt{8} \cdot 1 = \sqrt{8} ) and ( 1 \cdot a = a )).</td>
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<td>- The inverse property of addition (additive inverse property): states that the sum of a number and its additive inverse always equals zero (e.g., ( 5 + (-5) = 0 ); ( a + (-a) = 0 ) and ( (-a) + a = 0 )).</td>
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<td>- The inverse property of multiplication (multiplicative inverse property): states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., ( \frac{1}{2} \cdot \frac{2}{1} = 1 ) and ( \frac{1}{a} \cdot a = 1 )).</td>
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<td>- Zero has no multiplicative inverse.</td>
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<td>- The multiplicative property of zero: states that the product of any real number and zero is zero (e.g., ( 7 \cdot 0 = 0 ) and ( 0 \cdot 7 = 0 )); ( a \cdot 0 = 0 ) and ( 0 \cdot a = 0 ).</td>
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<td>- The <em>Addition property of inequality</em> states that any number must be added to both sides of an inequality to maintain the inequality if ( a &lt; b ), then ( a + c &lt; b + c ); if ( a &gt; b ), then ( a + c &gt; b + c ) (this property also applies to ( \leq ) and ( \geq )).</td>
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<td>- The <em>Subtraction property of inequality</em> states that any number must be subtracted from both sides of an inequality to maintain inequality if ( a &lt; b ), then ( a - c &lt; b - c ); if ( a &gt; b ), then ( a - c &gt; b - c ) (this property also applies to ( \leq ) and ( \geq )).</td>
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</tr>
<tr>
<td>- The <em>Multiplication property of inequality</em> states that if an inequality is multiplied on both sides by the same positive number the inequality symbol remains the same. But if an inequality is multiplied on both sides by the same negative number, the inequality symbol reverses. For example, to solve ( \frac{1}{2}x &gt; 9 ), multiply both sides of the inequality by 3, so ( x &gt; 27 ). To solve ( -\frac{1}{2}x &lt; 9 ), multiply both sides of the inequality by -3, so ( x &gt; -27 ).</td>
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<tr>
<td>- The <em>Division property of inequality</em> states that if an inequality is divided on both sides by the same positive number the inequality symbol remains the same. But if an inequality is divided on both sides by the same negative number, the inequality symbol reverses. For example, ( -4x &lt; 20 ). Divide both sides of the inequality by -4. So ( x &gt; -5 ).</td>
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<tr>
<td>- The <em>Substitution property of equality</em> states if two quantities are equivalent then one quantity can replace the other in an equation or inequality (e.g., if ( x = 5 ) then ( x = 1 ). Thus, ( 2 ) was substituted for ( 5 - x ) since they are equivalent) if ( a = b ) then ( b ) can be substituted for ( a ) in any</td>
<td></td>
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</tr>
</tbody>
</table>
6.141520 The student will
   a) represent a practical situation with a graph inequalities linear inequality in one variable; and
   b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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<tr>
<td>expression, equation or inequality.</td>
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</table>
Mathematics Standards of Learning

Curriculum Framework 2009 2016

Grade 7

Board of Education
Commonwealth of Virginia
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The Virginia Department of Education wishes to express sincere thanks to Deborah Kiger Bliss, Lois A. Williams, Ed.D., and Felicia Dyke, Ph.D., who assisted in the development of the 2009 Mathematics Standards of Learning Curriculum Framework.

**Acknowledgements**
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**NOTICE**
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

The 2009-2016 Mathematics Standards of Learning Curriculum Framework is a companion document to the 2009-2016 Mathematics Standards of Learning and amplifies the Mathematics Standards of Learning by further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. This provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instructionbackground information for the teacher (K-8). It contains content. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers’ knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that What each student should know and be able to demonstratedo in each standard is outlined. This is not meant to be an exhaustive list of student expectations nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise. [Reordered and rewritten]
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
**The Role of Instructional Technology**

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “...the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

**Computational Fluency**

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
In the middle grades, the focus of mathematics learning is to
- build on students’ concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students in the middle grades focus on mastering rational numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to most middle school mathematics topics.

- Students develop an understanding of integers and rational numbers by using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.

- Students develop an understanding of the properties of operations on real numbers through experiences with rational numbers and by applying the order of operations.

- Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

Mathematics instruction in grades 6 – 8 continues to focus on the development of number sense, with emphasis on rational and real numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to many middle school mathematics topics.

Students develop an understanding of integers and rational numbers using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.

Students develop an understanding of real numbers and the properties of operations on real numbers through experiences with rational and irrational numbers and apply the order of operations.

Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.
### STANDARD 7.1

**STAND: NUMBER AND NUMBER SENSE**

**GRADE LEVEL 7**

7.1 The student will

a) **investigate and describe the concept of negative exponents for powers of ten;**

b) **determine, compare and order scientific notation for numbers greater than zero written in scientific notation:**

*On the state assessment, items measuring this objective are assessed without the use of a calculator.*

c) **compare and order fractions, decimals, percents, and numbers written in scientific notation:**

d) **determine square roots of perfect squares:**

e) **identify and describe absolute value of rational numbers.**

---

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Negative exponents for powers of 10 are used to represent numbers between 0 and 1. [Same as middle column]
  
  \(10^{-3} = 0.001\).

- Negative exponents for powers of 10 can be investigated through patterns such as:
  
  \[
  \begin{align*}
  10^{-2} &= 0.1 \\
  10^{-1} &= 0.1 \\
  10^0 &= 1 \\
  10^1 &= 10 \\
  10^2 &= 100
  \end{align*}
  \]

  Percent means “per 100” or how many “out of 100”; percent is another name for hundredths.

- A percent is a ratio in which the denominator is 100. A number followed by a percent symbol (%) is equivalent to that number with a denominator of 100 (e.g., \(\frac{3}{5} = \frac{60}{100} = 0.60 = 60\%\)).

- Scientific notation should be used whenever the situation calls for use of very large or very small numbers. [Moved from middle column]

- A number written in scientific notation is the product of two factors — a decimal greater than or equal to 1 but less than 10, and a power of 10 (e.g., 3.1 \(\times 10^5 = 310,000\) and 2.85 \(\times 10^{-4} = 0.000285\)).

- The set of integers includes the set of whole numbers and their opposites, as well as zero. Integers can be represented in a number line regardless of direction.

---

### ESSENTIAL UNDERSTANDINGS

- **When should scientific notation be used?**
  
  Scientific notation should be used whenever the situation calls for use of very large or very small numbers. [Moved to US]

- **How are fractions, decimals and percents related?**
  
  Any rational number can be represented in fraction, decimal and percent form. [Moved to US]

- **What does a negative exponent mean when the base is 10?**
  
  A base of 10 raised to a negative exponent represents a number between 0 and 1. [Same as in US]

- **How is taking a square root different from squaring a number?**
  
  Squaring a number and taking a square root are inverse operations. [Moved to US]

- **Why is the absolute value of a number positive?**
  
  The absolute value of a number represents distance from zero on a number line regardless of direction.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Recognize powers of 10 with negative exponents by examining patterns. (a)

- Write a number greater than 0 in scientific notation. (Included in bullet below)

- Compare and determine equivalent relationships between numbers larger than 0 written in scientific notation. [Included in bullet below]

- Convert between a numbers greater than 0 written in scientific notation and as a fraction, decimals and/or percent forms. (b)

- Compare, order and determine equivalent relationships among rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents. Fractions and mixed numbers may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place, and percents are limited to the tenths place. Ordering is limited to no more than 4, five
### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- **opposites, \{-2, -1, 0, 1, 2\ldots\}. Zero has no opposite and is neither positive nor negative.**
- The opposite of a positive number is negative and the opposite of a negative number is positive.
- The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form \( \frac{a}{b} \) where \( a \) and \( b \) are integers and \( b \) does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of rational numbers are \( \sqrt{25}, \frac{1}{4}, -2.3, 82, 75\% \), and \( 0.59\overline{5} \). [Moved from 8.1]
- **Any rational number may be expressed as positive and negative can be represented in fractions or mixed numbers, positive and negative decimals, integers, and percents form.** [Moved from middle column]
- A proper fraction is a fraction whose numerator is less than the denominator. A proper fraction is a fraction that is always less than one.
- An improper fraction is a fraction whose numerator is greater than or equal to the denominator. An improper fraction is a fraction that is equal to or greater than one.
- Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction can be

### ESSENTIAL UNDERSTANDINGS

- Distance is positive. [Moved to US]

### ESSENTIAL KNOWLEDGE AND SKILLS

- **numbers**: Ordering may be in ascending or descending order. (c)
- **Compare and Order**: no more than 4-four numbers greater than 0 written in scientific notation. Ordering may be in ascending or descending order. (cb)
- Identify the perfect squares from 0 to 400. (d) [Moved from 8.5a]
- Determine the positive square root of a perfect square less than or equal from 0 to 400. (d)
- Demonstrate absolute value using a number line. (e)
- Determine the absolute value of a rational number. (e)
- Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle to solve practical problems. (e)

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*On the state assessment, items measuring this objective are assessed without the use of a calculator.*
7.1 The student will
a) investigate and describe the concept of negative exponents for powers of ten;
b) **determine compare and order scientific notation for numbers greater than zero written in scientific notation:**
c) compare and order fractions, decimals, percents and numbers written in scientific notation **rational numbers:**
d) **determine square roots of perfect squares:**
e) identify and describe absolute value of rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td><strong>expressed as a mixed number.</strong> A mixed number is written with two parts: a whole number and a proper fraction (e.g., $\frac{3}{4}$). Fractions can be positive or negative.</td>
</tr>
<tr>
<td><strong>Equivalent relationships among fractions, decimals, and percents can be determined by using manipulatives concrete materials and pictorial representations (e.g., fraction bars, base-10 blocks, fraction circles, colored counters, cubes, decimal squares, graph paper shaded figures, shaded grids, number lines and calculators).</strong></td>
</tr>
<tr>
<td><strong>Negative numbers lie to the left of zero and positive numbers lie to the right of zero on a number line.</strong></td>
</tr>
<tr>
<td><strong>Smaller numbers always lie to the left of larger numbers on the number line.</strong></td>
</tr>
<tr>
<td><strong>A perfect square is a whole number whose square root is an integer. Zero (a whole number) is a perfect square. (e.g., $36 = 6 \times 6 = 6^2$).</strong></td>
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<tr>
<td><strong>A square root of a number is a number which, when multiplied by itself, produces the given number (e.g., $\sqrt{121}$ is 11 since $11 \times 11 = 121$).</strong></td>
</tr>
<tr>
<td><strong>The symbol $\sqrt{\cdot}$ may be used to represent a non-negative (principal) square root. Students in grade 8 mathematics will explore the negative square root of a number, denoted $-\sqrt{\cdot}$.</strong></td>
</tr>
<tr>
<td><strong>The square root of a number can be represented geometrically as the length of a side of the a square.</strong></td>
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</tbody>
</table>
7.1 The student will
a) investigate and describe the concept of negative exponents for powers of ten;
b) determine compare and order scientific notation for numbers greater than zero written in scientific notation;*
c) compare and order fractions, decimals, percents and numbers written in scientific notation rational numbers;*
d) determine square roots of perfect squares;* and
e) identify and describe absolute value for of rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<tbody>
<tr>
<td>• Squaring a number and taking a square root are inverse operations. [Moved from middle column]</td>
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<tr>
<td>• The absolute value of a number is the distance from 0 on the number line regardless of direction. Distance is positive. [Moved from middle column] (e.g., $\left</td>
<td>\frac{-1}{2}\right</td>
<td>= \frac{1}{2}$).</td>
</tr>
<tr>
<td>• The absolute value of zero is zero.</td>
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</tbody>
</table>
7.2 The student will describe and represent arithmetic [Included in AI.7 EKS, AFDA.1 EKS and AII.5] and geometric sequences [Included in AFDA.1 EKS and AII.5] using variable expressions.

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<tr>
<td>- In the numeric pattern of an arithmetic sequence, students must determine the difference, called the common difference, between each succeeding number in order to determine what is added to each previous number to obtain the next number.</td>
<td>- When are variable expressions used? Variable expressions can express the relationship between two consecutive terms in a sequence.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:</td>
</tr>
</tbody>
</table>

| - In geometric sequences, students must determine what each number is multiplied by in order to obtain the next number in the geometric sequence. This multiplier is called the common ratio. Sample geometric sequences include 2, 4, 8, 16, 32, …; 1, 5, 25, 125, 625, …; and 80, 20, 5, 1.25, …. | - A variable expression can be written to express the relationship between two consecutive terms of a sequence. |
| - A variable expression can be written to express the relationship between two consecutive terms of a sequence. If \( n \) represents a number in the sequence 3, 6, 9, 12, …, the next term in the sequence can be determined using the variable expression \( n + 3 \). | - If \( n \) represents a number in the sequence 1, 5, 25, 125, …, the next term in the sequence can be determined by using the variable expression \( 5n \). | - Analyze arithmetic and geometric sequences to discover a variety of patterns. |
| - Identify the common difference in an arithmetic sequence. | - Identify the common ratio in a geometric sequence. |
| - Given an arithmetic or geometric sequence, write a variable expression to describe the relationship between two consecutive terms in the sequence. |
In the middle grades, the focus of mathematics learning is to
• build on students’ concrete reasoning experiences developed in the elementary grades;
• construct a more advanced understanding of mathematics through active learning experiences;
• develop deep mathematical understandings required for success in abstract learning experiences; and
• apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

• Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense.

• Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate.

• Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonableness of answers.

Computation and estimation in grades 6-8 focus on developing conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring an understanding as to why procedures work and make sense.

Students develop and refine estimation strategies based on an understanding of number concepts, properties and relationships. The development of problem solving, using operations with integers and rational numbers, builds upon the strategies developed in the elementary grades. Students will reinforce these skills and build on the development of proportional reasoning and more advanced mathematical skills.

Students learn to make sense of the mathematical tools available by making valid judgments of the reasonableness of answers. Students will balance the ability to make precise calculations through the application of the order of operations with knowing when calculations may require estimation to obtain appropriate solutions to practical problems.
7.23 The student will
   a) model addition, subtraction, multiplication and division of integers [Moved to 6.6a EKS] simplify numerical expressions involving rational numbers using the order of operations;[2] [Moved from 8.1a] and
   b) add, subtract, multiply, and divide integers [Moved to 6.6a] solve practical problems involving operations with rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

### UNDERSTANDING THE STANDARD
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<tr>
<td>The set of integers is the set of whole numbers and their opposites (e.g., …-3, -2, 1, 0, 1, 2, 3, …)</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>Integers are used in practical situations, such as temperature changes (above/below zero), balance in a checking account (deposit/withdrawals), and changes in altitude (above/below sea level)</td>
<td>Model addition, subtraction, multiplication and division of integers using pictorial representations of concrete manipulatives. [Moved to 6.6]</td>
</tr>
<tr>
<td>Concrete experiences in formulating rules for adding and subtracting integers should be explored by examining patterns using calculators, along a number line and using manipulatives, such as two-color counters, or by using algebra tiles. [Moved to 6.6]</td>
<td>Add, subtract, multiply, and divide integers. [Moved to 6.6]</td>
</tr>
<tr>
<td>Concrete experiences in formulating rules for multiplying and dividing integers should be explored by examining patterns with calculators, along a number line and using manipulatives, such as two-color counters, or by using algebra tiles. [Moved to 6.6]</td>
<td>Use the order of operations and apply the properties of real numbers to simplify numerical expressions involving addition, subtraction, multiplication and division of integers rational numbers expressed as integers, fractions, decimals, and percents. Exponents are limited to 1, 2, 3, and 4 and bases are limited to positive integers. Expressions should not include braces { } but may include brackets [ ] and absolute value</td>
</tr>
<tr>
<td>The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form ( \frac{a}{b} ) where ( a ) and ( b ) are integers and ( b ) does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of rational numbers are ( \sqrt{2}, \frac{1}{4}, -2.3, 82.75%, 4.59 ). [Moved from 8.1]</td>
<td>Fractions may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place. <em>(b)</em></td>
</tr>
<tr>
<td>Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction can be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., ( 3 \frac{2}{5} )). A fraction can have a positive or negative value.</td>
<td>Solve practical problems involving addition, subtraction, multiplication, and division with integers. Rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents. Fractions may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place. <em>(b)</em></td>
</tr>
</tbody>
</table>
### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Solving problems in the context of practical situations enhances interconnectedness and proficiency with estimation strategies.
- Practical problems involving rational numbers in grade 7 provide students the opportunity to use problem solving to apply computation skills involving positive and negative rational numbers expressed as integers, fractions, and decimals, along with the use of percents within practical situations.
- Some numbers can belong to more than one subset of the real numbers (e.g., 9 is a natural number, a whole number, an integer, and a rational number.)
- The order of operations is a convention that defines the computation order to follow in simplifying an expression. Having an established convention ensures that there is only one correct result when simplifying an expression. The order of operations is as follows:
  - First, complete all operations within grouping symbols¹. If there are grouping symbols within other grouping symbols, do the innermost operations first.
  - Second, evaluate all exponential expressions.
  - Third, multiply and/or divide in order from left to right.
  - Fourth, add and/or subtract in order from left to right.

  ¹ Parentheses ( ), brackets [ ], braces { }, absolute value | | (e.g., |3 (−5 + 2)| − 7), and the division bar — as in (e.g., ) should be treated as grouping symbols.

- Expressions are simplified using the order of operations and applying the properties of real numbers.
- The commutative property of addition states that changing the order of the addends does not affect the sum.

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<table>
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<tr>
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<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will</td>
<td></td>
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</tr>
<tr>
<td>a) model addition, subtraction, multiplication and division of integers [Moved to 6.6a EKS] simplify numerical expressions involving rational numbers using the order of operations;* [Moved from 8.1a] and</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) add, subtract, multiply, and divide integers [Moved to 6.6a] solve practical problems involving operations with rational numbers.</td>
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</tbody>
</table>

*On the state assessment, items measuring this objective are assessed without the use of a calculator.
STANDARD 7.23  STRAND: COMPUTATION AND ESTIMATION  GRADE LEVEL 7

7.23  The student will
a) model addition, subtraction, multiplication and division of integers [Moved to 6.6a EKS] simplify numerical expressions involving rational numbers using the order of operations,\[\text{Moved from 8.1a}\] and
b) add, subtract, multiply, and divide integers [Moved to 6.6a] solve practical problems involving operations with rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<tbody>
<tr>
<td>(e.g., $5.6 + 4 = 1 + 5.6$)</td>
<td></td>
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<tr>
<td>The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., $5.6 \cdot 4 = 4 \cdot 5.6$).</td>
<td></td>
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</tr>
<tr>
<td>The associative property of addition states that regrouping the addends does not change the sum (e.g., $5.6 + (4 + 3) = (5.6 + 4) + 3$).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The associative property of multiplication states that regrouping the factors does not change the product (e.g., $5.6 \cdot (4 + 3) = (5.6 \cdot 4) + 3$).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subtraction and division are neither commutative nor associative.</td>
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<td></td>
</tr>
</tbody>
</table>
| The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true. Distributive property examples:\
\[
5.2 \cdot (2 + 7) = (5.2 \cdot 2) + (5.2 \cdot 7)\text{ or } \\
5.2 \cdot (2 - 7) = (5.2 \cdot 2) - (5.2 \cdot 7)
\]
Converse of the distributive property examples:
\[
(5.2 \cdot 3) + (5.2 \cdot 7) = 5.2 \cdot (3 + 7)\text{ or } \\
(5.2 \cdot 3) - (5.2 \cdot 7) = 5.2 \cdot (3 - 7)
\] | | |
| The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division. | | |
| The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real | | |
7.23 The student will:

a) model addition, subtraction, multiplication and division of integers [Moved to 6.6a EKS] simplify numerical expressions involving rational numbers using the order of operations; and

b) add, subtract, multiply, and divide integers [Moved to 6.6a] solve practical problems involving operations with rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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- The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number (e.g., \( \sqrt{8} \cdot 1 = \sqrt{8} \)).
- The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division.
- Inverses are numbers that combine with other numbers and result in identity elements (e.g., \( 5 + (-5) = 0 \); \( \frac{1}{5} \cdot 5 = 1 \)).
- The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., \( 5.6 + (-5.6) = 0 \)).
- The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., \( -\frac{1}{4} \cdot -\frac{4}{1} = 1 \)).
- Zero has no multiplicative inverse.
- The multiplicative property of zero states that the product of any real number and zero is zero (e.g., \( 7 \cdot 0 = 0 \); \( 0 \cdot 7 = 0 \)).
- Division by zero is not a possible mathematical operation. It is undefined.
7.23 The student will
a) model addition, subtraction, multiplication and division of integers [Moved to 6.6a EKS] simplify numerical expressions involving rational numbers using the order of operations;* [Moved from 8.1a] and
b) add, subtract, multiply, and divide integers [Moved to 6.6a] solve practical problems involving operations with rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

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<td>(Background Information for Instructor Use Only)</td>
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<tr>
<td>• The substitution property of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality. (e.g., if ( x = 5 + 8 ) then ( x = 3 ). Thus, 3 was substituted for ( 5 + 8 ) since they are equivalent).</td>
<td></td>
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</table>
The student will solve single-step and multistep practical problems, using proportional reasoning.

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<tbody>
<tr>
<td>- A proportion is a statement of equality between two ratios.</td>
<td>- What makes two quantities proportional? Two quantities are proportional when one quantity is a constant multiple of the other. [Moved to US]</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>- A proportion can be written as ( \frac{a}{b} = \frac{c}{d} ), or ( a ) is to ( b ) as ( c ) is to ( d ). [Combined bullets]</td>
<td>- Write proportions that represent equivalent relationships between two sets of quantities. [Included in bullet below]</td>
<td>- Write proportions that represent equivalent relationships between two sets of quantities.</td>
</tr>
<tr>
<td>- Equivalent ratios arise by multiplying each value in a ratio pair by the same positive number constant value. For example, the ratio of 3:2 would be equivalent to the ratio 6:4 because each of the values in 3:2 can be multiplied by 2 to get 6:4.</td>
<td>- Given a proportional relationship between two quantities, create and use a ratio table of equivalent ratios to determine missing values.</td>
<td>- Given a proportional relationship between two quantities, create and use a ratio table of equivalent ratios to determine missing values.</td>
</tr>
<tr>
<td>- A ratio table is a table of values representing a proportional relationship that includes pairs of values that represent equivalent rates or ratios.</td>
<td>- Write and solve a proportion that represents a proportional relationship between two quantities to find a missing term value.</td>
<td>- Write and solve a proportion that represents a proportional relationship between two quantities to find a missing term value.</td>
</tr>
<tr>
<td>- A proportion can be solved by finding the product of the means and the product of the extremes. For example, in the proportion ( \frac{a}{b} = \frac{c}{d} ), ( a ) and ( d ) are the extremes and ( b ) and ( c ) are the means. If values are substituted for ( a ), ( b ), ( c ), and ( d ) such as ( 5:12 = 10:24 ), then the product of extremes ((5 \cdot \infty 4)) is equal to the product of the means ((12 \cdot \infty 10)).</td>
<td>- Apply proportional reasoning to convert units of measurement within and between the U.S. Customary System and the metric system when given the conversion factor. Calculators may be used. [Moved from 6.9]</td>
<td>- Apply proportional reasoning to convert units of measurement within and between the U.S. Customary System and the metric system when given the conversion factor. Calculators may be used.</td>
</tr>
<tr>
<td>- In a proportional situation, quantities increase or decrease together. [Combined with bullet below]</td>
<td>- Using 10% as a benchmark, compute 5%, 10%, 15%, or 20% of a given whole number.</td>
<td>- Using 10% as a benchmark, compute 5%, 10%, 15%, or 20% of a given whole number.</td>
</tr>
<tr>
<td>- In a proportional situation, two quantities increase multiplicatively. One quantity is a constant multiple of the other. [Included from US]</td>
<td>- Using 10% as a benchmark, mentally compute 5%, 10%, 15%, or 20% in a practical situation such as tips, tax and discounts.</td>
<td>- Using 10% as a benchmark, mentally compute 5%, 10%, 15%, or 20% in a practical situation such as tips, tax and discounts.</td>
</tr>
<tr>
<td>- A proportion is an equation which states that two ratios are equal. When solving a proportion, the ratios may first be written as fractions.</td>
<td>- Solve problems involving tips, tax, and discounts. Limit problems to only one percent computation per problem.</td>
<td>- Solve problems involving tips, tax, and discounts. Limit problems to only one percent computation per problem.</td>
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Example: A recipe for oatmeal cookies calls for 2 cups of flour for every 3 cups of oatmeal. How much flour is needed for a larger batch of cookies that uses 9 cups of oatmeal? To solve this problem, the ratio of flour to oatmeal could be written as a fraction in the proportion used to determine the amount of flour needed when 9 cups of oatmeal is used. To use a proportion to solve for the
### UNDERSTANDING THE STANDARD

<table>
<thead>
<tr>
<th>flour (cups)</th>
<th>2</th>
<th>4</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>oatmeal (cups)</td>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

To complete the table, we must create an equivalent ratio to 2:3. Just as 4:6 is equivalent to 2:3, then 6 cups of flour to 9 cups of oatmeal would create an equivalent ratio.

- A proportion can be solved by finding equivalent fractions.
- A rate is a ratio that compares two quantities measured in different units. A unit rate is a rate with a denominator of 1. Examples of rates include miles/hour and revolutions/minute.
- Proportions are used in everyday contexts, such as speed, recipe conversions, scale drawings, map reading, reducing and enlarging, comparison shopping, tips, tax, and discounts, and monetary conversions.
- A multistep problem is a problem that requires two or more steps to solve.
- Proportions can be used to convert length, weight (mass), and volume (capacity) within and between measurement systems. For example, if 2.54 cm is about 1 inch, how many inches are in 16 cm?

\[
\frac{1 \text{ inch}}{2.54 \text{ cm}} = \frac{x \text{ inch}}{16 \text{ cm}}
\]

\[
2.54x = 1 \cdot 16
\]

\[
2.54x = 16
\]

\[
x = \frac{16}{2.54}
\]
7.34 The student will solve single-step and multistep practical problems, using proportional reasoning.

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- The value of $x$ is $6.299$ or about 6.3 inches.
- Examples of conversions may include, but are not limited to:
  - Length: between feet and miles; miles and kilometers
  - Weight: between ounces and pounds; pounds and kilograms
  - Volume: between cups and fluid ounces; gallons and liters
- Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object’s mass, although they use the term weight (e.g., “How much does it weigh?” versus “What is its mass?”).
- When converting measurement units in practical situations, the precision of the conversion factor used will be based on the accuracy required within the context of the problem. For example, when converting from miles to kilometers, we may use a conversion factor of 1 mile ≈ 1.6 km or 1 mile ≈ 1.609 km, depending upon the accuracy needed.
- Estimation may be used prior to calculating conversions to evaluate the reasonableness of a solution.
- A percent is a special ratio in which the denominator is 100.
- Proportions can be used to represent percent problems as follows:

\[
\frac{\text{percent}}{100} = \frac{\text{part}}{\text{whole}}
\]
In the middle grades, the focus of mathematics learning is to
• build on students’ concrete reasoning experiences developed in the elementary grades;
• construct a more advanced understanding of mathematics through active learning experiences;
• develop deep mathematical understandings required for success in abstract learning experiences; and
• apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

• Students develop the measurement skills that provide a natural context and connection among many mathematics concepts. Estimation skills are developed in determining length, weight/mass, liquid volume/capacity, and angle measure. Measurement is an essential part of mathematical explorations throughout the school year.
• Students continue to focus on experiences in which they measure objects physically and develop a deep understanding of the concepts and processes of measurement. Physical experiences in measuring various objects and quantities promote the long-term retention and understanding of measurement. Actual measurement activities are used to determine length, weight/mass, and liquid volume/capacity.
• Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, pyramids, and cones.

[Moved from Geometry Strand Introduction and Rewritten]
Measurement and geometry in the middle grades provide a natural context and connection among many mathematical concepts. Students expand informal experiences with geometry and measurement in the elementary grades and develop a solid foundation for further exploration of these concepts in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.

Students develop measurement skills through exploration and estimation. Physical exploration to determine length, weight/mass, liquid volume/capacity, and angle measure are essential to develop a conceptual understanding of measurement. Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, square-based pyramids, and cones.

Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provide experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.

Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, square-based pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.

Students explore and develop an understanding of the Pythagorean Theorem. Understanding how the Pythagorean Theorem can be applied in practical situations has a far-reaching impact on subsequent mathematics learning and life experiences.
The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

**Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

**Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

**Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

**Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)

**Level 4: Deduction.** Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)
The student will

a) describe and determine the volume and surface area of rectangular prisms and [Moved from 7.5 EKS] cylinders; and

b) solve problems, including practical problems, involving the volume and surface area of rectangular prisms and cylinders;

c) describe how changing one measured attribute of a rectangular prism affects its volume and surface area. [Included in 8.6b]

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<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>How are volume and surface area related? Volume is a measure of the amount a container holds while surface area is the sum of the areas of the surfaces on the container. [Same as US]</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• The area of a rectangle is computed by multiplying the lengths of two adjacent sides.</td>
<td>How does the volume of a rectangular prism change when one of the attributes is increased? There is a direct relationship between the volume of a rectangular prism increasing when the length of one of the attributes of the prism is changed by a scale factor. [Moved to 8.6]</td>
<td>• Determine if a practical problem involving a rectangular prism or cylinder represents the application of volume or surface area. [Reordered]</td>
</tr>
<tr>
<td>• The area of a circle is computed by squaring the radius and multiplying that product by ( \pi ) ((A = \pi r^2), where ( \pi \approx 3.14 \text{ or } \frac{22}{7} )).</td>
<td></td>
<td>• Find Determine the surface area of a rectangular prism or cylinder using concrete objects, nets, diagrams, and formulas. (a)</td>
</tr>
<tr>
<td>• A polyhedron is a solid figure whose faces are all polygons.</td>
<td></td>
<td>• Determine the volume of rectangular prisms and cylinders using concrete objects, diagrams, and formulas. (a)</td>
</tr>
<tr>
<td>• A rectangular prism is a polyhedron in which all six faces are rectangles. A rectangular prism has 8 vertices and 12 edges.</td>
<td></td>
<td>• Determine if a practical problem involving a rectangular prism or cylinder represents the application of volume or surface area. (b) [Reordered]</td>
</tr>
<tr>
<td>• A cylinder is a solid figure formed by two congruent parallel faces called bases joined by a curved surface. In this grade level, cylinders are limited to right circular cylinders.</td>
<td></td>
<td>• Solve practical problems that require finding determining the surface area of a rectangular prism and cylinders. (b)</td>
</tr>
<tr>
<td>• A face is any flat surface of a solid figure. A base is a special face of a solid figure.</td>
<td></td>
<td>• Find the surface area of a cylinder. [Combined with another bullet]</td>
</tr>
<tr>
<td>• The surface area of a prism is the sum of the areas of all of its faces and is measured in square units.</td>
<td></td>
<td>• Solve practical problems that require finding the surface area of a cylinder. [Combined with another bullet]</td>
</tr>
<tr>
<td>• The volume of a three-dimensional figure is a measure of capacity and is measured in cubic units.</td>
<td></td>
<td>• Find the volume of a rectangular prism. [Combined with another bullet]</td>
</tr>
<tr>
<td>• Nets are two-dimensional representations of a three-dimensional figure that can be folded into a model of the three-dimensional figure.</td>
<td></td>
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</tbody>
</table>
The student will
a) describe and determine the volume and surface area of rectangular prisms and cylinders;
and
b) solve problems, including practical problems, involving the volume and surface area of rectangular prisms and cylinders;
and
c) describe how changing one measured attribute of a rectangular prism affects its volume and surface area.

UNDERSTANDING THE STANDARD
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<tr>
<td>• A cylinder can be represented on a flat surface as a net that contains two circles (the bases of the cylinder) and one rectangular region (the curved surface of the cylinder) whose length is the circumference of the circular base and whose width is the height of the cylinder. The surface area of the cylinder is the sum of the area of the two circles and the rectangle representing the curved surface ( SA = 2\pi r^2 + 2\pi rh ).</td>
<td></td>
</tr>
<tr>
<td>• The volume of a rectangular prism is computed by multiplying the area of the base, ( B ), (length times width) by the height of the prism ( V = lwh = Bh ).</td>
<td></td>
</tr>
<tr>
<td>• The volume of a cylinder is computed by multiplying the area of the base, ( \pi r^2 ) by the height of the cylinder ( V = \pi r^2 h = Bh ).</td>
<td></td>
</tr>
<tr>
<td>• The calculation of determining surface area and volume may vary depending upon the approximation for ( \pi ). Common approximations for ( \pi ) include 3.14, ( \frac{22}{7} ), or the ( \pi ) button on the calculator.</td>
<td></td>
</tr>
<tr>
<td>• There is a direct relationship between changing one measured attribute of a rectangular prism by a scale factor and its volume. For example, doubling the length of a prism will double its volume. This direct relationship does not hold true for surface area. [Moved to 8.6]</td>
<td></td>
</tr>
<tr>
<td>• Solve practical problems that require finding determining the volume of a rectangular prism and cylinders. (b).</td>
<td></td>
</tr>
<tr>
<td>• Find the volume of a cylinder. [Combined with another bullet]</td>
<td></td>
</tr>
<tr>
<td>• Solve practical problems that require finding the volume of a cylinder. [Combined with another bullet]</td>
<td></td>
</tr>
<tr>
<td>• Describe how the volume of a rectangular prism is affected when one measured attribute is multiplied by a scale factor. Problems will be limited to changing attributes by scale factors only. [Included in 8.6]</td>
<td></td>
</tr>
<tr>
<td>• Describe how the surface area of a rectangular prism is affected when one measured attribute is multiplied by a scale factor. Problems will be limited to changing attributes by scale factors only. [Included in 8.6]</td>
<td></td>
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</tbody>
</table>
7.56 The student will determine whether plane figures—quadrilaterals and triangles—are similar and write proportions to express the relationships between solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar figures quadrilaterals and triangles.

### UNDERSTANDING THE STANDARD

**Background Information for Instructor Use Only**

- Two similar polygons are similar if they have corresponding (matching) sides that are proportional and corresponding interior angles that are congruent and the lengths of corresponding sides are proportional. [Included from middle column]
- Similarity has practical applications in a variety of areas, including art, architecture, and the sciences.
- Similarity does not depend on the position or orientation of the figures.
- Congruent polygons have the same size and shape. Corresponding angles and sides are congruent. [Included from middle column]
- Congruent polygons are similar polygons for which the ratio of the corresponding sides is 1:1. However, similar polygons are not necessarily congruent. [Included from middle column]
- The symbol $\sim$ is used to represent similarity. For example, $\triangle ABC \sim \triangle DEF$.
- The symbol $\cong$ is used to represent congruence. For example, $\angle A \cong \angle B$.
- Similarity statements can be used to determine corresponding parts of similar figures such as:
  - Given: $\triangle ABC \sim \triangle DEF$
  - $\angle A$ corresponds to $\angle D$
  - $AB$ corresponds to $DE$ [Moved from EKS]
- A proportion representing corresponding sides of similar figures can be written as $\frac{a}{b} = \frac{c}{d}$.
  - Example: Given two similar quadrilaterals with corresponding angles labeled, write a proportion involving corresponding sides.

### ESSENTIAL UNDERSTANDINGS

- How do polygons that are similar compare to polygons that are congruent? Congruent polygons have the same size and shape. Similar polygons have the same shape, and corresponding angles between the similar figures are congruent. However, the lengths of the corresponding sides are proportional. All congruent polygons are considered similar with the ratio of the corresponding sides being 1:1. [Included in US]

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify corresponding sides and corresponding and congruent angles of similar figures—quadrilaterals and triangles using the traditional notation of curved lines for the angles.
- Given two similar quadrilaterals or triangles, write similarity statements using symbols. [Reordered]
- Write proportions to express the relationships between the lengths of corresponding sides of similar figures—quadrilaterals and triangles.
- Solve a proportion to determine a missing side length of similar quadrilaterals or triangles.
- Given the angle measures in a quadrilateral or triangle, determine the unknown angle measures of the corresponding angle in a similar quadrilateral or triangle.
- Determine if quadrilaterals or triangles are similar by examining congruence of corresponding angles and proportionality of corresponding sides. [Included in G.7]
- Given two similar figures, write similarity statements using symbols such as $\triangle ABC \sim \triangle DEF$, $\angle A$ corresponds to $\angle D$, and $AB$ corresponds to $DE$. [Reordered and example moved to US]
7.56 The student will determine whether plane figures—quadrilaterals and triangles—are similar and write proportions to express the relationships between solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar figures quadrilaterals and triangles.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- The traditional notation for marking congruent angles is to use a curve on each angle. Denote which angles are congruent with the same number of curved lines. For example, if \( \angle A \) is congruent to \( \angle C \), then both angles will be marked with the same number of curved lines.

- Congruent sides are denoted with the same number of hatch (or hash) marks on each congruent side. For example, a side on a polygon with 2 hatch marks is congruent to the side with 2 hatch marks on a congruent polygon or within the same polygon.

\[
\frac{5}{10} = \frac{2}{4} \quad \text{or} \quad \frac{5}{10} = \frac{3}{6} \quad \text{or} \quad \frac{1}{2} = \frac{2}{4}
\]

are some of the ways to express the proportional relationships that exist.

ESSENTIAL UNDERSTANDINGS

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal numbers of hatch marks indicate that sides are equal in length.</td>
</tr>
<tr>
<td>Equal numbers of arrows indicate that sides are parallel.</td>
</tr>
<tr>
<td>Equal numbers of angle curves indicate that angles have the same measure.</td>
</tr>
</tbody>
</table>
In the middle grades, the focus of mathematics learning is to
• build on students’ concrete reasoning experiences developed in the elementary grades;
• construct a more advanced understanding of mathematics through active learning experiences;
• develop deep mathematical understandings required for success in abstract learning experiences; and
• apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

• Students expand the informal experiences they have had with geometry in the elementary grades and develop a solid foundation for the exploration of geometry in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.

• Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provides experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.

• Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.

• Students explore and develop an understanding of the Pythagorean Theorem. Mastery of the use of the Pythagorean Theorem has far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

• Level 0: Pre-recognition. Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

• Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

• Level 2: Analysis. Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

• Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)
Level 4: Deduction. Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)

[Included in Measurement and Geometry Strand Introduction]
### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

| • A polygon is a closed plane figure composed of at least three line segments that do not cross. |
| • A quadrilateral is a closed plane (two-dimensional) figure polygon with four sides that are line segments. |
| • Properties of quadrilaterals include: number of parallel sides, angle measures, number of congruent sides, lines of symmetry, and the relationship between the diagonals. |
| • A diagonal is a segment in a polygon that connects two vertices but is not a side. |
| • To bisect means to divide into two equal parts. |
| • A line of symmetry divides a figure into two congruent parts each of which is the mirror image of the other. Lines of symmetry are not limited to horizontal and vertical lines. |
| • A parallelogram is a quadrilateral whose with both pairs of opposite sides are parallel and opposite angles are congruent. Properties of a parallelogram include the following:
  - opposite sides are parallel and congruent;
  - opposite angles are congruent; and
  - diagonals bisect each other and one diagonal divides the figure into two congruent triangles. |
| Parallelograms, with the exception of rectangles and rhombi, have no lines of symmetry. A rectangle and a rhombus have two lines of symmetry, with the exception of a square which has four lines of symmetry. A rhombus has two lines of symmetry. |

### ESSENTIAL UNDERSTANDINGS

- Why can some quadrilaterals be classified in more than one category? 
  Every quadrilateral in a subset has all of the defining attributes of the subset. For example, if a quadrilateral is a rhombus, it has all the attributes of a rhombus. However, if that rhombus also has the additional property of 4 right angles, then that rhombus is also a square. [Moved to US]

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Compare and contrast attributes, properties of the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid. (a)
- Sort and classify polygons as quadrilaterals, as parallelograms, rectangles, trapezoids, rhombi, and/or squares based on their properties. Properties include: number of parallel sides, angle measures, number of congruent sides, and the relationship between the diagonals. (a) [Moved from 6.13]
- Identify the classification(s) to which a quadrilateral belongs, using deductive reasoning and inference. (a) [Included in bullet above]
- Identify the sum of the measures of the interior angles of a quadrilateral as 360° and use that relationship. Given a diagram, to determine an unknown angle measure in a quadrilateral, using properties of quadrilaterals. (b) [Moved from 6.13]
- Given a diagram to determine an unknown side length in a quadrilateral using properties of quadrilaterals. (b)
7.67 The student will
a) compare and contrast the following quadrilaterals based on their properties: parallelogram, rectangle, square, rhombus, and trapezoid.
b) solve problems to determine unknown side lengths or angle measures of quadrilaterals.

UNDERSTANDING THE STANDARD

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>- The diagonals of a rectangle are the same length and bisect each other.</td>
<td>- A square is a rectangle quadrilateral that is a regular polygon with four congruent sides whose diagonals are perpendicular. A square is a rhombus with four right angles. Properties of a square include the following:</td>
<td>- Opposite sides are congruent and parallel; - all four angles are congruent and each angle measures 90(^\circ); and - diagonals are congruent and bisect each other at right angles.</td>
</tr>
<tr>
<td>• A square is a rectangle quadrilateral that is a regular polygon with four congruent sides whose diagonals are perpendicular. A square is a rhombus with four right angles. Properties of a square include the following:</td>
<td>- A rhombus is a parallelogram quadrilateral with four congruent sides whose diagonals bisect each other and intersect at right angles. Properties of a rhombus include the following:</td>
<td>- All sides are congruent; - opposite sides are parallel; - opposite angles are congruent; and - diagonals bisect each other at right angles.</td>
</tr>
<tr>
<td>• A square has four lines of symmetry. The diagonals of a square coincide with two of the lines of symmetry that can be drawn.</td>
<td>• A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called bases. The nonparallel sides of a trapezoid are called legs.</td>
<td>Example: Square with lines of symmetry shown:</td>
</tr>
</tbody>
</table>
| • An isosceles trapezoid with congruent nonparallel sides is called... |...
The student will

a) compare and contrast the following quadrilaterals based on their properties: parallelogram, rectangle, square, rhombus, and trapezoid.

b) solve problems to determine unknown side lengths or angle measures of quadrilaterals.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- An isosceles trapezoid has legs of equal length and congruent base angles. An isosceles trapezoid has one line of symmetry.

  Example: Isosceles trapezoid with line of symmetry shown:

![Isosceles Trapezoid](image)

- Quadrilaterals can be sorted according to common attributes and properties, using a variety of materials. A chart, graphic organizer, or Venn diagram can be made to organize quadrilaterals according to properties such as sides and/or angles.

- A chart, graphic organizer, or Venn diagram can be made to organize quadrilaterals according to attributes such as sides and/or angles. (Combined with bullet above)

- Quadrilaterals can be classified by the number of parallel sides: parallelogram, rectangle, rhombus, and square each have two pairs of parallel sides; a trapezoid has one pair of parallel sides; other quadrilaterals have no parallel sides.

- Quadrilaterals can be classified by the measures of their angles: a rectangle and a square have four 90° angles; a trapezoid may have zero or two 90° angles.

- Quadrilaterals can be classified by the number of congruent sides: a rhombus and a square have four congruent sides; a square has four congruent sides; a parallelogram and a rectangle each have two pairs of congruent sides, and an isosceles trapezoid has one pair of congruent sides.

- A square is a special type of both a rectangle and a rhombus.
7.67 The student will
   a) compare and contrast the following quadrilaterals based on their properties: parallelogram, rectangle, square, rhombus, and trapezoid.
   b) solve problems to determine unknown side lengths or angle measures of quadrilaterals.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)</th>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>which are special types of parallelograms, which are special types of quadrilaterals.</td>
<td></td>
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</tr>
<tr>
<td>• Any figure that has the properties of more than one subset of quadrilaterals can belong to more than one subset. For example, rectangles have opposite sides of equal length. Squares have all four sides of equal length thereby satisfying the properties of both subsets. [Moved from middle column]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The sum of the measures of the interior angles of a quadrilateral is 360°. Properties of quadrilaterals can be used to find unknown angle measures in a quadrilateral.</td>
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</tbody>
</table>
7.78 The student, given a polygon in the coordinate plane, will apply represent transformations (translations and reflections of right triangles or rectangles, dilations, rotations, and translations) by graphing in the coordinate plane. [Dilations included in 8.7 and G.3; Rotations included in G.3]

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- A transformation of a figure called the preimage changes the size, shape, or position of the figure to a new figure called the image.
- A translation of a geometric figure is a slide of the figure in which an image is formed by moving all the points the same distance in the same direction.
- A reflection is a transformation in which an image is formed by reflecting a figure across a line called the line of reflection. All corresponding points in the image and preimage are equidistant from the line of reflection.
- A dilation of a geometric figure is a transformation that changes the size of a figure by scale factor to create a similar figure. [Included in 8.7]
- The image of a polygon is the resulting polygon after the transformation. The preimage is the polygon before the transformation.
- A transformation of preimage point A can be denoted as the image A’ (read as “A prime”).
- The preimage of a figure that has been translated and then reflected over the x- or y-axis may result in a different transformation than the preimage of a figure that has been reflected over the x- or y-axis and then translated.

### ESSENTIAL UNDERSTANDINGS

- How does the transformation of a figure affect the size, shape and position of that figure?
- Translations, rotations and reflections do not change the size or shape of a figure. [Included in US and 8.7] A dilation of a figure and the original figure are similar. [Moved to 8.7] Reflections, translations and rotations usually change the position of the figure. [Included in US and 8.7]

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or rectangle that has been translated either vertically, horizontally, or a combination of a vertical and horizontal translation.
- Identify the coordinates of the image of a right triangle or rectangle that has been rotated 90° or 180° about the origin. [Included in 8.7 and G.3]
- Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or rectangle that has been reflected over the x- or y-axis.
- Identify the coordinates of a right triangle or rectangle that has been dilated. The center of the dilation will be the origin. [Included in 8.7 and G.3]
- Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or rectangle that has been translated and reflected over the x- or y-axis or reflected over the x- or y-axis and then translated.
- Sketch the image of a right triangle or rectangle that has been translated vertically, horizontally or a combination of both.
- Sketch the image of a right triangle or rectangle that has been rotated 90° or 180° about the origin. [Included in 8.7 and G.3]
- Sketch the image of a right triangle or rectangle that...
The student, given a polygon in the coordinate plane, will apply transformations (translations and reflections of right triangles or rectangles, dilations, rotations, and translations) by graphing in the coordinate plane. [Dilations included in 8.7 and G.3; Rotations included in G.3]

<table>
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<tbody>
<tr>
<td></td>
<td></td>
<td>has been reflected over the $x$- or $y$-axis.</td>
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<tr>
<td></td>
<td></td>
<td>- Sketch the image of a right triangle or rectangle that has been translated and reflected over the $x$- and $y$-axis or reflected over the $x$- or $y$-axis and then translated.</td>
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<tr>
<td></td>
<td></td>
<td>- Sketch the image of a dilation of a right triangle or rectangle limited to a scale factor of $rac{1}{4}$, $rac{1}{2}$, $2$, $3$, or $4$. [Moved to 8.7]</td>
</tr>
</tbody>
</table>
In the middle grades, the focus of mathematics learning is to
- build on students’ concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop an awareness of the power of data analysis and probability by building on their natural curiosity about data and making predictions.
- Students explore methods of data collection and use technology to represent data with various types of graphs. They learn that different types of graphs represent different types of data effectively. They use measures of center and dispersion to analyze and interpret data.
- Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability.
- Students explore experimental and theoretical probability through experiments and simulations by using concrete, active learning activities.

In the middle grades, students develop an awareness of the power of data analysis and the application of probability through fostering their natural curiosity about data and making predictions.

The exploration of various methods of data collection and representation allows students to become effective at using different types of graphs to represent different types of data. Students use measures of center and dispersion to analyze and interpret data.

Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability. Through experiments and simulations, students build on their understanding of the Fundamental Counting Principle from elementary mathematics to learn more about probability in the middle grades.
The student will:

- a) determine the theoretical and experimental probabilities of an event; and
- b) investigate and describe the difference between the experimental probability and theoretical probability of an event.

### UNDERSTANDING THE STANDARD

**ESSENTIAL UNDERSTANDINGS**

- In general, if all outcomes of an event are equally likely, the probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes in the sample space. [Moved from 6.16.
- The probability of an event occurring can be represented as a ratio or the equivalent fraction, decimal, or percent. [Moved from 6.16]
- The probability of an event occurring is a ratio between 0 and 1. [Moved from 6.16]
  - A probability of 0 means the event will never occur.
  - A probability of 1 means the event will always occur.
- The theoretical probability of an event is the expected probability and can be found determined with a formula.
- If all outcomes of an event are equally likely, the theoretical probability of an event = number of possible favorable outcomes / total number of possible outcomes
- The experimental probability of an event is determined by carrying out a simulation or an experiment.
- The experimental probability of an event = number of times desired outcomes occur / number of trials in the experiment
- In experimental probability, as the number of trials increases, the experimental probability gets closer to the theoretical probability (Law of Large Numbers).

**ESSENTIAL KNOWLEDGE AND SKILLS**

- What is the difference between the theoretical and experimental probability of an event? Theoretical probability of an event is the expected probability and can be found with a formula. The experimental probability of an event is determined by carrying out a simulation or an experiment. In experimental probability, as the number of trials increases, the experimental probability gets closer to the theoretical probability.
- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:
  - Determine the theoretical probability of an event. (a)
  - Determine the experimental probability of an event. (a)
  - Describe changes in the experimental probability as the number of trials increases. (b)
  - Investigate and describe the difference between the probability of an event found through experiment or simulation versus the theoretical probability of that same event. (b)
The student will determine the probability of compound events, using the Fundamental (Basic) Counting Principle. [Moved to 5.15]

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD (Teacher Notes)</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The Fundamental (Basic) Counting Principle is a computational procedure to determine the number of possible outcomes of several events. It is the product of the number of outcomes for each event that can be chosen individually (e.g., the possible outcomes or outfits of four shirts, two pants, and three shoes is $4 \cdot 2 \cdot 3 = 24$).</td>
<td></td>
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</tr>
<tr>
<td>• Tree diagrams are used to illustrate possible outcomes of events. They can be used to support the Fundamental (Basic) Counting Principle.</td>
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</tr>
<tr>
<td>• A compound event combines two or more simple events. For example, a bag contains 4 red, 3 green, and 2 blue marbles. What is the probability of selecting a green and then a blue marble?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• What is the Fundamental (Basic) Counting Principle? The Fundamental (Basic) Counting Principle is a computational procedure used to determine the number of possible outcomes of several events.</td>
<td></td>
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</tr>
<tr>
<td>• What is the role of the Fundamental (Basic) Counting Principle in determining the probability of compound events? The Fundamental (Basic) Counting Principle is used to determine the number of outcomes of several events. It is the product of the number of outcomes for each event that can be chosen individually.</td>
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</tr>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:</td>
<td></td>
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<tr>
<td>• Compute the number of possible outcomes by using the Fundamental (Basic) Counting Principle.</td>
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<td></td>
</tr>
<tr>
<td>• Determine the probability of a compound event containing no more than 2 events.</td>
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</tr>
</tbody>
</table>
The student, given data in a practical situation, will
a) construct and analyze data in a histogram; and
b) make observations and inferences about data represented in a histogram; and

7.911 The student, given data in a practical situation, will

a) construct and analyze data in a histogram; and

b) make observations and inferences about data represented in a histogram; and

c) compare and contrast histograms with other types of graphs presenting information from the same data represented in stem-and-leaf plots, line plots, and circle graphs.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- A histogram is a graph that provides a visual interpretation of numerical data by indicating the number of data points that lie within a range of values, called a class or a bin. The frequency of the data that falls in each class or bin is depicted by the use of a bar. Every element of the data set is not preserved when representing data in a histogram.

- All graphs tell a story and must include a title and labels that describe the data.

- Numerical data that can be characterized using consecutive intervals are best displayed in a histogram. (Moved from middle column)

- Teachers should be reasonable about the selection of data values. Students should have experiences constructing histograms, but a focus should be placed on the analysis of histograms.

- A histogram is a form of bar graph in which the categories are consecutive and equal intervals. The length or height of each bar is determined by the number of data elements (frequency) falling into a particular interval.

ESSENTIAL UNDERSTANDINGS

- What types of data are most appropriate to display in a histogram?

- Numerical data that can be characterized using consecutive intervals are best displayed in a histogram. (Moved to US)

ESSENTIAL KNOWLEDGE AND SKILLS

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Collect, analyze, organize, display, and interpret data represented in histogram. Data should be limited to 20 items or less. For collection and display of raw data, limit the data to 20 items. (a)

- Determine patterns and relationships within data sets (e.g., trends).

- Make observations and inferences, conjectures, and predictions based on analysis of a set of data represented in a histogram. (b)

- Describe the distribution of data represented in a histogram as skewed right, skewed left, or symmetric. (b)

- Compare and contrast data represented in histograms with the same data represented in line plots, circle graphs, and stem-and-leaf plots presenting information from the same data set. (c)
STANDARD 7.911 STRAND: PROBABILITY AND STATISTICS GRADE LEVEL 7

7.911 The student, given data in a practical situation, will
   a) construct and analyze represent data in a histogram; and
   b) make observations and inferences about data represented in a histogram; and
   c) compare and contrast histograms with other types of graphs presenting information from the same data set represented in stem-and-leaf plots, line plots, and circle graphs.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

<table>
<thead>
<tr>
<th>STUDENTS WHO READ GARFIELD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age Group</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>7-10</td>
</tr>
<tr>
<td>11-14</td>
</tr>
<tr>
<td>15-18</td>
</tr>
<tr>
<td>19-22</td>
</tr>
</tbody>
</table>

Number of Cappuccinos Made per Hour at the Cafe

<table>
<thead>
<tr>
<th>Number of Cups of Coffee</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 - 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 - 11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 - 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16 - 19</td>
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</tr>
</tbody>
</table>

To construct a histogram:

- Organize collected data into a table. Create one column for data range categories (bins), divided into equal intervals that will include all of your data (for example, 0-10, 11-20, 21-30), and another column for frequency.
  - Bins should be all the same size.
  - Bins should include all of the data.
  - Boundaries for bins should reflect the data values being represented.
  - Determine the number of bins based upon the data.
  - If possible, the number of bins created should be a factor of the number of data values (e.g., a histogram representing 20 data values might have 4 or 5 bins).
7.911 The student, given data in a practical situation, will
a) construct and analyze represent data in a histograms; and
b) make observations and inferences about data represented in a histogram; and
c) compare and contrast histograms with other types of graphs presenting information from the same data set represented in stem-and-leaf plots, line plots, and circle graphs.

## UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Create a graph. Mark the data range intervals on the x-axis (horizontal axis) with no space between the categories. Mark frequency on the y-axis (vertical axis), also in equal intervals.

- Plot your data. For each data range category (bin), draw a horizontal line at the appropriate frequency or marker. Then, create a vertical bar for that category reaching up to the marked frequency. Do this for each data range category (bin).

- Note: histograms may be drawn so that the bars are horizontal. To do this, interchange the x- and y-axis. Mark the data range intervals (bins) on the y-axis and the frequency on the x-axis. Draw the bars horizontally.

- Histograms give a sense of the density of the underlying distribution of the data.

- Comparisons, predictions and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. Data analysis helps describe data, recognize patterns or trends, and make predictions.

- The words used to describe some patterns in a histogram are: "symmetric", "skewed right" or "skewed left". Skewness is the measure of the asymmetry of a histogram.
7.9.11 The student, given data in a practical situation, will
a) construct and analyze data in a histogram; and
b) make observations and inferences about data represented in a histogram; and
c) compare and contrast histograms with other types of graphs presenting information from the same data set represented in stem-and-leaf plots, line plots, and circle graphs.

**UNDERSTANDING THE STANDARD**
(Background Information for Instructor Use Only)

<table>
<thead>
<tr>
<th></th>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
</tbody>
</table>

- When describing a histogram, it is important to talk about its shape or distribution, its center (such as mean or median) and its spread (including peaks). A symmetric histogram indicates that the mode, median, and mean are the same or are together in the center. The mode, median, and mean are near the center in a symmetric histogram. A skewed histogram indicates the mean and median have different values and do not lie in the center. The mean will be greater than the median in a histogram that is skewed right. The mean will be less than the median in a histogram that is skewed left. Skewed histograms with two modes or "peaks" are known as bi-modal.

- There are two types of data: categorical and numerical. Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights

[Body Lengths of 300 Weaver Ant Workers]
STANDARD 7.9.11 STRAND: PROBABILITY AND STATISTICS GRADE LEVEL 7

7.9.11 The student, given data in a practical situation, will
a) construct and analyze represent data in a histogram; and
b) make observations and inferences about data represented in a histogram; and

c) compare and contrast histograms with other types of graphs presenting information from the same data set represented in stem-and-leaf plots, line plots, and circle graphs.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

of fish caught would be numerical data. While students need to be aware of the differences, they do not have to know the terms for each type of data.

- Different types of graphs can be used to display categorical data. The way data is displayed is often dependent on what someone is trying to communicate.

- A line plot provides an ordered display of all values in a data set and shows the frequency of data on a number line. Line plots are used to show the spread of the data, to include clusters (groups of data points) and gaps (large spaces between data points), and quickly identify the range, mode, and any extreme data values (outliers).

- A circle graph is used for categorical and discrete numerical data. Circle graphs are used to show a relationship of the parts to a whole. Every element of the data set is not preserved when representing data in a circle graph.
The student, given data in a practical situation, will
a) construct and analyze represent data in a histograms; and
b) make observations and inferences about data represented in a histogram; and
c) compare and contrast histograms with other types of graphs presenting information from the same data set represented in stem-and-leaf plots, line plots, and circle graphs.

A stem and leaf plot is used for discrete numerical data and is used to show frequency of data distribution. A stem and leaf plot displays the entire data set and provides a picture of the distribution of data.

Different situations or contexts warrant different types of graphs, and it helps to have a good knowledge of what graphs are available. Students can determine which graph makes the most sense to use based on the type of data provided and which graph can
The student, given data in a practical situation, will
a) construct and analyze **represent data in a histograms**; and
b) make observations and **inferences** about data represented in a histogram; and
c) **compare and contrast** histograms with other types of graphs presenting information from the same data set represented in stem-and-leaf plots, line plots, and circle graphs.

<table>
<thead>
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<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>help them answer questions most easily.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Comparing different types of representations (charts and graphs) provide students an opportunity to learn how different graphs can show different things about the same data. Following construction of graphs, students benefit from discussions around what information each graph provides.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The information displayed in different graphs may be examined to determine how data are or are not related, ascertaining differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or “what could happen if” (inference).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In the middle grades, the focus of mathematics learning is to

- build on students’ concrete reasoning experiences developed in the elementary grades;
- construct through active learning experiences a more advanced understanding of mathematics;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

- Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students extend their knowledge of patterns developed in the elementary grades and through practical experiences by investigating and describing functional relationships.

- Students learn to use algebraic concepts and terms appropriately. These concepts and terms include variable, term, coefficient, exponent, expression, equation, inequality, domain, and range. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.

- Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.

- Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.

Patterns, functions and algebra become a larger mathematical focus in the middle grades as students extend their knowledge of patterns developed in the elementary grades.

Students make connections between the numeric concepts of ratio and proportion and the algebraic relationships that exist within a set of equivalent ratios. Students use variable expressions to represent proportional relationships between two quantities and begin to connect the concept of a constant of proportionality to rate of change and slope. Representation of relationships between two quantities using tables, graphs, equations, or verbal descriptions allow students to connect their knowledge of patterns to the concept of functional relationships. Graphing linear equations in two variables in the coordinate plane is a focus of the study of functions which continues in high school mathematics.

Students learn to use algebraic concepts and terms appropriately. These concepts and terms include variable, term, coefficient, exponent, expression, equation, inequality, domain, and range. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades. Students learn to solve equations by using concrete materials. They expand their skills from one-step to multistep equations and inequalities through their application in practical situation.
The student will represent relationships with tables, graphs, rules, and words.

**a)** determine if a proportional relationship exists between two quantities, given a table of values or a graph; [Moved to 6.12c, 2016]

**b)** identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations; [Moved to 6.12b, 2016]

**c)** a) determine the slope, \( m \), as rate of change in a proportional relationship between two quantities, given two ordered pairs, and write an equation in the form \( y = mx \) to represent the relationship;

**d)** b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in \( y = mx \) form where \( m \) represents the slope as rate of change.

**a)** determine the \( y \)-intercept, \( b \), in an additive relationship between two quantities and write an equation in the form \( y = x + b \) to represent the relationship;

**b)** graph a line representing an additive relationship between two quantities given the \( y \)-intercept and an ordered pair, or given the equation in the form \( y = x + b \), where \( b \) represents the \( y \)-intercept; and

**c)** make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

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<tr>
<td>Rules that relate elements in two sets can be represented by word sentences, equations, tables of values, graphs, or illustrated pictorially.</td>
<td>What are the different ways to represent the relationship between two sets of numbers? Rules that relate elements in two sets can be represented by word sentences, equations, tables of values, graphs, or illustrated pictorially.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>A relation is any set of ordered pairs. For each first member, there may be many second members.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A function is a relation in which there is one and only one second member for each first member.</td>
<td></td>
<td>Describe and represent relations and functions, using tables, graphs, rules and words. Given one representation, students will be able to represent the relation in another form.</td>
</tr>
<tr>
<td>As a table of values, a function has a unique value assigned to the second variable for each value of the first variable.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>As a graph, a function is any curve (including straight lines) such that any vertical line would pass through the curve only once.</td>
<td></td>
<td>Determine if a proportional relationship exists between two quantities given a table of values. (a) [Moved to 6.13, 2016]</td>
</tr>
<tr>
<td>Some relations are functions; all functions are relations.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>When two quantities, ( x ) and ( y ), vary in such a way that one of them is a constant multiple of the other, the two quantities are “proportional”. A model for that situation is ( y = kx ) where ( k ) is the slope or rate of change. Slope may also represent the unit rate of a proportional relationship between two quantities, also referred to as the constant of proportionality or the constant ratio of ( y ) to ( x ).</td>
<td></td>
<td>Determine if a proportional relationship exists between two quantities given a graph. (a) [Moved to 6.13, 2016]</td>
</tr>
<tr>
<td>The slope of a proportional relationship can be determined by finding the unit rate.</td>
<td></td>
<td>Identify the unit rate (constant of proportionality) of a proportional relationship represented by a table, graph, equation, or verbal description, including those represented in a practical situation. (b) [Moved to 6.13, 2016]</td>
</tr>
</tbody>
</table>

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 7 41
7.10 The student will represent relationships with tables, graphs, rules, and words.

a) determine if a proportional relationship exists between two quantities, given a table of values or a graph; [Moved to 6.12c, 2016]
b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations; [Moved to 6.12b, 2016]
c) determine the slope, \( m \), as rate of change in a proportional relationship between two quantities, given two ordered pairs, and write an equation in the form \( y = mx \) to represent the relationship;
d) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in \( y = mx \) form where \( m \) represents the slope as rate of change;
e) graph a line representing an additive relationship between two quantities given the \( y \)-intercept and an ordered pair, or given the equation in the form \( y = x + b \), where \( b \) represents the \( y \)-intercept; and
f) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

UNDERSTANDING THE STANDARD

Example: The ordered pairs (4, 2) and (6, 3) make up points that could be included on the graph of a proportional relationship. Determine the slope, or rate of change, of a line passing through these points. Write an equation of the line representing this proportional relationship.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

The slope, or rate of change, would be \( \frac{1}{2} \) or 0.5 since the \( y \)-coordinate of each ordered pair would result by multiplying \( \frac{1}{2} \) times the \( x \)-coordinate. This would also be the unit rate of this proportional relationship. The ratio of \( y \) to \( x \) is the same for each ordered pair. That is, \( \frac{\Delta y}{\Delta x} = \frac{3 - 2}{6 - 4} = \frac{1}{2} = 0.5 \).

The equation of a line representing this proportional relationship of \( y \) to \( x \) is \( y = \frac{1}{2} x \) or \( y = 0.5x \).

- The slope of a line is a rate of change, a ratio describing the vertical change to the horizontal change of the line.

\[
\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{vertical change}}{\text{horizontal change}}
\]
STANDARD 7.1012  STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA  GRADE LEVEL 7

The student will represent relationships with tables, graphs, rules, and words.

a) determine if a proportional relationship exists between two quantities, given a table of values or a graph; [Moved to 6.12c, 2016]
b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations; [Moved to 6.12b, 2016]
c) determine the slope, \( m \), as rate of change in a proportional relationship between two quantities, given two ordered pairs; and write an equation in the form \( y = mx \) to represent the relationship;
d) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in \( y = mx \) form where \( m \) represents the slope as rate of change;

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- A table of values representing a proportional relationship includes pairs of values that represent equivalent rates or ratios. All ratios are equivalent and a constant exists that can be multiplied by the measure of the one quantity to get the measure of the other quantity for every ratio pair. An example of a proportional relationship: Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges $8 for each medium pizza. This table represents the cost per number of pizza ordered.

<table>
<thead>
<tr>
<th>x (number of pizzas)</th>
<th>y (cost)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
</tr>
</tbody>
</table>

This relationship can be represented by \( y = 8x \) where \( y \) is the cost and \( x \) is the number of medium pizzas ordered. The constant of proportionality is evident with the table as the multiplicative relationship that exists within each \( x:y \) ratio. In this example, it is 8.

An example of a non-proportional relationship: Uptown Pizza sells medium pizzas for $7 each but charges a $3 delivery fee per order. This table represents the cost per number of pizzas ordered.

<table>
<thead>
<tr>
<th>x (number of pizzas)</th>
<th>y (cost)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
</tr>
</tbody>
</table>

Graph a line representing an additive relationship between two quantities, given the \( y \)-intercept and an ordered pair, or given the equation in the form \( y = x + b \), where \( b \) represents the \( y \)-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

ESSENTIAL UNDERSTANDINGS

- Graph a line representing an additive relationship \( y = x + b \), \( b \neq 0 \), to represent the relationship. (c)

ESSENTIAL KNOWLEDGE AND SKILLS

- Graph a line representing an additive relationship \( y = x + b \), \( b \neq 0 \) between two quantities, given an ordered pair on the line and the \( y \)-intercept \( b \). The \( y \)-intercept \( b \) is limited to integer values and slope is limited to 1. (d)

- Make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs. (c)
7.1012 The student will represent relationships with tables, graphs, rules, and words.

a) determine if a proportional relationship exists between two quantities, given a table of values or a graph; [Moved to 6.12c, 2016]
b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations; [Moved to 6.12b, 2016]
c) determine the slope, \( m \), as rate of change in a proportional relationship between two quantities, given two ordered pairs; and write an equation in the form \( y = mx \) to represent the relationship;
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e) determine the \( y \)-intercept, \( b \), in an additive relationship between two quantities and write an equation in the form \( y = x + b \) to represent the relationship;
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g) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

### UNDERSTANDING THE STANDARD
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<table>
<thead>
<tr>
<th>( x ) number of pizzas</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y ) total cost</td>
<td>16</td>
<td>17</td>
<td>24</td>
<td>31</td>
</tr>
</tbody>
</table>

The ratios represented in the table above are not equivalent. Other non-proportional relationships that are not linear will be studied in later courses.

- The graph of the line representing a proportional relationship is a straight line that passes through the origin (0, 0).
- A unit rate (constant of proportionality) is a ratio (represented as a fraction) with a denominator of 1. For example, the ratio of 3 cups of apple juice for every 2 cups of grape juice has a unit rate of \( \frac{3}{2} \) cups of apple juice for every 1 cup of grape juice. A unit rate can be described as “for each 1” or “for every 1.”
- In grade 6, proportional relationships were represented by the equation \( y = kx \), where \( k \) represents the constant of proportionality. A proportional relationship between two quantities can be modeled given a practical situation. Representations may include verbal descriptions, tables, equations, or graphs. Students may benefit from an informal discussion about independent and dependent variables when modeling practical situations. Grade 8 mathematics...
STANDARD 7.1012  STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA  GRADE LEVEL 7

7.1012  The student will represent relationships with tables, graphs, rules, and words.
   a) determine if a proportional relationship exists between two quantities, given a table of values or a graph; [Moved to 6.12c, 2016]
   b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations; [Moved to 6.12b, 2016]
   c) determine the slope, \( m \), as rate of change in a proportional relationship between two quantities, given two ordered pairs, and write an equation in the form \( y = mx \) to represent the relationship;
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<td>(Background Information for Instructor Use Only)</td>
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<td></td>
</tr>
<tr>
<td>formally addresses identifying dependent and independent variables.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Example (using a table of values): Cecil walks 2 meters every second (verbal description). If ( x ) represents the number of seconds and ( y ) represents the number of meters he walks, this proportional relationship can be represented using a table of values:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x ) seconds</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y ) meters</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

| This proportional relationship could be represented using the equation \( y = 2x \), since he walks 2 meters for each second of time. That is, \( \frac{y}{x} = \frac{2}{1} \) or \( 2 \). The same constant ratio of \( y \) to \( x \) of proportionality exists between the ordered pairs. This proportional relationship could be represented by the following graph. | | |

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STANDARD 7.1012  STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA  GRADE LEVEL 7

7.1012  The student will represent relationships with tables, graphs, rules, and words.
   a) determine if a proportional relationship exists between two quantities, given a table of values or a graph; [Moved to 6.12c, 2016]
   b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations; [Moved to 6.12b, 2016]
   c) determine the slope, \( m \), as rate of change in a proportional relationship between two quantities, given two ordered pairs; and write an equation in the form \( y = mx \) to represent the relationship;
   d) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in \( y = mx \) form where \( m \) represents the slope as rate of change.
   e) determine the \( y \)-intercept, \( b \), in an additive relationship between two quantities and write an equation in the form \( y = x + b \) to represent the relationship;
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<tbody>
<tr>
<td>Time Walked vs. Distance</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- A graph of a proportional relationship can be created by graphing ordered pairs generated in a table of values (as shown above), or by observing the rate of change or slope of the relationship and using slope triangles to graph ordered pairs that satisfy the relationship given.
7.1012 The student will represent relationships with tables, graphs, rules, and words.

a) determine if a proportional relationship exists between two quantities, given a table of values or a graph; [Moved to 6.12c, 2016]

b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations; [Moved to 6.12b, 2016]

c) determine the slope, \( m \), as rate of change in a proportional relationship between two quantities, given two ordered pairs; and write an equation in the form \( y = mx \) to represent the relationship;

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<tr>
<td>The slope of the graph of a proportional relationship is the constant of proportionality or the constant ratio of ( y ) to ( x ). The equation ( y = kx ) can be expressed as ( y = mx ) where ( m ) represents the slope or the constant of proportionality. Slope is the rate of change described by the multiplicative relationship between the ( y ) coordinate and the ( x ) coordinate. Two points are necessary to determine the value for ( m ) to describe the direction and steepness of the line.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The slope of a line is a rate of change, a ratio describing the vertical change to the horizontal change. Slope = ( \frac{\text{vertical change}}{\text{horizontal change}} ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Example (using slope triangles): Cecil walks 2 meters every second. If ( x ) represents the number of seconds and ( y ) represents the number of meters he walks, this proportional relationship can be represented graphically using slope triangles.</td>
<td></td>
<td></td>
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Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 7
7.1012 The student will represent relationships with tables, graphs, rules, and words.

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UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

For example, using this graph, determine the slope as a rate of change from two points on the graph.
The rate of change from (1, 2) to (2, 4) is 2 units up (the change in \( y \)) and 1 unit to the right (the change in \( x \)), \( \frac{2}{1} \) or 2. Thus, the slope of this line is 2. Slope
The student will represent relationships with tables, graphs, rules, and words.

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ea) determine the y-intercept, $b$, in an additive relationship between two quantities and write an equation in the form $y = x + b$ to represent the relationship;
b) graph a line representing an additive relationship between two quantities given the y-intercept and an ordered pair, or given the equation in the form $y = x + b$, where $b$ represents the y-intercept; and
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e) determine the \( y \)-intercept, \( b \), in an additive relationship between two quantities and write an equation in the form \( y = x + b \) to represent the relationship;

b) graph a line representing an additive relationship between two quantities given the \( y \)-intercept and an ordered pair, or given the equation in the form \( y = x + b \), where \( b \) represents the \( y \)-intercept; and
e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

ESSENTIAL UNDERSTANDINGS

ESSENTIAL KNOWLEDGE AND SKILLS

<table>
<thead>
<tr>
<th>Additive relationship:</th>
<th>Multiplicative relationship:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( y )</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

In the additive relationship, \( y \) is the result of adding 8 to \( x \).
In the multiplicative relationship, \( y \) is the result of multiplying 5 times \( x \). The ordered pair (2, 10) is a quantity in both relationships, however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative.

- Two quantities, \( x \) and \( y \), have an additive relationship when a constant value, \( b \), exists where \( y = x + b \); where \( b \neq 0 \). An additive relationship is not proportional and its graph does not pass through \((0, 0)\). Note that \( b \) can be a positive value or a
STANDARD 7.1012  STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA  GRADE LEVEL 7

7.1012  The student will represent relationships with tables, graphs, rules, and words.

a) determine if a proportional relationship exists between two quantities, given a table of values or a graph; [Moved to 6.12c, 2016]
b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations; [Moved to 6.12b, 2016]
c) determine the slope, \( m \), as rate of change in a proportional relationship between two quantities, given two ordered pairs; and write an equation in the form \( y = mx \) to represent the relationship;
d) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in \( y = mx \) form where \( m \) represents the slope as rate of change.
e) determine the \( y \)-intercept, \( b \), in an additive relationship between two quantities and write an equation in the form \( y = x + b \) to represent the relationship;
f) graph a line representing an additive relationship between two quantities given the \( y \)-intercept and an ordered pair, or given the equation in the form \( y = x + b \), where \( b \) represents the \( y \)-intercept; and
g) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

**UNDERSTANDING THE STANDARD**
(Background Information for Instructor Use Only)

negative value. When \( b \) is negative, the right side of the equation could be written using a subtraction symbol (e.g., if \( b \) is -5, then the equation \( y = x - 5 \) could be used).

Example:
Thomas is four years older than his sister, Amanda (verbal description). The following table shows the relationship between their ages at given points in time.

<table>
<thead>
<tr>
<th>Amanda’s Age</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thomas’ Age</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

The equation that represents the relationship between Thomas’ age and Amanda’s age is \( y = x + 4 \). A graph of the relationship between their ages would be:

**ESSENTIAL UNDERSTANDINGS**

**ESSENTIAL KNOWLEDGE AND SKILLS**
7.1012 The student will represent relationships with tables, graphs, rules, and words.

a) determine if a proportional relationship exists between two quantities, given a table of values or a graph; [Moved to 6.12c, 2016]

b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations; [Moved to 6.12b, 2016]

c) determine the slope, \( m \), as rate of change in a proportional relationship between two quantities, given two ordered pairs, and write an equation in the form \( y = mx \) to represent the relationship;

d) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in \( y = mx \) form where \( m \) represents the slope as rate of change.

e) determine the \( y \)-intercept, \( b \), in an additive relationship between two quantities and write an equation in the form \( y = x + b \) to represent the relationship;

f) graph a line representing an additive relationship between two quantities given the \( y \)-intercept and an ordered pair, or given the equation in the form \( y = x + b \), where \( b \) represents the \( y \)-intercept; and

g) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

#### ESSENTIAL UNDERSTANDINGS

- **Graphing a linear function** given an equation can be addressed using different methods. **One method** involves determining a table of ordered pairs by substituting into the equation values for one variable and solving for the other variable, plotting the ordered pairs in the coordinate plane, and connecting the points to form a straight line. **Another method** involves using slope triangles to determine points on the line.
7.1012 The student will represent relationships with tables, graphs, rules, and words.

a) determine if a proportional relationship exists between two quantities, given a table of values or a graph; [Moved to 6.12c, 2016]
b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations; [Moved to 6.12b, 2016]
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**UNDERSTANDING THE STANDARD**
(Background Information for Instructor Use Only)

- Example: Graph the equation \( y = x - 1 \).

  In order to graph the equation, we can create a table of values by substituting arbitrary values for \( x \) to determine coordinating values for \( y \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>( x - 1 )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>(-1) - 1</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>(0) - 1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>(1) - 1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>(2) - 1</td>
<td>1</td>
</tr>
</tbody>
</table>

  These values can then be plotted as the points (-1, -2), (0, -1), (1, 0), and (2, 1) on a graph.

  An equation written in \( y = x + b \) form provides information about the graph.

  If the equation is \( y = x - 1 \), then the slope, \( m \), of the line is 1 or \( \frac{1}{1} \) and the point where the line crosses the \( y \)-axis can be located at (0, -1). We also know,

  \[
  \text{slope} = m = \frac{\text{change in } y\text{-value}}{\text{change in } x\text{-value}} = \frac{+1}{+1}
  \]

  So we can plot some other points on the graph using this relationship between \( y \) and \( x \) values.

**ESSENTIAL UNDERSTANDINGS**

- An equation written in \( y = x + b \) form provides information about the graph.

**ESSENTIAL KNOWLEDGE AND SKILLS**
7.1012 The student will represent relationships with tables, graphs, rules, and words.

a) determine if a proportional relationship exists between two quantities, given a table of values or a graph; [Moved to 6.12c, 2016]

b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations; [Moved to 6.12b, 2016]

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e) determine the \( y \)-intercept, \( b \), in an additive relationship between two quantities and write an equation in the form \( y = x + b \) to represent the relationship;

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g) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

A table of values can be used to determine the graph of a line. The \( y \)-intercept is located on the \( y \)-axis which is where the \( x \)-coordinate is 0. The change in each \( y \)-value compared to the corresponding \( x \)-value can be verified by the patterns in the table of values.
7.1113 The student will

a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and [Included in 7.12 EKS]

b) evaluate algebraic expressions for given replacement values of the variables.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Teacher Notes)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- An expression is a name for a number.
- An expression that contains a variable is a variable expression.
- An expression that contains only numbers is a numerical expression.
- A verbal expression is a word phrase (e.g., “the sum of two consecutive integers”).
- A verbal sentence is a complete word statement (e.g., “The sum of two consecutive integers is five.”).
- An algebraic expression is a variable expression that contains at least one variable (e.g., \(2x - 5\)).
- An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., \(2x + 1 = 5\)).

To evaluate an algebraic expression, substitute a given replacement value for a variable and apply the order of operations. For example, if \(a = 3\) and \(b = -2\) then \(5a + b\) can be evaluated as: \(5(3) + (-2)\) and simplified using the order of operations to equal \(15 + (-2)\) which equals 13.

- Expressions are simplified by using the order of operations.

- The order of operations is a convention that defines the computation order to follow in simplifying an expression. It ensures that there is only one correct value. The order of operations is as follows:
  - First, complete all operations within grouping symbols\(^1\). If there are grouping symbols within other grouping symbols, do the innermost operations first.
  - Second, evaluate all exponential expressions.
  - Third, multiply and/or divide in order from left to right.
  - Fourth, add and/or subtract in order from left to right.

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Write verbal expressions as algebraic expressions. Expressions will be limited to no more than 2 operations. [Included in 7.12 EKS]

- Write verbal sentences as algebraic equations. Equations will contain no more than 1 variable term. [Included in 7.12 EKS]

- Translate algebraic expressions and equations to verbal expressions and sentences. Expressions and equations will be limited to no more than two operations. [Included in 7.12 EKS]

- Identify examples of expressions and equations.

- Represent algebraic expressions using concrete materials and pictorial representations. Concrete materials may include colored chips or algebra tiles.

- Apply the order of operations and apply the properties of real numbers to evaluate expressions for given replacement values of the variables. Exponents are limited to whole numbers 1, 2, 3, or 4 and bases are limited to positive integers. Expressions should not include braces \(\{}\) but may include brackets \([\) and absolute value \(\lvert\). Square roots are limited to perfect squares. Limit the number of replacements to no more than 3 per expression.
The student will:

a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and [Included in 7.12 EKS]

b) evaluate algebraic expressions for given replacement values of the variables.

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<tbody>
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<td><strong>(Teacher Notes)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Parentheses ( ), brackets [ ], braces { }, and the division bar should be treated as grouping symbols.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Expressions are simplified using the order of operations and applying the properties of real numbers. Students should utilize the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of a, b, or c in this standard).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 5.6 + 4 = 4 + 5.6).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 5.6 · -4 = -4 · 5.6).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The associative property of addition states that regrouping the addends does not change the sum (e.g., 5.6 + (4 + 3) = (5.6 + 4) + 3).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The associative property of multiplication states that regrouping the factors does not change the product (e.g., 5.6 · (4 · 3) = (5.6 · 4) · 3).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Subtraction and division are neither commutative nor associative.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Distributive property examples:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- 5.2 · (3 + 7) = (5.2 · 3) + (5.2 · 7), or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- 5.2 · (3 - 7) = (5.2 · 3) - (5.2 · 7)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Converse of the distributive property examples:</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7.1113 The student will
a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and [Included in 7.12 EKS]
b) evaluate algebraic expressions for given replacement values of the variables.

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<tr>
<th>UNDERSTANDING THE STANDARD (Teacher Notes)</th>
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<tbody>
<tr>
<td>- ((-5.2 \cdot 3) + (5.2 \cdot -7) = 5.2 \cdot (3 + -7)) or</td>
<td>- ((-5.2 \cdot 3) - (5.2 \cdot -7) = 5.2 \cdot (3 - -7))</td>
<td></td>
</tr>
<tr>
<td>- The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real number (e.g., (5 + 0 = 5)).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number (e.g., (\sqrt{8} \cdot 1 = \sqrt{8})).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inverses are numbers that combine with other numbers and result in identity elements. (e.g., (5 + (-5) = 0); (\frac{1}{4} \cdot 4 = 1).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., (5.6 + (-5.6) = 0)).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., (4 \cdot \frac{1}{4} = 1)).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Zero has no multiplicative inverse.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- The multiplicative property of zero states that the product of any real number and zero is zero (e.g., (7 \cdot 0 = 0)).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
STANDARD 7.1113 STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA GRADE LEVEL 7

7.1113 The student will
   a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and [Included in 7.12 EKS]
   b) evaluate algebraic expressions for given replacement values of the variables.

UNDERSTANDING THE STANDARD
(Teacher Notes)

ESSENTIAL UNDERSTANDINGS

ESSENTIAL KNOWLEDGE AND SKILLS

- Division by zero is not a possible mathematical operation. It is undefined.
- The substitution property of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality. (e.g., if \( x = 5 + 8 \) then \( x = 13 \). Thus, 3 was substituted for \( 5 + 8 \) since they are equivalent).
- Commutative property of addition: \( a + b = b + a \).
- Commutative property of multiplication: \( a \cdot b = b \cdot a \).
- Associative property of addition: \( (a + b) + c = a + (b + c) \).
- Associative property of multiplication: \( (a \cdot b) \cdot c = a \cdot (b \cdot c) \).
- Subtraction and division are neither commutative nor associative.
- Distributive property (over addition/subtraction): \( a \cdot (b + c) = a \cdot b + a \cdot c \) and \( a \cdot (b - c) = a \cdot b - a \cdot c \).
- The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division.
- Identity property of addition (additive identity property): \( a + 0 = a \) and \( 0 + a = a \).
- Identity property of multiplication (multiplicative identity property): \( a \cdot 1 = a \) and \( 1 \cdot a = a \).
- Inverses are numbers that combine with other numbers and result in identity elements [e.g., \( 5 + (-5) = 0; \frac{1}{2} \cdot 5 = 1 \)].
7.1113 The student will
a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and [Included in 7.12 EKS]
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<tbody>
<tr>
<td>- Inverse property of addition (additive inverse property): ( a + (-a) = 0 ) and ( (-a) + a = 0 ).</td>
<td></td>
<td></td>
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<tr>
<td>- Inverse property of multiplication (multiplicative inverse property): ( a \cdot \frac{1}{a} = 1 ) and ( \frac{1}{a} \cdot a = 1 ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Zero has no multiplicative inverse.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Multiplicative property of zero: ( a \cdot 0 = 0 ) and ( 0 \cdot a = 0 ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Division by zero is not a possible mathematical operation. It is undefined.</td>
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<tr>
<td>- Substitution property: if ( a = b ), then ( b ) can be substituted for ( a ) in any expression, equation, or inequality.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7.1214 The student will

a) solve one- and two-step linear equations in one variable; and, including

b) solve practical problems that requiring the solution of a one- and two-step linear equations in one variable.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- An equation is a mathematical sentence that states that two expressions are equal.

- A one-step equation is defined as an equation that requires the use of one operation to solve (e.g., \(x + 3 = 4\)).

- The solution to an equation is the value(s) that make it a true statement. Many equations have one solution and can be represented as a point on a number line.

- A variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable.

- The inverse operation for addition is subtraction, and the inverse operation for multiplication is division.

- A two-step equation is defined as an equation that requires the use of two operations to solve; may include, but not be limited to equations such as the following: \(2x + \frac{1}{2} = -5; -25 = 7(2x + 1); \frac{x - 7}{-3} = 4; 2(x + 4) = 8; \frac{3}{x} - 2 = 10; \frac{x - 7}{3} = 4\).

- An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an “equal sign (=)” (e.g., 3, 5x, 140 - 38.2, 18 : 21, 5 + x).

- An expression that contains a variable is a variable expression.

- A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used

ESSENTIAL UNDERSTANDINGS

- When solving an equation, why is it important to perform identical operations on each side of the equal sign? An operation that is performed on one side of an equation must be performed on the other side to maintain equality.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Represent and demonstrate steps for solving one and two-step linear equations in one variable using a variety of concrete materials, and pictorial representations and algebraic sentences.

- Apply properties of real numbers and properties of equality to solve one- and two-step linear equations in one variable. Coefficients and numerical terms will be rational.

- Confirm algebraic solutions to linear equations in one variable.

- Write verbal expressions and sentences as algebraic expressions and equations.

- Write algebraic expressions and equations as verbal expressions and sentences.

- Solve practical problems that require the solution of a one- or two-step linear equation.
7.1214 The student will

a) solve one- and two-step linear equations in one variable; and,

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- An algebraic expression is a variable expression that contains at least one variable (e.g., \(2x - 3\)). [Moved from 7.13]
- A verbal sentence is a complete word statement (e.g., “The sum of twice a number and two is fifteen.” could be represented by “\(2n + 2 = 15\)”.[Moved from 7.13]
- An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., \(2x - 8 = 7\)). [Moved from 7.13]

- Properties of real numbers and properties of equality can be used applied when to solving equations, and justifying solutions, and expressions simplification. Students should utilize the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of \(a, b,\) or \(c\) in this standard).

  - The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., \(5.6 + 4 = 4 + 5.6\)).
  - The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., \(5.6 \cdot 4 = 4 \cdot 5.6\)).
  - The associative property of addition states that regrouping the addends does not change the sum (e.g., \(5.6 + (4 + 3) = (5.6 + 4) + 3\)).
  - The associative property of multiplication states that regrouping the factors does not change the product (e.g., \(5.6 \cdot (4 \cdot 3) = (5.6 \cdot 4) \cdot 3\)).
  - Subtraction and division are neither commutative nor associative.

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 7
7.1214 The student will
   a) solve one- and two-step linear equations in one variable; and, including
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<td>$5.2 \cdot (3 + 7) = (5.2 \cdot 3) + (5.2 \cdot 7)$, or</td>
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<td>$(5.2 \cdot 3) - (5.2 \cdot 7) = 5.2 \cdot (3 - 7)$</td>
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<td>Inverses are numbers that combine with other numbers and result in identity elements (e.g., $5 + (\frac{5}{2}) = 0$; $\frac{1}{5} \cdot 5 = 1$).</td>
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| The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its
7.1214 The student will
a) solve one- and two-step linear equations in one variable; and, including
b) solve practical problems that requiring the solution of one- and two-step linear equations in one variable.

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<th>UNDERSTANDING THE STANDARD</th>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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</table>
| multiplicative inverse (reciprocal) equals one | Zero has no multiplicative inverse. | Commutative property of addition: \( a + b = b + a \).
| \( \frac{1}{4} \cdot \frac{4}{1} = 1 \) | The multiplicative property of zero states that the product of any real number and zero is zero (e.g., \( 7 \cdot 0 = 0 \) and \( 0 \cdot 7 = 0 \)). | Commutative property of multiplication: \( a \cdot b = b \cdot a \).
| Division by zero is not a possible mathematical operation. It is undefined. | The addition property of equality states that any number must be added to both sides of an equation to maintain equality. | Subtraction and division are not commutative.
| The subtraction property of equality states that any number must be subtracted from both sides of an equation to maintain equality. | The multiplication property of equality states that an equation must be multiplied on both sides by the same number to maintain equality. | The additive identity is zero (0) because any number added
| The division property of equality states that an equation must be divided on both sides by the same number to maintain equality. | The substitution property of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality. (e.g., if \( x = 5 + 3 \) then \( x = 3 \). Then, 3 was substituted for \( 5 + 3 \) since they are equivalent). |
The student will

a) solve one-and two-step linear equations in one variable; and,

b) solve practical problems that require the solution of a one-and two-step linear equations in one variable.

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<td>- Inverses are numbers that combine with other numbers and result in identity elements [e.g., ( 5 + (-5) = 0 ); ( \frac{1}{2} \cdot 2 = 1 )].</td>
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<td>- Inverse property of addition (additive inverse property): ( a + (-a) = 0 ) and ( (-a) + a = 0 ).</td>
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<td>- Inverse property of multiplication (multiplicative inverse property): ( a \cdot \frac{1}{a} = 1 ) and ( \frac{1}{a} \cdot a = 1 ).</td>
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<td>- Subtraction property of equality: if ( a = b ), then ( a - c = b - c ).</td>
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<td>- Multiplication property of equality: if ( a = b ), then ( a \cdot c = b \cdot c ).</td>
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<td>- Division property of equality: if ( a = b ) and ( c \neq 0 ), then ( \frac{a}{c} = \frac{b}{c} ).</td>
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7.1345 The student will:
  a) solve one- and two-step [two-step moved from 8.15b] linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division, and
  b) graph the solutions to inequalities on the number line.

### UNDERSTANDING THE STANDARD

**Background Information for Instructor Use Only**

- A one-step inequality may include, but not be limited to, inequalities such as the following: \(2x > 5; y - 3 \leq -6; \frac{1}{2}x < -3; a - (-4) \geq 11\) is defined as an inequality that requires the use of one operation to solve (e.g., \(x - 4 > 9\)).
- A two-step inequality may include, but not be limited to, inequalities such as the following: \(2x + 1 < -25; 2x + \frac{3}{2} \geq -5; 2 > 7.2x + 3; \frac{x - 7}{-3} \leq 4; 2(x - 4) > 8; 3\frac{3}{4}x - 2 \leq 10\).
- The solution set to an inequality is the set of all numbers that make the inequality true.
- The inverse operation for addition is subtraction, and the inverse operation for multiplication is division.
- The procedures for solving inequalities are the same as those to solve equations except for the case when an inequality is multiplied or divided on both sides by a negative number. Then the inequality sign is changed from less than to greater than, or greater than to less than. [Moved from middle column]
- When both expressions of an inequality are multiplied or divided by a negative number, the inequality symbol reverses (e.g., \(-3x < 15\) is equivalent to \(x > -5\)).
- Solutions to inequalities can be represented using a number line.
- In an inequality, there can be more than one value for the variable that makes the inequality true. There can be many solutions. (i.e., \(x + 4 > -3\) then the solution is \(x > -7\). This means that \(x\) can be any number greater than -7. A few solutions might be -6.5, -3, 0, 4, 25, etc.)

### ESSENTIAL UNDERSTANDINGS

- How are the procedures for solving equations and inequalities the same? The procedures are the same except for the case when an inequality is multiplied or divided on both sides by a negative number. Then the inequality symbol is changed from less than to greater than, or greater than to less than. [Moved to US]
- How is the solution to an inequality different from that of a linear equation? In an inequality, there can be more than one value for the variable that makes the inequality true. [Moved to US]

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

- Represent and solve one- and two-step inequalities in one variable, using a variety of concrete materials, pictorial representations, and algebraic sentences.
- Apply properties of real numbers and the multiplication and division properties of inequality to solve one-step inequalities in one variable, and the addition, subtraction, multiplication, and division properties of inequality to solve two-step inequalities in one variable. Coefficients and numeric terms will be rational.
- Graph representations to inequalities algebraically and graphically on the number line.
- Write verbal expressions and sentences as algebraic expressions and inequalities.
- Write algebraic expressions and inequalities as verbal expressions and sentences.
- Solve practical problems that require the solution of a one- or two-step inequality.
- Identify a numerical value(s) that satisfies the part of the solution set of a given inequality.
The student will

a) solve one- and two-step [two-step moved from 8.15b] linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division; and

b) graph the solutions to inequalities on the number line.

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<tr>
<td>Properties of real numbers and properties of inequality can be used to solve inequalities, justify solutions, and express simplification. Students should utilize the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of $a$, $b$, or $c$ in this standard):</td>
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<tr>
<td>- The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., $5.6 + 4 = 4 + 5.6$).</td>
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<td>- The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., $5.6 \cdot 4 = 4 \cdot 5.6$).</td>
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<td>- The associative property of addition states that regrouping the addends does not change the sum (e.g., $5.6 + (4 + 3) = (5.6 + 4) + 3$).</td>
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<td>- The associative property of multiplication states that regrouping the factors does not change the product (e.g., $5.6 \cdot (4 \cdot 3) = (5.6 \cdot 4) \cdot 3$).</td>
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<td>- Subtraction and division are neither commutative nor associative.</td>
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<td>- The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true. Distributive property examples:</td>
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<td>- $5.2 \cdot (3 + 7) = (5.2 \cdot 3) + (5.2 \cdot 7)$, or</td>
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The student will

a) solve one- and two-step [two-step moved from 8.15b] linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division, and

b) graph the solutions to inequalities on the number line.

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- The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division.

- The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real number (e.g., \( \frac{5}{2} + 0 = \frac{5}{2} \)).

- The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number (e.g., \( \sqrt{8} \cdot 1 = \sqrt{8} \)).

- Inverses are numbers that combine with other numbers and result in identity elements (e.g., \( 5 + (-5) = 0 \); \( \frac{1}{2} \cdot 2 = 1 \)).

- The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., \( 5.6 + (-5.6) = 0 \)).

- The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., \( -\frac{1}{4} \cdot -\frac{4}{1} = 1 \)).

- Zero has no multiplicative inverse.

- The multiplicative property of zero states that the product of any real number and zero is zero (e.g., \( 7 \cdot 0 = 0 \) and \( 0 \cdot 7 = 0 \)).
The student will

a) solve one- and two-step [two-step moved from 8.15b] linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division; and

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<td>The division property of inequality states that if an inequality is divided on both sides by the same positive number, the inequality symbol remains the same. But if an inequality is divided on both sides by the same negative number, the inequality symbol reverses. For example, ( 4x &lt; 20 ). Divide both sides of the inequality by 4, so ( x &lt; 5 ).</td>
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<td>The substitution property of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality. (e.g., if ( x = 5 + 8 ) then ( x = 13 ). Thus, 3 was substituted for ( 5 + 8 ) since they are equivalent).</td>
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<td>Commutative property of addition: ( a + b = b + a ).</td>
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7.1315 The student will
a) solve one- and two-step [two-step moved from 8.15b] linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division; and
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STANDARD 7.1315  STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA  GRADE LEVEL 7

7.1315 The student will
a) solve one- and two-step [two-step moved from 8.15b] linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division, and
b) graph the solutions to inequalities on the number line.

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<td><strong>b - c; if</strong> a &gt; b, then a - c &gt; b - c,</td>
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<tr>
<td>- <strong>Multiplication property of inequality (multiplication by a negative number):</strong> if a &lt; b and c &lt; 0, then a · c &gt; b · c; if a &gt; b and c &lt; 0, then a · c &lt; b · c.</td>
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<tr>
<td>- <strong>Division property of inequality:</strong> a &lt; b and c &gt; 0, then a/c &lt; b/c; if a &gt; b and c &gt; 0, then a/c &gt; b/c (this property also applies to less than or greater than).</td>
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7.16 The student will apply the following properties of operations with real numbers:

a) the commutative and associative properties for addition and multiplication;
b) the distributive property;
c) the additive and multiplicative identity properties;
d) the additive and multiplicative inverse properties; and
e) the multiplicative property of zero.

[Incorporated into 7.2, 7.11, 7.12, and 7.13]

**UNDERSTANDING THE STANDARD**

### Background Information for Instructor Use Only

#### ESSENTIAL UNDERSTANDINGS

- The commutative property for addition states that changing the order of the addends does not change the sum (e.g., 5 + 4 = 4 + 5).
- The commutative property for multiplication states that changing the order of the factors does not change the product (e.g., 5 · 4 = 4 · 5).
- The associative property of addition states that regrouping the addends does not change the sum [e.g., 5 + (4 + 3) = (5 + 4) + 3].
- The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5 · (4 · 3) = (5 · 4) · 3].
- Subtraction and division are neither commutative nor associative.
- The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number [e.g., 5 · (3 + 7) = (5 · 3) + (5 · 7), or 5 · (3 − 7) = (5 · 3) − (5 · 7)].
- Identity elements are numbers that combine with other numbers without changing the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). There are no identity elements for subtraction and division.

#### ESSENTIAL KNOWLEDGE AND SKILLS

- Why is it important to apply properties of operations when simplifying expressions?
  - Using the properties of operations with real numbers helps with understanding mathematical relationships.

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify properties of operations used in simplifying expressions.
- Apply the properties of operations to simplify expressions.
7.16 The student will apply the following properties of operations with real numbers:
   a) the commutative and associative properties for addition and multiplication;
   b) the distributive property;
   c) the additive and multiplicative identity properties;
   d) the additive and multiplicative inverse properties; and
   e) the multiplicative property of zero.

[Incorporated into 7.2, 7.11, 7.12, and 7.13]

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<td>▲ The additive identity property states that the sum of any real number and zero is equal to the given real number (e.g., $5 + 0 = 5$).</td>
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<td>▲ The multiplicative identity property states that the product of any real number and one is equal to the given real number (e.g., $8 \cdot 1 = 8$).</td>
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<td>▲ Inverses are numbers that combine with other numbers and result in identity elements [e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$].</td>
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<td>▲ The additive inverse property states that the sum of a number and its additive inverse always equals zero [e.g., $5 + (-5) = 0$].</td>
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<td>▲ The multiplicative inverse property states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., $4 \cdot \frac{1}{4} = 1$).</td>
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</table>
The 2009-2016 Mathematics Standards of Learning Curriculum Framework, is a companion document to the 2009-2016 Mathematics Standards of Learning, and amplifies the Mathematics Standards of Learning by further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students, provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction background information for the teacher (K-8). It contains content The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers’ knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that What each student should know and be able to demonstratedo in each standard is outlined. This is not meant to be an exhaustive list of student expectationsnor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student’s problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
In the middle grades, the focus of mathematics learning is to
• build on students’ concrete reasoning experiences developed in the elementary grades;
• construct a more advanced understanding of mathematics through active learning experiences;
• develop deep mathematical understandings required for success in abstract learning experiences; and
• apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

• Students in the middle grades focus on mastering rational numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to most middle school mathematics topics.

• Students develop an understanding of integers and rational numbers by using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.

• Students develop an understanding of the properties of operations on real numbers through experiences with rational numbers and by applying the order of operations.

• Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

Mathematics instruction in grades 6 – 8 continues to focus on the development of number sense, with emphasis on rational and real numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to many middle school mathematics topics.

Students develop an understanding of integers and rational numbers using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.

Students develop an understanding of real numbers and the properties of operations on real numbers through experiences with rational and irrational numbers and apply the order of operations.
8.1 The student will
   a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and [Combined with 7.28.14a]
   b) compare and order real numbers, decimals, fractions, percents, and numbers written in scientific notation. [Moved to EKS]

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Expression is a word used to designate any symbolic mathematical phrase that may contain numbers and/or variables. Expressions do not contain equal or inequality signs.
- The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form \( \frac{a}{b} \), where \( a \) and \( b \) are integers and \( b \) does not equal zero (e.g., \( \frac{1}{4} \), \( -2.3 \), \( 75\% \), \( \sqrt{25} \), \( 4.59 \)). [Included in 7.2 and 8.2]
- A rational number is any number that can be written in fraction form. [Included in 8.2]
- A numerical expression contains only numbers and the operations on those numbers. [Moved to 8.14]
- Expressions are simplified using the order of operations and the properties for operations with real numbers, i.e., associative, commutative, and distributive and inverse properties. [Moved to 8.14]
- The order of operations, a mathematical convention, is as follows: Complete all operations within grouping symbols. If there are grouping symbols within other grouping symbols (embedded), do the innermost operation first. Evaluate all exponential expressions. Multiply and/or divide in order from left to right. Add and/or subtract in order from left to right. Parentheses \((\), brackets \([\), braces \(\{\), the absolute value \(\mid\), division/fraction bar \(\div\), and the square root symbol \(\sqrt{\) should be treated as grouping symbols. [Moved to 7.2 and 8.14]
- A power of a number represents repeated multiplication of the number. For example, \((-5)^4\) means \((-5) \cdot (-5) \cdot (-5) \cdot (-5)\). The base is the number that is multiplied, and the exponent indicates how many times the base is used as a factor. [Moved to US]

ESSENTIAL UNDERSTANDINGS

- What is the role of the order of operations when simplifying numerical expressions? The order of operations prescribes the order to use to simplify a numerical expression. [Included in 7.2]
- How does the different ways rational numbers can be represented help us compare and order rational numbers? Numbers can be represented as decimals, fractions, percents, and in scientific notation. It is often useful to convert numbers to be compared and/or ordered to one representation (e.g., fractions, decimals or percents). [Moved to US]
- What is a rational number? A rational number is any number that can be written in fraction form. [Included in 8.2]
- When are numbers written in scientific notation? Scientific notation is used to represent very large and very small numbers. [Included in US]

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Simplify numerical expressions containing: 1) exponents (where the base is a rational number and the exponent is a positive whole number), 2) fractions, decimals, integers and square roots of perfect squares; and 3) grouping symbols (no more than 2 embedded grouping symbols). Order of operations and properties of operations with real numbers should be used. [Combined with 7.2 and 8.14]
- Compare and order no more than five real numbers, to include expressed as integers, fractions (proper or improper), decimals, mixed numbers, percents, and numbers written in scientific notation, radicals, and \( \pi \). Radicals may include both positive and negative square roots of values from 0 to 400, using positive and negative exponents. Ordering may be in ascending or descending order.
- Use rational approximations (to the nearest hundredth) of irrational numbers to compare and order, locating values on a number line. Radicals may include both positive and negative square roots of values from 0 to 400 yielding an irrational number. Values may be positive or negative.
8.1 The student will
a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and [Combined with 7.28.14a]
b) compare and order real numbers; decimals, fractions, percents, and numbers written in scientific notation. [Moved to EKS]

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<td>represents the number of times the base is used as a factor. In this example, ((-5)^4) is the base, and 4 is the exponent. The product is 625. Notice that the base appears inside the grouping symbols. The meaning changes with the removal of the grouping symbols. For example, (-5^4) means 5 (\times) 5 (\times) 5 (\times) 5 negated which results in a product of (-625). The expression (\text{(-}(5)^4)} means to take the opposite of 5 (\times) 5 (\times) 5 (\times) 5 which is (-625). Students should be exposed to all three representations. [Moved to 8.14]</td>
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- All state approved scientific calculators use algebraic logic (follow the order of operations). [Reordered]
- Real numbers can be represented as integers, fractions (proper or improper), decimals, percents, numbers written and in scientific notation, radicals, and \(\pi\). It is often useful to convert numbers to be compared and/or ordered to one representation (e.g., fractions, decimals or percents). [Moved from middle column]
- Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction can be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., \(3\frac{5}{8}\)). Fractions can have a positive or negative value.
- The density property states that between any two real numbers lies another real number. For example, between 3 and 5 we can find 4; between 4.0 and 4.2 we can find 4.16; between 4.16 and 4.17 we can find 4.165; between 4.165 and 4.166 we can find 4.1655, etc. Thus, we can always find another number in between two numbers. Students are not expected to know the term density property but the concept allows for a deeper
8.1 The student will
a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and [Combined with 7.28.14a]
b) compare and order real numbers, decimals, fractions, percents, and numbers written in scientific notation. [Moved to EKS]

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<td>(Background Information for Instructor Use Only)</td>
<td>understanding of the set of real numbers.</td>
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<tr>
<td>Scientific notation is used to represent very large or very small numbers. [Same as middle column]</td>
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<td>A number written in scientific notation is the product of two factors: a decimal greater than or equal to one but less than 10 multiplied by a power of 10 (e.g., $3.1 \times 10^5 = 310,000$ and $3.1 \times 10^{-5} = 0.000031$).</td>
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<td>Any real number raised to the zero power is 1. The only exception to this rule is zero itself. Zero raised to the zero power is undefined.</td>
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<tr>
<td>All state approved scientific calculators use algebraic logic (follow the order of operations). [Reordered]</td>
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8.2 The student will describe orally and in writing the relationships between the subsets of the real number system.

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<td>(Background Information for Instructor Use Only)</td>
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<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
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<td></td>
<td>Describe orally and in writing the relationships among the sets of natural or counting numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers. [Included in bullet below]</td>
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<td>Describe and illustrate the relationships among the subsets of the real number system by using representations (graphic organizers, number lines, etc.) such as Venn diagrams. Subsets include rational numbers, irrational numbers, integers, whole numbers, and natural or counting numbers.</td>
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<td>Identify the subsets of the real number system to which a given number belongs. [Included in bullet below]</td>
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<td>Determine whether a given number is as a member of a particular subset or subsets of the real number system, and explain why.</td>
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<td>Describe each subset of the set of real numbers and include examples and nonexamples.</td>
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<td></td>
<td>Recognize that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.†</td>
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- The subsets of real numbers includes natural numbers (counting numbers), whole numbers, integers, rational and irrational numbers.

- Some numbers can belong to more than one subset of the real numbers (e.g., 4 is a natural number, a whole number, an integer, and a rational number. The attributes of one subset can be contained in whole or in part in another subset. [Moved from the middle column] The relationships between the subsets of the real number system can be illustrated using graphic organizers (that may include, but not be limited to, Venn diagrams), number lines, and other representations.

- The set of natural numbers is the set of counting numbers \{1, 2, 3, 4, \ldots\}.

- The set of whole numbers includes the set of all the natural numbers or counting numbers and zero \{0, 1, 2, 3\ldots\}.

- The set of integers includes the set of whole numbers and their opposites \{-2, -1, 0, 1, 2\ldots\}. Zero has no opposite and is neither positive nor negative.

- The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form \(\frac{a}{b}\) where \(a\) and \(b\) are integers and \(b\) does not equal zero. [Moved from middle column 8.1] The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of rational numbers are (e.g., \(\sqrt{25}, \frac{1}{4}, -2.3, 75\%, \text{and } 4.5\bar{9}\)).

- The set of irrational numbers is the set of all nonrepeating, nonterminating decimals. An irrational number cannot be written in fraction form (e.g., \(\pi, \sqrt{2}, 1.232332333\ldots\)).

- The real number system is comprised of all rational and irrational numbers.

† Revised March 2011
8.3 The student will

a) estimate and determine the two consecutive integers between which a square root lies; and [Moved from Computation and Estimation strand 8.5b]

b) determine both the positive and negative square roots of a given perfect square. [Moved from Computation and Estimation strand 8.5 EKS]

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| - A perfect square is a whole number whose square root is an integer. [Moved from 8.5] | - The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to 
| - The square root of a given number is any number which, when multiplied times itself, equals the given number. [Moved from 8.5] | - Estimate and identify the two consecutive integers between which the positive or negative square root of a given whole number from 1 to 400 lies. Numbers are limited to natural numbers from 1 to 400. (a) [Moved from 8.5] | - Both the Whole numbers, except zero, have both positive and negative roots of whole numbers, except zero, can be determined. The square root of zero is zero. The value is neither positive nor negative. Zero (a whole number) is a perfect square. [Moved from 8.5] |
| - Both the Whole numbers, except zero, have both positive and negative roots of whole numbers, except zero, can be determined. The square root of zero is zero. The value is neither positive nor negative. Zero (a whole number) is a perfect square. [Moved from 8.5] | - Determine the positive or negative square root of a given perfect square from 1 to 400. (b) [Moved from 8.5] | - Any whole number other than a perfect square has a The positive and negative square root of any whole number other than a perfect square that lies between two consecutive whole numbers. [Moved from 8.5] (e.g., $\sqrt{57}$ lies between 7 and 8 since $7^2 = 49$ and $8^2 = 64$; $-\sqrt{11}$ lies between -4 and -3 since $(-4)^2 = 16$ and $(-3)^2 = 9$). |
| - Any whole number other than a perfect square has a The positive and negative square root of any whole number other than a perfect square that lies between two consecutive whole numbers. [Moved from 8.5] (e.g., $\sqrt{57}$ lies between 7 and 8 since $7^2 = 49$ and $8^2 = 64$; $-\sqrt{11}$ lies between -4 and -3 since $(-4)^2 = 16$ and $(-3)^2 = 9$). | - The symbol $\sqrt{}$ may be used to represent a positive (principal) root and $-\sqrt{}$ may be used to represent a negative root. |
| - The symbol $\sqrt{}$ may be used to represent a positive (principal) root and $-\sqrt{}$ may be used to represent a negative root. | - The square root of a whole number that is not a perfect square is an irrational number (e.g., $\sqrt{2}$ is an irrational number). An irrational number cannot be expressed exactly as a fraction $\frac{a}{b}$ where $b$ does not equal 0. [Moved from 8.5] |
| - The square root of a whole number that is not a perfect square is an irrational number (e.g., $\sqrt{2}$ is an irrational number). An irrational number cannot be expressed exactly as a fraction $\frac{a}{b}$ where $b$ does not equal 0. [Moved from 8.5] | - Square root symbols may be used to represent solutions to equations of the form $x^2 = p$. Examples may include: |
| - Square root symbols may be used to represent solutions to equations of the form $x^2 = p$. Examples may include: | - If $x^2 = 36$, then $x = \sqrt{36} = 6$ or $-\sqrt{36} = -6$. |
| - If $x^2 = 5$, then $x = \sqrt{5}$ or $-\sqrt{5}$. | - The square root of zero is zero. |
8.3 The student will
   a) estimate and determine the two consecutive integers between which a square root lies; and [Moved from Computation and Estimation strand 8.5b]
   b) determine both the positive and negative square roots of a given perfect square. [Moved from Computation and Estimation strand 8.5 EKS]

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<td>Students can use grid paper and estimation to determine what is needed to build a perfect square. [Moved from 8.5] The square root of a positive number is usually defined as the side length of a square with the area equal to the given number. If it is not a perfect square, the area provides a means for estimation. [Moved from middle column 8.5]</td>
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In the middle grades, the focus of mathematics learning is to
- build on students’ concrete reasoning experiences developed in the elementary grades;
- construct through active learning experiences a more advanced understanding of mathematics;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense.
- Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate.
- Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonableness of answers.
- Students reinforce skills with operations with whole numbers, fractions, and decimals through problem solving and application activities.

Computation and estimation in grades 6-8 focus on developing conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring an understanding as to why procedures work and make sense.

Students develop and refine estimation strategies based on an understanding of number concepts, properties and relationships. The development of problem solving, using operations with integers and rational numbers, builds upon the strategies developed in the elementary grades. Students will reinforce these skills and build on the development of proportional reasoning and more advanced mathematical skills.

Students learn to make sense of the mathematical tools available by making valid judgments of the reasonableness of answers. Students will balance the ability to make precise calculations through the application of the order of operations with knowing when calculations may require estimation to obtain appropriate solutions to practical problems.
The student will

\*a\* solve practical problems involving consumer applications, rational numbers, percents, ratios, and proportions; and

\*b\* determine the percent increase or decrease for a given situation.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Rational numbers may be expressed as whole numbers, integers, fractions, percents, and numbers written in scientific notation.

- Practical problems may include, but are not limited to, those related to economics, sports, science, social sciences, transportation, and health. Some examples include problems involving the amount of a paycheck per month, commissions, fees, the discount price on a product, temperature, simple interest, sales tax and installment buying.

- A percent is a special ratio with a denominator of 100. [Included from middle column]

- Reconciling an account is a process used to verify that two sets of records (usually the balances of two accounts) are in agreement. Reconciliation is used to ensure that the money leaving balance of an account matches the actual amount of money spent/deposited and/or withdrawn from the account.

- A discount is a percent of the original price. The discount price is the original price minus the discount.

- Simple interest for a number of years is determined by multiplying the product of the principle principal \((p)\), by the annual rate of interest \((r)\), by the number of years \((t)\) of the loan or investment using the formula \(I = prt\).

- The total value of an investment is equal to the sum of the original investment and the interest earned.

- The total cost of a loan is equal to the sum of the original cost and the interest paid.

- Percent increase and percent decrease are both percents of change measuring the percent a quantity increases or decreases. [Moved from middle column]

### ESSENTIAL UNDERSTANDINGS

- What is the difference between percent increase and percent decrease?

  - Percent increase and percent decrease are both percents of change measuring the percent a quantity increases or decreases. Percent increase shows a growing change in the quantity while percent decrease shows a lessening change. [Moved to US]

- What is a percent?

  - A percent is a special ratio with a denominator of 100. [Included in US]

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Write a proportion given the relationship of equality between two ratios. [Reordered]

- Solve practical problems involving consumer applications by using proportional reasoning and computation procedures for whole numbers, integers, fractions, rational numbers, percents, ratios, and proportions. Some problems may require the application of a formula. [Reordered]

- Maintain a checkbook and check registry for reconcile an account balance given a statement with five or fewer transactions. [Reordered]

- Compute a discount or markup and the resulting sale price for one discount or markup. [Reordered]

- Compute the percent increase or decrease for a one-step equation found in a real life situation. [Reworded and Reordered]

- Compute the sales tax or tip and resulting total. [Reordered]

- Substitute values for variables in given formulas. For example, use the simple interest formula \(I = prt\) to determine the value of any missing variable when given specific information.

- Compute the simple interest and new balance earned in an investment or on a loan for a given number of years given the principal amount, interest rate, and time period in years. [Reordered]

- Compute the percent increase or decrease found in a practical situation. [Reordered]
8.43 The student will
a) solve practical problems involving consumer applications, rational numbers, percents, ratios, and proportions; and
b) determine the percent increase or decrease for a given situation.

<table>
<thead>
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<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td></td>
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<tr>
<td>• Percent of change is the percent that a quantity increases or decreases. (Combined with bullet above)</td>
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</tbody>
</table>
| • Percent increase may determine the rate of growth and may be calculated using a ratio.  
  \[
  \text{Percent Increase} = \frac{\text{Change (new - original)}}{\text{original}}
  \] | | |
| • For percent increase, the change will result in a positive number. | | |
| • Percent decrease may determine the rate of decline and may be calculated using the same ratio as percent increase. However, the change will result in a negative number. | | |
8.4 The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables. [Moved to 8.14a]

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<tr>
<td>• Algebraic expressions use operations with algebraic symbols (variables) and numbers.</td>
<td>• What is the role of the order of operations when evaluating expressions? Using the order of operations assures only one correct answer for an expression.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Algebraic expressions are evaluated by substituting numbers for variables and applying the order of operations to simplify the resulting expression.</td>
<td></td>
<td>• Substitute numbers for variables in algebraic expressions and simplify the expressions by using the order of operations. Exponents are positive and limited to whole numbers less than 4. Square roots are limited to perfect squares.</td>
</tr>
<tr>
<td>• The order of operations is as follows: Complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols (embedded), do the innermost operation first. Evaluate all exponential expressions. Multiply and/or divide in order from left to right. Add and/or subtract in order from left to right.</td>
<td></td>
<td>• Apply the order of operations to evaluate formulas. Problems will be limited to positive exponents. Square roots may be included in the expressions but limited to perfect squares.</td>
</tr>
</tbody>
</table>

* Parentheses ( ), brackets [ ], braces { }, the absolute value |, division/fraction bar —, and the square root symbol \( \sqrt{ } \) should be treated as grouping symbols.
8.5 The student will

a) determine whether a given number is a perfect square; and \(\text{[Included in 7.1]}\)
b) find the two consecutive whole numbers between which a square root lies. \(\text{[Included in 8.3a]}\)

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<td>• A perfect square is a whole number whose square root is an integer (e.g., The square root of 25 is 5 and (-5); thus, 25 is a perfect square).</td>
<td>• How does the area of a square relate to the square of a number? The area determines the perfect square number. If it is not a perfect square, the area provides a means for estimation.</td>
<td>The student will use problem-solving, mathematical communication, mathematical reasoning, connections, and representations to:</td>
</tr>
<tr>
<td>• The square root of a number is any number which when multiplied by itself equals the number.</td>
<td>• Why do numbers have both positive and negative roots? The square root of a number is any number which when multiplied by itself equals the number. A product, when multiplying two positive factors, is always the same as the product when multiplying their opposites (e.g., (7 \cdot 7 = 49) and (-7 \cdot -7 = 49)).</td>
<td>- Identify the perfect squares from 0 to 400.</td>
</tr>
<tr>
<td>• Whole numbers have both positive and negative roots.</td>
<td></td>
<td>- Identify the two consecutive whole numbers between which the square root of a given whole number from 0 to 400 lies (e.g., (\sqrt{57}) lies between 7 and 8 since (7^2 = 49) and (8^2 = 64)).</td>
</tr>
<tr>
<td>• Any whole number other than a perfect square has a square root that lies between two consecutive whole numbers.</td>
<td></td>
<td>- Define a perfect square.</td>
</tr>
<tr>
<td>• The square root of a whole number that is not a perfect square is an irrational number (e.g., (\sqrt{2}) is an irrational number). An irrational number cannot be expressed exactly as a ratio.</td>
<td></td>
<td>- Find the positive or positive and negative square roots of a given whole number from 0 to 400. (Use the symbol (\sqrt{\text{__}}) to ask for the positive root and (-\sqrt{\text{__}}) when asking for the negative root.) (\text{[Moved to 8.3b]})</td>
</tr>
</tbody>
</table>

Students can use grid paper and estimation to determine what is needed to build a perfect square.
In the middle grades, the focus of mathematics learning is to
- build on students’ concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop the measurement skills that provide a natural context and connection among many mathematics concepts. Estimation skills are developed in determining length, weight/mass, liquid volume/capacity, and angle measure. Measurement is an essential part of mathematical explorations throughout the school year.

- Students continue to focus on experiences in which they measure objects physically and develop a deep understanding of the concepts and processes of measurement. Physical experiences in measuring various objects and quantities promote the long-term retention and understanding of measurement. Actual measurement activities are used to determine length, weight/mass, and liquid volume/capacity.

- Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, square-based pyramids, and cones.

Measurement and geometry in the middle grades provide a natural context and connection among many mathematical concepts. Students expand informal experiences with geometry and measurement in the elementary grades and develop a solid foundation for further exploration of these concepts in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.

Students develop measurement skills through exploration and estimation. Physical exploration to determine length, weight/mass, liquid volume/capacity, and angle measure are essential to develop a conceptual understanding of measurement. Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, square-based pyramids, and cones.

Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provide experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.

Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, square-based pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.

Students explore and develop an understanding of the Pythagorean Theorem. Understanding how the Pythagorean Theorem can be applied in practical situations has a far-reaching impact on subsequent mathematics learning and life experiences.
The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

**Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

**Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

**Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

**Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)

**Level 4: Deduction.** Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)
8.56 The student will **use**

a) verify by measuring and describe the relationships among **pairs of angles** that are vertical angles, adjacent angles, supplementary angles, and complementary angles; and

b) to determine the measure of unknown angles of less than 360°.

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- Vertical angles are a pair of (all nonadjacent angles) formed by two intersecting lines. Vertical angles are congruent and share a common vertex.  
- Complementary angles are any two angles such that the sum of their measures is 90°.  
- Supplementary angles are any two angles such that the sum of their measures is 180°.  
- Complementary and supplementary angles may or may not be adjacent. [Moved from middle column]  
- Reflex angles measure more than 180°.  
- Adjacent angles are any two non-overlapping angles that share a common side-ray and a common vertex.

### ESSENTIAL UNDERSTANDINGS

- How are vertical, adjacent, complementary and supplementary angles related?  
  Adjacent angles are any two non-overlapping angles that share a common side and a common vertex.  
  Vertical angles will always be nonadjacent angles. Supplementary and complementary angles may or may not be adjacent. [Included in US]

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Measure angles of less than 360° to the nearest degree, using appropriate tools.
- Identify and describe the relationships between angles formed by two intersecting lines. [Included in bullet below]
- Identify and describe the relationship between pairs of angles that are vertical, adjacent, supplementary, and complementary. [Combined bullets below]
- Identify and describe the relationship between pairs of angles that are supplementary. [Included in bullet above]
- Identify and describe the relationship between pairs of angles that are complementary. [Included in bullet above]
- Identify and describe the relationship between pairs of angles that are adjacent. [Included in bullet above]
- Use the relationships among supplementary, complementary, vertical, and adjacent angles to solve problems, including practical problems, involving the measure of unknown angles.

*Revised March 2011*
The student will:

- **a)** investigate and solve problems, including practical problems, involving volume and surface area of rectangular prisms, cylinders, [Included in 7.4] cones, and square-based pyramids; and
- **b)** describe how changing one measured attribute of a rectangular prism affects the volume and surface area.

## UNDERSTANDING THE STANDARD

### UNDERSTANDINGS

- A polyhedron is a solid figure whose faces are all polygons.
- A pyramid is a polyhedron with a base that is a polygon and other faces that are triangles with a common vertex. [Reordered]
- The area of the base of a pyramid is the area of the polygon which is the base.
- The total surface area of a pyramid is the sum of the areas of the triangular faces and the area of the base. [Reordered]
- Nets are two-dimensional representations of a three-dimensional figure that can be folded into a model of the three-dimensional figure. [Reordered]
- Surface area of a solid figure is the sum of the areas of the surfaces of the figure. [Moved from middle column]
- Volume is the amount a container holds. [Moved from middle column]
- A rectangular prism is a polyhedron that has a congruent pair of parallel rectangular bases and four faces that are rectangles. A rectangular prism has eight vertices and twelve edges. [Reordered] In this course, prisms are limited to right prisms with bases that are rectangles.
- The surface area of a rectangular prism is the sum of the areas of the faces and bases, found by using the formula \( S.A. = 2lw + 2lh + 2wh \). All six faces are rectangles. [Reordered]
- The volume of a rectangular prism is calculated by multiplying the length, width and height of the prism or by using the formula \( V = lwh \). [Reordered]
- A cube is a polyhedron rectangular prism with six congruent,

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

- Distinguish between situations that are applications of surface area and those that are applications of volume. (a)
- Investigate and compute the surface area of rectangular prisms, cylinders, cones, and square-based or triangular pyramids by finding the sum of the areas of the triangular faces and the base using concrete objects, nets, diagrams and formulas. (a) [Combined bullets below]
- Investigate and compute the surface area of a cone by calculating the sum of the areas of the side and the base, using concrete objects, nets, diagrams and formulas. [Included in bullet above]
- Investigate and compute the surface area of a right cylinder using concrete objects, nets, diagrams and formulas. [Included in bullet above]
- Investigate and compute the surface area of a rectangular prism using concrete objects, nets, diagrams and formulas. [Included in bullet above]
- Investigate and compute the volume of rectangular prisms, cylinders, cones, and square-based pyramids, using concrete objects, nets, diagrams, and formulas. (a)
- Solve practical problems involving volume and surface area of rectangular prisms, cylinders, cones, and square-
The student will

a) investigate and solve problems, including practical problems, involving volume and surface area of rectangular prisms, cylinders, [included in 7.4] cones, and square-based pyramids; and

b) describe how changing one measured attribute of a rectangular prism affects the volume and surface area.

### UNDERSTANDING THE STANDARD

#### (Background Information for Instructor Only)

- A cylinder is a solid figure formed by two congruent parallel faces called bases joined by a curved surface. In this course, cylinders are limited to right circular cylinders.
- The surface area of a right circular cylinder is found by using the formula: \( S.A. = 2\pi r^2 + 2\pi rh \), where \( r \) is the radius of a base, and \( h \) is the height.
- The volume of a cylinder is the area of the base of the cylinder multiplied by the height, found by using the formula: \( V = \pi r^2 h \), where \( h \) is the height and \( \pi r^2 \) is the area of the base. [Reordered]
- A cone is a solid figure formed by a face called a base that is joined to a vertex (apex) by a curved surface. In this course, cones are limited to right circular cones.
- The surface area of a right circular cone is found by using the formula, \( S.A. = \pi r^2 + \pi rl \), where \( l \) represents the slant height of the cone. The area of the base of a circular cone is \( \pi r^2 \). [Combined bullets]
- The volume of a cone is found by using \( V = \frac{1}{3} \pi r^2 h \), where \( h \) is the height and \( \pi r^2 \) is the area of the base. [Reordered]
- A square-based pyramid is a polyhedron with a square base and four faces that are triangles with a common vertex (apex) above the base. In this course level, pyramids are limited to right regular pyramids with a square base.
- The volume of a pyramid is \( \frac{1}{3} Bh \), where \( B \) is the area of the base and \( h \) is the height.

### ESSENTIAL UNDERSTANDINGS

- length, width or height of a prism by a factor greater than 1, the volume of the prism is also increased by that factor. [Included in US]

### ESSENTIAL KNOWLEDGE AND SKILLS

- Compare and contrast the volume and surface area of a prism with given set of attributes with the volume of a prism where one of the attributes has been increased by a factor of 2, 3, 5, or 10.
- Describe how the volume of a rectangular prism is affected when one measured attribute is multiplied by a factor of \( \frac{1}{10}, \frac{1}{3}, \frac{1}{4}, 2, 3, \text{ or } 4 \). (b)
- Describe how the surface area of a rectangular prism is affected when one measured attribute is multiplied by a factor of \( \frac{1}{2} \) or 2. (b)
- Describe the two-dimensional figures that result from slicing three-dimensional figures parallel to the base (e.g., as in plane sections of right rectangular prisms and right rectangular pyramids). [Revised March 2011]
The student will:

a) investigate and solve problems, including practical problems, involving volume and surface area of rectangular prisms, cylinders, cones, and square-based pyramids; and

b) describe how changing one measured attribute of a rectangular prism figure affects the volume and surface area.

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<tr>
<td>above]</td>
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<tr>
<td>• The surface area of a right circular cone is $\pi r^2 + \pi rl$, where $l$ represents the slant height of the cone. [Reordered]</td>
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<tr>
<td>• The volume of a cone is $\frac{1}{3}\pi r^2 h$, where $h$ is the height and $\pi r^2$ is the area of the base. [Reordered]</td>
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<tr>
<td>• The surface area of a right circular cylinder is $2\pi r^2 + 2\pi rh$.</td>
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<tr>
<td>• The volume of a cylinder is the area of the base of the cylinder multiplied by the height.</td>
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<tr>
<td>• The surface area of a rectangular prism is the sum of the areas of the six faces. [Reordered]</td>
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<tr>
<td>• The volume of a rectangular prism is calculated by multiplying the length, width and height of the prism. [Reordered]</td>
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<tr>
<td>• The surface area of a pyramid is the sum of the areas of the triangular faces and the area of the base, found by using the formula $S.A. = \frac{1}{2}lp + B$ where $l$ is the slant height, $p$ is the perimeter of the base and $B$ is the area of the base. [Reordered] [Moved from middle column]</td>
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<tr>
<td>• The volume of a pyramid is found by using the formula $V = \frac{1}{3}Bh$, where $B$ is the area of the base and $h$ is the height. [Same as middle column]</td>
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<tr>
<td>• The volume of prisms and cylinders are can be found by determining the area of the base and multiplying that by the height. [Moved from middle column]</td>
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<tr>
<td>• The formula for determining the volume of cones and cylinders are similar. For cones you are determining $\frac{1}{3}$ of the volume of the</td>
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The student will

a) investigate and solve problems, including practical problems, involving volume and surface area of rectangular prisms, cylinders, [included in 7.4] cones, and square-based pyramids; and

b) describe how changing one measured attribute of a rectangular prism figure affects the volume and surface area.

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<td>cylinder with the same size base and height. The volume of a cone is found by using ( V = \frac{1}{3} \pi r^2 h ). The volume of a cylinder is the area of the base of the cylinder multiplied by the height, found by using the formula, ( V = \pi r^2 h ), where ( h ) is the height and ( \pi r^2 ) is the area of the base. [Moved from middle column]</td>
<td>The calculation of determining surface area and volume may vary depending upon the approximation for ( \pi ). Common approximations for ( \pi ) include 3.14, ( \frac{22}{7} ), or the ( \pi ) button on the calculator.</td>
<td>• The volume of a prism is ( Bh ), where ( B ) is the area of the base and</td>
</tr>
<tr>
<td>• A prism is a solid figure that has a congruent pair of parallel bases and faces that are parallelograms. The surface area of a prism is the sum of the areas of the faces and bases. [Reordered]</td>
<td>• When the measurement of one attribute of a rectangular prism is changed through multiplication or division the volume increases by the same factor that the by which the attribute increased by. [Included from middle column] For example, if a prism has a volume of 2 x 3 x 4, the volume is 24 cubic units. However, if one of the attributes are is doubled, the volume doubles. That is, 2 x 3 x 8, the volume is 48 cubic units or 24 doubled.</td>
<td></td>
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<tr>
<td>• When one attribute of a rectangular prism is changed through multiplication or division, the surface area is affected differently than the volume. The formula for surface area of a rectangular prism is ( 2(lw) + 2(lh) + 2(wh) ) when the width is doubled then four faces are affected. For example, a rectangular prism with length = 7 in., width = 4 in., and height = 3 in. would have a surface area of ( 2(7 \cdot 4) + 2(7 \cdot 3) + 2(4 \cdot 3) ) or 122 square inches. If the height is doubled to 6 inches then the surface area would be found by ( 2(7 \cdot 4) + 2(7 \cdot 6) + 2(4 \cdot 6) ) or 188 square inches.</td>
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The student will

a) investigate and solve problems, including practical problems, involving volume and surface area of rectangular prisms, cylinders, cones, and square-based pyramids; and
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<td>h is the height of the prism. • Nets are two-dimensional representations that can be folded into three-dimensional figures. [Reordered]</td>
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In the middle grades, the focus of mathematics learning is to:

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students expand the informal experiences they have had with geometry in the elementary grades and develop a solid foundation for the exploration of geometry in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.

- Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, mirrors, and geometry software provides experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.

- Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.

- Students explore and develop an understanding of the Pythagorean Theorem. Mastery of the use of the Pythagorean Theorem has far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)

- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)
Level 4: Deduction. Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)
The student will
a) **given a polygon**, apply transformations, to include **translations**, rotations, reflections, and dilations, in the coordinate plane-figures; and
b) identify practical applications of transformations.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Translations, rotations, and reflections maintain congruence between the preimage and image but change location. Dilations by a scale factor other than 1 produce an image that is not congruent to the preimage but is similar. **Rotations and Reflections** change the orientation of the image. [Moved from middle column]
- A transformation of a figure, called preimage, changes the size, shape, and/or position of the figure to a new figure, called the image.
- A transformation of preimage point \(A\) can be denoted as the image \(A'\) (read as “A prime”). [Reordered]
- A **rotation** of a geometric figure is a clockwise or counterclockwise turn of the figure around a transformation in which an image is formed by rotating the preimage about a fixed point, called the center of rotation. The point center of rotation may or may not be a point on the figure-preimage. The fixed point is called the center of rotation.
- A reflection of a geometric figure moves all of the points of the figure across an axis in a transformation in which an image is formed by reflecting the preimage over a line called the line of reflection. Each point on the reflected figure image is the same distance from the axis line of reflection as the corresponding point in the original figure-preimage.
- A translation of a geometric figure is a transformation in which an image is formed by moving every point on the figure-preimage the same distance in the same direction.
- A dilation of a geometric figure is a transformation that changes the size of a figure by a scale factor to create a similar figure in which an image is formed by enlarging or reducing the preimage proportionally by a scale factor from the center of dilation. A dilation of a figure and the original figure are similar. The

### ESSENTIAL UNDERSTANDINGS

- How does the transformation of a figure on the coordinate grid affect the congruency, orientation, location and symmetry of an image? Translations, rotations and reflections maintain congruence between the preimage and image but change location. Dilations by a scale factor other than 1 produce an image that is not congruent to the preimage but is similar. Rotations and reflections change the orientation of the image. [Moved to US]

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Demonstrate the reflection of a polygon over the vertical or horizontal axis on a coordinate grid.
- Demonstrate 90°, 180°, 270°, and 360° clockwise and counterclockwise rotations of a figure on a coordinate grid. The center of rotation will be limited to the origin.
- Demonstrate the translation of a polygon on a coordinate grid.
- Demonstrate the dilation of a polygon from a fixed point on a coordinate grid.
- Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or a rectangle that has been rotated 90° clockwise or counterclockwise about the origin. (a) [Moved from 7.8]
- Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been translated vertically, horizontally, or a combination of both. (a)
- Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been reflected over the x- or y-axis. (a)
- Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or a rectangle that has been dilated. Scale factors are limited to \(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, 2, 3, \text{ or } 4\). The center of the dilation will be the origin. (a)
The student will

a) **given a polygon, apply transformations, to include translations, rotations, [Included in G.3] reflections, and dilations, in the coordinate plane-figures; and**

b) **identify practical applications of transformations.**

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

<table>
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<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
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</table>
| center of dilation may or may not be on the preimage.  

- The result of first translating and then reflecting over the x- or y-axis may not result in the same transformation of reflecting over the x- or y-axis and then translating.
- Practical applications may include, but are not limited to, the following:
  - A rotation of the hour hand of a clock from 2:00 to 3:00 shows a turn of 30° clockwise;
  - A reflection of a boat in water shows an image of the boat flipped upside down with the water line being the line of reflection;
  - A translation of a figure on a wallpaper pattern shows the same figure slid the same distance in the same direction; and
  - A dilation of a model airplane is the production model of the airplane.
- The image of a polygon is the resulting polygon after a transformation. The preimage is the original polygon before the transformation.
- A transformation of preimage point A can be denoted as the **image A’** (read as “A prime”). [Reordered]

- **Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been translated and reflected over the x- or y-axis, or reflected over the x- or y-axis and then translated.** (a)
- Sketch the image of a polygon that has been translated vertically, horizontally, or a combination of both. (a)
- Sketch the image of a polygon that has been translated 90° clockwise or counterclockwise about the origin. (a) [Moved from 7.8]
- Sketch the image of a polygon that has been reflected over the x- or y-axis. (a)
- Sketch the image of a dilation of a right triangle or a rectangle limited to a scale factor of $\frac{1}{4}, \frac{1}{2}, 2, 3,$ or $4$. The center of the dilation will be the origin. (a) [Moved from 7.8]
- Sketch the image of a polygon that has been translated and reflected over the x- or y-axis, or reflected over the x- or y-axis and then translated. (a)
- Identify the type of translation in a given example. (a, b)
- Identify practical applications of transformations including, but not limited to, tiling, fabric, and wallpaper designs, art, and scale drawings. (b)
- Identify the type of transformation in a given example.
The student will construct a three-dimensional model, given the top or bottom, side, and front views.

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<tbody>
<tr>
<td>(Background Information for Instructor Use Only)</td>
<td>How does knowledge of two-dimensional figures inform work with three-dimensional objects?</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>- A three-dimensional object can be represented as a two-dimensional model with views of the object from different perspectives. [Moved from middle column.]</td>
<td>It is important to know that a three-dimensional object can be represented as a two-dimensional model with views of the object from different perspectives. [Moved to US]</td>
<td>- Construct three-dimensional models, given the top or bottom, side, and front views.</td>
</tr>
<tr>
<td>- Three-dimensional models of geometric solids can be used to understand perspective and provide tactile experiences in determining two-dimensional perspectives.</td>
<td></td>
<td>- Identify three-dimensional models given a two-dimensional perspective.</td>
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<tr>
<td>- Three-dimensional models of geometric solids can be represented on isometric paper.</td>
<td></td>
<td>- Identify the two-dimensional perspective from the top or bottom, side and front view, given a three-dimensional model.</td>
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<tr>
<td>- The top view is a mirror image of the bottom view.</td>
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</table>
The student will
a) verify the Pythagorean Theorem; and
b) apply the Pythagorean Theorem.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Only)
- The Pythagorean Theorem is essential for solving problems involving right triangles.
- The relationship between the sides and angles of right triangles are useful in many applied fields.
- In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the legs (altitude and base). This relationship is known as the Pythagorean Theorem: \( a^2 + b^2 = c^2 \).

### ESSENTIAL UNDERSTANDINGS
- How can the area of squares generated by the legs and the hypotenuse of a right triangle be used to verify the Pythagorean Theorem? For a right triangle, the area of a square with one side equal to the measure of the hypotenuse equals the sum of the areas of the squares with one side each equal to the measures of the legs of the triangle. [ Included in US ]

### ESSENTIAL KNOWLEDGE AND SKILLS
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
- Identify the parts of a right triangle (the hypotenuse and the legs). [Moved to 5.13]
- Verify Determine if a triangle is a right triangle given the measures of its three sides. (ab)
- Verify the Pythagorean Theorem, using diagrams, concrete materials, and measurement. (a)
- Find Determine the measure of a side of a right triangle, given the measures of the other two sides. (b)
- Solve practical problems involving right triangles by using the Pythagorean Theorem. (b)
The student will
   a) verify the Pythagorean Theorem; and
   b) apply the Pythagorean Theorem.

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<td>Pythagorean triples.</td>
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<tr>
<td>• The hypotenuse of a right triangle is the side opposite the right angle.</td>
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<tr>
<td>• The hypotenuse of a right triangle is always the longest side of the right triangle.</td>
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<tr>
<td>• The legs of a right triangle form the right angle.</td>
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</table>
8.1011 The student will solve practical area and perimeter problems, including practical problems, involving composite plane figures.

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<tr>
<td>A plane figure is any two-dimensional shape that can be drawn in a plane.</td>
<td>How does knowing the areas of polygons assist in calculating the areas of composite figures?</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
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<tr>
<td>A polygon is a simple, closed plane figure with sides that are comprised of at least three line segments that do not cross.</td>
<td>The area of a composite figure can be found by subdividing the figure into triangles, rectangles, squares, trapezoids and semicircles, calculating their areas, and adding the areas together. [Moved to US]</td>
<td>Subdivide a plane figure into triangles, rectangles, squares, trapezoids, parallelograms, and semicircles. Estimate and determine the area of subdivisions and combine to determine the area of the composite plane figure.</td>
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<tr>
<td>The perimeter of a polygon is the path or distance around the any plane figure. The perimeter of a circle is called the circumference.</td>
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<td>Subdivide a plane figure into triangles, rectangles, squares, trapezoids, parallelograms, and semicircles. Use the attributes of the subdivisions to determine the perimeter and circumference of the composite plane figure.</td>
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<tr>
<td>Circumference is the distance around or “perimeter” of a circle.</td>
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<td>Apply perimeter, circumference, and area formulas to solve practical problems involving composite plane figures.</td>
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<tr>
<td>The area of a composite figure can be found by subdividing the figure into triangles, rectangles, squares, trapezoids, parallelograms, circles, and semicircles, calculating their areas, and combining the areas together by addition and/or subtraction based upon the given composite figure. [Moved from middle column]</td>
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<tr>
<td>The area of any composite figure is based upon knowing how to find the area of the composite parts such as triangles and rectangles. [Included in bullet above]</td>
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<tr>
<td>The area of a rectangle is computed by multiplying the lengths of two adjacent sides ( ( A = lw )).</td>
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<td>The area of a triangle is computed by multiplying the measure of its base by the measure of its height and dividing the product by 2 or multiplying by ( \frac{1}{2} ) (( A = \frac{bh}{2} ) or ( A = \frac{1}{2}bh )).</td>
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<td>The area of a parallelogram is computed by multiplying the measure of its base by the measure of its height ( ( A = bh )).</td>
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<td>The area of a trapezoid is computed by taking the average of the measures of the two bases and multiplying this average by the height ( A = \frac{1}{2}h(b_1 + b_2) ).</td>
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<tr>
<td>The area of a circle is computed by multiplying ( \pi ) times the</td>
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Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 8
8.1011 The student will solve practical area and perimeter problems, including practical problems, involving composite plane figures.

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<tr>
<td>radius squared ( A = \pi r^2 ).</td>
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<td>The circumference of a circle is found by multiplying ( \pi ) by the diameter or multiplying ( \pi ) by 2 times the radius ( C = \pi d ) or ( C = 2\pi r ).</td>
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<td>The area of a semicircle is half the area of a circle with the same diameter or radius.</td>
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<tr>
<td>An estimate of the area of a composite figure can be made by subdividing the polygon into triangles, rectangles, squares, trapezoids and semicircles, estimating their areas, and adding the areas together.</td>
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</table>
In the middle grades, the focus of mathematics learning is to
• build on students’ concrete reasoning experiences developed in the elementary grades;
• construct a more advanced understanding of mathematics through active learning experiences;
• develop deep mathematical understandings required for success in abstract learning experiences; and
• apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

• Students develop an awareness of the power of data analysis and probability by building on their natural curiosity about data and making predictions.
• Students explore methods of data collection and use technology to represent data with various types of graphs. They learn that different types of graphs represent different types of data effectively. They use measures of center and dispersion to analyze and interpret data.
• Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability.
• Students explore experimental and theoretical probability through experiments and simulations by using concrete, active learning activities.

In the middle grades, students develop an awareness of the power of data analysis and the application of probability through fostering their natural curiosity about data and making predictions.

The exploration of various methods of data collection and representation allows students to become effective at using different types of graphs to represent different types of data. Students use measures of center and dispersion to analyze and interpret data.

Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability. Through experiments and simulations, students build on their understanding of the Fundamental Counting Principle from elementary mathematics to learn more about probability in the middle grades.
8.11.12 The student will:
   a) compare and contrast the probability of independent and dependent events; and [Moved from 6.16] with and without replacement.
   b) determine probabilities for independent and dependent events.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Only)

- A simple event is one event (e.g., pulling one sock out of a drawer and examining the probability of getting one color). [Moved from 6.16]
- If all outcomes of an event are equally likely, the theoretical probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes in the sample space. [Moved from 6.16]
- The probability of an event occurring can be represented as a ratio or the equivalent fraction, decimal, or percent. [Moved from 6.16]
- The probability of an event occurring is a ratio between 0 and 1. A probability of 0 means the event will never occur. A probability of 1 means the event will always occur.
- Two events are either dependent or independent.
- If the outcome of one event does not influence the occurrence of the other event, they are called independent. If two events are independent, then the probability of the second event does not change regardless of whether or not the first occurs. For example, the first roll of a number cube does not influence the second roll of the number cube. Other examples of independent events are, but not limited to: flipping two coins; spinning a spinner and rolling a number cube; flipping a coin and selecting a card; and choosing a card from a deck, replacing the card and selecting again.
- The probability of three independent events is found by using the following formula: 
  \[ P(A \text{ and } B) = P(A) \cdot P(B) \]
  \[ P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C) \]
  - Example: When rolling a six-sided number cube and flipping a coin, simultaneously, what is the probability of rolling a 3 on one cube and getting a heads on the coin, a 4 on one cube, and a 5 on the third?

### ESSENTIAL UNDERSTANDINGS

- How are the probabilities of dependent and independent events similar? Different? If events are dependent then the second event is considered only if the first event has already occurred. If events are independent, then the second event occurs regardless of whether or not the first occurs. [Moved to US]

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

- Determine whether two events are independent or dependent. (a) [Moved from 6.16]
- Compare and contrast the probability of independent and dependent events. (a) [Moved from 6.16]
- Determine the probability of no more than three independent events. (b)
- Determine the probability of no more than two dependent events—without replacement. (b)
- Compare the outcomes of events with and without replacement.
8.1112 The student will:
   a) compare and contrast the probability of independent and dependent events; and [Moved from 6.16] with and without replacement.
   b) determine probabilities for independent and dependent events.

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<tr>
<td>[ P(3 \text{ and heads}) = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12} ]</td>
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<tr>
<td>[ P(3 \text{ and 4 and 5}) = P(3) \cdot P(4) \cdot P(5) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216} ]</td>
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<tr>
<td>• If the outcome of one event has an impact on the outcome of the other event, the events are called dependent. If events are dependent then the second event is considered only if the first event has already occurred. For example, if you choose a blue card from a set of nine different colored cards that has a total of four blue cards and you do not place that blue card back in the set before selecting a second card, the chance of selecting a blue card the second time is diminished because there are now only three blue cards remaining in the set. If you are dealt a King from a deck of cards and you do not place the King back into the deck before selecting a second card, the chance of selecting a King the second time is diminished because there are now only three Kings remaining in the deck. Other examples of dependent events include, but are not limited to: choosing two marbles from a bag but not replacing the first after selecting it; and picking a sock out of a drawer and then picking a second sock without replacing the first; determining the probability that it will snow and that school will be cancelled.</td>
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<tr>
<td>• The probability of two dependent events is found by using the following formula: [ P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A) ]</td>
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<tr>
<td>[ P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A) ]</td>
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<tr>
<td>- Example: You have a bag holding a blue ball, a red ball, and a yellow ball. What is the probability of picking a blue ball out of the bag on the first pick then without replacing the blue ball in the bag, picking a red ball on the second pick? [ P(\text{blue and red}) = P(\text{blue}) \cdot P(\text{red after blue}) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6} ]</td>
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<tr>
<td>[ P(\text{blue and red}) = P(\text{blue}) \cdot P(\text{red after blue}) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6} ]</td>
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</tbody>
</table>
8.12  The student will
a) represent numerical data in boxplots; [Moved from A.10]
b) make observations and inferences about data represented in boxplots; and [Moved from A.10]
c) compare and analyze two data sets using boxplots with the same data represented in histograms, stem and leaf plots, and line plots. [Moved from A.10]

**UNDERSTANDING THE STANDARD**
(Background Information for Instructor Use Only)

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<tr>
<td>A boxplot (box-and-whisker plot) is a convenient and informative way to represent single-variable (univariate) data.</td>
</tr>
<tr>
<td>Boxplots are effective at giving an overall impression of the shape, center, and spread of the data. It does not show a distribution in as much detail as a stem and leaf plot or a histogram.</td>
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<tr>
<td>A boxplot will allow you to quickly analyze a set of data by identifying key statistical measures (median and range) and major concentrations of data, and can be used to analyze a set of data.</td>
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<tr>
<td>A boxplot uses a rectangle to represent the middle half of a set of data and lines (whiskers) at both ends to represent the remainder of the data. The median is marked by a vertical line inside the rectangle.</td>
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<tr>
<td>The five critical points in a boxplot, commonly referred to as the five-number summary, are lower extreme (minimum), lower quartile, median, upper quartile, and upper extreme (maximum). Each of these points represents the bounds for the four quartiles. In the example below, the lower extreme is 15, the lower quartile is 19, the median is 21.5, the upper quartile is 25, and the upper extreme is 29.</td>
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<tr>
<td>The range is the difference between the upper extreme and the lower extreme. The interquartile range (IQR) is the difference between the upper quartile and the lower quartile. Using the example above, the range is 14 or 29 - 15. The interquartile range is 6 or 25 - 19.</td>
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<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
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<tr>
<td>Collect, analyze, and display, and interpret a numeric data set of no more than 20 items, using boxplots. (a)</td>
</tr>
<tr>
<td>Make observations and inferences about data represented in a boxplot. (b)</td>
</tr>
<tr>
<td>Given a data set represented in a boxplot, identify and describe the lower extreme (minimum), upper extreme (maximum), median, upper quartile, lower quartile, range, and interquartile range. (ab)</td>
</tr>
<tr>
<td>Given a data set the student will make inferences about the distribution (dispersion) of the data using a boxplot. (b)</td>
</tr>
<tr>
<td>Compare and analyze two data sets represented in boxplots, with the same data represented in a histogram, a stem and leaf plot and/or a line plot. (c)</td>
</tr>
</tbody>
</table>
8.12 The student will
a) represent numerical data in boxplots; [Moved from A.10]
b) make observations and inferences about data represented in boxplots; and [Moved from A.10]
c) compare and analyze two data sets using boxplots with the same data represented in histograms, stem and leaf plots, and line plots. [Moved from A.10]

### UNDERSTANDING THE STANDARD

**Background Information for Instructor Use Only**

**ESSENTIAL UNDERSTANDINGS**

- When there is an odd number of data values in a set of data, the median will not be considered when calculating the lower and upper quartiles.
  - Example: Calculate the median, lower quartile, upper quartile, and lower quartile extreme, and upper quartile extreme for the following data values:
    
    3 5 6 7 8 9 11 13 13

    Median: 8  
    Lower Quartile: 5.5  
    Upper Quartile: 12

- If all four sections of a boxplot are about the same length then the data is symmetric. A longer section indicates the data are more widely spread than those in shorter sections. If the median does not lie in the center of the box, the distribution can be considered skewed. For example:

  ![Boxplots illustrating skewness](image)

  - Each of the above boxplots illustrates a different skewness pattern. If most of the observations are concentrated on the low end of the scale, the distribution is skewed right. If most of the observations are concentrated on the high end of the scale, the distribution is skewed left. If a distribution is symmetric, the observations will be evenly split at the median, as shown above in the middle figure.

- In the pulse rate example, shown below, many students incorrectly interpret that longer sections contain more data and shorter ones contain less. It is important to remember that roughly
8.12 The student will

a) represent numerical data in boxplots; [Moved from A.10]
b) make observations and inferences about data represented in boxplots; and [Moved from A.10]
c) compare and analyze two data sets using boxplots with the same data represented in histograms, stem and leaf plots, and line plots. [Moved from A.10]

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- The same amount of data is in each section. The numbers in the left whisker (lowest of the data) are spread less widely than those in the right whisker.

- Comparisons, predictions, and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions.

- The information displayed in different graphs may be examined to determine how data are or are not related. Differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and or “what could happen if” (inferences).

- Boxplots are useful when comparing information about two data sets. This example compares the test scores for a college class offered at two different times.

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The student will:
a) represent numerical data in boxplots; 

b) make observations and inferences about data represented in boxplots; and

c) compare and analyze two data sets using boxplots with the same data represented in histograms, stem and leaf plots, and line plots. [Moved from A.10]

Using these boxplots, comparisons could be made about the two sets of data, such as comparing the median score of each class or the Interquartile Range (IQR) of each class.
8.13 The student will
a) represent data in scatterplots; make comparisons, predictions, and inferences, using information displayed in graphs; and
b) make observations and inferences about data represented in scatterplots; and
c) use a drawing to estimate the line of best fit for data represented in a scatterplot.

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- A scatterplot illustrates the relationship between two sets of numerical data represented by two variables (bivariate data). A scatterplot consists of points on the coordinate plane. The coordinates of the point represent the measures of the two attributes of the point. [Reordered]

- In a scatterplot, each point may represent an independent and dependent variable. The independent variable is graphed on the horizontal axis and the dependent is graphed on the vertical axis. [Reordered]

- Scatterplots can be used to predict linear trends and estimate a line of best fit. [Reordered]

- A line of best fit helps in making interpretations and predictions about the situation modeled in the data set. Lines and curves of best fit are explored more in Algebra I to make interpretations and predictions. [Moved to middle column]

- Comparisons, predictions, and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. [Included in US]

- The information displayed in different graphs may be examined to determine how data are or are not related, ascertaining differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inferences).

- Scatterplots can suggest various kinds of linear relationships between variables. For example, weight and height, where weight would be on y-axis and height would be on the x-axis. Linear relationships may be positive (rising), or negative (falling), or null (uncorrelated). If the pattern of points slopes from lower left to upper right, it indicates a positive linear relationship between the

### ESSENTIAL UNDERSTANDINGS

- Why do we estimate a line of best fit for a scatterplot?
  A line of best fit helps in making interpretations and predictions about the situation modeled in the data set. [Moved to US]

- What are the inferences that can be drawn from sets of data points having a positive relationship, a negative relationship, and no relationship?
  Sets of data points with positive relationships demonstrate that the values of the two variables are increasing. A negative relationship indicates that as the value of the independent variable increases, the value of the dependent variable decreases. [Included in US]

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Collect, organize, and interpret a data set of no more than 20 items using scatterplots. (a)

- Interpret data in a scatterplot. Predict from the trend an estimate of the line of best fit with a drawing for data represented in a scatterplot. (b,c)

- Interpret Make observations about a set of data points in a scatterplot as having a positive linear relationship, a negative linear relationship, or no relationship. (b)
8.13 The student will
a) represent data in scatterplots; make comparisons, predictions, and inferences, using information displayed in graphs; and
b) make observations and inferences about data represented in a scatterplot; and
c) use a drawing to estimate the line of best fit for data represented in a scatterplot.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
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<tbody>
<tr>
<td>variables being studied. If the pattern of points slopes from upper left to lower right, it indicates a negative correlation linear relationship. [Similar to middle column]</td>
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<tr>
<td>For example: The following scatterplots illustrate how patterns in data values may indicate linear relationships.</td>
<td></td>
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<tr>
<td>No correlation relationship Positive correlation relationship Negative correlation relationship</td>
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</table>

- **Correlation** A linear relationship between variables does not necessarily imply causation. For example, as the temperature at the beach increases, the sales at an ice cream store increase. If data were collected for these two variables, a positive correlation linear relationship would exist; however, there is no causal relationship between the variables (i.e., the temperature outside does not cause ice cream sales to increase, but there is a relationship between the two).

- A scatterplot illustrates the relationship between two sets of data. A scatterplot consists of points. The coordinates of the point represent the measures of the two attributes of the point.

- Scatterplots can be used to predict trends and estimate a line of best fit.

- In a scatterplot, each point is represented by an independent and
8.13 The student will
a) represent data in scatterplots; make comparisons, predictions, and inferences, using information displayed in graphs; and
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<tr>
<td>dependent variable. The independent variable is graphed on the horizontal axis and the dependent is graphed on the vertical axis.</td>
<td>[Reordered]</td>
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<tr>
<td>Correlation: The relationship between variables is not always linear, and may be modeled by other types of functions that are studied in high school and college level mathematics.</td>
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</table>
In the middle grades, the focus of mathematics learning is to
• build on students’ concrete reasoning experiences developed in the elementary grades;
• construct a more advanced understanding of mathematics through active learning experiences;
• develop deep mathematical understandings required for success in abstract learning experiences; and
• apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

• Students extend their knowledge of patterns developed in the elementary grades and through life experiences by investigating and describing functional relationships.

• Students learn to use algebraic concepts and terms appropriately. These concepts and terms include variable, term, coefficient, exponent, expression, equation, inequality, domain, and range. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.

• Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.

• Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.

Patterns, functions and algebra become a larger mathematical focus in the middle grades as students extend their knowledge of patterns developed in the elementary grades.

Students make connections between the numeric concepts of ratio and proportion and the algebraic relationships that exist within a set of equivalent ratios. Students use variable expressions to represent proportional relationships between two quantities and begin to connect the concept of a constant of proportionality to rate of change and slope. Representation of relationships between two quantities using tables, graphs, equations, or verbal descriptions allow students to connect their knowledge of patterns to the concept of functional relationships. Graphing linear equations in two variables in the coordinate plane is a focus of the study of functions which continues in high school mathematics.

Students learn to use algebraic concepts and terms appropriately. These concepts and terms include variable, term, coefficient, exponent, expression, equation, inequality, domain, and range. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades. Students learn to solve equations by using concrete materials. They expand their skills from one-step to multistep equations and inequalities through their application in practical situation.
8.14 The student will
a) apply the order of operations to evaluate an algebraic expression for given replacement values of the variables; and
b) simplify algebraic expressions in one variable.

**UNDERSTANDING THE STANDARD**
(Background Information for Instructor Use Only)

- An expression is a representation of a quantity. It contains numbers, variables, and/or operation symbols. It does not have an “equal sign (=)” (e.g., $\frac{3}{4}$, 5x, 140 - 38.2, -18 \cdot 21, (5 + 2x) \cdot 4). An expression cannot be solved. [Moved from 8.1]

- A numerical expression contains only numbers, the operations symbols, and grouping symbols. [Moved from 8.1]

- Expressions are simplified using the order of operations. [Included from 8.1]

- Simplifying an algebraic expression means to write the expression as a more compact and equivalent expression. This usually involves combining like terms.

- Like terms are terms that have the same variables and exponents. The coefficients do not need to match (e.g., 12x and -5x; 45 and $-\frac{2}{3}$; 9y, -51y and $\frac{4}{7}$y). [Reordered]

- Like terms may be added or subtracted using the distributive and other properties. For example:
  - $2(x - \frac{7}{3}) + 5x = 2x - \frac{14}{3} + 5x = 2x + 5x - 1 = 7x - 1$
  - $w + w - 2w = (1 + 1)w - 2w = 2w - 2w = (2 - 2)w = 0$
  - $w = 0$

- The order of operations is as follows:
  - First, complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols, do the innermost operation first.
  - Second, evaluate all exponential expressions.
  - Third, multiply and/or divide in order from left to right.
  - Fourth, add and/or subtract in order from left to right.

* Parentheses (), brackets [], braces {}, absolute value |

**ESSENTIAL UNDERSTANDINGS**

- What is the role of the order of operations when evaluating expressions?
  - Using the order of operations assures only one correct answer for an expression. [Moved to US]

**ESSENTIAL KNOWLEDGE AND SKILLS**

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
  - Use the order of operations and apply the properties of real numbers to evaluate algebraic expressions for the given replacement values of the variables. Substitute numbers for variables in algebraic expressions and simplify the expressions by using the order of operations. Exponents are positive and limited to whole numbers, and bases are limited to integers less than 4. Square roots are limited to perfect squares. Limit the number of replacements to no more than three per expression. (a) [Moved from 8.1a]

- Represent algebraic expressions using concrete materials and pictorial representations. Concrete materials may include colored chips or algebra tiles. (a)

- Simplify algebraic expressions in one variable. Expressions may need to be expanded (using the distributive property) or require combining like terms to simplify. Expressions will include only linear and numeric terms. Coefficients and numeric terms may be rational. (b)

- Apply the order of operations to evaluate formulas. Problems will be limited to positive exponents. Square roots may be included in the expressions but limited to perfect squares. [Incorporated into bullet above]
### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

(i.e., \(3(-5 + 2) - 7\)), and the division bar (i.e., \(\frac{3+4}{5+6}\)) should be treated as grouping symbols. [Reordered and rewritten]

- Properties of real numbers can be used to express simplification.
  Students should utilize the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of \(a\), \(b\), or \(c\) in this standard).

  - The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., \(5.6 + 4 = 4 + 5.6\)).
  
  - The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., \(5.6 \cdot -4 = -4 \cdot 5.6\)).
  
  - The associative property of addition states that regrouping the addends does not change the sum (e.g., \(5.6 + (4 + 3) = (5.6 + 4) + 3\)).
  
  - The associative property of multiplication states that regrouping the factors does not change the product (e.g., \(5.6 \cdot (4 \cdot 3) = (5.6 \cdot 4) \cdot 3\)).
  
  - Subtraction and division are neither commutative nor associative.

- The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true and vice versa.

  **Distributive property examples:**
  
  - \(-5(x + 7) = (-5 \cdot x) + (-5 \cdot 7) = -5x - 35\)
  
  - \(-5(x - 7) = (-5 \cdot x) - (-5 \cdot 7) = -5x + 35\)

  **Converse of the distributive property examples:**
  
  - \(-5x - 35 = (-5 \cdot x) + (-5 \cdot 7) = -5(x + 7)\), or
  
  - \(-5x + 35 = (-5 \cdot x) - (-5 \cdot 7) = -5(x - 7)\)
The student will

a) apply the order of operations to evaluate an algebraic expression for given replacement values of the variables; and

b) simplify algebraic expressions in one variable.

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<tr>
<td>The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division.</td>
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<tr>
<td>The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real number (e.g., ( \frac{5}{2} + 0 = \frac{5}{2} )).</td>
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<td>The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number (e.g., ( \sqrt{8} \cdot 1 = \sqrt{8} )).</td>
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<td>Inverses are numbers that combine with other numbers and result in identity elements. ( \text{e.g., } 5 + (-5) = 0; \frac{1}{5} \cdot 5 = 1 ).</td>
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<tr>
<td>The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., 5.6 + (-5.6) = 0).</td>
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<td>The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., ( \frac{1}{4} \cdot 4 = 1 )).</td>
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<tr>
<td>Zero has no multiplicative inverse.</td>
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<tr>
<td>The multiplicative property of zero states that the product of any real number and zero is zero (e.g., 7 ( \cdot ) 0 = 0 and 0 ( \cdot ) 7 = 0).</td>
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<tr>
<td>Division by zero is not a possible mathematical operation. It is undefined.</td>
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8.14 The student will
   a) apply the order of operations to evaluate an algebraic expression for given replacement values of the variables; and
   b) simplify algebraic expressions in one variable.

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<tr>
<td>- Commutative property of addition: ( a + b = b + a )</td>
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<tr>
<td>- Commutative property of multiplication: ( a \cdot b = b \cdot a )</td>
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<tr>
<td>- Associative property of addition: ( (a + b) + c = a + (b + c) )</td>
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<tr>
<td>- Associative property of multiplication: ( (a \cdot b) \cdot c = a \cdot (b \cdot c) )</td>
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<td>- Subtraction and division are neither commutative nor associative.</td>
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<tr>
<td>- Distributive property (over addition/subtraction): ( a \cdot (b + c) = a \cdot b + a \cdot c ) and ( a \cdot (b - c) = a \cdot b - a \cdot c )</td>
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<tr>
<td>- The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division.</td>
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<tr>
<td>- Identity property of addition (additive identity property): ( a + 0 = a ) and ( 0 + a = a )</td>
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<tr>
<td>- Identity property of multiplication (multiplicative identity property): ( a \cdot 1 = a ) and ( 1 \cdot a = a )</td>
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<tr>
<td>- Inverses are numbers that combine with other numbers and result in identity elements [e.g., ( 5 + (-5) = 0; \frac{1}{2} \cdot 5 = 1 )].</td>
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<tr>
<td>- Inverse property of addition (additive inverse property): ( a + (-a) = 0 ) and ( (-a) + a = 0 ).</td>
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<tr>
<td>- Inverse property of multiplication (multiplicative inverse property): ( a \cdot \frac{1}{a} = 1 ) and ( \frac{1}{a} \cdot a = 1 ).</td>
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<tr>
<td>- Zero has no multiplicative inverse.</td>
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<td>- Multiplicative property of zero: ( a \cdot 0 = 0 ) and ( 0 \cdot a = 0 ).</td>
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<td>- Division by zero is not a possible mathematical operation. It is undefined.</td>
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8.14 The student will
a) apply the order of operations to evaluate an algebraic expression for given replacement values of the variables; and
b) simplify algebraic expressions in one variable.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)
- Substitution property: if \( a = b \), then \( b \) can be substituted for \( a \) in any expression, equation, or inequality.

- A power of a number represents repeated multiplication of the number. For example, \((-5)^4\) means \((-5) \cdot (-5) \cdot (-5) \cdot (-5)\). The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In this example, \((-5)\) is the base, and 4 is the exponent. The product is 625. Notice that the base appears inside the grouping symbols. The meaning changes with the removal of the grouping symbols. For example, \((-5)^4\) means \((-5) \cdot (-5) \cdot (-5) \cdot (-5)\) negated which results in a product of -625. The expression \(-5^4\) means to take the opposite of \(5^4\) which is -625. Students should be exposed to all three representations.

- An algebraic expression is an expression that contains operations with algebraic symbols (variables) and numbers.

- Algebraic expressions are evaluated by substituting numbers for variables and applying the order of operations to simplify the resulting numeric expression.

- The order of operations is as follows: Complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols (embedded), do the innermost operation first. Evaluate all exponential expressions. Multiply and/or divide in order from left to right. Add and/or subtract in order from left to right.

- * Parentheses ( ), brackets [ ], braces { }, the absolute value \( | | \), division/fraction bar \( \div \), and the square root symbol \( \sqrt{ } \) should be treated as grouping symbols. [Reordered and rewritten]

ESSENTIAL UNDERSTANDINGS

ESSENTIAL KNOWLEDGE AND SKILLS
The student will make connections between any two representations (tables, graphs, words, and rules) of a given relationship.

a) determine whether a given relation is a function; and
b) determine the domain and range of a function. [Moved from 8.17]

### UNDERSTANDING THE STANDARD

(Background Information for Instructor Use Only)

- A relation is any set of ordered pairs. For each first member, there may be many second members.
- A function is a relation in which there is one and only one second member for each first member.
- A function is a relation between a set of inputs, called the domain, and a set of outputs, called the range, with the property that each input is related to exactly one output.
- As a table of values, a function has a unique value assigned to the second variable for each value of the first variable. In the “not a function” example, the input value “1” has two different output values, 5 and -3, assigned to it, so the example is not a function.

<table>
<thead>
<tr>
<th>function</th>
<th>not a function</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
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<tr>
<td>2</td>
<td>3</td>
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<tr>
<td>1</td>
<td>5</td>
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<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>-1</td>
<td>-3</td>
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</tbody>
</table>

- As a set of ordered pairs, a function has a unique or different y-value assigned to each x-value. For example, the set of ordered pairs, \{(1, 2), (2, 4), (3, 2), (4, 8)\}, is a function. This set of ordered pairs, \{(1, 2), (2, 4), (3, 2), (2, 3)\}, is not a function since the x-value of two “2” has two different y-values.
- As a graph of discrete points, a relation is a function if any curve (including straight lines) such that any when, for any value of x, a vertical line would pass through the curve no more than one point on the graph, only once.
- Some relations are functions; all functions are relations.

### ESSENTIAL UNDERSTANDINGS

- What is the relationship among tables, graphs, words, and rules in modeling a given situation? Any given relationship can be represented by all four. [Included in US]

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Determine whether a relation, represented by a set of no more than 10 ordered pairs, a table, or a graph of discrete points is a function. Sets are limited to no more than 10 ordered pairs. (a)
- Identify the domain and range of a function represented as a set of ordered pairs, a table, or a graph of discrete points. (b)
- Graph in a coordinate plane ordered pairs that represent a relation. [Combined with bullets above]
- Describe and represent relations and functions, using tables, graphs, words, and rules. Given one representation, students will be able to represent the relation in another form. [Combined with bullets above]
- Relate and compare different representations for the same relation. [Combined with bullets above]
The student will make connections between any two representations (tables, graphs, words, and rules) of a given relationship.

a) determine whether a given relation is a function; and

b) determine the domain and range of a function. [Moved from 8.17]

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>Graphs of functions can be discrete or continuous.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>In a discrete function graph there are separate, distinct points. You would not use a line to connect these points on a graph. The points between the plotted points have no meaning and cannot be interpreted. For example, the number of pets per household represents a discrete function since you cannot have a fraction of a pet.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>In a graph of a continuous function, every point in the domain can be interpreted therefore it is possible to connect the points on the graph with a continuous line as every point on the line answers the original question being asked. For example, the temperature of a liquid represents a continuous function.</strong> [Moved to 8.16]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Functions can be represented as ordered pairs, tables, graphs, equations, physical models, or in words. Any given relationship can be represented using multiple representations.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>As a graph, a function in grade 8 mathematics is any curve (including straight lines) such that any vertical line would pass through the function only once (vertical line test).</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>A discussion about determining whether a continuous graph of a relation is a function using the vertical line test should may occur in grade 8, but will be explored further in Algebra I.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>The domain is the set of all the input values for the independent variable or x-values (first number in an ordered pair). The domain of a continuous linear function is all real numbers.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>domain → function → range</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>The range is the set of all the output values for the dependent variable or y-values (second number in an ordered pair). The range of a continuous linear function is all real numbers.</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 8
The student will make connections between any two representations (tables, graphs, words, and rules) of a given relationship.

a) determine whether a given relation is a function; and

b) determine the domain and range of a function. [Moved from 8.17]

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<tr>
<td>(Background Information for Instructor Use Only)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>examples that follow, the domain is {-1, 0, 1, 2} and the range is {-3, 3, 5}.</td>
<td></td>
<td></td>
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<tr>
<td>• If a function is comprised of a discrete set of ordered pairs, then the domain is the set of all the x-coordinates, and the range is the set of all the y-coordinates. These sets of values can be determined given different representations of the function.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Example: The domain of a function is {-1, 1, 2, 3} and the range is {-3, 3, 5}. The following are representations of this function:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- A function represented as a table of values:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( x )</td>
<td>( y )</td>
</tr>
<tr>
<td></td>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>- A function represented as a set of ordered pairs: {(-1, 5), (1, -3), (2, 3), (3, 5)}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
8.1514 The student will make connections between any two representations (tables, graphs, words, and rules) of a given relationship. 

a) determine whether a given relation is a function; and 
b) determine the domain and range of a function. [Moved from 8.17]

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<tr>
<td>A function represented as a graph on a coordinate plane:</td>
<td></td>
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</tr>
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</table>

STANDARD 8.1514 STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA GRADE LEVEL 8

Mathematics Standards of Learning Curriculum Framework 2009-2016: Grade 8

49
8.16 The student will
a) **recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;**

b) **identify the slope and y-intercept of a linear function given a table of values, a graph or given the equation in** \( y = mx + b \) **form;**

c) **determine the independent and dependent variable, given a practical situation modeled by a linear function;**

d) **graph a linear function given the equation in** \( y = mx + b \) **form; and**

e) **distinguish between proportional and non-proportional situations modeled by linear functions.**

**make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.**

### UNDERSTANDING THE STANDARD (Background Information for Instructor Only)

- A linear function equation is an equation in two variables whose graph is a straight line, a type of continuous function (see SOL 8.14).
- A linear function equation represents a situation with a constant rate. For example, when driving at a rate of 35 mph, the distance increases as the time increases, but the rate of speed remains the same.
- Slope \( (m) \) represents the rate of change in a linear function or the "steepness" of the line. The slope of a line is a rate of change, a ratio describing the vertical change to the horizontal change.

\[
\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{vertical change}}{\text{horizontal change}}
\]

- A line is increasing if it rises from left to right. The slope is **positive**, i.e., \( m > 0 \).
- A line is decreasing if it falls from left to right. The slope is **negative**, i.e., \( m < 0 \).
- A horizontal line has zero slope, i.e., \( m = 0 \).

### ESSENTIAL UNDERSTANDINGS

- What types of real-life situations can be represented with linear equations? Any situation with a constant rate can be represented by a linear equation. [Included in US]

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- **Recognize and describe a line with a slope that is positive, negative, or zero (0). (a)**
- Construct a table of ordered pairs by substituting values for \( x \) in a linear equation to find values for \( y \).
- Plot in the coordinate plane ordered pairs \( (x, y) \) from a table.
- Connect the ordered pairs to form a straight line (a continuous function).
- Given a table of values for a linear function, identify the slope and y-intercept. The table will include the coordinate of the y-intercept. (b)
- Given a linear function in the form \( y = mx + b \), identify the slope and y-intercept. (b)
- Given the graph of a linear function, identify the slope and y-intercept. The value of the y-intercept will be limited to integers. The coordinates of the ordered pairs shown in the graph will be limited to integers. (b)
- Identify the dependent and independent variable, domain and range of given a practical situation modeled by a linear function. (c) [Moved from 8.17]
8.16 The student will

(a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;

(b) identify the slope and y-intercept of a linear function given a table of values, a graph or given the equation in \( y = mx + b \) form;

(c) determine the independent and dependent variable, given a practical situation modeled by a linear function;

(d) graph a linear function given the equation in \( y = mx + b \) form; and

e) distinguish between proportional and non-proportional situations modeled by linear functions and make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Only)

A line with a positive slope slants up to the right.
A line with a negative slope slants down to the right.
A line with a slope of 0 is horizontal.

A discussion about lines with undefined slope (vertical lines) should occur with students in grade 8 mathematics to compare undefined slope to lines with a defined slope. Further exploration of this concept will occur in Algebra I.

A linear function can be written in the form \( y = mx + b \), where \( m \) represents the slope or rate of change in \( y \) compared to \( x \), and \( b \) represents the y-intercept of the graph of the linear function. The y-intercept is the point at which the graph of the function intersects the y-axis and may be given as a single value, \( b \), or as the location of a point \((0, b)\).

Example: Given the equation of the linear function \( y = -3x + 2 \), the slope is \(-3\) or \(-\frac{3}{1}\) and the y-intercept is \(2\) or \((0, 2)\).

Example: The table of values represents a linear function. In the table, the point \((0, 2)\) represents the y-intercept. The slope is determined by observing the change in each y-value compared to the corresponding change in the x-value.

ESSENTIAL UNDERSTANDINGS

- Graph in a coordinate plane ordered pairs that represent a linear function. (b)
- Relate and compare different representations for the same linear function. (a, b)
- Given the equation of a linear function in the form \( y = mx + b \), graph the function. The value of the y-intercept will be limited to integers. (bd)
- Write the equation of a linear function in the form \( y = mx + b \) given values for the slope, \( m \), and the y-intercept or given a practical situation in which the slope, \( m \), and y-intercept are described verbally. (e)
- Make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs. (e)
- Recognize and describe a line with a slope that is positive, negative, or 0. (a)
- Identify, interpret, and compare the unit rate of the proportional and non-proportional relationships given its representation as a linear function. (e) Graphed as the slope of the graph, and compare two different proportional relationships represented in different ways. [Included in 6.13, 2016]

† Revised March 2011
8.16 The student will

a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;
b) identify the slope and \( y \)-intercept of a linear function given a table of values, the graph or given the equation in \( y = mx + b \) form;
c) determine the independent and dependent variable, given a practical situation modeled by a linear function;
d) graph a linear function given the equation in \( y = mx + b \) form; and two variables;
e) distinguish between proportional and non-proportional situations modeled by linear functions make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

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<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
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</thead>
<tbody>
<tr>
<td>-2</td>
<td>8</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
</tr>
</tbody>
</table>

\[
\text{slope } m = \frac{\text{change in } y\text{-value}}{\text{change in } x\text{-value}} = \frac{-3}{1} = -3
\]

- The slope, \( m \), and \( y \)-intercept of a linear function can be determined given the graph of the function.
  - Example: Given the graph of the linear function, determine the slope and \( y \)-intercept.
8.16 The student will
a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;
b) identify the slope and y-intercept of a linear function given a table of values, the graph or given the equation in \( y = mx + b \) form;
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</table>

Given the graph of a linear function, the y-intercept is found by determining where the line intersects the y-axis. The y-intercept would be 2 or located at the point (0, 2). The slope can be found by determining the change in each y-value compared to the change in each x-value. Here, we could use slope triangles to help visualize this:

\[
\text{slope} = m = \frac{\text{change in y-value}}{\text{change in x-value}} = \frac{-3}{1} = -3
\]

- Graphing a linear function given an equation can be addressed using different methods. One method requires involves...
8.16 The student will
a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;
b) identify the slope and \( y \)-intercept of a linear function given a table of values, a graph or given the equation in \( y = mx + b \) form;
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<tr>
<td>determining a table of ordered pairs by substituting into the equation values for one variable and solving for the other variable, plotting the ordered pairs in the coordinate plane, and connecting the points to form a straight line. Another method involves using slope triangles to determine points on the line.</td>
<td>- Example: Graph the linear function whose equation is ( y = 5x - 1 ). In order to graph the linear function, we can create a table of values by substituting arbitrary values for ( x ) to determining coordinating values for ( y ):</td>
<td></td>
</tr>
<tr>
<td>[ \begin{array}{ccc} x &amp; 5x - 1 &amp; y \ -1 &amp; 5(-1) - 1 &amp; -6 \ 0 &amp; 5(0) - 1 &amp; -1 \ 1 &amp; 5(1) - 1 &amp; 4 \ 2 &amp; 5(2) - 1 &amp; 9 \end{array} ]</td>
<td>The values can then be plotted as points on a graph.</td>
<td>Knowing the equation of a linear function written in ( y = mx + b ) provides information about the slope and ( y )-intercept of the function. If the equation is ( y = 5x - 1 ), then the slope, ( m ), of the line is ( 5 ) or ( \frac{5}{1} ) and the ( y )-intercept is ( -1 ) and can be located at the point ((0, -1)). We can graph the line by first plotting the ( y )-intercept. We also know:</td>
</tr>
</tbody>
</table>
8.16 The student will

a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;

b) identify the slope and \( y \)-intercept of a linear function given a table of values, the graph or given the equation in \( y = mx + b \) form;

c) determine the independent and dependent variable, given a practical situation modeled by a linear function;

d) graph a linear function given the equation in \( y = mx + b \) form; and

e) distinguish between proportional and non-proportional situations modeled by linear functions make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

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</tr>
<tr>
<td>Other points can be plotted on the graph using the relationship between the ( y ) and ( x ) values. Slope triangles can be used to help locate the other points.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- A table of values can be used in conjunction with using slope triangles to verify the graph of a linear function. The \( y \)-intercept is located on the \( y \)-axis which is where the \( x \)-coordinate is 0. The change in each \( y \)-value compared to the corresponding \( x \)-value can be verified by the patterns in the table of values.
8.16 The student will
a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;
b) identify the slope and y-intercept of a linear function given a table of values, the graph or given the equation in \( y = mx + b \) form;
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e) distinguish between proportional and non-proportional situations modeled by linear functions and make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

### UNDERSTANDING THE STANDARD

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1</td>
<td>-5</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>+1</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>+1</td>
<td>9</td>
</tr>
</tbody>
</table>

- The axes of a coordinate plane are generally labeled \( x \) and \( y \); however, any letters may be used that are appropriate for the function.
- A function has values that represent the input (\( x \)) and values that represent the output (\( y \)). The independent variable is the input value.
- The dependent variable depends on the independent variable and is the output value.
- Below is a table of values for finding the circumference of circles, \( C = \pi d \), where the value of \( \pi \) is approximated as 3.14.

<table>
<thead>
<tr>
<th>Diameter</th>
<th>Circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 in.</td>
<td>3.14 in.</td>
</tr>
<tr>
<td>2 in.</td>
<td>6.28 in.</td>
</tr>
<tr>
<td>3 in.</td>
<td>9.42 in.</td>
</tr>
<tr>
<td>4 in.</td>
<td>12.56 in.</td>
</tr>
</tbody>
</table>
8.16 The student will
   a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;
   b) identify the slope and y-intercept of a linear function given a table of values, the graph or given the equation in \( y = mx + b \) form;
   c) determine the independent and dependent variable, given a practical situation modeled by a linear function;
   d) graph a linear function given the equation in \( y = mx + b \) form; and two variables;
   e) distinguish between proportional and non-proportional situations modeled by linear functions; make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

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<tr>
<td>The independent variable, or input, is the diameter of the circle. The values for the diameter make up the domain.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The dependent variable, or output, is the circumference of the circle. The set of values for the circumference makes up the range.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>In a graph of a continuous function every point in the domain can be interpreted therefore it is possible to connect the points on the graph with a continuous line as every point on the line answers the original question being asked. [Moved from 8.14]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The context of a problem may determine if it is appropriate for ordered pairs representing a linear relationship to be connected by a straight line. If the independent variable ( x ) represents a discrete quantity (e.g., number of people, number of tickets, etc.) then it is not appropriate to connect the ordered pairs with a straight line when graphing. If the independent variable ( x ) represents a continuous quantity (e.g., amount of time, temperature, etc.) then it is appropriate to connect the ordered pairs with a straight line when graphing.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Example: The function ( y = 7x ) represents the cost in dollars ( y ) for ( x ) tickets to an event. The domain of this function would be discrete and would be represented by discrete points on a graph. Not all values for ( x ) could be represented and connecting the points would not be appropriate.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Example: The function ( y = -2.5x + 20 ) represents the number of gallons of water ( y ) remaining in a 20-gallon tank being drained for ( x ) number of minutes. The domain in this function would be continuous, There would be an ( x )-value</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
8.16 The student will
a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;
b) identify the slope and \( y \)-intercept of a linear function given a table of values, the graph or given the equation in \( y = mx + b \) form;
c) determine the independent and dependent variable, given a practical situation modeled by a linear function;
d) graph a linear function given the equation in \( y = mx + b \) form; and two variables.
e) distinguish between proportional and non-proportional situations modeled by linear functions make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Only)
representing any point in time until the tank is drained so connecting the points to form a straight line would be appropriate (note: the context of the problem limits the values that \( x \) can represent).

- Functions can be represented as ordered pairs, tables, graphs, equations, physical models, or in words. Any given relationship can be represented using multiple representations. [Moved from 8.14]

- Interpret The equation \( y = mx + b \) as defining a linear function whose graph (solution) is a straight line. The equation of a linear function can be determined given the slope, \( m \), and the \( y \)-intercept, \( b \). Verbal descriptions of practical situations that can be modeled by a linear function can also be represented using an equation.
  - Example: Write the equation of a linear function whose slope is \( \frac{3}{4} \) and \( y \)-intercept is -4, or located at the point (0, -4).
    
    The equation of this line can be found by substituting the values for the slope, \( m = \frac{3}{4} \), and the \( y \)-intercept, \( b = -4 \), into the general form of a linear function \( y = mx + b \).
    
    Thus, the equation would be \( y = \frac{3}{4}x - 4 \).
  
  - Example: John charges a $30 flat fee to trouble shoot a jet-ski that is not working properly and $50 per hour needed for any repairs. Write a linear function that represents the total cost, \( y \), of a jet-ski repair, based on
8.16 The student will
a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;
b) identify the slope and y-intercept of a linear function given a table of values, the graph or given the equation in \( y = mx + b \) form;
c) determine the independent and dependent variable, given a practical situation modeled by a linear function;
d) graph a linear function given the equation in \( y = mx + b \) form; and

two variables.
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<td>the number of hours, ( x ), needed to repair it. Assume that there is no additional charge for parts. In this practical situation, the y-intercept, ( b ), would be $30, to represent the initial flat fee to troubleshoot the jet ski. The slope, ( m ), would be $50, since that would represent the rate per hour. The equation to represent this situation would be ( y = 50x + 30 ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The y-intercept (( b )) is the point at which the graph of a function intersects with the ( y )-axis. It is represented as an ordered pair, ((0, b)). For example, in the function ( y = 3x + 2 ), the slope is ( 3 ) or ( \frac{3}{1} ) and the y-intercept is ( 2 ) or ((0, 2)), a point on the ( y )-axis.</td>
<td></td>
<td></td>
</tr>
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</table>
| • Slope (\( m \)) represents the rate of change in a linear function or the “steepness” and direction of the line. The slope of a line is a rate of change, a ratio describing the vertical change to the horizontal change: \[
\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{vertical change}}{\text{horizontal change}}
\] |
| • A line is increasing if it goes up from left to right. The slope is positive, i.e., \( m > 0 \). |
| • A line is decreasing if it goes down from left to right. The slope is negative, i.e., \( m < 0 \). |
| • A horizontal line has zero slope, i.e., \( m = 0 \). |
8.16 The student will

a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;

b) identify the slope and y-intercept of a linear function given a table of values, a graph or given the equation in \( y = mx + b \) form;

c) determine the independent and dependent variable, given a practical situation modeled by a linear function;

d) graph a linear function given the equation in \( y = mx + b \) form; and two variables;

e) distinguish between proportional and non-proportional situations modeled by linear functions and make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>• A discussion about lines with undefined slope (vertical lines) should occur with students in grade 8 mathematics to compare undefined slope to lines with a defined slope. Further exploration of this concept will occur in Algebra I.</td>
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<tr>
<td>• A proportional relationship between two variables can be represented by a linear function ( y = mx ) that passes through the point ((0, 0)) and thus has a y-intercept of 0. The variable ( y ) results from ( x ) being multiplied by ( m ), the rate of change or slope.</td>
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<tr>
<td>• The linear function ( y = x + b ) represents a linear function that is a non-proportional additive relationship. The variable ( y ) results from the value ( b ) being added to ( x ). In this linear relationship, there is a y-intercept of ( b ) and the constant rate of change or slope would be 1. When there is In a linear function with a slope other than 1, there is a coefficient in front of the ( x ) term, which represents the constant rate of change, or slope.</td>
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<tr>
<td>• Proportional relationships and additive relationships between two quantities are special cases of linear functions that are discussed in grade 7 mathematics.</td>
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</table>
STANDARD 8.1715 STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA GRADE LEVEL 8

The student will

a) solve multistep linear equations in one variable with the variable on one and both two sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable;

b) solve two-step linear inequalities and graph the results on a number line; and [Moved to 8.18.13]

c) identify properties of operations used to solve an equation. [Incorporated into 8.14, 8.17 and 8.18 EKS]

UNDERSTANDING THE STANDARD
(Background Information for Instructor Only)

- A multistep equation may include, but not be limited to equations such as the following: is an equation that requires more than one different mathematical operation to solve.

\[ 2x + 1 = \frac{2}{3}x - 3(2x + 7) = \frac{1}{2}x; 2x + 7 - 5x = 27; -5x - (x + 3) = -12. \]

- A two-step inequality is defined as an inequality that requires the use of two different operations to solve (e.g., \(3x - 4 > 9\)). [Moved to 8.17]

- An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an “equal sign (=)” (e.g., \(\frac{2}{3}x, 140 - 38.2, 18 \cdot 21, 5 + x\))

- An expression that contains a variable is a variable expression. A variable expression is like a phrase: as a phrase does not have a verb, so an expression does not have an “equal sign (=)”. An expression cannot be solved.

- A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used when the numbers are unknown. For example, the verbal expression “a number multiplied by 5” could be represented by the variable expression “\(5n\)” or “\(5n\)”.

- An algebraic expression is a variable expression that contains at least one variable (e.g., \(2x - 3\)).

- A verbal sentence is a complete word statement (e.g., “The sum of two consecutive integers is thirty-five.”) could be represented by “\(n + (n + 1) = 35\)”.

- An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., \(2x + 3 = -4x + 1\)).

- In an equation, the “equal sign (=)” indicates that the value of the expression on the left is the same as equivalent to the value of the expression on the right.

ESSENTIAL UNDERSTANDINGS

- How does the solution to an equation differ from the solution to an inequality?

- While a linear equation has only one replacement value for the variable that makes the equation true, an inequality can have more than one. [Moved to 8.18 US]

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Solve two- to four-step linear equations in one variable using concrete materials, pictorial representations, and paper and pencil illustrating the steps performed. Represent and solve multistep linear equations in one variable (up to four steps) using a variety of concrete materials and pictorial representations.

- Apply properties of real numbers and properties of equality to solve multistep linear equations in one variable (up to four steps). Coefficients and numeric terms will be rational. Equations may contain expressions that need to be expanded (using the distributive property) or require collecting like terms to solve.

- Solve two-step inequalities in one variable by showing the steps and using algebraic sentences. [Moved to 8.18.13a]

- Graph solutions to two-step linear inequalities on a number line. [Moved to 8.18.13b]

- Write verbal expressions and sentences as algebraic expressions and equations.

- Write algebraic expressions and equations as verbal expressions and sentences.

- Solve practical problems that require the solution of a multistep linear equation.
8.1715 The student will:

a) solve multistep linear equations in one variable with the variable on one and both two sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable;

b) solve two-step linear inequalities and graph the results on a number line; and [Moved to 8.18.13]

c) identify properties of operations used to solve an equation. [Incorporated into 8.14, 8.17 and 8.18 EKS]

UNDERSTANDING THE STANDARD
(Background Information for Instructor Only)

right.

To maintain equality, an operation that is performed on one side of an equation must be performed on the other side.

- Like terms are terms that have the same variables and exponents. The coefficients do not need to match (e.g., 12x and -5x; 45 and $-\frac{2}{3}y$; $-\frac{9}{5}y$ and $\frac{4}{y}$). [Reordered]

- Like terms may be added or subtracted using the distributive and other properties. For example:

  \[4.6y - 5y = (-4.6 - 5)y = -9.6y\]
  
  \[w + w - 2w = (1 + 1)w - 2w = 2w - 2w = (2 - 2)w = 0\]

- When both expressions of an inequality are multiplied or divided by a negative number, the inequality sign reverses. [Moved to 8.18]

Real-world problems can be interpreted, represented, and solved using linear equations in one variable.

Properties of real numbers and properties of equality can be used to solve equations, justify solutions and express simplification. Students should utilize the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of a, b, or c in this standard).

- The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., \(5 + 4 = 4 + 5\)).

- The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., \(5 \cdot 4 = 4 \cdot 5\)).

- The associative property of addition states that regrouping the addends does not change the sum.

- The associative property of multiplication states that regrouping the factors does not change the product (e.g., \(5 \cdot (4 \cdot 5) = (5 \cdot 4) \cdot 5\)).

- The distributive property states that multiplying a sum by a number is the same as doing each multiplication separately and then adding (e.g., \(a(b + c) = ab + ac\)).

- The identity property states that multiplying by 1 does not change a number (e.g., \(1 \cdot a = a\)).

- The multiplicative inverse property states that multiplying a number by its reciprocal results in 1 (e.g., \(\frac{1}{a} \cdot a = 1\)).

ESSENTIAL UNDERSTANDINGS

- Confirm algebraic solutions to linear equations in one variable.

- Identify properties of operations used to solve an equation from among:
  - the commutative properties of addition and multiplication;
  - the associative properties of addition and multiplication;
  - the distributive property;
  - the identity properties of addition and multiplication;
  - the zero property of multiplication;
  - the additive inverse property; and
  - the multiplicative inverse property. [Incorporated into 8.14, 8.17, and 8.18]
8.1715 The student will

a) solve multistep linear equations in one variable with the variable on one and both two-sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable;

b) solve two-step linear inequalities and graph the results on a number line; and [Moved to 8.187.13]

c) identify properties of operations used to solve an equation. [Incorporated into 8.14, 8.17 and 8.18 EKS]

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<td>[e.g., 5 + (1 + 2) = (5 + 1) + 2].</td>
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<td>o The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5.6 · (4 · 3) = (5.6 · 4) · 3].</td>
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<td>o Subtraction and division are neither commutative nor associative.</td>
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<tr>
<td>o The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true, and vice versa. Distributive property examples:</td>
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<tr>
<td>* 5(x + 7) = (5 · x) + (5 · 7) = 5x + 35 * 5(x - 7) = (5 · x) - (5 · 7) = 5x - 35</td>
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<td>Converse of the distributive property examples:</td>
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<tr>
<td>* 5x + 35 = (5 · x) + (5 · 7) = 5(x + 7), or * 5x - 35 = (5 · x) - (5 · 7) = 5(x - 7)</td>
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<tr>
<td>o The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division.</td>
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<tr>
<td>o The identity property of addition (additive identity property) states that the sum of any real number and zero is equal to the given real number (e.g., (\frac{5}{2} + 0 = \frac{5}{2})).</td>
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<tr>
<td>o The identity property of multiplication (multiplicative identity property) states that the product of any real number and one is equal to the given real number (e.g., (\sqrt{8} \cdot 1 = \sqrt{8})).</td>
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<tr>
<td>o Inverses are numbers that combine with other numbers and result in identity elements.</td>
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The student will:

a) solve multistep linear equations in one variable with the variable on one and both two-sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable;  

b) solve two-step linear inequalities and graph the results on a number line; and [Moved to 8.18.7.13] 
c) identify properties of operations used to solve an equation. [Incorporated into 8.14, 8.17 and 8.18 EKS]

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<tr>
<td>{ e.g., 5 + ( -5 ) = 0; 5 - 5 = 1 }</td>
<td>The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., ( 5 - (-5) = 0 )).</td>
<td>[ e.g., 5 + ( -5 ) = 0; 5 - 5 = 1 ]</td>
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<tr>
<td>[ e.g., 5 + ( -5 ) = 0; ] [ 5 - 5 = 1 ]</td>
<td>The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., ( \frac{1}{2} \cdot \frac{2}{1} = 1 )).</td>
<td>Zero has no multiplicative inverse.</td>
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<tr>
<td>The multiplicative property of zero states that the product of any real number and zero is zero (e.g., ( 7 \cdot 0 = 0 ) and ( 0 \cdot 7 = 0 )).</td>
<td>Division by zero is not a possible mathematical arithmetic operation. It is undefined.</td>
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<tr>
<td>The addition property of equality states that any number must be added to both sides of an equation to maintain equality.</td>
<td>The subtraction property of equality states that any number must be subtracted from both sides of an equation to maintain equality.</td>
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<tr>
<td>The multiplication property of equality states that an equation must be multiplied on both sides by the same number to maintain equality.</td>
<td>The division property of equality states that an equation must be divided on both sides by the same number to maintain equality.</td>
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<tr>
<td>The substitution property of equality states if two quantities are equivalent then one quantity can replace the other in an equation or inequality (e.g., if ( x = 5 + 8 ) then ( x = 3 ). Thus, 3 was substituted for 5 + 8 since they are equivalent).</td>
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</tbody>
</table>
8.1715 The student will
a) solve multistep linear equations in one variable with the variable on one and both two-sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable;

b) solve two-step linear inequalities and graph the results on a number line; and [Moved to 8.18.13]
c) identify properties of operations used to solve an equation. [Incorporated into 8.14, 8.17 and 8.18 EKS]

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<td>- Commutative property of addition: ( a + b = b + a )</td>
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<tr>
<td>- Commutative property of multiplication: ( a \cdot b = b \cdot a )</td>
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<td>- Associative property of addition: ( (a + b) + c = a + (b + c) )</td>
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<tr>
<td>- Associative property of multiplication: ( (a \cdot b) \cdot c = a \cdot (b \cdot c) )</td>
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<tr>
<td>- Subtraction and division are neither commutative nor associative</td>
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<tr>
<td>- Distributive property (over addition/subtraction): ( a \cdot (b + c) = a \cdot b + a \cdot c ) and ( a \cdot (b - c) = a \cdot b - a \cdot c )</td>
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<tr>
<td>- The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division.</td>
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<td>- Identity property of addition (additive identity property): ( a + 0 = a ) and ( 0 + a = a )</td>
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<td>- Identity property of multiplication (multiplicative identity property): ( a \cdot 1 = a ) and ( 1 \cdot a = a )</td>
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<tr>
<td>- Inverses are numbers that combine with other numbers and result in identity elements [e.g., ( 5 + (-5) = 0 ), ( \frac{1}{2} \cdot 5 = 1 )]</td>
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<tr>
<td>- Inverse property of addition (additive inverse property): ( a + (-a) = 0 ) and ( (-a) + a = 0 )</td>
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<td>- Inverse property of multiplication (multiplicative inverse property): ( a \cdot \frac{1}{a} = 1 ) and ( \frac{1}{a} \cdot a = 1 )</td>
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<td>- Zero has no multiplicative inverse</td>
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<td>- Multiplicative property of zero: ( a \cdot 0 = 0 ) and ( 0 \cdot a = 0 )</td>
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8.1715 The student will
a) solve multistep linear equations in one variable with the variable on one and both two sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable;

b) solve two-step linear inequalities and graph the results on a number line; and [Moved to 8.18.13]
c) identify properties of operations used to solve an equation. [Incorporated into 8.14, 8.17 and 8.18 EKS]

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<td>- Division by zero is not a possible mathematical operation. It is undefined.</td>
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<tr>
<td>- Substitution property: if ( a = b ), then ( b ) can be substituted for ( a ) in any expression, equation, or inequality.</td>
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<td>- Addition property of equality: if ( a = b ), then ( a + c = b + c ).</td>
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<td>- Subtraction property of equality: if ( a = b ), then ( a - c = b - c ).</td>
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<tr>
<td>- Multiplication property of equality: if ( a = b ), then ( a \cdot c = b \cdot c ).</td>
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<tr>
<td>- Division property of equality: if ( a = b ) and ( c \neq 0 ), then ( \frac{a}{c} = \frac{b}{c} ).</td>
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<tr>
<td>- Combining like terms means to combine terms that have the same variable and the same exponent (e.g., ( 8x + 11 - 3x ) can be ( 5x + 11 ) by combining the like terms of ( 8x ) and ( -3x )). [Reordered and combined in other bullets]</td>
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</table>
8.18 The student will solve multistep linear inequalities in one variable with the variable on one and both sides of the inequality symbol, including practical problems, and graph the results on a number line. [Moved from 8.15b]

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| • A multistep inequality may include, but not be limited to inequalities such as the following:  
  \[2x + 1 \geq \frac{x}{2} - 3(2x + 7) \leq \frac{1}{2}x; 2x + 7 - 5x < 27; -5x - (x + 3) > -12.\]  
• When both expressions of an inequality are multiplied or divided by a negative number, the inequality sign reverses. [Moved from 8.15]  
• A solution to an inequality is the value or set of values that can be substituted to make the inequality true.  
• In an inequality, there can be more than one value for the variable that makes the inequality true. There can be many solutions. (i.e., \(x + 4 > 7\) then the solution is \(x > 3\). This means that \(x\) can be any number greater than 3. A few solutions might be 6.5, 3, 0, 4, 25, etc.)  
• Real-world problems can be modeled and solved using linear inequalities.  
• The properties of real numbers and properties of inequality can be used to solve inequalities, justify solutions, and express simplification. Students should utilize the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of \(a\), \(b\), or \(c\) in this standard).  
  The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 5.6 + 4 = 4 + 5.6).  
  The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 5.6 · 4 = 4 · 5.6).  
  The associative property of addition states that regrouping | • Apply properties of real numbers and properties of inequality to solve multistep linear inequalities (up to four steps) in one variable. Coefficients and numeric terms in these equations will be rational. Inequalities may contain expressions that need to be expanded (using the distributive property) or require collecting like terms to solve.  
• Graph solutions to multistep linear inequalities on a number line.  
• Translate verbal statements, including practical situations, into algebraic inequalities.  
• Write verbal expressions and sentences as algebraic expressions and inequalities.  
• Write algebraic expressions and inequalities as verbal expressions and sentences.  
• Solve practical problems involving that require the solution of a multistep linear inequality in one variable.  
• Identify a numerical value(s) that is part of the solution set of a given inequality. |
The student will solve multistep linear inequalities in one variable with the variable on one and both sides of the inequality symbol, including practical problems, and graph the solution on a number line. [Moved from 8.15b]

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<td><strong>number line.</strong></td>
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The associative property of multiplication states that regrouping the factors does not change the product:

\[ (a \cdot b) \cdot c = a \cdot (b \cdot c) \]

The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number. The converse of this property is also true:

**Distributive property examples:**
- \(5(x + 7) = (5 \cdot x) + (5 \cdot 7) = 5x + 35\)
- \(5(x - 7) = (5 \cdot x) - (5 \cdot 7) = 5x - 35\)

**Converse of the distributive property examples:**
- \(5x + 35 = (5 \cdot x) + (5 \cdot 7) = 5(x + 7)\) or
- \(5x - 35 = (5 \cdot x) - (5 \cdot 7) = 5(x - 7)\)

The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division.

**The identity property of addition (additive identity property)** states that the sum of any real number and zero is equal to the given real number (e.g., \(x + 0 = x\)).

**The identity property of multiplication (multiplicative identity property)** states that the product of any real number and one is equal to the given real number (e.g., \(\sqrt{2} \cdot 1 = \sqrt{2}\)).

Inverses are numbers that combine with other numbers and result in identity elements.
The student will solve multistep linear inequalities in one variable with the variable on one and both sides of the inequality symbol, including practical problems, and graph the solution on a number line. [Moved from 8.15b]

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<td>[5 + (-5) = 0] [5 + (-5) = 0]</td>
<td>The inverse property of addition (additive inverse property) states that the sum of a number and its additive inverse always equals zero (e.g., 5.6 + (-5.6) = 0).</td>
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</tr>
<tr>
<td>The inverse property of multiplication (multiplicative inverse property) states that the product of a number and its multiplicative inverse (reciprocal) equals one (e.g., [\frac{1}{4} \times 4 = 1]).</td>
<td>Zero has no multiplicative inverse.</td>
<td></td>
</tr>
<tr>
<td>Division by zero is not a possible mathematical operation. It is undefined.</td>
<td>The addition property of inequality states that any number must be added to both sides of an inequality to maintain the inequality.</td>
<td></td>
</tr>
<tr>
<td>The subtraction property of inequality states that any number must be subtracted from both sides of an inequality to maintain inequality.</td>
<td>The multiplication property of inequality states that if an inequality is multiplied on both sides by the same positive number the inequality symbol remains the same. But if an inequality is multiplied on both sides by the same negative number, the inequality symbol reverses. For example, to solve [\frac{x}{2} &gt; 9], multiply both sides of the inequality by 3, so [x &gt; 27]. To solve [-\frac{x}{2} &gt; 9], multiply both sides of the inequality by 3, so [x &lt; -27].</td>
<td></td>
</tr>
<tr>
<td>The division property of inequality states that if an</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### UNDERSTANDING THE STANDARD

<table>
<thead>
<tr>
<th>(Background Information for Instructor Only)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inequality is divided on both sides by the same positive number, the inequality symbol remains the same. But if an inequality is divided on both sides by the same negative number, the inequality symbol reverses. For example: $-4y &lt; 20$. Divide both sides of the inequality by $-4$. So $y &gt; 5$.</td>
</tr>
<tr>
<td>- Commutative property of addition: $a + b = b + a$.</td>
</tr>
<tr>
<td>- Commutative property of multiplication: $a \cdot b = b \cdot a$.</td>
</tr>
<tr>
<td>- Associative property of addition: $(a + b) + c = a + (b + c)$.</td>
</tr>
<tr>
<td>- Associative property of multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.</td>
</tr>
<tr>
<td>- Subtraction and division are neither commutative nor associative.</td>
</tr>
<tr>
<td>- Distributive property (over addition/subtraction): $a \cdot (b + c) = a \cdot b + a \cdot c$ and $a \cdot (b - c) = a \cdot b - a \cdot c$.</td>
</tr>
<tr>
<td>- The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by 1 is the number. There are no identity elements for subtraction and division.</td>
</tr>
<tr>
<td>- Identity property of addition (additive identity property): $a + 0 = a$ and $0 + a = a$.</td>
</tr>
<tr>
<td>- Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a$ and $1 \cdot a = a$.</td>
</tr>
<tr>
<td>- Inverses are numbers that combine with other numbers and result in identity elements [e.g., $5 + (-5) = 0$; $\frac{1}{2} \cdot 5 = 1$].</td>
</tr>
<tr>
<td>- Inverse property of addition (additive inverse property): $a + (-a) = 0$ and $(-a) + a = 0$.</td>
</tr>
<tr>
<td>- Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$.</td>
</tr>
</tbody>
</table>

### ESSENTIAL UNDERSTANDINGS

### ESSENTIAL KNOWLEDGE AND SKILLS
The student will solve multistep linear inequalities in one variable with the variable on one and both sides of the inequality symbol, including practical problems, and graph the results on a number line. [Moved from 8.15b]

### UNDERSTANDING THE STANDARD
(Background Information for Instructor Only)

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplicative property of zero:</td>
<td>$a \cdot 0 = 0 \text{ and } 0 \cdot a = 0$.</td>
</tr>
<tr>
<td>Division by zero</td>
<td>Is not a possible mathematical operation. It is undefined.</td>
</tr>
<tr>
<td>Substitution property</td>
<td>If $a = b$, then $b$ can be substituted for $a$ in any expression, equation, or inequality.</td>
</tr>
<tr>
<td>Addition property of inequality</td>
<td>If $a &lt; b$, then $a + c &lt; b + c$; if $a &gt; b$, then $a + c &gt; b + c$.</td>
</tr>
<tr>
<td>Subtraction property of inequality</td>
<td>If $a &lt; b$, then $a - c &lt; b - c$; if $a &gt; b$, then $a - c &gt; b - c$.</td>
</tr>
<tr>
<td>Multiplication property of inequality</td>
<td>If $a &lt; b$ and $c &gt; 0$, then $a \cdot c &lt; b \cdot c$; if $a &gt; b$ and $c &gt; 0$, then $a \cdot c &gt; b \cdot c$.</td>
</tr>
<tr>
<td>Multiplication property of inequality (multiplication by a negative number)</td>
<td>If $a &lt; b$ and $c &lt; 0$, then $a \cdot c &gt; b \cdot c$; if $a &gt; b$ and $c &lt; 0$, then $a \cdot c &lt; b \cdot c$.</td>
</tr>
<tr>
<td>Division property of inequality</td>
<td>If $a &lt; b$ and $c &gt; 0$, then $\frac{a}{c} &lt; \frac{b}{c}$; if $a &gt; b$ and $c &gt; 0$, then $\frac{a}{c} &gt; \frac{b}{c}$.</td>
</tr>
<tr>
<td>Division property of inequality (division by a negative number)</td>
<td>If $a &lt; b$ and $c &lt; 0$, then $\frac{a}{c} &gt; \frac{b}{c}$; if $a &gt; b$ and $c &lt; 0$, then $\frac{a}{c} &lt; \frac{b}{c}$.</td>
</tr>
</tbody>
</table>

- **The substitution property of equality states** if two quantities are equivalent then one quantity can replace the other in an equation or inequality (e.g., if $x = 5 + 8$ then $x = 13$. Thus, 13 was substituted for $5 + 8$ since they are equivalent).
STANDARD 8.17  STRAND: PATTERNS, FUNCTIONS, AND ALGEBRA  GRADE LEVEL 8

8.17 The student will identify the domain, range, independent variable, or dependent variable in a given situation.

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The domain is the set of all the input values for the independent variable in a given situation.</td>
<td>• What are the similarities and differences among the terms domain, range, independent variable and dependent variable? The value of the dependent variable changes as the independent variable changes. The domain is the set of all input values for the independent variable. The range is the set of all possible values for the dependent variable.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• The range is the set of all the output values for the dependent variable in a given situation.</td>
<td></td>
<td>• Apply the following algebraic terms appropriately: domain, range, independent variable, and dependent variable.</td>
</tr>
<tr>
<td>• The independent variable is the input value.</td>
<td></td>
<td>• Identify examples of domain, range, independent variable, and dependent variable.</td>
</tr>
<tr>
<td>• The dependent variable depends on the independent variable and is the output value.</td>
<td></td>
<td>• Determine the domain of a function.</td>
</tr>
<tr>
<td>• Below is a table of values for finding the circumference of circles, $C = \pi d$, where the value of $\pi$ is approximated as 3.14.</td>
<td></td>
<td>• Determine the range of a function.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Diameter</th>
<th>Circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 in.</td>
<td>3.14 in.</td>
</tr>
<tr>
<td>2 in.</td>
<td>6.28 in.</td>
</tr>
<tr>
<td>3 in.</td>
<td>9.42 in.</td>
</tr>
<tr>
<td>4 in.</td>
<td>12.56 in.</td>
</tr>
</tbody>
</table>

• The independent variable, or input, is the diameter of the circle. The values for the diameter make up the domain.

• The dependent variable, or output, is the circumference of the circle. The set of values for the circumference makes up the range.
Algebra I

Board of Education
Commonwealth of Virginia
Introduction

The 2009-2016 Mathematics Standards of Learning Curriculum Framework is a companion document to the 2009-2016 Mathematics Standards of Learning and amplifies the Mathematics Standards of Learning by further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. It contains background information for the teacher (K-8). The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that What each student should know and be able to demonstrate in each standard is outlined. This is not meant to be an exhaustive list of student expectations nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise. [Reordered and rewritten]
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
**Algebra Readiness**

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

**Equity**

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
**ALGEBRA I**

**STANDARD A.1**

The student will

a) represent verbal quantitative situations algebraically; and

b) evaluate these algebraic expressions for given replacement values of the variables.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Algebra is a tool for reasoning about quantitative situations so that relationships become apparent. [Moved to introduction]</td>
<td></td>
</tr>
<tr>
<td>• Algebra is a tool for describing and representing patterns and relationships. [Moved to introduction]</td>
<td></td>
</tr>
<tr>
<td>• Mathematical modeling involves creating algebraic representations of quantitative real-world practical situations.</td>
<td></td>
</tr>
<tr>
<td>• The numerical value of an expression is dependent upon the values of the replacement set for the variables.</td>
<td></td>
</tr>
<tr>
<td>• There are a variety of ways to compute the value of a numerical expression and evaluate an algebraic expression using order of operations.</td>
<td></td>
</tr>
<tr>
<td>• The operations and the magnitude of the numbers in an expression impact the choice of an appropriate computational technique (mental mathematics, calculator, or paper and pencil).</td>
<td></td>
</tr>
</tbody>
</table>

An appropriate computational technique could be mental mathematics, calculator, or paper and pencil.

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

• Translate between verbal quantitative situations into and algebraic expressions and equations and vice versa. (a) |

• Model real-world practical situations with algebraic expressions in a variety of representations (concrete, pictorial, symbolic, verbal). (a) |

• Evaluate algebraic expressions using the order of operations that include absolute value, square roots, and cube roots for a given replacement set of values to include rational numbers, without rationalizing the denominator. (b) |

• Evaluate expressions that contain absolute value, square roots, and cube roots.
**ALGEBRA I**

**STANDARD A.2**

The student will perform operations on polynomials, including
- a) applying the laws of exponents to perform operations on expressions;
- b) adding, subtracting, multiplying, and dividing polynomials; and
- c) factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations. [Moved to EKS]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>- The laws of exponents can be investigated using inductive reasoning.</td>
<td></td>
</tr>
<tr>
<td>- A relationship exists between the laws of exponents and scientific notation. [Rewritten and reordered]</td>
<td></td>
</tr>
<tr>
<td>- Operations with polynomials can be represented concretely, pictorially, and symbolically.</td>
<td></td>
</tr>
<tr>
<td>- Polynomial expressions can be used to model real-world practical situations.</td>
<td></td>
</tr>
<tr>
<td>- The distributive property is the unifying concept for polynomial operations.</td>
<td></td>
</tr>
<tr>
<td>- Factoring reverses polynomial multiplication.</td>
<td></td>
</tr>
<tr>
<td>- Trinomials may be factored by various methods including factoring by grouping.</td>
<td></td>
</tr>
<tr>
<td>- Example of factoring by grouping</td>
<td></td>
</tr>
<tr>
<td>[2x^2 + 5x - 3]</td>
<td></td>
</tr>
<tr>
<td>[2x^2 + 6x - x - 3]</td>
<td></td>
</tr>
<tr>
<td>[2x(x + 3) - (x + 3)]</td>
<td></td>
</tr>
<tr>
<td>[(x + 3)(2x - 1)]</td>
<td></td>
</tr>
<tr>
<td>- Some polynomials are prime polynomials and cannot be factored</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>- Simplify monomial expressions and ratios of monomial expressions in which the exponents are integers, using the laws of exponents. (a)</td>
</tr>
<tr>
<td>- Model sums, differences, products, and quotients of polynomials with concrete objects and their related pictorial and symbolic representations. (b)</td>
</tr>
<tr>
<td>- Relate concrete and pictorial manipulations that model polynomial operations to their corresponding symbolic representations.</td>
</tr>
<tr>
<td>- Find-Determine sums and differences of polynomials. (b)</td>
</tr>
<tr>
<td>- Find-Determine products of polynomials. The factors will have no more than five total terms should be limited to five or fewer terms (i.e., ((4x^2+2)(3x+5)) represents four terms and ((x+1)(2x^2+x+3)) represents five terms). (b)</td>
</tr>
<tr>
<td>- Find-Determine the quotient of polynomials, using a monomial or binomial divisor, or a completely factored divisor. (b)</td>
</tr>
<tr>
<td>- Factor completely first- and second-degree polynomials in one...</td>
</tr>
</tbody>
</table>
# ALGEBRA I  
## STANDARD A.2  

The student will perform operations on polynomials, including  
- a) applying the laws of exponents to perform operations on expressions;  
- b) adding, subtracting, multiplying, and dividing polynomials; and  
- c) factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations. [Moved to EKS]

## ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

| | over the set of real numbers, integers into two or more factors, each of lesser degree than the original polynomial.  
| | Polynomial expressions can be used to define functions and these functions can be represented graphically.  
| | There is a relationship between the factors of any polynomial and the x-intercepts of the graph of its related function. [Moved to A.7 US]  
| | The laws of exponents can be applied to perform operations involving numbers written in scientific notation. [Rewritten and reordered]  
| | For division of polynomials in this standard, instruction on the use of long or synthetic division is not required, but students may benefit from experiences with these methods, which become more useful and prevalent in the study of advanced levels of algebra. |

## ESSENTIAL KNOWLEDGE AND SKILLS

| | variable with integral coefficients. After factoring out the greatest common factor (GCF), leading coefficients should have no more than four factors. (c)  
| | Factor and verify algebraic factorizations of polynomials with a graphing utility. (c) [Moved from standard]  
| | Identify prime polynomials.  
| | Use the x-intercepts from the graphical representation of the polynomial to determine and confirm its factors. [Moved to A.7d EKS] |
ALGEBRA I
STANDARD A.3

The student will simplify express the
a) square roots of non-negative rational whole numbers and monomial algebraic expressions;
b) cube roots of integers whole rational numbers; and the square root of a monomial algebraic expression in simplest radical form

c) numerical expressions containing square or cube roots.

### ESSENTIAL UNDERSTANDINGS

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A radical expression in Algebra I contains the square root symbol (\sqrt{_}) or the cube root symbol (\sqrt[3]{_}).</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>A square root of a number (a) is a number (y) such that (y^2 = a).</td>
<td>Express the square roots of a whole non-negative rational whole number in simplest form. (a)</td>
</tr>
<tr>
<td>A cube root of a number (b) is a number (y) such that (y^3 = b).</td>
<td>Express the principal square root of a monomial algebraic expression in simplest form where variables are assumed to have positive values. (a) [Reordered]</td>
</tr>
<tr>
<td>A square root in simplest form is one in which the radicand (argument) has no perfect square factors other than one.</td>
<td>Express the cube root of an integer whole rational number in simplest form. (b)</td>
</tr>
<tr>
<td>The inverse of squaring a number is determining the square root.</td>
<td>Simplify a numerical expression containing square or cube roots. (c)</td>
</tr>
<tr>
<td>Any non-negative number other than a perfect square has a principal square root that lies between two consecutive whole numbers.</td>
<td>Add, subtract, and multiply, and divide two monomial radical expressions limited to a numerical radicand without requiring rationalizing the denominator. (c)</td>
</tr>
<tr>
<td>A cube root in simplest form is one in which the radicand (argument) has no perfect cube factors other than one.</td>
<td>Express the principal square root of a monomial algebraic expression in simplest form where variables are assumed to have positive values. [Reordered]</td>
</tr>
<tr>
<td>The cube root of a perfect cube is an integer.</td>
<td></td>
</tr>
<tr>
<td>The cube root of a nonperfect cube lies between two consecutive integers.</td>
<td></td>
</tr>
<tr>
<td>The inverse of cubing a number is determining the cube root.</td>
<td></td>
</tr>
</tbody>
</table>

In the real number system, the argument of a square root must be nonnegative while the argument of a cube root may be any real number.
**ALGEBRA I**

**STANDARD A.4**

The student will solve multistep linear and quadratic equations in two variables, including:

- **a)** multistep linear equations in one variable algebraically; solving literal equations (formulas) for a given variable;
- **b)** justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets; solving quadratic equations in one variable algebraically and graphically;
- **c)** solving literal equations for a specified variable; multistep linear equations algebraically and graphically;
- **d)** solving systems of two linear equations in two variables algebraically and graphically; and
- **e)** solving real-world practical problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions. [Included in EKS]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>• A solution to an equation is the value or set of values that can be substituted to make the equation true.</td>
<td></td>
</tr>
<tr>
<td>• Each point on the graph of a linear or quadratic equation in two variables is a solution of the equation.</td>
<td></td>
</tr>
<tr>
<td>• The solution of an equation in one variable can be found by graphing the expression on each side of the equation separately and finding the x-coordinate of the point of intersection.</td>
<td></td>
</tr>
<tr>
<td>• Real-world practical problems may be interpreted, represented, and solved using linear and quadratic equations.</td>
<td></td>
</tr>
<tr>
<td>• The process of solving linear and quadratic equations can be modeled in a variety of ways, using concrete, pictorial, and symbolic representations.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:</td>
</tr>
<tr>
<td>• Determine if a linear equation in one variable has one, an infinite number, or no solutions. (a)</td>
</tr>
<tr>
<td>• Solve a literal equation (formula) for a specified variable. [Reordered]</td>
</tr>
<tr>
<td>• Simplify expressions and solve equations, using field properties of the real numbers and properties of equality to justify simplification. Simplify expressions and solve equations (a,b). [Moved from A.4b]</td>
</tr>
<tr>
<td>• Solve multistep linear equations in one variable algebraically. (a)[Reordered]</td>
</tr>
<tr>
<td>• Solve quadratic equations in one variable algebraically. Solutions may be rational or irrational and given in simplest form. (b)</td>
</tr>
</tbody>
</table>
## ALGEBRA I
### STANDARD A.4

The student will solve multistep linear and quadratic equations in two variables, including

- **a)** multistep linear equations in one variable algebraically [Reordered];
- **b)** justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets [Moved to EKS];
- **c)** solving quadratic equations in one variable algebraically and graphically;
- **d)** solving literal equations for a specified variable [Reordered];
- **e)** solving systems of two linear equations in two variables algebraically and graphically; and
- **f)** solving real-world practical problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions. [Included in EKS]

### ESSENTIAL UNDERSTANDINGS

**UNDERSTANDING THE STANDARD**

- Properties of real numbers and properties of equality can be used to justify equation solutions and expression simplification.
- Field Properties of Real Numbers:
  - **Associative Property of Addition**
  - **Associative Property of Multiplication**
  - **Commutative Property of Addition**
  - **Commutative Property of Multiplication**
  - **Identity Property of Addition (Additive Identity)**
  - **Identity Property of Multiplication (Multiplicative Identity)**
  - **Inverse Property of Addition (Additive Inverse)**
  - **Inverse Property of Multiplication (Multiplicative Inverse)**
  - **Distributive Property**

### ESSENTIAL KNOWLEDGE AND SKILLS

- Confirm algebraic solutions to quadratic equations graphically. (b) [Reordered]
- Identify the roots or zeros of a quadratic function over the real number system as the solution(s) to the quadratic equation that is formed by setting the given quadratic expression equal to zero. [Included in A.7c]
- Solve multistep linear equations in one variable. [Reordered]
- Confirm algebraic solutions to linear and quadratic equations using a graphing calculator. [Reordered] [Included in A.7d]
- Solve a literal equation for a specified variable. (c) [Reordered]
- Given a system of two linear equations in two variables that has a unique solution, solve the system by substitution or elimination to find the ordered pair which satisfies both equations. (d)
- Given a system of two linear equations in two variables that has a
ALGEBRA I
STANDARD A.4

The student will solve multistep linear and quadratic equations in two variables, including

a) multistep linear equations in one variable algebraically; solving literal equations (formulas) for a given variable;

b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;

c) solving quadratic equations in one variable algebraically and graphically;

d) solving literal equations for a specified variable;

e) solving systems of two linear equations in two variables algebraically and graphically; and

f) solving real-world practical problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

ESSENTIAL UNDERSTANDING

UNDERSTANDING THE STANDARD

- Properties of Equality
  - Multiplicative Property of Zero
  - Zero Product Property
  - Reflexive Property
  - Symmetric Property
  - Transitive Property of Equality
  - Addition Property of Equality
  - Subtraction Property of Equality
  - Multiplication Property of Equality
  - Division Property of Equality
  - Substitution

- Quadratic equations in one variable may be solved algebraically by factoring and applying properties of equality or by using the quadratic formula over the set of real numbers (Algebra I) or set of complex numbers (Algebra II).

ESSENTIAL KNOWLEDGE AND SKILLS

- unique solution, solve the system graphically by identifying the point of intersection. (d)

- Solve and confirm algebraic solutions to a system of two linear equations using a graphing utility. (d)

- Determine whether a system of two linear equations has one solution, an infinite number, or no solutions, or infinite solutions. (d)

- Write a system of two linear equations that models a real-world practical situation. (e)

- Interpret and determine the reasonableness of the algebraic or graphical solution of a system of two linear equations that models a real-world practical situation. (e)

- Solve practical problems involving equations and systems of equations. (e)
**ALGEBRA I**

**STANDARD A.4**

The student will solve **multistep linear and quadratic equations in two variables**, including:

a) **multistep linear equations in one variable algebraically** [Reordered] solving literal equations (formulas) for a given variable [Reordered];

b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets [Moved to EKS];

c) solving quadratic equations **in one variable** algebraically and graphically;

d) solving literal equations for a specified variable [Reordered] multistep linear equations algebraically and graphically [Reordered];

e) solving systems of two linear equations in two variables algebraically and graphically; and

f) solving real-world practical problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions. [Included in EKS]

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**ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD**

- The zero(s), root(s), or the x-intercept(s) of a quadratic function are the real root(s) or solution(s) of the quadratic equation that is formed by setting the given algebraic quadratic expression of the function equal to zero.

- The zero(s), root(s), or x-intercept(s) of a function are the values of x when f(x) = 0.

- Literal equations include formulas.

- A system of linear equations with exactly one solution is characterized by the graphs of two lines whose intersection is a single point, and the coordinates of this point satisfy both equations.

- A system of two linear equations with no solution is characterized by the graphs of two lines that are parallel that do not intersect.

- A system of two linear equations having infinite solutions is

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**ESSENTIAL KNOWLEDGE AND SKILLS**

- Determine if a linear equation in one variable has one, an infinite number, or no solutions. [Reordered]²

---²Revised March 2011
**ALGEBRA I**

**STANDARD A.4**

The student will solve multistep linear and quadratic equations in two variables, including

- a) multistep linear equations in one variable algebraically [Reordered];
- b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets [Moved to EKS];
- c) solving quadratic equations in one variable algebraically and graphically;
- d) solving literal equations for a specified variable [Reordered];
- e) solving systems of two linear equations in two variables algebraically and graphically; and
- f) solving real-world practical problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions. [Included in EKS]

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**ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD**

- characterized by two graphs that coincide (the graphs will appear to be the graph of one line), and the coordinates of all points on the line satisfy both equations. These lines will have the same slope and y-intercept.

- Systems of two linear equations can be used to model two real-world practical conditions that must be satisfied simultaneously.

- Equations and systems of equations can be used as mathematical models for real-world practical situations.

- The domain of a function may be restricted algebraically, graphically, or by the practical situation modeled by a function.

- Solutions and intervals may be expressed in different formats, including set notation or using equations and inequalities.

Examples may include:
ALGEBRA I
STANDARD A.4

The student will solve multistep linear and quadratic equations in two variables, including
a) multistep linear equations in one variable algebraically; solving literal equations (formulas) for a given variable; solving literal equations (formulas) for a given variable;
b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;
c) solving quadratic equations in one variable algebraically and graphically;
d) solving literal equations for a specified variable; multistep linear equations algebraically and graphically;
ed) solving systems of two linear equations in two variables algebraically and graphically; and
f) solving real-world practical problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

<table>
<thead>
<tr>
<th>Equation/ Inequality</th>
<th>Set Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>x = 3</td>
<td>{3}</td>
</tr>
<tr>
<td>x = 3 or x = 5</td>
<td>{3, 5}</td>
</tr>
<tr>
<td>0 ≤ x &lt; 3</td>
<td>{x</td>
</tr>
<tr>
<td>y ≥ 3</td>
<td>{y</td>
</tr>
<tr>
<td>Empty (null) set Ø</td>
<td>{}</td>
</tr>
</tbody>
</table>
**ALGEBRA I**  
**STANDARD A.5**

The student will solve multistep linear inequalities in two variables, including:

- a) solving multistep linear inequalities in one variable algebraically and represent the solution graphically;
- b) represent the solution of linear inequalities in two variables algebraically [Moved from SOL stem] and graphically [Moved from A.6] justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets [Included in EKS];
- c) solving real-world practical problems involving inequalities; and
- d) represent the solution to solving systems of inequalities 

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<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
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</thead>
<tbody>
<tr>
<td>• A solution to an inequality is the value or set of values that can be substituted to make the inequality true.</td>
<td></td>
</tr>
<tr>
<td>• The graph of the solutions of a linear inequality is a half-plane bounded by the graph of its related linear equation. Points on the boundary are included unless the inequality contains only &lt; or &gt; (no equality condition) it is a strict inequality.</td>
<td></td>
</tr>
<tr>
<td>• Real-world practical problems can be modeled and solved using linear inequalities.</td>
<td></td>
</tr>
<tr>
<td>• Properties of inequality and order can be used to solve inequalities.</td>
<td></td>
</tr>
<tr>
<td>• Set builder notation may be used to represent solution sets of inequalities.</td>
<td></td>
</tr>
<tr>
<td>• The domain of a function may be restricted algebraically, graphically, or by the practical situation modeled by a function.</td>
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<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
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<tr>
<td>• Solve multistep linear inequalities in one variable algebraically and represent the solution graphically. (a)</td>
</tr>
<tr>
<td>• Apply the field properties of real numbers and properties of inequality to solve multistep linear inequalities in one variable algebraically. (a) Justify steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers.</td>
</tr>
<tr>
<td>• Represent the solution of a linear inequality in two variables algebraically and graphically. (b)</td>
</tr>
<tr>
<td>• Solve real-world practical problems involving linear inequalities. (c)</td>
</tr>
<tr>
<td>• Determine if a coordinate pair is a solution of a linear inequality or a system of linear inequalities. (c)</td>
</tr>
<tr>
<td>• Represent the solution of solving systems of two linear inequalities algebraically and graphically. (including practical problems)</td>
</tr>
</tbody>
</table>
ALGEBRA I
STANDARD A.5

The student will solve multistep linear inequalities in two variables, including
a) solving multistep linear inequalities in one variable algebraically and represent the solution graphically;
b) represent the solution of linear inequalities in two variables algebraically and graphically [Moved from SOL stem] justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets [Included in EKS];
c) solving real-world practical problems involving inequalities; and
d) represent the solution to a solving systems of inequalities algebraically and graphically.

ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

- Solutions and intervals may be expressed in different formats, including set notation or using equations and inequalities. Examples may include:

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<td>Empty (null) set Ø</td>
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</table>

- Field Properties of Real Numbers:
  - Associative Property of Addition
  - Associative Property of Multiplication
  - Commutative Property of Addition
  - Commutative Property of Multiplication
  - Identity Property of Addition (Additive Identity)
  - Identity Property of Multiplication (Multiplicative Identity)
  - Inverse Property of Addition (Additive Inverse)
  - Inverse Property of Multiplication (Multiplicative Inverse)

ESSENTIAL KNOWLEDGE AND SKILLS

- Determine and verify algebraic solutions using a graphing utility. (a,b,c,d)
## ALGEBRA I
### STANDARD A.5

The student will solve multistep linear inequalities in two variables, including:

a) solving multistep linear inequalities in one variable algebraically and represent the solution graphically;
b) represent the solution of linear inequalities in two variables algebraically (Moved from SOL stem) and graphically [Moved from A.6] justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets [Included in EKS];
c) solving real-world practical problems involving inequalities; and
d) represent the solution to a solving systems of inequalities algebraically and graphically.

### ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

- Distributive Property
- Properties of Inequality
  - Transitive Property of Inequality
  - Addition Property of Inequality
  - Subtraction Property of Inequality
  - Multiplication Property of Inequality
  - Division Property of Inequality
  - Substitution

### ESSENTIAL KNOWLEDGE AND SKILLS
## ALGEBRA I

### STANDARD A.6

The student will graph linear equations and linear inequalities in two variables, including [Linear equations moved to A.6c and linear inequalities moved to A.5b]

**a)** determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. 
Slope will be described as rate of change and will be positive, negative, zero, or undefined [Included in US and EKS]; and

**b)** writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line; and

**c)** graph linear equations in two variables. [Moved from SOL stem]

### ESSENTIAL UNDERSTANDINGS

**UNDERSTANDING THE STANDARD**

- Changes in slope may be described by dilations or reflections or both.
- Changes in the y-intercept may be described by translations.
- Linear equations can be graphed using slope, x- and y-intercepts, and/or transformations of the parent function.
- The slope of a line represents a constant rate of change in the dependent variable when the independent variable changes by a constant amount.
- The equation of a line defines the relationship between two variables.
- The graph of a line represents the set of points that satisfies the equation of a line.
- A line can be represented by its graph or by an equation. *Students should have experiences writing equations of lines in various forms, including* written in $\text{Standard F}orm$, $\text{Slope-intercept F}orm$, or $\text{Point-Slope F}orm$.
- The graph of the solutions of a linear inequality is a half-plane bounded by the graph of its related linear equation. Points on the

### ESSENTIAL KNOWLEDGE AND SKILLS

**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**

- Graph linear equations and inequalities in two variables, including those that arise from a variety of real-world situations. [Reordered]
- Use the parent function $y=x$ and describe transformations defined by changes in the slope or y-intercept. [Reordered]
- Find Determine the slope of the line, given the equation of a linear function. (a)
- Find Determine the slope of a line, given the coordinates of two points on the line. (a)
- Find Determine the slope of a line, given the graph of a line. (a)
- Recognize and describe a line with a slope or rate of change that is positive, negative, zero, or undefined. (a)
- Use transformational graphing to investigate effects of changes in equation parameters on the graph of the equation.
- Write an equation of a line when given the graph of a line. (b)
### ALGEBRA I

**STANDARD A.6**

The student will graph linear equations and linear inequalities in two variables, including [Linear equations moved to A.6c and linear inequalities moved to A.5b]

- a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined [Included in US and EKS]; and
- b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line; and
- c) graph linear equations in two variables. [Moved from SOL stem]

### ESSENTIAL UNDERSTANDINGS

**UNDERSTANDING THE STANDARD**

<table>
<thead>
<tr>
<th>Boundary are included unless it is a strict inequality. [Moved to A.5 US]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel lines have equal slopes.</td>
</tr>
<tr>
<td>The product of the slopes of perpendicular lines is -1 unless one of the lines has an undefined slope.</td>
</tr>
<tr>
<td>Slope can be described as a rate of change and will be positive, negative, zero, or undefined. [Moved from A.6a]</td>
</tr>
</tbody>
</table>

### ESSENTIAL KNOWLEDGE AND SKILLS

- Write an equation of a line when given two points on the line whose coordinates are integers. (b)
- Write an equation of a line when given the slope and a point on the line whose coordinates are integers. (b)
- Write an equation of a vertical line as \( x = a \). (b)
- Write the equation of a horizontal line as \( y = c \). (b)
- Write the equation of a line parallel or perpendicular to a given line through a given point. (b)
- Graph a linear equation in two variables, including those that arise from a variety of practical situations. (c) [Reordered]
- Use the parent function \( y = x \) and describe transformations defined by changes in the slope or \( y \)-intercept. (c) [Reordered]
ALGEBRA I
STANDARD A.7

The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including:

- determining whether a relation is a function;
- domain and range;
- zeros of a function;
- \( x \)- and \( y \)-intercepts;
- finding the values of a function for elements in its domain; and
- making connections between any two and among multiple representations of functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic.

<table>
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<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
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<tbody>
<tr>
<td>A relation is a function if and only if each element in the domain is paired with a unique element of the range. [Reordered]</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to determine whether a relation, represented by a set of ordered pairs, a table, a mapping, or a graph is a function. (a)</td>
</tr>
<tr>
<td>Functions describe the relationship between two variables where each input is paired to a unique output.</td>
<td>Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically. Domains may be limited by problem context or in graphical representations. (b, c, d)</td>
</tr>
<tr>
<td>Function families consist of a parent function and all transformations of the parent function.</td>
<td>Use the ( x )-intercepts from the graphical representation of a quadratic function to determine and confirm its factors. (c, d) [Moved from A.2 EKS]</td>
</tr>
<tr>
<td>The domain of a function is the set of all possible values of the independent variable.</td>
<td>For each any value ( x ) in the domain of ( f ), find determine ( f(x) ). (e)</td>
</tr>
<tr>
<td>The range of a function is the set of all possible values of the dependent variable.</td>
<td>Represent relations and functions using verbal descriptions, tables, equations, and graphs concrete, verbal, numeric, graphic, and algebraic forms. Given one representation, students will be able to represent the relation in another form. (f)</td>
</tr>
<tr>
<td>For each ( x ) in the domain of ( f ), ( x ) is a member of the input of the function ( f ), ( f(x) ) is a member of the output of ( f ), and the ordered pair ( (x, f(x)) ) is a member of ( f ).</td>
<td>Detect patterns in data and represent arithmetic and geometric</td>
</tr>
</tbody>
</table>
ALGEBRA I
STANDARD A.7

The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including:

- determining whether a relation is a function;
- domain and range;
- zeros of a function;
- \( x \)- and \( y \)-intercepts;
- finding the values of a function for elements in its domain; and
- making connections between any two and among multiple representations of functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic.

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</table>
| Given a polynomial function \( f(x) \), the following statements are equivalent for any real number, \( k \), such that \( f(k) = 0 \):
  - \( k \) is a zero of the polynomial function \( f(x) \), located at \((k,0)\);
  - \((x - k)\) is a factor of \( f(x) \);
  - \( k \) is a solution or root of the polynomial equation \( f(x) = 0 \);
  - and
  - the point \((k,0)\) is an \( x \)-intercept for the graph of \( y = f(x) \).
| Investigate and analyze characteristics and multiple representations of functions with a graphing utility. (a,b,c,d,e,f) |
| The \( x \)-intercept is the point at which the graph of a relation or function intersects with the \( x \)-axis. It can be expressed as a value or a coordinate. |
| The \( y \)-intercept is the point at which the graph of a relation or function intersects with the \( y \)-axis. It can be expressed as a value or a coordinate. |
| The domain of a function may be restricted by the practical situation modeled by a function. |
| Solutions and intervals may be expressed in different formats, including set notation or using equations and inequalities. | patterns algebraically. (f) [Included in AFDA.1 and AII.5] |

Mathematics Standards of Learning Curriculum Framework 2009-2016: Algebra I
ALGEBRA I
STANDARD A.7

The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including:

a) determining whether a relation is a function;
b) domain and range;
c) zeros of a function;
d) $x$- and $y$-intercepts;
e) finding the values of a function for elements in its domain; and
f) making connections between any two and among multiple representations of functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic.

### ESSENTIAL UNDERSTANDINGS

**UNDERSTANDING THE STANDARD**

Examples may include:

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</tr>
<tr>
<td>Empty (null) set $\emptyset$</td>
<td>${}$</td>
</tr>
</tbody>
</table>

- In an arithmetic pattern, the common difference is the value that is added to obtain the next value in the pattern.
- A set of data may be characterized by patterns, and those patterns can be represented in multiple ways.
- Graphs can be used as visual representations to investigate relationships between quantitative data.
- Inductive reasoning may be used to make conjectures about characteristics of function families.
- Each element in the domain of a relation is the abscissa of a point of the graph of the relation.

### ESSENTIAL KNOWLEDGE AND SKILLS
### ALGEBRA I

**STANDARD A.7**

The student will investigate and analyze function—linear and quadratic—families and their characteristics both algebraically and graphically, including:

- a) determining whether a relation is a function;
- b) domain and range;
- c) zeros of a function;
- d) \(x\)- and \(y\)-intercepts;
- e) finding the values of a function for elements in its domain; and
- f) making connections between any two and among multiple and among multiple representations of functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic.

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<tbody>
<tr>
<td>• Each element in the range of a relation is the ordinate of a point of the graph of the relation.</td>
<td></td>
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<tr>
<td>• A relation is a function if and only if each element in the domain is paired with a unique element of the range. [Reordered]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The values of (f(x)) are the ordinates of the points of the graph of (f).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The object (f(x)) is the unique object in the range of the function (f) that is associated with the object (x) in the domain of (f).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• For each (x) in the domain of (f), (x) is a member of the input of the function (f), (f(x)) is a member of the output of (f), and the ordered pair ([x, f(x)]) is a member of (f).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• An object (x) in the domain of (f) is an (x)-intercept or a zero of a function (f) if and only if (f(x) = 0). [Reordered]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Set builder notation may be used to represent domain and range of a relation.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
ALGEBRA I
STANDARD A.8

The student, given a data set or practical situation, in a real-world context, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Practical problems may be represented and solved by using direct variation or inverse variations.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• A direct variation represents a proportional relationship between two quantities. The statement “y is directly proportional to x” is translated as $y = kx$.</td>
<td>• Given a data set or practical situation, including a real-world situation, determine whether a direct variation exists.</td>
</tr>
<tr>
<td>• The constant of proportionality ($k$) in a direct variation is represented by the ratio of the dependent variable to the independent variable and can be referred to as the constant of variation.</td>
<td>• Given a data set or practical situation, including a real-world situation, determine whether an inverse variation exists.</td>
</tr>
<tr>
<td>• A direct variation can be represented by a line passing through the origin.[Reordered]</td>
<td>• Given a data set or practical situation, write an equation for a direct variation, given a set of data.</td>
</tr>
<tr>
<td>• An inverse variation represents an inversely proportional relationship between two quantities. The statement “y is inversely proportional to x” is translated as $y = \frac{k}{x^2}$.</td>
<td>• Given a data set or practical situation, write an equation for an inverse variation, given a set of data.</td>
</tr>
<tr>
<td>• The constant of proportionality ($k$) in an inverse variation is represented by the product of the dependent variable and the independent variable and can be referred to as the constant of variation.</td>
<td>• Given a data set or practical situation, graph an equation representing a direct variation, given a set of data.</td>
</tr>
<tr>
<td>• The value of the constant of proportionality is typically positive when applied in practical situations.</td>
<td></td>
</tr>
<tr>
<td>• A direct variation can be represented by a line passing through the origin.</td>
<td></td>
</tr>
</tbody>
</table>

Mathematics Standards of Learning Curriculum Framework 2009-2016: Algebra I
**ALGEBRA I**  
**STANDARD A.8**

The student, given a **data set or practical situation, in a real-world context**, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGSUNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real-world problems may be modeled using direct and/or inverse variations.</td>
<td>Reordered</td>
</tr>
</tbody>
</table>

origin. [Reordered]
ALGEBRA I
STANDARD A.9 [Included in AFDA.7 and Algebra II.11]
The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Descriptive statistics may include measures of center and dispersion.</td>
<td>The student will use problem solving, mathematical communication,</td>
</tr>
<tr>
<td>• Variance, standard deviation, and mean absolute deviation measure the dispersion of the data.</td>
<td>mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• The sum of the deviations of data points from the mean of a data set is 0.</td>
<td>• Analyze descriptive statistics to determine the implications for the real-world situations from which the data derive.</td>
</tr>
<tr>
<td>• Standard deviation is expressed in the original units of measurement of the data.</td>
<td>• Given data, including data in a real-world context, calculate and interpret the mean absolute deviation of a data set.</td>
</tr>
<tr>
<td>• Standard deviation addresses the dispersion of data about the mean.</td>
<td>• Given data, including data in a real-world context, calculate variance and standard deviation of a data set and interpret the standard deviation.</td>
</tr>
<tr>
<td>• Standard deviation is calculated by taking the square root of the variance.</td>
<td>• Given data, including data in a real-world context, calculate and interpret z-scores for a data set.</td>
</tr>
<tr>
<td>• The greater the value of the standard deviation, the further the data tend to be dispersed from the mean.</td>
<td>• Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.</td>
</tr>
<tr>
<td>• For a data distribution with outliers, the mean absolute deviation may be a better measure of dispersion than the standard deviation or variance.</td>
<td>• Compare and contrast mean absolute deviation and standard deviation in a real-world context.</td>
</tr>
<tr>
<td>• A z-score (standard score) is a measure of position derived from the mean and standard deviation of data.</td>
<td></td>
</tr>
<tr>
<td>• A z-score derived from a particular data value tells how many standard deviations that data value is above or below the mean of the data set. It is positive if the data value lies above the mean and negative if the data value lies below the mean.</td>
<td></td>
</tr>
</tbody>
</table>
ALGEBRA I
STANDARD A.10 [Moved to 8.12]

The student will compare and contrast multiple univariate data sets, using box-and-whisker plots.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Statistical techniques can be used to organize, display, and compare sets of data.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning,</td>
</tr>
<tr>
<td>• Box-and-whisker plots can be used to analyze data.</td>
<td>connections, and representations to</td>
</tr>
<tr>
<td></td>
<td>• Compare, contrast, and analyze data, including data from real-world situations displayed</td>
</tr>
<tr>
<td></td>
<td>in box-and-whisker plots.</td>
</tr>
</tbody>
</table>
The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve real-world practical problems, using mathematical models of linear and quadratic functions. Mathematical models will include linear and quadratic functions.

### ESSENTIAL UNDERSTANDING UNDERSTANDING THE STANDARD

- Data and scatterplots may indicate patterns that can be modeled with an algebraic equation.
- Determining the curve of best fit for a relationship among a set of data points is a tool for algebraic analysis of data. In Algebra I, curves of best fit are limited to linear or quadratic functions.
- The curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate. [Reordered]
- Knowledge of transformational graphing using parent functions can be used to generate/verify a mathematical model from a scatterplot that approximates the data.
- The graphing calculator utilities can be used to collect, organize, represent, and generate determine the equation of a curve of best fit for a set of data.
- The curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate. [Reordered]
- Many problems can be solved by using a mathematical model as an interpretation of a real-world practical situation. The solution must then refer to the original real-world practical situation.
- Considerations such as sample size, randomness, and bias should...

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Write Determine an equation for a curve of best fit, using a graphing utility, given a set of no more than twenty data points in a table, a graph, or real-world practical situation.
- Make predictions about unknown outcomes, using data, scatterplots, or the equation of the curve of best fit.
- Solve practical problems involving an equation of the curve of best fit.
- Design experiments and collect data to address specific, real-world questions. [Included in AFDA.8]
- Evaluate the reasonableness of a mathematical model of a real-world situation.
- Evaluate the reasonableness of a mathematical model of a practical situation.
**TOPIC STRAND: STATISTICS**

**ALGEBRA I**  
**STANDARD A.9-11**

The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve real-world practical problems, using mathematical models of linear and quadratic functions. Mathematical models will include linear and quadratic functions.

<table>
<thead>
<tr>
<th><strong>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</strong></th>
<th><strong>ESSENTIAL KNOWLEDGE AND SKILLS</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>affect experimental design.</td>
<td></td>
</tr>
<tr>
<td>• Data that fit linear ((y = mx + b)) and quadratic ((y = ax^2 + bx + c)) functions arise from practical situations.</td>
<td></td>
</tr>
<tr>
<td>• Rounding that occurs during intermediate steps of problem solving may reduce the accuracy of the final answer.</td>
<td></td>
</tr>
<tr>
<td>• Evaluation of the reasonableness of a mathematical model of a practical situation involves asking questions including:</td>
<td></td>
</tr>
<tr>
<td>- “Is there another linear or quadratic curve that better fits the data?”</td>
<td></td>
</tr>
<tr>
<td>- “Does the curve of best fit make sense?”</td>
<td></td>
</tr>
<tr>
<td>- “Could the curve of best fit be used to make reasonable predictions?”</td>
<td></td>
</tr>
</tbody>
</table>
Mathematics Standards of Learning

Curriculum Framework 2009 2016

Geometry

Board of Education
Commonwealth of Virginia
Introduction

The 2009-2016 Mathematics Standards of Learning Curriculum Framework is a companion document to the 2009-2016 Mathematics Standards of Learning, and amplifies the Mathematics Standards of Learning by further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. The curriculum framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

**Essential Understandings**
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

**Understanding the Standard**
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. Background information for the teacher (K-8). It contains content. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers’ knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

**Essential Knowledge and Skills**
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that What each student should know and be able to demonstrate so in each standard is outlined. This is not meant to be an exhaustive list of student expectations or a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving
Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication
Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning
Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections
Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations
Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “...the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student’s problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
**TOPIC STRAND: REASONING, LINES, AND TRANSFORMATIONS**

**GEOMETRY**

**STANDARD G.1**

The student will use deductive reasoning to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include:

- identifying the converse, inverse, and contrapositive of a conditional statement;
- translating a short verbal argument into symbolic form; and
- determining the validity of a logical argument. [Added from EKS] using Venn diagrams to represent set relationships [Moved to DM.12]; and
- using deductive reasoning.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDING</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductive reasoning, deductive reasoning, and proof are critical in establishing general claims.</td>
<td></td>
</tr>
<tr>
<td>Deductive reasoning is the method that uses logic to draw conclusions based on definitions, postulates, and theorems.</td>
<td></td>
</tr>
<tr>
<td>Valid forms of deductive reasoning include the law of syllogism, the law of contrapositive, the law of detachment, and the identification of a counterexample. [Moved from EKS]</td>
<td></td>
</tr>
<tr>
<td>Symbolic notation is used to represent logical arguments, including the use of →, ↔, ¬, ∨, ∧, and ⊻.</td>
<td></td>
</tr>
<tr>
<td>The law of syllogism states that if ( p \rightarrow q ) is true and ( q \rightarrow r ) is true, then ( p \rightarrow r ) is true.</td>
<td></td>
</tr>
<tr>
<td>The law of contrapositive states that if ( p \rightarrow q ) is true and ( \neg q ) is true, then ( \neg p ) is true.</td>
<td></td>
</tr>
<tr>
<td>The law of detachment states that if ( p \rightarrow q ) is true and ( p ) is true, then ( q ) is true.</td>
<td></td>
</tr>
<tr>
<td>A counterexample is used to show an argument is false.</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>Identify the converse, inverse, and contrapositive of a conditional statement. (a)</td>
</tr>
<tr>
<td>Translate verbal arguments into symbolic form, such as ( (p \rightarrow q) ) and ( (\neg p \rightarrow \neg q) ) using the symbols of formal logic ( \rightarrow, \neg, \land, \lor ). (b) [Reordered]</td>
</tr>
<tr>
<td>Determine the validity of a logical argument. [Reordered]</td>
</tr>
<tr>
<td>Determine the validity of a logical argument using valid forms of deductive reasoning, including the law of syllogism, the law of the contrapositive, the law of detachment, and counterexamples. (c) [Moved to US]</td>
</tr>
<tr>
<td>Determine that an argument is false using a counterexample. (c)</td>
</tr>
<tr>
<td>Select and use various types of reasoning and methods of proof, as appropriate. [Moved to US]</td>
</tr>
<tr>
<td>Use Venn diagrams to represent set relationships, such as intersection and union. [Moved to DM.12]</td>
</tr>
</tbody>
</table>
**TOPIC STRAND: REASONING, LINES, AND TRANSFORMATIONS**

**GEOMETRY**

**STANDARD G.1**

The student will use **deductive reasoning** to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include:

a) identifying the converse, inverse, and contrapositive of a conditional statement;

b) translating a short verbal argument into symbolic form; and

c) determining the validity of a logical argument. [Added from EKS] using Venn diagrams to represent set relationships [Moved to DM.12]; and

d) using deductive reasoning.

**ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD**

- Inductive reasoning is the method of drawing conclusions from a limited set of observations.

- Proof is a justification that is logically valid and based on initial assumptions, definitions, postulates, and theorems, and/or properties of equality.

- Logical arguments consist of a set of premises or hypotheses and a conclusion.

- Euclidean geometry is an axiomatic system based on undefined terms (point, line and plane), postulates, and theorems.

- When a conditional \((p \rightarrow q)\) and its converse \((q \rightarrow p)\) are true, the statements can be written as a biconditional, i.e., \(p \iff q\) or \(p\) if and only if \(q\); or \(p \leftrightarrow q\).

- Logical arguments that are valid may not be true. Truth and validity are not synonymous.

- Conditional statements can be represented with Venn diagrams.

- Formal proofs utilize symbols of formal logic to determine validity of a logical argument. [Moved from EKS]

**ESSENTIAL KNOWLEDGE AND SKILLS**

- Interpret Venn diagrams. [Moved to DM.12]

- Recognize and use the symbols of formal logic [Reordered], which include \(\rightarrow, \iff, \neg, \land, \land\), and \(\lor\). [Moved to US]
### GEOMETRY

**STANDARD G.2**

The student will use the relationships between angles formed by two lines cut intersected by a transversal to

- **a)** determine whether prove two or more lines are parallel; and
- **b)** verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
- **c)** solve real-world problems, including practical problems, involving angles formed when parallel lines are cut intersected by a transversal.

---

### ESSENTIAL UNDERSTANDINGS

**UNDERSTANDING THE STANDARD**

- Deductive or inductive reasoning is used in mathematical proofs. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods.
- Parallel lines intersected by a transversal form angles with specific relationships.
- Some angle relationships may be used when proving two lines intersected by a transversal are parallel.
- If two parallel lines are intersected by a transversal, then:
  - corresponding angles are congruent;
  - alternate interior angles are congruent;
  - alternate exterior angles are congruent;
  - same-side (consecutive) interior angles are supplementary; and
  - same-side (consecutive) exterior angles are supplementary.
- Deductive proofs can be used to show that two or more lines are parallel.

---

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Use algebraic and coordinate methods as well as deductive proofs to verify whether two lines are parallel. [Included in G.3b]
- Prove two or more lines are parallel given angle measurements expressed numerically or algebraically. (a)
- Prove two lines parallel using deductive proofs given relationships between and among angles. (a)
- Solve problems by using the relationships between pairs of angles formed by the intersection of two parallel lines and a transversal including corresponding angles, alternate interior angles, alternate exterior angles, and same-side (consecutive) interior angles, and same-side (consecutive) exterior angles. (b)
- Solve real-world problems, including practical problems, involving intersecting and parallel lines in a plane. (b)
## GEOMETRY

### STANDARD G.2

The student will use the relationships between angles formed by two lines cut\(\text{intersected}\) by a transversal to

- a) determine whether prove two or more lines are parallel; and
- b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
- c) solve real-world problems, including practical problems, involving angles formed when parallel lines are cut\(\text{intersected}\) by a transversal.

### ESSENTIAL UNDERSTANDINGS

**UNDERSTANDING THE STANDARD**

- The construction of the line parallel to a given line through a point not on the line can be justified using the angle relationships formed when two lines are intersected by a transversal.

The Parallel Postulate differentiates Euclidean from non-Euclidean geometries such as spherical geometry and hyperbolic geometry.

### ESSENTIAL KNOWLEDGE AND SKILLS
GEOMETRY

STANDARD G.3

The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include:

a) investigating and using formulas for determining distance, midpoint, and slope;
b) applying slope to verify and determine whether lines are parallel or perpendicular;
c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transforms and combinations of transformations can be used to describe movement of objects in a plane. [Reordered]</td>
<td></td>
</tr>
<tr>
<td>Symmetry and transformations can be explored with computer software, paper folding, and coordinate methods.</td>
<td></td>
</tr>
<tr>
<td>The distance formula is an application of the Pythagorean Theorem.</td>
<td></td>
</tr>
<tr>
<td>Geometric figures can be represented in the coordinate plane.</td>
<td></td>
</tr>
<tr>
<td>Techniques for investigating symmetry may include paper folding, coordinate methods, and dynamic geometry software. [Combined]</td>
<td></td>
</tr>
<tr>
<td>Parallel lines have the same slope.</td>
<td></td>
</tr>
<tr>
<td>The product of the slopes of perpendicular lines is -1 unless one of the lines has an undefined slope.</td>
<td></td>
</tr>
<tr>
<td>A transformation of a figure called a preimage changes the size, shape, and/or position of the figure to a new figure called the image. [Reordered]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>Determine the coordinates of the midpoint or endpoint of a segment, using the midpoint formula. (a)</td>
</tr>
<tr>
<td>Use a formula to find the slope of a line. (a)</td>
</tr>
<tr>
<td>Apply the distance formula to determine the length of a line segment when given the coordinates of the endpoints. (a) [Reordered]</td>
</tr>
<tr>
<td>Compare the slopes to determine whether two lines are parallel, perpendicular, or neither. (b)</td>
</tr>
<tr>
<td>Determine whether a figure has point symmetry, line symmetry, both, or neither. (c)</td>
</tr>
</tbody>
</table>
**TOPIC: REASONING, LINES, AND TRANSFORMATIONS**

**STANDARD G.3**

The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include:

- a) investigating and using formulas for determining distance, midpoint, and slope;
- b) applying slope to verify and determine whether lines are parallel or perpendicular;
- c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
- d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
</table>
| • Transformations and combinations of transformations can be used to describe movement of objects in a plane. [Reordered] | • Given an image and preimage, identify the transformation or combination of transformations that has/have taken place as a reflection, rotation, dilation, or translation occurred. Included transformations include:
  - a translation;
  - a reflection over any horizontal or vertical line or the lines $y = x$ or $y = -x$;
  - a clockwise or counter clockwise rotation of 90°, 180°, 270°, or 360° on a coordinate grid where the center of rotation is limited to the origin; and
  - a dilation from a fixed point on a coordinate grid. (d) |
| • The image of an object or function graph after an isomorphic transformation is congruent to the preimage of the object. | • Apply the distance formula to find the length of a line segment when given the coordinates of the endpoints. [Reordered] |
| - A rotation is an isomorphic transformation in which an image is formed by rotating the preimage about a point called the center of rotation. The center of rotation may or may not be on the preimage. Rotations may be more than 180°. | |
| - A reflection is an isomorphic transformation in which an image is formed by reflecting the preimage over a line called the line of reflection. All corresponding points in the image are equidistant from the line of reflection. | |
| - A translation is an isomorphic transformation in which an image is formed by moving every point on the preimage the same distance in the same direction. | |
| • A dilation is a transformation in which an image is formed by enlarging or reducing the preimage proportionally by a scale factor from the center of dilation. The center of dilation may or may not be on the preimage. The image is similar to the preimage. | |
## GEOMETRY

### STANDARD G.4

The student will construct and justify the constructions of

- a line segment congruent to a given line segment;
- the perpendicular bisector of a line segment;
- a perpendicular to a given line from a point not on the line;
- a perpendicular to a given line at a given point on the line;
- the bisector of a given angle;
- an angle congruent to a given angle; and
- a line parallel to a given line through a point not on the given line;
- an equilateral triangle, a square, and a regular hexagon inscribed in a circle. [Moved from EKS]

### ESSENTIAL UNDERSTANDINGS

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Construction techniques are used to solve real-world practical problems in engineering, architectural design, and building construction.</td>
</tr>
<tr>
<td>• Construction techniques include using a straightedge and compass, paper folding, and dynamic geometry software.</td>
</tr>
<tr>
<td>• Geometric constructions assist in justifying, verifying, and visually reinforcing geometric relationships.</td>
</tr>
<tr>
<td>• There are multiple methods to most geometric constructions. Students would benefit from experiences with more than one method and should be able to justify each step of geometric constructions.</td>
</tr>
<tr>
<td>• Individual steps of constructions can be justified using angle relationships, properties of quadrilaterals, congruent triangles, and/or circles.</td>
</tr>
</tbody>
</table>

- The construction for a line segment congruent to a given line segment can be justified using properties of a circle.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Construct and justify the constructions of
  - a line segment congruent to a given line segment; (a)
  - the perpendicular bisector of a line segment; (b)
  - a perpendicular to a given line from a point not on the line; (c)
  - a perpendicular to a given line at a given point on the line; (d)
  - the bisector of a given angle; (e)
  - an angle congruent to a given angle; and (f)
  - a line parallel to a given line through a point not on the given line; (g) and
  - an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (h)
- Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. [Reordered]
- Construct the inscribed and circumscribed circles of a triangle.
- Construct a tangent line from a point outside a given circle to the
The student will construct and justify the constructions of:
   a) a line segment congruent to a given line segment;
   b) the perpendicular bisector of a line segment;
   c) a perpendicular to a given line from a point not on the line;
   d) a perpendicular to a given line at a given point on the line;
   e) the bisector of a given angle;
   f) an angle congruent to a given angle; and
   g) a line parallel to a given line through a point not on the given line; and
   h) an equilateral triangle, a square, and a regular hexagon inscribed in a circle. [Moved from EKS]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The construction for the perpendicular bisector of a line segment can be justified using the properties of quadrilaterals or congruent triangles.</td>
<td>circle.*</td>
</tr>
<tr>
<td>The constructions for a perpendicular to a given line from a point on, or not on, the line can be justified using the properties of quadrilaterals or congruent triangles.</td>
<td>*Revised March 2011</td>
</tr>
<tr>
<td>The constructions for the bisector of a given angle and an angle congruent to a given angle can be justified using the properties of quadrilaterals or congruent triangles.</td>
<td></td>
</tr>
<tr>
<td>The construction for a line parallel to a given line through a point not on the line can be justified using the angle relationships formed when two lines are intersected by a transversal.</td>
<td></td>
</tr>
<tr>
<td>The constructions for an equilateral triangle, square, or regular hexagon inscribed in a circle can be justified using properties of circles.</td>
<td></td>
</tr>
</tbody>
</table>
### Mathematics Standards of Learning Curriculum Framework 2009-2016: Geometry

**TOPIC STRAND:** REASONING, LINES, AND TRANSFORMATIONS

**GEOMETRY STANDARD G.4**

The student will construct and justify the constructions of

- a) a line segment congruent to a given line segment;
- b) the perpendicular bisector of a line segment;
- c) a perpendicular to a given line from a point not on the line;
- d) a perpendicular to a given line at a given point on the line;
- e) the bisector of a given angle;
- f) an angle congruent to a given angle; and
- g) a line parallel to a given line through a point not on the given line; and
- h) an equilateral triangle, a square, and a regular hexagon inscribed in a circle. [Moved from EKS]

### ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

- Constructions can be completed within the context of complex figures.

![Example](image)

**Given inscribed ΔABC with diameter AC, complete a construction to identify the center of the circle.**
GEOMETRY
STANDARD G.5

The student, given information concerning the lengths of sides and/or measures of angles in triangles, will solve problems, including practical problems. This will include:

- a) ordering the sides by length, given the-angle measures;
- b) ordering the angles by degree measure, given the-side lengths;
- c) determining whether a triangle exists; and
- d) determining the range in which the length of the third side must lie.

These concepts will be considered in the context of real-world situations. [Moved to SOL stem]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
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</tr>
</thead>
<tbody>
<tr>
<td>• The longest side of a triangle is opposite the largest angle of the triangle and the shortest side is opposite the smallest angle.</td>
<td></td>
</tr>
<tr>
<td>• In a triangle, the lengths of two sides and the included angle determine the length of the side opposite the angle.</td>
<td></td>
</tr>
<tr>
<td>• In order for a triangle to exist, the length of each side must be within a range that is determined by the lengths of the other two sides.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Given information about the lengths of sides and/or measures of angles in triangles, solve problems, including practical problems. (a,b,c,d) [Reordered]</td>
</tr>
<tr>
<td>• Order the sides of a triangle by their lengths when given information about the measures of the angles. (a)</td>
</tr>
<tr>
<td>• Order the angles of a triangle by their measures when given information about the lengths of the sides. (b)</td>
</tr>
<tr>
<td>• Given the lengths of three segments, determine whether a triangle could be formed. (c)</td>
</tr>
<tr>
<td>• Given the lengths of two sides of a triangle, determine the range in which the length of the third side must lie. (d)</td>
</tr>
<tr>
<td>• Solve real-world problems given information about the lengths of sides and/or measures of angles in triangles. [Reordered]</td>
</tr>
</tbody>
</table>
## GEOMETRY
### STANDARD G.6

The student, given information in the form of a figure or statement, will prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Deductive or inductive reasoning is used in mathematical proofs. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods. [Moved from EKS]</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>- Congruence has real-world practical applications in a variety of areas, including art, architecture, and the sciences.</td>
<td></td>
</tr>
<tr>
<td>- Congruence does not depend on the position of the triangles.</td>
<td></td>
</tr>
<tr>
<td>- Congruent triangles are a result of rigid isomorphic transformations.</td>
<td></td>
</tr>
<tr>
<td>- Concepts of logic can demonstrate congruence or similarity.</td>
<td></td>
</tr>
<tr>
<td>- Congruent figures are also similar, but similar figures are not necessarily congruent.</td>
<td></td>
</tr>
<tr>
<td>- Corresponding parts of congruent triangles are congruent.</td>
<td></td>
</tr>
</tbody>
</table>
| - Two triangles can be proven congruent using:  
  - Side-Angle-Side (SAS);  
  - Side-Side-Side (SSS);  
  - Angle-Angle-Side (AAS); and  
  - Angle-Side-Angle (ASA). |   |

[Moved from EKS]
GEOMETRY

STANDARD G.6

The student, given information in the form of a figure or statement, will prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Two right triangles can be proven congruent using Hypotenuse-Leg (HL).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Triangle congruency can be explored using geometric constructions such as an angle congruent to a given angle or a line segment congruent to a given line segment.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The construction for the bisector of a given angle can be justified using congruent triangles.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The construction for an angle congruent to a given angle can be justified using congruent triangles.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The construction of the perpendicular to a given line from a point on the line can be justified using congruent triangles.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• The construction of the perpendicular to a given line from a point not on the line can be justified using congruent triangles.</td>
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</tr>
</tbody>
</table>
**TOPICSTRAND: TRIANGLES**

**GEOMETRY**  
**STANDARD G.7**

The student, given information in the form of a figure or statement, will prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.

### Essential Understanding

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deductive or inductive reasoning is used in mathematical proofs. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods. [Moved from EKS]</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:</td>
</tr>
<tr>
<td>Similarity has real-world practical applications in a variety of areas, including art, architecture, and the sciences.</td>
<td>- Prove two triangles similar given relationships among angles and sides of triangles expressed numerically or algebraically. [Rewritten and reordered]</td>
</tr>
<tr>
<td>Similarity does not depend on the position of the triangles.</td>
<td>- Prove two triangles similar given representations in the coordinate plane and using coordinate methods (distance formula and slope formula). [Rewritten and reordered]</td>
</tr>
<tr>
<td>Similar triangles are created using dilations.</td>
<td>- Use direct proofs definitions, postulates, and theorems to prove triangles similar.</td>
</tr>
<tr>
<td>Congruent figures are also similar, but similar figures are not necessarily congruent.</td>
<td>- Use algebraic methods to prove that triangles are similar. [Reordered]</td>
</tr>
<tr>
<td>Corresponding sides of similar triangles are proportional.</td>
<td>- Use coordinate methods, such as the distance formula, to prove two triangles are similar. [Reordered]</td>
</tr>
<tr>
<td>Corresponding angles of similar triangles are congruent.</td>
<td></td>
</tr>
</tbody>
</table>
GEOMETRY
STANDARD G.8

The student will solve real-world problems, including practical problems, involving right triangles. This will include applying by using the

a) the Pythagorean Theorem and its converse;

b) properties of special right triangles; and

c) right-triangle-trigonometry trigonometric ratios.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The Pythagorean Theorem is essential for solving problems involving right triangles.</td>
<td></td>
</tr>
<tr>
<td>• Many historical and algebraic proofs of the Pythagorean Theorem exist.</td>
<td></td>
</tr>
<tr>
<td>• The relationships between the sides and angles of right triangles are useful in many applied fields.</td>
<td></td>
</tr>
<tr>
<td>• Some practical problems can be solved by choosing an efficient representation of the problem.</td>
<td></td>
</tr>
<tr>
<td>• Another formula for the area of a triangle is ( A = \frac{1}{2}ab \sin C ).</td>
<td></td>
</tr>
<tr>
<td>• The converse of the Pythagorean Theorem can be used to determine if a triangle is a right triangle.</td>
<td></td>
</tr>
<tr>
<td>• 45°-45°-90° and 30°-60°-90° triangles are special right triangles because their side lengths can be specified as exact values using radicals rather than decimal approximations.</td>
<td></td>
</tr>
</tbody>
</table>

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Solve problems, including practical problems, using right triangle trigonometry and properties of special right triangles. (a,b,c) [Reordered]
- Determine whether a triangle formed with three given lengths is a right triangle. (a)
- Solve for missing lengths in geometric figures, using properties of 45°-45°-90° triangles where rationalizing denominators may be necessary. (b)
- Solve for missing lengths in geometric figures, using properties of 30°-60°-90° triangles where rationalizing denominators may be necessary. (b).
- Solve problems, including practical problems, involving right triangles with missing side lengths or angle measurements, using sine, cosine, and tangent ratios. (c)
- Solve real-world problems, using right triangle trigonometry and properties of right triangles. [Reordered]
GEOMETRY
STANDARD G.8

The student will solve real-world problems, including practical problems, involving right triangles. This will include applying by using the

a) the Pythagorean Theorem and its converse;

b) properties of special right triangles; and

c) right triangle trigonometry trigonometric ratios.

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<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
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</tr>
</thead>
<tbody>
<tr>
<td>• The ratios of side lengths in similar right triangles (adjacent/hypotenuse or opposite/hypotenuse) are independent of the scale factor and depend only on the angle the hypotenuse makes with the adjacent side, thus justifying the definition and calculation of trigonometric functions using the ratios of side lengths for similar right triangles. The sine of an acute angle in a right triangle is equal to the cosine of its complement. [Moved from EKS]</td>
<td>• Explain and use the relationship between the sine and cosine of complementary angles. † [Moved to US]</td>
</tr>
</tbody>
</table>

†Revised March 2011
GEOMETRY
STANDARD G.9

The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems, including practical problems.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The terms characteristics and properties can be used interchangeably to describe quadrilaterals. The term characteristics is used in elementary and middle school mathematics.</td>
<td></td>
</tr>
<tr>
<td>• Deductive or inductive reasoning is used in mathematical proofs. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods.</td>
<td></td>
</tr>
<tr>
<td>• Quadrilaterals have a hierarchical nature based on the relationships between their sides, angles, and diagonals.</td>
<td></td>
</tr>
<tr>
<td>• Characteristics and properties of quadrilaterals can be used to identify the quadrilateral and to determine the measures of sides and angles.</td>
<td></td>
</tr>
<tr>
<td>• Given coordinate representations of quadrilaterals, the distance, slope, and midpoint formulas may be used to verify that quadrilaterals have specific properties.</td>
<td></td>
</tr>
<tr>
<td>• The angle relationships formed when parallel lines are intersected by a transversal can be used to prove properties of quadrilaterals.</td>
<td></td>
</tr>
<tr>
<td>• Congruent triangles can be used to prove properties of quadrilaterals.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Solve problems, including real-world problems, using the properties specific to parallelograms, rectangles, rhombi, squares, isosceles trapezoids, and trapezoids.</td>
</tr>
<tr>
<td>• Prove that quadrilaterals have specific properties, using coordinate and algebraic methods, such as the distance formula, slope, and midpoint formula.</td>
</tr>
<tr>
<td>• Prove the characteristics and properties of quadrilaterals, using direct proofs, deductive reasoning, algebraic, and coordinate methods.</td>
</tr>
<tr>
<td>• Prove properties of angles for a quadrilateral inscribed in a circle. [Moved to G.11 US]</td>
</tr>
</tbody>
</table>

*Revised March 2011
## Mathematics Standards of Learning Curriculum Framework 2009-2016: Geometry

### Geometry

#### STANDARD G.9

The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems, including practical problems.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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</thead>
</table>
| A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Properties of a parallelogram include the following:  
- opposite sides are congruent;  
- opposite angles are congruent;  
- consecutive angles are supplementary; and  
- diagonals bisect each other.  |
| A rectangle is a quadrilateral with four right angles. Properties of a rectangle include the following:  
- opposite sides are parallel and congruent; and  
- diagonals are congruent and bisect each other. |
| A rhombus is a quadrilateral with four congruent sides. Properties of a rhombus include the following:  
- all sides are congruent;  
- opposite sides are parallel;  
- opposite angles are congruent;  
- diagonals are perpendicular bisectors of each other;  
- diagonals bisect opposite angles; and  
- diagonals divide the rhombus into four congruent right triangles. |
**GEOMETRY**

**STANDARD G.9**

The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems, including practical problems.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
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</tr>
</thead>
<tbody>
<tr>
<td>• A square is a quadrilateral that is a regular polygon with four congruent sides and four right angles. Properties of a square include the following:</td>
<td></td>
</tr>
<tr>
<td>– opposite sides are parallel;</td>
<td></td>
</tr>
<tr>
<td>– diagonals are congruent;</td>
<td></td>
</tr>
<tr>
<td>– diagonals are perpendicular bisectors of each other; and</td>
<td></td>
</tr>
<tr>
<td>– diagonals divide the square into four congruent 45°-45°-90° triangles.</td>
<td></td>
</tr>
<tr>
<td>• A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called bases. The nonparallel sides of a trapezoid are called legs.</td>
<td></td>
</tr>
<tr>
<td>• An isosceles trapezoid has the following properties:</td>
<td></td>
</tr>
<tr>
<td>– nonparallel sides are congruent;</td>
<td></td>
</tr>
<tr>
<td>– diagonals are congruent; and</td>
<td></td>
</tr>
<tr>
<td>– base angles are congruent.</td>
<td></td>
</tr>
<tr>
<td>• The construction of the perpendicular bisector of a line segment can be justified using the properties of quadrilaterals.</td>
<td></td>
</tr>
<tr>
<td>• The construction of the perpendicular to a given line from a point on, or not on, the line can be justified using the properties of quadrilaterals.</td>
<td></td>
</tr>
<tr>
<td>• The construction of the perpendicular to a given line from a point on the line can be justified using the properties of quadrilaterals.</td>
<td></td>
</tr>
</tbody>
</table>
## GEOMETRY

### STANDARD G.9

The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems, including practical problems.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The construction of a bisector of a given angle can be justified using the properties of quadrilaterals.</td>
<td></td>
</tr>
<tr>
<td>• The construction of a square inscribed in a circle can be justified using the properties of squares.</td>
<td></td>
</tr>
</tbody>
</table>
GEOMETRY
STANDARD G.10

The student will solve real-world problems, including practical problems, involving angles of convex polygons. This will include determining the

a) determining the sum of the interior and/or exterior angles;
b) determining the measure of an interior and/or exterior angle; and
c) determining the number of sides of a regular polygon.

ESSENTIAL UNDERSTANDING
UNDERSTANDING THE STANDARD

- A regular polygon will tessellate the plane if the measure of an interior angle is a factor of 360°. [Reordered]
- Both regular and nonregular polygons can tessellate the plane. [Reordered]
- In convex polygons, each interior angle has a measure less than 180°.
- In concave polygons, one or more interior angles have a measure greater than 180°.
- Two intersecting lines form angles with specific relationships.
- An exterior angle is formed by extending a side of a polygon.
- The exterior angle and the corresponding interior angle form a linear pair.
- The sum of the measures of the interior angles of a convex polygon may be found by dividing the interior of the polygon into nonoverlapping triangles.
- Both regular and nonregular polygons can tessellate the plane. [Reordered]
- A regular polygon will tessellate the plane if the measure of an interior angle is a factor of 360°. [Reordered]

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Solve real-world problems, including practical problems, involving the measures of interior and exterior angles of convex polygons. (a, b, c)
- Identify tessellations in art, construction, and nature. [Moved to US]
- Find the sum of the measures of the interior and exterior angles of a convex polygon. (a)
- Find the measure of each interior and exterior angle of a regular polygon. (b)
- Determine angle measures of a regular polygon in a tessellation. (b)
- Find the number of sides of a regular polygon, given the measures of interior or exterior angles of the polygon. (c)
GEOMETRY
STANDARD G.10

The student will solve real-world problems, including practical problems, involving angles of convex polygons. This will include:

- determining the sum of the interior and/or exterior angles;
- determining the measure of an interior and/or exterior angle; and
- determining the number of sides of a regular polygon.

ESSENTIAL UNDERSTANDINGS
UNDERSTANDING THE STANDARD

- interior angle is a factor of 360. [Reordered]
- The sum of the measures of the angles around a point in a tessellation is 360°.
- Tessellations can be found in art, construction and nature. [Moved from EKS]

ESSENTIAL KNOWLEDGE AND SKILLS
**GEOMETRY**

**STANDARD G.11**

The student will solve problems, including practical problems, by applying properties of circles, use angles, arcs, chords, tangents, and secants to. This will include determining:

- a) investigate, verify, and apply properties of circles angle measures formed by intersecting chords, secants, and/or tangents;
- b) solve real-world problems involving properties of circles lengths of segments formed by intersecting chords, secants, and/or tangents; and
- c) find arc lengths; and
d) and areas of sectors in circles.

**ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD**

- All circles are similar.
- A chord is a line segment that joins any two points on a circle. A chord is part of a segment of a secant.
- Arcs can be measured in degrees or in units of length.
- Real-world applications of the properties of circles may be drawn from architecture, art, and construction.
- Properties of circles can be verified using deductive reasoning, algebraic, and coordinate methods. [Moved from EKS]
- Inscribed quadrilaterals have opposite angles that are supplementary. [Moved from G.9 EKS]
- Properties associated with segment lengths can be verified using similar triangles.
- The ratio of the central angle to 360° is proportional to the ratio of the arc length to the circumference of the circle.
- The ratio of the central angle to 360° is proportional to the ratio of the area of the sector to the area of the circle.

**ESSENTIAL KNOWLEDGE AND SKILLS**

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

- Solve problems, including practical problems, by applying properties of circles. (a,b,c,d) [Reordered]
- Find determine lengths, angle measures, and arc measures, and arc measures associated with:
  - two intersecting chords;
  - two intersecting secants;
  - an intersecting secant and tangent;
  - two intersecting tangents; and
  - central and inscribed angles. (a)
- Determine segment lengths associated with:
  - two intersecting chords;
  - two intersecting secants;
  - an intersecting secant and tangent; and
  - two intersecting tangents. (b)
GEOMETRY
STANDARD G.11
The student will solve problems, including practical problems, by applying properties of circles, use angles, arcs, chords, tangents, and secants to determine

- investigate, verify, and apply properties of circles' angle measures formed by intersecting chords, secants, and/or tangents;
- solve real-world problems involving properties of circles' lengths of segments formed by intersecting chords, secants, and/or tangents; and
- find arc lengths; and
- find areas of sectors in circles.

ESSENTIAL UNDERSTANDING
UNDERSTANDING THE STANDARD

- The construction for an inscribed equilateral triangle, square, and regular hexagon can be justified using properties of a circle.

ESSENTIAL KNOWLEDGE AND SKILLS

- Calculate the area of a sector and the length of an arc of a circle in degrees or units of length, using proportions. (c)
- Calculate the area of a sector. (d)
- Solve real-world problems associated with circles, using properties of angles, lines, and arcs. [Reordered]
- Verify properties of circles, using deductive reasoning, algebraic, and coordinate methods. [Moved to US]
**GEOMETRY**

**STANDARD G.12**

The student will solve problems involving, given the coordinates of the center of a circle and a point on the circle, will write the equations of the circles.

### ESSENTIAL UNDERSTANDING UNDERSTANDING THE STANDARD

- A circle is a locus of points equidistant from a given point, the center.
- The distance between any point on the circle and the center is the length of the radius. [Moved from EKS]
- Standard form for the equation of a circle is 
  \[(x-h)^2 + (y-k)^2 = r^2\], where the coordinates of the center of the circle are \((h,k)\) and \(r\) is the length of the radius.
- The equation of a circle gives the coordinates of every point, \((x,y)\), on the circle.
- The midpoint formula and distance formula are important when determining the equation of a circle.
- The equation of a circle with a given center and radius can be derived using the Pythagorean Theorem. [Moved from EKS]
- The midpoint of the diameter is the center of the circle.
- The circle is a conic section.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Given a graph or the equation of a circle in standard form, identify the coordinates of the center of the circle.
- Given the coordinates of the endpoints of a diameter of a circle, determine the coordinates of the center of the circle.
- Given a graph or the equation of a circle in standard form, identify the length of the radius or diameter of the circle.
- Given the coordinates of the endpoints of the diameter of a circle, determine the length of the radius or diameter of the circle.
- Given the coordinates of the center and the coordinates of a point on the circle, determine the length of the radius or diameter of the circle.
- Use the distance formula to find the radius of a circle. [Moved to US]
- Given the coordinates of the center and length of the radius of the circle, identify the coordinates of a point(s) on the circle.
- Determine the equation of a circle given:
  - a graph of a circle with a center with coordinates that are integers;
  - coordinates of the center and a point on the circle;
  - coordinates of the center and the length of the radius or diameter;
GEOMETRY
STANDARD G.12

The student will solve problems involving, given the coordinates of the center of a circle and a point on the circle, will write the equations of the circles.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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</table>
| or
- coordinates of the endpoints of a diameter.
- Given the equation of a circle in standard form, identify the coordinates of the center and find the radius of the circle.
- Given the coordinates of the endpoints of a diameter, find the equation of the circle. [Reordered]
- Given the coordinates of the center and a point on the circle, find the equation of the circle. [Combined]
- Recognize that the equation of a circle of given center and radius is derived using the Pythagorean Theorem. [Moved to US]

*Revised March 2011
### ESSENTIAL UNDERSTANDING UNDERSTANDING THE STANDARD

- A cylinder is a solid figure formed by two congruent parallel faces called bases joined by a curved surface. In this course, cylinders are limited to right circular cylinders.
- A cone is a solid figure formed by a face called a base that is joined to a vertex (apex) by a curved surface. In this course, cones are limited to right circular cones.
- A prism is a polyhedron that has a congruent pair of parallel bases and faces that are parallelograms. In this course, prisms are limited to right prisms with bases that are triangles, rectangles, or regular hexagons.
- A rectangular prism is a polyhedron in which all six faces are rectangles.
- A triangular prism is a polyhedron that has a congruent pair of parallel triangular bases and faces that are parallelograms.
- A pyramid is a polyhedron with a base that is a polygon and three or more faces that are triangles with a common vertex.
- A regular pyramid is a pyramid with a base that is a regular polygon. In this course, pyramids are limited to right regular pyramids with triangular, square, or hexagonal bases.
- A regular polygon has congruent sides and congruent interior angles.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Determine the total surface area of cylinders, prisms, pyramids, cones, hemispheres, and spheres, using the appropriate formulas.
- Calculate the volume of cylinders, prisms, pyramids, cones, hemispheres, and spheres, using the appropriate formulas.
- Solve problems, including real-world problems, involving total surface area and volume of cylinders, prisms, pyramids, cones, hemispheres, and spheres, as well as combinations of composite three-dimensional figures.
- Solve problems, including practical problems, involving the lateral surface area of circular cylinders, prisms, and regular pyramids.
- Given information about a three-dimensional figure such as length of a side, area of a face, or volume, determine missing information.
- Calculators may be used to find decimal approximations for results. [Moved to US]
**GEOMETRY**

**STANDARD G.13**

The student will use formulas for surface area and volume of three-dimensional objects to solve real-world practical problems.

<table>
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<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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<tbody>
<tr>
<td>• Subdivision of polygons may assist in determining the area of regular polygons.</td>
<td>• Calculators may be used to determine decimal approximations for</td>
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<tr>
<td>• The surface area of a prism or pyramid three-dimensional object is the sum of the areas of all its faces.</td>
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<tr>
<td>• The surface area of a cylinder, cone, or hemisphere is the sum of the areas of the curved surface and bases.</td>
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<tr>
<td>• The lateral surface area of a cylinder is the area of the curved surface of the cylinder, not including the parallel bases.</td>
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<tr>
<td>• The lateral surface area of a rectangular-based prism is the sum of the areas of all faces, not including the parallel bases.</td>
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</tr>
<tr>
<td>• The lateral surface area of a triangular-based prism is the sum of the areas of all faces, not including the triangular-shaped, parallel bases.</td>
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<tr>
<td>• The volume of a three-dimensional object figure is the number of unit cubes that would fill the object figure.</td>
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<tr>
<td>• Composite figures consist of two or more three-dimensional figures. The surface area of a composite figure may not be equal to the sum of the surface areas of the individual figures.</td>
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<tr>
<td>• Volume and surface area of spheres, cones and cylinders should be considered in terms of ( \pi ) or as a decimal representation approximation.</td>
<td></td>
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<tr>
<td>• Calculators may be used to determine decimal approximations for</td>
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GEOMETRY  
STANDARD G.13

The student will use formulas for surface area and volume of three-dimensional objects to solve real-world practical problems.

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<tr>
<td>results. [Moved from EKS]</td>
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GEOMETRY
STANDARD G.14

The student will apply the concepts of similarity geometric objects into two- or three-dimensional geometric figures. This will include:

1. Comparing ratios between side-lengths, perimeters, areas, and volumes of similar figures;
2. Determining how changes in one or more dimensions of an object figure affect area and/or volume of the object figure;
3. Determining how changes in area and/or volume of an object figure affect one or more dimensions of the object figure; and
4. Solving real-world problems, including practical problems, about similar geometric objects figures.

### ESSENTIAL UNDERSTANDING

**UNDERSTANDING THE STANDARD**

- A change in one dimension of an object figure results in predictable changes in area and/or volume. The resulting figure may or may not be similar to the original figure.
- A constant ratio, the scale factor, exists between corresponding lengths of sides of similar figures.
- If the ratio between dimensions of similar figures is $a:b$ then:
  - The ratio of their areas is $a^2:b^2$.
  - The ratio of their volumes is $a^3:b^3$.
- Proportional reasoning is integral important when comparing attribute measures in similar objects figures.

**ESSENTIAL KNOWLEDGE AND SKILLS**

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:
  - Compare ratios between side lengths, perimeters, areas, and volumes, given two similar figures. (a)
  - Describe how changes in one or more dimensions affect other derived measures (perimeter, area, total surface area, and volume) of an object figure. (b)
  - Describe how changes in one or more measures (perimeter, area, total surface area, and volume) affect other measures of an object figure. (c)
  - Solve real-world problems involving measured attributes of similar object figures. (d)
Mathematics Standards of Learning

Curriculum Framework 2009 2016

Algebra, Functions, and Data Analysis

Board of Education
Commonwealth of Virginia
The 2009-2016 Mathematics Standards of Learning Curriculum Framework is a companion document to the 2009-2016 Mathematics Standards of Learning, and amplifies the Mathematics Standards of Learning by further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. It provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instructionbackground information for the teacher (K-8). It contains content—The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers’ knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate in each standard is outlined. This is not meant to be an exhaustive list of student expectations nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations—physical, visual, symbolic, verbal, and contextual—and recognize that representation is both a process and a product.
**The Role of Instructional Technology**

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

**Computational Fluency**

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student’s problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of and the ability to apply the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.1

The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) function families and their characteristics. Key concepts include:

- a) domain and range
- b) continuity
- c) intervals on which a function is increasing or decreasing
- d) local and absolute maxima and minima
- e) domain and range
- f) values of a function for elements in its domain
- g) intervals in which the function is increasing/decreasing
- h) zeros
- i) intercepts
- j) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic
- k) end behaviors
- l) vertical and horizontal asymptotes

ESSENTIAL UNDERSTANDING THE STANDARD

- A relation is a function if and only if each element in the domain is paired with a unique element of the range.
- Functions are used to model practical phenomena.
- Functions describe the relationship between two variables where each input is paired to a unique output.
- Function families consist of a parent function and all transformations of the parent function.
- The domain of a function is the set of all possible values of the independent variable.
- The range of a function is the set of all possible values of the dependent variable.
- The domain of a function consists of the first coordinates of the ordered pairs that are elements of a function. Each element in the

ESSENTIAL KNOWLEDGE AND SKILLS

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
- Identify the domain and range for a relation, given a set of ordered pairs, a table, or a graph.
- For each $x$ in the domain of $f$, find $f(x)$.
- Identify the zeros of the function algebraically and confirm them, using the graphing calculator.
- Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically. Domains may be limited by problem context or in graphical representations.
- Identify intervals on which the function is increasing or decreasing.
- Identify the location and value of the absolute maximum and absolute minimum of a function over the domain of the function.
ALGEBRA, FUNCTIONS AND DATA ANALYSIS

TOPIC: ALGEBRA AND FUNCTIONS

STANDARD AFDA.1

The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) function families and their characteristics. Key concepts include:

a) **domain and range** continuity; [Reordered]

b) **intervals on which a function is increasing or decreasing** local and absolute maxima and minima [Reordered];

c) **domain and range** absolute maximum and minimum [Reordered];

d) zeros;

e) intercepts;

f) **values of a function for elements in its domain** intervals in which the function is increasing/decreasing [Reordered];

g) **connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs** including concrete, verbal, numeric, graphic, and algebraic;

h) **end behaviors**; and

hi) **vertical and horizontal asymtotes**.

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**ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD**

- **domain** is an input into the independent variable of the function.

- The **range** of a function consists of the second coordinates of the ordered pairs that are elements of a function. Each element in the range is an output in the dependent variable of a function.

- For each x in the domain of f, x is a member of the input of the function f, f(x) is a member of the output of f, and the ordered pair (x, f(x)) is a member of f.

- A value x in the domain of f is an x-intercept or a zero of a function f if and only if f(x) = 0. [Reordered]

- Functions describe the relationship between two variables where each input is paired to a unique output. [Reordered]

- Functions are used to model real-world phenomena. [Reordered]

- The domain of a function may be restricted algebraically.

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**ESSENTIAL KNOWLEDGE AND SKILLS**

- graphically or by using a graphing utility. (c)

- For any x value in the domain of f, determine f(x). (f) [Reordered]

- Represent relations and functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic forms. Given one representation, represent the relation in another form. (g)

- Detect patterns in data and represent arithmetic and geometric patterns algebraically. (g)

- Describe the end behavior of a function. (h)

- Determine the equations of the horizontal asymptote of an exponential function and the vertical asymptote of a logarithmic function. (i)

- Investigate and analyze characteristics and multiple representations of functions with a graphing utility.
ALGEBRA, FUNCTIONS AND DATA ANALYSIS

STANDARD AFDA.1

The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include:

a) **domain and range**
   - continuity; [Reordered]

b) intervals on which a function is increasing or decreasing
   - local and absolute maxima and minima [Reordered];

c) **domain and range**
   - maximum and minimum [Reordered];

d) zeros;

e) intercepts;

f) values of a function for elements in its domain
   - intervals in which the function is increasing/decreasing [Reordered];

g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;
   - including concrete, verbal, numeric, graphic, and algebraic;

h) end behaviors; and

i) vertical and horizontal asymptotes.

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ESSENTIAL UNDERSTANDINGS

UNDERSTANDING THE STANDARD

- A function can be described on an interval as increasing, decreasing, or constant over a specified interval or over the entire domain of the function.

- A function, \( f(x) \), is increasing over an interval if the values of \( f(x) \) consistently increase over the interval as the \( x \) values increase.

- A function, \( f(x) \), is decreasing over an interval if the values of \( f(x) \) consistently decrease over the interval as the \( x \) values increase.

- A function, \( f(x) \), is constant over an interval if the values of \( f(x) \) remain constant over the interval as the \( x \) values increase.

- A function is increasing on an interval if its graph, as read from left to right, is rising in that interval.

- A function is decreasing on an interval if its graph, as read from

ESSENTIAL KNOWLEDGE AND SKILLS

(a,b,c,d,e,f,g,h,i)

- Recognize restricted/discontinuous domains and ranges.

- Recognize graphs of parent functions for linear, quadratic, exponential and logarithmic functions. [Included in AFDA.2]

- Identify \( x \)-intercepts (zeros), \( y \)-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, points of discontinuity, end behavior, and maximum and minimum points, given a graph of a function. [Reordered and split]

- Describe continuity of a function on its domain or at a point.

- Express intervals using correct interval notation and/or a compound inequality. [Moved to US]
**TOPIC: ALGEBRA AND FUNCTIONS**

**STANDARD AFDA.1**

The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include:

a) **domain** and **range** continuity; [Reordered]

b) intervals on which a function is increasing or decreasing local and absolute maxima and minima [Reordered];

c) **domain** and **range** absolute maxima and minima [Reordered];

d) zeros;

e) intercepts;

f) **values** of a function for elements in its domain intervals in which the function is increasing/decreasing [Reordered];

g) **connections** between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic;

h) end behaviors; and

hi) vertical and horizontal asymptotes.

**ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD**

- Exponential and logarithmic functions are either strictly increasing or strictly decreasing.

- A function is continuous on an interval if the function is defined for every value in the interval and there are no breaks in the graph. A continuous function can be drawn without lifting the pencil.

- A turning point is a point on a continuous interval where the graph changes from increasing to decreasing or from decreasing to increasing.

- A function, \( f \), has a **local maximum** in some interval at \( x = a \) if \( f(a) \) is the largest value of \( f \) in that interval over its domain.

- A function, \( f \), has a **local minimum** in some interval at \( x = a \) if \( f(a) \) is the smallest value of \( f \) in that interval over its domain.
ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.1
The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) function families and their characteristics. Key concepts include:

a) domain and range;

b) intervals on which a function is increasing or decreasing;

c) local and absolute maxima and minima;

d) zeros;

e) intercepts;

f) values of a function for elements in its domain;

g) intervals in which the function is increasing/decreasing;

h) end behaviors; and

ii) vertical and horizontal asymptotes.

ESSENTIAL UNDERSTANDING: UNDERSTANDING THE STANDARD

- Solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation. Examples may include:

<table>
<thead>
<tr>
<th>Equation/ Inequality</th>
<th>Set Notation</th>
<th>Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>x = 3</td>
<td>{3}</td>
<td></td>
</tr>
<tr>
<td>x = 3 or x = 5</td>
<td>{3, 5}</td>
<td></td>
</tr>
<tr>
<td>0 ≤ x &lt; 3</td>
<td>[x</td>
<td>0 ≤ x &lt; 3]</td>
</tr>
<tr>
<td>y ≥ 3</td>
<td>{y : y ≥ 3}</td>
<td>[3, ∞)</td>
</tr>
<tr>
<td>Empty (null) set Ø</td>
<td>{}</td>
<td></td>
</tr>
</tbody>
</table>

- Asymptotes can be used to describe local behavior and end behavior of graphs. They are lines or other curves that approximate the graphical behavior of a function. [Reordered]

- A value x in the domain of f is an x-intercept or a zero of a function f if and only if f(x) = 0.

- The x-intercept is the point at which the graph of a relation or
ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.1
The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) function families and their characteristics. Key concepts include:

a) **domain and range**
   - continuity [Reordered]

b) **intervals on which a function is increasing or decreasing**
   - local and absolute maxima and minima [Reordered]

c) **domain and range**
   - absolute maximum and minimum [Reordered]

d) **zeros**

f) **intercepts**

g) **values of a function for elements in its domain**
   - intervals in which the function is increasing/decreasing [Reordered]

h) **connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs**
   - including concrete, verbal, numeric, graphic, and algebraic

i) **end behaviors**

ii) **vertical and horizontal asymptotes**

---

**ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD**

- A function intersects with the x-axis. It can be expressed as a value or a coordinate.

- The y-intercept is the point at which the graph of a relation or function intersects with the y-axis. It can be expressed as a value or a coordinate.

- Given a polynomial function \( f(x) \), the following statements are equivalent for any real number, \( k \), such that \( f(k) = 0 \):
  - \( k \) is a zero of the polynomial function \( f(x) \) located at \( (k, 0) \);
  - \( k \) is a solution or root of the polynomial equation \( f(x) = 0 \);
  - the point \( (k, 0) \) is an x-intercept for the graph of the polynomial \( f(x) = 0 \); and
  - \( (x – k) \) is a factor of the polynomial \( f(x) \).

- Connections between multiple representations (graphs, tables, and equations) of a function can be made.

---

**ESSENTIAL KNOWLEDGE AND SKILLS**
ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.1

The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include:

a) **domain and range**, continuity; [Reordered]
b) **intervals on which a function is increasing or decreasing**, local and absolute maxima and minima [Reordered];
c) **domain and range**, absolute maxima and minima [Reordered];
d) **zeros**;
e) **intercepts**;
f) **values of a function for elements in its domain**, intervals in which the function is increasing/decreasing [Reordered];
g) **connections between and among multiple any two representations of functions using verbal descriptions, tables, equations, and graphs**, including concrete, verbal, numeric, graphic, and algebraic;
h) **end behaviors**; and

i) **vertical and horizontal asymptotes**.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>In an arithmetic pattern, the common difference is the value that is added to obtain the next value in the pattern.</td>
<td>In geometric number patterns, the common ratio is a multiplier used to obtain the next value in the pattern.</td>
</tr>
<tr>
<td>In geometric number patterns, the common ratio is a multiplier used to obtain the next value in the pattern.</td>
<td>End behavior describes a function’s values as x approaches positive or negative infinity.</td>
</tr>
<tr>
<td>End behavior describes a function’s values as x approaches positive or negative infinity.</td>
<td>Asymptotes can be used to describe local behavior and end behavior of graphs. They are lines or other curves that approximate the graphical behavior of a function. [Reordered]</td>
</tr>
</tbody>
</table>
### ALGEBRA, FUNCTIONS AND DATA ANALYSIS

**STANDARD AFDA.2**

The student will use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic) function.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Knowledge of transformational graphing using parent functions can be used to generate/verify a mathematical model from a scatterplot that approximates the data.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• The graph of a parent function is an anchor graph from which other graphs are derived using transformations.</td>
<td>• Write an equation of a line when given the graph of a line.</td>
</tr>
<tr>
<td>• Transformations of graphs include:</td>
<td>• Recognize graphs of parent functions for linear, quadratic, exponential and logarithmic functions.</td>
</tr>
<tr>
<td>- Translations (horizontal and vertical shifting of a graph);</td>
<td>• Write the equation of a linear, quadratic, exponential, or logarithmic function in ((h, k)_{\text{vertex}}) form given the graph of the parent function and transformation information.</td>
</tr>
<tr>
<td>- Reflections (over the x- and y-axis); and</td>
<td>• Describe the transformation from the parent function given the equation written in ((h, k)_{\text{vertex}}) form or the graph of the function.</td>
</tr>
<tr>
<td>- Dilations (stretching and compressing graphs); and</td>
<td>• Given the equation of a function, recognize the parent function and transformation to graph the given function.</td>
</tr>
<tr>
<td>- Rotations</td>
<td>• Recognize the vertex of a parabola given a quadratic equation in ((h, k)_{\text{vertex}}) form or graphed.</td>
</tr>
<tr>
<td>• The equation of a line can be determined by two points on the line or by the slope and a point on the line.</td>
<td>• Describe the parent function represented by a scatterplot.</td>
</tr>
</tbody>
</table>
### ALGEBRA, FUNCTIONS AND DATA ANALYSIS

#### STANDARD AFDA.3

The student will collect and analyze data, and generate and determine the equation for the curve of best fit in order to make predictions, and solve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world practical problems or applications using models of linear, quadratic, and exponential functions. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

#### ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

- The regression equation modeling a set of data points can be used to make predictions where appropriate. [Rewritten and reordered]
- Data and scatterplots may indicate patterns that can be modeled with a function.
- The curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate.
- Knowledge of transformational graphing using parent functions can be used to generate a mathematical model from a scatterplot that approximates the data.
- Graphing calculators/utilities can be used to collect, organize, picture, represent, and create an equation of a curve of best fit algebraic model of the for a set of data.
- Data that fit linear \(y = mx + b\), quadratic \(y = ax^2 + bx + c\), and exponential \(y = ab^x\), and logarithmic models arise from practical situations.
- Two variables may be strongly associated without a cause-and-effect relationship existing between them.
- Each data point may be considered to be comprised of two parts: fit (the part explained by the model) and residual (the result of chance variation or of variables not measured).

#### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Write/Determine an equation for the line/curve of best fit, given a set of no more than twenty data points in a table, on a graph, or from a practical situation.
- Make predictions about unknown outcomes, using data, scatterplots, or the equation of the line/curve of best fit.
- Solve practical problems involving an equation of the curve of best fit.
- Evaluate the reasonableness of a mathematical model of a practical situation.
- Collect and analyze data to make decisions and justify conclusions. [Combined]
- Investigate scatterplots to determine if patterns exist, and identify the patterns. [Combined]
- Find an equation for the curve of best fit for data, using a graphing calculator. Models will include linear, quadratic, exponential, and logarithmic functions. [Combined]
- Make predictions, using data, scatterplots, or equation of curve of best fit. [Combined]
**TOPIC: ALGEBRA AND FUNCTIONS**

**STANDARD AFDA.3**

The student will collect and analyze data, and determine the equation of the curve of best fit in order to make predictions, and solve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world practical problems or applications using models of linear, quadratic, and exponential functions. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Residual = Actual – Fitted</td>
<td></td>
</tr>
<tr>
<td>• Least squares regression generates the equation of the line that minimizes the sum of the squared distances between the data points and the line.</td>
<td></td>
</tr>
<tr>
<td>• A correlation coefficient measures the degree of association between two variables that are related linearly.</td>
<td></td>
</tr>
<tr>
<td>• Rounding that occurs during intermediate steps of problem solving may reduce the accuracy of the final answer.</td>
<td></td>
</tr>
<tr>
<td>• Evaluation of the reasonableness of a mathematical model of a practical situation involves asking questions including:</td>
<td></td>
</tr>
<tr>
<td>- “Is there another curve (linear, quadratic, or exponential) that better fits the data?”</td>
<td></td>
</tr>
<tr>
<td>- “Does the curve of best fit make sense?”</td>
<td></td>
</tr>
<tr>
<td>- “Could the curve of best fit be used to make reasonable predictions?”</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Given a set of data, determine the model that would best describe the data. [Combined]</td>
</tr>
<tr>
<td>• Describe the errors inherent in extrapolation beyond the range of the data.</td>
</tr>
<tr>
<td>• Estimate the correlation coefficient when given data and/or scatterplots.</td>
</tr>
</tbody>
</table>
### ALGEBRA, FUNCTIONS AND DATA ANALYSIS

#### TOPIC: ALGEBRA AND FUNCTIONS

**STANDARD AFDA.4**

The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate multiple representations of functions for analysis, interpretation, and prediction.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDING</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UNDERSTANDING THE STANDARD</strong></td>
<td><strong>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</strong></td>
</tr>
<tr>
<td>The most appropriate representation of a function depends on the questions to be answered and/or the analysis to be done.</td>
<td>Given an equation, graph a linear, quadratic, exponential or logarithmic function with the aid of a graphing calculator.</td>
</tr>
<tr>
<td>Given data may be represented as discrete points or as a continuous graph with respect to the real-world practical context.</td>
<td>Make predictions given a table of values, a graph, or an algebraic formula.</td>
</tr>
<tr>
<td>Real-world practical data may best be represented as a table, a graph, or as a formula.</td>
<td>Describe relationships between data represented in a table, in a scatterplot, and as elements of a function.</td>
</tr>
<tr>
<td></td>
<td>Determine the appropriate representation of data derived from real-world situations.</td>
</tr>
<tr>
<td></td>
<td>Analyze and interpret the data in context of the real-world practical situation.</td>
</tr>
<tr>
<td></td>
<td>Use a graphing utility to graph, analyze, interpret, and make predictions.</td>
</tr>
</tbody>
</table>
# ALGEBRA, FUNCTIONS AND DATA ANALYSIS

## STANDARD AFDA.5

The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

## ESSENTIAL UNDERSTANDINGS

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Linear programming models an optimization process.</td>
</tr>
<tr>
<td>• A linear programming model consists of a system of constraints and an objective quantity that can be maximized or minimized.</td>
</tr>
<tr>
<td>• Any maximum or minimum value will occur at a corner point of a feasible region.</td>
</tr>
</tbody>
</table>

## ESSENTIAL KNOWLEDGE AND SKILLS

<table>
<thead>
<tr>
<th>Knowledge and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Model practical problems with systems of linear inequalities.</td>
</tr>
<tr>
<td>• Solve systems of no more than four linear inequalities with pencil and paper and using a graphing calculator utility.</td>
</tr>
<tr>
<td>• Solve systems of no more than four equations algebraically and graphically.</td>
</tr>
<tr>
<td>• Identify the feasibility region of a system of linear inequalities.</td>
</tr>
<tr>
<td>• Identify the coordinates of the corner points of a feasibility region.</td>
</tr>
<tr>
<td>• Find and describe the maximum or minimum value for the function defined over the feasibility region.</td>
</tr>
<tr>
<td>• Describe the meaning of the absolute maximum or absolute minimum value within its context. [Combined with bullet above]</td>
</tr>
</tbody>
</table>
ALGEBRA, FUNCTIONS AND DATA ANALYSIS  
STANDARD AFDA.6  
The student will calculate probabilities. Key concepts include:  
   a) conditional probability;  
   b) dependent and independent events;  
   c) addition and multiplication rules mutually exclusive events;  
   d) counting techniques (permutations and combinations); and  
   e) Law of Large Numbers.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The Fundamental Counting Principle states that if one decision can be made $n$ ways and another can be made $m$ ways, then the two decisions can be made $nm$ ways.</td>
<td></td>
</tr>
<tr>
<td>• <strong>Permutations</strong> are used to calculate the number of possible arrangements of objects. [Reordered]</td>
<td></td>
</tr>
<tr>
<td>• <strong>Combinations</strong> are used to calculate the number of possible selections of objects without regard to the order selected. [Reordered]</td>
<td></td>
</tr>
<tr>
<td>• A sample space is the set of all possible mutually exclusive outcomes of a random experiment.</td>
<td></td>
</tr>
<tr>
<td>• An event is a subset of the sample space.</td>
<td></td>
</tr>
<tr>
<td>• $P(E)$ is a way to represent the probability that the event $E$ occurs.</td>
<td></td>
</tr>
<tr>
<td>• Mutually exclusive events are events that cannot both occur simultaneously.</td>
<td></td>
</tr>
<tr>
<td>• Mutually exclusive events are calculated using the addition or multiplication rules.</td>
<td></td>
</tr>
<tr>
<td>• If $A$ and $B$ are mutually exclusive then $P(A \cup B) = P(A) + P(B)$.</td>
<td></td>
</tr>
<tr>
<td>• The complement of event $A$ consists of all outcomes in which</td>
<td></td>
</tr>
<tr>
<td>• The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
<td></td>
</tr>
<tr>
<td>• Analyze, interpret and make predictions based on theoretical probability within practical context. (a,b,c,e) [Reordered]</td>
<td></td>
</tr>
<tr>
<td>• Determine conditional probabilities for dependent, independent, and mutually exclusive events. (a,b,c) [Reordered]</td>
<td></td>
</tr>
<tr>
<td>• Represent and calculate probabilities using Venn diagrams and probability trees. (a) [Reordered]</td>
<td></td>
</tr>
<tr>
<td>• Compare and contrast permutations and combinations. [Reordered]</td>
<td></td>
</tr>
<tr>
<td>• Calculate the number of permutations of $n$ objects taken $r$ at a time. [Reordered]</td>
<td></td>
</tr>
<tr>
<td>• Calculate the number of combinations of $n$ objects taken $r$ at a time. [Reordered]</td>
<td></td>
</tr>
<tr>
<td>• Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events. (b,c)</td>
<td></td>
</tr>
<tr>
<td>• Given two or more events in a problem setting, determine if the events are complementary, dependent, independent, and/or mutually exclusive. (b,c)</td>
<td></td>
</tr>
</tbody>
</table>
ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.6
The student will calculate probabilities. Key concepts include:
1) conditional probability;
2) dependent and independent events;
3) addition and multiplication rules of mutually exclusive events;
4) counting techniques (permutations and combinations); and
5) Law of Large Numbers.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>event A does not occur.</td>
<td>Find conditional probabilities for dependent, independent, and mutually exclusive events. [Reordered]</td>
</tr>
<tr>
<td>• $P(B</td>
<td>A)$ is the probability that B will occur given that A has already occurred. $P(B</td>
</tr>
<tr>
<td>• Venn diagrams may be used to examine conditional probabilities.</td>
<td>• Analyze, interpret and make predictions based on theoretical probability within real-world context. [Reordered]</td>
</tr>
</tbody>
</table>

- $P(B|A) = \frac{P(A \cap B)}{P(A)}$
- $P(A \cap B) = P(A)P(B|A)$

- Two events, A and B, are independent if the occurrence of one does not affect the probability of the occurrence of the other. If A and B are not independent, then they are said to be dependent.
- If A and B are independent events, then $P(A \cap B) = P(A)P(B)$.
- Permutations are used to calculate the number of possible arrangements of objects when the order matters. [Reordered]

- Given a real-world practical situation, determine when to use permutations or combinations. (d) [Combined with bullet above]
ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.6

The student will calculate probabilities. Key concepts include:
   a) conditional probability;
   b) dependent and independent events;
   c) addition and multiplication rules; mutually exclusive events;
   d) counting techniques (permutations and combinations); and
   e) Law of Large Numbers.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Combinations are used to calculate the number of possible selections of objects without regard to the order selected. [Reordered]</td>
<td></td>
</tr>
<tr>
<td>- A permutation is the number of possible ways to arrange a group of objects without repetition and when order matters (e.g., the outcome 1,2,3 is different from the outcome 3,2,1 when order matters; therefore, both arrangements would be included in the possible outcomes).</td>
<td></td>
</tr>
<tr>
<td>- A combination is the number of possible ways to select or arrange objects when there is no repetition and order does not matter (e.g., the outcome 1,2,3 is the same as the outcome 3,2,1 when order does not matter; therefore, both arrangements would not be included in the possible outcomes).</td>
<td></td>
</tr>
<tr>
<td>- The Law of Large Numbers states that as a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.</td>
<td></td>
</tr>
</tbody>
</table>

ESSENTIAL KNOWLEDGE AND SKILLS
ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.7

The student will analyze the normal distribution. Key concepts include:

a) describe the distribution of a univariate data set using descriptive statistics characterizing of normally distributed data;
b) identify and describe properties of a normal distribution; percentiles;
c) interpret and compare z-scores for normally distributed data; normalizing data using z-scores; and
d) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve and probability.

ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

- Analysis of the descriptive statistical information generated by a univariate data set includes the relationships between central tendency, dispersion, and position.
- The normal distribution curve is a family of symmetrical curves defined by the mean and the standard deviation. [Reordered]
- Areas under the curve represent probabilities associated with continuous distributions.
- The focus of this standard is on the interpretation of descriptive statistics, z-scores, probabilities, and their relationship to the normal curve in the context of a data set.
- Descriptive statistics include measures of center (mean, median, mode) and dispersion or spread (variance and standard deviation).
- Variance ($\sigma^2$) and standard deviation ($\sigma$) measure the spread of data about the mean in a data set.
- Standard deviation is expressed in the original units of measurement of the data.
- The greater the value of the standard deviation, the further the data tend to be dispersed from the mean.
- In order to develop an understanding of standard deviation as a

ESSENTIAL KNOWLEDGE AND SKILLS

- Describe the distribution of a univariate data set using descriptive statistics including measures of center and dispersion (spread). (a)
- Interpret mean, median, mode, range, interquartile range, variance, and standard deviation of a univariate data set in terms of the problem’s context.
- Explain the influence of outliers on a univariate data set.
- Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.
- Identify the properties of a normal probability distribution. (ba)
- Describe how the standard deviation and the mean affect the graph of the normal distribution. (ba)
- Given standard deviation and mean, calculate and interpret z-score for a data point. (eb)
- Compare two sets of normally distributed data using a standard normal distribution and z-scores, given mean and standard deviation. (eb)
- Represent probability as area under the curve of a standard normal distribution. (dc)
ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.7

The student will analyze the normal distribution. Key concepts include:

a) Describe the distribution of a univariate data set using descriptive statistics; characteristics of normally distributed data;

b) Identify and describe properties of a normal distribution; percentiles;

c) Interpret and compare z-scores for normally distributed data; normalizing data using z-scores; and

d) Apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve; area under the standard normal curve and probability.

ESSENTIAL UNDERSTANDINGS
UNDERSTANDING THE STANDARD

- Measure of dispersion (spread), students should have experiences analyzing the formulas for and the relationship between variance and standard deviation.

- The normal distribution curve is the family of symmetrical, bell-shaped curves defined by the mean and the standard deviation of a data set. The arithmetic mean ($\mu$) is located on the line of symmetry of the curve and is approximately equivalent to the median and mode of the data set.

- The normal curve is a probability distribution and the total area under the curve is 1.

- For a normal distribution, approximately 68 percent of the data fall within one standard deviation of the mean, approximately 95 percent of the data fall within two standard deviations of the mean, and approximately 99.7 percent of the data fall within three standard deviations of the mean. This is often referred to as the Empirical Rule or the 68-95-99.7 rule.

ESSENTIAL KNOWLEDGE AND SKILLS

- Use a graphing utility or a table of Standard Normal Probabilities to determine probabilities associated with areas under the standard normal curve based on z-scores. (d)

- Use a graphing utility to represent curves of a normally distributed data set. (a,b,c,d) [Moved to US]

- Use a graphing utility to investigate, represent, and determine relationships between a normally distributed data set and its descriptive statistics. (a,b,c,d)

  Determine the probability of a given event, using the normal distribution.
ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.7

The student will analyze the normal distribution. Key concepts include:

a) describe the distribution of a univariate data set using descriptive statistics characteristics of normally distributed data;

b) identify and describe properties of a normal distribution percentiles;

c) interpret and compare z-scores for normally distributed data normalizing data using z-scores; and

d) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve area under the standard normal curve and probability.

ESSENTIAL UNDERSTANDING

UNDERSTANDING THE STANDARD

Note: This chart illustrates percentages that correspond to subdivisions in one standard deviation increments. Percentages for other subdivisions require the table of Standard Normal Probabilities or a graphing utility.

- The mean and standard deviation of a normal distribution affect the location and shape of the curve. The vertical line of symmetry of the normal distribution falls at the mean. The greater the standard deviation, the wider ("flatter" or "less peaked") the distribution of the data.

- A z-score derived from a particular data value tells how many
ALGEBRA, FUNCTIONS AND DATA ANALYSIS

STANDARD AFDA.7

The student will analyze the normal distribution. Key concepts include:

- a) describe the distribution of a univariate data set using descriptive statistics characteristics of normally distributed data;
- b) identify and describe properties of a normal distribution percentiles;
- c) interpret and compare z-scores for normally distributed data using z-scores; and
- d) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve under the standard normal curve and probability.

ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard deviations that data value falls above or below the mean of the data set. It is positive if the data value lies above the mean and negative if the data value lies below the mean.</td>
</tr>
<tr>
<td>A standard normal distribution is the set of all z-scores. The mean of the data in a standard normal distribution density function is 0 and the standard deviation is 1. This allows for the comparison of unlike normal data.</td>
</tr>
<tr>
<td>The table of Standard Normal Probabilities and graphing utilities may be used to determine normal distribution probabilities.</td>
</tr>
<tr>
<td>Given a z-score (z), the table of Standard Normal Probabilities (z-table) shows the area under the curve to the left of z. This area represents the proportion of observations with a z-score less than the one specified. Table rows show the z-score’s whole number and tenths place. Table columns show the hundredths place.</td>
</tr>
<tr>
<td>Graphing utilities can be used to represent a normally distributed data set and explore relationships between the data set and its descriptive statistics.</td>
</tr>
<tr>
<td>The amount of data that falls within 1, 2, or 3 standard deviations</td>
</tr>
</tbody>
</table>
**ALGEBRA, FUNCTIONS AND DATA ANALYSIS**

**STANDARD AFDA.7**

The student will analyze the normal distribution. Key concepts include:

- **a)** describe the distribution of a univariate data set using descriptive statistics characteristics of normally distributed data;
- **b)** identify and describe properties of a normal distribution percentiles;
- **c)** interpret and compare $z$-scores for normally distributed data normalizing data using $z$-scores; and
- **d)** apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve and probability.

### ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>of the mean is constant and the basis of $z$-score data normalization.</td>
</tr>
</tbody>
</table>
ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.8

The student will design and conduct an experiment/survey. Key concepts include:
   a) sample size;
   b) sampling technique;
   c) controlling sources of bias and experimental error;
   d) data collection; and
   e) data analysis and reporting.

---

**ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD**

- The value of a sample statistic may vary from sample to sample, even if the simple random samples are taken repeatedly from the population of interest.
- Poor data collection can lead to misleading and meaningless conclusions.
- Considerations such as sample size, randomness, and bias affect experimental design.
- The purpose of sampling is to provide sufficient information so that population characteristics may be inferred.
- Inherent bias diminishes as sample size increases.
- Experiments must be carefully designed in order to detect a cause-and-effect relationship between variables.
- Principles of experimental design include comparison with a control group, randomization, and blindness.
- The precision, accuracy and reliability of data collection can be analyzed and described.

---

**ESSENTIAL KNOWLEDGE AND SKILLS**

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
- Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling. (a,b) [Reordered]
- Determine which sampling technique is best, given a particular context. (b) [Reordered]
- Compare and contrast controlled experiments and observational studies and the conclusions one may draw from each. [Reordered]
- Identify biased sampling methods. (c)
- Given a plan for a survey, identify possible sources of bias, and describe ways to reduce bias. (c) [Reordered]
- Plan and conduct an experiment or survey. The experimental design should address control, randomization, and minimization of experimental error. (a,b,c,d) [Reordered]
- Compare and contrast controlled experiments and observational studies and the conclusions one may draw from each. (e) [Reordered]
- Write a report describing the experiment/survey and the resulting
### ALGEBRA, FUNCTIONS AND DATA ANALYSIS

#### STANDARD AFDA.8

The student will design and conduct an experiment/survey. Key concepts include:

- a) sample size;
- b) sampling technique;
- c) controlling sources of bias and experimental error;
- d) data collection; and
- e) data analysis and reporting.

#### ESSENTIAL UNDERSTANDINGS

**UNDERSTANDING THE STANDARD**

- data and analysis. (e) [Reordered]
  - Select a data collection method appropriate for a given context.
  - Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling. [Reordered]
  - Determine which sampling technique is best, given a particular context. [Reordered]
  - Plan and conduct an experiment or survey. The experimental design should address control, randomization, and minimization of experimental error. (a,b,c,d)-[Reordered]
  - Design a survey instrument.
  - Given a plan for a survey, identify possible sources of bias, and describe ways to reduce bias. [Reordered]
  - Write a report describing the experiment/survey and the resulting data and analysis. [Reordered]
The 2009-2016 Mathematics Standards of Learning Curriculum Framework is a companion document to the 2009-2016 Mathematics Standards of Learning and amplifies the Mathematics Standards of Learning by further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

**Essential Understandings**
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

**Understanding the Standard**
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

**Essential Knowledge and Skills**
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate in each standard. This is not meant to be an exhaustive list of student expectations nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving
Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication
Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning
Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections
Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations
Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Computational development of conceptual understanding and computational skills can enhance student’s problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
ALGEBRA II
STANDARD AII.1

The student, given rational, radical, or polynomial expressions, will
a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions
containing rational exponents;
c) write radical expressions as expressions containing rational exponents and vice versa; and
d) factor polynomials completely in one or two variables.

ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

- Computational skills applicable to numerical fractions also apply to rational expressions involving variables.
- Radical expressions can be written and simplified using rational exponents.
- Only radicals with a common radicand and index can be added or subtracted, which may require rewriting a radical with a lower base and different index.
- A relationship exists among arithmetic complex fractions, algebraic complex fractions, and rational numbers.
- The complete factorization of polynomials has occurred when each factor is a prime polynomial.
- Pattern recognition can be used to determine complete factorization of a polynomial.
- Polynomials may be factored in various ways, including, but not limited to grouping or applying general patterns including such as difference of squares, sum and difference of cubes, and perfect square trinomials.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Add, subtract, multiply, and divide rational algebraic expressions. (a)
- Simplify a rational algebraic expression with common monomial or binomial factors. Algebraic expressions should be limited to linear and quadratic expressions. (a)
- Recognize a complex algebraic fraction, and simplify it as a quotient or product of simple algebraic fractions. (a)
- Simplify radical expressions containing positive rational numbers and variables. (b)
- Convert from between radical expressions notation to and expressions containing rational exponents notation, and vice versa. (b)
- Add and subtract radical expressions. (b)
- Multiply and divide radical expressions. Simplification may include not requiring rationalizing the denominators. (b)
### ALGEBRA II

#### STANDARD AII.1

The student, given rational, radical, or polynomial expressions, will

- a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
- b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;
- c) write radical expressions as expressions containing rational exponents and vice versa [Moved to EKS]; and
- d) factor polynomials completely in one or two variables.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Factor polynomials in one or two variables with no more than four terms completely over the set of integers by applying general patterns including difference of squares, sum and difference of cubes, and perfect square trinomials. Factors of the polynomial should be constant, linear, or quadratic. (c)</td>
<td></td>
</tr>
<tr>
<td>• Factor polynomials completely over the integers. [Combined]</td>
<td></td>
</tr>
<tr>
<td>• Verify polynomial identities including the difference of squares, sum and difference of cubes, and perfect square trinomials. (c)²</td>
<td></td>
</tr>
</tbody>
</table>

²Revised March 2011
The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world practical problems, including writing the first $n$ terms, finding determining the $n^{th}$ term, and evaluating summation formulas. Notation will include $\Sigma$ and $a_n$.

**ESSENTIAL UNDERSTANDING**

- Sequences and series arise from real-world practical situations.
- The study of sequences and series is an application of the investigation of patterns.
- A sequence is a function whose domain is the set of natural numbers.
- Sequences can be defined explicitly and recursively.

**ESSENTIAL KNOWLEDGE AND SKILLS**

- Distinguish between a sequence and a series.
- Generalize patterns in a sequence using explicit and recursive formulas.
- Use and interpret the notations $\Sigma$, $n$, $n^{th}$ term, and $a_n$.
- Given the formula, find determine $a_n$ (the $n^{th}$ term) for an arithmetic or a geometric sequence.
- Given formulas, write the first $n$ terms and find determine the sum, $S_n$, of the first $n$ terms of an arithmetic or geometric series.
- Given the formula, find determine the sum of a convergent infinite series.
- Model real-world practical situations using sequences and series.
### ALGEBRA II
### STANDARD AII.23

The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of $i$, and identify field properties that are valid for the complex numbers.

#### ESSENTIAL UNDERSTANDINGS

**UNDERSTANDING THE STANDARD**

- Complex numbers can be classified as are organized into a hierarchy of subsets: either real or imaginary numbers.
- A complex number multiplied by its conjugate is a real number.
- Equations having no real number solutions may have solutions in the set of complex numbers.
- Field Algebraic properties apply to complex numbers as well as real numbers.
- All complex numbers can be written in the form $a + bi$ where $a$ and $b$ are real numbers and $i$ is the imaginary unit that satisfies the equation $i^2 = -1$ (e.g., $3 + 2i$; $\pm \sqrt{-9} = 0 \pm 3i$; $5 = 5 + 0i$).

#### ESSENTIAL KNOWLEDGE AND SKILLS

**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**

- Recognize that the square root of $-1$ is represented as $i$.
- Determine which field properties apply to the complex number system.
- Simplify radical expressions containing negative rational numbers and express in $a + bi$ form.
- Simplify powers of $i$.
- Add, subtract, and multiply complex numbers.
- Place the following sets of numbers in a hierarchy of subsets: complex, pure imaginary, real, rational, irrational, integers, whole, and natural.
- Write a real number in $a+bi$ form.
- Write a pure imaginary number in $a+bi$ form.
**TOPIC: EQUATIONS AND INEQUALITIES**

**ALGEBRA II**

**STANDARD AII.34**

The student will solve, algebraically and graphically,

a) absolute value **linear** equations and inequalities;

b) quadratic equations over the set of complex numbers;

c) equations containing rational algebraic expressions; and

d) equations containing radical expressions.

Graphing calculators will be used for solving and for confirming the algebraic solutions. [Moved to EKS]

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**ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD**

- A quadratic function whose graph does not intersect the x-axis has roots with imaginary components.
- The quadratic formula can be used to solve any quadratic equation.
- The quadratic formula can be derived by applying the completion of squares to any quadratic equation in standard form. [Moved from EKS]
- The value of the discriminant of a quadratic equation can be used to describe the number and type of real and complex solutions.
- Solutions of quadratic equations are real or a sum or difference of a real and imaginary component.
- Imaginary/Complex solutions occur in conjugate pairs.
- Quadratic equations with exactly one real root can be referred to as having one distinct root with a multiplicity of two. For instance, the quadratic equation, \( x^2 - 4x + 4 \), has two identical factors, giving one real root with a multiplicity of two.

**ESSENTIAL KNOWLEDGE AND SKILLS**

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Solve absolute value **linear** equations or inequalities in one variable algebraically. (a)
- Represent solutions to absolute value linear inequalities in one or two variables graphically. (a)
- Solve absolute value **linear** equations and inequalities algebraically and graphically. (a)
- Solve a quadratic equation over the set of complex numbers using an appropriate strategy algebraically. (b)
- Calculate the discriminant of a quadratic equation to determine the number and type of real and complex solutions. (b)
- Solve rational equations with real solutions containing rational, factorable algebraic expressions with monomial or binomial denominators algebraically and graphically. Algebraic expressions should be limited to linear and quadratic expressions. (c)
ALGEBRA II
STANDARD AII.34

The student will solve, algebraically and graphically,
  a) absolute value linear equations and inequalities;
  b) quadratic equations over the set of complex numbers;
  c) equations containing rational algebraic expressions; and
  d) equations containing radical expressions.

Graphing calculators will be used for solving and for confirming the algebraic solutions. [Moved to EKS]

**ESSENTIAL UNDERSTANDINGSTHIS UNDERSTANDING THE STANDARD**

| • The definition of absolute value (for any real numbers $a$ and $b,$ where $b \geq 0,$ if $|a| = b,$ then $a = b$ or $a = -b$) is used in solving absolute value equations and inequalities. |
| • Absolute value inequalities in one variable can be solved algebraically or by using a compound statement. |
| • Compound statements representing solutions of an inequality in one variable can be represented graphically on a number line. |
| • Real-world problems can be interpreted, represented, and solved using equations and inequalities. |
| • The process of solving radical or rational equations can lead to extraneous solutions. |
| • An extraneous solution is a solution of the simplified form of an equation that does not satisfy the original equation. |
| • Equations can be solved in a variety of ways. |
| • The zeros of the function are the root(s) or solution(s) of the equation that is formed by setting the given expression equal to zero. |

**ESSENTIAL KNOWLEDGE AND SKILLS**

| • Solve an equation containing no more than one radical expression algebraically and graphically. (d) |
| • Solve equations and verify possible algebraic solutions to an equation using a graphing utility containing rational or radical expressions. (a,b,c,d) [Moved from standard] |
| • Apply an appropriate equation to solve a real-world problem. |
| • Recognize that the quadratic formula can be derived by applying the completion of squares to any quadratic equation in standard form.² [Moved to US] |

²Revised March 2011
ALGEBRA II
STANDARD AII.34

The student will solve, algebraically and graphically,
a) absolute value linear equations and inequalities;
b) quadratic equations over the set of complex numbers;
c) equations containing rational algebraic expressions; and
d) equations containing radical expressions.

Graphing calculators will be used for solving and for confirming the algebraic solutions. [Moved to EKS]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>UNDERSTANDING THE STANDARD</th>
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</thead>
<tbody>
<tr>
<td>• The zeros, roots, or solutions of a function are the values of x that make f(x) = 0</td>
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</tr>
<tr>
<td>• The real zeros of a function are the x-intercepts of that function.</td>
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<tr>
<td>• Radical expressions may be converted to expressions using rational exponents.</td>
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<tr>
<td>• The equation of an inverse variation is a rational function.</td>
<td></td>
</tr>
<tr>
<td>• Set builder notation may be used to represent solution sets of equations and inequalities.</td>
<td></td>
</tr>
<tr>
<td>• Solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation. Examples may include:</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation/Inequality</th>
<th>Set Notation</th>
<th>Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>x = 3</td>
<td>{3}</td>
<td></td>
</tr>
<tr>
<td>x = 3 or x = 5</td>
<td>{3, 5}</td>
<td></td>
</tr>
<tr>
<td>0 ≤ x &lt; 3</td>
<td>{x: 0 ≤ x &lt; 3}</td>
<td>[0, 3)</td>
</tr>
<tr>
<td>y ≥ 3</td>
<td>{y: y ≥ 3}</td>
<td>[3, ∞)</td>
</tr>
<tr>
<td>Empty (null) set ∅</td>
<td>{}</td>
<td></td>
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</table>
ALGEBRA II
STANDARD AII.45

The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic equations, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions. [Moved to EKS]

<table>
<thead>
<tr>
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<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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<tbody>
<tr>
<td>• Quadratic equations included in this standard will only include those that can be represented as parabolas of the form $y = ax^2 + bx + c$ where $a \neq 0$.</td>
<td>• Predict Determine the number of solutions to a nonlinear-linear-quadratic and quadratic-quadratic system of two equations in two variables.</td>
</tr>
<tr>
<td>• Solutions of a nonlinear system of equations are numerical values that satisfy every equation in the system.</td>
<td>• Solve a linear-quadratic system of two equations in two variables algebraically and graphically.</td>
</tr>
<tr>
<td>• A linear-quadratic system of equations may have 0, 1, or 2 solutions.</td>
<td>• Solve a quadratic-quadratic system of two equations in two variables algebraically and graphically.</td>
</tr>
<tr>
<td>• A quadratic-quadratic system of equations may have 0, 1, 2, or infinite number of solutions.</td>
<td>• Solve systems of equations and verify solutions of systems of equations with a graphing utility. [Moved from standard]</td>
</tr>
<tr>
<td>• The coordinates of points of intersection in any system of equations are solutions to the system.</td>
<td></td>
</tr>
<tr>
<td>• Real-world Practical problems can be interpreted, represented, and solved using systems of equations.</td>
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</tr>
</tbody>
</table>
**ALGEBRA II**  
**STANDARD AII.6**  
For absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions, the student will

a) recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families; and will

b) use knowledge of transformations to convert between graphic and symbolic forms of equations and the corresponding graph of functions.

A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions. [Moved to EKS]

### ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

- The transformation of a function called a pre-image changes the size, shape, and/or position of the function to a new function called the image.
- The graphs/equations for a family of functions can be determined using a transformational approach.
- The graph of a parent function is an anchor graph from which other graphs are derived using transformations. [Reordered]
- Transformations of functions may require the domain to be restricted.
- Transformations of graphs include
  - Translations (horizontal and/or vertical shifting of a graph);
  - Reflections (over the x-axis and/or y-axis); and
  - Dilations (horizontal or vertical stretching and compressing graphs).
- A parent graph is an anchor graph from which other graphs are derived with transformations. [Reordered]
- The reflection of a function over the line $y = x$ represents the

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Recognize the general shape of function families. (a)
- Recognize graphs of parent functions. (a)
- Identify the equation from a graph of a function. (b)
- Identify the graph of a function from the equation. (b)
- Write the equation of a function given the graph. (b)
- Graph a transformation of a parent function, given the equation. (b)
- Identify the transformation(s) of a function, from a graph or from the equation. Transformations of exponential and logarithmic functions, given a graph, should be limited to a single transformation. (b)
- Investigate and verify transformations of functions using a graphing utility. (a, b) [Moved from standard]
- Given a transformation of a parent function, identify the graph of the transformed function. [Rewritten and combined]
ALGEBRA II  
STANDARD AII.6

For absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions, the student will

a) recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families; and will

b) use knowledge of transformations to convert between graphic and symbolic forms of equations and the corresponding graph of functions.

A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions. [Moved to EKS]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>inverse of a function.</td>
<td>• Given the equation and using a transformational approach, graph a function. [Rewritten and combined]</td>
</tr>
<tr>
<td></td>
<td>• Given the graph of a function, identify the parent function. [Rewritten and combined]</td>
</tr>
<tr>
<td></td>
<td>• Given the graph of a function, identify the transformations that map the preimage to the image in order to determine the equation of the image. [Rewritten and combined]</td>
</tr>
<tr>
<td></td>
<td>• Using a transformational approach, write the equation of a function given its graph. [Rewritten and combined]</td>
</tr>
</tbody>
</table>
ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions families algebraically and graphically. Key concepts include

a) domain, and-range, and continuity including limited and discontinuous domains and ranges [Moved to EKS];
b) intervals in which a function is increasing or decreasing; [Reordered]
c) maxima and minima extrema;
d) zeros;
e) x- and y-intercepts;
f) intervals in which a function is increasing or decreasing values of a function for elements in its domain;
ge) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic asymptotes;
h) end behavior;
i) vertical and horizontal asymptotes; [Reordered]
j) inverse of a function; and
k) composition of multiple functions algebraically and graphically.

Graphing calculators will be used as a tool to assist in investigation of functions. [Moved to EKS]

---

**ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD**

- Functions may be used to model practical situations. [reordered]
- Functions describe the relationship between two variables where each input is paired to a unique output.
- Function families consist of a parent function and all transformations of the parent function.
- The domain of a function is the set of all possible values of the independent variable.
- The range of a function is the set of all possible values of the dependent variable.

**ESSENTIAL KNOWLEDGE AND SKILLS**

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify the domain, range, zeros, and intercepts of a function (linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) presented algebraically or graphically, including graphs with discontinuities. (a,d,e)
- Describe a function as continuous or discontinuous. (a)
- Describe restricted/discontinuous domains and ranges.
- Given the graph of a function, identify intervals on which the function (linear, quadratic, absolute value, square root, cube root, polynomial, exponential, and logarithmic) is increasing and/or
ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions families algebraically and graphically. Key concepts include

- domain, and-range, and continuity including limited and discontinuous domains and ranges; [Moved to EKS];
- intervals in which a function is increasing or decreasing; [Reordered]
- maxima and minima; extremum;
- zeros;
- x– and y–intercepts;
- intervals in which a function is increasing or decreasing values of a function for elements in its domain;
- connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic asymptotes;
- end behavior;
- vertical and horizontal asymptotes; [Reordered]
- inverse of a function; and
- composition of multiple functions algebraically and graphically.

Graphing calculators will be used as a tool to assist in investigation of functions. [Moved to EKS]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>• For each ( x ) in the domain of ( f ), ( x ) is a member of the input of the function ( f ), ( f(x) ) is a member of the output of ( f ), and the ordered pair ((x, f(x))) is a member of ( f ).</td>
<td>• Identify the location and value of absolute maxima and absolute minima of a function over the domain of the function graphically or by using a graphing utility. (c)</td>
</tr>
<tr>
<td>• Functions may be used to model real-world situations. [Reordered]</td>
<td>• Identify the location and value of relative maxima or relative minima of a function over some interval of the domain graphically or by using a graphing utility. (c)</td>
</tr>
<tr>
<td>• A function is said to be continuous on an interval if its graph has no breaks, jumps, or holes in that interval.</td>
<td>• For any ( x ) value in the domain of ( f ), determine ( f(x) ). (f)</td>
</tr>
<tr>
<td>• The domain and range of a function may be restricted algebraically, graphically, or by the real-world practical situation modeled by the function.</td>
<td>• Represent relations and functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic forms. Given one representation, represent the relation in</td>
</tr>
</tbody>
</table>
ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions families algebraically and graphically. Key concepts include:

- **a)** domain, and range, and continuity including limited and discontinuous domains and ranges [Moved to EKS];
- **b)** intervals in which a function is increasing or decreasing; [Reordered]
- **c)** maxima and minima extrema;
- **d)** zeros;
- **e)** $x$- and $y$-intercepts;
- **f)** intervals in which a function is increasing or decreasing values of a function for elements in its domain;
- **g)** connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic asymptotes;
- **h)** end behavior;
- **i)** vertical and horizontal asymptotes; [Reordered]
- **j)** inverse of a function; and
- **k)** composition of multiple functions algebraically and graphically.

Graphing calculators will be used as a tool to assist in investigation of functions. [Moved to EKS]

### ESSENTIAL UNDERSTANDINGS

- Discontinuous domains and ranges include those with removable (holes) and nonremovable (asymptotes) discontinuities.
- A function can be described on an interval as increasing, decreasing, or constant over a specified interval or over the entire domain of the function.
- A function, $f(x)$, is increasing over an interval if the values of $f(x)$ consistently increase over the interval as the $x$ values increase.
- A function, $f(x)$, is decreasing over an interval if the values of $f(x)$ consistently decrease over the interval as the $x$ values increase.
- A function, $f(x)$, is constant over an interval if the values of $f(x)$ remain constant over the interval as the $x$ values increase.

### ESSENTIAL KNOWLEDGE AND SKILLS

- Describe the end behavior of a function. (h) [Reordered]
- Find the equations of vertical and horizontal asymptotes of functions (rational, exponential, and logarithmic). (i)
- Describe the end behavior of a function. [Reordered]
- Find the inverse of a function (linear, quadratic, cubic, square root, and cube root). (i)
- Graph the inverse of a function as a reflection across the line $y = x$. (i)
ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions families algebraically and graphically. Key concepts include

a) domain, and range, and continuity including limited and discontinuous domains and ranges [Moved to EKS];

b) intervals in which a function is increasing or decreasing; [Reordered]

c) maxima and minima extrema;

d) zeros;

e) x- and y-intercepts;

d) intervals in which a function is increasing or decreasing values of a function for elements in its domain;

e) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic asymptotes;

f) end behavior;

i) vertical and horizontal asymptotes; [Reordered]

j) inverse of a function; and

k) composition of multiple functions algebraically and graphically.

Graphing calculators will be used as a tool to assist in investigation of functions. [Moved to EKS]

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### ESSENTIAL UNDERSTANDINGS

**UNDERSTANDING THE STANDARD**

- Solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation. Examples may include:

<table>
<thead>
<tr>
<th>Equation/ Inequality</th>
<th>Set Notation</th>
<th>Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = 3 )</td>
<td>{3}</td>
<td></td>
</tr>
<tr>
<td>( x = 3 ) or ( x = 5 )</td>
<td>{3,5}</td>
<td></td>
</tr>
<tr>
<td>( 0 \leq x &lt; 3 )</td>
<td>{x \mid 0 \leq x &lt; 3}</td>
<td>[0, 3)</td>
</tr>
<tr>
<td>( y \geq 3 )</td>
<td>{y \mid y \geq 3}</td>
<td>[3, \infty)</td>
</tr>
<tr>
<td>Empty (null) set ( \emptyset )</td>
<td>{ }</td>
<td></td>
</tr>
</tbody>
</table>

- A function, \( f \), has an absolute maximum located at \( x = a \) if \( f(a) \) is the largest value of \( f \) over its domain.

---

### ESSENTIAL KNOWLEDGE AND SKILLS

- Investigate exponential and logarithmic functions, using the graphing calculator.

- Convert between logarithmic and exponential forms of an equation with bases consisting of natural numbers.

- Find-Determine the composition of two functions algebraically and graphically. (k)

- Use algebraic composition of functions to verify two functions are inverses. (k) [Included in US]

- Investigate and analyze characteristics and multiple representations of functions with a graphing utility. \( a,b,c,d,e,f,g,h,i,j,k \) [Moved from standard]
ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions families algebraically and graphically. Key concepts include

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- h) end behavior;
- i) vertical and horizontal asymptotes; [Reordered]
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- k) composition of multiple functions algebraically and graphically.

Graphing calculators will be used as a tool to assist in investigation of functions. [Moved to EKS]

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<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
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<tbody>
<tr>
<td>• A function, $f$, has an absolute minimum located at $x = a$ if $f(a)$ is the smallest value of $f$ over its domain.</td>
<td></td>
</tr>
<tr>
<td>• Relative maximum points occur where the function changes from increasing to decreasing.</td>
<td></td>
</tr>
<tr>
<td>• A function, $f$, has a relative maximum located at $x = a$ over some interval of the domain if $f(a)$ is the largest value of $f$ on the interval.</td>
<td></td>
</tr>
<tr>
<td>• Relative minimum points occur where the function changes from decreasing to increasing.</td>
<td></td>
</tr>
</tbody>
</table>
**TOPIC STRAND:** FUNCTIONS AND STATISTICS

**ALGEBRA II**  
**STANDARD AII.7**

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions families algebraically and graphically. Key concepts include:

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- d) zeros;
- e) $x$– and $y$-intercepts;
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Graphing calculators will be used as a tool to assist in investigation of functions. [Moved to EKS]

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<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
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<tbody>
<tr>
<td>• A function, $f$, has a relative minimum located at $x = a$ over some interval of the domain if $f(a)$ is the smallest value of $f$ on the interval.</td>
<td></td>
</tr>
<tr>
<td>• A value $x$ in the domain of $f$ is an $x$-intercept or a zero of a function $f$ if and only if $f(x) = 0$.</td>
<td></td>
</tr>
<tr>
<td>• Given a polynomial function $f(x)$, the following statements are equivalent for any real number $k$, such that $f(k) = 0$;</td>
<td></td>
</tr>
<tr>
<td>- $k$ is a zero of the polynomial function $f(x)$ located at $(k, 0)$;</td>
<td></td>
</tr>
<tr>
<td>- $k$ is a solution or root of the polynomial equation $f(x) = 0$;</td>
<td></td>
</tr>
<tr>
<td>- the point $(k, 0)$ is an $x$-intercept for the graph of $f(x) = 0$; and</td>
<td></td>
</tr>
<tr>
<td>- $(x - k)$ is a factor of $f(x)$.</td>
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ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions families algebraically and graphically. Key concepts include

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d) zeros;
e) x– and y−intercepts;
f) intervals in which a function is increasing or decreasing values of a function for elements in its domain;
eg) connections between and among multiple any two representations of functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic asymptotes;
f) end behavior;
i) vertical and horizontal asymptotes; [Reordered]
j) inverse of a function; and
k) composition of multiple functions algebraically and graphically.

Graphing calculators will be used as a tool to assist in investigation of functions. [Moved to EKS]

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<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
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</tr>
</thead>
<tbody>
<tr>
<td>• Connections between multiple representations (graphs, tables, and equations) of a function can be made.</td>
<td></td>
</tr>
<tr>
<td>• End behavior describes the values of a function as x approaches positive or negative infinity. [reordered]</td>
<td></td>
</tr>
<tr>
<td>• If (a, b) is an element of a function, then (b, a) is an element of the inverse of the function. [reordered]</td>
<td></td>
</tr>
<tr>
<td>• The reflection of a function over the line y = x represents the inverse of the reflected function.</td>
<td></td>
</tr>
<tr>
<td>• A function is invertible if its inverse is also a function. For an inverse of a function to be a function, the domain of the function may need to be restricted.</td>
<td></td>
</tr>
</tbody>
</table>
ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions families algebraically and graphically. Key concepts include

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- c) maxima and minima extrema;
- d) zeros;
- e) $x$- and $y$-intercepts;
- f) intervals in which a function is increasing or decreasing values of a function for elements in its domain;
- g) connections between and among multiple two representations of functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic asymptotes;
- h) end behavior;
- i) vertical and horizontal asymptotes; [Reordered]
- j) inverse of a function; and
- k) composition of multiple functions algebraically and graphically.

Graphing calculators will be used as a tool to assist in investigation of functions. [Moved to EKS]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
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</tr>
</thead>
<tbody>
<tr>
<td>- Exponential and logarithmic functions are inverses of each other. [Reordered]</td>
<td></td>
</tr>
<tr>
<td>- Functions can be combined using composition of functions. [Reordered]</td>
<td></td>
</tr>
<tr>
<td>- Two functions, $f(x)$ and $g(x)$, are inverses of each other if $f(g(x)) = g(f(x)) = x$.</td>
<td></td>
</tr>
<tr>
<td>- Asymptotes may describe both local and global behavior of functions. [Reordered]</td>
<td></td>
</tr>
<tr>
<td>- End behavior describes a function as $x$ approaches positive and negative infinity. [Reordered]</td>
<td></td>
</tr>
<tr>
<td>- A zero of a function is a value of $x$ that makes $f(x)$ equal zero.</td>
<td></td>
</tr>
</tbody>
</table>
ALGEBRA II
STANDARD AII.7

The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions families algebraically and graphically. Key concepts include

a) domain, and-range, and continuity including limited and discontinuous domains and ranges [Moved to EKS];
b) intervals in which a function is increasing or decreasing; [Reordered]
c) maxima and minima extrema;
d) zeros;
e) \(x\)- and \(y\)-intercepts;
f) intervals in which a function is increasing or decreasing values of a function for elements in its domain;

ey) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs including concrete, verbal, numeric, graphic, and algebraic

i) asymptotes;

j) end behavior;

k) vertical and horizontal asymptotes; [Reordered]

l) inverse of a function; and

m) composition of multiple functions algebraically and graphically.

Graphing calculators will be used as a tool to assist in investigation of functions. [Moved to EKS]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Combined]</td>
<td></td>
</tr>
<tr>
<td>• If ((a, b)) is an element of a function, then ((b, a)) is an element of the inverse of the function. [Reordered]</td>
<td></td>
</tr>
<tr>
<td>• Exponential ((y = a^x)) and logarithmic ((y = \log_a x)) functions are inverses of each other. [Reordered]</td>
<td></td>
</tr>
<tr>
<td>• Functions can be combined using composition of functions. [Reordered]</td>
<td></td>
</tr>
</tbody>
</table>

Mathematics Standards of Learning Curriculum Framework 2009-2016: Algebra II
ALGEBRA II
STANDARD AII.8
The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDING</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>- The Fundamental Theorem of Algebra states that, including complex and repeated solutions, an ( n )th degree polynomial equation has exactly ( n ) roots (solutions).</td>
<td></td>
</tr>
<tr>
<td>- Solutions of polynomial equations may be real, imaginary, or a combination of real and imaginary.</td>
<td></td>
</tr>
<tr>
<td>- Imaginary solutions occur in conjugate pairs.</td>
<td></td>
</tr>
<tr>
<td>- Given a polynomial function ( f(x) ), the following statements are equivalent for any real number ( k ), such that ( f(k) = 0 ):</td>
<td></td>
</tr>
<tr>
<td>- ( k ) is a zero of the polynomial function ( f(x) ) located at ( (k, 0) );</td>
<td></td>
</tr>
<tr>
<td>- ( k ) is a solution or root of the polynomial equation ( f(x) = 0 );</td>
<td></td>
</tr>
<tr>
<td>- the point ( (k, 0) ) is an x-intercept for the graph of polynomial ( f(x) = 0 ); and</td>
<td></td>
</tr>
<tr>
<td>- ( (x - k) ) is a factor of polynomial ( f(x) ).</td>
<td></td>
</tr>
<tr>
<td>- Polynomial equations may have fewer distinct roots than the order of the polynomial. In these situations, a root may have “multiplicity.” For instance, the polynomial equation ( y = x^3 - 6x^2 + 9x ) has two identical factors, ( (x - 3) ), and one other factor, ( x ). This polynomial equation has two distinct, real roots, one with a multiplicity of 2.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>- Describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.</td>
</tr>
<tr>
<td>- Define a polynomial function in factored form, given its zeros.</td>
</tr>
<tr>
<td>- Determine a factored form of a polynomial expression from the x-intercepts of the graph of its corresponding function.</td>
</tr>
<tr>
<td>- For a function, identify zeros of multiplicity greater than 1 and describe the effect of those zeros on the graph of the function.</td>
</tr>
<tr>
<td>- Given a polynomial equation, determine the number of real and type of solutions and nonreal solutions.</td>
</tr>
</tbody>
</table>
# Algebra II
## Standard AII.9

The student will collect and analyze data, determine the equation of the curve of best fit, in order to make predictions, and solve real-world practical problems, using mathematical models of linear, quadratic, and exponential functions. Mathematical models will include polynomial, exponential, and logarithmic functions.

### Essential Understandings

**Understanding the Standard**

- Data and scatterplots may indicate patterns that can be modeled with an algebraic equation.
- The curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate.
- Knowledge of transformational graphing using parent functions can be used to generate a mathematical model from a scatterplot that approximates the data.
- Graphing calculator utilities can be used to collect, organize, picture, represent, and create an equation of a curve of best fit algebraic model of the for a set of data.

**Essential Knowledge and Skills**

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDING OF UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data that fit linear (( y = mx + b )), quadratic (( y = ax^2 + bx + c )), and exponential (( y = ab^x )) models arise from real-world practical situations.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td></td>
<td>- Collect and analyze data.</td>
</tr>
<tr>
<td></td>
<td>- Investigate scatterplots to determine if patterns exist and then identify the patterns.</td>
</tr>
<tr>
<td></td>
<td>- Find an equation for of the curve of best fit for data, using a graphing calculator utility, given a set of no more than twenty data points in a table, graph, or practical situation. Models will include polynomial, exponential, and logarithmic functions.</td>
</tr>
<tr>
<td></td>
<td>- Make predictions, using data, scatterplots, or the equation of the curve of best fit.</td>
</tr>
<tr>
<td></td>
<td>- Solve practical problems involving an equation of the curve of best fit.</td>
</tr>
<tr>
<td></td>
<td>- Given a set of data, determine the model that would best describe the data.</td>
</tr>
<tr>
<td></td>
<td>- Evaluate the reasonableness of a mathematical model of a practical situation.</td>
</tr>
<tr>
<td>Rounding that occurs during intermediate steps of problem solving may reduce the accuracy of the final answer.</td>
<td></td>
</tr>
<tr>
<td>Evaluation of the reasonableness of a mathematical model of a practical situation involves asking questions including:</td>
<td></td>
</tr>
<tr>
<td>- “Is there another curve (quadratic or exponential) that better</td>
<td></td>
</tr>
</tbody>
</table>
## ALGEBRA II
### STANDARD AII.9

The student will collect and analyze data, determine the equation of the curve of best fit, in order to make predictions, and solve real-world practical problems, using mathematical models of linear, quadratic, and exponential functions. Mathematical models will include polynomial, exponential, and logarithmic functions.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Does the curve of best fit make sense?”</td>
<td></td>
</tr>
<tr>
<td>“Could the curve of best fit be used to make reasonable predictions?”</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
</table>
**ALGEBRA II**
**STANDARD AII.10**

The student will identify, represent, create, and solve real-world problems, including practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.

<table>
<thead>
<tr>
<th><strong>UNDERSTANDING THE STANDARD</strong></th>
<th><strong>ESSENTIAL KNOWLEDGE AND SKILLS</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Real-world practical problems can be modeled, represented, and solved by using direct variation, inverse variation, joint variation, and a combination of direct and inverse variations.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• A direct variation represents a proportional relationship between two quantities. The statement “y is directly proportional to x” is translated as ( y = kx ).</td>
<td>• Translate “y varies jointly as x and z” as ( y = kxz ). [Moved to US]</td>
</tr>
<tr>
<td>• The constant of proportionality ((k)) in a direct variation is represented by the ratio of the dependent variable to the independent variable and can be referred to as the constant of variation.</td>
<td>• Translate “y is directly proportional to x” as ( y = kx ). [Moved to US]</td>
</tr>
<tr>
<td>• A direct variation can be represented by a line passing through the origin.</td>
<td>• Translate “y is inversely proportional to x” as ( y = \frac{k}{x} ). [Moved to US]</td>
</tr>
<tr>
<td>• An inverse variation represents an inversely proportional relationship between two quantities. The statement “y is inversely proportional to x” is translated as ( y = \frac{k}{x^2} ).</td>
<td>• Given a situation, determine the value of the constant of proportionality. [Combined]</td>
</tr>
<tr>
<td>• The constant of proportionality ((k)) in an inverse variation is represented by the product of the dependent variable and the independent variable and can be referred to as the constant of variation.</td>
<td>• Given a data set or practical situation, write the equation for an inverse variation.</td>
</tr>
<tr>
<td>• The graph of an inverse variation is a rational function.</td>
<td>• Given a data set or practical situation, write the equation for a joint variation.</td>
</tr>
</tbody>
</table>
| • Joint variation is a combination of direct variations. | • Set up and solve problems, including real-world practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.
ALGEBRA II
STANDARD AII.10

The student will identify, represent, create, and solve real-world problems, including practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.

**ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD**

- statement “y varies jointly as x and z” is translated as $y = kxz$.
- The value of the constant of proportionality is typically positive when applied in practical situations.

**ESSENTIAL KNOWLEDGE AND SKILLS**
ALGEBRA II
STANDARD AII.11

The student will
a) describe the distribution of a univariate data set using descriptive statistics;
ba) identify and describe properties of a normal distribution;
 eb) interpret and compare $z$-scores for normally distributed data; and
dc) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

- The focus of this standard is on the interpretation of descriptive statistics, $z$-scores, probabilities, and their relationship to the normal curve in the context of a data set.
- Descriptive statistics include measures of center (mean, median, mode) and dispersion or spread (variance and standard deviation).
- Variance ($\sigma^2$) and standard deviation ($\sigma$) measure the spread of data about the mean in a data set.
- Standard deviation is expressed in the original units of measurement of the data.
- The greater the value of the standard deviation, the further the data tend to be dispersed from the mean.
- In order to develop an understanding of standard deviation as a measure of dispersion (spread), students should have experiences analyzing the formulas for and the relationship between variance and standard deviation.
- A normal distribution curve is the family of symmetrical, bell-shaped curves defined by the mean and the standard deviation of a data set. The arithmetic mean ($\mu$) is located on the line of symmetry of the curve and is approximately equivalent to the median and mode of the data set.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Describe the distribution of a univariate data set using descriptive statistics including measures of center and dispersion (spread). (a)
- Identify the properties of a normal probability distribution. (bg)
- Describe how the standard deviation and the mean affect the graph of the normal distribution. (bg)
- Solve problems involving the relationship of the mean, standard deviation, and $z$-score of a normally distributed data set. (eb)
- Compare two sets of normally distributed data using a standard normal distribution and $z$-scores, given the mean and standard deviation. (eb)
- Represent probability as area under the curve of a standard normal probability distribution. (dc)
- Use the graphing calculator utility or a table of standard normal probability table to determine probabilities associated with areas under the standard normal curve or percentiles based on $z$-scores. (dc)
**ALGEBRA II**  
**STANDARD AII.11**

The student will

a) describe the distribution of a univariate data set using descriptive statistics;  
b) identify and describe properties of a normal distribution;  
eb) interpret and compare z-scores for normally distributed data; and  
dc) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

**ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD**

- Areas under the curve represent probabilities associated with continuous distributions.  
- The normal curve is a probability distribution and the total area under the curve is 1.  
- For a normal distribution, approximately 68 percent of the data fall within one standard deviation of the mean, approximately 95 percent of the data fall within two standard deviations of the mean, and approximately 99.7 percent of the data fall within three standard deviations of the mean. This is often referred to as the Empirical Rule or the 68-95-99.7 rule.

**ESSENTIAL KNOWLEDGE AND SKILLS**

- Use a graphing utility to represent curves of a normally distributed data set. (a,b,c,d) [Moved to US]
- Use a graphing utility to investigate, represent, and determine relationships between a normally distributed data set and its descriptive statistics. (a,b,c,d)
ALGEBRA II
STANDARD AII.11

The student will

a) describe the distribution of a univariate data set using descriptive statistics;
b) identify and describe properties of a normal distribution;
ec) interpret and compare $z$-scores for normally distributed data; and
dc) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NOTE:</strong> This chart illustrates percentages that correspond to subdivisions in one standard deviation increments. Percentages for other subdivisions require the table of Standard Normal Probabilities or a graphing utility.</td>
<td></td>
</tr>
<tr>
<td>• The mean and standard deviation of a normal distribution affect the location and shape of the curve. The vertical line of symmetry of the normal distribution falls at the mean. The greater the standard deviation, the wider (“flatter” or “less peaked”) the distribution of the data.</td>
<td></td>
</tr>
<tr>
<td>• A $z$-score derived from a particular data value tells how many standard deviations that data value falls above or below the mean of the data set. It is positive if the data value lies above the mean and negative if the data value lies below the mean.</td>
<td></td>
</tr>
<tr>
<td>• A standard normal distribution is the set of all $z$-scores. The mean of the data in a standard normal distribution is 0 and the standard deviation is 1. This allows for the comparison of unlike normal data.</td>
<td></td>
</tr>
<tr>
<td>• The table of Standard Normal Probabilities and graphing utilities may be used to determine normal distribution probabilities.</td>
<td></td>
</tr>
<tr>
<td>• Given a $z$-score ($z$), use the table of Standard Normal Probabilities</td>
<td></td>
</tr>
</tbody>
</table>
**STRAND: FUNCTIONS AND STATISTICS**

**ALGEBRA II**  
**STANDARD AII.11**

The student will

- describe the distribution of a univariate data set using descriptive statistics;
- identify and describe properties of a normal distribution;
- interpret and compare \( z \)-scores for normally distributed data; and
- apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
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</tr>
</thead>
<tbody>
<tr>
<td>(( z )-table) shows the area under the curve to the left of ( z ). This area represents the proportion of observations with a ( z )-score less than the one specified. Table rows show the ( z )-score’s whole number and tenths place. Table columns show the hundredths place.</td>
<td></td>
</tr>
<tr>
<td>- Graphing utilities can be used to represent a normally distributed data set and explore relationships between the data set and its descriptive statistics.</td>
<td></td>
</tr>
<tr>
<td>- The standard normal curve allows for the comparison of data from different normal distributions.</td>
<td></td>
</tr>
<tr>
<td>- A ( z )-score is a measure of position derived from the mean and standard deviation of data. [Combined]</td>
<td></td>
</tr>
<tr>
<td>- A ( z )-score expresses, in standard deviation units, how far an element falls from the mean of the data set. [Combined]</td>
<td></td>
</tr>
<tr>
<td>- A ( z )-score is a derived score from a given normal distribution. [Combined]</td>
<td></td>
</tr>
<tr>
<td>- A standard normal distribution is the set of all ( z )-scores. [Combined]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Combined]</td>
</tr>
</tbody>
</table>
# ALGEBRA II

## STANDARD AII.12

The student will compute and distinguish between permutations and combinations and use technology for applications.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDING</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The <strong>Fundamental Counting Principle</strong> states that if one decision can be made ( n ) ways and another can be made ( m ) ways, then the two decisions can be made ( nm ) ways.</td>
<td></td>
</tr>
<tr>
<td>• <strong>Permutations</strong> are used to calculate the number of possible arrangements of objects.</td>
<td></td>
</tr>
<tr>
<td>• <strong>Combinations</strong> are used to calculate the number of possible selections of objects without regard to the order selected.</td>
<td></td>
</tr>
<tr>
<td>• A permutation is the number of possible ways to arrange a group of objects <strong>without repetition</strong> and when order matters (e.g., the outcome 1,2,3 is different from the outcome 3,2,1 when order matters; therefore, both arrangements would be included in the possible outcomes).</td>
<td></td>
</tr>
<tr>
<td>• A combination is the number of possible ways to select or arrange objects <strong>when there is no repetition and order does not matter</strong> (e.g., the outcome 1,2,3 is the same as the outcome 3,2,1 when order does not matter; therefore, both arrangements would not be included in the possible outcomes).</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Compare and contrast permutations and combinations.</td>
</tr>
<tr>
<td>• Calculate the number of permutations of ( n ) objects taken ( r ) at a time.</td>
</tr>
<tr>
<td>• Calculate the number of combinations of ( n ) objects taken ( r ) at a time.</td>
</tr>
<tr>
<td>• Use permutations and combinations as counting techniques to solve real-world practical problems.</td>
</tr>
<tr>
<td>• Calculate and verify permutations and combinations using the graphing utility. [Moved from standard]</td>
</tr>
</tbody>
</table>
Mathematics Standards of Learning

Curriculum Framework 2009

Trigonometry

Board of Education
Commonwealth of Virginia
The 2009-2016 Mathematics Standards of Learning Curriculum Framework, is a companion document to the 2009-2016 Mathematics Standards of Learning, and amplifies the Mathematics Standards of Learning by and further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students, providing additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

**Essential Understandings**
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

**Understanding the Standard**
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instructionbackground information for the teacher (K-8). It contains content background information for the teacher (K-8). The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers’ knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

**Essential Knowledge and Skills**
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that What each student should know and be able to demonstrate in each standard is outlined. This is not meant to be an exhaustive list of student expectationsnor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student’s problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
**TOPIC STRAND:** TRIGONOMETRIC FUNCTIONS

**TRIGONOMETRY STANDARD T.1**

The student, given a point other than the origin on the terminal side of an angle, will determine the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
</table>
| • Triangular trigonometric function definitions are related to circular trigonometric function definitions.  
  • Both degrees and radians are units for measuring angles.  
  • Drawing an angle in standard position will force the terminal side to lie in a specific quadrant or axis.  
  • A point on the terminal side of an angle determines a reference triangle from which the values of the six trigonometric functions may be derived.  
  • If one trigonometric function value is known, then a triangle can be formed to use in determining the other five trigonometric function values. [Moved from T.2] | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to  
  • Define the six trigonometric functions of an angle in a right triangle.  
  • Define the six circular trigonometric functions of an angle in standard position. [Moved to T.2 EKS]  
  • Draw a reference right triangle when given a point on the terminal side of the angle in standard position.  
  • Draw a reference right triangle when given the value of a trigonometric function of the angle.  
  • Determine the value of any trigonometric function when given a point on the terminal side of an angle in standard position. [reordered]  
  • Given one trigonometric function value, determine the other five trigonometric function values. [Moved from T.2]  
  • Make the connection between the triangular and circular trigonometric functions.  
  • Recognize and draw an angle in standard position. [Combined]  
  • Show how a point on the terminal side of an angle determines a }
TRIGONOMETRY
STANDARD T.1

The student, given a point other than the origin on the terminal side of an angle in standard position, or the value of the trigonometric function of the angle, will use determine the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>reference triangle, [Combined]</td>
<td></td>
</tr>
</tbody>
</table>
## Topic: Triangular and Circular Trigonometric Functions

### Trigonometry Standard T.2

The student, given the value of one trigonometric function, will find the values of the other trigonometric functions, using the definitions and properties of the trigonometric functions. [Combined into T.1]

The student will develop and apply the properties of the unit circle in degrees and radians. [Moved and rewritten from T.3]

<table>
<thead>
<tr>
<th><strong>Essential Understandings</strong></th>
<th><strong>Essential Knowledge and Skills</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>- Triangular trigonometric function definitions are related to circular trigonometric function definitions.</td>
<td>- Define the six circular trigonometric functions of an angle in standard position. [Moved from T.1]</td>
</tr>
<tr>
<td>- If one trigonometric function value is known, then a triangle can be formed to use in finding the other five trigonometric function values. [Moved to T.1 US]</td>
<td>- Apply the properties of the unit circle to determine trigonometric function values of special angles and their related angles in both degrees and radians without using a calculator. [Moved from T.3 EKS]</td>
</tr>
<tr>
<td>Knowledge of the unit circle is a useful tool for finding all six trigonometric values for special angles (30°, 45°, 60°, and 90°).</td>
<td>- Apply the properties of the unit circle to convert between special angles expressed in radians and degrees, without using a calculator.</td>
</tr>
<tr>
<td>The relationships between the angles measures and side lengths of special right triangles (30°-60°-90° and 45°-45°-90°) are widely used in mathematics.</td>
<td>- Given one trigonometric function value, find the other five trigonometric function values. [Moved to T.1 EKS]</td>
</tr>
<tr>
<td>Special right triangles may be used to develop the unit circle.</td>
<td>- Develop the unit circle, using both degrees and radians. [Reordered]</td>
</tr>
<tr>
<td>Unit circle properties will allow special angle and related angle trigonometric values to be found without the aid of a calculator.</td>
<td>- Solve problems, using the circular function definitions and the properties of the unit circle.</td>
</tr>
<tr>
<td>Degrees and radians are units of angle measure.</td>
<td>- Recognize the connections between the coordinates of points on a unit circle and</td>
</tr>
<tr>
<td>A radian is the measure of the central angle that is determined by an arc whose length is the same as the radius of the circle.</td>
<td></td>
</tr>
<tr>
<td>There is a connection between sides and angles of special right triangles, the unit circle, and the coordinate plane.</td>
<td></td>
</tr>
</tbody>
</table>
### Standard T.2

The student, given the value of one trigonometric function, will find the values of the other trigonometric functions, using the definitions and properties of the trigonometric functions. [Combined into T.1]

The student will develop and apply the properties of the unit circle in degrees and radians. [Moved and rewritten from T.3]

<table>
<thead>
<tr>
<th>Essential Understandings of Understanding the Standard</th>
<th>Essential Knowledge and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>— coordinate geometry;</td>
</tr>
<tr>
<td></td>
<td>— cosine and sine values; and</td>
</tr>
<tr>
<td></td>
<td>— lengths of sides of special right triangles (30°–60°–90° and 45°–45°–90°).</td>
</tr>
</tbody>
</table>
TOPIC STRAND: TRIANGULAR AND CIRCULAR APPLICATIONS OF TRIGONOMETRIC FUNCTIONS

TRIGONOMETRY

STANDARD T.93

The student will find, without the aid of a calculator, the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting angle measures from radians to degrees and vice versa. [Moved to T.2]

The student will solve problems, including practical problems, involving
a) arc length and area of sectors in circles using radians and degrees; and
b) linear and angular velocity.

ESSENTIAL UNDERSTANDINGS

UNDERSTANDING THE STANDARD

- Special angles are widely used in mathematics. [Moved to T.2 US]
- Unit circle properties will allow special angle and related angle trigonometric values to be found without the aid of a calculator. [Moved to T.2 US]
- Degrees and radians are units of angle measure.
- A radian is the measure of the central angle that is determined by an arc whose length is the same as the radius of the circle.
- The relationship between the radian measure of an angle and the length of the intercepted arc can be represented by \( s = r\theta \)
  where \( s \) is the arc length, \( r \) is the length of the radius, and \( \theta \) is the measure of the angle.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Convert between any angle expressed in radians and degrees without using a calculator. (a) [reordered]
- Derive the relationship between the radian measure of an angle and the length of the intercepted arc. (b)
- Calculate the length of an arc in radians. (a)
- Calculate the area of sectors in circles. (b)
- Solve practical problems involving linear and angular velocity. (c)
- Find trigonometric function values of special angles and their related angles in both degrees and radians. [Moved to T.2 EKS]
- Apply the properties of the unit circle without using a calculator. [Combined and moved to T.2 EKS]
- Use a conversion factor to convert from radians to degrees and vice versa without using a calculator. [Reordered]
**TOPIC STRAND: INVERSE TRIGONOMETRIC FUNCTIONS**

**EQUATIONS AND IDENTITIES**

### TRIGONOMETRY

**STANDARD T.74**

The student will find, with the aid of a calculator, the value of any trigonometric function and inverse trigonometric function.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The trigonometric function values of any angle can be found by using a calculator.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• The inverse trigonometric functions can be used to find, determine angle measures whose trigonometric function values are known.</td>
<td>• Use a calculator to find, determine the trigonometric function values of any angle in either degrees or radians.</td>
</tr>
<tr>
<td>• Calculations of inverse trigonometric function values can be related to the triangular definitions of the trigonometric functions.</td>
<td>• Define inverse trigonometric functions.</td>
</tr>
<tr>
<td>• Trigonometric functions are not invertible, because they are periodic. Domain restrictions on trigonometric functions are necessary in order to determine the inverse trigonometric function.</td>
<td>• Find, determine angle measures by using the inverse trigonometric functions when the trigonometric function values are given.</td>
</tr>
</tbody>
</table>
### Mathematics Standards of Learning Curriculum Framework 2009

#### TRIGONOMETRY

**STANDARD T.5**

The student will verify basic trigonometric identities and make substitutions, using the basic identities.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understanding the Standard</td>
<td></td>
</tr>
<tr>
<td>• Trigonometric identities can be used to simplify trigonometric expressions, equations, or identities.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Trigonometric identity substitutions can help solve trigonometric equations, verify another identity, or simplify trigonometric expressions.</td>
<td>• Use trigonometric identities to make algebraic substitutions to simplify and verify trigonometric identities. The basic trigonometric identities include</td>
</tr>
<tr>
<td></td>
<td>– reciprocal identities;</td>
</tr>
<tr>
<td></td>
<td>– Pythagorean identities;</td>
</tr>
<tr>
<td></td>
<td>– sum and difference identities;</td>
</tr>
<tr>
<td></td>
<td>– double-angle identities; and</td>
</tr>
<tr>
<td></td>
<td>– half-angle identities.</td>
</tr>
</tbody>
</table>
TOPIC STRAND: TRIGONOMETRIC EQUATIONS, GRAPHS, AND PRACTICAL PROBLEMS OF TRIGONOMETRIC FUNCTIONS

TRIGONOMETRY STANDARD T.36

The student, given one of the six trigonometric functions in standard form, will
   a) state the domain and the range of the function;
   b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;
   c) sketch the graph of the function by using transformations for at least a two-period interval; and
   d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.

ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

- The domain and range of a trigonometric function determine the scales of the axes for the graph of the trigonometric function.
- The amplitude, period, phase shift, and vertical shift are important characteristics of the graph of a trigonometric function, and each has a specific purpose in applications using trigonometric equations.
- The graph of a trigonometric function can be used to display information about the periodic behavior of a real-world situation, such as wave motion or the motion of a Ferris wheel.
- Standard form of the trigonometric functions may be written in multiple ways [e.g., \( y = A \sin(Bx + C) + D \) or \( y = A \sin[B(x + C)] + D \)] [Moved from EKS]

ESSENTIAL KNOWLEDGE AND SKILLS

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
  - State the domain and the range of a trigonometric function written in standard form. [Reordered]
  - Determine the amplitude, period, phase shift, and vertical shift, and asymptotes of a trigonometric function from the equation of the function and from the graph of the function.
  - Describe the effect of changing \( A, B, C, \) or \( D \) in the standard form of a trigonometric equation {e.g., \( y = A \sin(Bx + C) + D \) or \( y = A \cos[B(x + C)] + D \)}. [Moved to US]
  - Sketch the graph of a function written in standard form by using transformations for at least a two-period interval, including both positive and negative values for the domain. [Reordered]
  - State the domain and the range of a function written in standard form {e.g., \( y = A \sin(Bx + C) + D \)
    - or \( y = A \cos[B(x + C)] + D \)}. [Reordered, Moved to US]
  - Sketch the graph of a function written in standard form {e.g., 
    - \( y = A \sin(Bx + C) + D \) or \( y = A \cos[B(x + C)] + D \) by using...
**TOPIC STRAND:** TRIGONOMETRIC EQUATIONS, GRAPHS, AND PRACTICAL PROBLEMS OF TRIGONOMETRIC FUNCTIONS

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDING</th>
<th>UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student, given one of the six trigonometric functions in standard form, will</td>
<td>transformations for at least one period or one cycle. [Reordered, Moved to US]</td>
<td></td>
</tr>
<tr>
<td>a) state the domain and the range of the function;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) sketch the graph of the function by using transformations for at least a two-period interval; and</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**TOPIC STRAND: INVERSE GRAPHS OF TRIGONOMETRIC FUNCTIONS**

**TRIGONOMETRY STANDARD T.47**

The student will graph the six inverse trigonometric functions, identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included. [Included in EKS]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDING</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trigonometric functions are not invertible, because they are periodic. Domain restrictions on trigonometric functions are necessary in order to determine the inverse trigonometric function.</td>
</tr>
<tr>
<td></td>
<td>Restrictions on the domains of some inverse-trigonometric functions exist.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>Find the domain and range of the inverse trigonometric functions.</td>
</tr>
<tr>
<td>Use the restrictions on the domains of the inverse trigonometric functions in finding the values of the inverse trigonometric functions.</td>
</tr>
<tr>
<td>Identify the graphs of the inverse trigonometric functions.</td>
</tr>
</tbody>
</table>
**TOPIC STRAND: TRIGONOMETRIC EQUATIONS, GRAPHS, AND IDENTITIES**  
**PRACTICAL PROBLEMS**

**TRIGONOMETRY**  
**STANDARD T.68**

The student will solve trigonometric equations and inequalities that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solutions for trigonometric equations will depend on the domains.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to solve trigonometric equations with and without restricted domains algebraically and by using a graphing utility.</td>
</tr>
<tr>
<td>A calculator graphing utility can be used to find the solution of a trigonometric equation, as the points of intersection of the graphs when one side of the equation is entered in the calculator as ( Y_1 ) and the other side is entered as ( Y_2 ).</td>
<td>Solve trigonometric inequalities equations with infinite solutions algebraically and by using a graphing utility.</td>
</tr>
<tr>
<td></td>
<td>Check for reasonableness of results, and verify algebraic solutions, using a graphing utility.</td>
</tr>
</tbody>
</table>

Mathematics Standards of Learning Curriculum Framework 2009-16: Trigonometry
### Mathematics Standards of Learning Curriculum Framework 2009-16: Trigonometry

#### TOPIC: APPLICATIONS OF TRIGONOMETRIC FUNCTIONS

#### STRAND: EQUATIONS, GRAPHS, AND PRACTICAL PROBLEMS

### TRIGONOMETRY

#### STANDARD T.89

The student will identify, create, and solve real-world practical problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines. [Included in EKS]

### ESSENTIAL UNDERSTANDINGS

**UNDERSTANDING THE STANDARD**

- A real-world practical problem may be solved by using one of a variety of techniques associated with triangles.

### ESSENTIAL KNOWLEDGE AND SKILLS

**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**

- Write Create and solve a real-world practical problems involving triangles.
- Solve real-world problems involving triangles. [Combined]
- Use the trigonometric functions to model real-world practical situations.
- Identify a solution technique associated with triangles that could be used with a given problem.
- Prove Apply the sum and difference identities addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.†

†Revised March 2011
Mathematics Standards of Learning

Curriculum Framework 2009 2016

Computer Mathematics

Board of Education
Commonwealth of Virginia
Acknowledgements
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Acknowledgements
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NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

The 2009-2016 Mathematics Standards of Learning Curriculum Framework is a companion document to the 2009-2016 Mathematics Standards of Learning, and amplifies the Mathematics Standards of Learning by further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

**Essential Understandings**
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

**Understanding the Standard**
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction background information for the teacher (K-8). It contains content. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

**Essential Knowledge and Skills**
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that What each student should know and be able to demonstratedo in each standard is outlined. This is not meant to be an exhaustive list of student expectations or a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving
Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication
Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning
Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections
Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations
Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student’s problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
**Algebra Readiness**

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

**Equity**

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners. Deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
# COMPUTER MATHEMATICS

## STANDARD COM.1

The student will **design and apply computer programming techniques and skills to solve practical real-world problems in mathematics arising from consumer, business, and other applications in mathematics.** Problems will include opportunities for students to analyze data in charts, graphs, and tables and to use their knowledge of equations, formulas, and functions to solve these problems.

## ESSENTIAL UNDERSTANDING

**UNDERSTANDING THE STANDARD**

- The computer is an essential tool for mathematical problem solving in consumer-related problems.
- A subtask that has been solved previously may be reused again and again. [Moved to COM.5]
- Programming languages require the use of particular structures to express algorithms as programs.
- Designing algorithms is in the problem solving phase of computer programming.
- Real-world problems that can be modeled mathematically can be solved with a computer program.
- Data arising from probability and statistics applications can be displayed in tables and graphs and analyzed within the structure of a computer program.
- Sample mathematical problems may include:
  - the bisection method for a solution to a polynomial equation;
  - use of recursion to solve the Tower of Hanoi problem;
  - implementing a cellular automaton such as Conway’s Game of Life;
  - implementing a maze-traversing or map-coloring algorithm; and
  - determining if an input number is prime.

## ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Design and implement computer programs to solve consumer practical problems.
- Analyze and interpret graphs, charts, and tables in the design and implementation of a computer program.
- Design and implement computer programs to
  - solve practical problems arising from business; and
  - solve mathematical problems, using formulas, equations, and functions;"
### COMPUTER MATHEMATICS

**STANDARD COM.2**

The student will design, write, document, test, and debug, and document a computer program. Programming documentation will include preconditions and postconditions of program segments, input/output specifications, the step-by-step plan, the test data, a sample run, and the program listing with appropriately placed comments. [Moved to US]

### ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

- The successful completion of a structured program requires problem solving skills. Programming documentation includes the step-by-step plan, the test data, a sample run, and the program listing with appropriately placed comments.
- Documentation enables programmers to use modules written by others without having to read and understand their code.
- Comments communicate a programmer's original intent and are useful for debugging incorrect code.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Describe what a computer program is.
- Design, write, document, test, and debug a complete computer program.
- List and describe the stages processes involved in writing a computer program.
- Describe the function of an algorithm.
- Describe the interplay between hardware and software in program execution. [Moved to COM.16 EKS]
- Compare and contrast compiling and executing a program.
- Provide required documentation for a program. [Reordered]
- Determine what a given output statement will print.
- Debug a program.
- Provide required documentation for a program. [Reordered]
**TOPIC**: STRAND: PROGRAM DESIGN

### COMPUTER MATHEMATICS

**STANDARD COM.3**

The student will write program specifications that define the constraints of a given problem. These specifications will include descriptions of preconditions, postconditions, the desired output, analysis of the available input, and an indication as to whether or not the problem is solvable under the given conditions.

### ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

- A programmer begins the programming process by analyzing the problem and developing a general solution (algorithm).
- The successful completion of a structured computer program requires problem solving skills.
- Program specifications include descriptions of preconditions, postconditions, the desired input and output, analysis of the available input, and an indication as to whether or not the problem is solvable using a computer program.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Write program specifications that define the constraints of a given problem. [Reordered]
- For a given problem, describe the preconditions, postconditions, and desired input and output of a given problem.
- Determine whether or not a given problem is solvable using a computer program.
- Write program specifications that define the constraints of a problem. [Reordered]
**TOPIC** STRAND: PROGRAM DESIGN

**COMPUTER MATHEMATICS**
**STANDARD COM.4**

The student will design an **step-by-step plan (algorithm)** to solve a given problem. The plan will be in the form of a **program flowchart**, **pseudo code**, hierarchy chart, and/or data-flow diagram. [Moved to US]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• All programs are implementations of algorithms.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Algorithms used to solve problems may include flowcharts, pseudo code, and/or data-flow diagrams.</td>
<td>• Design a step-by-step plan-algorithm to solve a problem. Utilize the following problem-solving formats: flowchart; pseudo code; hierarchy chart; and data-flow diagram.</td>
</tr>
</tbody>
</table>

## COMPUTER MATHEMATICS
### STANDARD COM.5

The student will divide a given problem into manageable sections (modules) by task and implement the solution. The modules will include an appropriate user-defined functions, subroutines, and procedures. Enrichment topics might include user-defined libraries (units) and object-oriented programming.

### ESSENTIAL UNDERSTANDINGS

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Functional decomposition is a way to develop a program in which the problem is divided into sub-problems whose solutions comprise the solution to the original problem.</td>
</tr>
<tr>
<td>• A subtask that has been solved previously may be reused in other programs.</td>
</tr>
</tbody>
</table>

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Subdivide a problem into modules by task.</td>
</tr>
<tr>
<td>• Implement the solution of the problem. [Reordered]</td>
</tr>
</tbody>
</table>
| • Write task-oriented modules, including which may include
  - a user-defined function;
  - subroutines; and or
  - procedures. |
| • Determine the need for a subroutine or user-defined function. |
| • Determine the difference between and the need for internal and external subroutines and functions. |
| • Implement the solution of the problem. [Reordered] |
The student will translate a-mathematical expressions into a computer-programming statement, which involves by declaring variables, writing assignment statements, and using the order of operations.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• A variable identifies a location in memory where a data value that can be changed is stored.</td>
<td></td>
</tr>
<tr>
<td>• An assignment statement stores the value of an expression into a variable.</td>
<td></td>
</tr>
<tr>
<td>• The order of operations is</td>
<td></td>
</tr>
<tr>
<td>— Parentheses;</td>
<td></td>
</tr>
<tr>
<td>— exponents;</td>
<td></td>
</tr>
<tr>
<td>— multiplication and division in order from left to right; and</td>
<td></td>
</tr>
<tr>
<td>— addition and subtraction in order from left to right.</td>
<td></td>
</tr>
<tr>
<td>• Variable assignment statements will differ depending upon the programming language used.</td>
<td></td>
</tr>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
<td></td>
</tr>
<tr>
<td>• Translate a-mathematical expressions into a computer-programming statement.</td>
<td></td>
</tr>
<tr>
<td>• Use the order of operations to simplify expressions. [Reordered]</td>
<td></td>
</tr>
<tr>
<td>• Declare appropriately-named variables to store values used in computations.</td>
<td></td>
</tr>
<tr>
<td>• Write variable assignment statements.</td>
<td></td>
</tr>
<tr>
<td>• Use the order of operations to simplify expressions. [Reordered]</td>
<td></td>
</tr>
<tr>
<td>• Construct and evaluate expressions that include multiple arithmetic operations.</td>
<td></td>
</tr>
</tbody>
</table>
**TOPIC STRAND: PROGRAM DESIGN**

**COMPUTER MATHEMATICS**

**STANDARD COM.743**

The student will select and implement call built-in (library) functions into processing data, as appropriate.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
</table>
| • The argument of a library function is a value or expression associated with the independent variable.  
• A library function is a subroutine. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to  
• Use library functions in designing programs to process data.  
• Use library functions that are arithmetic or string operations.  
• Invoke a value-returning library function. |
**TOPIC STRAND: PROGRAM DESIGN**

**COMPUTER MATHEMATICS**
**STANDARD COM.8.14**

The student will implement conditional statements that include “if/then” statements, “if/then/else” statements, case statements, and Boolean logic.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Boolean logic is a system using variables with only two values: TRUE and FALSE.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• The “if” statement is the fundamental control structure that allows branches in the flow of control.</td>
<td>• Construct a simple logical (Boolean) expression to evaluate a given condition.</td>
</tr>
<tr>
<td>• Case statements may be used to simplify a series of “if/then/else” statements.</td>
<td>• Construct an “if/then” statement to perform a specific task.</td>
</tr>
<tr>
<td></td>
<td>• Construct an “if/then/else” statement to perform a specific task.</td>
</tr>
<tr>
<td></td>
<td>• Construct a case statement to perform a specific task.</td>
</tr>
<tr>
<td></td>
<td>• Use conditional statements to incorporate decision making into programs.</td>
</tr>
</tbody>
</table>
**COMPUTER MATHEMATICS**  
**STANDARD COM.917**  
The student will implement pre-existing defined algorithms, including sort routines, search routines, and simple animation routines.

<table>
<thead>
<tr>
<th><strong>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</strong></th>
<th><strong>ESSENTIAL KNOWLEDGE AND SKILLS</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Arranging values into an order is known as sorting.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• A sequential search algorithm starts at the beginning of a list and examines each data value in sequence.</td>
<td>• Implement pre-existing defined algorithms into a program.</td>
</tr>
<tr>
<td>• A search algorithm finds an item with specified properties among a collection of items.</td>
<td>• Implement a sort routine on a one-dimensional array.</td>
</tr>
<tr>
<td>• Implementation of animation routines will differ depending upon the programming language used.</td>
<td>• Implement a sequential search routine on a one-dimensional array.</td>
</tr>
</tbody>
</table>

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:  
- Implement pre-existing defined algorithms into a program.  
- Implement a sort routine on a one-dimensional array.  
- Implement a sequential search routine on a one-dimensional array.  
- Implement a binary search routine on a one-dimensional array.  
- Implement a simple animation routine.
**TOPIC: PROGRAM IMPLEMENTATION**

**STANDARD COM.106**

The student will design and implement the input phase of a program, which will include designing screen layout and getting information into the program by way of user interaction, data statements, and/or file input, and validating input. The input phase will also include methods of filtering out invalid data (error trapping).

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDING UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• A program needs data on which to operate.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• A file is a named area in secondary storage that holds a collection of information.</td>
<td>• Design a screen layout to facilitate input.</td>
</tr>
<tr>
<td></td>
<td>• Design program information input by user interaction and file input.</td>
</tr>
<tr>
<td></td>
<td>- user interaction;</td>
</tr>
<tr>
<td></td>
<td>- data statements (BASIC); and</td>
</tr>
<tr>
<td></td>
<td>- file input.</td>
</tr>
<tr>
<td></td>
<td>• Filter out invalid Validate data, using a variety of methods (error trapping).</td>
</tr>
<tr>
<td></td>
<td>• Construct input statements to read values into a program.</td>
</tr>
<tr>
<td></td>
<td>• Determine the contents of variables that have been assigned values by input statements.</td>
</tr>
</tbody>
</table>
**TOPIC: PROGRAM IMPLEMENTATION**

**COMPUTER MATHEMATICS STANDARD COM.117**

The student will design and implement the output phase of a computer program, which will include designing output layout, accessing a variety of available output devices, using output statements, and labeling results.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Output is dependent on input and processing.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Implementation of the output portion of a program includes designing the output layout and displaying the output in the desired format.</td>
<td>• Design an output layout.</td>
</tr>
<tr>
<td></td>
<td>• Access various available output devices.</td>
</tr>
<tr>
<td></td>
<td>• Use output statements.</td>
</tr>
<tr>
<td></td>
<td>• Label results.</td>
</tr>
</tbody>
</table>
The student will design and implement computer graphics to enhance output, which will include topics appropriate for the available programming environment as well as student background. Students will use graphics as an end in itself, as an enhancement to other output, and as a vehicle for reinforcing programming techniques.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Work with computer graphics is specific to the computer operating system, programming language.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td></td>
<td>• Design and implement computer graphics using various techniques such as,</td>
</tr>
<tr>
<td></td>
<td>• plotting points or shapes;</td>
</tr>
<tr>
<td></td>
<td>• determining and set window or screen dimensions;</td>
</tr>
<tr>
<td></td>
<td>• determining and set screen or background colors;</td>
</tr>
<tr>
<td></td>
<td>• using box commands.</td>
</tr>
<tr>
<td></td>
<td>• Implement computer graphics.</td>
</tr>
<tr>
<td></td>
<td>• Plot points and areas.</td>
</tr>
<tr>
<td></td>
<td>• Determine and set window or screen dimensions.</td>
</tr>
<tr>
<td></td>
<td>• Determine and set screen and background colors.</td>
</tr>
<tr>
<td></td>
<td>• Use box commands. [Rewritten]</td>
</tr>
<tr>
<td></td>
<td>• Describe the role of graphics in the computer environment.</td>
</tr>
</tbody>
</table>
The student will implement various mechanisms for performing iteration with an algorithm, loops, including iterative loops. Other topics will include single entry point, single exit point, preconditions, and postconditions.

### Essential Understandings

**Understanding the Standard**

- An iterative algorithm executes a sequence of statements repeatedly.
- Iterative algorithms execute a fixed number of times.
- Iterative algorithms can be nested.
- Iterative algorithms include:
  - iterative loops;
  - indefinite loops;
  - pretest loops;
  - posttest loops; and
  - nested loops.

### Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

- Determine when an iterative algorithm is needed in a computer program.
- Implement loops into programs. Include:
  - iterative loops;
  - pretest loops; and
  - posttest loops. [Moved to US]
- Incorporate single entry point, single exit point, preconditions, and postconditions into iterative algorithms.
**COMPUTER MATHEMATICS**

**STANDARD COM.1416**

The student will select and implement appropriate data structures, including arrays (one- and/or two-dimensional-and/or multidimensional), files, and records. Implementation will include creating the data structure, putting information into the structure, and retrieving information from the structure.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDING: UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
</table>
| • Structured data types: Data structures are collections of components that are given a single name and whose organization is characterized by the method used to access the individual components.  
• Multi: Two-dimensional arrays may be viewed as arrays of one-dimensional arrays.  
• Data files may be used to store information needed each time the program is executed.  
• An object is a basic data structure used to organize data of different types and referenced by name. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to  
• Declare. Implement a one-dimensional or two-dimensional array for a given problem.  
  - Choose an appropriate component data type for an array.  
  - Assign a value to an array component element.  
  - Fill an array with data, and process the data in the array.  
  - Access a particular component element of a two-dimensional array.  
  - Process a two-dimensional array by rows and by columns.  
  - Retrieve data from an array.  
• Use data files in computer programs, both as a source of input data and as a way to save data for the next program execution.  
• Implement objects to consolidate related information of different data types. |
**TOPIC:** STRAND: DATA MANIPULATION

**COMPUTER MATHEMATICS**
**STANDARD COM.159**

The student will define simple and use appropriate variable data types that include integer, real (fixed and scientific notation), character, string, and Boolean, and object.

**STANDARD COM.10**

The student will use appropriate variable data types, including integer, real (fixed and scientific notation), character, string, and Boolean. This will also include variables representing structured data types. [Combined with COM.15]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• A data type is defined as a set of values and a set of operations on the values.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Boolean data has only two literal constants, and they represent TRUE and FALSE.</td>
<td>• Define and use variables using data types, including</td>
</tr>
<tr>
<td>• A string is an array of character data for which there exists an aggregate name constant.</td>
<td>– integer;</td>
</tr>
</tbody>
</table>

• Write numeric and string variables, using valid names.
• Use standard naming conventions to create variable names.
**COMPUTER MATHEMATICS**  
**STANDARD COM.1644**

The student will describe the way the computer stores, accesses, and processes variables, including the following topics: the use of variables versus constants, variables’ addresses, pointers, parameter passing, scope of variables, and local versus global variables.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Computers consist of hardware components that interact with software.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Program design must identify each piece of data as a constant, local variable, or global variable depending on the scope of the data in the program.</td>
<td>• Determine when the use of a variable is appropriate.</td>
</tr>
<tr>
<td>• Parameters are used to provide information to other parts of the program (functions, methods, subroutines, or procedures).</td>
<td>• Describe how a computer stores, accesses, and processes variables.</td>
</tr>
<tr>
<td>• Parameter passing may include using variables’ addresses or pointers or equivalent features.</td>
<td>• Incorporate variable addresses, pointers, and parameter passing into programs.</td>
</tr>
<tr>
<td>• Variable addresses and pointers are used to access data stored in computer memory.</td>
<td>• Differentiate between local and global variables, and describe their appropriate use.</td>
</tr>
<tr>
<td></td>
<td>• Compare and contrast variables and constants.</td>
</tr>
<tr>
<td></td>
<td>• Describe the basic interplay between hardware and software in program execution. [Moved from COM.2 EKS]</td>
</tr>
</tbody>
</table>
# Mathematics Standards of Learning Curriculum Framework 2009-2016

## COMPUTER MATHEMATICS STANDARD COM.1718

The student will test a program, using an appropriate set of data. The set of test data should include boundary cases and test all branches of a program. Be appropriate and complete for the type of program being tested.

<table>
<thead>
<tr>
<th>Essential Understandings: Understanding the Standard</th>
<th>Essential Knowledge and Skills</th>
</tr>
</thead>
</table>
| • A program test can reveal problems (bugs) in the program.  
• Testing a program for bugs is part of problem solving.  
• Various forms of data can be used to debug a program. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to  
• Test a program, using an appropriate and complete set of data.  
• Demonstrate that a set of data tests all branches of a program. |
# COMPUTER MATHEMATICS

## STANDARD COM.1819

The student will debug a program, using appropriate techniques (e.g., appropriately placed controlled breaks, the printing of intermediate results, and other debugging tools available in the programming environment), and identify the difference between among syntax errors, runtime errors, and logic errors.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Debugging a program is problem solving is a methodical process of finding and correcting errors so the computer program executes as intended.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td></td>
<td>• Debug a program, using controlled breaks, the printing of intermediate results, and other debugging tools.</td>
</tr>
<tr>
<td></td>
<td>• Identify the differences among syntax errors, runtime errors, and logic errors.</td>
</tr>
</tbody>
</table>
**TOPIC: PROGRAM TESTING**

**COMPUTER MATHEMATICS**

**STANDARD COM.20**

The student will design, write, test, debug, and document a complete structured program that requires the synthesis of many of the concepts contained in previous standards. [Included in COM.2]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>- The successful completion of a structured program requires problem solving skills.</td>
<td>- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td></td>
<td>- Design, write, test, debug, and document a complete structured program.</td>
</tr>
</tbody>
</table>
Probability and Statistics
The Virginia Department of Education wishes to express sincere thanks to Deborah Kiger Bliss, Lois A. Williams, Ed.D., and Felicia Dyke, Ph.D.—who assisted in the development of the 2009 Mathematics Standards of Learning Curriculum Framework.

The Virginia Department of Education wishes to express sincere thanks to Michael Bolling, who assisted in the development of the 2016 Mathematics Standards of Learning and 2016 Mathematics Standards of Learning Curriculum Framework.

NOTICE
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Introduction

The 2009–2016 Mathematics Standards of Learning Curriculum Framework is a companion document to the 2009–2016 Mathematics Standards of Learning and amplifies the Mathematics Standards of Learning by further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings and Essential Knowledge and Skills. The purpose of each column is explained below.

### Essential Understandings

This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Understanding the Standard

This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. It contains background information for the teacher (K–8). The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers’ knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

### Essential Knowledge and Skills

Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that What each student should know and be able to demonstrate in each standard is outlined. This is not meant to be an exhaustive list of student expectations nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving
Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication
Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning
Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections
Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations
Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
### PROBABILITY AND STATISTICS

**STANDARD PS.1**

The student will analyze graphical displays of univariate data, including dotplots, stemplots, boxplots, cumulative frequency graphs, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays. [Moved to EKS]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Data are collected for a purpose and have meaning in a context.</td>
<td><strong>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</strong></td>
</tr>
<tr>
<td>• Measures of central tendency describe how the data cluster or group.</td>
<td>• Create and interpret graphical displays of data, including dotplots, stem and leaf stemplots, boxplots, cumulative frequency graphs, and histograms, using appropriate technology.</td>
</tr>
<tr>
<td>• Measures of dispersion describe how the data spread (disperse) around the center of the data.</td>
<td>• Examine graphs of data for clusters and gaps, and relate those phenomena to the data in context.</td>
</tr>
<tr>
<td>• Graphical displays of data may be analyzed informally.</td>
<td>• Examine graphs of data for outliers, and explain the outlier(s) within the context of the data.</td>
</tr>
<tr>
<td>• Data analysis must take place within the context of the problem.</td>
<td>• Examine graphs of data and identify the central tendency of the data as well as the spread.</td>
</tr>
<tr>
<td></td>
<td>• Explain the central tendency and the spread of the data within the context of the data.</td>
</tr>
</tbody>
</table>

† Standard should be included in a one-semester course in Probability and Statistics.
**PROBABILITY AND STATISTICS**

**STANDARD PS.2**

The student will analyze numerical characteristics of univariate data sets to describe patterns and departure from patterns, using mean, median, mode, variance, standard deviation, interquartile range, range, and outliers.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Data are collected for a purpose and have meaning within a context.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>2. Analysis of the descriptive statistical information generated by a univariate data set should include the interplay between central tendency and dispersion as well as among specific measures.</td>
<td>* Interpret mean, median, mode, range, interquartile range, variance, and standard deviation of a univariate data set in terms of the problem’s context.</td>
</tr>
<tr>
<td>3. Data points identified algorithmically as outliers should not be excluded from the data unless sufficient evidence exists to show them to be in error.</td>
<td>* Identify possible outliers, using an algorithm.</td>
</tr>
<tr>
<td></td>
<td>* Explain the influence of outliers on a univariate data set.</td>
</tr>
<tr>
<td></td>
<td>* Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.</td>
</tr>
</tbody>
</table>

† Standard should be included in a one-semester course in Probability and Statistics.
## PROBABILITY AND STATISTICS

### STANDARD PS.3

The student will compare distributions of two or more univariate data sets, **numerically and graphically,** analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data are collected for a purpose and have meaning in a context.</td>
<td></td>
</tr>
<tr>
<td>Statistical tendency refers to typical cases but not necessarily to individual cases.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Compare and contrast two or more univariate data sets, <strong>numerically and graphically,</strong> by analyzing measures of center and spread within a contextual framework.</td>
</tr>
<tr>
<td>• Describe any unusual features of the data, such as clusters, gaps, or outliers, within the context of the data.</td>
</tr>
<tr>
<td>• Analyze in context kurtosis and skewness in conjunction with measures of center and spread in a contextual framework with other descriptive measures.</td>
</tr>
</tbody>
</table>

† Standard should be included in a one-semester course in Probability and Statistics.
**PROBABILITY AND STATISTICS**  
**STANDARD PS.4**

The student will analyze scatterplots to identify and describe the relationship between two variables, using shape; strength of relationship; clusters; positive, negative, or no association; outliers; and influential points.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• A scatterplot serves two purposes:</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>– to determine if there is a useful relationship</td>
<td>• Examine scatterplots of data, and describe skewness, kurtosis, and</td>
</tr>
<tr>
<td>between two variables, and</td>
<td>correlation within the context of the data.</td>
</tr>
<tr>
<td>– to determine the family of equations that</td>
<td>• Describe and explain any unusual features of the data, such as</td>
</tr>
<tr>
<td>describes the relationship.</td>
<td>clusters, gaps, or outliers, within the context of the data.</td>
</tr>
<tr>
<td>• Data are collected for a purpose and have</td>
<td>• Identify influential data points (observations that have great effect on</td>
</tr>
<tr>
<td>meaning in a context.</td>
<td>a line of best fit because of extreme ( x )-values) and describe the effect</td>
</tr>
<tr>
<td>• Association between two variables considers</td>
<td>of the influential points.</td>
</tr>
<tr>
<td>both the direction and strength of the association.</td>
<td></td>
</tr>
<tr>
<td>• The strength of an association between two</td>
<td></td>
</tr>
<tr>
<td>variables reflects how accurately the value of</td>
<td></td>
</tr>
<tr>
<td>one variable can be predicted based on the</td>
<td></td>
</tr>
<tr>
<td>value of the other variable.</td>
<td></td>
</tr>
<tr>
<td>• Outliers are observations with large residuals</td>
<td></td>
</tr>
<tr>
<td>and do not follow the pattern apparent in the</td>
<td></td>
</tr>
<tr>
<td>other data points.</td>
<td></td>
</tr>
</tbody>
</table>

† Standard should be included in a one-semester course in Probability and Statistics.
### PROBABILITY AND STATISTICS

**STANDARD PS.5**

The student will find determine and interpret linear correlation, use the method of least squares regression to model the linear relationship between two variables, and use the residual plots to assess linearity.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data are collected for a purpose and have meaning in a context.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>Least squares regression generates the equation of the line that minimizes the sum of the squared distances from the data points to the line.</td>
<td>- Calculate a correlation coefficient, $r$.</td>
</tr>
<tr>
<td>Each data point may be considered to be comprised of two parts: fit (the part explained by the model) and residual (the result of chance variation or of variables not measured).</td>
<td>- Explain how the correlation coefficient, $r$, measures association by looking at its formula.</td>
</tr>
<tr>
<td>Residual = Actual – Fitted</td>
<td>- Interpret the coefficient of determination, $r^2$, in a contextual framework.</td>
</tr>
<tr>
<td>A correlation coefficient measures the degree of association between two variables that are related linearly.</td>
<td>- Use regression lines to make predictions, and identify the limitations of the predictions.</td>
</tr>
<tr>
<td>Two variables may be strongly associated without a cause-and-effect relationship existing between them.</td>
<td>- Use residual plots to determine if a linear model is satisfactory for describing the relationship between two variables.</td>
</tr>
<tr>
<td>- Describe the errors inherent in extrapolation beyond the range of the data.</td>
<td>- Describe the errors inherent in extrapolation beyond the range of the data.</td>
</tr>
<tr>
<td>- Use least squares regression to find determine the equation of the line of best fit for a set of data.</td>
<td>- Use least squares regression to find determine the equation of the line of best fit for a set of data.</td>
</tr>
<tr>
<td>- Interpret the slope and $y$-intercept of the least squares regression line in a contextual framework.</td>
<td>- Interpret the slope and $y$-intercept of the least squares regression line in a contextual framework.</td>
</tr>
<tr>
<td>- Explain how least squares regression generates the equation of the line of best fit by examining the formulas used in computation.</td>
<td>- Explain how least squares regression generates the equation of the line of best fit by examining the formulas used in computation.</td>
</tr>
</tbody>
</table>
PROBABILITY AND STATISTICS
STANDARD PS.6

The student will make logarithmic and power transformations to achieve linearity.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• A logarithmic transformation reduces positive skewness because it compresses the upper tail of the distribution while stretching the lower tail.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Nonlinear transformations do not preserve relative spacing between data points.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Apply a logarithmic transformation to data.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Explain how a logarithmic transformation works to achieve a linear relationship between variables.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Apply a power transformation to data.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Explain how a power transformation works to achieve a linear relationship between variables.</td>
</tr>
</tbody>
</table>
# Probability and Statistics

## Standard PS.7

The student, using two-way tables and other graphical displays, will analyze categorical data to describe patterns and departure from patterns and to find marginal frequency and relative frequencies, including conditional frequencies.

<table>
<thead>
<tr>
<th>Essential Understandings</th>
<th>Essential Knowledge and Skills</th>
</tr>
</thead>
</table>
| • Data are collected for a purpose and have meaning in a context.  
• Simpson’s paradox refers to the fact that aggregate proportions can reverse the direction of the relationship seen in the individual parts.  
• Two categorical variables are independent if the conditional frequencies of one variable are the same for every category of the other variable. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to  
• Produce a two-way table as a summary of the information obtained from two categorical variables.  
• Create and interpret graphical displays of categorical data including bar charts.  
• Calculate marginal, relative, and conditional frequencies in a two-way table.  
• Use marginal, relative, and conditional frequencies to analyze data in two-way tables within the context of the data. |

1 Standard should be included in a one-semester course in Probability and Statistics.
The student will describe the methods of data collection in a census, sample survey, experiment, and observational study and identify an appropriate method of solution for a given problem setting.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The value of a sample statistic varies from sample to sample if the simple random samples are taken repeatedly from the population of interest.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Poor data collection can lead to misleading and meaningless conclusions.</td>
<td>• Compare and contrast controlled experiments and observational studies and the conclusions one can draw from each.</td>
</tr>
<tr>
<td></td>
<td>• Compare and contrast population and sample, and parameter and statistic.</td>
</tr>
<tr>
<td></td>
<td>• Identify biased sampling methods.</td>
</tr>
<tr>
<td></td>
<td>• Describe simple random sampling.</td>
</tr>
<tr>
<td></td>
<td>• Select a data collection method appropriate for a given context.</td>
</tr>
</tbody>
</table>

1 Standard should be included in a one-semester course in Probability and Statistics.
### PROBABILITY AND STATISTICS

**STANDARD PS.9**

The student will plan and conduct a survey. The plan will address sampling techniques (e.g., simple random and stratified) [Moved to EKS] and methods to reduce bias.

<table>
<thead>
<tr>
<th><strong>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</strong></th>
<th><strong>ESSENTIAL KNOWLEDGE AND SKILLS</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• The purpose of sampling is to provide sufficient information so that population characteristics may be inferred.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Inherent bias diminishes as sample size increases.</td>
<td>• Distinguish between a population and a sample. [Moved from standard]</td>
</tr>
<tr>
<td></td>
<td>• Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling.</td>
</tr>
<tr>
<td></td>
<td>• Determine which sampling technique is best, given a particular context.</td>
</tr>
<tr>
<td></td>
<td>• Plan a survey to answer a question or address an issue.</td>
</tr>
<tr>
<td></td>
<td>• Given a plan for a survey, identify possible sources of bias, and describe ways to reduce bias.</td>
</tr>
<tr>
<td></td>
<td>• Design a survey instrument.</td>
</tr>
<tr>
<td></td>
<td>• Conduct a survey.</td>
</tr>
</tbody>
</table>

† Standard should be included in a one-semester course in Probability and Statistics.
## PROBABILITY AND STATISTICS

**STANDARD PS.10**

The student will plan and conduct a well-designed experiment. The plan will address control, randomization, replication, blinding, and measurement of experimental error.

### ESSENTIAL UNDERSTANDINGS

**UNDERSTANDING THE STANDARD**

- Experiments must be carefully well-designed in order to detect a cause-and-effect relationship between variables.
- Principles of experimental design include comparison with a control group, randomization, and blindness.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Plan and conduct a well-designed experiment. The experimental design should address control, randomization, replication, blinding and minimization of experimental error.
- Identify treatments, levels, factors, control groups, and experimental units in an experimental design.
- Identify sources of bias and confounding, including the placebo effect.
- Identify a situation when a block design, including matched pairs, would reduce the effects of confounding variables.

---

1 Standard should be included in a one-semester course in Probability and Statistics.
# Mathematics Standards of Learning Curriculum Framework 2016

## PROBABILITY AND STATISTICS

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### STANDARD PS.11†

The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The complement of event $A$ consists of all outcomes in which event $A$ does not occur.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Two events, $A$ and $B$, are independent if the occurrence of one does not affect the probability of the occurrence of the other. If $A$ and $B$ are not independent, then they are said to be dependent.</td>
<td></td>
</tr>
<tr>
<td>• Events $A$ and $B$ are mutually exclusive if they cannot occur simultaneously.</td>
<td>• Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events.</td>
</tr>
<tr>
<td></td>
<td>• Given two or more events in a problem setting, determine if the events are complementary, dependent, independent, and/or mutually exclusive.</td>
</tr>
</tbody>
</table>

†Standard should be included in a one-semester course in Probability and Statistics.
The student will determine probabilities (relative frequency and theoretical), including conditional probabilities for events that are either dependent or independent, by applying the Law of Large Numbers concept, the addition rule, and the multiplication rule.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
</table>
| • Data are collected for a purpose and have meaning in a context.  
• Venn diagrams may be used to determine conditional probabilities.  
• The Law of Large Numbers states that as a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to  
• Calculate relative frequency and expected frequency.  
• Find conditional probabilities for dependent, independent, and mutually exclusive events. |

1 Standard should be included in a one-semester course in Probability and Statistics.
**TOPIC STRAND: PROBABILITY**

**STANDARD PS.13**

The student will develop, interpret, and apply the binomial and geometric probability distributions for discrete random variables, including computing the mean and standard deviation for the binomial and geometric variables.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• A probability distribution is a complete listing of all possible outcomes of an experiment together with their probabilities.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• In a binomial distribution, the procedure has a fixed number of independent trials.</td>
<td>• Develop the binomial and geometric probability distributions within a real-world practical context.</td>
</tr>
<tr>
<td>• In a geometric distribution, the procedure is repeated until the first success.</td>
<td>• Calculate the mean and standard deviation for the binomial and geometric variables.</td>
</tr>
<tr>
<td>• A random variable assumes different values depending on the event outcome.</td>
<td>• Use the binomial and geometric distributions to calculate probabilities associated with experiments for which there are only two possible outcomes.</td>
</tr>
<tr>
<td>• A probability distribution combines descriptive statistical techniques and probabilities to form a theoretical model of behavior.</td>
<td></td>
</tr>
</tbody>
</table>
## PROBABILITY AND STATISTICS

**STANDARD PS.14**

The student will simulate probability distributions, including binomial and geometric.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>• A probability distribution combines descriptive methods and probabilities to form a theoretical model of behavior.</td>
<td></td>
</tr>
<tr>
<td>• A probability distribution gives the probability for each value of the random variable.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Design and conduct an experiment a simulation of a binomial distribution that simulates a binomial distribution.</td>
</tr>
<tr>
<td>• Design and conduct a simulation of an experiment that simulates a geometric distribution.</td>
</tr>
<tr>
<td>• Calculate probabilities resulting from simulations of binomial and geometric distributions.</td>
</tr>
</tbody>
</table>
**TOPIC**: Probability

**STRAND**: Probability and Statistics

**STANDARD PS.15**

The student will identify random variables as independent or dependent and find the mean and standard deviations for random variables and sums and differences of independent random variables.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS: UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• A random variable is a variable that has a single numerical value, determined by chance, for each outcome of a procedure.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td></td>
<td>• Compare and contrast independent and dependent random variables.</td>
</tr>
<tr>
<td></td>
<td>• Determine the mean (expected value) and standard deviation for a random variable and linear transformation of a random variable.</td>
</tr>
<tr>
<td></td>
<td>• Determine the mean (expected value) for sums and differences of random variables.</td>
</tr>
<tr>
<td></td>
<td>• Find the standard deviation for sums and differences of independent random variables.</td>
</tr>
</tbody>
</table>
**TOPIC**

**STRAND:** PROBABILITY

---

**STANDARD PS.16**

The student will identify properties of a normal distribution and apply the normal distribution to determine probabilities, using a table or graphing calculator [Moved to EKS].

---

### ESSENTIAL UNDERSTANDINGS

Understanding the Standard

- The normal distribution curve is a family of symmetrical curves defined by the mean and the standard deviation.
- Areas under the curve represent probabilities associated with continuous distributions.
- The normal curve is a probability distribution and the total area under the curve is 1.

---

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify the properties of a normal probability distribution.
- Describe how the standard deviation and the mean affect the graph of the normal distribution.
- Calculate and interpret the z-score of a given data value from a normal distribution.
- Determine the probability of a given event, using the normal distribution.
- Use a graphing utility calculator and a table of Standard Normal Probabilities to determine probabilities. [Moved from standard]

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\* Standard should be included in a one-semester course in Probability and Statistics.
### PROBABILITY AND STATISTICS

**STANDARD PS.17**

The student, given data from a large sample, will find, determine, and interpret appropriate point estimates and confidence intervals for parameters. The parameters will include proportion and mean, difference between two proportions, and difference between two means (independent and paired), and slope of a least-squares regression line.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UNDERSTANDING THE STANDARD</strong></td>
<td><strong>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</strong></td>
</tr>
<tr>
<td>- A primary goal of sampling is to estimate the value of a parameter based on a statistic.</td>
<td>- Construct confidence intervals to estimate a population parameter, such as a proportion or the difference between two proportions; or a mean or the difference between two means; or slope of a least-squares regression line.</td>
</tr>
<tr>
<td>- Confidence intervals use the sample statistic to construct an interval of values that one can be reasonably certain contains the true (unknown) parameter.</td>
<td>- Select a value for alpha (Type I error) for the confidence level of a confidence interval.</td>
</tr>
<tr>
<td>- Confidence intervals and tests of significance are complementary procedures.</td>
<td>- Interpret confidence intervals and confidence levels in the context of the data.</td>
</tr>
<tr>
<td>- Paired comparisons experimental design allows control for possible effects of extraneous variables.</td>
<td>- Explain the importance of random sampling for confidence intervals.</td>
</tr>
<tr>
<td></td>
<td>- Explain how changes in confidence level and sample size effect width of the confidence interval and margin of error.</td>
</tr>
<tr>
<td></td>
<td>- Calculate point estimates for parameters and discuss the limitations of point estimates.</td>
</tr>
</tbody>
</table>
# Probability and Statistics

**Standard PS.18**

The student will apply and interpret the logic of an appropriate hypothesis-testing procedure. Tests will include large sample test for proportion, mean, difference between two proportions, and difference between two means (independent and paired); and Chi-squared tests for goodness of fit, homogeneity of proportions, and independence; and slope of a least-squares regression line.

## Essential Understandings

- Confidence intervals and tests of significance are complementary procedures.
- Paired comparisons experimental design allows control for possible effects of extraneous variables.
- Tests of significance assess the extent to which sample data support a hypothesis about a population parameter.
- The purpose of a goodness of fit test is to decide if the sample results are consistent with results that would have been obtained if a random sample had been selected from a population with a known distribution.
- Practical significance and statistical significance are not necessarily congruent.

## Essential Knowledge and Skills

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:
  - Use the Chi-squared test for goodness of fit to decide if the population being analyzed fits a particular distribution pattern.
  - Use hypothesis-testing procedures to determine whether or not to reject the null hypothesis. The null hypothesis may address proportion, mean, difference between two proportions or two means, goodness of fit, homogeneity of proportions, and independence, and the slope of a least-squares regression line.
  - Compare and contrast Type I and Type II errors.
  - Explain how and why the hypothesis-testing procedure allows one to reach a statistical decision.
## PROBABILITY AND STATISTICS
### STANDARD PS.19

The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means.

### ESSENTIAL UNDERSTANDING

**The Central Limit Theorem states:**
- The mean of the sampling distribution of means is equal to the population mean.
- If the sample size is sufficiently large, the sampling distribution approximates the normal probability distribution.
- If the population is normally distributed, the sampling distribution is normal regardless of sample size.

**Sampling distributions have less variability with larger sample sizes.**

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:
- Describe the use of the Central Limit Theorem for drawing inferences about a population parameter based on a sample statistic.
- Describe the effect of sample size on the sampling distribution and on related probabilities.
- Use the normal approximation to calculate probabilities of sample statistics falling within a given interval.
- Identify and describe the characteristics of a sampling distribution of a sample proportion, mean, difference between two sample proportions, or difference between two sample means.
## PROBABILITY AND STATISTICS

**STANDARD PS.20**

The student will identify properties of a \( t \)-distribution and apply \( t \)-distributions to single-sample and two-sample (independent and matched pairs) \( t \)-procedures, using tables or graphing calculators.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
<th>UNDERSTANDING THE STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Paired comparisons experimental design allows control for possible effects of extraneous variables.</td>
<td></td>
</tr>
<tr>
<td>- The sampling distribution of means with a small sample size follows a ( t )-distribution.</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>- Identify the properties of a ( t )-distribution.</td>
</tr>
<tr>
<td>- Compare and contrast a ( t )-distribution and a normal distribution.</td>
</tr>
<tr>
<td>- Use a ( t )-test for single-sample and two-sample data.</td>
</tr>
</tbody>
</table>
Discrete Mathematics

Board of Education
Commonwealth of Virginia
The 2009-2016 Mathematics Standards of Learning Curriculum Framework, is a companion document to the 2009-2016 Mathematics Standards of Learning, and amplifies the Mathematics Standards of Learning by and further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students, provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

**Essential Understandings**
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

**Understanding the Standard**
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. Background information for the teacher (K-8). It contains content. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

**Essential Knowledge and Skills**
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that What each student should know and be able to demonstrate in each standard is outlined. This is not meant to be an exhaustive list of student expectations, nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student’s problem-solving skills.

Computational fluency refers to having flexible, efficient, and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand, and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
**DISCRETE MATHEMATICS**
**STANDARD DM.1**

The student will model problems, using vertex-edge graphs. The concepts of valence, connectedness, paths, planarity, and directed graphs will be investigated. Adjacency matrices and matrix operations will be used to solve problems (e.g., food chains, number of paths).[Included in US]

### ESSENTIAL UNDERSTANDINGS
#### UNDERSTANDING THE STANDARD
- A tournament is a digraph that results from giving directions to the edges of a complete graph.
- Adjacent vertices are connected by an edge.
- In a connected graph, every pair of vertices is adjacent.
- Graphs can be used to solve problems including food chains and number of paths. [Moved from standard]

### ESSENTIAL KNOWLEDGE AND SKILLS
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
- Find the valence of each vertex in a graph.
- Use graphs to model situations in which the vertices represent objects, and edges (drawn between vertices) represent a particular relationship between objects.
- Represent the vertices and edges of a graph as an adjacency matrix, and use the matrix to solve problems.
- Investigate and describe valence and connectedness.
- Determine whether a graph is planar or nonplanar.
- Use directed graphs (digraphs) to represent situations with restrictions in traversal possibilities.

† Standard should be included in a one-semester course in Discrete Mathematics.
The student will solve problems through investigation and application of circuits, cycles, Euler Paths, Euler Circuits, Hamilton Paths, and Hamilton Circuits. Optimal solutions will be sought using existing algorithms and student-created algorithms.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
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</thead>
<tbody>
<tr>
<td>• Euler’s Theorem states: If $G$ is a connected graph and all its valences are even, then $G$ has an Euler Circuit.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Pairs of routes (circuits) correspond to the same Hamilton Circuit because one route can be obtained from the other by traversing the vertices in reverse order. There are $\frac{(n - 1)!}{2}$ Hamilton Circuits.</td>
<td>• Determine if a graph has an Euler Circuit or Path, and find determine it, if it exists.</td>
</tr>
<tr>
<td>• A multigraph is connected if there is a path between every pair of vertices.</td>
<td>• Determine if a graph has a Hamilton Circuit or Path, and find determine it, if it exists.</td>
</tr>
</tbody>
</table>

*Standard should be included in a one-semester course in Discrete Mathematics.*
### DISCRETE MATHEMATICS
#### STANDARD DM.3
The student will apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization. Graph coloring and chromatic number will be used. [Moved to EKS]

<table>
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<tbody>
<tr>
<td>• Every planar graph has a chromatic number that is less than or equal to four (the four-color-map theorem).</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• A graph can be colored with two colors if and only if it contains no cycle of odd length.</td>
<td>• Model projects consisting of several subtasks, using a graph.</td>
</tr>
<tr>
<td>• The chromatic number of a graph cannot exceed one more than the maximum number of degrees of the vertices of the graph.</td>
<td>• Use graphs to resolve conflicts that arise in scheduling.</td>
</tr>
<tr>
<td></td>
<td>• Determine the chromatic number of a graph. [Moved from standard]</td>
</tr>
</tbody>
</table>

† Standard should be included in a one-semester course in Discrete Mathematics.
### DISCRETE MATHEMATICS

**STANDARD DM.4**

The student will apply algorithms, such as Kruskal’s, Prim’s, or Dijkstra’s, [Included in EKS] relating to trees, networks, and paths. Appropriate technology will be used to determine the number of possible solutions and generate solutions when a feasible number exists.

### ESSENTIAL UNDERSTANDINGS

<table>
<thead>
<tr>
<th>UNDERSTANDING THE STANDARD</th>
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</tr>
</thead>
<tbody>
<tr>
<td>• A spanning tree of a connected graph $G$ is a tree that is a subgraph of $G$ and contains every vertex of $G.$</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td></td>
<td>• Use Kruskal’s Algorithm to find/ determine the shortest spanning tree of a connected graph.</td>
</tr>
<tr>
<td></td>
<td>• Use Prim’s Algorithm to find/ determine the shortest spanning tree of a connected graph.</td>
</tr>
<tr>
<td></td>
<td>• Use Dijkstra’s Algorithm to find/ determine the shortest spanning tree of a connected graph.</td>
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</tbody>
</table>
### DISCRETE MATHEMATICS

#### STANDARD DM.57$^\dagger$

The student will analyze and describe the issue of fair division in discrete and continuous cases, (e.g., cake cutting, estate division[Included in EKS]). Algorithms for continuous and discrete cases will be applied.

<table>
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<tbody>
<tr>
<td>• Group decision making combines the wishes of many to yield a single fair result.</td>
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<tr>
<td>• A fair division problem may be discrete or continuous.</td>
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<tr>
<td>• The success of the estate division algorithm requires that each heir be capable of placing a value on each object in the estate.</td>
<td></td>
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<tr>
<td>• A fair division problem consists of $n$ individuals (players) who must partition some set of goods, $s$, into $n$ disjoint sets.</td>
<td></td>
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<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
<td></td>
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<tr>
<td>• Investigate and describe situations involving discrete division (e.g., estate division).</td>
<td></td>
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<tr>
<td>• Use an algorithm for fair division for a group of indivisible objects.</td>
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</tr>
<tr>
<td>• Investigate and describe situations involving continuous division of an infinitely divisible set (e.g., cake cutting).</td>
<td></td>
</tr>
<tr>
<td>• Use an algorithm for fair division of an infinitely divisible set.</td>
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</tbody>
</table>

$^\dagger$Standard should be included in a one-semester course in Discrete Mathematics.
The student will investigate and describe weighted voting and the results of various election methods. These may include approval and preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners.

<table>
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<tbody>
<tr>
<td>• Historically, popular voting methods have often led to counterintuitive results.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• A candidate who wins over every other candidate in a one-on-one ballot is a Condorcet winner.</td>
<td>• Determine in how many different ways a voter can rank choices.</td>
</tr>
<tr>
<td>• A Borda count assigns points in descending order to each voter’s subsequent ranking and then adds these points to arrive at a group’s final ranking.</td>
<td>• Investigate and describe the following voting procedures:</td>
</tr>
<tr>
<td>• To select a voting system is to compromise between the shortcomings inherent in each system.</td>
<td>– weighted voting;</td>
</tr>
<tr>
<td></td>
<td>– plurality;</td>
</tr>
<tr>
<td></td>
<td>– majority;</td>
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<tr>
<td></td>
<td>– sequential (winners run off);</td>
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<tr>
<td></td>
<td>– sequential (losers are eliminated);</td>
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<tr>
<td></td>
<td>– Borda count; and</td>
</tr>
<tr>
<td></td>
<td>– Condorcet winner.</td>
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<tr>
<td></td>
<td>• Compare and contrast different voting procedures.</td>
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<td></td>
<td>• Describe the possible effects of approval voting, insincere and sincere voting, a preference schedule, and strategic voting on the election outcome.</td>
</tr>
</tbody>
</table>

† Standard should be included in a one-semester course in Discrete Mathematics.
The student will identify apportionment inconsistencies that apply to issues such as salary caps in sports and allocation of representatives to Congress. Historical and current methods will be compared.

<table>
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<tbody>
<tr>
<td>• The apportionment of Congressional representatives is based on the latest census.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td></td>
<td>• Compare and contrast the Hamilton and Jefferson methods of political apportionment with the Hill-Huntington method (currently in use in the U.S. House of Representatives) and the Webster-Willcox method.</td>
</tr>
<tr>
<td></td>
<td>• Solve allocation problems, using apportionment methods.</td>
</tr>
<tr>
<td></td>
<td>• Investigate and describe how salary caps affect apportionment.</td>
</tr>
</tbody>
</table>
### Discrete Mathematics

**Standard DM.84**

The student will describe and apply sorting algorithms and coding algorithms used in sorting, processing, and communicating information. These will include:

- a) bubble sort, merge sort, and network sort; and
- b) ISBN, UPC, Zip, and banking codes. [Included in EKS]

### Essential Understandings

**Understanding the Standard**

- A bubble sort orders elements of an array by comparing adjacent elements.
- A merge sort combines two sorted lists into a single sorted list.
- Coding algorithms must account for the number of possible codes within the constraints of the coding system.

### Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

- Select and apply a sorting algorithm, such as a
  - bubble sort;
  - merge sort; and
  - network sort.

- Describe and apply a coding algorithm, such as
  - ISBN numbers;
  - UPC codes;
  - Zip codes; and
  - banking codes.
The student will select, justify, and apply an appropriate technique to solve a logic problem. Techniques will include Venn diagrams, truth tables, and matrices. [Included in EKS]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS</th>
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<tbody>
<tr>
<td>• Two-valued (Boolean) algebra serves as a workable method for interpreting the logical truth and falsity of compound statements.</td>
<td></td>
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<tr>
<td>• Venn diagrams provide pictures of topics in set theory, such as intersection and union, mutually exclusive sets, and the empty set.</td>
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</table>

<table>
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<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Generate truth tables that encode the truth and falsity of two or more statements.</td>
</tr>
<tr>
<td>• Use Venn diagrams to represent set relationships, such as intersection and union. [Moved from G.1]</td>
</tr>
<tr>
<td>• Interpret Venn diagrams. [Moved from G.1]</td>
</tr>
<tr>
<td>• Use Venn diagrams to codify and solve logic problems.</td>
</tr>
<tr>
<td>• Use matrices as arrays of data to solve logic problems.</td>
</tr>
</tbody>
</table>

† Standard should be included in a one-semester course in Discrete Mathematics.
## DISCRETE MATHEMATICS
### STANDARD DM.105

The student will use algorithms to schedule tasks in order to determine a minimum project time. The algorithms will include critical path analysis, the list-processing algorithm, and student-created algorithms.

<table>
<thead>
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<tbody>
<tr>
<td>• Critical path scheduling sometimes yields optimal solutions.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
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<tr>
<td></td>
<td>• Specify in a digraph the order in which tests are to be performed.</td>
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<tr>
<td></td>
<td>• Identify the critical path to determine the earliest completion time (minimum project time).</td>
</tr>
<tr>
<td></td>
<td>• Use the list-processing algorithm to determine an optimal schedule.</td>
</tr>
<tr>
<td></td>
<td>• Create and test scheduling algorithms.</td>
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</table>
The student will solve linear programming problems. Appropriate technology will be used to facilitate the use of matrices, graphing techniques, and the Simplex method of determining solutions. [Included in EKS]

<table>
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<tbody>
<tr>
<td><strong>UNDERSTANDING THE STANDARD</strong></td>
<td><strong>THE STUDENT WILL USE PROBLEM SOLVING, MATHEMATICAL COMMUNICATION, MATHEMATICAL REASONING, CONNECTIONS, AND REPRESENTATIONS TO</strong></td>
</tr>
<tr>
<td>• Linear programming models an optimization process.</td>
<td>• Model real-world practical problems with systems of linear inequalities.</td>
</tr>
<tr>
<td>• A linear programming model consists of a system of constraints and an objective quantity that can be maximized or minimized.</td>
<td>• Identify the feasibility region of a system of linear inequalities with no more than four constraints.</td>
</tr>
<tr>
<td>• Any maximum or minimum value for a system of inequalities will occur at a corner point of a feasible region.</td>
<td>• Identify the coordinates of the corner points of a feasibility region.</td>
</tr>
<tr>
<td></td>
<td>• Determine the maximum or minimum value of the system.</td>
</tr>
<tr>
<td></td>
<td>• Describe the meaning of the maximum or minimum value in terms of the original problem.</td>
</tr>
</tbody>
</table>
DISCRETE MATHEMATICS
STANDARD DM.1240

The student will use the recursive process and difference equations with the aid of appropriate technology to generate
   a) compound interest;
   b) sequences and series;
   c) fractals;
   d) population growth models; and
   e) the Fibonacci sequence.

ESSENTIAL UNDERSTANDINGS
UNDERSTANDING THE STANDARD

• Recursion is a process that creates new objects from existing objects that were created by the same process.
• A fractal is a figure whose dimension is not a whole number.
• Fractals are self-similar.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

• Use finite differences and recursion to model compound interest and population growth situations.
• Model arithmetic and geometric sequences and series recursively.
• Compare and contrast the recursive process, and create fractals.
• Compare and contrast the recursive process and the Fibonacci sequence.
• Find a recursive relationship that generates the Fibonacci sequence.
DISCRETE MATHEMATICS
STANDARD DM.13

The student will apply the formulas of combinatorics in the areas of
a) the Fundamental (Basic) Counting Principle;
 b) knapsack and bin-packing problems;
c) permutations and combinations; and
d) the pigeonhole principle.

ESSENTIAL UNDERSTANDINGS
UNDERSTANDING THE STANDARD

- The branch of mathematics that addresses the number of ways objects can be arranged or combined is combinatorics.
- If \( n \) and \( r \) are positive integers and \( n \geq r \),
  \[
  n \ P \ r = \frac{n!}{(n-r)!} \quad \text{and} \quad n \ C \ r = \frac{n!}{r! \ (n-r)!}.
  \]
- A bin-packing problem determines the minimum number of containers of fixed volume (bins) required to hold a set of objects.
- A knapsack problem determines the most valuable set of objects that fit into a container (knapsack) of fixed volume.
- Bin packing and knapsack packing are optimization techniques.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Find the number of combinations possible when subsets of \( r \) elements are selected from a set of \( n \) elements without regard to order.
- Use the Fundamental (Basic) Counting Principle to determine the number of possible outcomes of an event.
- Use the knapsack and bin-packing algorithms to solve real-world practical problems.
- Find the number of permutations possible when \( r \) objects selected from \( n \) objects are ordered.
- Use the pigeonhole principle to solve packing problems to facilitate proofs.
Mathematics Standards of Learning

Curriculum Framework 2009-2016

Mathematical Analysis

Board of Education
Commonwealth of Virginia
The 2009-2016 Mathematics Standards of Learning Curriculum Framework, is a companion document to the 2009-2016 Mathematics Standards of Learning, and amplifies the Mathematics Standards of Learning by and further defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students, providing additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

The Curriculum Framework also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

**Essential Understandings**
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

**Understanding the Standard**
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instructionbackground information for the teacher (K-8). It contains content—The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s) that may extend the teachers’ knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

**Essential Knowledge and Skills**
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that What each student should know and be able to demonstratedo in each standard is outlined. This is not meant to be an exhaustive list of student expectationsnor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.
Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include actual real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.
The Role of Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships, or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships, or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades 4-7, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must simultaneously develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student’s problem-solving skills.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.
**Algebra Readiness**

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade 8. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher level content. “Algebra readiness” describes students’ mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of “algebra readiness” should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

**Equity**

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities. Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, Mathematics instruction should address the individual needs of all learners— including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.
MATHEMATICAL ANALYSIS
STANDARD MA.1

The student will investigate and identify the characteristics/properties of polynomial, rational, piecewise, and step and rational functions and use these to sketch their graphs of the functions. This will include determining zeros, upper and lower bounds, y-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics.[ Included in EKS]

ESSENTIAL UNDERSTANDINGS
UNDERSTANDING THE STANDARD

- The graphs of polynomial, and rational, piecewise, and step functions can be determined by exploring characteristics/properties and components of the functions.
- A variety of notations should be used, including such as set builder notation and interval notation.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify a polynomial, rational, piecewise, and step function, given an equation or graph.
- Identify rational functions, given an equation or graph.
- Given a graph or equation of a polynomial, rational, piecewise, or step function, identify:
  - Domain and range,
  - Zeros, upper and lower bounds,
  - Y-intercepts,
  - Symmetry,
  - Asymptotes (horizontal, vertical, and oblique/slab);
  - Points of discontinuity;
  - Intervals for which the function is increasing, decreasing or constant, decreasing, points of discontinuity,
  - End behavior;
  - Relative and/or absolute maximum and minimum points, given a graph of a function.
- Sketch the graph of a polynomial, rational, piecewise, or step function.
- Sketch the graph of a rational function.
- Investigate and verify characteristics of a polynomial, or rational, function piecewise, and step function, using a graphing
### MATHEMATICAL ANALYSIS

**STANDARD MA.1**

The student will investigate and identify the characteristics/properties of polynomial, rational, piecewise, and step and rational functions and use these to sketch their graphs of the functions of the functions. This will include determining zeros, upper and lower bounds, $y$-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics. [Included in EKS]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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<tbody>
<tr>
<td></td>
<td>calculator utility.</td>
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<td>• Rationalize the denominator of a rational function.</td>
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</table>
The student will investigate and identify the characteristics of exponential and logarithmic functions to graph the function, in order to graph these functions and solve equations, and solve real-world practical problems. This will include the role of e, natural and common logarithms, laws of exponents and logarithms, and the solution of logarithmic and exponential equations. [Included in EKS]

### ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

- Exponential and logarithmic functions are inverse functions.
- Some examples of appropriate models or situations for exponential and logarithmic functions are:
  - Population growth;
  - Compound interest;
  - Depreciation/appreciation;
  - Richter scale; and
  - Radioactive decay.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify exponential functions from an equation or a graph.
- Identify logarithmic functions from an equation or a graph.
- Define e, and know its approximate value.
- Write and convert between logarithmic equations written in logarithmic and exponential form and vice versa.
- Identify common and natural logarithms, given an equation or practical situation.
- Use laws of exponents and logarithms to solve equations and simplify expressions.
- Model real-world practical problems, using exponential and logarithmic functions.
- Graph exponential and logarithmic functions, using a graphing utility, and identify asymptotes, end behavior, intercepts, domain, and range.
The student will apply compositions of functions and inverses of functions to real-world practical situations. Analytical methods and graphing utilities will be used to and investigate and verify the domain and range of resulting functions.

### ESSENTIAL UNDERSTANDINGS

- In composition of functions, a function serves as input for another function.
- The composition of functions $f(x)$ and $g(x)$ can be determined using the graphs of $f(x)$ and $g(x)$.
- A graph of a function and its inverse are symmetric about the line $y = x$.
- $$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$$

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Find Determine the composition of functions algebraically and graphically.
- Find Determine the inverse of a function algebraically and graphically.
- Determine the domain and range of the composite functions algebraically and graphically.
- Determine the domain and range of the inverse of a function algebraically and graphically.
- Verify the accuracy of sketches of functions, using a graphing utility. [Included in MA.1]
The student will **find** the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution. [Included in EKS]

### Essential Understandings
- The limit of a function is the value approached by $f(x)$ as $x$ approaches a given value or infinity.

### Essential Knowledge and Skills
- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
  - Verify intuitive reasoning estimates about the limit of a function, using a graphing utility.
  - Find the limit of a function algebraically, and verify with a graphing utility.
  - Find the limit of a function numerically, and verify with a graphing utility.
  - Use limit notation when describing end behavior of a function.
The student will investigate and describe the continuity of functions, using graphs and algebraic methods. [Moved to EKS]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
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<tbody>
<tr>
<td>• Continuous and discontinuous functions can be identified by their equations or graphs.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>• Discontinuity can be described as point (removable), jump or infinite.</td>
<td>• Describe continuity of a function.</td>
</tr>
<tr>
<td>• A function, ( f ), is continuous at a point ( c ) if and only if</td>
<td>• Investigate the continuity of functions including absolute value, step, rational, and piece-wise-defined functions, using graphical and algebraic methods.</td>
</tr>
<tr>
<td>- ( f(c) \text{ exists} )</td>
<td>• Classify types of discontinuity.</td>
</tr>
<tr>
<td>- ( \lim_{x \to c} f(x) \text{ exists} )</td>
<td>• Prove continuity at a point, using the definition of limits.</td>
</tr>
<tr>
<td>- ( f(c) = \lim_{x \to c} f(x) )</td>
<td>• Use transformations to sketch absolute value, step, and rational functions. [Included in MA2.6]</td>
</tr>
<tr>
<td></td>
<td>• Verify the accuracy of sketches of functions, using a graphing utility. [Moved to MA.1]</td>
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### MATHEMATICAL ANALYSIS

**STANDARD MA.68**

The student will investigate, graph, and identify the characteristics properties of conic sections from equations in \((h, k)\) vertex and standard forms. Transformations in the coordinate plane will be used to graph conic sections. [Included in EKS]

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDING</th>
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<tbody>
<tr>
<td>Matrices can be used to represent transformations of figures in the plane.</td>
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<tr>
<td>A conic section is a figure formed by the intersection of a plane with a right circular cone. Conic sections include the parabola, circle, ellipse, and hyperbola.</td>
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<tr>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
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<tr>
<td>Given a translation or rotation matrix, find determine an equation for the transformed function or conic section.</td>
</tr>
<tr>
<td>Investigate and verify graphs of transformed conic sections, using a graphing utility.</td>
</tr>
<tr>
<td>Graph conic sections from equations written in vertex or standard form using transformations.</td>
</tr>
<tr>
<td>Identify properties of conic sections.</td>
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</table>
TOPIC: STRAND: ANALYTICAL GEOMETRY

MATHEMATICAL ANALYSIS
STANDARD MA.7.14

The student will perform operations with vectors in the coordinate plane and solve real-world practical problems using vectors. This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components. [Included in EKS]

ESSENTIAL UNDERSTANDINGS
UNDERSTANDING THE STANDARD

- Every vector has an equal vector that has its initial point at the origin.
- The magnitude and direction of a vector with the origin as the initial point are completely determined by the coordinates of its terminal point.
- Every nonzero vector has a corresponding unit vector, which has the same direction as the vector, but a magnitude of 1. [Moved from EKS]

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Use vector notation.
- Perform the operations of addition, subtraction, scalar multiplication, and inner (dot) product on vectors.
- Graph vectors and resultant vectors.
- Express complex numbers in vector notation.
- Define unit vector, and find the unit vector in the same direction as a given vector. [Moved to US]
- Identify properties of vector addition, scalar multiplication, and dot product.
- Find the components of a vector components.
- Find the norm (magnitude) of a vector.
- Use vectors in simple geometric proofs.
- Solve real-world problems, including practical problems, using vectors.
## MATHEMATICAL ANALYSIS

**STANDARD MA.8.13**

The student will identify, create, and solve real-world practical problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines. [Included in the EKS.]

### ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD

- Real-world practical problems can be modeled using trigonometry and vectors.
- Triangles can be formed from vectors.

### ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Solve and create problems, including practical problems, using trigonometric functions.
- Solve and create problems, including practical problems, using the Pythagorean Theorem.
- Solve and create problems, including practical problems, using the Law of Sines and the Law of Cosines.
- Solve real-world problems, including practical problems, where triangles are formed from using vectors.
The student will investigate and identify the characteristics of the graphs of polar equations, using graphing utilities. This will include classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations. [Included in EKS]

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<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
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<tbody>
<tr>
<td>The real number system is represented geometrically on the number line, and the complex number system is represented geometrically on the plane where (a + bi) corresponds to the point ((a, b)) in the plane.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>Recognize Classify polar equations (rose, cardioid, limaçon, lemniscate, spiral, and circle), given the graph or the equation.</td>
<td>Recognize Classify polar equations (rose, cardioid, limaçon, lemniscate, spiral, and circle), given the graph or the equation.</td>
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<tr>
<td>Determine the effects of changes in the parameters of polar equations on the graph, using a graphing utility.</td>
<td>Determine the effects of changes in the parameters of polar equations on the graph, using a graphing utility.</td>
</tr>
<tr>
<td>Convert between complex numbers from written in rectangular form and polar form and vice versa.</td>
<td>Convert between complex numbers from written in rectangular form and polar form and vice versa.</td>
</tr>
<tr>
<td>Find Determine and verify the intersection of the graphs of two polar equations, using a graphing utility.</td>
<td>Find Determine and verify the intersection of the graphs of two polar equations, using a graphing utility.</td>
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MATHEMATICAL ANALYSIS
STANDARD MA.1042

The student will use parametric equations to model and solve application problems.

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<tr>
<th>ESSENTIAL UNDERSTANDING(S)</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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<tbody>
<tr>
<td><strong>UNDERSTANDING THE STANDARD</strong></td>
<td><strong>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</strong></td>
</tr>
<tr>
<td>• Parametric equations are used to express two dependent variables, $x$ and $y$, in terms of an independent variable (parameter), $t$.</td>
<td>• Graph parametric equations, using a graphing utility.</td>
</tr>
<tr>
<td>• Some curves cannot be represented as a function, $f(x)$. Parametric graphing enables the representation of these curves in terms of functions.</td>
<td>• Use parametric equations to model practical problems, including motion over time.</td>
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<td></td>
<td>• Determine solutions to parametric equations graphically, using a graphing utility.</td>
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<td></td>
<td>• Compare and contrast traditional solution methods with parametric methods.</td>
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<td></td>
<td>• Use a graphing utility to graph and analyze parametric equations.</td>
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**MATHEMATICAL ANALYSIS**  
**STANDARD MA.1144**

The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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</table>
| • Matrices are a convenient shorthand form may be used to solving systems of equations.  
• Matrices can model a variety of linear systems.  
• Solutions of a linear system are values that satisfy every equation in the system.  
• Matrices can be used to model and solve real-world problems. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to  
• Multiply matrices by a scalar. [Reordered]  
• Add, subtract, and multiply matrices, and multiply matrices by a scalar.  
• Model problems with a system of no more than three linear equations.  
• Express a system of linear equations as a matrix equation.  
• Solve a system of equations using matrices.  
• Find the inverse of a 2 x 2 or 3 x 3 matrix using paper-pencil.  
• Verify two matrices are inverses using matrix multiplication.  
• Verify the commutative and associative properties for matrix addition and multiplication. |
**MATHEMATICAL ANALYSIS**
**STANDARD MA.124**

The student will expand binomials having positive integral exponents through the use of the Binomial Theorem, the formula for combinations, and Pascal’s Triangle. [Included in EKS]

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</table>
| • The Binomial Theorem provides a formula for calculating the product 
\[(a + b)^n\] for any positive integer \(n\). | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to |
| • Pascal’s Triangle is a triangular array of binomial coefficients. | • Expand binomials having positive integral exponents. |
| | • Use the Binomial Theorem, the formula for combinations, and Pascal’s Triangle to expand binomials. |
The student will **find** determine the sum (sigma notation included) of finite and infinite convergent series, which will lead to an intuitive approach to a limit.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Examination of infinite sequences and series may lead to a limiting process.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</td>
</tr>
<tr>
<td>Arithmetic sequences have a common difference between any two consecutive terms.</td>
<td>Use and interpret the notation: $\sum$, $n$, $n$th, and $a_n$.</td>
</tr>
<tr>
<td>Geometric sequences have a common factor, ratio between any two consecutive terms.</td>
<td>Derive the formulas associated with arithmetic and geometric sequences and series.</td>
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<td></td>
<td>Given the formula, <strong>find</strong> determine the nth term, $a_n$, for an arithmetic or geometric sequence.</td>
</tr>
<tr>
<td></td>
<td>Given the formula, <strong>find</strong> determine the sum, $S_n$, if it exists, of an arithmetic or geometric series.</td>
</tr>
<tr>
<td></td>
<td>Model and solve problems, using sequence and series information.</td>
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<td></td>
<td>Distinguish between a convergent and divergent series.</td>
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<td></td>
<td>Discuss convergent series in relation to the concept of a limit.</td>
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</table>
The student will use mathematical induction to prove formulas and mathematical statements.

**MATHEMATICAL ANALYSIS**  
**STANDARD MA.146**

<table>
<thead>
<tr>
<th>ESSENTIAL UNDERSTANDINGS UNDERSTANDING THE STANDARD</th>
<th>ESSENTIAL KNOWLEDGE AND SKILLS</th>
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</table>
| - Mathematical induction is a method of proof that depends on a recursive process.  
  - Mathematical induction allows reasoning from specific true values of the variable to general values of the variable. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to  
  - Compare inductive and deductive reasoning.  
  - Prove formulas and mathematical statements, using mathematical induction. |