Mathematics *Standards of Learning* for Virginia Public Schools

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Mathematics *Standards of Learning* for Virginia Public Schools

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Virginia 2023 Mathematics *Standards of Learning* Guiding Principles

In today's data-driven world, mastery of mathematical knowledge and skills is foundational for success. Virginia's robust and growing economy runs on workers with advanced mathematical skills. By setting clear, rigorous learning standards that are aligned to the needs of the world beyond high school, we are centering our K-12 education system on excellence. *How* we approach the implementation of these standards is just as important as defining the standards. The following Guiding Principles will help shape the policies and practices that will ensure that every Virginia student graduates high school with the mathematical knowledge and skills needed to be ready for college, career, and life.

Raise the Floor; Remove the Ceiling

Algebra is the gateway to higher education and promising careers. Therefore, every student in Virginia must continue their mathematics studies beyond Algebra before they graduate. We must also ensure there are no limits to how far or fast students can go in their mathematical studies. School divisions must clarify the pathways for the study of mathematics beyond Algebra and ensure that families are part of the decision-making process around which pathways best meet student needs and goals. The study of mathematics in Virginia will not be a "one size fits all" system. Due to the growing number of options for accessing advanced courses through dual and concurrent enrollment opportunities, Advanced Placement, Cambridge, and International Baccalaureate, there should be no limit to how far a student can progress in their mathematics journey by the time they leave high school.

All students will have the support and extra time to be able to access these rigorous pathways. The Department will partner with school divisions to ensure additional time is available for all students to access these courses. The Department will partner with school divisions to ensure extra time is available for all students to have supports such as tutoring and intervention.

Ensure Every Student Builds Strong Mathematics Foundational Skills

Students must possess strong foundational skills while also being able to apply these to real-world situations. Foundational skills, such as addition, subtraction, multiplication, division, fractions, and percentages are essential building blocks for upper-level mathematics.

What makes our foundation skill building best in class:

- Our kindergarten and first grade students are working with repeating patterns and counting by 5s and 10s while many other states are not doing this until much later. We are getting the foundations done much earlier.
- Our kindergarten and first grade students are grouping collections of objects in sets of tens and ones to strengthen their conceptual understanding of place value. Many other states do not include this in standards for kindergarten and first grade.

What makes our accelerated pathways the best in class:

- Our students are learning advanced algebraic models in middle school while in most states students are not exposed to this until high school.

- Our standards have students addressing greatest common factor in grade 4 and prime factorization of numbers and least common multiples in grade 5. Many other states do not start this work until middle school.

Master Critical Content

No student should move on to a new concept nor a new course without mastery of the prerequisite material and skills. Schools need to build processes that allow student mastery to drive how time is used rather than schedules dictating student coursework. This means rethinking the development of personalized mathematics pathways which ensure students are in courses that are challenging to them, yet manageable, as well as encouraging students who have mastered prerequisite material to enroll in more advanced courses and content beyond their grade level.

Schools, supported by their divisions and the Virginia Department of Education (VDOE), should prioritize meeting students where they are and supporting their individual academic needs—whether spending more time on mastering a difficult topic, filling a knowledge gap, or providing access to enrichment programs.

Integrate Mathematics Across All Content Areas

Mathematics appears in all content areas including a balanced equation in Chemistry, a measure in music, or a historic timeline in social studies. The application of mathematics in other content areas provides an opportunity for students to think critically and problem solve as students navigate complex data sets, design principles, and use technology in and across other content areas. Students will have the opportunity to learn mathematics in all content areas and be more informed citizens who see the connections between mathematics and the world around them.

Prepare Teachers to Teach Mathematics Accurately and Effectively

To have great mathematic students, we must have great mathematic teachers. Our educators will receive training that helps them convey content accurately and effectively to their students. By training teachers well, our students will master the basics of mathematics, deepen their mathematical knowledge and be able to apply mathematics across content areas and to real world situations. Teachers must have strong instructional materials, be trained in how to use these materials, and be able to support struggling students as well as provide advanced opportunities to those students who learn mathematics more quickly.

Apply Mathematics to Better Use Technology

Students will learn foundational mathematical skills and master computational skills before using technology as a substitute. As students' progress in their mathematical knowledge, they will apply mathematics in a variety of experiential contexts to learn and use technology appropriately. Students will learn how technology is a tool that facilitates complex mathematical thinking, requires students to solve complex problems and allows students to simulate real-world scenarios that integrates the application of mathematical reasoning and critical thinking.

Virginia 2023 Mathematics *Standards of Learning* – Background on the Revisions and a Roadmap for Successful Implementation of the Standards

Background and Overview of the Revision Process

The *Standards of Learning* provide a framework for instructional content to raise the academic achievement of all students in Virginia and to prepare them for college and career. Pursuant to legislation from the 2000 Virginia General Assembly, the Board of Education established a seven-year cycle for review of the *Standards of Learning*. As a result, the 1995 Mathematics *Standards of Learning* were reviewed in 2001, 2009, 2016, and 2023, the results of which are contained in this document.

The 2023 *Mathematics Standards* were revised with input from parents, teachers, the business community, school administrators, representatives from higher education and state mathematics education organizations. The *Standards* set clear and rigorous academic expectations for students. Parents and families are encouraged to work with their children, their children's teachers, and their children's schools to help them achieve these academic standards.

The following resources were central to informing this revision and ensuring that these standards are best in class:

- National Assessment of Educational Progress (NAEP) Framework (2026);
- National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics (2000);
- NCTM Focus in High School Mathematics: Reasoning and Sense Making (2009);
- Pre-K–12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II) report (2020) from the American Statistical Association;
- Mathematics and College Career Readiness Standards (ACT, 2022), SAT Suite of Assessments (2014) - Mathematics Test Details.

Changes from Prior Standards

There are several notable changes from the most recent set of standards. These include the following:

1) *An emphasis on the mastery of basic facts* - The mathematics standards establish foundational skills at each grade level and course for which students will develop proficiency. Building automaticity with basic mathematics facts is critical to ensure that students establish a firm foundation to learn more complex mathematical concepts in upper grade levels.
2) *Coherence* - Content standards for mathematics must be developmentally appropriate within a grade level and vertically coherent across the grade levels and courses to ensure that students master skills at each grade level. The focus of the standards concurrently emphasizes conceptual understanding, procedural knowledge, and application of mathematics content to provide building blocks from grade level to grade level and course to course.
3) *Mastery of Skills, Understanding of Concepts, and Application of Both:* The mathematics standards foster the application of the five mathematical process goals including reasoning, communication, problem solving, connections, and representation, and set students up to recognize and see mathematics in real-world applications. These processes support students in becoming problem solvers and to be able to make mathematical connections using mathematical representations to model and interpret contextual situations. Preparing Virginia's students to pursue higher education, to compete in a modern workforce, and to be

informed citizens requires rigorous mathematical knowledge and skills. Students must gain a basic of mathematical skills while also gaining an understanding of ideas. The approach taken in these standards ensures students can apply mathematical concepts.

4) *Students will have personalized mathematics pathways based on mastery and readiness*- Throughout a student's mathematics schooling from kindergarten through grade eight, specific content strands are included. These content strands are Number and Number Sense; Computation and Estimation; Measurement and Geometry; Probability and Statistics; and Patterns, Functions, and Algebra. The *Standards of Learning* and knowledge and skills within each strand progress in complexity throughout the grade levels and into high school course content. The *Standards* are organized by content strand and arranged in a numeric order, however local curricula and pacing guides should determine the instructional sequence of the content that is best suited to meet the needs and goals of the student. The standards reinforce foundational mathematics skills and facts to ensure that students can accelerate at their own pace after they have mastered a concept or skill. These standards are designed to ensure mastery from grade to grade, providing an opportunity for students to demonstrate competency before learning the next skill.

The Virginia Mathematics *Standards of Learning* are built to ensure that our standards:

1) Are best in class, and are some of the most challenging in the nation as they are benchmarked against NAEP, ACT, and SAT;
2) Reinforce foundational mathematics skills and facts to ensure that students can accelerate at their own pace;
3) Ensure students can apply mathematical concepts in their career and college pathways; and
4) Are designed to ensure mastery from grade to grade and provide an opportunity for students to demonstrate competency before the next skill.

Implementation

These standards are best in class. They are benchmarked against the National Assessment for Educational Progress (NAEP), the ACT and SAT and are therefore among the most challenging mathematics standards in the nation. Successful implementation of these rigorous standards will ensure that every Virginia high school graduate has the mathematical skills, knowledge and competencies to pursue higher education, to compete in a modern workforce, and to be informed citizens.

To ensure these standards are implemented successfully, the following actions should be taken:

VDOE will:

1) Implement the Standards of Quality by providing opportunities for students to take Algebra and other advanced coursework in middle school;
2) Provide instructional guides to school divisions on how to best implement the 2023 Mathematics *Standards of Learning* to ensure a seamless transition to more rigorous expectations;
3) Partner with educators in regional hubs across the Commonwealth to network and collaboratively support teachers with implementation;
4) Ensure that teachers are equipped with high quality instructional materials and tutoring resources be able to effectively teach the new standards;
5) Become shoulder to shoulder partners with School Divisions to develop the capacity of teachers, Central Office leaders, and principals through professional learning opportunities in virtual, in person, and year-round opportunities;

6) Provide differentiated supports to school divisions who are accredited with conditions in partnership with the Office of School Quality and the Office of Student Supports;
7) Create regional supports and professional learning for teachers to include scaffolds and supports for English Language Learners, Gifted and Talented, and Special Education students; and
8) Design parent resources and supports to increase parent engagement in their students' education and mathematics achievement.

The State Board of Education will:

1) Review the graduation requirements to ensure students have four years of credit-bearing mathematics experiences;
2) Set the definition of mastery (also known as proficiency) at every grade level so that students are on track to be prepared to meet to the needs of higher education, the military, and the workplace. This proficiency cut score on assessments will be benchmarked to NAEP and the most rigorous state definitions of proficiency;
3) Design a state accountability system so that:
    a) parents, teachers, policymakers have a clear picture of how every school is or is not ensuring every child is making progress to and attaining mastery of the standards; and
    b) all students not making progress to and attaining mastery of the standards benefit from targeted supports and interventions.

Local School Divisions will:

1) Provide teachers with resources, training, and support to effectively implement the 2023 Mathematics *Standards of Learning*;
2) Ensure students, parents, and teachers have timely information on student growth and mastery of mathematics content;
3) Provide tutoring and interventions as just in time supports for those students struggling to reach grade level mastery;
4) Clearly communicate the pathways available to students within their middle school and help ready students to pursue accelerated pathways in high school; and
5) Highlight and increase the number of enrichment opportunities available to students through dual enrollment, summer camps, and higher education programming.

Mathematics *Standards of Learning* for Virginia Public Schools

2023 Kindergarten

The Kindergarten *Standards* place emphasis on developing the concept of number by counting, recognizing, representing, and comparing quantities; recognizing and describing with fluency part-part-whole relationships for numbers up to 5; as well as modeling and solving addition and subtraction problems within 10. Students will recognize, describe, and create simple repeating patterns; and recognize and describe plane figures according to their characteristics; and construct plane figures using a variety of materials. Students will investigate measurement of length, height, weight, volume, and time using direct comparisons. Students will engage with the data cycle using object graphs and picture graphs. The data cycle includes formulating questions to be explored with data; collecting or acquiring data; organizing and representing data; and analyzing data and communicating results.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem-solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

K.NS.1 The student will utilize flexible counting strategies to determine and describe quantities up to 100.

Students will demonstrate the following Knowledge and Skills:

a) Use one-to-one correspondence to determine how many are in a given set containing 30 or fewer concrete objects (e.g., cubes, pennies, balls), and describe the last number named as the total number of objects counted.
b) Recognize and explain that the number of objects remains the same regardless of the arrangement or the order in which the objects are counted.
c) Represent forward counting by ones using a variety of tools, including five-frames, ten-frames, and number paths (a prelude to number lines).
d) Count forward orally by ones from 0 to 100.
e) Count forward orally by ones, within 100, starting at any given number.
f) Count backward orally by ones when given any number between 1 and 20.
g) State the number after, without counting, when given any number between 0 and 30.
h) State the number before, without counting, when given any number between 1 and 20.

i) Use objects, drawings, words, or numbers to compose and decompose numbers 11-19 into a ten and some ones.

j) Group a collection of up to 100 objects (e.g., counters, pennies, cubes) into sets of ten and count by tens to determine the total (e.g., there are 3 groups of ten and 6 leftovers, 36 total objects).

K.NS.2 The student will identify, represent, and compare quantities up to 30.

Students will demonstrate the following Knowledge and Skills:

a) Read, write, and identify the numerals 0 through 30.

b) Construct a set of objects that corresponds to a given numeral within 30, including an empty set.

c) Determine and write the numeral that corresponds to the total number of objects in a given set of 30 or fewer concrete objects or pictorial models.

d) Given a set of up to 30 objects, construct another set which has more, fewer, or the same number of objects using concrete or pictorial models.

e) Given a numeral up to 30, construct a set which has more, fewer, or the same number of objects using concrete or pictorial models.

f) Compare two sets containing up to 30 concrete objects or pictorial models, using the terms *more*, *fewer*, or the *same as* (*equal to*).

g) Compare numbers up to 30, to the benchmarks of 5 and 10 using various models (e.g., five frames, ten frames, number paths [a prelude to number lines], beaded racks, hands) using the terms *greater than*, *less than*, or the *same as* (*equal to*).

Computation and Estimation

K.CE.1 The student will model and solve single-step contextual problems using addition and subtraction with whole numbers within 10.

Students will demonstrate the following Knowledge and Skills:

a) Use objects, drawings, words, or numbers to compose and decompose numbers less than or equal to 5 in multiple ways.

b) Recognize and describe with fluency part-part-whole relationships for numbers up to 5 in a variety of configurations.

c) Model and identify the number that makes 5 when added to a given number less than or equal to 5.

d) Use objects, drawings, words, or numbers to compose and decompose numbers less than or equal to 10 in multiple ways.

e) Model and identify the number that makes 10 when added to a given number less than or equal to 10.

f) Model and solve single-step contextual problems (join, separate, and part-part-whole) using 10 or fewer concrete objects.

Measurement and Geometry

K.MG.1 The student will reason mathematically by making direct comparisons between two objects or events using the attributes of length, height, weight, volume, and time.

Students will demonstrate the following Knowledge and Skills:

a) Use direct comparisons to compare, describe, and justify the:
   i) lengths of two objects using the terms longer or shorter;
   ii) heights of two objects using the terms taller or shorter;
   iii) weights of two objects using the terms heavier or lighter;
   iv) volumes of two containers using the terms more or less; and
   v) amount of time spent on two events using the terms longer or shorter.

K.MG.2 The student will identify, describe, name, compare, and construct plane figures (circles, triangles, squares, and rectangles).

Students will demonstrate the following Knowledge and Skills:

a) Identify and name concrete and pictorial representations of circles, triangles, squares, and rectangles regardless of their orientation in space.
b) Describe triangles, squares, and rectangles to include the number of sides and number of vertices.
c) Describe a circle using terms such as *round* and *curved*.
d) Distinguish between examples and nonexamples of identified plane figures (circles, triangles, squares, and rectangles).
e) Compare and contrast two plane figures using characteristics to describe similarities and differences.
f) Construct plane figures (circles, triangles, squares, and rectangles) using a variety of materials (e.g., straws, sticks, pipe cleaners).

K.MG.3 The student will describe the units of time represented in a calendar.

Students will demonstrate the following Knowledge and Skills:

a) Identify a calendar as a tool used to measure time.
b) Name the days of the week and state that there are seven days in one week.
c) Determine the day before and after a given day (e.g., yesterday, today, tomorrow).
d) Name the twelve months of the year and state that there are twelve months in one year.
e) Distinguish between days of the week and months of the year.

Probability and Statistics

K.PS.1 The student will apply the data cycle (pose questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on object graphs and picture graphs.

Students will demonstrate the following Knowledge and Skills:

a) Sort and classify concrete objects into appropriate subsets (categories) based on one attribute (e.g., size, shape, color, thickness).
b) Describe and label attributes (e.g., size, color, shape) of a set of objects (e.g., coins, counters, buttons) that has been sorted.
c) Pose questions, given a predetermined context, that require the collection of data (limited to 25 or fewer data points for no more than four categories).

d) Determine the data needed to answer a posed question, and collect the data using various methods (e.g., counting objects, drawing pictures).
e) Organize and represent a data set (vertically or horizontally) by sorting concrete objects into organized groups to form a simple object graph.
f) Organize and represent a data set (vertically or horizontally) using pictures to form a simple picture graph.
g) Analyze data represented in object graphs and picture graphs and communicate results:
    i) ask and answer questions about the data represented in object graphs and picture graphs (e.g., how many in each category, which categories have the greatest, least, or the same amount of data); and
    ii) draw conclusions about the data and make predictions based on the data.

Patterns, Functions, and Algebra

K.PFA.1 The student will identify, describe, extend, and create simple repeating patterns using various representations.

Students will demonstrate the following Knowledge and Skills:

a) Identify and describe the core found in repeating patterns.
b) Extend a repeating pattern by adding at least two complete repetitions of the core to the pattern.
c) Create and describe a repeating pattern using objects, colors, sounds, movements, or pictures.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 1

The Grade 1 *Standards* place emphasis on counting, representing, comparing, and ordering sets of objects up to 120; recognizing, describing, and transferring repeating and growing patterns; and analyzing and sorting plane figures. Students' understanding of number is expanded through recognizing and describing part-part-whole relationships for numbers up to 10, recalling addition and subtraction facts to 10 with automaticity, as well as solving problems using addition and subtraction within 20. Students will begin the study of fractions by solving problems that involve partitioning models into two and four equal-sized parts. Students will use nonstandard units to measure and compare objects by length, weight, and volume. Students will engage with the data cycle using object graphs, picture graphs, and tables. The data cycle includes formulating questions to be explored with data; collecting or acquiring data; organizing and representing data; and analyzing data and communicating results.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem-solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

1.NS.1 The student will utilize flexible counting strategies to determine and describe quantities up to 120.

Students will demonstrate the following Knowledge and Skills:

a) Count forward orally by ones from 0 to 120 starting at any number between 0 and 120.
b) Count backward orally by ones when given any number between 1 and 30.
c) Represent forward counting patterns when counting by groups of 5 and groups of 10 up to 120 using a variety of tools (e.g., objects, coins, 120 chart).
d) Represent forward counting patterns when counting by groups of 2 up to at least 30 using a variety of tools (e.g., beaded number strings, number paths [a prelude to number lines], 120 chart).
e) Group a collection of up to 120 objects into tens and ones, and count to determine the total (e.g., 5 groups of ten and 6 ones is equal to 56 total objects).
f) Identify a penny, nickel, and dime by their attributes and describe the number of pennies equivalent to a nickel and a dime.

g) Count by ones, fives, or tens to determine the value of a collection of like coins (pennies, nickels, or dimes), whose total value is 100 cents or less.

1.NS.2 The student will represent, compare, and order quantities up to 120.

Students will demonstrate the following Knowledge and Skills:

a) Read and write numerals 0-120 in sequence and out of sequence.
b) Estimate the number of objects (up to 120) in a given collection and justify the reasonableness of an answer.
c) Create a concrete or pictorial representation of a number using tens and ones and write the corresponding numeral up to 120 (e.g., 47 can be represented as 47 ones or it can be grouped into 4 tens with 7 ones left over).
d) Describe the number of groups of tens and ones when given a two-digit number and justify reasoning.
e) Compare two numbers between 0 and 120 represented pictorially or with concrete objects using the terms *greater than*, *less than*, or *equal to*.
f) Order three sets, each set containing up to 120 objects, from least to greatest, and greatest to least.

1.NS.3 The student will use mathematical reasoning and justification to solve contextual problems that involve partitioning models into two and four equal-sized parts.

Students will demonstrate the following Knowledge and Skills:

a) Represent equal shares of a whole with two or four sharers, when given a contextual problem.
b) Represent and name halves and fourths of a whole, using a region/area model (e.g., pie pieces, pattern blocks, paper folding, drawings) and a set model (e.g., eggs, marbles, counters) limited to two or four items.
c) Describe and justify how shares are equal pieces or equal parts of the whole (limited to halves, fourths) when given a contextual problem.

Computation and Estimation

1.CE.1 The student will recall with automaticity addition and subtraction facts within 10 and represent, solve, and justify solutions to single-step problems, including those in context, using addition and subtraction with whole numbers within 20.

Students will demonstrate the following Knowledge and Skills:

a) Recognize and describe with fluency part-part-whole relationships for numbers up to 10 in a variety of configurations.
b) Demonstrate fluency with addition and subtraction within 10 by applying reasoning strategies (e.g., count on/count back, one more/one less, doubles, make ten).
c) Recall with automaticity addition and subtraction facts within 10.
d) Investigate, recognize, and describe part-part-whole relationships for numbers up to 20 in a variety of configurations (e.g., beaded racks, double ten frames).
e) Solve addition and subtraction problems within 20 using various strategies (e.g., inverse relationships: if 9 + 3 = 12 then 12 - 3 = 9; decomposition using known sums/differences: 9 + 7 can be thought of as 9 decomposed into 2 and 7, then use doubles, 7 + 7 = 14; 14 + 2 = 16 or decompose the 7 into 1 and 6; make a ten: 1 + 9 = 10; 10 + 6 = 16).

f) Represent, solve, and justify solutions to single-step addition and subtraction problems (join, separate, and part-part-whole) within 20, including those in context, using words, objects, drawings, or numbers.

g) Determine the unknown whole number that will result in a sum or difference of 10 or 20 (e.g., 14 - __ = 10 or 15 + __ = 20).

h) Identify and use (+) as a symbol for addition and (-) as a symbol for subtraction.

i) Describe the equal symbol (=) as a balance representing an equivalent relationship between expressions on either side of the equal symbol (e.g., 6 and 1 is the same as 4 and 3; 6 + 1 is balanced with 4 + 3; 6 + 1 = 4 + 3).

j) Use concrete materials to model, identify, and justify when two expressions are not equal (e.g., 10 - 3 is not equal to 3 + 5).

k) Use concrete materials to model an equation that represents the relationship of two expressions of equal value.

l) Write an equation that could be used to represent the solution to an oral, written, or picture problem.

Measurement and Geometry

1.MG.1 The student will reason mathematically using nonstandard units to measure and compare objects by length, weight, and volume.

Students will demonstrate the following Knowledge and Skills:

a) Use nonstandard units to measure the:

   i) lengths of two objects (units laid end to end with no gaps or overlaps) and compare the measurements using the terms longer/shorter, taller/shorter, or the same as;

   ii) weights of two objects (using a balance scale or a pan scale) and compare the measurements using the terms lighter, heavier, or the same as; and

   iii) volumes of two containers and compare the measurements using the terms more, less, or the same as.

b) Measure the length, weight, or volume of the same object or container with two different units and describe how and why the measurements differ.

1.MG.2 The student will describe, sort, draw, and name plane figures (circles, triangles, squares, and rectangles), and compose larger plane figures by combining simple plane figures.

Students will demonstrate the following Knowledge and Skills:

a) Describe triangles, squares, and rectangles using the terms sides, vertices, and angles. Describe a circle using terms such as *round* and *curved*.

b) Sort plane figures based on their characteristics (e.g., number of sides, vertices, angles, curved).

c) Draw and name the plane figure (circle, square, rectangle, triangle) when given information about the number of sides, vertices, and angles.

d) Identify, name, and describe representations of circles, squares, rectangles, and triangles, regardless of orientation, in different environments and explain reasoning.

e) Recognize and name the angles found in rectangles and squares as right angles.

f) Compose larger plane figures by combining two or three simple plane figures (triangles, squares, and/or rectangles).

1.MG.3 The student will demonstrate an understanding of the concept of passage of time (to the nearest hour and half-hour) and the calendar.

Students will demonstrate the following Knowledge and Skills:

a) Identify different tools to measure time including clocks (analog and digital) and calendar.
b) Describe the units of time represented on a clock as minutes and hours.
c) Tell time to the hour and half-hour, using analog and digital clocks.
d) Describe the location of the hour hand relative to time to the hour and half-hour on an analog clock.
e) Describe the location of the minute hand relative to time to the hour and half-hour on an analog clock.
f) Match the time shown on a digital clock to an analog clock to the hour and half-hour.
g) Identify specific days/dates on a calendar (e.g., What date is Saturday? How many Fridays are in October?).
h) Use ordinal numbers first through tenth to describe the relative position of specific days/dates (e.g., What is the first Monday in October? What day of the week is May 6th?).
i) Determine the day/date before and after a given day/date (e.g., Today is the 8th, so yesterday was the ?), and a date that is a specific number of days/weeks in the past or future (e.g., Tim's birthday is in 10 days, what will be the date of his birthday?).

Probability and Statistics

1.PS.1 The student will apply the data cycle (pose questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on object graphs, picture graphs, and tables.

Students will demonstrate the following Knowledge and Skills:

a) Sort and classify concrete objects into appropriate subsets (categories) based on one or two attributes, such as size, shape, color, and/or thickness (e.g., sort a set of objects that are both red and thick).
b) Describe and label attributes of a set of objects that has been sorted.
c) Pose questions, given a predetermined context, that require the collection of data (limited to 25 or fewer data points for no more than four categories).
d) Determine the data needed to answer a posed question and collect the data using various methods (e.g., counting objects, drawing pictures, tallying).
e) Organize and represent a data set by sorting the collected data using various methods (e.g., tallying, T-charts).
f) Represent a data set (vertically or horizontally) using object graphs, picture graphs, and tables.
g) Analyze data represented in object graphs, picture graphs, and tables and communicate results:
    i) ask and answer questions about the data represented in object graphs, picture graphs, and tables (e.g., total number of data points represented, how many in each category, how many more or less are in one category than another); and

ii) draw conclusions about the data and make predictions based on the data.

Patterns, Functions, and Algebra

1.PFA.1 The student will identify, describe, extend, create, and transfer repeating patterns and increasing patterns using various representations.

Students will demonstrate the following Knowledge and Skills:

a) Identify and describe repeating and increasing patterns.
b) Analyze a repeating or increasing pattern and generalize the change to extend the pattern using objects, colors, movements, pictures, or geometric figures.
c) Create a repeating or increasing pattern using objects, pictures, movements, colors, or geometric figures.
d) Transfer a repeating or increasing pattern from one form to another.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 2

The Grade 2 *Standards* extend the study of number and spatial sense to include three-digit whole numbers and solid geometric figures. Students will demonstrate fluency and recall with automaticity addition and subtraction facts within 20. Students will also solve single-step and multistep problems, including those in context, involving addition or subtraction of whole numbers where addends or minuends do not exceed 100. Students will begin to use U.S. Customary units to measure length, weight, and liquid volume to the nearest whole unit. Students will engage with the data cycle using pictographs and bar graphs. The data cycle includes formulating questions to be explored with data; collecting or acquiring data; organizing and representing data; and analyzing data and communicating results.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem-solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

2.NS.1 The student will utilize flexible counting strategies to determine and describe quantities up to 200.

Students will demonstrate the following Knowledge and Skills:

a) Represent forward counting patterns when counting by groups of 2 up to at least 50, starting at various multiples of 2 and using a variety of tools (e.g., objects, number lines, hundreds charts).

b) Represent forward counting patterns created when counting by groups of 5s, 10s, and 25s starting at various multiples up to at least 200 using a variety of tools (e.g., objects, number lines, hundreds charts).

c) Describe and use patterns in skip counting by multiples of 2 (to at least 50), and multiples of 5, 10, and 25 (to at least 200) to justify the next number in the counting sequence.

d) Represent forward counting patterns when counting by groups of 100 up to at least 1,000 starting at 0 using a variety of tools (e.g., objects, number lines, calculators, one thousand charts).

e) Represent backward counting patterns when counting by groups of 10 from 200 or less using a variety of tools including objects, number lines, calculators, and hundreds charts.

f) Describe and use patterns in skip counting backwards by 10s (from at least 200) to justify the next number in the counting sequence.

g) Choose a reasonable estimate up to 1,000 when given a contextual problem (e.g., What would be the best estimate for the number of students in our school – 5, 50, or 500?).

h) Represent even numbers (up to 50) with concrete objects, using two equal groups or two equal addends.

i) Represent odd numbers (up to 50) with concrete objects, using two equal groups with one leftover or two equal addends plus 1.

j) Determine whether a number (up to 50) is even or odd using concrete objects and justify reasoning (e.g., dividing collections of objects into two equal groups, pairing objects).

2.NS.2 The student will demonstrate an understanding of the ten-to-one relationships of the base 10 number system to represent, compare, and order whole numbers up to 999.

Students will demonstrate the following Knowledge and Skills:

a) Write the three-digit whole number represented by a given model (e.g., concrete objects, pictures of base 10 blocks).

b) Read, write, and represent three-digit numbers in standard form, expanded form, and word form, using concrete or pictorial representations.

c) Apply patterns within the base 10 system to determine and communicate, orally and in written form, the place (ones, tens, hundreds) and value of each digit in a three-digit whole number (e.g., in 352, the 5 represents 5 tens and its value is 50).

d) Investigate and explain the ten-to-one relationships among ones, tens, and hundreds, using models.

e) Compose and decompose whole numbers up to 200 by making connections between a variety of models (e.g., base 10 blocks, place value cards, presented orally, in expanded or standard form) and counting strategies (e.g., 156 can be 1 hundred, 5 tens, 6 ones; 1 hundred, 4 tens, 16 ones; 15 tens, 6 ones).

f) Plot and justify the position of a given number up to 100 on a number line with pre-marked benchmarks of 1s, 2s, 5s, 10s, or 25s.

g) Compare two whole numbers, each 999 or less, represented concretely, pictorially, or symbolically, using words (greater than, less than, or equal to) and symbols (>, <, or =). Justify reasoning orally, in writing, or with a model.

h) Order up to three whole numbers, each 999 or less, represented concretely, pictorially, or symbolically from least to greatest and greatest to least.

2.NS.3 The student will use mathematical reasoning and justification to solve contextual problems that involve partitioning models into equal-sized parts (halves, fourths, eighths, thirds, and sixths).

Students will demonstrate the following Knowledge and Skills:

a) Model and describe fractions as representing equal-size parts of a whole.

b) Describe the relationship between the number of fractional parts needed to make a whole and the size of the parts (i.e., as the whole is divided into more parts, each part becomes smaller).

c) Compose the whole for a given fractional part and its value (in context) for halves, fourths, eighths, thirds, and sixths (e.g., when given $\frac{1}{4}$, determine how many pieces would be needed to make $\frac{4}{4}$).

d) Using same-size fraction pieces, from a region/area model, count by unit fractions up to two wholes (e.g., zero one-fourths, one one-fourth, two one-fourths, three one-fourths, four one-fourths, five one-fourths; or zero-fourths, one-fourth, two-fourths, three-fourths, four-fourths, five-fourths).

e) Given a context, represent, name, and write fractional parts of a whole for halves, fourths, eighths, thirds, and sixths using:

   i) region/area models (e.g., pie pieces, pattern blocks, geoboards);

   ii) length models (e.g., paper fraction strips, fraction bars, rods, number lines); and

   iii) set models (e.g., chips, counters, cubes).

f) Compare unit fractions for halves, fourths, eighths, thirds, and sixths using words (greater than, less than or equal to) and symbols (>, <, =), with region/area and length models.

2.NS.4 The student will solve problems that involve counting and representing money amounts up to $2.00.

Students will demonstrate the following Knowledge and Skills:

a) Identify a quarter and its value and determine multiple ways to represent the value of a quarter using pennies, nickels, and/or dimes.

b) Count by ones, fives, tens, and twenty-fives to determine the value of a collection of mixed coins and one-dollar bills whose total value is $2.00 or less.

c) Construct a set of coins and/or bills to total a given amount of money whose value is $2.00 or less.

d) Represent the value of a collection of coins and one-dollar bills (limited to $2.00 or less) using the cent (¢) and dollar ($) symbols and decimal point (.).

Computation and Estimation

2.CE.1 The student will recall with automaticity addition and subtraction facts within 20 and estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction with whole numbers where addends or minuends do not exceed 100.

Students will demonstrate the following Knowledge and Skills:

a) Apply strategies (e.g., rounding to the nearest 10, compatible numbers, other number relationships) to estimate a solution for single-step addition or subtraction problems, including those in context, where addends and minuends do not exceed 100.

b) Apply strategies (e.g., the use of concrete and pictorial models, place value, properties of addition, the relationship between addition and subtraction) to determine the sum or difference of two whole numbers where addends or minuends do not exceed 100.

c) Represent, solve, and justify solutions to single-step and multistep contextual problems (e.g., join, separate, part-part-whole, comparison) involving addition or subtraction of whole numbers where addends or minuends do not exceed 100.

d) Demonstrate fluency with addition and subtraction within 20 by applying reasoning strategies (e.g., doubles, near doubles, make-a-ten, compensations, inverse relationships).

e) Recall with automaticity addition and subtraction facts within 20.

f) Use patterns, models, and strategies to make generalizations about the algebraic properties for fluency (e.g., 4 + 3 is equal to 3 + 4; 0 + 8 = 8).

g) Determine the missing number in an equation (number sentence) through modeling and justification with addition and subtraction within 20 (e.g., 3 + __ = 5 or __ + 2 = 5; 5 – __ = 3 or 5 – 2 = __).

h) Use inverse relationships to write all related facts connected to a given addition or subtraction fact model within 20 (e.g., given a model for 3 + 4 = 7, write 4 + 3 = 7, 7 – 4 = 3, and 7 – 3 = 4).

i) Describe the not equal symbol (≠) as representing a relationship where expressions on either side of the not equal symbol represent different values and justify reasoning.

j) Represent and justify the relationship between values and expressions as equal or not equal using appropriate models and/or symbols (e.g., 9 + 24 = 10 + 23; 45 - 9 = 46 - 10; 15 +16 ≠ 31 +15).

Measurement and Geometry

2.MG.1 The student will reason mathematically using standard units (U.S. Customary) with appropriate tools to estimate, measure, and compare objects by length, weight, and liquid volume to the nearest whole unit.

Students will demonstrate the following Knowledge and Skills:

a) Explain the purpose of various measurement tools and how to use them appropriately by:
   i) identifying a ruler as an instrument to measure length;
   ii) identifying different types of scales as instruments to measure weight; and
   iii) identifying different types of measuring cups as instruments to measure liquid volume.

b) Use U.S. Customary units to estimate, measure, and compare the two for reasonableness:
   i) the length of an object to the nearest inch, using a ruler;
   ii) the weight of an object to the nearest pound, using a scale; and
   iii) the liquid volume of a container to the nearest cup, using a measuring cup.

2.MG.2 The student will demonstrate an understanding of the concept of time to the nearest five minutes, using analog and digital clocks.

Students will demonstrate the following Knowledge and Skills:

a) Identify the number of minutes in an hour (60 minutes) and the number of hours in a day (24 hours).

b) Determine the unit of time (minutes, hours, days, or weeks) that is most appropriate when measuring a given activity or context and explain reasoning (e.g., Would you measure the time it takes to brush your teeth in minutes or hours?).

c) Show, tell, and write time to the nearest five minutes, using analog and digital clocks.

d) Match a written time (e.g., 1:35, 6:20, 9:05) to the time shown on an analog clock to the nearest five minutes.

2.MG.3 The student will identify, describe, and create plane figures (including circles, triangles, squares, and rectangles) that have at least one line of symmetry and explain its relationship with congruency.

Students will demonstrate the following Knowledge and Skills:

a) Explore a figure using a variety of tools (e.g., paper folding, geoboards, drawings) to show and justify a line of symmetry, if one exists.
b) Create figures with at least one line of symmetry using various concrete and pictorial representations.
c) Describe the two resulting figures formed by a line of symmetry as being congruent (having the same shape and size).

2.MG.4 The student will describe, name, compare, and contrast plane and solid figures (circles/spheres, squares/cubes, and rectangles/rectangular prisms).

Students will demonstrate the following Knowledge and Skills:

a) Trace faces of solid figures (cubes and rectangular prisms) to create the set of plane figures related to the solid figure.
b) Compare and contrast models and nets (cutouts) of cubes and rectangular prisms (e.g., number and shapes of faces, edges, vertices).
c) Given a concrete or pictorial model, name and describe the solid figure (sphere, cube, and rectangular prism) by its characteristics (e.g., number of edges, number of vertices, shapes of faces).
d) Compare and contrast plane and solid figures (circles/spheres, squares/cubes, and rectangles/rectangular prisms) according to their characteristics (e.g., number and shapes of their faces, edges, vertices).

Probability and Statistics

2.PS.1 The student will apply the data cycle (pose questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on pictographs and bar graphs.

Students will demonstrate the following Knowledge and Skills:

a) Pose questions, given a predetermined context, that require the collection of data (limited to 25 or fewer data points for no more than six categories).
b) Determine the data needed to answer a posed question and collect the data using various methods (e.g., voting; creating lists, tables, or charts; tallying).
c) Organize and represent a data set using a pictograph where each symbol represents up to 2 data points. Determine and use a key to assist in the analysis of the data.
d) Organize and represent a data set using a bar graph with a title and labeled axes (limited to 25 or fewer data points for up to six categories, and limit increments of scale to multiples of 1 or 2).
e) Analyze data represented in pictographs and bar graphs and communicate results:
    i) ask and answer questions about the data represented in pictographs and bar graphs (e.g., total number of data points represented, how many in each category, how many more or less are in one category than another). Pictograph keys will be limited to symbols

representing 1, 2, 5, or 10 pieces of data and bar graphs will be limited to scales with increments in multiples of 1, 2, 5, or 10; and

ii) draw conclusions about the data and make predictions based on the data.

Patterns, Functions, and Algebra

2.PFA.1 The student will describe, extend, create, and transfer repeating and increasing patterns (limited to addition of whole numbers) using various representations.

Students will demonstrate the following Knowledge and Skills:

a) Identify and describe repeating and increasing patterns.

b) Analyze a repeating or increasing pattern and generalize the change to extend the pattern using objects, pictures, and numbers.

c) Create a repeating or increasing pattern using various representations (e.g., objects, pictures, numbers).

d) Transfer a given repeating or increasing pattern from one form to another (e.g., objects, pictures, numbers) and explain the connection between the two patterns.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 3

The Grade 3 *Standards* place emphasis on developing an understanding of the concepts of multiplication and division. Students will recall with automaticity multiplication facts through 10 × 10 and the corresponding division facts and will use this knowledge to solve contextual problems with multiplication and division. Students will extend knowledge of place value to six-digit whole numbers. Concrete models and pictorial representations will be used to deepen understanding of fraction concepts, including comparing, composing, and decomposing. Students will use standard units (U.S. Customary and metric) to measure length, weight/mass, and liquid volume. Students will be introduced to solving contextual problems involving elapsed time. Properties of shapes will be explored, and students will identify polygons, combine and subdivide polygons, and name the resulting polygon(s). Students will engage with the data cycle using pictographs and bar graphs. The data cycle includes formulating questions to be explored with data; collecting or acquiring data; organizing and representing data; and analyzing data and communicating results. Students will work with a variety of growing patterns using various representations.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem-solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

3.NS.1 The student will use place value understanding to read, write, and determine the place and value of each digit in a whole number, up to six digits, with and without models.

Students will demonstrate the following Knowledge and Skills:

a) Read and write six-digit whole numbers in standard form, expanded form, and word form.
b) Apply patterns within the base 10 system to determine and communicate, orally and in written form, the place and value of each digit in a six-digit whole number (e.g., in 165,724, the 5 represents 5 thousands and its value is 5,000).
c) Compose, decompose, and represent numbers up to 9,999 in multiple ways, according to place value (e.g., 256 can be 1 hundred, 14 tens, 16 ones, but also 25 tens, 6 ones), with and without models.

3.NS.2 The student will demonstrate an understanding of the base 10 system to compare and order whole numbers up to 9,999.

Students will demonstrate the following Knowledge and Skills:

a) Compare two whole numbers, each 9,999 or less, using symbols (>, <, =, ≠) and/or words (*greater than*, *less than*, *equal to*, *not equal to*), with and without models.
b) Order up to three whole numbers, each 9,999 or less, represented with and without models, from least to greatest and greatest to least.

3.NS.3 The student will use mathematical reasoning and justification to represent and compare fractions (proper and improper) and mixed numbers with denominators of 2, 3, 4, 5, 6, 8, and 10), including those in context.

Students will demonstrate the following Knowledge and Skills:

a) Represent, name, and write a given fraction (proper or improper) or mixed number with denominators of 2, 3, 4, 5, 6, 8, and 10 using:
   i) region/area models (e.g., pie pieces, pattern blocks, geoboards);
   ii) length models (e.g., paper fraction strips, fraction bars, rods, number lines); and
   iii) set models (e.g., chips, counters, cubes).
b) Identify a fraction represented by a model as the sum of unit fractions.
c) Use a model of a fraction greater than one to count the fractional parts to name and write it as an improper fraction and as a mixed number (e.g., $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4} = 1\frac{1}{4}$).
d) Compose and decompose fractions (proper and improper) with denominators of 2, 3, 4, 5, 6, 8, and 10 in multiple ways (e.g., $\frac{7}{4} = \frac{4}{4} + \frac{3}{4}$ or $\frac{4}{6} = \frac{3}{6} + \frac{1}{6} = \frac{2}{6} + \frac{2}{6}$) with models.
e) Compare a fraction, less than or equal to one, to the benchmarks of 0, $\frac{1}{2}$, and 1 using area/region models, length models, and without models.
f) Compare two fractions (proper or improper) and/or mixed numbers with like numerators of 2, 3, 4, 5, 6, 8, and 10 (e.g., $\frac{2}{3} > \frac{2}{8}$) using words (*greater than*, *less than*, *equal to*) and/or symbols (>, <, =), using area/region models, length models, and without models.
g) Compare two fractions (proper or improper) and/or mixed numbers with like denominators of 2, 3, 4, 5, 6, 8, and 10 (e.g., $\frac{3}{6} < \frac{4}{6}$) using words (*greater than*, *less than*, *equal to*) and/or symbols (>, <, =), using area/region models, length models, and without models.
h) Represent equivalent fractions with denominators of 2, 3, 4, 5, 6, 8, or 10, using region/area models and length models.

3.NS.4 The student will solve problems, including those in context, that involve counting, comparing, representing, and making change for money amounts up to $5.00.

Students will demonstrate the following Knowledge and Skills:

a) Determine the value of a collection of bills and coins whose total is $5.00 or less.
b) Construct a set of bills and coins to total a given amount of money whose value is $5.00 or less.
c) Compare the values of two sets of coins or two sets of bills and coins, up to $5.00, with words (*greater than*, *less than*, *equal to)* and/or symbols (>, <, =) using concrete or pictorial models.
d) Solve contextual problems to make change from $5.00 or less by using counting on or counting back strategies with concrete or pictorial models.

Computation and Estimation

3.CE.1 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction with whole numbers where addends and minuends do not exceed 1,000.

Students will demonstrate the following Knowledge and Skills:

a) Determine and justify whether an estimate or an exact answer is appropriate when solving single-step and multistep contextual problems involving addition and subtraction, where addends and minuends do not exceed 1,000.

b) Apply strategies (e.g., rounding to the nearest 10 or 100, using compatible numbers, using other number relationships) to estimate a solution for single-step or multistep addition or subtraction problems, including those in context, where addends or minuends do not exceed 1,000.

c) Apply strategies (e.g., place value, properties of addition, other number relationships) and algorithms, including the standard algorithm, to determine the sum or difference of two whole numbers where addends and minuends do not exceed 1,000.

d) Identify and use the appropriate symbol to distinguish between expressions that are equal and expressions that are not equal (e.g., $256 - 13 = 220 + 23$; $457 + 100 \neq 557 + 100$).

e) Represent, solve, and justify solutions to single-step and multistep contextual problems involving addition and subtraction with whole numbers where addends and minuends do not exceed 1,000.

3.CE.2 The student will recall with automaticity multiplication and division facts through 10 × 10; and represent, solve, and justify solutions to single-step contextual problems using multiplication and division with whole numbers.

Students will demonstrate the following Knowledge and Skills:

a) Represent multiplication and division of whole numbers through 10 × 10, including in a contextual situation, using a variety of approaches and models (e.g., repeated addition/subtraction, equal-sized groups/sharing, arrays, equal jumps on a number line, using multiples to skip count).

b) Use inverse relationships to write the related facts connected to a given model for multiplication and division of whole numbers through 10 × 10.

c) Apply strategies (e.g., place value, the properties of multiplication and/or addition) when multiplying and dividing whole numbers.

d) Demonstrate fluency with multiplication facts through 10 × 10 by applying reasoning strategies (e.g., doubling, add-a-group, subtract-a-group, near squares, and inverse relationships).

e) Represent, solve, and justify solutions to single-step contextual problems that involve multiplication and division of whole numbers through 10 × 10.

f) Recall with automaticity the multiplication facts through 10 × 10 and the corresponding division facts.

g) Create an equation to represent the mathematical relationship between equivalent expressions using multiplication and/or division facts through 10 × 10 (e.g., $4 \times 3 = 14 - 2$, $35 \div 5 = 1 \times 7$).

Measurement and Geometry

3.MG.1 The student will reason mathematically using standard units (U.S. Customary and metric) with appropriate tools to estimate and measure objects by length, weight/mass, and liquid volume to the nearest half or whole unit.

Students will demonstrate the following Knowledge and Skills:

a) Justify whether an estimate or an exact measurement is needed for a contextual situation and choose an appropriate unit.

b) Estimate and measure:

   i) length of an object to the nearest U.S. Customary unit ($\frac{1}{2}$ inch, inch, foot, yard) and metric unit (centimeter, meter);

   ii) weight/mass of an object to the nearest U.S. Customary unit (pound) and metric unit (kilogram); and

   iii) liquid volume to the nearest U.S. Customary unit (cup, pint, quart, gallon) and metric unit (liter).

c) Compare estimates of length, weight/mass, or liquid volume with the actual measurements.

3.MG.2 The student will use multiple representations to estimate and solve problems, including those in context, involving area and perimeter (in both U.S. Customary and metric units).

Students will demonstrate the following Knowledge and Skills:

a) Solve problems, including those in context, involving area:

   i) describe and give examples of area as a measurement in contextual situations; and

   ii) estimate and determine the area of a given surface by counting the number of square units, describe the measurement (using the number and unit) and justify the measurement.

b) Solve problems, including those in context, involving perimeter:

   i) describe and give examples of perimeter as a measurement in contextual situations;

   ii) estimate and measure the distance around a polygon (with no more than six sides) to determine the perimeter and justify the measurement; and

   iii) given the lengths of all sides of a polygon (with no more than six sides), determine its perimeter and justify the measurement.

3.MG.3 The student will demonstrate an understanding of the concept of time to the nearest minute and solve single-step contextual problems involving elapsed time in one-hour increments within a 12-hour period.

Students will demonstrate the following Knowledge and Skills:

a) Tell and write time to the nearest minute, using analog and digital clocks.

b) Match a written time (e.g., 4:38, 7:09, 12:51) to the time shown on analog and digital clocks to the nearest minute.

c) Solve single-step contextual problems involving elapsed time in one-hour increments, within a 12-hour period (within a.m. or within p.m.) when given:

   i) the starting time and the ending time, determine the amount of time that has elapsed;

ii) the starting time and amount of elapsed time in one-hour increments, determine the ending time; or

iii) the ending time and the amount of elapsed time in one-hour increments, determine the starting time.

3.MG.4 The student will identify, describe, classify, compare, combine, and subdivide polygons.

Students will demonstrate the following Knowledge and Skills:

a) Describe a polygon as a closed plane figure composed of at least three line segments that do not cross.

b) Classify figures as polygons or not polygons and justify reasoning.

c) Identify and describe triangles, quadrilaterals, pentagons, hexagons, and octagons in various orientations, with and without contexts.

d) Identify and name examples of polygons (triangles, quadrilaterals, pentagons, hexagons, octagons) in the environment.

e) Classify and compare polygons (triangles, quadrilaterals, pentagons, hexagons, octagons).

f) Combine no more than three polygons, where each has three or four sides, and name the resulting polygon (triangles, quadrilaterals, pentagons, hexagons, octagons).

g) Subdivide a three-sided or four-sided polygon into no more than three parts and name the resulting polygons.

Probability and Statistics

3.PS.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on pictographs and bar graphs.

Students will demonstrate the following Knowledge and Skills:

a) Formulate questions that require the collection or acquisition of data.

b) Determine the data needed to answer a formulated question and collect or acquire existing data (limited to 30 or fewer data points for no more than eight categories) using various methods (e.g., polls, observations, tallies).

c) Organize and represent a data set using pictographs that include an appropriate title, labeled axes, and key. Each pictograph symbol should represent 1, 2, 5 or 10 data points.

d) Organize and represent a data set using bar graphs with a title and labeled axes, with and without the use of technology tools. Determine and use an appropriate scale (increments limited to multiples of 1, 2, 5 or 10).

e) Analyze data represented in pictographs and bar graphs, and communicate results orally and in writing:

i) describe the categories of data and the data as a whole (e.g., data were collected on preferred ways to cook or prepare eggs - scrambled, fried, hard boiled, and egg salad);

ii) identify parts of the data that have special characteristics, including categories with the greatest, the least, or the same (e.g., most students prefer scrambled eggs);

iii) make inferences about data represented in pictographs and bar graphs;

iv) use characteristics of the data to draw conclusions about the data and make predictions based on the data (e.g., it is unlikely that a third grader would like hard boiled eggs); and

v) solve one- and two-step addition and subtraction problems using data from pictographs and bar graphs.

Patterns, Functions, and Algebra

3.PFA.1 The student will identify, describe, extend, and create increasing and decreasing patterns (limited to addition and subtraction of whole numbers), including those in context, using various representations.

Students will demonstrate the following Knowledge and Skills:

a) Identify and describe increasing and decreasing patterns using various representations (e.g., objects, pictures, numbers, number lines).

b) Analyze an increasing or decreasing pattern and generalize the change to extend the pattern or identify missing terms using various representations.

c) Solve contextual problems that involve identifying, describing, and extending patterns.

d) Create increasing and decreasing patterns using objects, pictures, numbers, and number lines.

e) Investigate and explain the connection between two different representations of the same increasing or decreasing pattern.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 4

The Grade 4 *Standards* place emphasis on multiplication and division with whole numbers and solving problems involving addition and subtraction of decimals and fractions with like denominators. Students will recall with automaticity multiplication through 12×12 and the corresponding division facts as they become proficient in multiplying and dividing larger numbers. Students will apply knowledge of place value and the properties of addition and multiplication as strategies for solving problems. Students will identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices. Students will describe and compare characteristics of plane and solid figures. Concrete models and pictorial representations will be used to solve problems involving perimeter and area, patterns, probability, and equivalence of fractions and decimals. Students will engage with the data cycle using line graphs. The data cycle includes formulating questions to be explored with data; collecting or acquiring data; organizing and representing data; and analyzing data and communicating results.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem-solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

4.NS.1 The student will use place value understanding to read, write, and identify the place and value of each digit in a nine-digit whole number.

Students will demonstrate the following Knowledge and Skills:

a) Read nine-digit whole numbers, presented in standard form, and represent the same number in written form.

b) Write nine-digit whole numbers in standard form when the numbers are presented orally or in written form.

c) Apply patterns within the base 10 system to determine and communicate, orally and in written form, the place and value of each digit in a nine-digit whole number (e.g., in 568,165,724, the 8 represents 8 millions and its value is 8,000,000).

4.NS.2 The student will demonstrate an understanding of the base 10 system to compare and order whole numbers up to seven digits.

Students will demonstrate the following Knowledge and Skills:

a) Compare two whole numbers up to seven digits each, using words (*greater than, less than, equal to, not equal to*) and/or using symbols (>, <, =, ≠).
b) Order up to four whole numbers up to seven digits each, from least to greatest or greatest to least.

4.NS.3 The student will use mathematical reasoning and justification to represent, compare, and order fractions (proper, improper, and mixed numbers with denominators 12 or less), with and without models.

Students will demonstrate the following Knowledge and Skills:

a) Compare and order no more than four fractions (proper or improper), and/or mixed numbers, with like denominators by comparing the number of parts (numerators) using fractions with denominators of 12 or less (e.g., $\frac{1}{5} < \frac{3}{5}$). Justify comparisons orally, in writing, or with a model.*
b) Compare and order no more than four fractions (proper or improper), and/or mixed numbers, with like numerators and unlike denominators by comparing the size of the parts using fractions with denominators of 12 or less (e.g., $\frac{3}{8} < \frac{3}{5}$). Justify comparisons orally, in writing, or with a model.*
c) Use benchmarks (e.g., 0, $\frac{1}{2}$, or 1) to compare and order no more than four fractions (proper or improper), and/or mixed numbers, with like and unlike denominators of 12 or less. Justify comparisons orally, in writing, or with a model.*
d) Compare two fractions (proper or improper) and/or mixed numbers using fractions with denominators of 12 or less, using the symbols >, <, and = (e.g., $\frac{2}{3} > \frac{1}{7}$). Justify comparisons orally, in writing, or with a model.*
e) Represent equivalent fractions with denominators of 12 or less, with and without models.*
f) Compose and decompose fractions (proper and improper) and/or mixed numbers with denominators of 12 or less, in multiple ways, with and without models.*
g) Represent the division of two whole numbers as a fraction given a contextual situation and a model (e.g., $\frac{3}{5}$ means the same as 3 divided by 5 or $\frac{3}{5}$ represents the amount of muffin each of five children will receive when sharing three muffins equally).

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

4.NS.4 The student will use mathematical reasoning and justification to represent, compare, and order decimals through thousandths, with and without models.

Students will demonstrate the following Knowledge and Skills:

a) Investigate and describe the ten-to-one place value relationship for decimals through thousandths, using concrete models (e.g., place value mats/charts, decimal squares, base 10 blocks).
b) Represent and identify decimals expressed through thousandths, using concrete, pictorial, and numerical representations.
c) Read and write decimals expressed through thousandths, using concrete, pictorial, and numerical representations.
d) Identify and communicate, both orally and in written form, the place and value of each digit in a decimal through thousandths (e.g., given 0.385, the 8 is in the hundredths place and has a value of 0.08).

e) Compare using symbols (<, >, =) and/or words (*greater than, less than, equal to*) and order (least to greatest and greatest to least), a set of no more than four decimals expressed through thousandths, using multiple strategies (e.g., benchmarks, place value, number lines). Justify comparisons with a model, orally, and in writing.

4.NS.5 The student will reason about the relationship between fractions and decimals (limited to halves, fourths, fifths, tenths, and hundredths) to identify and represent equivalencies.

Students will demonstrate the following Knowledge and Skills:

a) Represent fractions (proper or improper) and/or mixed numbers as decimals through hundredths, using multiple representations, limited to halves, fourths, fifths, tenths, and hundredths.*

b) Identify and model equivalent relationships between fractions (proper or improper) and/or mixed numbers and decimals, using halves, fourths, fifths, tenths, and hundredths.*

c) Write the decimal and fraction equivalent for a given model (e.g., $\frac{1}{4}$ = 0.25 or 0.25 = $\frac{1}{4}$; 1.25 = $\frac{5}{4}$ or $1\frac{1}{4}$; 1.02 = $\frac{102}{100}$ or $1\frac{2}{100}$).*

* **On the state assessment, items measuring this objective are assessed without the use of a calculator.**

Computation and Estimation

4.CE.1 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction with whole numbers.

Students will demonstrate the following Knowledge and Skills:

a) Determine and justify whether an estimate or an exact answer is appropriate when solving contextual problems involving addition and subtraction with whole numbers. Refine estimates by adjusting the final amount, using terms such as *closer to*, *between*, and *a little more than*.

b) Apply strategies (e.g., rounding to the nearest 100 or 1,000, using compatible numbers, other number relationships) to estimate a solution for single-step or multistep addition or subtraction problems with whole numbers, where addends or minuends do not exceed 10,000.*

c) Apply strategies (e.g., place value, properties of addition, other number relationships) and algorithms, including the standard algorithm, to determine the sum or difference of two whole numbers, where addends and minuends do not exceed 10,000.*

d) Estimate, represent, solve, and justify solutions to single-step and multistep contextual problems involving addition and subtraction with whole numbers where addends and minuends do not exceed 1,000,000.

***On the state assessment, items measuring this objective are assessed without the use of a calculator.**

4.CE.2 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using multiplication with whole numbers, and single-step problems, including those in context, using division with whole numbers; and recall with automaticity the multiplication facts through 12 × 12 and the corresponding division facts.

Students will demonstrate the following Knowledge and Skills:

a) Determine and justify whether an estimate or an exact answer is appropriate when solving contextual problems involving multiplication and division of whole numbers. Refine estimates by adjusting the final amount, using terms such as *closer to*, *between*, and *a little more than*.

b) Recall with automaticity the multiplication facts through 12 × 12 and the corresponding division facts.*

c) Create an equation using addition, subtraction, multiplication, and division to represent the relationship between equivalent mathematical expressions (e.g., 4 × 3 = 2 × 6; 10 + 8 = 36 ÷ 2; 12 × 4 = 60 − 12).

d) Identify and use the appropriate symbol to distinguish between expressions that are equal and expressions that are not equal, using addition, subtraction, multiplication, and division (e.g., 4 × 12 = 8 × 6 and 64 ÷ 8 ≠ 8 × 8).

e) Determine all factor pairs for a whole number 1 to 100, using concrete, pictorial, and numerical representations.

f) Determine common factors and the greatest common factor of no more than three numbers.

g) Apply strategies (e.g., rounding, place value, properties of multiplication and/or addition) and algorithms, including the standard algorithm, to estimate and determine the product of two whole numbers when given:

   i) a two-digit factor and a one-digit factor;*

   ii) a three-digit factor and a one-digit factor;* or

   iii) a two-digit factor and a two-digit factor.*

h) Estimate, represent, solve, and justify solutions to single-step and multistep contextual problems that involve multiplication with whole numbers.

i) Apply strategies (e.g., rounding, compatible numbers, place value) and algorithms, including the standard algorithm, to estimate and determine the quotient of two whole numbers, given a one-digit divisor and a two- or three-digit dividend, with and without remainders.*

j) Estimate, represent, solve, and justify solutions to single-step contextual problems involving division with whole numbers.

k) Interpret the quotient and remainder when solving a contextual problem.

* **On the state assessment, items measuring this objective are assessed without the use of a calculator.**

4.CE.3 The student will estimate, represent, solve, and justify solutions to single-step problems, including those in context, using addition and subtraction of fractions (proper, improper, and mixed numbers with like denominators of 2, 3, 4, 5, 6, 8, 10, and 12), with and without models; and solve single-step contextual problems involving multiplication of a whole number (12 or less) and a unit fraction, with models.

Students will demonstrate the following Knowledge and Skills:

a) Estimate and determine the sum or difference of two fractions (proper or improper) and/or mixed numbers, having like denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12 (e.g., $\frac{3}{8} + \frac{3}{8}$, $2\frac{1}{5} + \frac{4}{5}$, $\frac{7}{4} - \frac{5}{4}$) and simplify the resulting fraction. Addition and subtraction with fractions may include regrouping.*

b) Estimate, represent, solve, and justify solutions to single-step contextual problems using addition and subtraction with fractions (proper or improper) and/or mixed numbers, having like denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction. Addition and subtraction with fractions may include regrouping.

c) Solve single-step contextual problems involving multiplication of a whole number, limited to 12 or less, and a unit fraction (e.g., $6 \times \frac{1}{3}$, $\frac{1}{5} \times 8$, $2 \times \frac{1}{10}$), with models.*

d) Apply the inverse property of multiplication in models (e.g., use a visual fraction model to represent $\frac{4}{4}$ or 1 as the product of $4 \times \frac{1}{4}$).

* **On the state assessment, items measuring this objective are assessed without the use of a calculator.**

4.CE.4 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction of decimals through the thousandths, with and without models.

Students will demonstrate the following Knowledge and Skills:

a) Apply strategies (e.g., rounding to the nearest whole number, using compatible numbers) and algorithms, including the standard algorithm, to estimate and determine the sum or difference of two decimals through the thousandths, with and without models, in which:*
   i) decimals do not exceed the thousandths; and
   ii) addends, subtrahends, and minuends are limited to four digits.

b) Estimate, represent, solve, and justify solutions to single-step and multistep contextual problems using addition and subtraction of decimals through the thousandths.

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

Measurement and Geometry

4.MG.1 The student will reason mathematically to solve problems, including those in context, that involve length, weight/mass, and liquid volume using U.S. Customary and metric units.

Students will demonstrate the following Knowledge and Skills:

a) Determine an appropriate unit of measure to use when measuring:
   i) length in both U.S. Customary (inch, foot, yard, mile) and metric units (millimeter, centimeter, meter);
   ii) weight/mass in both U.S. Customary (ounce, pound) and metric units (gram, kilogram); and
   iii) liquid volume in both U.S. Customary (cup, pint, quart, gallon) and metric units (milliliter, liter).

b) Estimate and measure:
   i) length of an object to the nearest U.S. Customary unit ($\frac{1}{2}$ inch, $\frac{1}{4}$ inch, $\frac{1}{8}$ inch, foot, yard) and nearest metric unit (millimeter, centimeter, or meter);
   ii) weight/mass of an object to the nearest U.S. Customary unit (ounce, pound) and nearest metric unit (gram, kilogram); and
   iii) liquid volume to the nearest U.S. Customary unit (cup, pint, quart, gallon) and nearest metric unit (milliliter, liter).

c) Compare estimates of length, weight/mass, or liquid volume with the actual measurements.

d) Given the equivalent measure of one unit, solve problems, including those in context, by determining the equivalent measures within the U.S. Customary system for:
   i) length (inches and feet, feet and yards, inches and yards);

ii) weight/mass (ounces and pounds); and

iii) liquid volume (cups, pints, quarts, and gallons).

4.MG.2 The student will solve single-step and multistep contextual problems involving elapsed time (limited to hours and minutes within a 12-hour period).

Students will demonstrate the following Knowledge and Skills:

a) Solve single-step and multistep contextual problems involving elapsed time in hours and minutes, within a 12-hour period (within a.m., within p.m., and across a.m. and p.m.) when given:

   i) the starting time and the ending time, determine the amount of time that has elapsed in hours and minutes;

   ii) the starting time and amount of elapsed time in hours and minutes, determine the ending time; or

   iii) the ending time and the amount of elapsed time in hours and minutes, determine the starting time.

4.MG.3 The student will use multiple representations to develop and use formulas to solve problems, including those in context, involving area and perimeter limited to rectangles and squares (in both U.S. Customary and metric units).

Students will demonstrate the following Knowledge and Skills:

a) Use concrete materials and pictorial models to develop a formula for the area and perimeter of a rectangle (including a square).

b) Determine the area and perimeter of a rectangle when given the measure of two adjacent sides (in whole number units), with and without models.

c) Determine the area and perimeter of a square when given the measure of one side (in whole number units), with and without models.

d) Use concrete materials and pictorial models to explore the relationship between area and perimeter of rectangles.

e) Identify and represent rectangles with the same perimeter and different areas or with the same area and different perimeters.

f) Solve contextual problems involving area and perimeter of rectangles and squares.

4.MG.4 The student will identify, describe, and draw points, rays, line segments, angles, and lines, including intersecting, parallel, and perpendicular lines.

Students will demonstrate the following Knowledge and Skills:

a) Identify and describe points, lines, line segments, rays, and angles, including endpoints and vertices.

b) Describe endpoints and vertices in relation to lines, line segments, rays, and angles.

c) Draw representations of points, line segments, rays, angles, and lines, using a ruler or straightedge.

d) Identify parallel, perpendicular, and intersecting lines and line segments in plane and solid figures, including those in context.

e) Use symbolic notation to name points, lines, line segments, rays, angles, and to describe parallel and perpendicular lines.

4.MG.5 The student will classify and describe quadrilaterals (parallelograms, rectangles, squares, rhombi, and/or trapezoids) using specific properties and attributes.

Students will demonstrate the following Knowledge and Skills:

a) Develop definitions for parallelograms, rectangles, squares, rhombi, and trapezoids through the exploration of properties and attributes.
b) Identify and describe points, line segments, angles, and vertices in quadrilaterals.
c) Identify and describe parallel, intersecting, perpendicular, and congruent sides in quadrilaterals.
d) Compare, contrast, and classify quadrilaterals (parallelograms, rectangles, squares, rhombi, and/or trapezoids) based on the following properties and attributes:
   i) parallel sides;
   ii) perpendicular sides;
   iii) congruence of sides; and
   iv) number of right angles.
e) Denote properties of quadrilaterals and identify parallel sides, congruent sides, and right angles by using geometric markings.
f) Use symbolic notation to name line segments and angles in quadrilaterals.

4.MG.6 The student will identify, describe, compare, and contrast plane and solid figures according to their characteristics (number of angles, vertices, edges, and the number and shape of faces), with and without models.

Students will demonstrate the following Knowledge and Skills:

a) Identify concrete models and pictorial representations of solid figures (cube, rectangular prism, square pyramid, sphere, cone, and cylinder).
b) Identify and describe solid figures (cube, rectangular prism, square pyramid, and sphere) according to their characteristics (number of angles, vertices, edges, and by the number and shape of faces).
c) Compare and contrast plane and solid figures (limited to circles, squares, triangles, rectangles, spheres, cubes, square pyramids, and rectangular prisms) according to their characteristics (number of sides, angles, vertices, edges, and the number and shape of faces).

Probability and Statistics

4.PS.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on line graphs.

Students will demonstrate the following Knowledge and Skills:

a) Formulate questions that require the collection or acquisition of data.
b) Determine the data needed to answer a formulated question and collect or acquire existing data (limited to 10 or fewer data points) using various methods (e.g., observations, measurements, experiments).
c) Organize and represent a data set using line graphs with a title and labeled axes with whole number increments, with and without the use of technology tools.

d) Analyze data represented in line graphs and communicate results orally and in writing:
   i) describe the characteristics of the data represented in a line graph and the data as a whole (e.g., the time period when the temperature increased the most);
   ii) identify parts of the data that have special characteristics and explain the meaning of the greatest, the least, or the same (e.g., the highest temperature shows the warmest day);
   iii) make inferences about data represented in line graphs;
   iv) draw conclusions about the data and make predictions based on the data to answer questions; and
   v) solve single-step and multistep addition and subtraction problems using data from line graphs.

4.PS.2 The student will model and determine the probability of an outcome of a simple event.

Students will demonstrate the following Knowledge and Skills:

a) Describe probability as the degree of likelihood of an outcome occurring using terms such as *impossible*, *unlikely*, *equally likely*, *likely*, and *certain*.
b) Model and determine all possible outcomes of a given simple event where there are no more than 24 possible outcomes, using a variety of manipulatives (e.g., coins, two-sided counters, number cubes, spinners).
c) Write the probability of a given simple event as a fraction between 0 and 1, where there are no more than 24 possible outcomes.
d) Determine the likelihood of an event occurring and relate it to its whole number or fractional representation (e.g., impossible or zero; equally likely; certain or one).
e) Create a model or contextual problem to represent a given probability.

Patterns, Functions, and Algebra

4.PFA.1 The student will identify, describe, extend, and create increasing and decreasing patterns (limited to addition, subtraction, and multiplication of whole numbers), including those in context, using various representations.

Students will demonstrate the following Knowledge and Skills:

a) Identify, describe, extend, and create increasing and decreasing patterns using various representations (e.g., objects, pictures, numbers, number lines, input/output tables, and function machines).
b) Analyze an increasing or decreasing single-operation numerical pattern found in lists, input/output tables, or function machines and generalize the change to identify the rule, extend the pattern, or identify missing terms.
c) Given a rule, create increasing and decreasing patterns using numbers and input/output tables (including function machines).
d) Solve contextual problems that involve identifying, describing, and extending increasing and decreasing patterns using single-operation input and output rules.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 5

The Grade 5 *Standards* place emphasis on solving problems, including those in context, with whole numbers, fractions, and decimals. Students will develop an understanding of prime and composite numbers, determine the prime factorization of a whole number up to 100, and solve problems using order of operations. Students will also solve problems involving area, perimeter, and volume. Students will determine the probability of an outcome by constructing a model of a sample space and using the Fundamental (Basic) Counting Principle. Students will engage with the data cycle using line plots and stem-and-leaf plots. The data cycle includes formulating questions to be explored with data; collecting or acquiring data; organizing and representing data; and analyzing data and communicating results. Students will be introduced to expressions with a variable to support the development of the algebraic concepts necessary for success in the middle grades.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem-solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

5.NS.1 The student will use reasoning and justification to identify and represent equivalency between fractions (with denominators that are thirds, eighths, and factors of 100) and decimals; and compare and order sets of fractions (proper, improper, and/or mixed numbers having denominators of 12 or less) and decimals (through thousandths).

Students will demonstrate the following Knowledge and Skills:

a) Use concrete and pictorial models to represent fractions with denominators that are thirds, eighths, and factors of 100 in their equivalent decimal form.*

b) Use concrete and pictorial models to represent decimals in their equivalent fraction form (thirds, eighths, and factors of 100).*

c) Identify equivalent relationships between decimals and fractions with denominators that are thirds, eighths, and factors of 100 in their equivalent decimal form, with and without models.*

d) Compare (using symbols <, >, =) and order (least to greatest and greatest to least) a set of no more than four decimals and fractions (proper, improper) and/or mixed numbers using multiple strategies (e.g., benchmarks, place value, number lines). Justify solutions orally, in writing, or with a model.*

* **On the state assessment, items measuring this objective are assessed without the use of a calculator.**

5.NS.2 The student will demonstrate an understanding of prime and composite numbers, and determine the prime factorization of a whole number up to 100.

Students will demonstrate the following Knowledge and Skills:

a) Given a whole number up to 100, create a concrete or pictorial representation to demonstrate whether the number is prime or composite, and justify reasoning.
b) Classify, compare, and contrast whole numbers up to 100 using the characteristics prime and composite.
c) Determine the prime factorization for a whole number up to 100.

Computation and Estimation

5.CE.1 The student will estimate, represent, solve, and justify solutions to single-step and multistep contextual problems using addition, subtraction, multiplication, and division with whole numbers.

Students will demonstrate the following Knowledge and Skills:

a) Estimate the sum, difference, product, and quotient of whole numbers in contextual problems.
b) Represent, solve, and justify solutions to single-step and multistep contextual problems by applying strategies (e.g., estimation, properties of addition and multiplication) and algorithms, including the standard algorithm, involving addition, subtraction, multiplication, and division of whole numbers, with and without remainders, in which:
   i) sums, differences, and products do not exceed five digits;
   ii) factors do not exceed two digits by three digits;
   iii) divisors do not exceed two digits; or
   iv) dividends do not exceed four digits.
c) Interpret the quotient and remainder when solving a contextual problem.

5.CE.2 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction of fractions with like and unlike denominators (with and without models), and solve single-step contextual problems involving multiplication of a whole number and a proper fraction, with models.

Students will demonstrate the following Knowledge and Skills:

a) Determine the least common multiple of two numbers to find the least common denominator for two fractions.
b) Estimate and determine the sum or difference of two fractions (proper or improper) and/or mixed numbers, having like and unlike denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12 (e.g., $\frac{5}{8}+\frac{1}{4}$, $\frac{4}{5}-\frac{2}{3}$, $3\frac{3}{4}+2\frac{5}{12}$), and simplify the resulting fraction.*
c) Estimate and solve single-step and multistep contextual problems involving addition and subtraction with fractions (proper or improper) and/or mixed numbers having like and unlike denominators, with and without models. Denominators should be limited to 2, 3, 4, 5, 6, 8, 10, and 12. Answers should be expressed in simplest form.

d) Solve single-step contextual problems involving multiplication of a whole number, limited to 12 or less, and a proper fraction (e.g., $9 \times \frac{2}{3}$, $8 \times \frac{3}{4}$), with models. The denominator will be a factor of the whole number and answers should be expressed in simplest form.*

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

5.CE.3 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition, subtraction, multiplication, and division with decimal numbers.

Students will demonstrate the following Knowledge and Skills:

a) Apply estimation strategies (e.g., rounding to the nearest whole number, tenth or hundredth; compatible numbers, place value) to determine a reasonable solution for single-step and multistep contextual problems involving addition, subtraction, and multiplication of decimals, and single-step contextual problems involving division of decimals.

b) Estimate and determine the product of two numbers using strategies and algorithms, including the standard algorithm, when given:

   i) a two-digit factor and a one-digit factor (e.g., 2.3×4; 0.08×0.9; $.16 \times 5$);*

   ii) a three-digit factor and a one-digit factor (e.g., 0.156×4, 3.28×7, 8.09×0.2);* and

   iii) a two-digit factor and a two-digit factor (e.g., 0.85×3.7, 14×1.6, 9.2×3.5).*

   (Products will not exceed the thousandths place, and leading zeroes will not be considered when counting digits.)

c) Estimate and determine the quotient of two numbers using strategies and algorithms, including the standard algorithm, in which:*

   i) quotients do not exceed four digits with or without a decimal point;

   ii) quotients may include whole numbers, tenths, hundredths, or thousandths;

   iii) divisors are limited to a single digit whole number or a decimal expressed as tenths; and

   iv) no more than one additional zero will need to be annexed.

d) Solve single-step and multistep contextual problems involving addition, subtraction, and multiplication of decimals by applying strategies (e.g., estimation, modeling) and algorithms, including the standard algorithm.

e) Solve single-step contextual problems involving division with decimals by applying strategies (e.g., estimation, modeling) and algorithms, including the standard algorithm.

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

5.CE.4 The student will simplify numerical expressions with whole numbers using the order of operations.

Students will demonstrate the following Knowledge and Skills:

a) Use order of operations to simplify numerical expressions with whole numbers, limited to addition, subtraction, multiplication, and division in which:*

   i) expressions may contain no more than one set of parentheses;

   ii) simplification will be limited to five whole numbers and four operations in any combination of addition, subtraction, multiplication, or division;

   iii) whole numbers will be limited to two digits or less; and

   iv) expressions should not include braces, brackets, or fraction bars.

b) Given a whole number numerical expression involving more than one operation, describe which operation is completed first, which is second, and which is third.*

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

Measurement and Geometry

5.MG.1 The student will reason mathematically to solve problems, including those in context, that involve length, mass, and liquid volume using metric units.

Students will demonstrate the following Knowledge and Skills:

a) Determine the most appropriate unit of measure to use in a contextual problem that involves metric units:
   i) length (millimeters, centimeters, meters, and kilometers);
   ii) mass (grams and kilograms); and
   iii) liquid volume (milliliters and liters).

b) Estimate and measure to solve contextual problems that involve metric units:
   i) length (millimeters, centimeters, and meters);
   ii) mass (grams and kilograms); and
   iii) liquid volume (milliliters and liters).

c) Given the equivalent metric measure of one unit, in a contextual problem, determine the equivalent measurement within the metric system:
   i) length (millimeters, centimeters, meters, and kilometers);
   ii) mass (grams and kilograms); and
   iii) liquid volume (milliliters and liters).

5.MG.2 The student will use multiple representations to solve problems, including those in context, involving perimeter, area, and volume.

Students will demonstrate the following Knowledge and Skills:

a) Investigate and develop a formula for determining the area of a right triangle.

b) Estimate and determine the area of a right triangle, with diagrams, when the base and the height are given in whole number units, in metric or U.S. Customary units, and record the solution with the appropriate unit of measure (e.g., 16 square inches).

c) Describe volume as a measure of capacity and give examples of volume as a measurement in contextual situations.

d) Investigate and develop a formula for determining the volume of rectangular prisms using concrete objects.

e) Solve problems, including those in context, to estimate and determine the volume of a rectangular prism using concrete objects, diagrams, and formulas when the length, width, and height are given in whole number units. Record the solution with the appropriate unit of measure (e.g., 12 cubic inches).

f) Identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation.

g) Solve contextual problems that involve perimeter, area, and volume in standard units of measure.

5.MG.3 The student will classify and measure angles and triangles, and solve problems, including those in context.

Students will demonstrate the following Knowledge and Skills:

a) Classify angles as right, acute, obtuse, or straight and justify reasoning.
b) Classify triangles as right, acute, or obtuse and equilateral, scalene, or isosceles and justify reasoning.
c) Identify congruent sides and right angles using geometric markings to denote properties of triangles.
d) Compare and contrast the properties of triangles.
e) Identify the appropriate tools (e.g., protractor, straightedge, angle ruler, available technology) to measure and draw angles.
f) Measure right, acute, obtuse, and straight angles, using appropriate tools, and identify measures in degrees.
g) Use models to prove that the sum of the interior angles of a triangle is 180 degrees and use the relationship to determine an unknown angle measure in a triangle.
h) Solve addition and subtraction contextual problems to determine unknown angle measures on a diagram.

Probability and Statistics

5.PS.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on line plots (dot plots) and stem-and-leaf plots.

Students will demonstrate the following Knowledge and Skills:

a) Formulate questions that require the collection or acquisition of data.
b) Determine the data needed to answer a formulated question and collect or acquire existing data (limited to 30 or fewer data points) using various methods (e.g., polls, observations, measurements, experiments).
c) Organize and represent a data set using a line plot (dot plot) with a title, labeled axes, and a key, with and without the use of technology tools. Lines plots (dot plots) may contain whole numbers, fractions, or decimals.
d) Organize and represent numerical data using a stem-and-leaf plot with a title and key, where the stems are listed in ascending order and the leaves are in ascending order, with or without commas between the leaves.
e) Analyze data represented in line plots (dot plots) and stem-and-leaf plots and communicate results orally and in writing:
    i) describe the characteristics of the data represented in a line plot (dot plot) and stem-and-leaf plot as a whole (e.g., the shape and spread of the data);
    ii) make inferences about data represented in line plots (dot plots) and stem-and-leaf plots (e.g., based on a line plot (dot plot) of the number of books students in a bus line have in their backpack, every student will have from two to four books in their backpack);
    iii) identify parts of the data that have special characteristics and explain the meaning of the greatest, the least, or the same (e.g., the stem-and-leaf plot shows that the same number of students scored in the 90s as scored in the 70s);

iv) draw conclusions about the data and make predictions based on the data to answer questions; and

v) solve single-step and multistep addition and subtraction problems using data from line plots (dot plots) and stem-and-leaf plots.

5.PS.2 The student will solve contextual problems using measures of center and the range.

Students will demonstrate the following Knowledge and Skills:

a) Describe mean as fair share.

b) Describe and determine the mean of a set of data values representing data from a given context as a measure of center.

c) Describe and determine the median of a set of data values representing data from a given context as a measure of center.

d) Describe and determine the mode of a set of data values representing data from a given context as a measure of center.

e) Describe and determine the range of a set of data values representing data from a given context as a measure of spread.

5.PS.3 The student will determine the probability of an outcome by constructing a model of a sample space and using the Fundamental (Basic) Counting Principle.

Students will demonstrate the following Knowledge and Skills:

a) Determine the probability of an outcome by constructing a sample space (with a total of 24 or fewer equally likely possible outcomes), using a tree diagram, list, or chart to represent and determine all possible outcomes.

b) Determine the number of possible outcomes by using the Fundamental (Basic) Counting Principle.

Patterns, Functions, and Algebra

5.PFA.1 The student will identify, describe, extend, and create increasing and decreasing patterns with whole numbers, fractions, and decimals, including those in context, using various representations.

Students will demonstrate the following Knowledge and Skills:

a) Identify, describe, extend, and create increasing and decreasing patterns using various representations (e.g., objects, pictures, numbers, number lines, input/output tables, function machines).

b) Analyze an increasing or decreasing single-operation numerical pattern found in lists, input/output tables, and function machines, and generalize the change to identify the rule, extend the pattern, or identify missing terms. (Patterns will be limited to addition, subtraction, multiplication, and division of whole numbers; addition and subtraction of fractions with like denominators of 12 or less; and addition and subtraction of decimals expressed in tenths or hundredths).

c) Solve contextual problems that involve identifying, describing, and extending increasing and decreasing patterns using single-operation input and output rules (limited to addition, subtraction, multiplication, and division of whole numbers; addition and subtraction of

fractions with like denominators of 12 or less; and addition and subtraction of decimals expressed in tenths or hundredths).

5.PFA.2 The student will investigate and use variables in contextual problems.

Students will demonstrate the following Knowledge and Skills:

a) Describe the concept of a variable (presented as a box, letter, or other symbol) as a representation of an unknown quantity.
b) Write an equation (with a single variable that represents an unknown quantity and one operation) from a contextual situation, using addition, subtraction, multiplication, or division.
c) Use an expression with a variable to represent a given verbal expression involving one operation (e.g., "5 more than a number" can be represented by $y + 5$).
d) Create and write a word problem to match a given equation with a single variable and one operation.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 6

The Grade 6 *Standards* provide a transition from the emphasis placed on whole number arithmetic in the elementary grades to an introduction to algebraic thinking. The *Standards* include a focus on single-step and multistep problems involving operations with integers and positive rational numbers. Students will determine equivalency, compare, and order decimals, fractions, and percents. Students will solve problems involving area and perimeter and begin to graph in a coordinate plane. In addition, students will continue using the data cycle by applying it to circle graphs and develop concepts regarding measures of center. Students will solve linear equations in one variable, write inequality statements, and use algebraic terminology. Students will use ratios to compare two quantities and represent proportional relationships as a precursor to the development of the concept of linear functions.

The building of mathematical knowledge can be enhanced by using technology tools when used strategically and to complement further learning during grades 6-8. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and the use of instructional technology. Graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools support mathematical problem solving, reasoning, and justification, as well as application in science, engineering, business and industry, government, and practical affairs.

In grades 6-8, the acquisition of specialized mathematical vocabulary and language builds as students continue to deepen their understanding of mathematics through communication and problem solving.

Problem solving increases in complexity as students progress into middle and high school. Students continue to build upon their mathematical knowledge by participating in problem-solving activities that promote higher levels of critical thinking and analysis.

Number and Number Sense

6.NS.1 The student will reason and use multiple strategies to express equivalency, compare, and order numbers written as fractions, mixed numbers, decimals, and percents.

Students will demonstrate the following Knowledge and Skills:

a) Estimate and determine the percent represented by a given model (e.g., number line, picture, verbal description), including percents greater than 100% and less than 1%.*

b) Represent and determine equivalencies among decimals (through the thousandths place) and percents incorporating the use of number lines, and concrete and pictorial models.*

c) Represent and determine equivalencies among fractions (proper or improper) and mixed numbers that have denominators that are 12 or less or factors of 100 and percents incorporating the use of number lines, and concrete and pictorial models.*

d) Represent and determine equivalencies among decimals, percents, fractions (proper or improper), and mixed numbers that have denominators that are 12 or less or factors of 100 incorporating the use of number lines, and concrete and pictorial models.*

e) Use multiple strategies (e.g., benchmarks, number line, equivalency) to compare and order no more than four positive rational numbers expressed as fractions (proper or improper), mixed numbers, decimals, and percents (decimals through thousandths, fractions with denominators of 12 or less or factors of 100) with and without models. Justify solutions orally, in writing or with a model. Ordering may be in ascending or descending order.*

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

6.NS.2 The student will reason and use multiple strategies to represent, compare, and order integers.

Students will demonstrate the following Knowledge and Skills:

a) Represent integers (e.g., number lines, concrete materials, pictorial models), including models derived from contextual situations, and identify an integer represented by a point on a number line.
b) Compare and order integers using a number line.
c) Compare integers, using mathematical symbols (<, >, =).
d) Identify and describe the absolute value of an integer as the distance from zero on the number line.

6.NS.3 The student will recognize and represent patterns with whole number exponents and perfect squares.

Students will demonstrate the following Knowledge and Skills:

a) Recognize and represent patterns with bases and exponents that are whole numbers.
b) Recognize and represent patterns of perfect squares not to exceed 20^2, by using concrete and pictorial models.
c) Justify if a number between 0 and 400 is a perfect square through modeling or mathematical reasoning.
d) Recognize and represent powers of 10 with whole number exponents by examining patterns in place value.

Computation and Estimation

6.CE.1 The student will estimate, demonstrate, solve, and justify solutions to problems using operations with fractions and mixed numbers, including those in context.

Students will demonstrate the following Knowledge and Skills:

a) Demonstrate/model multiplication and division of fractions (proper or improper) and mixed numbers using multiple representations.*
b) Multiply and divide fractions (proper or improper) and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form.*
c) Investigate and explain the effect of multiplying or dividing a fraction, whole number, or mixed number by a number between zero and one.*
d) Estimate, determine, and justify the solution to single-step and multistep problems in context that involve addition and subtraction with fractions (proper or improper) and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less. Answers are expressed in simplest form.

e) Estimate, determine, and justify the solution to single-step and multistep problems in context that involve multiplication and division with fractions (proper or improper) and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form.

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

6.CE.2 The student will estimate, demonstrate, solve, and justify solutions to problems using operations with integers, including those in context.

Students will demonstrate the following Knowledge and Skills:

a) Demonstrate/model addition, subtraction, multiplication, and division of integers using pictorial representations or concrete manipulatives.*

b) Add, subtract, multiply, and divide two integers.*

c) Simplify an expression that contains absolute value bars | | and an operation with two integers (e.g., $-|5 - 8|$ or $\frac{|-12|}{8}$) and represent the result on a number line.

d) Estimate, determine, and justify the solution to one and two-step contextual problems, involving addition, subtraction, multiplication, and division with integers.

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

Measurement and Geometry

6.MG.1 The student will identify the characteristics of circles and solve problems, including those in context, involving circumference and area.

Students will demonstrate the following Knowledge and Skills:

a) Identify and describe chord, diameter, radius, circumference, and area of a circle.

b) Investigate and describe the relationship between:

   i) diameter and radius;

   ii) radius and circumference; and

   iii) diameter and circumference.

c) Develop an approximation for pi (3.14) by gathering data and comparing the circumference to the diameter of various circles, using concrete manipulatives or technological models.

d) Develop the formula for circumference using the relationship between diameter, radius, and pi.

e) Solve problems, including those in context, involving circumference and area of a circle when given the length of the diameter or radius.

6.MG.2 The student will reason mathematically to solve problems, including those in context, that involve the area and perimeter of triangles and parallelograms.

Students will demonstrate the following Knowledge and Skills:

a) Develop the formula for determining the area of parallelograms and triangles using pictorial representations and concrete manipulatives (e.g., two-dimensional diagrams, grid paper).

b) Solve problems, including those in context, involving the perimeter and area of triangles and parallelograms.

6.MG.3 The student will describe the characteristics of the coordinate plane and graph ordered pairs.

Students will demonstrate the following Knowledge and Skills:

a) Identify and label the axes, origin, and quadrants of a coordinate plane.
b) Identify and describe the location (quadrant or the axis) of a point given as an ordered pair. Ordered pairs will be limited to coordinates expressed as integers.
c) Graph ordered pairs in the four quadrants and on the axes of a coordinate plane. Ordered pairs will be limited to coordinates expressed as integers.
d) Identify ordered pairs represented by points in the four quadrants and on the axes of the coordinate plane. Ordered pairs will be limited to coordinates expressed as integers.
e) Relate the coordinates of a point to the distance from each axis and relate the coordinates of a single point to another point on the same horizontal or vertical line. Ordered pairs will be limited to coordinates expressed as integers.
f) Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to determine the length of a side joining points with the same first coordinate or the same second coordinate. Ordered pairs will be limited to coordinates expressed as integers. Apply these techniques in the context of solving contextual and mathematical problems.

6.MG.4 The student will determine congruence of segments, angles, and polygons.

Students will demonstrate the following Knowledge and Skills:

a) Identify regular polygons.
b) Draw lines of symmetry to divide regular polygons into two congruent parts.
c) Determine the congruence of segments, angles, and polygons given their properties.
d) Determine whether polygons are congruent or noncongruent according to the measures of their sides and angles.

Probability and Statistics

6.PS.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graphs.

Students will demonstrate the following Knowledge and Skills:

a) Formulate questions that require the collection or acquisition of data with a focus on circle graphs.
b) Determine the data needed to answer a formulated question and collect the data (or acquire existing data) using various methods (e.g., observations, measurement, surveys, experiments).
c) Determine the factors that will ensure that the data collected is a sample that is representative of a larger population.
d) Organize and represent data using circle graphs, with and without the use of technology tools. The number of data values should be limited to allow for comparisons that have denominators of 12 or less or those that are factors of 100 (e.g., in a class of 20 students, 7 choose apples as a favorite fruit, so the comparison is 7 out of 20, $\frac{7}{20}$, or 35%).
e) Analyze data represented in a circle graph by making observations and drawing conclusions.

f) Compare data represented in a circle graph with the same data represented in other graphs, including but not limited to bar graphs, pictographs, and line plots (dot plots), and justify which graphical representation best represents the data.

6.PS.2 The student will represent the mean as a balance point and determine the effect on statistical measures when a data point is added, removed, or changed.

Students will demonstrate the following Knowledge and Skills:

a) Represent the mean of a set of data graphically as the balance point represented in a line plot (dot plot).
b) Determine the effect on measures of center when a single value of a data set is added, removed, or changed.
c) Observe patterns in data to identify outliers and determine their effect on mean, median, mode, or range.

Patterns, Functions, and Algebra

6.PFA.1 The student will use ratios to represent relationships between quantities, including those in context.

Students will demonstrate the following Knowledge and Skills:

a) Represent a relationship between two quantities using ratios.
b) Represent a relationship in context that makes a comparison by using the notations $\frac{a}{b}$, $a{:}b$, and a to b.
c) Represent different comparisons within the same quantity or between different quantities (e.g., part to part, part to whole, whole to whole).
d) Create a relationship in words for a given ratio expressed symbolically.
e) Create a table of equivalent ratios to represent a proportional relationship between two quantities, when given a ratio.
f) Create a table of equivalent ratios to represent a proportional relationship between two quantities, when given a contextual situation.

6.PFA.2 The student will identify and represent proportional relationships between two quantities, including those in context (unit rates are limited to positive values).

Students will demonstrate the following Knowledge and Skills:

a) Identify the unit rate of a proportional relationship represented by a table of values, a contextual situation, or a graph.
b) Determine a missing value in a ratio table that represents a proportional relationship between two quantities using a unit rate.
c) Determine whether a proportional relationship exists between two quantities, when given a table of values, context, or graph.
d) When given a contextual situation representing a proportional relationship, find the unit rate and create a table of values or a graph.
e) Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs.

6.PFA.3 The student will write and solve one-step linear equations in one variable, including contextual problems that require the solution of a one-step linear equation in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Identify and develop examples of the following algebraic vocabulary: equation, variable, expression, term, and coefficient.

b) Represent and solve one-step linear equations in one variable, using a variety of concrete manipulatives and pictorial representations (e.g., colored chips, algebra tiles, weights on a balance scale).

c) Apply properties of real numbers and properties of equality to solve a one-step equation in one variable. Coefficients are limited to integers and unit fractions. Numeric terms are limited to integers.

d) Confirm solutions to one-step linear equations in one variable using a variety of concrete manipulatives and pictorial representations (e.g., colored chips, algebra tiles, weights on a balance scale).

e) Write a one-step linear equation in one variable to represent a verbal situation, including those in context.

f) Create a verbal situation in context given a one-step linear equation in one variable.

6.PFA.4 The student will represent a contextual situation using a linear inequality in one variable with symbols and graphs on a number line.

Students will demonstrate the following Knowledge and Skills:

a) Given the graph of a linear inequality in one variable on a number line, represent the inequality in two equivalent ways (e.g., $x < -5$ or $-5 > x$) using symbols. Symbols include $<$, $>$, $\leq$, $\geq$.

b) Write a linear inequality in one variable to represent a given constraint or condition in context or given a graph on a number line.

c) Given a linear inequality in one variable, create a corresponding contextual situation or create a number line graph.

d) Use substitution or a number line graph to justify whether a given number in a specified set makes a linear inequality in one variable true.

e) Identify a numerical value(s) that is part of the solution set of a given inequality in one variable.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 7

The Grade 7 *Standards* continue to emphasize the foundations of Algebra. The *Standards* address the representation and comparison of rational numbers using exponents, scientific notation, and square roots. Students continue to develop proficiency in operations with rational numbers and solving problems in context by expanding their study from Grade 6. Students will build on the concept of ratios to solve problems involving proportional reasoning, which is emphasized throughout the Grade 7 *Standards.* Students will solve problems involving volume and surface area and focus on the relationships among the properties of quadrilaterals. In addition, students will continue to apply the data cycle and extend the application to histograms. Probability is investigated through comparing experimental results to theoretical expectations. Students continue to develop their understanding of solving linear equations and inequalities in one variable by applying the properties of real numbers. Students investigate proportional relationships and begin to develop a concept of slope as rate of change.

The building of mathematical knowledge can be enhanced by using technology tools when used strategically and to complement further learning during grades 6-8. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and the use of instructional technology. Graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools support mathematical problem solving, reasoning, and justification, as well as application in science, engineering, business and industry, government, and practical affairs.

In grades 6-8, the acquisition of specialized mathematical vocabulary and language builds as students continue to deepen their understanding of mathematics through communication and problem solving.

Problem solving increases in complexity as students progress into middle and high school. Students continue to build upon their mathematical knowledge by participating in problem-solving activities that promote higher levels of critical thinking and analysis.

Number and Number Sense

7.NS.1 The student will investigate and describe the concept of exponents for powers of ten and compare and order numbers greater than zero written in scientific notation.

Students will demonstrate the following Knowledge and Skills:

a) Investigate and describe powers of 10 with negative exponents by examining patterns.
b) Represent a power of 10 with a negative exponent in fraction and decimal form.
c) Convert between numbers greater than 0 written in scientific notation and decimals.*
d) Compare and order no more than four numbers greater than 0 written in scientific notation. Ordering may be in ascending or descending order.*

*** On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.**

7.NS.2 The student will reason and use multiple strategies to compare and order rational numbers.

Students will demonstrate the following Knowledge and Skills:

a) Use multiple strategies (e.g., benchmarks, number line, equivalency) to compare (using symbols <, >, =) and order (a set of no more than four) rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents. Fractions and mixed numbers may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place. Ordering may be in ascending or descending order. Justify solutions orally, in writing or with a model.*

*** On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.**

7.NS.3 The student will recognize and describe the relationship between square roots and perfect squares.

Students will demonstrate the following Knowledge and Skills:

a) Determine the positive square root of a perfect square from 0 to 400.*
b) Describe the relationship between square roots and perfect squares.*

*** On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.**

Computation and Estimation

7.CE.1 The student will estimate, solve, and justify solutions to multistep contextual problems involving operations with rational numbers.

Students will demonstrate the following Knowledge and Skills:

a) Estimate, solve, and justify solutions to contextual problems involving addition, subtraction, multiplication, and division with rational numbers expressed as integers, fractions (proper or improper), mixed numbers, and decimals. Fractions may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place.

7.CE.2 The student will solve problems, including those in context, involving proportional relationships.

Students will demonstrate the following Knowledge and Skills:

a) Given a proportional relationship between two quantities, create and use a ratio table to determine missing values.
b) Write and solve a proportion that represents a proportional relationship between two quantities to find a missing value, including problems in context.
c) Apply proportional reasoning to solve problems in context, including converting units of measurement, when given the conversion factor.
d) Estimate and determine the percentage of a given whole number, including but not limited to the use of benchmark percentages.

Measurement and Geometry

7.MG.1 The student will investigate and determine the volume formula for right cylinders and the surface area formulas for rectangular prisms and right cylinders and apply the formulas in context.

Students will demonstrate the following Knowledge and Skills:

a) Develop the formulas for determining the volume of right cylinders and solve problems, including those in contextual situations, using concrete objects, diagrams, and formulas.

b) Develop the formulas for determining the surface area of rectangular prisms and right cylinders and solve problems, including those in contextual situations, using concrete objects, two-dimensional diagrams, nets, and formulas.

c) Determine if a problem in context, involving a rectangular prism or right cylinder, represents the application of volume or surface area.

d) Describe how the volume of a rectangular prism is affected when one measured attribute is multiplied by a factor of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, 2, 3, or 4, including those in contextual situations.

e) Describe how the surface area of a rectangular prism is affected when one measured attribute is multiplied by a factor of $\frac{1}{2}$ or 2, including those in contextual situations.

7.MG.2 The student will solve problems and justify relationships of similarity using proportional reasoning.

Students will demonstrate the following Knowledge and Skills:

a) Identify corresponding congruent angles of similar quadrilaterals and triangles, through the use of geometric markings.

b) Identify corresponding sides of similar quadrilaterals and triangles.

c) Given two similar quadrilaterals or triangles, write similarity statements using symbols.

d) Write proportions to express the relationships between the lengths of corresponding sides of similar quadrilaterals and triangles.

e) Recognize and justify if two quadrilaterals or triangles are similar using the ratios of corresponding side lengths.

f) Solve a proportion to determine a missing side length of similar quadrilaterals or triangles.

g) Given angle measures in a quadrilateral or triangle, determine unknown angle measures in a similar quadrilateral or triangle.

h) Apply proportional reasoning to solve problems in context including scale drawings. Scale factors shall have denominators no greater than 12 and decimals no less than tenths.

7.MG.3 The student will compare and contrast quadrilaterals based on their properties and determine unknown side lengths and angle measures of quadrilaterals.

Students will demonstrate the following Knowledge and Skills:

a) Compare and contrast properties of the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid:

   i) parallel/perpendicular sides and diagonals;

   ii) congruence of angle measures, side, and diagonal lengths; and

   iii) lines of symmetry.

b) Sort and classify quadrilaterals as parallelograms, rectangles, trapezoids, rhombi, and/or squares based on their properties:
   i) parallel/perpendicular sides and diagonals;
   ii) congruence of angle measures, side, and diagonal lengths; and
   iii) lines of symmetry.
c) Given a diagram, determine an unknown angle measure in a quadrilateral, using properties of quadrilaterals.
d) Given a diagram, determine an unknown side length in a quadrilateral using properties of quadrilaterals.

7.MG.4 The student will apply dilations of polygons in the coordinate plane.

Students will demonstrate the following Knowledge and Skills:

a) Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been dilated. Scale factors are limited to $\frac{1}{4}$, $\frac{1}{2}$, 2, 3, or 4. The center of the dilation will be the origin.
b) Sketch the image of a dilation of a polygon limited to a scale factor of $\frac{1}{4}$, $\frac{1}{2}$, 2, 3, or 4. The center of the dilation will be the origin.
c) Identify and describe dilations in context including, but not limited to, scale drawings and graphic design.

Probability and Statistics

7.PS.1 The student will use statistical investigation to determine the probability of an event and investigate and describe the difference between the experimental and theoretical probability.

Students will demonstrate the following Knowledge and Skills:

a) Determine the theoretical probability of an event.
b) Given the results of a statistical investigation, determine the experimental probability of an event.
c) Describe changes in the experimental probability as the number of trials increases.
d) Investigate and describe the difference between the probability of an event found through experiment or simulation versus the theoretical probability of that same event.

7.PS.2 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on histograms.

Students will demonstrate the following Knowledge and Skills:

a) Formulate questions that require the collection or acquisition of data with a focus on histograms.
b) Determine the data needed to answer a formulated question and collect the data (or acquire existing data) using various methods (e.g., observations, measurement, surveys, experiments).
c) Determine how sample size and randomness will ensure that the data collected is a sample that is representative of a larger population.

d) Organize and represent numerical data using histograms with and without the use of technology
e) Investigate and explain how using different intervals could impact the representation of the data in a histogram.
f) Compare data represented in histograms with the same data represented in other graphs, including but not limited to line plots (dot plots), circle graphs, and stem-and-leaf plots, and justify which graphical representation best represents the data.
g) Analyze data represented in histograms by making observations and drawing conclusions. Determine how histograms reveal patterns in data that cannot be easily seen by looking at the corresponding given data set.

Patterns, Functions, and Algebra

7.PFA.1 The student will investigate and analyze proportional relationships between two quantities using verbal descriptions, tables, equations in $y = mx$ form, and graphs, including problems in context.

Students will demonstrate the following Knowledge and Skills:

a) Determine the slope, m, as the rate of change in a proportional relationship between two quantities given a table of values, graph, or contextual situation and write an equation in the form $y = mx$ to represent the direct variation relationship. Slope may include positive or negative values (slope will be limited to positive values in a contextual situation).
b) Identify and describe a line with a slope that is positive, negative, or zero (0), given a graph.
c) Graph a line representing a proportional relationship, between two quantities given an ordered pair on the line and the slope, m, as rate of change. Slope may include positive or negative values.
d) Graph a line representing a proportional relationship between two quantities given the equation of the line in the form $y = mx$, where m represents the slope as rate of change. Slope may include positive or negative values.
e) Make connections between and among representations of a proportional relationship between two quantities using problems in context, tables, equations, and graphs. Slope may include positive or negative values (slope will be limited to positive values in a contextual situation).

7.PFA.2 The student will simplify numerical expressions, simplify and generate equivalent algebraic expressions in one variable, and evaluate algebraic expressions for given replacement values of the variables.

Students will demonstrate the following Knowledge and Skills:

a) Use the order of operations and apply the properties of real numbers to simplify numerical expressions. Exponents are limited to 1, 2, 3, or 4 and bases are limited to positive integers. Expressions should not include braces { } but may include brackets [ ] and absolute value bars | |. Square roots are limited to perfect squares.*
b) Represent equivalent algebraic expressions in one variable using concrete manipulatives and pictorial representations (e.g., colored chips, algebra tiles).
c) Simplify and generate equivalent algebraic expressions in one variable by applying the order of operations and properties of real numbers. Expressions may require combining like terms to simplify. Expressions will include only linear and numeric terms. Coefficients and numeric terms may be positive or negative rational numbers.*

d) Use the order of operations and apply the properties of real numbers to evaluate algebraic expressions for given replacement values of the variables. Exponents are limited to 1, 2, 3, or 4 and bases are limited to positive integers. Expressions should not include braces { } but may include brackets [ ] and absolute value bars | |. Square roots are limited to perfect squares. Limit the number of replacements to no more than three per expression. Replacement values may be positive or negative rational numbers.

*** On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.**

7.PFA.3 The student will write and solve two-step linear equations in one variable, including problems in context, that require the solution of a two-step linear equation in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Represent and solve two-step linear equations in one variable using a variety of concrete materials and pictorial representations.
b) Apply properties of real numbers and properties of equality to solve two-step linear equations in one variable. Coefficients and numeric terms will be rational.
c) Confirm algebraic solutions to linear equations in one variable.
d) Write a two-step linear equation in one variable to represent a verbal situation, including those in context.
e) Create a verbal situation in context given a two-step linear equation in one variable.
f) Solve problems in context that require the solution of a two-step linear equation.

7.PFA.4 The student will write and solve one- and two-step linear inequalities in one variable, including problems in context, that require the solution of a one- and two-step linear inequality in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Apply properties of real numbers and the addition, subtraction, multiplication, and division properties of inequality to solve one- and two-step inequalities in one variable. Coefficients and numeric terms will be rational.
b) Investigate and explain how the solution set of a linear inequality is affected by multiplying or dividing both sides of the inequality statement by a rational number less than zero.
c) Represent solutions to one- or two-step linear inequalities in one variable algebraically and graphically using a number line.
d) Write one- or two-step linear inequalities in one variable to represent a verbal situation, including those in context.
e) Create a verbal situation in context given a one or two-step linear inequality in one variable.
f) Solve problems in context that require the solution of a one- or two-step inequality.
g) Identify a numerical value(s) that is part of the solution set of as given one- or two-step linear inequality in one variable.
h) Describe the differences and similarities between solving linear inequalities in one variable and linear equations in one variable.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Grade 8

The Grade 8 *Standards* continue to build on the concepts needed for success in high school level Algebra, Geometry, and Statistics. Students will explore real numbers and the subsets of the real number system. Proportional reasoning is expounded upon as students solve a variety of problems. Students find the volume and surface area of more complex three-dimensional figures and apply transformations to geometric shapes in the coordinate plane. Students will verify and apply the Pythagorean Theorem creating a foundation for further study of triangular relationships in Geometry. Students will represent data, both univariate and bivariate, and continue to apply the data cycle and extend the application to boxplots and scatterplots. Students build upon the algebraic concepts developed in the Grade 6 and 7 *Standards*, which include simplifying algebraic expressions, solving multistep linear equations and inequalities in one variable, and graphing linear functions. The Grade 8 *Standards* are vital to providing a solid foundation in Algebra 1.

The building of mathematical knowledge can be enhanced by using technology tools when used strategically and to complement further learning during grades 6-8. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and the use of instructional technology. Graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools support mathematical problem solving, reasoning, and justification, as well as application in science, engineering, business and industry, government, and practical affairs.

In grades 6-8, the acquisition of specialized mathematical vocabulary and language builds as students continue to deepen their understanding of mathematics through communication and problem solving.

Problem solving increases in complexity as students progress into middle and high school. Students continue to build upon their mathematical knowledge by participating in problem-solving activities that promote higher levels of critical thinking and analysis.

Number and Number Sense

8.NS.1 The student will compare and order real numbers and determine the relationships between real numbers.

Students will demonstrate the following Knowledge and Skills:

a) Estimate and identify the two consecutive natural numbers between which the positive square root of a given number lies and justify which natural number is the better approximation. Numbers are limited to natural numbers from 1 to 400.

b) Use rational approximations (to the nearest hundredth) of irrational numbers to compare, order, and locate values on a number line. Radicals may include both positive and negative square roots of values from 0 to 400 yielding an irrational number.

c) Use multiple strategies (e.g., benchmarks, number line, equivalency) to compare and order no more than five real numbers expressed as integers, fractions (proper or improper), decimals, mixed numbers, percents, numbers written in scientific notation, radicals, and π. Radicals may

include both positive and negative square roots of values from 0 to 400. Ordering may be in ascending or descending order. Justify solutions orally, in writing or with a model.

8.NS.2 The student will investigate and describe the relationship between the subsets of the real number system.

Students will demonstrate the following Knowledge and Skills:

a) Describe and illustrate the relationships among the subsets of the real number system by using representations (e.g., graphic organizers, number lines). Subsets include rational numbers, irrational numbers, integers, whole numbers, and natural numbers.
b) Classify and explain why a given number is a member of a particular subset or subsets of the real number system.
c) Describe each subset of the set of real numbers and include examples and non-examples.

Computation and Estimation

8.CE.1 The student will estimate and apply proportional reasoning and computational procedures to solve contextual problems.

Students will demonstrate the following Knowledge and Skills:

a) Estimate and solve contextual problems that require the computation of one discount or markup and the resulting sale price.
b) Estimate and solve contextual problems that require the computation of the sales tax, tip and resulting total.
c) Estimate and solve contextual problems that require the computation of the percent increase or decrease.

Measurement and Geometry

8.MG.1 The student will use the relationships among pairs of angles that are vertical angles, adjacent angles, supplementary angles, and complementary angles to determine the measure of unknown angles.

Students will demonstrate the following Knowledge and Skills:

a) Identify and describe the relationship between pairs of angles that are vertical, adjacent, supplementary, and complementary.
b) Use the relationships among supplementary, complementary, vertical, and adjacent angles to solve problems, including those in context, involving the measure of unknown angles.

8.MG.2 The student will investigate and determine the surface area of square-based pyramids and the volume of cones and square-based pyramids.

Students will demonstrate the following Knowledge and Skills:

a) Determine the surface area of square-based pyramids by using concrete objects, nets, diagrams, and formulas.
b) Determine the volume of cones and square-based pyramids, using concrete objects, diagrams, and formulas.

c) Examine and explain the relationship between the volume of cones and cylinders, and the volume of rectangular prisms and square based pyramids.
d) Solve problems in context involving volume of cones and square-based pyramids and the surface area of square-based pyramids.

8.MG.3 The student will apply translations and reflections to polygons in the coordinate plane.

Students will demonstrate the following Knowledge and Skills:

a) Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been translated vertically, horizontally, or a combination of both.
b) Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been reflected over the x- or *y*-axis
c) Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been translated and reflected over the x- or *y*-axis or reflected over the *x*- or *y*-axis and then translated.
d) Sketch the image of a polygon that has been translated vertically, horizontally, or a combination of both.
e) Sketch the image of a polygon that has been reflected over the *x*- or *y*-axis.
f) Sketch the image of a polygon that has been translated and reflected over the *x*- or *y*-axis, or reflected over the *x*- or *y*-axis and then translated.
g) Identify and describe transformations in context (e.g., tiling, fabric, wallpaper designs, art).

8.MG.4 The student will apply the Pythagorean Theorem to solve problems involving right triangles, including those in context.

Students will demonstrate the following Knowledge and Skills:

a) Verify the Pythagorean Theorem using diagrams, concrete materials, and measurement.
b) Determine whether a triangle is a right triangle given the measures of its three sides.
c) Identify the parts of a right triangle (the hypotenuse and the legs) given figures in various orientations.
d) Determine the measure of a side of a right triangle, given the measures of the other two sides.
e) Apply the Pythagorean Theorem, and its converse, to solve problems involving right triangles in context.

8.MG.5 The student will solve area and perimeter problems involving composite plane figures, including those in context.

Students will demonstrate the following Knowledge and Skills:

a) Subdivide a plane figure into triangles, rectangles, squares, trapezoids, parallelograms, circles, and semicircles. Determine the area of subdivisions and combine to determine the area of the composite plane figure.
b) Subdivide a plane figure into triangles, rectangles, squares, trapezoids, parallelograms, and semicircles. Use the attributes of the subdivisions to determine the perimeter of the composite plane figure.
c) Apply perimeter, circumference, and area formulas to solve contextual problems involving composite plane figures.

Probability and Statistics

8.PS.1 The student will use statistical investigation to determine the probability of independent and dependent events, including those in context.

Students will demonstrate the following Knowledge and Skills:

a) Determine whether two events are independent or dependent and explain how replacement impacts the probability.
b) Compare and contrast the probability of independent and dependent events.
c) Determine the probability of two independent events.
d) Determine the probability of two dependent events.

8.PS.2 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on boxplots.

Students will demonstrate the following Knowledge and Skills:

a) Formulate questions that require the collection or acquisition of data with a focus on boxplots.
b) Determine the data needed to answer a formulated question and collect the data (or acquire existing data) using various methods (e.g., observations, measurement, surveys, experiments).
c) Determine how statistical bias might affect whether the data collected from the sample is representative of the larger population.
d) Organize and represent a numeric data set of no more than 20 items, using boxplots, with and without the use of technology.
e) Identify and describe the lower extreme (minimum), upper extreme (maximum), median, upper quartile, lower quartile, range, and interquartile range given a data set, represented by a boxplot.
f) Describe how the presence of an extreme data point (outlier) affects the shape and spread of the data distribution of a boxplot.
g) Analyze data represented in a boxplot by making observations and drawing conclusions.
h) Compare and analyze two data sets represented in boxplots.
i) Given a contextual situation, justify which graphical representation (e.g., pictographs, bar graphs, line graphs, line plots/dot plots, stem-and-leaf plots, circle graphs, histograms, and boxplots) best represents the data.
j) Identify components of graphical displays that can be misleading.

8.PS.3 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on scatterplots.

Students will demonstrate the following Knowledge and Skills:

a) Formulate questions that require the collection or acquisition of data with a focus on scatterplots.
b) Determine the data needed to answer a formulated question and collect the data (or acquire existing data) of no more than 20 items using various methods (e.g., observations, measurement, surveys, experiments).

c) Organize and represent numeric bivariate data using scatterplots with and without the use of technology.
d) Make observations about a set of data points in a scatterplot as having a positive linear relationship, a negative linear relationship, or no relationship
e) Analyze and justify the relationship of the quantitative bivariate data represented in scatterplots.
f) Sketch the line of best fit for data represented in a scatterplot.

Patterns, Functions, and Algebra

8.PFA.1 The student will represent, simplify, and generate equivalent algebraic expressions in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Represent algebraic expressions using concrete manipulatives or pictorial representations (e.g., colored chips, algebra tiles), including expressions that apply the distributive property.
b) Simplify and generate equivalent algebraic expressions in one variable by applying the order of operations and properties of real numbers. Expressions may need to be expanded (using the distributive property) or require combining like terms to simplify. Expressions will include only linear and numeric terms. Coefficients and numeric terms may be rational.

8.PFA.2 The student will determine whether a given relation is a function and determine the domain and range of a function.

Students will demonstrate the following Knowledge and Skills:

a) Determine whether a relation, represented by a set of ordered pairs, a table, or a graph of discrete points is a function. Sets are limited to no more than 10 ordered pairs.
b) Identify the domain and range of a function represented as a set of ordered pairs, a table, or a graph of discrete points.

8.PFA.3 The student will represent and solve problems, including those in context, by using linear functions and analyzing their key characteristics (the value of the *y*-intercept (*b*) and the coordinates of the ordered pairs in graphs will be limited to integers).

Students will demonstrate the following Knowledge and Skills:

a) Determine how adding a constant (b) to the equation of a proportional relationship $y = mx$ will translate the line on a graph.
b) Describe key characteristics of linear functions including slope (m), y-intercept (b), and independent and dependent variables.
c) Graph a linear function given a table, equation, or a situation in context.
d) Create a table of values for a linear function given a graph, equation in the form of $y = mx + b$, or context.
e) Write an equation of a linear function in the form $y = mx + b$, given a graph, table, or a situation in context.
f) Create a context for a linear function given a graph, table, or equation in the form $y = mx + b$.

8.PFA.4 The student will write and solve multistep linear equations in one variable, including problems in context that require the solution of a multistep linear equation in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Represent and solve multistep linear equations in one variable with the variable on one or both sides of the equation (up to four steps) using a variety of concrete materials and pictorial representations.

b) Apply properties of real numbers and properties of equality to solve multistep linear equations in one variable (up to four steps). Coefficients and numeric terms will be rational. Equations may contain expressions that need to be expanded (using the distributive property) or require combining like terms to solve.

c) Write a multistep linear equation in one variable to represent a verbal situation, including those in context.

d) Create a verbal situation in context given a multistep linear equation in one variable.

e) Solve problems in context that require the solution of a multistep linear equation.

f) Interpret algebraic solutions in context to linear equations in one variable.

g) Confirm algebraic solutions to linear equations in one variable.

8.PFA.5 The student will write and solve multistep linear inequalities in one variable, including problems in context that require the solution of a multistep linear inequality in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Apply properties of real numbers and properties of inequality to solve multistep linear inequalities (up to four steps) in one variable with the variable on one or both sides of the inequality. Coefficients and numeric terms will be rational. Inequalities may contain expressions that need to be expanded (using the distributive property) or require combining like terms to solve.

b) Represent solutions to inequalities algebraically and graphically using a number line.

c) Write multistep linear inequalities in one variable to represent a verbal situation, including those in context.

d) Create a verbal situation in context given a multistep linear inequality in one variable.

e) Solve problems in context that require the solution of a multistep linear inequality in one variable.

f) Identify a numerical value(s) that is part of the solution set of a given inequality.

g) Interpret algebraic solutions in context to linear inequalities in one variable.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Algebra 1

The successful mastery of Algebra 1 is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra 1 *Standards of Learning.* The progression of algebraic content includes patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. All students are expected to achieve proficiency with the Algebra 1 *Standards*. The study of Algebra 1 assists students in generalizing patterns and representing relevant, contextual situations with algebraic models. To assist students in developing meaning and connecting algebraic concepts to geometry and statistics, consideration should be given to the sequential development of concepts and skills by using concrete materials to support the transition from the numeric to the symbolic. Connections between Algebra 1 and other subject areas through contextual applications may help students attach meaning to the abstract concepts of algebra.

These *Standards* require students to use algebra as a tool for representing and solving a variety of contextual problems. Tables and graphs will be used to interpret algebraic expressions, equations, and inequalities and to analyze behaviors of functions. These *Standards* include a transformational approach to graphing functions and writing equations when given the graph of the equation. Transformational graphing builds a strong connection between algebraic and graphic representations of functions.

Technology tools will be used to assist in teaching and learning. Graphing technologies facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving problems, verifying solutions, and making connections.

Expressions and Operations

A.EO.1 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.

Students will demonstrate the following Knowledge and Skills:

a) Translate between verbal quantitative situations and algebraic expressions, including contextual situations.

b) Evaluate algebraic expressions which include absolute value, square roots, and cube roots for given replacement values to include rational numbers, without rationalizing the denominator.

A.EO.2 The student will perform operations on and factor polynomial expressions in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Determine sums and differences of polynomial expressions in one variable, using a variety of strategies, including concrete objects and their related pictorial and symbolic models.

b) Determine the product of polynomial expressions in one variable, using a variety of strategies, including concrete objects and their related pictorial and symbolic models, the application of the distributive property, and the use of area models. The factors should be limited to five or fewer terms (e.g., $(4x + 2)(3x + 5)$ represents four terms and $(x + 1)(2x^2 + x + 3)$ represents five terms).

c) Factor completely first- and second-degree polynomials in one variable with integral coefficients. After factoring out the greatest common factor (GCF), leading coefficients should have no more than four factors.
d) Determine the quotient of polynomials, using a monomial or binomial divisor, or a completely factored divisor.
e) Represent and demonstrate equality of quadratic expressions in different forms (e.g., concrete, verbal, symbolic, and graphical).

A.EO.3 The student will derive and apply the laws of exponents.

Students will demonstrate the following Knowledge and Skills:

a) Derive the laws of exponents through explorations of patterns, to include products, quotients, and powers of bases.
b) Simplify multivariable expressions and ratios of monomial expressions in which the exponents are integers, using the laws of exponents.

A.EO.4 The student will simplify and determine equivalent radical expressions involving square roots of whole numbers and cube roots of integers.

Students will demonstrate the following Knowledge and Skills:

a) Simplify and determine equivalent radical expressions involving the square root of a whole number in simplest form.
b) Simplify and determine equivalent radical expressions involving the cube root of an integer.
c) Add, subtract, and multiply radicals, limited to numeric square and cube root expressions.
d) Generate equivalent numerical expressions and justify their equivalency for radicals using rational exponents, limited to rational exponents of $\frac{1}{2}$ and $\frac{1}{3}$ (e.g., $\sqrt{5} = 5^{\frac{1}{2}}$; $\sqrt[3]{8} = 8^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} = 2$).

Equations and Inequalities

A.EI.1 The student will represent, solve, explain, and interpret the solution to multistep linear equations and inequalities in one variable and literal equations for a specified variable.

Students will demonstrate the following Knowledge and Skills:

a) Write a linear equation or inequality in one variable to represent a contextual situation.
b) Solve multistep linear equations in one variable, including those in contextual situations, by applying the properties of real numbers and/or properties of equality.
c) Solve multistep linear inequalities in one variable algebraically and graph the solution set on a number line, including those in contextual situations, by applying the properties of real numbers and/or properties of inequality.
d) Rearrange a formula or literal equation to solve for a specified variable by applying the properties of equality.
e) Determine if a linear equation in one variable has one solution, no solution, or an infinite number of solutions.

f) Verify possible solution(s) to multistep linear equations and inequalities in one variable algebraically, graphically, and with technology to justify the reasonableness of the answer(s). Explain the solution method and interpret solutions for problems given in context.

A.EI.2 The student will represent, solve, explain, and interpret the solution to a system of two linear equations, a linear inequality in two variables, or a system of two linear inequalities in two variables.

Students will demonstrate the following Knowledge and Skills:

a) Create a system of two linear equations in two variables to represent a contextual situation.
b) Apply the properties of real numbers and/or properties of equality to solve a system of two linear equations in two variables, algebraically and graphically.
c) Determine whether a system of two linear equations has one solution, no solution, or an infinite number of solutions.
d) Create a linear inequality in two variables to represent a contextual situation.
e) Represent the solution of a linear inequality in two variables graphically on a coordinate plane.
f) Create a system of two linear inequalities in two variables to represent a contextual situation.
g) Represent the solution set of a system of two linear inequalities in two variables, graphically on a coordinate plane.
h) Verify possible solution(s) to a system of two linear equations, a linear inequality in two variable, or a system of two linear inequalities algebraically, graphically, and with technology to justify the reasonableness of the answer(s). Explain the solution method and interpret solutions for problems given in context.

A.EI.3 The student will represent, solve, and interpret the solution to a quadratic equation in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Solve a quadratic equation in one variable over the set of real numbers with rational or irrational solutions, including those that can be used to solve contextual problems.
b) Determine and justify if a quadratic equation in one variable has no real solutions, one real solution, or two real solutions.
c) Verify possible solution(s) to a quadratic equation in one variable algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.

Functions

A.F.1 The student will investigate, analyze, and compare linear functions algebraically and graphically, and model linear relationships.

Students will demonstrate the following Knowledge and Skills:

a) Determine and identify the domain, range, zeros, slope, and intercepts of a linear function, presented algebraically or graphically, including the interpretation of these characteristics in contextual situations.
b) Investigate and explain how transformations to the parent function $y = x$ affect~~s~~ the rate of change (slope) and the y-intercept of a linear function.

c) Write equivalent algebraic forms of linear functions, including slope-intercept form, standard form, and point-slope form, and analyze and interpret the information revealed by each form.

d) Write the equation of a linear function to model a linear relationship between two quantities, including those that can represent contextual situations. Writing the equation of a linear function will include the following situations:

   i) given the graph of a line;

   ii) given two points on the line whose coordinates are integers;

   iii) given the slope and a point on the line whose coordinates are integers;

   iv) vertical lines as $x = a$; and

   v) horizontal lines as $y = c$.

e) Write the equation of a line parallel or perpendicular to a given line through a given point.

f) Graph a linear function in two variables, with and without the use of technology, including those that can represent contextual situations.

g) For any value, x, in the domain of f, determine $f(x)$, and determine x given any value $f(x)$ in the range of f, given an algebraic or graphical representation of a linear function.

h) Compare and contrast the characteristics of linear functions represented algebraically, graphically, in tables, and in contextual situations.

A.F.2 The student will investigate, analyze, and compare characteristics of functions, including quadratic, and exponential functions, and model quadratic and exponential relationships.

Students will demonstrate the following Knowledge and Skills:

a) Determine whether a relation, represented by a set of ordered pairs, a table, a mapping, or a graph is a function; for relations that are functions, determine the domain and range.

b) Given an equation or graph, determine key characteristics of a quadratic function including x-intercepts (zeros), y-intercept, vertex (maximum or minimum), and domain and range (including when restricted by context); interpret key characteristics as related to contextual situations, where applicable.

c) Graph a quadratic function, $f(x)$, in two variables using a variety of strategies, including transformations $f(x) + k$ and $kf(x)$, where k is limited to rational values.

d) Make connections between the algebraic (standard and factored forms) and graphical representation of a quadratic function.

e) Given an equation or graph of an exponential function in the form $y = ab^x$ (where b is limited to a natural number), interpret key characteristics, including y-intercepts and domain and range; interpret key characteristics as related to contextual situations, where applicable.

f) Graph an exponential function, $f(x)$, in two variables using a variety of strategies, including transformations $f(x) + k$ and $kf(x)$, where k is limited to rational values.

g) For any value, x, in the domain of f, determine $f(x)$ of a quadratic or exponential function. Determine x given any value $f(x)$ in the range of f of a quadratic function. Explain the meaning of x and $f(x)$ in context.

h) Compare and contrast the key characteristics of linear functions ($f(x) = x$), quadratic functions ($f(x) = x^2$), and exponential functions ($f(x) = b^x$) using tables and graphs.

Statistics

A.ST.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit using linear and quadratic functions.

Students will demonstrate the following Knowledge and Skills:

a) Formulate investigative questions that require the collection or acquisition of bivariate data.

b) Determine what variables could be used to explain a given contextual problem or situation or answer investigative questions.

c) Determine an appropriate method to collect a representative sample, which could include a simple random sample, to answer an investigative question.

d) Given a table of ordered pairs or a scatterplot representing no more than 30 data points, use available technology to determine whether a linear or quadratic function would represent the relationship, and if so, determine the equation of the curve of best fit.

e) Use linear and quadratic regression methods available through technology to write a linear or quadratic function that represents the data where appropriate and describe the strengths and weaknesses of the model.

f) Use a linear model to predict outcomes and evaluate the strength and validity of these predictions, including through the use of technology.

g) Investigate and explain the meaning of the rate of change (slope) and y-intercept (constant term) of a linear model in context.

h) Analyze relationships between two quantitative variables revealed in a scatterplot.

i) Make conclusions based on the analysis of a set of bivariate data and communicate the results.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Geometry

Geometry is a course designed for students who have successfully completed the *Standards* for Algebra 1. The course includes an emphasis on developing reasoning skills through the exploration of geometric relationships including properties of geometric figures, trigonometric relationships, and mathematical proofs. In this course, students use various types of reasoning, justification, and methods of direct and indirect proof and interpret and determine the validity of conditional statements. Venn diagrams are used to represent set relationships.

This set of standards includes emphasis on two- and three-dimensional reasoning skills, coordinate and transformational geometry, and the use of geometric models to solve problems. A variety of applications and some general problem-solving techniques, including algebraic skills, should be used to implement these *Standards*. Technology tools and dynamic geometry applications will be used to assist in teaching and learning. Graphing technologies facilitate visualizing, analyzing, and understanding algebraic behaviors and geometric patterns and provide a powerful tool for visualizing and verifying solutions.

Reasoning, Lines and Transformations

G.RLT.1 The student will translate logic statements, identify conditional statements, and use and interpret Venn diagrams.

Students will demonstrate the following Knowledge and Skills:

a) Translate propositional statements and compound statements into symbolic form, including negations ($\sim p$, read "*not p*"), conjunctions ($p \wedge q$, read "*p and q*"), disjunctions ($p \vee q$, read "*p or q*"), conditionals ($p \rightarrow q$, read "*if p then q*"), and biconditionals ($p \leftrightarrow q$, read "*p if and only if q*"), including statements representing geometric relationships.

b) Identify and determine the validity of the converse, inverse, and contrapositive of a conditional statement, and recognize the connection between a biconditional statement and a true conditional statement with a true converse, including statements representing geometric relationships.

c) Use Venn diagrams to represent set relationships, including union, intersection, subset, and negation.

d) Interpret Venn diagrams, including those representing contextual situations.

G.RLT.2 The student will analyze, prove, and justify the relationships of parallel lines cut by a transversal.

Students will demonstrate the following Knowledge and Skills:

a) Prove and justify angle pair relationships formed by two parallel lines and a transversal, including:
   i) corresponding angles;
   ii) alternate interior angles;
   iii) alternate exterior angles;
   iv) same-side (consecutive) interior angles; and
   v) same-side (consecutive) exterior angles.

b) Prove two or more lines are parallel given angle measurements expressed numerically or algebraically.
c) Solve problems by using the relationships between pairs of angles formed by the intersection of two parallel lines and a transversal.

G.RLT.3 The student will solve problems, including contextual problems, involving symmetry and transformation.

Students will demonstrate the following Knowledge and Skills:

a) Locate, count, and draw lines of symmetry given a figure, including figures in context.
b) Determine whether a figure has point symmetry, line symmetry, both, or neither, including figures in context.
c) Given an image or preimage, identify the transformation or combination of transformations that has/have occurred. Transformations include:
   i) translations;
   ii) reflections over any horizontal or vertical line or the lines $y = x$ or $y = -x$;
   iii) clockwise or counterclockwise rotations of 90°, 180°, 270°, or 360° on a coordinate grid where the center of rotation is limited to the origin; and
   iv) dilations, from a fixed point on a coordinate grid.

Triangles

G.TR.1 The student will determine the relationships between the measures of angles and lengths of sides in triangles, including problems in context.

Students will demonstrate the following Knowledge and Skills:

a) Given the lengths of three segments, determine whether a triangle could be formed.
b) Given the lengths of two sides of a triangle, determine the range in which the length of the third side must lie.
c) Order the sides of a triangle by their lengths when given information about the measures of the angles.
d) Order the angles of a triangle by their measures when given information about the lengths of the sides.
e) Solve for interior and exterior angles of a triangle, when given two angles.

G.TR.2 The student will, given information in the form of a figure or statement, prove and justify two triangles are congruent using direct and indirect proofs, and solve problems involving measured attributes of congruent triangles.

Students will demonstrate the following Knowledge and Skills:

a) Use definitions, postulates, and theorems (including Side-Side-Side (SSS); Side-Angle-Side (SAS); Angle-Side-Angle (ASA); Angle-Angle-Side (AAS); and Hypotenuse-Leg (HL)) to prove and justify two triangles are congruent.
b) Use algebraic methods to prove that two triangles are congruent.
c) Use coordinate methods, such as the slope formula and the distance formula, to prove two triangles are congruent.

d) Given a triangle, use congruent segment, congruent angle, and/or perpendicular line constructions to create a congruent triangle (SSS, SAS, ASA, AAS, and HL).

G.TR.3 The student will, given information in the form of a figure or statement, prove and justify two triangles are similar using direct and indirect proofs, and solve problems, including those in context, involving measured attributes of similar triangles.

Students will demonstrate the following Knowledge and Skills:

a) Use definitions, postulates, and theorems (including Side-Angle-Side (SAS); Side-Side-Side (SSS); and Angle-Angle (AA)) to prove and justify that triangles are similar.
b) Use algebraic methods to prove that triangles are similar.
c) Use coordinate methods, such as the slope formula and the distance formula, to prove two triangles are similar.
d) Describe a sequence of transformations that can be used to verify similarity of triangles located in the same plane.
e) Solve problems, including those in context involving attributes of similar triangles.

G.TR.4 The student will model and solve problems, including those in context, involving trigonometry in right triangles and applications of the Pythagorean Theorem.

Students will demonstrate the following Knowledge and Skills:

a) Determine whether a triangle formed with three given lengths is a right triangle.
b) Find and verify trigonometric ratios using right triangles.
c) Model and solve problems, including those in context, involving right triangle trigonometry (sine, cosine, and tangent ratios).
d) Solve problems using the properties of special right triangles.
e) Solve for missing lengths in geometric figures, using properties of 45°-45°-90° triangles, where rationalizing denominators may be necessary.
f) Solve for missing lengths in geometric figures, using properties of 30°-60°-90° triangles, where rationalizing denominators may be necessary.
g) Solve problems, including those in context, involving right triangles using the Pythagorean Theorem and its converse, including recognizing Pythagorean Triples.

Polygons and Circles

G.PC.1 The student will prove and justify theorems and properties of quadrilaterals, and verify and use properties of quadrilaterals to solve problems, including the relationships between the sides, angles, and diagonals.

Students will demonstrate the following Knowledge and Skills:

a) Solve problems, using the properties specific to parallelograms, rectangles, rhombi, squares, isosceles trapezoids, and trapezoids.
b) Prove and justify that quadrilaterals have specific properties, using coordinate and algebraic methods, such as the slope formula, the distance formula, and the midpoint formula.
c) Prove and justify theorems and properties of quadrilaterals using deductive reasoning.
d) Use congruent segment, congruent angle, angle bisector, perpendicular line, and/or parallel line constructions to verify properties of quadrilaterals.

G.PC.2 The student will verify relationships and solve problems involving the number of sides and measures of angles of convex polygons.

Students will demonstrate the following Knowledge and Skills:

a) Solve problems involving the number of sides of a regular polygon given the measures of the interior and exterior angles of the polygon.
b) Justify the relationship between the sum of the measures of the interior and exterior angles of a convex polygon and solve problems involving the sum of the measures of the angles.
c) Justify the relationship between the measure of each interior and exterior angle of a regular polygon and solve problems involving the measures of the angles.

G.PC.3 The student will solve problems, including those in context, by applying properties of circles.

Students will demonstrate the following Knowledge and Skills:

a) Determine the proportional relationship between the arc length or area of a sector and other parts of a circle.
b) Solve for arc measures and angles in a circle formed by central angles.
c) Solve for arc measures and angles in a circle involving inscribed angles.
d) Calculate the length of an arc of a circle.
e) Calculate the area of a sector of a circle.
f) Apply arc length or sector area to solve for an unknown measurement of the circle including the radius, diameter, arc measure, central angle, arc length, or sector area.

G.PC.4 The student will solve problems in the coordinate plane involving equations of circles.

Students will demonstrate the following Knowledge and Skills:

a) Derive the equation of a circle of given the center and radius using the Pythagorean Theorem.
b) Solve problems in the coordinate plane involving equations of circles:
    i) given a graph or the equation of a circle in standard form, identify the coordinates of the center of the circle;
    ii) given the coordinates of the endpoints of a diameter of a circle, determine the coordinates of the center of the circle.
    iii) given a graph or the equation of a circle in standard form, identify the length of the radius or diameter of the circle.
    iv) given the coordinates of the endpoints of the diameter of a circle, determine the length of the radius or diameter of the circle.
    v) given the coordinates of the center and the coordinates of a point on the circle, determine the length of the radius or diameter of the circle; and
    vi) given the coordinates of the center and length of the radius of a circle, identify the coordinates of a point(s) on the circle.
c) Determine the equation of a circle given:
    i) a graph of a circle with a center with coordinates that are integers;
    ii) coordinates of the center and a point on the circle;
    iii) coordinates of the center and the length of the radius or diameter; and

iv) coordinates of the endpoints of a diameter.

Two- and Three-Dimensional Figures

G.DF.1 The student will create models and solve problems, including those in context, involving surface area and volume of rectangular and triangular prisms, cylinders, cones, pyramids, and spheres.

Students will demonstrate the following Knowledge and Skills:

a) Identify the shape of a two-dimensional cross section of a three-dimensional figure.
b) Create models and solve problems, including those in context, involving surface area of three-dimensional figures, as well as composite three-dimensional figures.
c) Solve multistep problems, including those in context, involving volume of three-dimensional figures, as well as composite three-dimensional figures.
d) Determine unknown measurements of three-dimensional figures using information such as length of a side, area of a face, or volume.

G.DF.2 The student will determine the effect of changing one or more dimensions of a three-dimensional geometric figure and describe the relationship between the original and changed figure.

Students will demonstrate the following Knowledge and Skills:

a) Describe how changes in one or more dimensions of a figure affect other derived measures (perimeter, area, total surface area, and volume) of the figure.
b) Describe how changes in surface area and/or volume of a figure affect the measures of one or more dimensions of the figure.
c) Solve problems, including those in context, involving changing the dimensions or derived measures of a three-dimensional figure.
d) Compare ratios between side lengths, perimeters, areas, and volumes of similar figures.
e) Recognize when two- and three-dimensional figures are similar and solve problems, including those in context, involving attributes of similar geometric figures.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Algebra, Functions, and Data Analysis

Algebra, Functions, and Data Analysis is a course designed for students who have successfully completed the *Standards* for Algebra 1 and may benefit from additional support in their transition to Algebra 2. Within the context of mathematical modeling and data analysis, students will study functions and their behaviors, systems of inequalities, probability, experimental design and implementation, and analysis of data. Data will be generated through practical applications arising from science, business, and finance. Students will solve problems that require the formulation of linear, quadratic, exponential, or piecewise-defined equations or a system of equations.

Through the investigation of mathematical models and interpretation/analysis of data from relevant, applied contexts and situations, students will strengthen conceptual understandings in mathematics and further develop connections between algebra and statistics. Students should use the language and symbols of mathematics in representations and communication, both orally and in writing, throughout the course.

These *Standards* include a transformational approach to graphing functions and writing equations when given the graph of the equation. Transformational graphing builds a strong connection between algebraic and graphic representations of functions.

Technology tools will be used to assist in teaching and learning. Graphing technologies facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Algebra and Functions

AFDA.AF.1 The student will investigate, analyze, and compare linear, quadratic, and exponential function families, algebraically and graphically, using transformations.

Students will demonstrate the following Knowledge and Skills:

a) Identify graphs and equations of parent functions for linear, quadratic, and exponential function families.

b) Describe the transformation from the parent function given the equation or the graph of the function.

c) Determine and analyze whether a linear, quadratic, or exponential function best models a given representation, including those in context.

d) Write the equation of a linear, quadratic, or exponential function, given a graph, using transformations of the parent function.

e) Use a graphical or algebraic representation of a function to solve problems within a context, graphically and algebraically, when appropriate.

f) Graph a function given the equation of a function, using transformations of the parent function. Use technology to verify transformations of functions.

g) Compare and contrast linear, quadratic, and exponential functions using multiple representations (e.g., graphs, tables, equations, verbal descriptions).

AFDA.AF.2 The student will investigate and analyze characteristics of the graphs of linear, quadratic, exponential, and piecewise-defined functions.

Students will demonstrate the following Knowledge and Skills:

a) Determine the domain and range of a function given a graphical representation, including those limited by contexts.
b) Identify intervals on a graph for which a function is increasing, decreasing, or constant.
c) Given a graph, identify the location and value of the absolute maximum and absolute minimum of a function over the domain of a function.
d) Given a graph, determine the zeros and intercepts of a function.
e) Describe and recognize the connection between points on the graph and the value of a function.
f) Describe the end behavior of a function given its graph.
g) Identify horizontal and/or vertical asymptotes from the graph of a function, if they exist.
h) Describe and relate the characteristics of the graphs of linear, quadratic, exponential, and piecewise-defined functions, including those in contextual situations.

AFDA.AF.3 The student will represent and interpret contextual situations with constraints that require optimization using linear programming techniques, including systems of linear equations or inequalities, solving graphically and when appropriate, algebraically.

Students will demonstrate the following Knowledge and Skills:

a) Represent and interpret contextual problems requiring optimization with systems of linear equations or inequalities.
b) Solve systems of no more than four equations or inequalities graphically and when appropriate, algebraically.
c) Identify the feasible region of a system of linear inequalities.
d) Identify the coordinates of the vertices of a feasible region.
e) Determine and describe the maximum or minimum value for the function defined over a feasible region.
f) Interpret the validity of possible solution(s) algebraically, graphically, using technology, and in context and justify the reasonableness of the answer(s) or the solution method in context.

Data Analysis

AFDA.DA.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit using linear, quadratic, and exponential functions.

Students will demonstrate the following Knowledge and Skills:

a) Formulate investigative questions that require the collection or acquisition of bivariate data, where exactly two of the variables are quantitative.
b) Collect or acquire bivariate data from a representative sample to answer an investigative question.

c) Represent bivariate data with a scatterplot using technology and describe how the variables are related in terms of the given context.
d) Make predictions, decisions, and critical judgments using data, scatterplots, or the equation(s) of the mathematical model.

AFDA.DA.2 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on the design and implementation of an experiment and/or observational study.

Students will demonstrate the following Knowledge and Skills:

a) Formulate questions that can be addressed with data and assess the type of data relevant to the question (e.g., quantitative versus categorical).
b) Investigate, describe, and determine best sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling.
c) Plan and conduct an experiment and/or observational study. The experimental design should address control, randomization, and minimization of experimental error.
d) Collect or acquire data to answer a statistical question.
e) Recognize that data may contain errors, have missing values, or may be biased, and make decisions about how to account for these issues.
f) Identify biased sampling methods.
g) Given a plan for an observational study, identify possible sources of bias, and describe ways to reduce bias.
h) Select, create, and use appropriate visual representations of data to brainstorm solutions.
i) Use appropriate statistical methods to analyze data.
j) Communicate the description of an experiment and/or observational study, the resulting data, analysis, and the validity of the conclusions.

AFDA.DA.3 The student will calculate and interpret probabilities, including those in contextual situations.

Students will demonstrate the following Knowledge and Skills:

a) Analyze, interpret, and make predictions based on theoretical probability.
b) Calculate conditional probabilities for dependent, independent, and mutually exclusive events.
c) Represent and calculate probabilities using Venn diagrams, probability trees, organized lists, two-way tables, simulations, or other probability models.
d) Interpret probabilities from simulations or experiments to make informed decisions and justify the rationale.
e) Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events.
f) Given two or more events in a problem setting, determine whether the events are complementary, dependent, independent, and/or mutually exclusive.
g) Compare and contrast permutations and combinations, including those in contextual situations.
h) Calculate the number of permutations of n objects taken r at a time, without repetition.
i) Calculate the number of combinations of n objects taken r at a time, without repetition.

AFDA.DA.4 The student will describe and apply the properties of normal distribution, including those in contextual situations.

Students will demonstrate the following Knowledge and Skills:

a) Identify and describe the properties of a normal distribution.
b) Determine when the normal distribution is a reasonable representation of the data.
c) Describe how the mean and the standard deviation affect the graph of the normal distribution.
d) Calculate and interpret the z-score for a data point, given the mean and the standard deviation.
e) Compare two sets of normally distributed data using a standard normal distribution and z-scores, given the mean and the standard deviation.
f) Represent probability as the area under the curve of a standard normal distribution.
g) Determine probabilities associated with areas under the standard normal curve, using technology or a table of Standard Normal Probabilities.
h) Investigate, represent, and determine relationships between a normally distributed data set and its descriptive statistics.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Algebra 2

Students enrolled in Algebra 2 are assumed to have mastered the concepts outlined in the Algebra 1 *Standards.* Students earning an Advanced Studies Diploma are assumed to have mastered the content in the Algebra 2 *Standards of Learning*. A thorough treatment of advanced algebraic concepts will be provided through the study of functions, equations, inequalities, systems of equations, polynomials, rational and radical equations, complex numbers, and curves of best fit. Emphasis will be placed on contextual applications and modeling throughout the course of study. Oral and written communication concerning the language of algebra, logic of procedures, and interpretation of results should also permeate the course.

These *Standards* include a transformational approach to graphing functions. Transformational graphing uses translation, reflection, dilation, and rotation to generate a "family of functions" from a given "parent" function and builds a strong connection between algebraic and graphic representations of functions. Students will vary the coefficients and constants of an equation, observe the changes in the graph of the equation, and make generalizations that can be applied to many graphs.

Technology tools will be used to assist in teaching and learning. Graphing technologies facilitate visualizing, analyzing, making connections, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Expressions and Operations

A2.EO.1 The student will perform operations on and simplify rational expressions.

Students will demonstrate the following Knowledge and Skills:

a) Add, subtract, multiply, or divide rational algebraic expressions, simplifying the result.
b) Justify and determine equivalent rational algebraic expressions with monomial and binomial factors. Algebraic expressions should be limited to linear and quadratic expressions.
c) Recognize a complex algebraic fraction and simplify it as a product or quotient of simple algebraic fractions.
d) Represent and demonstrate equivalence of rational expressions written in different forms.

A2.EO.2 The student will perform operations on and simplify radical expressions.

Students will demonstrate the following Knowledge and Skills:

a) Simplify and determine equivalent radical expressions that include numeric and algebraic radicands.
b) Add, subtract, multiply, and divide radical expressions that include numeric and algebraic radicands, simplifying the result. Simplification may include rationalizing the denominator.
c) Convert between radical expressions and expressions containing rational exponents.

A2.EO.3 The student will perform operations on polynomial expressions and factor polynomial expressions in one and two variables.

Students will demonstrate the following Knowledge and Skills:

a) Determine sums, differences, and products of polynomials in one and two variables.

b) Factor polynomials completely in one and two variables with no more than four terms over the set of integers.
c) Determine the quotient of polynomials in one and two variables, using monomial, binomial, and factorable trinomial divisors.
d) Represent and demonstrate equality of polynomial expressions written in different forms and verify polynomial identities including the difference of squares, sum and difference of cubes, and perfect square trinomials.

A2.EO.4 The student will perform operations on complex numbers.

Students will demonstrate the following Knowledge and Skills:

a) Explain the meaning of i.
b) Identify equivalent radical expressions containing negative rational numbers and expressions in $a + bi$ form.
c) Apply properties to add, subtract, and multiply complex numbers.

Equations and Inequalities

A2.EI.1 The student will represent, solve, and interpret the solution to absolute value equations and inequalities in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Create an absolute value equation in one variable to model a contextual situation.
b) Solve an absolute value equation in one variable algebraically and verify the solution graphically.
c) Create an absolute value inequality in one variable to model a contextual situation.
d) Solve an absolute value inequality in one variable and represent the solution set using set notation, interval notation, and using a number line.
e) Verify possible solution(s) to absolute value equations and inequalities in one variable algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.

A2.EI.2 The student will represent, solve, and interpret the solution to quadratic equations in one variable over the set of complex numbers and solve quadratic inequalities in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Create a quadratic equation or inequality in one variable to model a contextual situation.
b) Solve a quadratic equation in one variable over the set of complex numbers algebraically.
c) Determine the solution to a quadratic inequality in one variable over the set of real numbers algebraically.
d) Verify possible solution(s) to quadratic equations or inequalities in one variable algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.

A2.EI.3 The student will solve a system of equations in two variables containing a quadratic expression.

Students will demonstrate the following Knowledge and Skills:

a) Create a linear-quadratic or quadratic-quadratic system of equations to model a contextual situation.
b) Determine the number of solutions to a linear-quadratic and quadratic-quadratic system of equations in two variables.
c) Solve a linear-quadratic and quadratic-quadratic system of equations algebraically and graphically, including situations in context.
d) Verify possible solution(s) to linear-quadratic or quadratic-quadratic system of equations algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.

A2.EI.4 The student will represent, solve, and interpret the solution to an equation containing rational algebraic expressions.

Students will demonstrate the following Knowledge and Skills:

a) Create an equation containing a rational expression to model a contextual situation.
b) Solve rational equations with real solutions containing factorable algebraic expressions algebraically and graphically. Algebraic expressions should be limited to linear and quadratic expressions.
c) Verify possible solution(s) to rational equations algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.
d) Justify why a possible solution to an equation containing a rational expression might be extraneous.

A2.EI.5 The student will represent, solve, and interpret the solution to an equation containing a radical expression.

Students will demonstrate the following Knowledge and Skills:

a) Solve an equation containing no more than one radical expression algebraically and graphically.
b) Verify possible solution(s) to radical equations algebraically, graphically, and with technology, to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.
c) Justify why a possible solution to an equation with a square root might be extraneous.

A2.EI.6 The student will represent, solve, and interpret the solution to a polynomial equation.

Students will demonstrate the following Knowledge and Skills:

a) Determine a factored form of a polynomial equation, of degree three or higher, given its zeros or the x-intercepts of the graph of its related function.
b) Determine the number and type of solutions (real or imaginary) of a polynomial equation of degree three or higher.
c) Solve a polynomial equation over the set of complex numbers.
d) Verify possible solution(s) to polynomial equations of degree three or higher algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions in context.

Functions

A2.F.1 The student will investigate, analyze, and compare square root, cube root, rational, exponential, and logarithmic function families, algebraically and graphically, using transformations.

Students will demonstrate the following Knowledge and Skills:

a) Distinguish between the graphs of parent functions for square root, cube root, rational, exponential, and logarithmic function families.

b) Write the equation of a square root, cube root, rational, exponential, and logarithmic function, given a graph, using transformations of the parent function, including $f(x) + k$; $f(kx)$; $f(x + k)$; and $kf(x)$, where k is limited to rational values. Transformations of exponential and logarithmic functions, given a graph, should be limited to a single transformation.

c) Graph a square root, cube root, rational, exponential, and logarithmic function, given the equation, using transformations of the parent function including $f(x) + k$; $f(kx)$; $f(x + k)$; and $kf(x)$, where k is limited to rational values. Use technology to verify transformations of the functions.

d) Determine when two variables are directly proportional, inversely proportional, or neither, given a table of values. Write an equation and create a graph to represent a direct or inverse variation, including situations in context.

e) Compare and contrast the graphs, tables, and equations of square root, cube root, rational, exponential, and logarithmic functions.

A2.F.2 The student will investigate and analyze characteristics of square root, cube root, rational, polynomial, exponential, logarithmic, and piecewise-defined functions algebraically and graphically.

Students will demonstrate the following Knowledge and Skills:

a) Determine and identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically, including graphs with discontinuities.

b) Compare and contrast the characteristics of square root, cube root, rational, polynomial, exponential, logarithmic, and piecewise-defined functions.

c) Determine the intervals on which the graph of a function is increasing, decreasing, or constant.

d) Determine the location and value of absolute (global) maxima and absolute (global) minima of a function.

e) Determine the location and value of relative (local) maxima or relative (local) minima of a function.

f) For any value, x, in the domain of f, determine $f(x)$ using a graph or equation. Explain the meaning of x and $f(x)$ in context, where applicable.

g) Describe the end behavior of a function.

h) Determine the equations of any vertical and horizontal asymptotes of a function using a graph or equation (rational, exponential, and logarithmic).

i) Determine the inverse of a function algebraically and graphically, given the equation of a linear or quadratic function (linear, quadratic, and square root). Justify and explain why two functions are inverses of each other.

j) Graph the inverse of a function as a reflection over the line $y = x$.

k) Determine the composition of two functions algebraically and graphically.

Statistics

A2.ST.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on univariate quantitative data represented by a smooth curve, including a normal curve.

Students will demonstrate the following Knowledge and Skills:

a) Formulate investigative questions that require the collection or acquisition of a large set of univariate quantitative data or summary statistics of a large set of univariate quantitative data and investigate questions using a data cycle.

b) Collect or acquire univariate data through research, or using surveys, observations, scientific experiments, polls, or questionnaires.

c) Examine the shape of a data set (skewed versus symmetric) that can be represented by a histogram, and sketch a smooth curve to model the distribution.

d) Identify the properties of a normal distribution.

e) Describe and interpret a data distribution represented by a smooth curve by analyzing measures of center, measures of spread, and shape of the curve.

f) Calculate and interpret the z-score for a value in a data set.

g) Compare two data points from two different distributions using z-scores.

h) Determine the solution to problems involving the relationship of the mean, standard deviation, and z-score of a data set represented by a smooth or normal curve.

i) Apply the Empirical Rule to answer investigative questions.

j) Compare multiple data distributions using measures of center, measures of spread, and shape of the distributions.

A2.ST.2 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit using linear, quadratic, exponential, or a combination of these functions.

Students will demonstrate the following Knowledge and Skills:

a) Formulate investigative questions that require the collection or acquisition of bivariate data and investigate questions using a data cycle.

b) Collect or acquire bivariate data through research, or using surveys, observations, scientific experiments, polls, or questionnaires.

c) Represent bivariate data with a scatterplot using technology.

d) Determine whether the relationship between two quantitative variables is best approximated by a linear, quadratic, exponential, or a combination of these functions.

e) Determine the equation(s) of the function(s) that best models the relationship between two variables using technology. Curves of best fit may include a combination of linear, quadratic, or exponential (piecewise-defined) functions.

f) Use the correlation coefficient to designate the goodness of fit of a linear function using technology.
g) Make predictions, decisions, and critical judgments using data, scatterplots, or the equation(s) of the mathematical model.
h) Evaluate the reasonableness of a mathematical model of a contextual situation.

A2.ST.3 The student will compute and distinguish between permutations and combinations.

Students will demonstrate the following Knowledge and Skills:

a) Compare and contrast permutations and combinations to count the number of ways that events can occur.
b) Calculate the number of permutations of n objects taken r at a time.
c) Calculate the number of combinations of n objects taken r at a time.
d) Use permutations and combinations as counting techniques to solve contextual problems.
e) Calculate and verify permutations and combinations using technology.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Trigonometry

The standards below outline the content for a one-semester course in Trigonometry. Trigonometry includes the study of trigonometric definitions, applications, graphing, and solving trigonometric equations and inequalities. Emphasis should also be placed on using connections between right triangle ratios, trigonometric functions, and circular functions. In addition, the application of trigonometric concepts should be included throughout the course of study. Oral and written communication concerning the language of mathematics, logic of procedure, and interpretation of results should also permeate the course.

Technology tools will be used to assist in teaching and learning. Graphing technologies facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Triangle Trigonometry

T.TT.1 The student will determine the sine, cosine, tangent, cotangent, secant, and cosecant of the acute angles in a right triangle and use these ratios to solve for missing sides and angle measures, including application in contextual problems.

Students will demonstrate the following Knowledge and Skills:

a) Define and represent the six triangular trigonometric ratios (sine, cosine, tangent, cosecant, secant, and cotangent) of an angle in a right triangle.

b) Describe the relationships between side lengths in special right triangles (30°-60°-90° and 45°-45°-90°).

c) Use the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines to solve contextual problems.

d) Represent and solve contextual problems involving right triangles, including problems involving angles of elevation and depression.

T.TT.2 The student will find the area of any triangle and solve for the lengths of the sides and measures of the angles in a non-right triangle using the Law of Sines and the Law of Cosines.

Students will demonstrate the following Knowledge and Skills:

a) Apply the Law of Sines, and the Law of Cosines, as appropriate, to find missing sides and angles in non-right triangles.

b) Recognize the ambiguous case when applying the Law of Sines and the potential for two triangle solutions in some situations.

c) Solve problems that integrate the use of the Law of Sines and the Law of Cosines and the triangle area formula (Area = $\frac{1}{2}absinC$, where a and b are triangle sides and C is the included angle) to find the area of any triangle, including those in contextual problems.

Circular Trigonometry

T.CT.1 The student will determine the degree and radian measure of angles; sketch angles in standard position on a coordinate plane; and determine the sine, cosine, tangent,

cosecant, secant, and cotangent of an angle, given a point on the terminal side of an angle in standard position or the value of a trigonometric function of the angle.

Students will demonstrate the following Knowledge and Skills:

a) Define a radian as a unit of angle measure and determine the relationship between the radian measure of an angle and the length of the intercepted arc in a circle.
b) Determine the degree and radian measure of angles to include both negative and positive rotations in the coordinate plane.
c) Find both positive and negative coterminal angles for a given angle.
d) Identify the quadrant or axis in/on which the terminal side of an angle lies.
e) Draw a reference right triangle when given a point on the terminal side of an angle in standard position.
f) Draw a reference right triangle when given the value of a trigonometric function of an angle (sine, cosine, tangent, cosecant, secant, and cotangent).
g) Determine the value of any trigonometric function (sine, cosine, tangent, cosecant, secant, and cotangent) when given a point on the terminal side of an angle in standard position.
h) Given one trigonometric function value, determine the other five trigonometric function values.
i) Calculate the length of an arc of a circle in radians.
j) Calculate the area of a sector of a circle.

T.CT.2 The student will develop and apply the properties of the unit circle in degrees and radians.

Students will demonstrate the following Knowledge and Skills:

a) Convert between radian and degree measure of special angles of the unit circle without the use of technology.
b) Define the six circular trigonometric functions of an angle in standard position on the unit circle.
c) Apply knowledge of right triangle trigonometry, special right triangles, and the properties of the unit circle to determine trigonometric functions values of special angles (0°, 30°, 45°, 60°, and 90°) and their related angles in degree and radians without the use of technology.

Graphs of Trigonometric Functions

T.GT.1 The student will graph and analyze trigonometric functions and apply trigonometric functions to represent periodic phenomena.

Students will demonstrate the following Knowledge and Skills:

a) Sketch the graph of the six parent trigonometric functions (sine, cosine, tangent, cosecant, secant, and cotangent) for at least a two-period interval.
b) Determine the domain and range, amplitude, period, and asymptote locations for a trigonometric function, given a graph or an equation.
c) Describe the effects of changing the parameters (A, B, C, or D in the standard form of a trigonometric equation) on the graph of the function using graphing technology.

d) Sketch the graph of a transformed sine, cosine, and tangent function written in standard form by using transformations for at least a two-period interval, including both positive and negative values for the domain.
e) Apply trigonometric functions and their graphs to represent periodic phenomena.

T.GT.2 The student will graph the six inverse trigonometric functions.

Students will demonstrate the following Knowledge and Skills:

a) Determine the domain and range of the inverse trigonometric functions.
b) Use the restrictions on the domain of an inverse trigonometric function to determine a value of the inverse trigonometric function.
c) Graph inverse trigonometric functions.

Identities and Equations

T.IE.1 The student will evaluate expressions involving the six trigonometric functions and the inverse sine, cosine, and tangent functions.

Students will demonstrate the following Knowledge and Skills:

a) Determine the values of trigonometric functions, with and without graphing technology.
b) Determine angle measures by using the inverse trigonometric functions, with and without a graphing technology.
c) Evaluate composite functions that involve trigonometric functions and inverse trigonometric functions.

T.IE.2 The student will use basic trigonometric identity substitutions to simplify and verify trigonometric identities.

Students will demonstrate the following Knowledge and Skills:

a) Use trigonometric identities to make algebraic substitutions to simplify and verify trigonometric identities. The basic trigonometric identities include
   i) reciprocal identities;
   ii) Pythagorean identities;
   iii) sum and difference identities;
   iv) double-angle identities; and
   v) half-angle identities.
b) Apply the sum, difference, and half-angle identities to evaluate trigonometric function values of angles that are not integer multiples of the special angles to solve problems, including contextual situations.

T.IE.3 The student will solve trigonometric equations and inequalities.

Students will demonstrate the following Knowledge and Skills:

a) Solve trigonometric equations with and without restricted domains algebraically and graphically.
b) Solve trigonometric inequalities algebraically and graphically.

c) Verify and justify algebraic solutions to trigonometric equations and inequalities, using graphing technology.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Computer Mathematics

This course is intended to provide students with experiences in using computer programming techniques and skills to solve problems involving mathematical models. Students enrolled in Computer Mathematics are assumed to have studied the concepts and skills in Algebra 1 and beginning Geometry.

Even though computer ideas should be introduced in the context of mathematical concepts, problem solving should be developed in the most general sense, allowing the techniques to be applied by students in many other environments. Strategies include defining the problem; developing, refining, and implementing a plan; and testing and revising the solution. Programming, ranging from simple programs involving only a few lines to complex programs involving subprograms, should permeate the entire course. Programming concepts, problem-solving strategies, and mathematical applications should be integrated throughout the course.

These standards identify fundamental principles and concepts in the field of computer science that will be used within the context of mathematical problem solving in a variety of applications. As students develop and refine skills in logic, organization, and precise expression, they will apply those skills to enhance learning in all disciplines.

Data Representation and Storage

CM.DRS.1 The student will represent data and convert data between different number systems.

Students will demonstrate the following Knowledge and Skills:

a) Represent data in different number systems, including binary, decimal, and hexadecimal.
b) Convert data between number systems (e.g., binary to decimal, decimal to hexadecimal).

CM.DRS.2 The student will differentiate between variable data types based upon their characteristics.

Students will demonstrate the following Knowledge and Skills:

a) Describe the characteristics of different variable data types, including
   i) Boolean;
   ii) character;
   iii) integer;
   iv) decimal (double/float); and
   v) string.
b) Differentiate between variable data types to determine the data type needed based upon intended use (e.g., character versus string, integer versus double/float).

CM.DRS.3 The student will represent data using appropriate data structures.

Students will demonstrate the following Knowledge and Skills:

a) Given a specific task or problem, determine the appropriate data structure (e.g., lists, arrays, objects) to represent data.

b) Perform tasks related to lists or arrays (one-dimensional or two-dimensional), including
   i) declare a list or array (one-dimensional or two-dimensional);
   ii) choose an appropriate data type for a list or an array; and
   iii) fill the list or array with data.
c) Access and manipulate a particular element of a list or an array.
d) Implement predefined objects to consolidate related information of different data types.

Components of Programming

CM.CP.1 The student will design a step-by-step plan to perform a task or solve a problem, including those arising from mathematical or interdisciplinary contexts.

Students will demonstrate the following Knowledge and Skills:

a) Design a step-by-step plan to perform a task or solve a problem using a flowchart or pseudocode that outlines the subtasks needed.
b) Define the variables needed to perform a task or solve a problem.
c) Define the constraints of a task or problem (e.g., pre-conditions, post-conditions) to determine the desired input and output.

CM.CP.2 The student will construct Boolean expressions and implement conditional statements.

Students will demonstrate the following Knowledge and Skills:

a) Write and implement Boolean expressions using logical and relational operators (e.g., !, &&, ||, ==, <, >, >=, <=, !=).
b) Write and implement "if" conditional statements.
c) Write and implement "if/else" conditional statements.
d) Write and implement compound conditional statements (e.g., nested conditionals, chained conditional statements).
e) Determine which parts of an algorithm are executed based on a condition being true or false.

CM.CP.3 The student will perform iteration with loops.

Students will demonstrate the following Knowledge and Skills:

a) Write and implement "while" and "for" loops.
b) Differentiate between loops that run a fixed number of times and loops that run an indefinite number of times (e.g., stopping dependent on variable conditions).
c) Identify conditions that cause infinite loops.
d) Determine the outcome of code segments that include loops.

CM.CP.4 The student will write and implement the output phase of a computer program.

Students will demonstrate the following Knowledge and Skills:

a) Write and implement the output phase of a computer program, which may include:
   i) formatting output in text-based environments;

ii) displaying output through a graphical user interface; and
iii) sending output to a physical device (e.g., speakers, robots, LED lights).

b) Write output to a file.

CM.CP.5 The student will write and implement the input phase of a computer program.

Students will demonstrate the following Knowledge and Skills:

a) Write and implement input statements to store user given values into a program.
b) Validate input data using exception coding (e.g., using a "while" loop to control valid input by a user).
c) Determine what output a program will produce given a specific input.

CM.CP.6 The student will implement library functions.

Students will demonstrate the following Knowledge and Skills:

a) Implement library functions to process data.
b) Implement library functions to perform mathematical operations (e.g., random, absolute value, square root, power).
c) Implement void library functions and return library functions.
d) Implement overloaded library functions.

CM.CP.7 The student will write and implement user-defined functions.

Students will demonstrate the following Knowledge and Skills:

a) Write and implement a user-defined function to complete a task or sub-task.
b) Write and implement void functions and return functions.
c) Write and implement functions that accept parameters.

CM.CP.8 The student will implement pre-defined algorithms, including search routines and sort routines.

Students will demonstrate the following Knowledge and Skills:

a) Differentiate between types of search routines.
b) Differentiate between types of sort routines.
c) Implement pre-defined algorithms.
d) Implement a search routine on a one-dimensional list or an array, including sequential search and binary search.
e) Implement a sort routine on a one-dimensional list or an array (e.g., selection sort, insertion sort, merge sort).

Applications of Programming

CM.AP.1 The student will write and implement programs using sequencing, selection, and iteration to perform a specific task or solve a problem, including those arising from mathematical and interdisciplinary contexts.

Students will demonstrate the following Knowledge and Skills:

a) Determine what components of programming are needed to implement a step-by-step plan to perform a specific task or solve a problem.
b) Write a computer program that includes sequencing, selection (conditionals), and iteration (loops).
c) Write and implement computer programs to solve mathematical problems using
   i) formulas and equations;
   ii) functions;
   iii) probability and statistics; and
   iv) data-analysis.

CM.AP.2 The student will create documentation using written comments to annotate the intended purpose of the components of a user-created program.

Students will demonstrate the following Knowledge and Skills:

a) Create documentation using written comments to:
   i) describe the overall purpose of a program;
   ii) align a previously created step-by-step plan to a written program;
   iii) describe pre-conditions and post-conditions; and
   iv) improve the readability of a program.

CM.AP.3 The student will verify how programs access and process variables.

Students will demonstrate the following Knowledge and Skills:

a) Verify that the variable types are aligned to the purpose of the algorithm.
b) Verify that global variables are set to constant values before run time.
c) Differentiate between the scopes of variables (e.g., global scope versus local scope) and verify the intended use.

CM.AP.4 The student will translate a mathematical expression or statement into computer code.

Students will demonstrate the following Knowledge and Skills:

a) Declare, initialize, and assign variables to represent mathematical expressions or statements.
b) Implement order of operations, including logical and relational operators.
c) Translate a mathematical expression or statement into a programming statement(s).

CM.AP.5 The student will trace existing code to interpret the intended purpose.

Students will demonstrate the following Knowledge and Skills:

a) Trace existing code of an algorithm to
   i) identify values at each stage of an algorithm; and
   ii) predict return values of functions given specific arguments.
b) Use tracing to describe the intended purpose of existing code for an algorithm.

Evaluation of Programming

CM.EP.1 The student will test a program to match a sample output, using a set of data.

Students will demonstrate the following Knowledge and Skills:

a) Produce a given output by entering a data set.
b) Test a program including boundary cases and inaccurate data types to verify the intended outcomes.

CM.EP.2 The student will identify errors and debug a program using various techniques.

Students will demonstrate the following Knowledge and Skills:

a) Differentiate among syntax errors, runtime errors, and logic errors.
b) Debug a program using various techniques:
   i) interpret syntax and runtime error messages;
   ii) place controlled breaks;
   iii) output intermediate results;
   iv) disable a section of code by converting it into a comment;
   v) trace code to identify logic errors; and
   vi) use debugging tools available in the programming environment.

CM.EP.3 The student will compare and contrast the efficiency of computer programs.

Students will demonstrate the following Knowledge and Skills:

a) Compare and contrast the efficiency of computer programs in terms of
   i) complexity of algorithms with the same intended outcomes;
   ii) memory space used; and
   iii) run time.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Probability and Statistics

The following standards outline the content of a one-year course in Probability and Statistics. If a one-semester course is desired, the standards with a dagger (†) would apply. The purpose of the course is to present basic concepts and techniques for collecting and analyzing data, drawing conclusions, and making predictions.

Technology tools will be used to assist in teaching and learning. Graphing technologies facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Data in Context

PS.DC.1† The student will use a statistical cycle to formulate questions, describe types of data, data sources, and constraints within the context of a problem.

Students will demonstrate the following Knowledge and Skills:

a) Define the stages of the statistical cycle and how each stage relates to the others.
b) Formulate questions and conclusions based on context.
c) Understand the type of data relevant to the question at hand (e.g., quantitative versus categorical).
d) Compare and contrast population and sample, and parameter and statistic.
e) Identify and explain constraints of the statistical approach.

PS.DC.2† The student will compare and contrast data collection methods to plan and conduct an observational study.

Students will demonstrate the following Knowledge and Skills:

a) Investigate and describe sampling techniques (e.g., simple random sampling, stratified sampling, systematic sampling, cluster sampling).
b) Determine which sampling technique is best, given a particular context.
c) Investigate and explain biased influences inherent within sampling methods and various forms of response bias.
d) Use the statistical cycle to plan and conduct an observational study to answer a question or address a problem.

PS.DC.3† The student will utilize the principles of experimental design to plan and conduct a well-designed experiment.

Students will demonstrate the following Knowledge and Skills:

a) Describe the principles of experimental design, including:
   i) treatment/control groups;
   ii) blinding/placebo effects;
   iii) experimental units/subjects; and
   iv) blocking/matched pairs and completely randomized designs.

b) Evaluate the principles of experimental design to address comparison, randomization, replication, and control within the context of the problem.
c) Compare and contrast controlled experiments and observational studies and the conclusions that may be drawn from each.
d) Use the statistical cycle to plan and conduct a well-designed experiment to answer a question or address a problem.
e) Select a data collection method appropriate for a given context.

Descriptive Statistics

PS.DS.1† The student will represent and analyze data visualizations of univariate quantitative data, including dot plots, stemplots, boxplots, cumulative frequency graphs, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers, within the context of a problem.

Students will demonstrate the following Knowledge and Skills:

a) Create and interpret graphical displays of data, including dot plots, stemplots, boxplots, cumulative frequency graphs, and histograms, using appropriate technology.
b) Examine the graphs within the context of the problem by analyzing:
   i) shape;
   ii) measures of center;
   iii) spread; and
   iv) unusual features of the data (e.g., outliers, clusters, gaps).

PS.DS.2† The student will represent and analyze numerical characteristics of univariate quantitative data sets to describe patterns and departures from patterns within the context of a problem.

Students will demonstrate the following Knowledge and Skills:

a) Interpret measures of central tendency: mean, median, and mode.
b) Interpret measures of spread: range, interquartile range, variance, and standard deviation.
c) Identify possible outliers, using an algorithm.
d) Investigate and explain the influence of outliers on a univariate data set.
e) Investigate and explain ways in which standard deviation addresses variability by examining the formula for standard deviation.

PS.DS.3† The student will represent, compare, and analyze distributions of two or more univariate quantitative data sets, numerically and graphically.

Students will demonstrate the following Knowledge and Skills:

a) Create graphical displays of data, including back-to-back stemplots, parallel dot plots, parallel boxplots, and histograms, using appropriate technology.
b) Compare and contrast two or more univariate data sets, numerically and graphically, within the context of a problem by analyzing:
   i) shape;
   ii) measures of center;

iii) measures of spread; and

iv) unusual features of the data (e.g., clusters, gaps, outliers).

PS.DS.4 The student will represent and analyze categorical data, using two-way tables and other graphical displays, to describe patterns and relationships.

Students will demonstrate the following Knowledge and Skills:

a) Create and interpret graphical displays of univariate categorical data, including bar graphs within the context of the problem, using appropriate technology.

b) Create and interpret graphical displays comparing distributions of two or more univariate categorical data sets including segmented and side-by-side bar graphs within the context of the problem, using appropriate technology.

c) Generate and interpret a two-way table as a summary of the information obtained from two categorical variables.

d) Calculate and interpret marginal, relative, and conditional frequencies to analyze data in a two-way table within the context of a problem.

PS.DS.5 The student will represent and analyze quantitative bivariate data with scatterplots to identify and describe the relationship between two variables.

Students will demonstrate the following Knowledge and Skills:

a) Create scatterplots, using appropriate technology.

b) Examine and interpret scatterplots in the context of the problem by analyzing:

i) the form of relationship for linear and nonlinear trends;

ii) the direction of the relationship for positive, negative, or no association;

iii) the strength of the relationship such as strong, moderate, or weak; and

iv) the presence of unusual features within the data (e.g., clusters, gaps, influential points, outliers).

PS.DS.6 The student will create and interpret a linear model using the least squares regression method to assess the relationship between two quantitative variables.

Students will demonstrate the following Knowledge and Skills:

a) Create the least squares regression model using technology to interpret the contextual meaning of the slope and y-intercept.

b) Using technology, calculate and interpret the correlation coefficient, r, within the context of a problem.

c) Using technology, calculate and interpret the coefficient of determination, r^2, within the context of a problem.

d) Use regression lines to make predictions, and identify the limitations of the predictions, such as extrapolation.

e) Calculate and interpret a residual to understand the error of a prediction.

f) Using technology, calculate and interpret the standard deviation of the residuals, s.

Probability

PS.P.1† The student will organize information and apply probability rules to compute probabilities of events within the context of a problem.

Students will demonstrate the following Knowledge and Skills:

a) Given two or more events, determine whether the events are complementary, dependent, independent, and/or mutually exclusive, and compute the probability of those events.
b) Represent and calculate probabilities using Venn diagrams, tree diagrams, and two-way tables.
c) Apply the addition rule, the multiplication rule, and complementary rule to calculate probabilities.
d) Calculate conditional probabilities to determine the association or independence of two events.

PS.P.2 The student will represent and interpret situations using discrete random distributions, including binomial distributions.

Students will demonstrate the following Knowledge and Skills:

a) Identify discrete random variables and create a table to represent valid discrete probability distributions within the context of a problem.
b) Calculate and interpret the mean (expected value) and standard deviation for a discrete random variable within the context of a problem.
c) Determine if a discrete random variable satisfies the conditions for a binomial distribution.
d) Design and conduct a simulation of a binomial distribution.
e) Calculate and interpret probabilities from a binomial distribution within the context of a problem.
f) Calculate the mean and standard deviation for binomial distributions.
g) Describe the center, shape, and spread of a discrete random variable within the context of a problem.

PS.P.3† The student will represent and interpret situations using normal distributions.

Students will demonstrate the following Knowledge and Skills:

a) Compare and contrast discrete and continuous distributions.
b) Represent probability as the area under the curve of a normal distribution using the Empirical Rule and graphing technology.
c) Describe the center, shape, and spread of normal distributions within the context of a problem.
d) Compare and contrast two or more sets of normally distributed data using z-scores, percentiles, or probabilities within the context of a problem.
e) Standardize a data value from a normal distribution and interpret the z-score within the context of a problem.
f) Calculate and interpret probabilities of a normal distribution using technology within the context of a problem.

Inferential Statistics

PS.IS.1 The student will apply properties of sampling distributions and inference procedures to make decisions about population proportions.

Students will demonstrate the following Knowledge and Skills:

a) Describe the shape, center, and spread of the sampling distribution of a proportion within the context of a problem.

b) Given a problem, construct a one sample *z* confidence interval:
   i) identify the basic conditions for inference: random sample, independence, and normality;
   ii) calculate a confidence interval using technology; and
   iii) interpret the interval within the context of the problem.

c) Explain how changes in confidence level and sample size affect width of the confidence interval and margin of error.

d) Calculate and interpret a point estimate and margin of error of a confidence interval for a proportion within the context of the problem.

e) Explain how and why the hypothesis testing procedure allows one to reach a statistical decision.

f) Given a problem, apply the one sample *z* hypothesis testing procedures:
   i) construct appropriate null and alternate hypotheses;
   ii) identify the basic conditions for inference: random sample; independence, and normality;
   iii) calculate and interpret the *p*-value using technology;
   iv) determine and justify whether to reject the null hypothesis; and
   v) interpret the results within the context of the problem.

g) Use the statistical cycle to plan and conduct a statistical study about a proportion to answer a question or address a problem with inference.

PS.IS.2 The student will apply properties of sampling distributions and inference procedures to make decisions about populations.

Students will demonstrate the following Knowledge and Skills:

a) Describe the shape, center, and spread of the sampling distribution of a mean within the context of a problem.

b) Calculate and interpret a point estimate and a margin of error for a confidence interval of a mean within the context of a problem.

c) Describe the use of the Central Limit Theorem in satisfying the assumptions and conditions for inference about a mean.

d) Identify the properties of a *t* distribution.

e) Given a problem, construct a one sample *t* confidence interval:
   i) identify the basic conditions for inference: random sample, independence, and approximate normality;
   ii) calculate a confidence interval using technology; and
   iii) interpret the interval within the context of the problem.

f) Given a problem, apply the one sample t hypothesis testing procedures:
   i) construct appropriate null and alternate hypotheses;

ii) identify the basic conditions for inference: random sample, independence, and approximate normality;
iii) calculate and interpret the *p* value using technology;
iv) determine and justify whether to reject the null hypothesis; and
v) interpret the results within the context of the problem.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Discrete Mathematics

The following standards outline the content of a one-year course in Discrete Mathematics. If a one-semester course is desired, the standards with a dagger (†) would apply.

Discrete Mathematics may be described as the study of mathematical properties of sets and systems that have a countable (discrete) number of elements. With the advent of modern technology, discrete (discontinuous) models have become as important as continuous models. In this course, the main focus is problem solving in a discrete setting. Techniques that are not considered in the current traditional courses of algebra, geometry, and calculus will be utilized. As students solve problems, they will analyze and determine whether a solution exists (existence problems), investigate how many solutions exist (counting problems), and focus on finding the best solution (optimization problems). Connections will be made to other disciplines. The importance of discrete mathematics has been influenced by technology.

Technology tools will be used to assist in teaching and learning. Graphing technologies facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Logical Reasoning

DM.LR.1† The student will use reasoning to develop and apply logical arguments.

Students will demonstrate the following Knowledge and Skills:

a) Use Venn diagrams to codify and solve logic problems.
b) Express logical statements in symbolic form.
c) Represent a conditional statement as its converse, inverse, and contrapositive.
d) Describe how symbolic logic can be used to map the processes of computer applications.
e) Construct a truth table to display all possible input combinations and their outputs.
f) Identify the rules of inference and model basic logical statements including De Morgan's Law.
g) Apply logical reasoning to model contextual situations and make decisions.

DM.LR.2† The student will apply logic and proof techniques in the construction of a sound argument.

Students will demonstrate the following Knowledge and Skills:

a) Apply informal logical reasoning to contextual problems (e.g., predicting the behavior of software, solving puzzles).
b) Outline the basic structure of a proof technique (e.g., direct proof, proof by contradiction, induction).
c) Deduce the best type of proof for a given problem.
d) Use the rules of inference to construct direct proofs and proofs by contradiction.
e) Construct induction proofs involving summations and inequalities.
f) Use a truth table to prove the logical equivalence of statements.

DM.LR.3† The student will apply Boolean algebra to represent and analyze the function of logical gates and circuits.

Students will demonstrate the following Knowledge and Skills:

a) Explain basic properties of Boolean algebra: duality, complements, and standard forms.
b) Represent verbal statements as Boolean expressions.
c) Apply Boolean algebra to prove identities and simplify expressions.
d) Generate truth tables that encode the truth and falsity of two or more statements.
e) Explain the operation of discrete logic gates.
f) Describe the relationship between Boolean algebra and electronic circuits.
g) Analyze a combinational network using Boolean expressions.
h) Design simple combinational networks that use NAND (AND followed by NOT), NOR (OR followed by NOT), and XOR (exclusive-OR) gates.

DM.LR.4 The student will use mathematical induction to prove formulas and mathematical statements.

Students will demonstrate the following Knowledge and Skills:

a) Compare and contrast inductive and deductive reasoning.
b) Explain the relationship between weak and strong induction.
c) Construct induction proofs involving a divisibility argument.
d) Prove the Binomial Theorem through mathematical induction.

Set and Number Theory

DM.SNT.1† The student will identify and use the properties of sets and set operations.

Students will demonstrate the following Knowledge and Skills:

a) Compare and contrast sets, relations, and functions.
b) Express relationships between sets using Venn diagrams.
c) Describe a set using set-builder notation.
d) Construct new sets using the set operations intersection, union, difference, and complement.
e) Identify the laws of set theory (e.g., associative, commutative, distributive, De Morgan's Law).
f) Use the principle of inclusion and exclusion to determine the size of a set.
g) Use the properties of set operations to prove set equality.

DM.SNT.2† The student will apply the formulas of combinatorics.

Students will demonstrate the following Knowledge and Skills:

a) Create a tree diagram to represent relationships between independent events.
b) Use the Fundamental (Basic) Counting Principle to determine the number of possible outcomes of an event.
c) Determine the number of combinations possible when subsets of r elements are selected from a set of n elements without regard to order.
d) Determine the number of permutations possible when r objects selected from n objects are ordered.
e) Use the pigeonhole principle to solve packing problems to facilitate proofs.
f) Construct a proof by induction using principles of combinatorics.

DM.SNT.3 The student will use Pascal's Triangle to analyze numerical patterns and relationships.

Students will demonstrate the following Knowledge and Skills:

a) Construct Pascal's Triangle.
b) Expand binomials having positive integral exponents, using the Binomial Theorem and Pascal's Triangle.
c) Compare the binomial coefficient to the calculation of combinations.
d) Identify the Fibonacci numbers within Pascal's Triangle.

Graph Theory

DM.GT.1† The student will represent problems using vertex-edge graphs. The concepts of degree, connectedness, paths, planarity, and directed graphs will be analyzed.

Students will demonstrate the following Knowledge and Skills:

a) Illustrate the basic terminology of graph theory (e.g., vertex, edge, graph, degree of a vertex).
b) Use graphs to map situations in which the vertices represent objects, and edges represent a particular relationship between objects.
c) Identify and describe degree and connectedness.
d) Determine whether a graph is planar or nonplanar.
e) Analyze the relationship between faces, edges, and vertices using Euler's formula ($F = E - V + 2$).
f) Use directed graphs (digraphs) to represent situations with restrictions in traversal possibilities.
g) Determine when graphs are trees.

DM.GT.2† The student will solve problems through analysis and application of circuits, cycles, Euler paths, Euler circuits, Hamilton paths, and Hamilton circuits. Optimal solutions will be determined using existing algorithms and student-created algorithms.

Students will demonstrate the following Knowledge and Skills:

a) Determine whether a graph has an Euler circuit or path, and determine the circuit or path, if it exists.
b) Determine whether a graph has a Hamilton circuit or path, and determine the circuit or path, if it exists.
c) Count the number of Hamilton circuits for a complete graph with n vertices.
d) Use an Euler circuit algorithm to solve optimization problems.

DM.GT.3† The student will apply graphs to conflict-resolution problems, such as graph coloring, scheduling, matching, and optimization.

Students will demonstrate the following Knowledge and Skills:

a) Model projects consisting of several subtasks, using a graph.
b) Use graphs to resolve conflicts that arise in scheduling.
c) Use graph coloring to determine the chromatic number of a graph.

DM.GT.4 The student will recognize and apply algorithms to solve configuration, conflict-resolution, and sorting problems.

Students will demonstrate the following Knowledge and Skills:

a) Recognize algorithms such as nearest neighbor, brute force, and cheapest link as they apply to graphs.
b) Use Kruskal's algorithm to determine the shortest spanning tree of a connected graph.
c) Use Prim's algorithm to determine the shortest spanning tree of a connected graph.
d) Use Dijkstra's algorithm to determine the shortest spanning tree of a connected graph.

DM.GT.5 The student will use algorithms to schedule tasks to determine a minimum project time.

Students will demonstrate the following Knowledge and Skills:

a) Specify in a digraph the order in which tests are to be performed.
b) Identify the critical path to determine the earliest completion time (minimum project time).
c) Use the list-processing algorithm to determine an optimal schedule.
d) Create and test scheduling algorithms.

Computational Methods

DM.CM.1† The student will describe and apply sorting and searching algorithms used in processing and communicating information.

Students will demonstrate the following Knowledge and Skills:

a) Select and apply a sorting algorithm, such as a bubble sort, merge sort, or network sort.
b) Describe the advantages and disadvantages of various sorting algorithms.
c) Analyze the knapsack and bin-packing problems.
d) Select and apply search algorithms to analyze problems.
e) Determine the average, best-case, and worst-case reasoning for different searches.

DM.CM.2† The student will use recursive processes.

Students will demonstrate the following Knowledge and Skills:

a) Compare and contrast iterative and recursive processes.
b) Use recursive processes to model growth and decay.
c) Use recursive processes to create fractals.
d) Use recursive processes to generate the Fibonacci sequence.
e) Determine if a recursive solution is more efficient than an iterative solution.

DM.CM.3 The student will identify and apply cryptographic methods.

Students will demonstrate the following Knowledge and Skills:

a) Compare and contrast ciphers and codes.
b) Describe the evolution of cipher systems.
c) Identify the Fundamental Theorem of Arithmetic.
d) Describe how the complexity of prime factorization is used in cryptography.
e) Describe modular arithmetic in context (e.g., clocks, days of the week, measures of time).
f) Analyze the relationship between divisibility and modulus.
g) Determine congruence within modular arithmetic.
h) Perform operations within modular arithmetic.
i) Apply modular arithmetic to problems in context (e.g., cryptography, International Standard Book Number (ISBN), International Bank Account Number (IBAN)).

DM.CM.4 The student will analyze the limitations of algorithms and their contextual relationships in computing.

Students will demonstrate the following Knowledge and Skills:

a) Describe maximum complexity of an algorithm using Big O notation.
b) Describe Turing machines and how they are used to test the limits of computation.
c) Describe the halting problem and explain how it characterizes the fundamental limitations of computation and undecidability.
d) Explain the P versus NP problem and defend a justification for equality, inequality, or undecidability.
e) Analyze how the equivalence of P- and NP-class problems might impact society.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Mathematical Analysis

Students enrolled in Mathematical Analysis are assumed to have mastered Geometry and Algebra 2 concepts. Mathematical Analysis develops students' understanding of algebraic and transcendental functions, parametric and polar equations, sequences and series, and vectors. The content of this course serves as appropriate preparation for a calculus course.

Technology tools will be used to assist in teaching and learning. Graphing technologies facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Characteristics of Functions

MA.CF.1 The student will identify and analyze the properties of polynomial, rational, piecewise-defined, absolute value, radical, and step functions and sketch the graphs of the functions.

Students will demonstrate the following Knowledge and Skills:

a) Use mathematical reasoning to identify polynomial, rational, piecewise-defined, absolute value, radical, and step functions, given an equation or graph.
b) Given multiple representations of a polynomial, rational, piecewise-defined, absolute value, radical, and step function, analyze:
   i) domain and range;
   ii) roots (including complex roots);
   iii) intercepts;
   iv) symmetry (including even and odd functions);
   v) asymptotes (horizontal, vertical, and oblique/slant;
   vi) points of discontinuity;
   vii) intervals for which the function is increasing, decreasing or constant;
   viii) end behavior; and
   ix) relative and/or absolute maximum and minimum points.
c) Sketch the graph of a polynomial, rational, piecewise-defined, absolute value, radical, and step function.

MA.CF.2 The student will determine the limit of a function if it exists.

Students will demonstrate the following Knowledge and Skills:

a) Verify estimates about the limit of a function using graphing technology.
b) Determine the limit of a function algebraically and verify with graphing technology.
c) Determine the limit of a function numerically and verify with graphing technology.
d) Use proper limit notation, including when describing the end behavior of a function.
e) As the variable approaches a finite number,
   i) determine the limit of a function numerically by direct substitution;
   ii) determine the limit of a function using algebraic manipulation;
   iii) estimate the limit of a function using a table; and

iv) determine the limit of a function from a given graph.

f) As the variable approaches positive or negative infinity, analyze the limit of a function to describe the end behavior.

MA.CF.3 The student will analyze and describe the continuity of functions.

Students will demonstrate the following Knowledge and Skills:

a) Describe continuity of a function.

b) Use mathematical notation to communicate and describe the continuity of functions including polynomial, rational, piecewise, absolute value, radical, and step function, using graphical and algebraic methods.

c) Prove continuity at a point, using the definition.

d) Classify types of discontinuity based on which condition of continuity is violated.

Functional Relationships

MA.FR.1 The student will analyze compositions of functions to determine and verify inverses of functions.

Students will demonstrate the following Knowledge and Skills:

a) Construct the composition of functions algebraically and graphically.

b) Determine the domain and range of composite functions algebraically and graphically.

c) Develop the inverse of a function algebraically and graphically.

d) Compare the domain and range of the inverse of a function with the original function, both algebraically and graphically.

e) Use mathematical reasoning to generalize and communicate the criteria for an inverse function to exist.

MA.FR.2 The student will analyze the characteristics of exponential and logarithmic functions, and sketch the graphs of the functions.

Students will demonstrate the following Knowledge and Skills:

a) Generalize characteristics of exponential and logarithmic functions from an equation or a graph.

b) Define *e* and estimate its value.

c) Convert between equations written in logarithmic and exponential form.

d) Use laws of exponents and properties of logarithms to solve equations and simplify expressions.

e) Represent contextual problems, using exponential and logarithmic functions, to include common and natural logarithms.

f) Sketch the graph of exponential and logarithmic functions and identify asymptotes, end behavior, intercepts, domain, and range.

MA.FR.3 The student will analyze sequences and finite series, and model and solve problems in context using sequences and series.

Students will demonstrate the following Knowledge and Skills:

a) Use and interpret the notation: $\sum$, n, n^{th}, and a_n.
b) Derive the formulas associated with arithmetic and geometric sequences and series.
c) Determine the nth term, a_n, for an arithmetic or geometric sequence.
d) Determine the sum, S_n, if it exists, of an arithmetic or geometric series.
e) Model and solve problems in context, using sequences and series.
f) Distinguish between a convergent and divergent series.
g) Describe convergent series in relation to the concept of a limit.

Analytic Geometry

MA.AG.1 The student will identify and analyze the properties of conic sections and sketch a graph given an equation.

Students will demonstrate the following Knowledge and Skills:

a) Given a translation or rotation matrix, determine an equation for the transformed function or conic section.
b) Convert between standard and general forms of conic equations by completing the square.
c) Graph conic sections from equations written in general or standard form using transformations.
d) Identify characteristics of conic sections including center, vertices, axes, symmetry, foci, directrix, eccentricity, and asymptotes.
e) Represent applications of conic sections.

MA.AG.2 The student will use parametric equations to model and solve problems in context.

Students will demonstrate the following Knowledge and Skills:

a) Graph and analyze parametric equations and use the graph to determine solutions.
b) Use parametric equations to model contextual problems, including motion over time.

MA.AG.3 The student will perform operations with vectors in the coordinate plane.

Students will demonstrate the following Knowledge and Skills:

a) Use vector notation.
b) Perform the operations of addition, subtraction, and scalar multiplication, graphically and algebraically on vectors.
c) Find the dot (inner) product of two vectors and use it to determine the angle between two vectors.
d) Determine if two vectors are orthogonal.
e) Express complex numbers in vector notation.
f) Verify properties of the dot product.
g) Determine the components of a vector.
h) Determine the norm (magnitude) of a vector.
i) Find a unit vector in the same direction of a given vector.
j) Apply vectors to problems in context.

MA.AG.4 The student will investigate and identify the characteristics of the graphs of polar equations.

Students will demonstrate the following Knowledge and Skills:

a) Classify polar equations (rose, cardioid, limaçon, lemniscate, spiral, and circle), given the graph or the equation.
b) Determine the effects of changes in the parameters of polar equations on the graph, using graphing technology.
c) Convert between polar and rectangular forms of coordinates.
d) Convert between complex numbers written in rectangular form and polar form.
e) Convert equations between polar and rectangular forms.
f) Determine and verify the intersection of the graphs of two polar equations, using graphing technology.

MA.AG.5 The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.

Students will demonstrate the following Knowledge and Skills:

a) Multiply matrices by a scalar.
b) Add, subtract, and multiply matrices.
c) Represent problems with a system of no more than three linear equations.
d) Express a system of linear equations as a matrix equation.
e) Solve a system of equations using matrices.
f) Determine the inverse of a two-by-two or three-by-three matrix using paper and pencil.
g) Verify two matrices are inverses using matrix multiplication.
h) Verify the commutative and associative properties for matrix addition and multiplication.

Mathematics *Standards of Learning* for Virginia Public Schools 2023 Data Science

The following standards outline the content of a one-year course in Data Science. If a one-semester course is desired, the standards with a dagger (†) would apply. The Data Science *Standards of Learning* provide an introduction to the learning principles associated with analyzing big data.

Through the use of open-source technology tools, students will identify and explore problems that involve the use of relational database concepts and data-intensive computing to find solutions and make generalizations. Students will engage in a data science problem-solving structure to interact with large data sets as a means to formulate problems, collect and clean data, visualize data, model using data, and communicate effectively about data formulated solutions.

Data in Context

DS.1† The student will identify specific examples of real-world problems that can be effectively addressed using data science.

Students will demonstrate the following Knowledge and Skills:

a) Identify and explain characteristics that best lend themselves to a data driven approach to problem solving.
b) Formulate questions based on context.
c) Understand the type of data relevant to the context of the question at hand.
d) Define relationships between variables and constant relationships.
e) Create a hypothesis of interest in terms of measurable data.
f) Define the stages of the data cycle and how each stage is related to the other.
g) Identify and explain constraints of the data-driven approach.

DS.2 The student will be able to formulate a top-down plan for data collection and analysis, with quantifiable results, based on the context of a problem.

Students will demonstrate the following Knowledge and Skills:

a) Design a data project plan, which is aligned with the data science cycle, that includes the following components:
   i) definition of the goal of the project as it pertains to a real-world problem;
   ii) identification of the various parameters of the problem and stakeholders;
   iii) a timeline for the project with deliverables;
   iv) Key Performance Indicators (KPI) for the successful data project deliverables;
   v) resource needs and tools for the project;
   vi) bias considerations for the sampling process of the project; and
   vii) limitations of the project.
b) Given the context and parameters of a problem, choose from among various sampling techniques, which may include
   i) simple random;
   ii) systematic;
   iii) stratified; and

iv) cluster;

to justify the sampling methodology of the project design and implementation.

Data Bias

DS.3† The student will recognize the importance of data literacy and develop an awareness of how the analysis of data can be used in problem solving to effect change and create innovative solutions.

Students will demonstrate the following Knowledge and Skills:

a) Formulate relevant/clarifying questions to identify potential data biases presented in existing analyses/visualizations.
b) Effectively read data summaries and visualizations and explain/translate into nontechnical terms in proper context.
c) Identify potential data biases in terms of data presented and discuss the potential effects of such biases in terms of how they could affect data analysis and decision making.
d) Identify privacy and consumer protection issues that might be a result of how data is presented.
e) Describe the types of data that business, industry, and government entities collect and possible ways the data is used.

DS.4 The student will be able to identify data biases in the data collection process and understand the implications and privacy issues surrounding data collection and processing.

Students will demonstrate the following Knowledge and Skills:

a) Identify data biases in the data collection process that include, but are not limited to, confirmation, selection, outliers, overfitting / under fitting, and confounding and describe mitigation strategies for these biases.
b) Provide examples of sampling biases in terms of data collection and the potential effects.
c) Identify and describe data biases as a producer as well as a consumer/decision maker of data.
d) Describe how the data collection process should be focused, relevant, and limited to the scope of the data project plan.
e) Describe privacy considerations in the collection of data as both a consumer and producer.

Data and Communication

DS.5† The student will use storytelling as a strategy to effectively communicate with data.

Students will demonstrate the following Knowledge and Skills:

a) Define storytelling and explain the importance of storytelling as a strategy to communicate the idea behind and results of a data science project effectively.
b) Explain the steps involved in data storytelling and how it relates to the data cycle.
c) Effectively identify a story worth telling based on the data (looking for trends, correlations, outliers) and by asking a question or forming a hypothesis based on insight and audience.
d) Effectively select visualizations that simplify the information, highlight the most important data, and communicate key points quickly.

e) Effectively simplify the information presented to make it more concise and focus the audience's attention on the key parameters that support the student's hypothesis.
f) Effectively form a narrative based on data available to provide context, insight, and interpretation to make the analysis more relevant to a given audience.
g) Explain how data storytelling should include complete and accurate information, and consistent visuals for effective communication.

DS.6† The student will justify the design, use, and effectiveness of different forms of data visualizations.

Students will demonstrate the following Knowledge and Skills:

a) Conduct exploratory data analysis using visualization.
b) Formulate questions from exploration of a data set to consider how data will communicate a story.
c) Determine the effectiveness of different data visualization choices based on the data context from conventional statistical charts to unconventional/emerging data visualizations to more complex visualizations.
d) Create a visualization of a data set and summarize the representation using the context of the data.
e) Compare two or more different representations to ensure the design communicates the features and behavior of data sets.
f) Justify design choices (based on data set type, size, context, and audience) of data visualizations to highlight important features, trends, and insights.

Data Modeling

DS.7 The student will be able to assess reliability of source data in preparation for mathematical modeling.

Students will demonstrate the following Knowledge and Skills:

a) Explain why determining the reliability of big data sources is a key skill that data scientists use to build data trust across an organization.
b) Describe the difference between reliability of a data source compared to statistical reliability and validity in research analysis. Assess processing source data for reliability based on validity, completeness, and uniqueness.

DS.8† The student will be able to acquire and prepare big data sets for modeling and analysis.

Students will demonstrate the following Knowledge and Skills:

a) Explain the pros and cons of collecting data versus acquiring it from existing sources.
b) Apply matrix operations using algebraic methods (with the support of technology tools) to:
   i) wrangle the data (sort, select, filter, and replace);
   ii) clean the data;
   iii) format and enrich the data; and
   iv) combine and store the data.
c) Read data from different sources for preparation and analysis.

d) Identify important parameters about a big data set based on the context of data collected/acquired.
e) Define and document the process of ingesting, formatting, and cleaning data for future decision making by:
   i) making data more easily understood by a wider audience; and
   ii) connecting data with existing contextual data.

DS.9† The student will select and analyze data models to make predictions, while assessing accuracy and sources of uncertainty.

Students will demonstrate the following Knowledge and Skills:

a) Identify factors that contribute to the overall behavior of a data set (e.g., true values, bias, and noise).
b) Fit models based on the behavior of the data, (e.g., models of univariate and bivariate data), in order to make predictions.
c) Distinguish between linear and nonlinear associations between variables using visualizations.
d) Identify models that are overly complex and therefore fitting to random noise which decreases their predictive accuracy.
e) Use regression techniques to perform selection of optimal features.
f) Recognize the potential implications of removing features.
g) Select the optimal model for a data set from among a large collection of models, using technological tools.

DS.10† The student will be able to summarize and interpret data represented in both conventional and emerging visualizations.

Students will demonstrate the following Knowledge and Skills:

a) Apply descriptive statistics to explain measures of central tendency and measures of variability/dispersion to describe center and spread in visualizations of distributions.
b) Define emerging visualizations and describe summarization of characteristics and relationships based on audience and purpose which may include:
   i) a heat map, which uses color to show changes and magnitude of a third variable to a two-dimensional plot; and
   ii) a bubble chart, which is a multivariate graph that is both a scatterplot and a proportional area chart. Typically, each plotted point then represents a third variable by the area of its circle.
c) Interpret various emerging visualizations by describing patterns, trends, and relationships between and among the variables.

DS.11 The student will select statistical models and use goodness of fit testing to extract actionable knowledge directly from data.

Students will demonstrate the following Knowledge and Skills:

a) Calculate the theoretical probability of random events and compare them to the observed frequencies.
b) Describe the normal curve determined by the mean and standard deviation of a univariate data set.

c) Fit nonlinear models to data sets and use these models to predict unobserved data values.
d) Select pairs of variables that identify meaningful clusters of data.
e) Select an appropriate statistical distribution and test its goodness of fit based on the context of the data being analyzed. Statistical distributions may include, but are not limited to
    i) Normal;
    ii) Binomial; and
    iii) Poisson.

Data and Computing

DS.12† The student will be able to select and utilize appropriate technological tools and functions within those tools to process and prepare data for analysis.

Students will demonstrate the following Knowledge and Skills:

a) Utilize technology tools to be able to access data effectively from multiple sources (e.g., tables, column separated values, spreadsheets, documents, databases).
b) Utilize tools and functions (in tools) to effectively explore the data for issues and errors before beginning to process it.
c) Define the (tools and technological) process to optimally ingest data and to export data after processing.
d) Utilize tools and their functions to clean and validate data by:
    i) removing data that are incomplete, incorrect, or duplicated;
    ii) removing extraneous data or outliers; and
    iii) standardizing data to conform to contextual norms (e.g., privacy, sensitive data).
e) Utilize tools and their functions to combine and store data by:
    i) merging multiple data sets for efficiency purposes; and
    ii) optimizing storage of data based on volume, velocity, and variety.
f) Utilize tools to format and store the data appropriately to allow for effective analysis.

DS.13† The student will be able to select and utilize appropriate technological tools and functions within those tools to analyze and communicate data effectively.

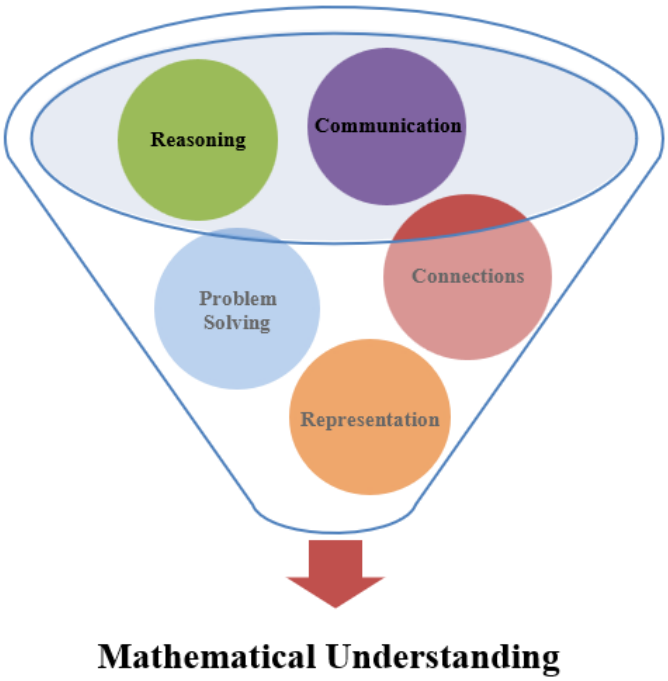
Students will demonstrate the following Knowledge and Skills:

a) Select and utilize technology tools to effectively generate conventional and unconventional visualizations of data to explore patterns and/or analyze a large data set.
b) Utilize specific functions in technology tools to perform descriptive and inferential statistical analysis.
c) Utilize coding to store and extract data more effectively for data analysis.
d) Select and apply features of technology tools effectively to organize, summarize and gain insight from data.
e) Select the appropriate visualization based on context and audience and create it using technology tools to effectively communicate an idea.

Mathematics *Standards of Learning* for Virginia Public Schools Appendix

Mathematical Process Goals for Students

Students learn and apply the five mathematical process goals as they work to achieve the content of the Mathematics *Standards*. These processes support students in becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret contextual situations. Contextual situations include real-world problems and problems that model real-world situations.



Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible when learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical

statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

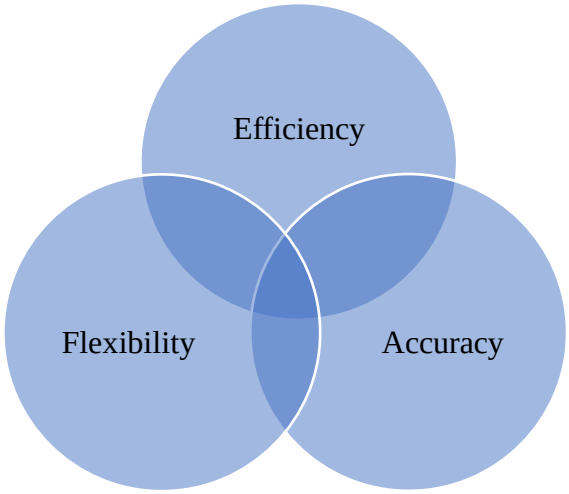
The strategic use of technological tools can support both the learning of mathematical procedures and skills as well as the development of advanced mathematical proficiencies, such as problem solving, reasoning, and justifying.

State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, the Knowledge and Skills that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

The mathematics standards emphasize procedural understanding through the development of computational fluency. Computational fluency is the ability to use flexible, efficient, and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose and apply and can explain why their actions make sense. Students demonstrate effective use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems. Students share their ideas collaboratively, discuss how to solve problems in different ways and produce accurate answers efficiently.

Computational Fluency



The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades.

Rigorous mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

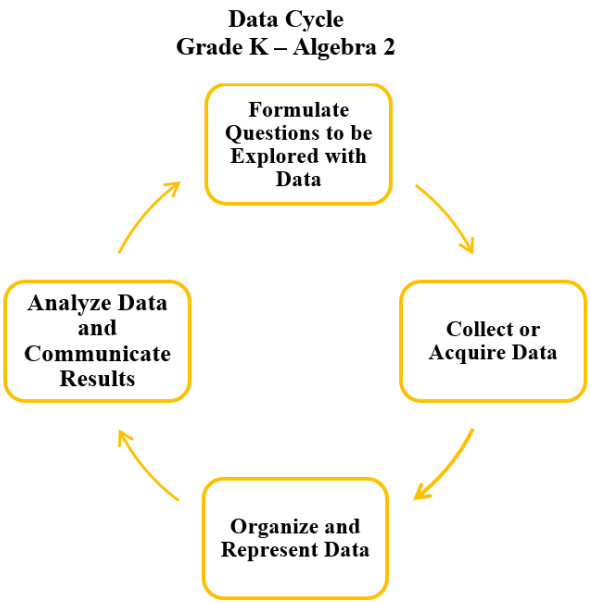
Algebraic Thinking and Readiness

The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra 1 *Standards of Learning*. The preparation of students for Algebra 1 includes the mastery of, and the ability to apply, the Mathematics *Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. Included in the progression of algebraic content in kindergarten through grade eight is patterning; generalization of arithmetic concepts; proportional reasoning; representing mathematical relationships using tables, symbols, and graphs; and algebraic equations and inequalities. The K-8 Mathematics *Standards of Learning* form a progression of content knowledge and develop the foundation and reasoning necessary to be well-prepared for mathematics courses beyond Algebra 1, including Geometry and Statistics. Divisions have local autonomy to compact content in the mathematics standards of learning when creating accelerated curriculum for students taking Algebra 1 in middle school. Algebra 1 is a course that all students should have the opportunity to take in Grade 8 or earlier. School divisions should ensure processes are in place to create accelerated coursework and to support individual readiness.

Data and Data Analysis

Analyzing data requires the ability to read, write, and communicate about data in context. The skills needed to analyze data are integrated in the mathematics standards and derived from and build upon a strong mathematical foundation. A process for data analysis is included in the standards as a Data Cycle. The cycle includes asking meaningful questions, collecting, or acquiring appropriate data, and analyzing, interpreting, and communicating about the data. The standards include direct reference to mathematical skills needed to analyze and interpret data. Data and data analysis are necessary for many jobs such as those in science, technology, business, and engineering, but also to ensure students

can develop problem solving skills and navigate as a citizen in a world in which data plays a vital role.



Formal Mathematics Vocabulary

The development of mathematics vocabulary supports students' conceptual understanding, abstract reasoning, and ability to communicate effectively. Teachers should facilitate student connections between formal mathematical vocabulary and their current conceptual understanding. Teacher and student use of formal mathematics vocabulary is a focus of instruction in all mathematics classrooms.

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