Instructional Guide to Support 2023 Grade 6 Mathematics *Standards of Learning*

Virginia Department of Education

Superintendent of Public Instruction
Emily Anne Gullickson, M.Ed. J.D.

Office of Math and Science
Dr. Anne Petersen, Director of Math and Science
Dr. Angela Byrd-Wright, Director of Humanities (Former Mathematics Coordinator)
Ms. Victoria Bohidar, Mathematics Coordinator
Dr. Jessica Brown, Elementary Mathematics Specialist
Dr. Regina Mitchell, Mathematics and Special Education Specialist
Mrs. Donna Snyder, Mathematics Division Support Specialist

NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

Table of Contents

Introduction

The Mathematics Instructional Guide, a companion document to the 2023 Mathematics *Standards of Learning*, amplifies the Standards of Learning by defining the core knowledge and skills in practice, supporting teachers and their instruction, and serving to transition classroom instruction from the 2016 *Mathematics Standards of Learning* to the newly adopted 2023 Mathematics *Standards of Learning*. Instructional supports are accessible in #GoOpenVA and support the decisions local school divisions must make concerning local curriculum development and how best to help students meet the goals of the standards. The local curriculum should include a variety of information sources, readings, learning experiences, and forms of assessment selected at the local level to create a rigorous instructional program.

For a complete list of the changes by standard, the 2023 Virginia Mathematics *Standards of Learning – Overview of Revisions* is available and delineates in greater specificity the changes for each grade level and course.

The Instructional Guide is divided into three sections: Understanding the Standard, Skills in Practice, and Concepts and Connections aligned to the Standard. The purpose of each is explained below.

Understanding the Standard

This section includes mathematics understandings and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, or examples regarding information sources that support the content. They describe what students should know (core knowledge) as a result of the instruction specific to the course/grade level and include evidence-based practices to approaching the Standard. There are also possible misconceptions and common student errors for each standard to help teachers plan their instruction.

Skills in Practice

This section outlines supporting questions and skills that are specifically linked to the standard. They frame student inquiry, promote critical thinking, and assist in learning transfer. Curriculum writers and teachers should use them to plan instruction to deepen understanding of the broader unit and course objectives. This is not meant to be an exhaustive list of student expectations.

Concepts and Connections

This section outlines concepts that transcend grade levels and thread through the K through 12 mathematics program as appropriate at each level. Concept connections reflect connections to prior grade-level concepts as content and practices build within the discipline as well as potential connections across disciplines.

Grade 6

Number and Number Sense

In K-12 mathematics, numbers and number sense form the foundation building blocks for mathematics understanding. Students develop a foundational understanding of numbers, their properties, and the relationships between them as they learn to compare and order numbers, understand place value, and perform numerical operations. Number sense includes an understanding of patterns and the relationships between numbers and being able to apply this knowledge to real world problems. Throughout K-12, students build on their number sense to develop a deeper understanding of mathematical concepts and reasoning.

In Grade 6, there are multiple representations of numbers and relationships among numbers that provide meaning and structure that allow students to make sense of the world around them. At this grade level, students express equivalency; compare and order numbers written as fractions, mixed numbers, decimals, and percents; represent, compare, and order integers; and recognize and represent patterns with exponents and perfect squares.

6.NS.1

The student will reason and use multiple strategies to express equivalency, compare, and order numbers written as fractions, mixed numbers, decimals, and percents.

Students will demonstrate the following Knowledge and Skills:

a) Estimate and determine the percent represented by a given model (e.g., number line, picture, verbal description), including percents greater than 100% and less than 1%.*

b) Represent and determine equivalencies among decimals (through the thousandths place) and percents incorporating the use of number lines, and concrete and pictorial models.*

c) Represent and determine equivalencies among fractions (proper or improper) and mixed numbers that have denominators that are 12 or less or factors of 100 and percents incorporating the use of number lines, and concrete and pictorial models.*

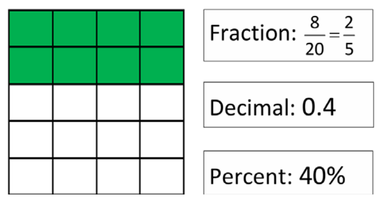
d) Represent and determine equivalencies among decimals, percents, fractions (proper or improper), and mixed numbers that have denominators that are 12 or less or factors of 100 incorporating the use of number lines, and concrete and pictorial models.*

e) Use multiple strategies (e.g., benchmarks, number line, equivalency) to compare and order no more than four positive rational numbers expressed as fractions (proper or improper), mixed numbers, decimals, and percents (decimals through thousandths, fractions with denominators of 12 or less or factors of 100) with and without models. Justify solutions orally, in writing or with a model. Ordering may be in ascending or descending order.*

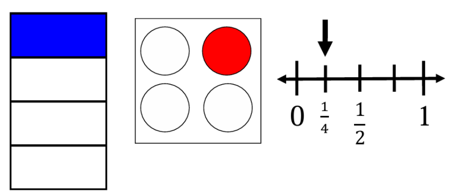
*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

Understanding the Standard

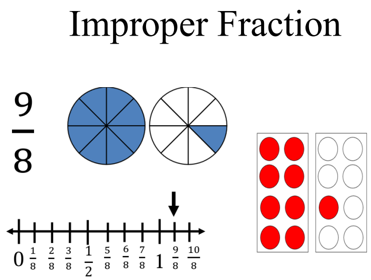
- Fractions, decimals, and percents are three different ways to express the same number. Any number that can be written as a fraction can be expressed as a terminating or repeating decimal or a percent.
- Percent means "per 100" or how many "out of 100"; percent is another name for hundredths.
- A number followed by a percent symbol (%) is equivalent to a fraction with that number as the numerator and with 100 as the denominator (e.g., $30\% = \frac{30}{100}$; $139\% = \frac{139}{100}$).
- Percents can be expressed as fractions or decimals (e.g., $38\% = \frac{38}{100} = 0.38$; $139\% = \frac{139}{100} = 1.39$).
- Percents are used to solve contextual problems including sales, tax, and discounts.
- When estimating a percent, students should consider benchmarks of 0%, 25%, 50%, 75%, and 100%.
- For percents less than 1, focus on benchmarks that are less than 1% (like 0.5% or $\frac{3}{4}$%).
- Equivalent relationships among fractions, decimals, and percents may be determined by using concrete materials and pictorial representations (e.g., fraction bars, base 10 blocks, fraction circles, number lines, colored counters, cubes, decimal squares, shaded figures, shaded grids, or calculators).

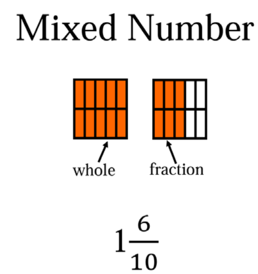


- Some fractions can be rewritten as equivalent fractions with denominators of powers of 10 and can be represented as decimals or percents (e.g., $\frac{3}{5} = \frac{6}{10} = \frac{60}{100}$ = 0.60 = 60%). Fractions, decimals, and percents can be represented by using an area model, a set model, or a measurement model. For example, the fraction $\frac{1}{4}$ is shown below using each of the three models.



- Proper fractions, improper fractions, and mixed numbers are terms used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$).





- Some fractions have a decimal representation that is a terminating decimal, which means it has a finite number of digits (e.g., $\frac{1}{8} = 0.125$). Other fractions have a decimal representation that does not terminate but continues to repeat (e.g., $\frac{2}{9}$ = 0.222...) The repeating decimal can be written with ellipses (three dots) as in 0.222... or denoted with a bar above the digits that repeat as in $0.\overline{2}$.
- The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where *a* and *b* are integers, and *b* does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of positive rational numbers are: 0.275, $\frac{1}{4}$, 82, 75%, $\frac{22}{5}$, $4.\overline{59}$.

Equivalent
Relationships

$$56\% = \frac{56}{100} = \frac{14}{25} = 0.56$$

$$2\tfrac{4}{9} = 2.444... = 244.\overline{4}\%$$

$$1.8 = 180\% = \frac{180}{100} = 1\tfrac{4}{5}$$

- Students are not expected to know the names of the subsets of the real numbers until Grade 8.

- Strategies using 0, $\frac{1}{2}$, and 1 as benchmarks can be used to compare fractions and decimals. For example: Which is greater: $\frac{4}{7}$ or 0.4? $\frac{4}{7}$ is greater than $\frac{1}{2}$ because 4, the numerator, represents more than half of 7, the denominator. 0.4 is less than $\frac{1}{2}$ because 0.4 is less than 0.5 which is equivalent to $\frac{1}{2}$. Therefore, $\frac{4}{7} > 0.4$.
- When comparing two fractions close to 1, the distance from 1 can be used as the benchmark. For example: Which is greater, $\frac{6}{7}$ or $\frac{8}{9}$? $\frac{6}{7}$ is $\frac{1}{7}$ away from 1 whole. $\frac{8}{9}$ is $\frac{1}{9}$ away from 1 whole. Since, $\frac{1}{9} < \frac{1}{7}$, then $\frac{6}{7}$ is a greater distance away from 1 whole than $\frac{8}{9}$. Therefore, $\frac{6}{7} < \frac{8}{9}$.
- Students may justify their reasoning by using benchmarks, number lines, equivalence, pictures, etc.

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Connections: Number lines are a way to visualize parts of a whole**.** Number lines can be used to represent, compare, and order fractions as well as determine equivalencies among them. Number lines are helpful in teaching benchmark fractions. Prior to Grade 6, elementary students worked with number lines to understand concepts of whole number place value, fractions (proper, improper, and mixed numbers), decimals, computation, and probability. Number lines are beneficial tools for making benchmark comparisons.

- Have students make a human number line. Give students pieces of paper with fractions, decimals, and percents on them, and have students arrange themselves in order.

- Have students place fractions and decimals in the correct location on a number line. For example –

  - $\frac{10}{8}$
  - 1.2
  - 0.556
  - 0.8%

Students may create a number line labeled zero to one, not recognizing that some of the numbers are greater than one. Another common error for some students is to place 0.8% as if it were 8% or 80%. Some students may focus only on the digit when interpreting percents rather than the relationship to the whole. Teachers can help students connect their prior understanding of fractions and decimals to percent. Students should determine the location to place numbers on a number line in terms of their relationships to the benchmarks of 0, $\frac{1}{2}$, 1 whole, and greater than 1 whole. Multiple experiences and discussions centered on benchmarking with fractions, decimals, and percents can help strengthen these connections.

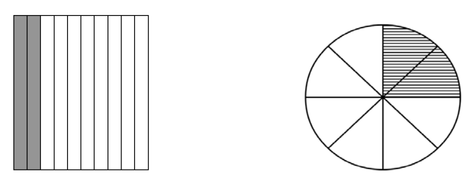
Mathematical Reasoning: Encourage students to justify their reasoning orally, in writing, or with a model. When doing so, students are to apply the appropriate vocabulary such as ascending and descending. The following questions may elicit students' understanding –

- What strategies are used to compare fractions and mixed numbers?
- How are fractions, decimals, and percents alike and different?
- How can fractions, decimals, and percents be represented in various ways?
- Why is it necessary to have multiple forms of numbers?
- When ordering fractions, decimals, and percents, detail a strategy used and discuss how you justified your solution.

- An activity to support students' understanding of equivalencies as well as comparing/ordering fractions, decimals, and percents is as follows –
  - Create sets of playing cards for each group by writing various fractions, decimals, and percents on index cards.
  - Put students in groups of two to four and give each group a set of cards. Provide concrete materials for students to use as they justify and discuss their reasoning.
  - Have students shuffle the cards and place them face down on the table. In turn, each player draws two cards, places them face up, and compares the numbers. If the two numbers are equal, the cards are placed one on top of the other. Correct comparisons earn a player one point.
  - As a variation, each player can draw three (or four) cards and order the numbers least to greatest.

Mathematical Representations: Some students may not understand that a fraction is part of a whole and that decimals and percents are other ways to represent fractions. Concrete materials and pictorial representations assist students in understanding that a fraction, decimal and percent can represent the same value. Several examples with misconceptions follow –

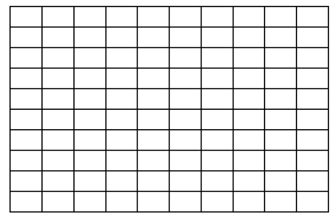
- Ask students to name the fraction, decimal, and percent shown in each model below and then compare the two models:



Students may think both models represent the same amount since two portions are shaded in each model. Some students may have the misconception that decimals and percents can only be modeled on a grid, as in the first model. Students may be able to identify the correct fraction in the second model, but unable to write an equivalent decimal or percent.

Students will benefit from practice with a variety of pictorial models of percents, beginning with the 100-grid and transitioning to other representations. Some possible models may include fraction circles, number lines, colored counters, and shaded figures or grids. As students gain more understanding of the meaning of percent, transition to examples of other fractional representations that are not out of 100.

- Ask students to represent $\frac{13}{20}$ on a grid like the one shown below:



  - What decimal is equivalent to the fraction $\frac{13}{20}$?
  - What percent is equivalent to the fraction $\frac{13}{20}$?

  Students may shade in 13 of the squares on the grid, not understanding the relationship of the numerator to the denominator or how they relate to the whole in the model of 100 squares. Students may not understand $\frac{13}{20}$ is equivalent to $\frac{65}{100}$ or 0.65. Students may need more experience modeling fraction and decimal equivalency using grids, numbers lines, or money.

- Have students place the following numbers in order greatest to least: $\frac{17}{4}$, $1\frac{1}{4}$, $4.75\%, 4.748$

  Some students may place 4.75% after $\frac{17}{4}$ and 4.748. Other students may also place 4.75% after $1\frac{1}{4}$. These students may not understand the decimal number as part of a percentage. Students would benefit from a variety of experiences with modeling fractions, decimals, and percents that are less than 1% and greater than 100%. Begin student experiences with concrete modeling, then transition to pictorial models and number lines to assist students with conceptualizing the relative sizes of fractions, decimals, and percents.

- Have students place the following numbers in order least to greatest: $\frac{2}{3}$, 66%, $\frac{3}{4}$, 0.67

  Because $\frac{2}{3}$ is a repeating decimal, some students may struggle with where to place it in relation to 66% and 0.67. This may indicate these students need more experience working with fractions that are repeating decimals. Teachers may wish to have students use base 10 blocks or shade decimal models to help students visualize the relative size comparison. Pairing this visual with placement on a number line would help students understand the difference that exists between these values.

Concepts and Connections

Concepts

There are multiple representations of numbers and relationships among numbers that provide meaning and structure and allow us to make sense of the world around us.

Connections

- *Within the grade level/course*:
  - 6.NS.2 – The student will reason and use multiple strategies to represent, compare, and order integers.
- *Vertical Progression*:
  - 5.NS.1 – The student will use reasoning and justification to identify and represent equivalency between fractions (with denominators that are thirds, eighths, and factors of 100) and decimals; and compare and order sets of fractions (proper, improper, and/or mixed numbers having denominators of 12 or less) and decimals (through thousandths).
  - 7.NS.2 – The student will reason and use multiple strategies to compare and order rational numbers.

Across Content Areas [Theme – Modeling]:

- *Computer Science*:
  - Logic and Conditional Statements concepts include conditional statements and Boolean logic.
  - Geometry and Transformations concepts include manipulating objects in computer graphics; applying geometric calculations in game development and simulations; and developing user interfaces with symmetrical and proportional layouts.
  - Ratios, Proportions, and Scaling concepts include image and game object scaling and normalizing data for machine learning models.
  - Ratios and Proportional Reasoning highlights include scaling in graphics and simulations – scale images in graphic design, adjusting resolution in digital media, and resizing elements for UX design; algorithm efficiency and performance analysis –

utilized in application to define of runtime or memory usage changes as input size grows; data encoding and compression – proportional reasoning helps in lossy compression techniques.
    - Geometry highlights include game development and graphics programming – 2D and 3D coordinate systems used in positioning objects, animations, and rendering; computer vision and image processing – shape detection, edge recognition, and transformations (rotation, scaling, and translation); and geographic information systems (GIS) – coordinate systems are used to map and analyze real-world data.
- *Science:*
    - 6.2 The student will investigate and understand that the solar system is organized and the various bodies in the solar system interact. To meet this standard, students are expected to make a model of solar system through Jupiter.
    - 6.3 The student will investigate and understand that there is a relationship between the sun, Earth, and the moon. To meet this standard, students are expected to develop and use a model to explain the moon phases and eclipses.
    - 6.4 The student will investigate and understand that there are basic sources of energy, and that energy can be transformed. To meet this standard, students are expected to use a model to explain the source of energy sources.
    - 6.7 The student will investigate and understand that air has properties and that Earth's atmosphere has structure and is dynamic. To meet this standard, students are expected to use models to explain air movement and weather conditions.
    - 6.8 The student will investigate and understand that land and water have roles in watershed systems. To meet this standard, students are expected to use models to identify components of watershed systems.

Resources to Support Local Curriculum

- A list of approved textbooks and instructional materials is posted on the VDOE website.
- Middle School Mathematics Formula Sheet (PDF)

6.NS.2

The student will reason and use multiple strategies to represent, compare, and order integers.

Students will demonstrate the following Knowledge and Skills:

a) Represent integers (e.g., number lines, concrete materials, pictorial models), including models derived from contextual situations, and identify an integer represented by a point on a number line.

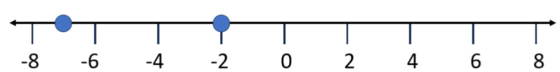
b) Compare and order integers using a number line.

c) Compare integers, using mathematical symbols (<, >, =).

d) Identify and describe the absolute value of an integer as the distance from zero on the number line.

Understanding the Standard

- The set of integers includes the set of whole numbers and their opposites {…-2, -1, 0, 1, 2, …}. Zero has no opposite and is an integer that is neither positive nor negative.
- The opposite of a positive number is negative, and the opposite of a negative number is positive.
- Positive integers are greater than zero.
- Negative integers are less than zero.
- A negative integer is always less than a positive integer.
- On a conventional number line, a smaller number is always located to the left of a larger number (e.g., -7 lies to the left of -2, thus -7 < -2).



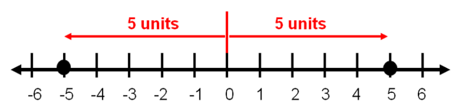
- When comparing two negative integers using a number line, the negative integer that is closer to zero is greater. On the number line below, negative four is closer to zero than negative five, so negative four is greater than negative five.

-5 < 1 or 1 > -5

-5 < -4 or -4 > -5

- Integers are used in contextual situations, such as temperature (above/below zero degrees), deposits/withdrawals in a checking account, golf (above/below par), timelines, football yardage, positive and negative electrical charges, and altitude (above/below sea level).
- Integers should be explored by modeling on number lines, both horizontal and vertical, and using manipulatives, such as two-color counters, drawings, or algebra tiles.
- The absolute value of a number is the distance of a number from zero on the number line regardless of direction. Absolute value is represented using the symbol | | (e.g., $|-6| = 6$ and $|6| = 6$). Absolute value is always positive.
- The absolute value of zero is zero.
- An integer and its opposite are the same distance from zero on a number line. Thus, they have the same absolute value. For example: The opposite of 5 is -5, and $|-5| = 5$ and $|5| = 5$.

$$|5| = 5 \quad |-5| = 5$$

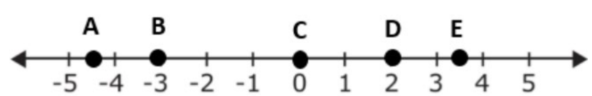


Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Connections:

- Number lines are a way to visualize integers (whole numbers and their opposites) and the distance from zero on the number line (absolute value). For example, when given the following number line, there are several misconceptions that students may experience –



One common misconception for some students is not understanding that integers are whole numbers and their opposites. Some students may select the points that represent fraction/decimal values. These students recognize that integers differ from whole numbers, but do not recognize they represent a "whole," and do not include "parts" of numbers. Other students may not select zero as an integer, thinking it is not included as part of this set. Students with these misconceptions may benefit from a visual representation in the form of a diagram of the sets of numbers. In addition, a discussion of whole numbers and what they represent as well as a discussion of the meaning of "opposite" would provide more clarity for some students. It is important for students to realize that sets of numbers in mathematics overlap, and that numbers can belong to more than one set.

- Use painter's tape to make extra-large number lines on the wall or floor or make them interactive and have students use sticky notes to label the tick marks or show different points on the line.

- Have students make a human number line. Give students pieces of paper with integers on them and they can arrange themselves in order. This reinforces comparing integers as well. Some students believe that absolute value means to take the opposite of the number instead of understanding it represents the distance away from zero. While engaging in this activity, ask students to determine the absolute value of -4 and 4. Students should state that these numbers are the same distance away from zero. Extensions of this activity include –

    - Have two students stand back-to-back at zero. What is the distance between each student? Have each student walk 3 steps in opposite directions. What is the distance between the two students now? What expression could be used to model the steps, |-3 – 3| or |3 – (-3)|? Why is the distance not zero? How could this activity relate to absolute value?

    - Select two more students and have them stand at zero and face the same direction. Have one student walk 5 steps and one student walk 3 steps both in a positive direction. What is the distance between the two students now? What expression could be used to model the steps, |5 – 3| or |3 – 5|? How could this activity relate to absolute value?

    - Select two more students and have them stand at zero and face the same direction. Have one student walk 2 steps and one student walk 7 steps, both in a negative direction. What is the distance between the two students now? What expression could be used to model the steps, |-2 – (-7)| or |-7 – (-2)|? How could this activity relate to absolute value?

- Provide students with the following scenario: Henry and Jenny were comparing two integers. Henry said, "My integer is greater than your integer." Jenny said, "That may be true, but the absolute value of my integer is greater than your integer." Locate Henry's and Jenny's integers on a number line and explain your reasoning. Ask students to then compare their answers with each other. Ask students to report out their observations as well as variations in responses.

Mathematical Reasoning: Encourage students to justify their reasoning and understanding of integers orally, in writing, or with a model. When doing so, students are to use the appropriate vocabulary such as positive, negative, integer, absolute value, order, and compare. The following questions may elicit students' understanding of concepts related to this standard –

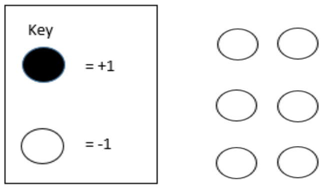
- What are some examples of integers in the real world?
- Why do we need negative numbers? Give specific examples.
- What is the importance of the zero when comparing integers?
- Explain the meaning of zero in one of the following situations: temperature, elevation, sea level, or money.
- Which number is greater, –13 or –10? Explain your reasoning.
- Can the absolute value be a negative number? Why or why not? Explain your reasoning.
- Is the opposite of a number the same as the absolute value of a number? Why or why not? Explain your reasoning.
- Is the absolute value of zero equal to zero? Why or why not? Explain your reasoning.
- Is –|3| equivalent to |–3|? Why or why not? Explain your reasoning.

Mathematical Representations: Making connections between different representations may help students deepen their understanding of integer representations. Examples with common misconceptions follow –

- Consider the following situation: The deep end of the pool is 6 feet below ground. What integer does this situation represent? Create a model to explain your response.

  A common error for some students is to create a model showing a positive integer rather than the negative integer from the situation. Students may struggle with creating a model where the outcome is a negative. These students may benefit from connecting these situations to a vertical number line that provides a visual reference for negative numbers.

- Use the key below to determine what integer is represented.



  A common misconception is thinking that counters always represent positive integers. Some students struggle with the idea that an object (counter) can represent a negative number. Students with this misconception may benefit from modeling practical situations using counters to represent positive and negative numbers. As students record their thinking, they should use a key that represents the counters used in the model. Another possible strategy is to model integers using two-color counters, with one color representing positive integers and the other color representing negative integers. Students could then connect these integer models to number line models.

Concepts and Connections

Concepts

There are multiple representations of numbers and relationships among numbers that provide meaning and structure and allow us to make sense of the world around us.

Connections

- *Within the grade level/course*:
  - 6.NS.1 – The student will reason and use multiple strategies to express equivalency, compare, and order numbers written as fractions, mixed numbers, decimals, and percents.
  - 6.CE.2 – The student will estimate, demonstrate, solve, and justify solutions to problems using operations with integers, including those in context.
- *Vertical Progression*:
  - 5.NS.1 – The student will use reasoning and justification to identify and represent equivalency between fractions (with denominators that are thirds, eighths, and factors of 100) and decimals; and compare and order sets of fractions (proper, improper, and/or mixed numbers having denominators of 12 or less) and decimals (through thousandths).
  - 7.NS.2 – The student will reason and use multiple strategies to compare and order rational numbers.

Across Content Areas

Reference 6.NS.1

Resources to Support Local Curriculum

- A list of approved textbooks and instructional materials is posted on the VDOE website.
- Middle School Mathematics Formula Sheet (PDF)

6.NS.3

The student will recognize and represent patterns with whole number exponents and perfect squares.

Students will demonstrate the following Knowledge and Skills:

a) Recognize and represent patterns with bases and exponents that are whole numbers.

b) Recognize and represent patterns of perfect squares not to exceed 20^2, by using concrete and pictorial models.

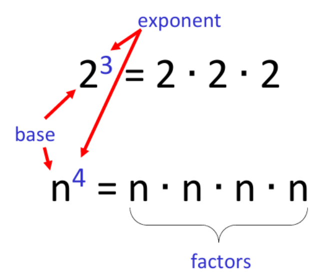
c) Justify if a number between 0 and 400 is a perfect square through modeling or mathematical reasoning.

d) Recognize and represent powers of 10 with whole number exponents by examining patterns in place value.

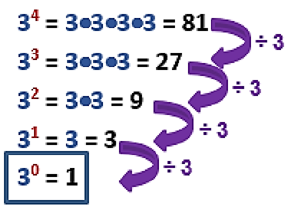
Understanding the Standard

- The symbol · can be used in Grade 6 in place of "x" to indicate multiplication.
- In exponential notation, the base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. For example, in 8^3, 8 is the base and 3 is the exponent (e.g., $8^3 = 8 \cdot 8 \cdot 8$).

Exponential Form



- Any real number other than zero raised to zero power is 1. Patterns can be used to foster this understanding for students. See the example below.

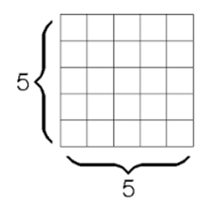


- Zero raised to zero power (0^0) is undefined according to some calculators. Other calculators will return a value of 1 when 0 is raised to the 0 power. There is debate among mathematicians surrounding this value. Students should not be expected to provide a direct value for this quantity (0^0).
- An integer that can be expressed as the square of another integer is called a perfect square (e.g., $36 = 6 \cdot 6 = 6^2$). Zero (a whole number) is a perfect square.

Perfect Squares

$0^2 = 0 \cdot 0 =$ **0**
$1^2 = 1 \cdot 1 =$ **1**
$2^2 = 2 \cdot 2 =$ **4**
$3^2 = 3 \cdot 3 =$ **9**
$4^2 = 4 \cdot 4 =$ **16**
$5^2 = 5 \cdot 5 =$ **25**
$6^2 = 6 \cdot 6 =$ **36**
$7^2 = 7 \cdot 7 =$ **49**
$8^2 = 8 \cdot 8 =$ **64**
$9^2 = 9 \cdot 9 =$ **81**
$10^2 = 10 \cdot 10 =$ **100**

- Perfect squares may be represented geometrically as the areas of squares whose side lengths are whole numbers (e.g., $1 \cdot 1$, $2 \cdot 2$, $3 \cdot 3$). This can be modeled with grid paper, tiles, geoboards, and virtual manipulatives.



- The examination of patterns in place value of the powers of 10 in Grade 6 leads to the development of scientific notation in Grade 7.

Powers of Ten

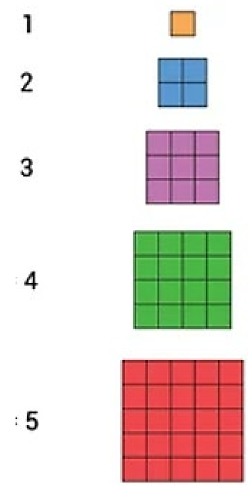
Power of Ten	Meaning	Value
10^5	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	100,000 *One hundred thousand*
10^4	$10 \cdot 10 \cdot 10 \cdot 10$	10,000 *Ten thousand*
10^3	$10 \cdot 10 \cdot 10$	1,000 *One thousand*
10^2	$10 \cdot 10$	100 *One hundred*
10^1	10	10 *Ten*
10^0	1	1 *One*

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Representations:

- Allow students to derive meaning through the construction of squares using concrete manipulatives and pictorial representations to observe the perfect squares. Perfect squares can be modeled with grid paper, tiles, geoboards, and virtual manipulatives. For example –



- Question students about their observations as they are building perfect squares, exploring the connection between perfect squares and the area model. Have students use a multiplication chart to model and depict other "square relationships".

- Facilitate a discussion with students as to why some numbers are considered perfect squares (e.g., 25, 36) and why others (e.g., 15, 18) are not. This difference can be modeled using grid paper or square tiles. Help students recognize that the areas of these numbers are determined by their factors. They should be able to describe and explain why a number is or isn't a perfect square.

- Use a place value chart to represent 10^5.

Ten Millions	Millions	Hundred Thousands	Ten thousands	Thousands	Hundreds	Tens	Ones

A common student error is writing the number in standard form by writing 10 and then using the exponent to determine the number of zeroes written after the 10. In this example, students may write 1,000,000 because they place 5 zeroes after the initial 10. This may result from a procedural focus on using the exponent to determine the number of zeroes rather than a conceptual focus on base ten understanding. Students may understand the place value of numbers but may not be able to relate it to powers of 10.

Students may benefit from using concrete materials like base ten blocks to build conceptual understanding by building 10^2 as 10 groups of 10, then building 10 groups of 10^2 (100) to show 10^3, etc. Then students can record the patterns as number sentences as they explore powers of 10, beginning with thinking of 10^2 as 10 × 10 = 100, then 10^3 as 10 × 10 × 10, etc. As students build these patterns, they begin to see the connections between powers of 10 and place value.

Mathematical Reasoning:

- Have students create squares using tiles or draw representations of perfect squares on grid paper. Questions to elicit understanding of these representations may include –
  - How can the area model be used to relate perfect squares to multiplication?
  - Using square tiles or grid paper, model how you know the numbers 6 and 12 are *not* perfect squares. Explain and justify your reasoning.
  - Using square tiles or grid paper, model how you know the numbers 16 and 49 are perfect squares. Explain and justify your reasoning.
  - Is zero to the zero power (0^0) a perfect square?
  - Why is any real number other than zero raised to the zero-power equivalent to 1?

- Ask students to determine the value of 6^5 based on the pattern:

$$6^1$$
$$6^2$$
$$6^3$$

  Some students may find it challenging to recognize and extend a pattern that includes bases and exponents. To build student understanding, emphasize that exponents represent how many times the base is used as a factor. Encourage students to write out the exponential expression in expanded form to assist with the understanding that bases are being multiplied.

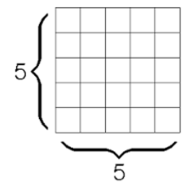
- Give students the following scenario: Derrick stated, “The number 225 is not a perfect square because it is an odd number.” Sarah stated, “You are incorrect. The number 225 is a perfect square. There can be even and odd numbers that are perfect squares.” Encourage students to explain who is correct in this scenario and justify their reasoning using a concrete representation, pictorial representation, examples, and/or counterexamples.

  - To develop students’ understanding of this concept, provide them with values (such as 0, 50, 125, 200, and 365) and ask them to determine whether the values are perfect squares. A common misconception students may make from the given values is assuming that even numbers are perfect squares because they can be divided by 2 to get a whole number. For example, the values of 50 and 200, when divided by 2, provide students with whole numbers, however they are not perfect squares.

Mathematical Connections:

- The square root of a number can be represented geometrically as the length of a side of a square. Squaring a number and finding the square root of a number are inverse operations.

$5^2 = 5 \cdot 5 = \mathbf{25}$



$\sqrt{25} = \sqrt{5 \cdot 5} = \sqrt{5^2} = 5$

It is important to note that students in Grade 6 are not expected to work with square roots. The relationship between perfect squares and square roots will be a focus in Grade 7.

- Students can begin to explore the connection between perfect squares and square roots by investigating side lengths and areas of geometric squares using arrays, grid paper, square tiles, etc. Having students complete a table like the one below as they explore building geometric squares with manipulatives can help solidify this connection for students.

Side Length	1		8	12		20
Perfect Square	1	16	64		256	

  - A possible misconception is thinking that a perfect square can be divided by 2 (or cut in half) to find the side length. For example, students might know the number 16 is a perfect square but think the side length is 8. When given the area of a square and being asked to find the side length, students may divide by 4 instead of finding the square root of the area.

- The examination of patterns in place value of the powers of 10 in Grade 6 leads to the development of scientific notation in Grade 7. For example –

  - Suppose you are given 10^2, 10^3, and 10^4. Provide the next three terms in the pattern and explain how you arrived at your answer.
  - What is the equivalent of 3^4?

A common student error is to multiply the base by the exponent. Students may have the misconception that exponents are another way to write basic multiplication facts. Students will benefit from practice with exponential notation by evaluating numbers written in exponential form and writing the repeated factors out to obtain the product. Students can also complete activities such as matching expanded form with exponential notation.

Across Content Areas

Reference 6.NS.1

Concepts and Connections

Concepts

There are multiple representations of numbers and relationships among numbers that provide meaning and structure and allow us to make sense of the world around us.

Connections

- *Within the grade level/course*:
  - There are no horizontal connections as this is the students' first formal exposure to whole number exponents and perfect squares.
- *Vertical Progression*:
  - There are no formal standards that address patterns with whole number exponents and perfect square in previous grade levels.
  - 7.NS.3 – The student will recognize and describe the relationship between square roots and perfect squares.

Resources to Support Local Curriculum

- A list of approved textbooks and instructional materials is posted on the VDOE website.
- Middle School Mathematics Formula Sheet (PDF)
- Whole-Number Exponents and Perfect Squares Exemplar Mathematics Instructional Plan (Word | PDF)

Computation and Estimation

In K-12 mathematics, computation and estimation are integral to developing mathematical proficiency. Computation refers to the process of performing numerical operations, such as addition, subtraction, multiplication, and division. It involves applying strategies and algorithms to solve arithmetic problems accurately and efficiently. Estimation is the process of obtaining a reasonable or close approximation of a result without performing an exact calculation. Estimation is particularly useful when an exact answer is not required, especially in real-world situations, and when assessing the reasonableness of an answer. Computation and estimation are used to support problem solving, critical thinking, and mathematical reasoning by providing students with a range of approaches to solve mathematics problems quickly and accurately.

In Grade 6, students learn how estimation and the operations (addition, subtraction, multiplication, and division) allow them to model, represent, and solve different types of problems with rational numbers. At this grade level, students represent and solve problems using operations with fractions, mixed numbers, and integers.

It is critical at this grade level that while students build their conceptual understanding through concrete and pictorial representations, that students also apply standard algorithms when performing operations as specified within the parameters of each standard of this strand. At this grade level, students become more efficient with problem solving strategies. Fluent use of standard algorithms not only depends on automatic retrieval (automaticity learned during the elementary grades), but also reinforces them. Automaticity further grounds students' entry points and the structures needed to solve contextual problems and execute mathematical procedures with rational numbers.

6.CE.1

The student will estimate, demonstrate, solve, and justify solutions to problems using operations with fractions and mixed numbers, including those in context.

Students will demonstrate the following Knowledge and Skills:

a) Demonstrate/model multiplication and division of fractions (proper or improper) and mixed numbers using multiple representations.*

b) Multiply and divide fractions (proper or improper) and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form.*

c) Investigate and explain the effect of multiplying or dividing a fraction, whole number, or mixed number by a number between zero and one.*

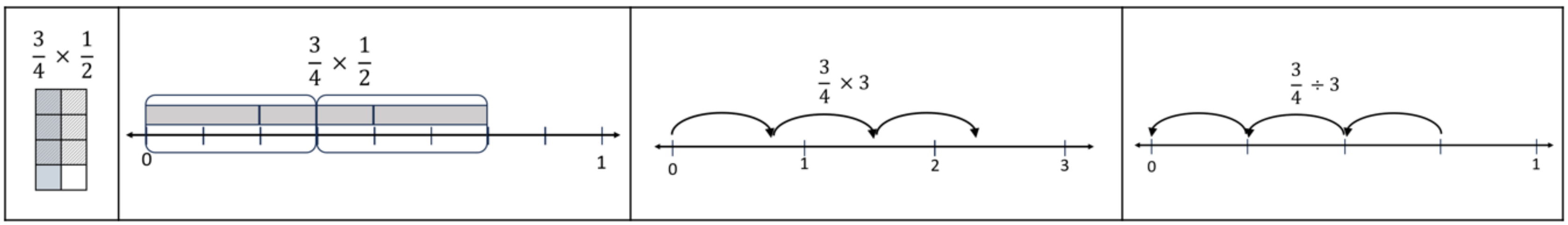
d) Estimate, determine, and justify the solution to single-step and multistep problems in context that involve addition and subtraction with fractions (proper or improper) and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less. Answers are expressed in simplest form.

e) Estimate, determine, and justify the solution to single-step and multistep problems in context that involve multiplication and division with fractions (proper or improper) and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form.

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

Understanding the Standard

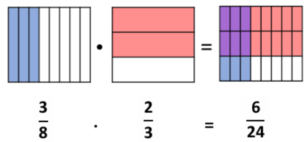
- A fraction can be expressed in simplest form (simplest equivalent fraction) by dividing the numerator and denominator by their greatest common factor.
- When the numerator and denominator have no common factors other than 1, then the fraction is in simplest form.
- Addition and subtraction are inverse operations. Multiplication and division are inverse operations.
- Models for representing multiplication and division of fractions may include arrays, paper folding, repeated addition, repeated subtraction, fraction strips, fraction rods, pattern blocks, or area models.



- It is helpful to use estimation to develop computational strategies and determine the reasonableness of a solution. For example: $2\frac{7}{8} \cdot \frac{3}{4}$ is about $\frac{3}{4}$ of 3, so the answer is between 2 and 3.
- When multiplying a whole number by a fraction such as $3 \cdot \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole.
- When multiplying a fraction by a whole number such as $\frac{1}{3} \cdot 6$, we are trying to determine a part of the whole: $\frac{1}{3}$ of six wholes.
- When multiplying a fraction by a fraction such as $\frac{1}{2} \cdot \frac{3}{4}$, the problem is asking for part of a part: one-half of $\frac{3}{4}$.

Fraction Multiplication

How much is $\frac{3}{8}$ of $\frac{2}{3}$?

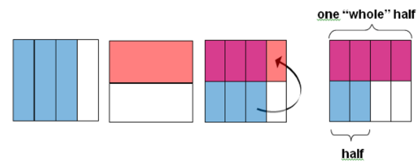


$$\frac{3}{8} \cdot \frac{2}{3} = \frac{6}{24} = \frac{1}{4}$$

Fraction Division

$$\frac{3}{4} \div \frac{1}{2}$$

How many halves are in three-fourths?



$$\frac{3}{4} \div \frac{1}{2} = 1\,\frac{1}{2}$$

- It is helpful to use benchmark fractions or decimals to explore the effect of multiplying or dividing a fraction, whole number, or mixed number by a number between zero and one. Students should understand that multiplying by a number between zero and one will decrease the value of the original number and dividing by a number between zero and one will increase the value of the original number.
- Solving multistep problems in the context of contextual situations enhances proficiency with estimation strategies.
- Students may justify their reasoning by using estimation strategies, models, benchmarks, etc.

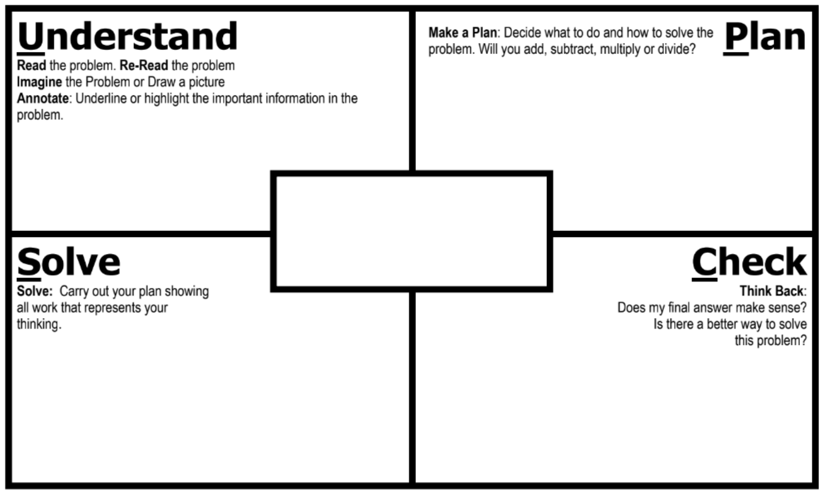
Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Problem Solving: Problem solving requires both an ability to correctly define a problem and find a solution to it. Model effective practices that demonstrate for students how to arrive at their solutions through estimation, exactness, and justification. To unpack contextual problems, guide students through the problem-solving process by taking care to annotate *essential vocabulary* (not "key words") related to applied operations.
For example –

- Understand the problem by reading and then re-reading the problem; imagining the problem or drawing a picture; underlining or highlighting the important information and essential vocabulary within the problem.

- Create a plan by deciding what to do and how to solve the problem using the applicable operation(s).
- Carry out the plan by showing all work to represent thinking (e.g., using symbols, words, concrete manipulatives, pictorial representations).
- Check the solution by determining if the solution answers the question asked in the contextual problem. During this phase of problem solving, it is essential that students are exposed to strategies that allow them to determine the reasonableness of their responses and solutions; and to support multiple mathematically sound and accurate ways of arriving at a solution.
- Graphic organizers such as the four-square model (Polya method) can help students to organize their thinking as they solve contextual problems –



- Prior to solving problems, students should be encouraged to estimate the solution. When estimating students may use a variety of strategies including rounding and compatible numbers where any or all numbers may be adjusted.
- As students solve contextual problems, ask questions such as –
  - Are there multiple ways to solve a single problem?
  - How do you know that you have provided a reasonable answer?
  - What role does estimation play in solving contextual problems?

Mathematical Communication: Recall multiple problem types learned at the elementary grades and apply to contextual problems as students advance in their understanding of more complex contextual problems and structures. Teach students a solution method for solving each problem type. Introduce a solution method using a worked-out example. Talk through the problem-solving process and connect the relevant problem information to the worked-out example. Say out loud the decisions that were made to solve the problem at each step. Then demonstrate how to apply the solution method by solving a similar problem with students using that method. Discuss each decision made and ask students guiding questions to engage them as problems are solved.

- Melissa has $\frac{4}{5}$ of a bag of dog food. Her dog eats $\frac{1}{10}$ of a bag each day. With the dog food that Melissa has, how many days will she have enough food for her dog?

  A common mistake that students make is multiplying to solve this problem. Giving students multiple examples of real-world context may help. Real conversations like, "What is happening with the bag of food? Are you adding pieces together when you feed the dog? Are you taking away food? Are you putting food in multiple times? Are you breaking the food into pieces?" Conversations about the relationship between repeated subtraction and division may benefit struggling students. Teachers may also wish to encourage students to estimate. Many times, when students think about real world context, they realize their answer is not reasonable.

Mathematical Reasoning: Solving problems in contextual situations enhances proficiency with estimation strategies. Reasoning skills are essential for solving problems effectively and strategically. Reasoning involves analyzing information, identifying assumptions, evaluating evidence, and drawing logical conclusions. When solving problems in context, students may have difficulty understanding which operation(s) to use and may use the wrong operation(s) when solving. Students may not realize that all numbers in a contextual problem are not always needed. To help students derive meaning of which operation to apply, ask students how thinking about the meaning of addition can help when solving word problems. Give students a chance to explain individually and share their thoughts with each other. Repeat this process for subtraction, multiplication, and division. *Have students explain the action of the word to move away from a reliance on "key words."* For example –

- Addition:
  - Finding the total quantity of separate quantities
  - Combining two or more quantities
- Subtraction:
  - Finding how much more or how much less
  - Finding how much further
  - Finding the difference between two quantities
  - Determining a quantity when taking one amount from another
- Multiplication:
  - Finding the quantity needed for *x* number of people or *x* number of something
  - Having equal groups and finding the total of all groups
  - Finding a part (fraction) of a whole number

- Taking a part of a part (fraction of a fraction)

- Division:
  - Dividing an item (or quantity) into equal sized pieces
  - Dividing a quantity into equal groups
  - Using an equal amount of something over time
  - Determining how many fractional groups can be made from a quantity

Mathematical Connections: Students must connect fluency of facts, concepts, procedures, and mathematical language through mathematical reasoning to effectively solve contextual problems. Fluency with rational numbers at this grade level will help with improving logical reasoning skills, which will then lend themselves to solving contextual problems.

In the following example, students must first use their previous understanding of computation with whole numbers and fractions to arrive at an exact solution. Next, students must apply their understanding of mathematical language to determine which operation(s) to use, and with this information, design a plan to solve the solution. Lastly, students must determine the reasonableness of their solution. Common misconceptions are given below –

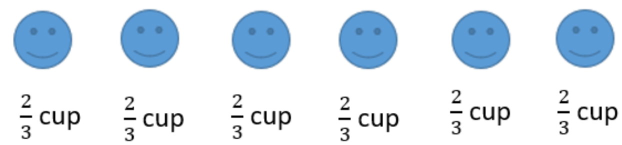
- Marcus has $3\frac{1}{6}$ feet of string and he buys a piece that is $2\frac{7}{12}$ feet long. He is making necklaces that each use $1\frac{1}{2}$ feet of string. How many complete necklaces can Marcus make with both pieces of string?

  When there are multistep word problems, students often have a hard time knowing which operations to use and when to use them. A hands-on activity may help students understand the context of the problem. Breaking students into small groups and giving them $3\frac{1}{6}$ ft. and $2\frac{7}{12}$ ft. (total of $5\frac{9}{12} = 5\frac{3}{4}$ feet) of string is a good activity to reinforce this skill. Students can combine the string and then see how much string they have all together. They can then create “necklaces” that are each $1\frac{1}{2}$ feet long.

  Cutting string repeatedly into equal parts will reinforce the need to use division or repeated subtraction. Allowing students to come up with the correct answer with the activity and then completing the computation will show them if there is a discrepancy. If students combine the string, the final quotient is $3\frac{5}{6}$ necklaces. Some students may round that to 4 necklaces.

  Students who think about the question holistically and do not think that the string would not be tied together when making necklaces may see that they can make two necklaces from the first piece of string and only one necklace from the second piece of string. Students can be reminded about the word *complete* so that they understand that the partial section of string cannot count toward a complete necklace. Relating this concept to real life often clears up misconceptions.

- Tiffany is having 5 friends over for a birthday party. Each serving of ice cream is $\frac{2}{3}$ of a cup. She would like to double that amount for each serving and have enough ice cream for her and her friends. How much ice cream will she need?



  One common mistake is that students skip over words in the question that are related to operations, such as double, triple, half, etc. It may be helpful to encourage students to draw a picture or model what is happening in the contextual situation. There are six children, and each child gets $\frac{2}{3}$ of a cup of ice cream (see picture above). This may be a good time to have rich conversations about when the doubling should happen. Ask students, *"Will you get the same answer if you double the $\frac{2}{3}$ first and then multiply by 6, or double the 6 and then multiply by $\frac{2}{3}$, or multiply the 6 by $\frac{2}{3}$, and then double the answer?"* Having students share and discuss different strategies also promotes greater understanding.

Concepts and Connections

Concepts

Estimation and the operations of addition, subtraction, multiplication, and division, allow us to model, represent, and solve different types of problems with rational numbers.

Connections

- *Within the grade level/course*:
  - 6.CE.2 – The student will estimate, demonstrate, solve, and justify solutions to problems using operations with integers, including those in context.
- *Vertical Progression*:
  - 5.CE.1 – The student will estimate, represent, solve, and justify solutions to single-step and multistep contextual problems using addition, subtraction, multiplication, and division with whole numbers.
  - 5.CE.2 – The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction of fractions with like and unlike denominators (with and without models), and solve single-step contextual problems involving multiplication of a whole number and a proper fraction, with models.
  - 5.CE.3 – The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition, subtraction, multiplication, and division with decimal numbers.

- 7.CE.1 – The student will estimate, solve, and justify solutions to multistep contextual problems involving operations with rational numbers.
- 7.CE.2 – The student will solve problems, including those in context, involving proportional relationships.

Resources to Support Local Curriculum

- A list of approved textbooks and instructional materials is posted on the VDOE website.
- Middle School Mathematics Formula Sheet (PDF)

6.CE.2

The student will estimate, demonstrate, solve, and justify solutions to problems using operations with integers, including those in context.

Students will demonstrate the following Knowledge and Skills:

a) Demonstrate/model addition, subtraction, multiplication, and division of integers using pictorial representations or concrete manipulatives.*

b) Add, subtract, multiply, and divide two integers.*

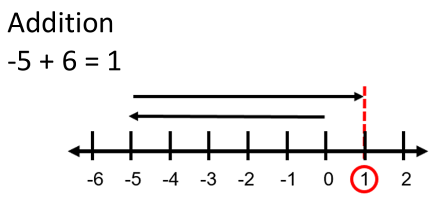
c) Simplify an expression that contains absolute value bars | | and an operation with two integers (e.g., –|5 – 8| or $\frac{|-12|}{8}$) and represent the result on a number line.

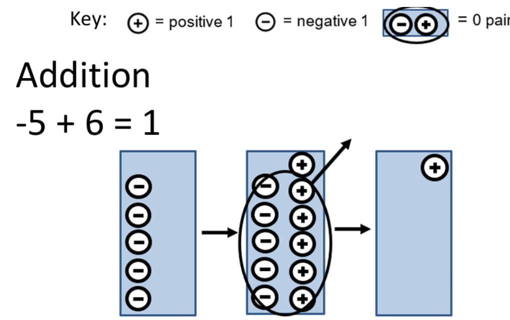
d) Estimate, determine, and justify the solution to one- and two-step contextual problems, involving addition, subtraction, multiplication, and division with integers.

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

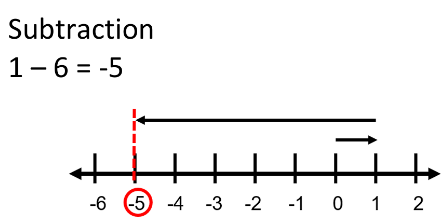
Understanding the Standard

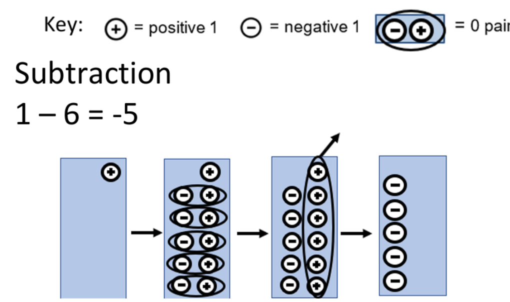
- The set of integers is the set of whole numbers and their opposites {...-2, -1, 0, 1, 2, ...}. Zero has no opposite and is neither positive nor negative.
- Integers are used in contextual situations, such as temperature (above/below zero degrees), deposits/withdrawals in a checking account, golf (above/below par), timelines, football yardage, positive and negative electrical charges, and altitude (above/below sea level).
- Concrete experiences in formulating rules for adding, subtracting, multiplying, and dividing integers should be explored by examining patterns using calculators, a number line, and manipulatives, such as two-color counters, drawings, or algebra tiles.
- Sums, differences, products, and quotients of integers are either positive, negative, undefined or zero. This may be demonstrated using patterns and models.
- When determining the sum of:
  - a positive integer and a positive integer, the sum will be positive (e.g., 7 + 3 = 10).
  - a positive integer and a negative integer, the sum may be positive or negative (e.g., 7 + -1 = 6; 3 + -5 = -2).
  - a negative integer and a negative integer, the sum will be negative (e.g., -3 + (-4) = -7).





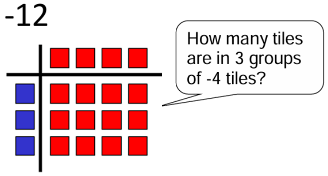
- When determining the difference of:
  - a positive integer and a positive integer, the difference may be positive or negative (e.g., 7 – 3 = 4; 2 – 5 = -3).
  - a positive integer and a negative integer, the difference will be positive (e.g., 7 – (-1) = 8).
  - a negative integer and a positive integer, the difference will be negative (e.g., -3 – 1) = -4).
  - a negative integer and a negative integer, the difference may be positive or negative (e.g., -1 – (-2) = 1; -6 – (-3) = -3).





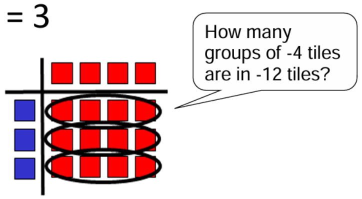
- When determining the product of:
  - a positive integer and a positive integer, the product will be positive (e.g., 7 · 2 = 14).
  - a positive integer and a negative integer, the product will be negative (e.g., 6 · (-3) = -18).
  - a negative integer and a negative integer, the product will be positive (e.g., -5 · (-4) = 20).

Multiplication
$3 \cdot (-4) = -12$



- When determining the quotient of:
  - a positive integer and a positive integer, the quotient will be positive (e.g., 14 ÷ 2 = 7).
  - a positive integer and a negative integer, the quotient will be negative (e.g., 18 ÷ (-3) = -6).
  - a negative integer and a positive integer, the quotient will be negative (e.g., -15 ÷ 3 = -5).
  - a negative integer and a negative integer, the quotient will be positive (e.g., -20 ÷ (-2) = 10).

Division
$-12 \div -4 = 3$



- Students may justify their reasoning by using estimation strategies, models, benchmarks, etc.

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Problem Solving: As students estimate, determine, and justify the solution to one- and two-step contextual problems, involving addition, subtraction, multiplication, and division with integers, it is important that multiple ways of solving problems are modeled to include graphic organizers, step-by-step processes, or modeling through concrete or pictorial representations and then make the transition to standard algorithms with each operation. Further, examining the reasonableness of the solution is a critical element to problem solving as students should go back to the contextual problem and determine whether their solution answered the question posed. As teachers provide support to students by helping them develop their problem-solving skills, consider the following –

- Ask students to discuss the following questions:
  - What do effective problem solvers do, and what do they do when they get stuck?
  - How does your strategy for solving a problem affect the solution that you get?
  - How do you most effectively communicate your mathematical ideas so that others can understand?
  - How does the nature of the problem help you to determine the most appropriate way to solve it?
  - Does creating a plan help you to become a better problem solver? Why or why not?
  - Were there any other methods that you used to arrive at your answer?
  - Is your answer reasonable? How do you know?
  - When using concrete or pictorial representations, what is the most effective model for this problem, and what can we learn from it?

- Students need practice with all integer operations in practical situations to assist with determining which operation to use. Examples with misconceptions are included –

  - The temperature outside currently is 12 degrees and is dropping at an average rate of 3 degrees per hour. If the temperature continues to drop at this rate throughout the evening, what will the temperature be in 8 hours?

    Students using "key words" instead of reading to understand the context of the problem may subtract 3 from 12 and stop there because of the word "dropping". Students may answer -24 indicating the temperature dropped 24 degrees instead of what the temperature will be in 8 hours.

  - During a 12-hour period, the temperature rose by 48°F. The temperature rose the same number of degrees each hour. How many degrees did the temperature rise each hour?

    One common error is that students perform the wrong operation and subtract getting the answer of 36°F. Extra instruction and practice with problem solving and making sense of "each hour" or "per hour" will help students understand division. Students may also write their answer as a negative quotient because they are working with integers. Vocabulary such as "rise" (rose) and fall can be demonstrated and reinforced on a number line. Teachers are encouraged to use number lines and manipulatives to help with problem solving.

Mathematical Reasoning:

- Common misconceptions when completing integer operations include –

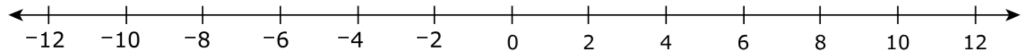
- **Adding Integers:** Students may ignore the signs and just add the integers, or they may believe that adding two negative integers results in a positive answer.
- **Subtracting Integers:** Students may ignore the signs and just subtract the integers. Students may think that subtracting two negative integers results in a negative answer. Students may think that subtraction is commutative and start with the "largest" number first instead of subtracting in the order given.
- **Multiplying Integers:** Students may ignore the signs and just multiply the integers. Students may think that multiplying two negative integers results in a negative answer. Students may think that if the signs of the factors are different and the larger factor is positive, the answer is positive. Students may think that if the signs of the factors are different and the larger factor is negative, the answer is negative.
- **Dividing Integers:** Students may ignore the signs and just divide the integers. Students may think that dividing two negative integers results in a negative answer. Students may think that if the signs different and the larger integer is positive, the answer is positive. Students may think that if the signs are different and the larger integer is negative, the answer is negative.

- Questions to elicit understanding of integer operations may include –
  - How do you add two integers if they have the same sign?
  - How do you add two integers if they have different signs?
  - How do you subtract integers?
  - How are the rules for multiplying and dividing integers different from the rules for adding and subtracting?
  - When multiplying or dividing integers, how do you know what the sign of your answer will be?
  - When you multiply three positive integers, what sign does the product have? Is this the same if you multiply three negative integers? Give an example to prove your answer.
  - Explain how to determine the sign of the product if the multiplication problem contains more than two factors.

- Students in Grade 6 are expected to simplify expressions that contain absolute value bars and one operation with two integers, as well as represent the result on a number line. This work with integer operations will prepare students for evaluating more complex numerical expressions using the order of operations in Grade 7.

  A common error students may make is thinking absolute value means the opposite of the value in the absolute value bars. This may indicate students need additional time to explore absolute value using a number line to visualize that the absolute value of a number is the distance from 0 on the number line regardless of direction; therefore, distance is positive. The following activities can help students explore and develop reasoning skills to better understand absolute value and integer operations.

- Have students plot the value of $|-4+12|$ on a number line.

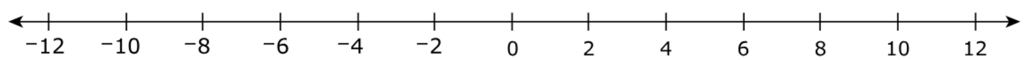


- Give students a number line with positive and negative integers. Students can place the end of a piece of string that has been cut to represent the length of a given number (such as 8) on zero. Keeping one end of the string at zero, rotate the piece of string to the other side of zero. The string has the same length and distance from zero, representing $|-8|$ and $|8|$.

- Place a large number line on the floor and hand each student a number line and two counters to use at their desk. Select two students to come to the front to model on the large number line. Everyone else will model on their number line. Have the two students stand back-to-back at zero. Discuss the distance between the two students (zero). Have the students walk 3 steps in opposite directions. Ask students:
  - What is the distance between the two students now?
  - Why is the distance not zero?
  - What expression could be used to model the steps?” $|-3-3|$ or $|3-(-3)|$

- Select two students and have them stand side by side at zero on a large number line and face the same direction. Have one student walk 5 steps and one student walk 3 steps, both in a positive direction. Discuss how the number of steps and the distance between the students correlates to absolute value. Have students write the expressions that could be used to model the steps: $|5-3|$ or $|3-5|$

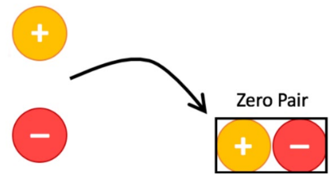
- Have students plot the value of $\frac{|-20|}{-5}$ on a number line.



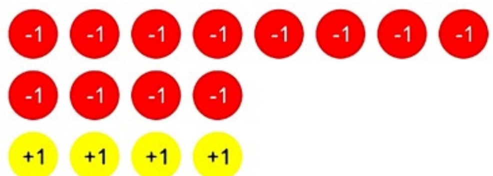
Mathematical Representations:

- Concrete experiences in formulating rules for adding, subtracting, multiplying, and dividing integers should be explored by examining patterns using calculators, number lines, and manipulatives (e.g., two-color counters, drawings, algebra tiles). Dynamic software can also be used to examine these patterns.

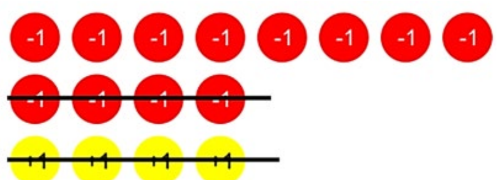
  - **When using two-color counters, students must understand the concept of zero pairs:** When modeling integers, one color can represent a positive number and another color can represent a negative number. In this instance and throughout the following examples, a yellow counter will represent a positive integer and a red counter will represent a negative integer. A zero pair is the pair of the positive and negative form of the same number.



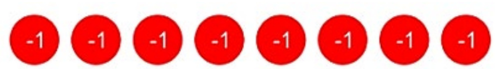
- **When modeling addition with two-color counters:** For example, add -12 + 4.
  - Model the addends using counters. If the addend is positive, use yellow counters. If the addend is negative, use red counters.



  - Equal amounts of red and yellow counters create zero pairs and cancel each other out.



  - Determine the amount and color of the remaining counters and write the corresponding integer: -8

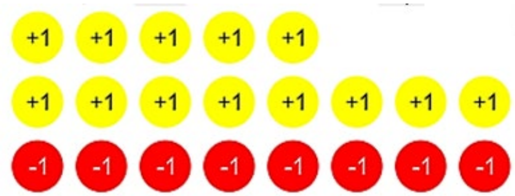


- **When modeling subtraction with two-color counters:** For example, subtract 5 – (-8).
  - If the minuend is positive, use yellow counters. If the minuend is negative, use red counters. Remove as many counters as needed, depending on the integer being subtracted. If enough colored counters are not present, create zero pairs and then subtract.

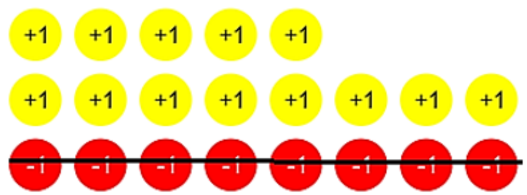
  For example:

The yellow counters represent positive 1. Use yellow counters to represent 5.

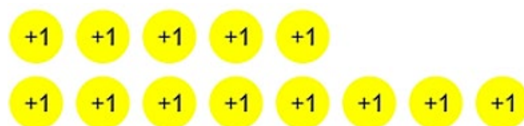
Remove 8 red counters. However, there are no red counters. Add 8 zero pairs (this is “adding the opposite”).



Remove 8 red counters.



Count the remaining counters to find the solution. Because there are thirteen positive yellow counters left, the answer is 13.



- **When modeling multiplication with two-color counters:**
  - **Multiplying two positive integers (Case 1):** Create as many groups of the second number using yellow counters as indicated by the first number.
  - **Multiplying two negative integers (Case 2)**: Create as many groups of the second number using red counters as indicated by the first number. Inverse the final answer as negative groups are the opposite of positive groups.
  - **Multiplying one positive and one negative integer (Case 3):** If the first number is positive and second is negative, create as many groups of the second number using red chips as indicated by the first number. If the first number is negative and second is positive, create as many groups of the second number using yellow chips as indicated by the first number. Inverse the final answer as negative groups are the opposite of positive groups.

- **Example (Case 2): Multiplying two negative integers:** For example, multiply -2 × -3.

Create as many groups of the second number using red chips as indicated by the first number. Create -2 groups of -3. As we cannot create negative groups, create 2 groups of -3.

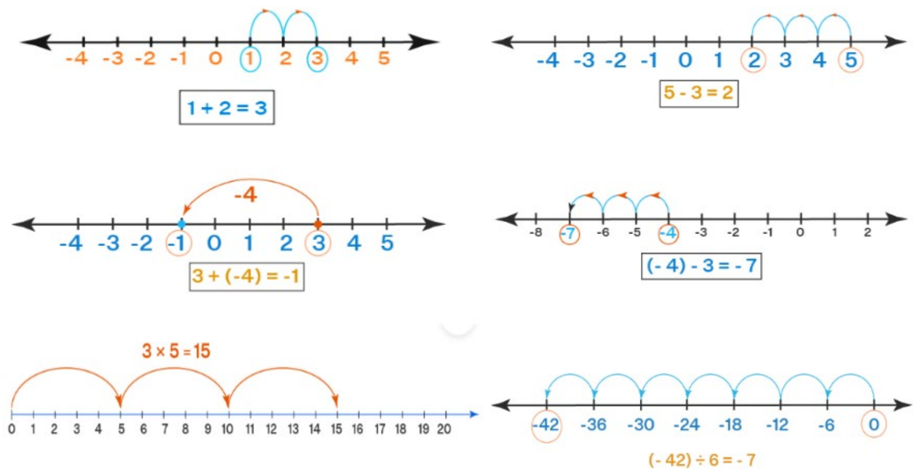
Create the inverse of the final answer (as negative groups are the opposite of positive groups). The result is 6.

- **When modeling division with two-color counters:**
  - **Dividing two positive integers (Case 1):** Use yellow integer counters to represent as many counters as indicated by the first number. Then, divide the set of counters equally into groups as specified by the second number.
  - **Dividing two negative integers (Case 2):** Use red integer counters to represent as many counters as indicated by the first number. Divide the set of counters equally into groups as specified by the second number. Inverse the groups created as negative groups are the opposite of positive groups.
  - **Dividing one positive and one negative integer (Case 3):** *If the dividend is positive and the divisor is negative:* Use yellow integer counters to represent as many counters as indicated by the first number. Divide the set of counters equally into groups specified by the second number. Inverse the groups created as negative groups are the opposite of positive groups. *If the dividend is positive and the divisor is positive:* Use red integer counters to represent as many counters as indicated by the first number. Divide the set of counters equally into groups specified by the second number.
    - For example, divide: -10 ÷ 2

Use red integer counters to represent as many counters as indicated by the first number.

Divide the set of counters equally into groups specified by the second number. The result is -5.

- **When completing operations on a number line:** Key points to remember when completing operations on a number line – move to the right side, if the number is positive; and move to the left side, if the number is negative. Some examples of integer operations using number lines are pictured below.



- ○ Ask students to create four multiple integer operations problems on an index card. One of the problems must use a concrete representation, the second a pictorial representation, the third a number line, and lastly, an algorithm. Allow the students to engage in a "give one, get one" exchange to solve one another's problems. Then, allow the students to present at least one of their problems to the class for whole group discussion.

- ○ Use a number line to determine the sum or difference of two integers. Create a large number line on the floor with tape. Students can walk the number line to solve the problem. Ask students how their class number line would differ when multiplying and dividing integers and how they could physically model these two operations using their class number line.

Reference 6.CE.1 Skills in Practice for guidance related to* ***Mathematical Problem Solving, Mathematical Communication, Mathematical Reasoning**, and* ***Mathematical Connections*** *when solving contextual problems.*

Concepts and Connections

CONCEPTS

Estimation and the operations of addition, subtraction, multiplication, and division, allow us to model, represent, and solve different types of problems with rational numbers.

CONNECTIONS

- *Within the grade level/course*:
  - ○ 6.NS.2 – The student will reason and use multiple strategies to represent, compare, and order integers.

   - 6.CE.1 – The student will estimate, demonstrate, solve, and justify solutions to problems using operations with fractions and mixed numbers, including those in context.
- *Vertical Progression*:
   - 5.CE.1 – The student will estimate, represent, solve, and justify solutions to single-step and multistep contextual problems using addition, subtraction, multiplication, and division with whole numbers.
   - 5.CE.2 – The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction of fractions with like and unlike denominators (with and without models), and solve single-step contextual problems involving multiplication of a whole number and a proper fraction, with models.
   - 5.CE.3 – The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition, subtraction, multiplication, and division with decimal numbers.
   - 7.CE.1 – The student will estimate, solve, and justify solutions to multistep contextual problems involving operations with rational numbers.
   - 7.CE.2 – The student will solve problems, including those in context, involving proportional relationships.

Resources to Support Local Curriculum

- A list of approved textbooks and instructional materials is posted on the VDOE website.
- Middle School Mathematics Formula Sheet (PDF)

Measurement and Geometry

In K-12 mathematics, measurement and geometry allow students to gain understanding of the attributes of shapes as well as develop an understanding of standard units of measurement. Measurement and geometry are imperative as students develop a spatial understanding of the world around them. The standards in the measurement and geometry strand aim to develop students' special reasoning, visualization, and ability to apply geometric and measurement concepts in real-world situations.

In Grade 6, students analyze and describe geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes. At this grade level, students solve problems involving area and circumference of circles and solve problems involving the area and perimeter of triangles and parallelograms. In addition, students describe characteristics of the coordinate plane, graph ordered pairs, and determine congruence of segments, angles, and polygons.

6.MG.1

The student will identify the characteristics of circles and solve problems, including those in context, involving circumference and area.

Students will demonstrate the following Knowledge and Skills:

a) Identify and describe chord, diameter, radius, circumference, and area of a circle.

b) Investigate and describe the relationship between:
   i) diameter and radius;
   ii) radius and circumference; and
   iii) diameter and circumference.

c) Develop an approximation for pi (3.14) by gathering data and comparing the circumference to the diameter of various circles, using concrete manipulatives or technological models.

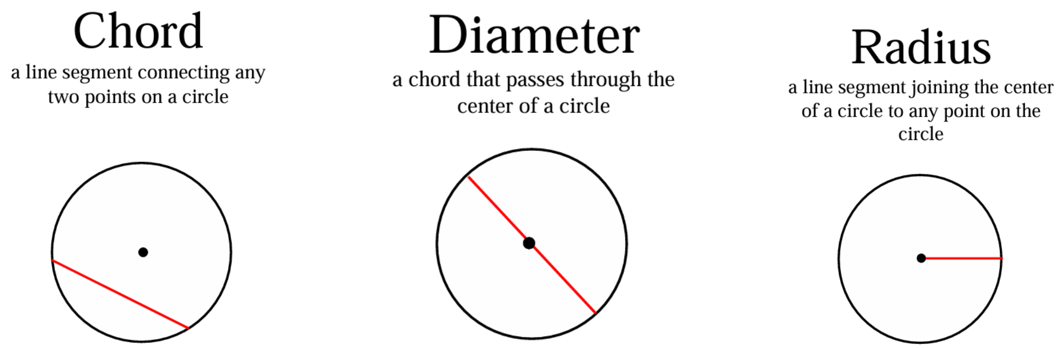
d) Develop the formula for circumference using the relationship between diameter, radius, and pi.

e) Solve problems, including those in context, involving circumference and area of a circle when given the length of the diameter or radius.

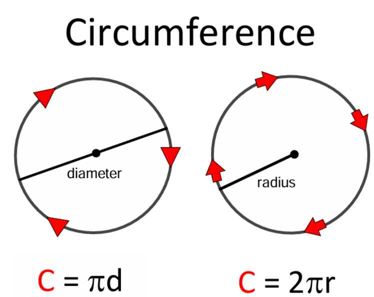
Understanding the Standard

- A chord is a line segment connecting any two points on a circle. A chord may or may not go through the center of a circle. The diameter is the longest chord of a circle.

- A diameter is a chord that goes through the center of a circle. The length of the diameter of a circle is twice the length of the radius.
- A radius is a line segment connecting the center of a circle to any point on the circle. Two radii end to end form a diameter of a circle.

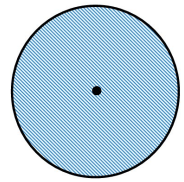


- Perimeter is the path or distance around any plane figure. The perimeter of a circle is called the circumference.
- The circumference of a circle is about three times the measure of its diameter.
- The value of pi (π) is the ratio of the circumference of a circle to its diameter. Thus, the circumference of a circle is proportional to its diameter.
- The circumference of a circle is computed using $C = \pi d$ or $C = 2\pi r$, where d is the diameter and r is the radius of the circle.



- The area of a closed curve is the number of nonoverlapping square units required to fill the region enclosed by the curve.
- The area of a circle is computed using the formula A = πr2, where r is the radius of the circle.

Area of a Circle



$A = \pi r^2$

- When determining area and circumference of a circle, the calculation may vary depending upon the approximation for pi that is used. Common approximations for π include 3.14, $\frac{22}{7}$, or the pi (π) button on a calculator.

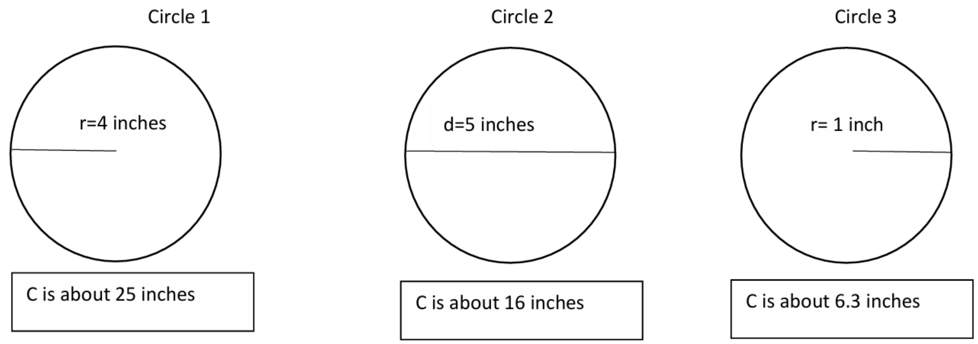
Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Problem Solving: Students may not be able to explain that for any size circle, the ratio of its circumference to its diameter is always pi, or just a little bit more than 3. Encourage them to notice patterns across all three ratios. Additionally, having them complete hands-on activities where they measure the diameter and circumference of circular objects such as lids and cans might make the abstract concept of pi more concrete.

The *Middle School Mathematics Formula Sheet* should be used in tandem with this standard to help students use the appropriate formula when solving stand-alone problems or problems in context.

- Have students examine the circles below. Each one lists the circumference and radius or diameter. How could students use this information to obtain an approximation of pi?



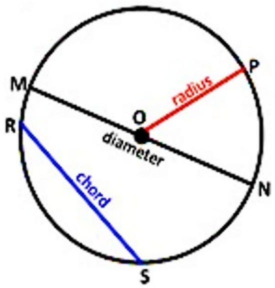
- Consider the following situation –
  - Billy looked at the tires on his bike. He noticed that the radius of each tire was 6 inches, and the circumference of each tire was about 38 inches. Use this information to find an approximate value for pi.

  Students may not remember to find the diameter before creating ratios to find an approximation for pi. Additionally, students may have a difficult time grasping the concept of pi and that it can be calculated by dividing a circle's circumference by its diameter. Some students may need a visual to help understand the problem. Encourage students to draw a picture labeling the circumference and diameter. Hands-on activities where the students actively measure the circumference and diameter of circular objects such as lids or cans will help show them that you do not always get an exact answer, but rather an approximation.

- Questions to further elicit students' understanding of this standard are –
  - What is the term for the distance around a circle?
  - How do you determine the circumference and area of a circle?
  - What is the relationship between the diameter and the radius of a circle?
  - What is the relationship between the circumference of a circle and its diameter or radius?
  - How can the approximation for pi (π) be derived?

Mathematical Connections: Students must identify and describe the concepts of diameter, radius, chord, and circumference of a circle. Through this standard, students learn that circumference is the distance around, or "perimeter" of, a circle; an approximation for circumference is about three times the diameter of a circle; and an approximation for circumference is about six times the radius of a circle. Students learn to investigate and describe the relationship between (a) diameter and radius; (b) diameter and chord; (c) radius and circumference; and (d) diameter and circumference. Through these understandings, students must derive pi (π), and connect and apply their understanding of circumference and area of a circle by using the following formulas –

- $C = 2\pi r$ or $C = \pi d$, where *C* is the circumference, *d* is the diameter, and *r* is the radius of the circle.
- $A = \pi r^2$, where *A* is the area and *r* is the radius of the circle.



- To help students develop connections between circumference, area, radius, diameter, and pi (π) –
  - Discuss the definitions of *circumference, chord, diameter,* and *radius*. Tell students that they will now investigate how the circumference of a circle compares to the circle's diameter (and radius).
  - Give each student cut-out circles of various sizes, a ruler, and a 3-foot length of yarn. Direct students to use the yarn to measure the distance around each circle, cutting the exact length of yarn needed for each circle. Then, have students use the ruler to measure the length of each piece of yarn. Emphasize that this is the circumference of each circle.
  - Have students fold each circle at some point, but *not* in half. Share with students that the line created is a chord, because the two endpoints both lie on the circle. Next, have students fold each circle in half, crease it, unfold it, and draw a line along the crease. Direct students to use their rulers to measure the length of this line across the center of each circle. Emphasize that this is the diameter of each circle and that a diameter is also a chord.
  - Have students divide the diameter of each circle in half. Emphasize that this is the radius of each circle.
  - Have students divide Length of Yarn by Length of Line for each circle. Advise students that they will determine if there is a relationship between the length of yarn (circumference) and the length of line (diameter)—the ratio of the circumference of a circle to its diameter. Ask them what they observe about the circumference divided by the diameter of each circle. They should notice that each ratio is the whole number 3 followed by different numbers in the decimal places. Point out that they have discovered that the circumference of a circle is a little more than three times larger than the diameter of the same circle.
  - Display the formula for circumference, $C = 2\pi r$, and explain each aspect of it as follows:
    - *C* = circumference (length of yarn)

        - π or pi = the ratio of the circumference of a circle to its diameter (ratio of length of yarn to length of line, or length of yarn divided by length of line)
        - $2r$ = radius multiplied by 2, which is the diameter (length of line)
    - Have students use the Desmos calculator to find the exact circumference of each circle by substituting the known values into the formula and performing the indicated operation. Discuss with students how their results could differ slightly when using either the π button located on the calculator to arrive at their solutions or using the approximation for pi (3.14).
    - Next, have students use unit squares to fill in each circle without going beyond the edges. This will enable them to estimate the area of each circle. Considering that a square does not accommodate rounded edges, point out to students that they will have to estimate the amount of some of the squares being used. Share with students that the area of a closed curve is the number of non-overlapping square units required to fill the regions enclosed by the curve. Overlay graph paper over the circles, use small linking cubes, or dynamic geometry software to simulate square units.
    - After students have completed their estimates of the area of each circle, introduce the formula for the area of a circle, $A = \pi r^2$, where r is the radius of the circle. Have students use the Desmos calculator to find the exact area of each circle by substituting the known values into the formula and performing the indicated operation. Discuss with students how their results could differ slightly when using either the π button located on the calculator to arrive at their solutions or using the approximation for pi (3.14).

Mathematical Representations: Because students are expected to understand the components of a circle as well as connect the relationships between them, providing students with real-world applications to represent these relationships is key to grounding their understanding. For example –

- Given the proportional relationship between circumference and diameter, explain why $\frac{C}{d} = \pi$. Would the results of this proportional relationship be the same as $\frac{C}{2r} = \pi$? Explain why or why not.
- Have students to bring in circular objects and measure the distance around (circumference) and across (diameter) each object. Engage in a discussion regarding which object has the largest circumference and area (or smallest circumference and area). Allow students to work in small groups to derive pi (π), and to compare the circumference to the diameter. Allow students to exchange their items to verify each other's work.
- Ask students to create lengths of radii such that the area will be larger than the circumference or area will be smaller than the circumference.

Concepts and Connections

Concepts

Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.

Connections

- *Within the grade level/course*:
  - 6.MG.2 – The student will reason mathematically to solve problems, including those in context, that involve the area and perimeter of triangles and parallelograms.
- *Vertical Progression*:
  - 5.MG.2 – The student will use multiple representations to solve problems, including those in context, involving perimeter, area, and volume.
  - 7.MG.1 – The student will investigate and determine the volume formula for right cylinders and the surface area formulas for rectangular prisms and right cylinders and apply the formulas in context.
- *Digital Learning Integration*
  - 6-8 CT.A. Students create, identify, explore, and solve problems using technology-assisted methods such as data analysis, modeling, or algorithmic thinking.
  - 6-8 CT.B. Students find or organize data and use appropriate technologies to interpret, analyze, and represent data to construct models, predict outcomes, solve problems, and make evidence-based decisions.

Across Content Areas [Theme – Graphing]

- *Computer Science:*
  - Coordinate Plane and Graphing concepts include placing and moving objects in a 2D or 3D coordinate system; and graphing algorithms.
- *Science:*
  - 6.2 The student will investigate and understand that the solar system is organized and the various bodies in the solar system interact. To meet this standard, students are expected to make a model of solar system through Jupiter.
  - 6.3 The student will investigate and understand that there is a relationship between the sun, Earth, and the moon. To meet this standard, students are expected to develop and use a model to explain the moon phases and eclipses.
  - 6.4 The student will investigate and understand that there are basic sources of energy, and that energy can be transformed. To meet this standard, students are expected to use a model to explain the source of energy sources.

- 6.7 The student will investigate and understand that air has properties and that Earth’s atmosphere has structure and is dynamic. To meet this standard, students are expected to use models to explain air movement and weather conditions.
- 6.8 The student will investigate and understand that land and water have roles in watershed systems. To meet this standard, students are expected to use models to identify components of watershed systems.

Resources to Support Local Curriculum

- A list of approved textbooks and instructional materials is posted on the VDOE website.
- Middle School Mathematics Formula Sheet (PDF)
- Circles Exemplar Mathematics Instructional Plan (Word | PDF)
- Going the Distance Exemplar Mathematics Instructional Plan (Word | PDF)
- Human Circles Classroom Activity (Word)

6.MG.2

The student will reason mathematically to solve problems, including those in context, that involve the area and perimeter of triangles and parallelograms.

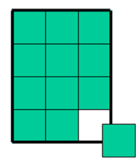
Students will demonstrate the following Knowledge and Skills:

a) Develop the formula for determining the area of parallelograms and triangles using pictorial representations and concrete manipulatives (e.g., two-dimensional diagrams, grid paper).

b) Solve problems, including those in context, involving the perimeter and area of triangles and parallelograms.

Understanding the Standard

- Experiences in developing the formulas for area and perimeter using manipulatives such as tiles, one-inch cubes, graph paper, geoboards, or tracing paper, promote an understanding of the formulas and their use.
- Area is the surface included within a plane figure. Area is measured by the number of square units needed to cover a surface or plane figure (e.g., the area of the book cover is 90 square inches or 90 in.2).



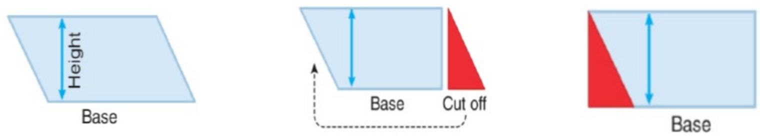
$l \times w$

$4 \times 3 = 12$

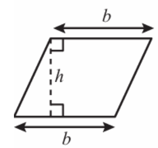
Area = 12 square units

- The formula for the area of a rectangle is one of the first that is developed and is usually given as A = l x w, or "area equals length times width". Thinking ahead, an equivalent but more unifying idea is A = b x h, or "area equals base times height". The base-times-height formulation can be generalized to all parallelograms (not just rectangles) and is useful in developing the area formulas for triangles and trapezoids.
- Any side of a figure can be called a base. For each base that a figure has, there is a corresponding height. The height is the perpendicular distance to the base. The formula A = b x h generates the same area through the commutative property regardless of which side is considered the base.
- Furthermore, the same approach can be extended to three dimensions. For example, the volume of a rectangular prism can be found by multiplying length x width x height (as in the formula V = l x w x h), or by multiplying the area of the base (B) times the height (h) (as in the formula V = Bh). Volumes of cylinders – explored in Grade 7 – are given in terms of the area of the base (B) times the height (h), or V = Bh. Therefore, base times height connects a large family of formulas that otherwise must be mastered independently.

- If a parallelogram is subdivided into two congruent right triangles and one rectangle, one of the right triangles from the original parallelogram can be repositioned to form a rectangle. The original parallelogram is now in the shape of a rectangle.

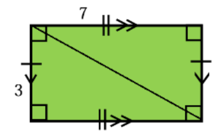


- By decomposing and recomposing the shape, a parallelogram can always be transformed into a rectangle with the same base, the same height, and the same area. Therefore, the formula for the area of a parallelogram is exactly the same as the area for a rectangle: A = bh.

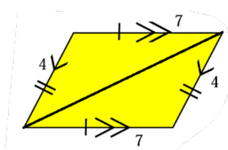


$$A = bh$$

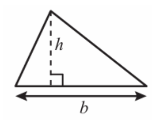
- Any rectangle can be subdivided into two congruent right triangles by drawing a diagonal. Since the two resulting triangles are congruent, each triangle is exactly half of the area of the original rectangle.



- Two copies of any triangle will always form a parallelogram with the same base and height. Therefore, the triangle has an area of half of the parallelogram.

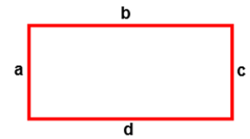


- Hence, the area of a triangle can be determined using the formula $A = \frac{1}{2}bh$.

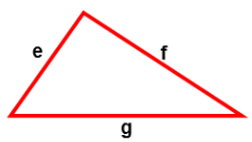


$$A = \frac{1}{2}bh$$

- Perimeter is the path or distance around any plane figure. Perimeter is a measure of length (e.g., the perimeter of the book cover is 38 inches). The perimeter of a polygon can be determined by computing the sum of the side lengths.



P = a + b + c + d



P = e + f + g

Skills in Practice

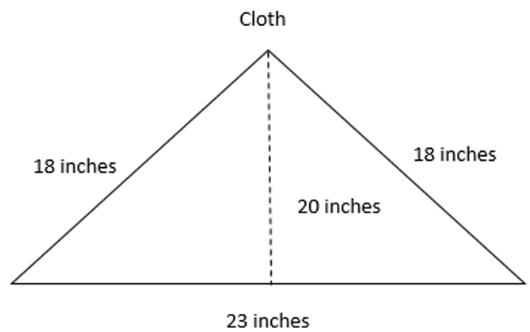
While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Problem Solving: While students might be adept at finding the perimeter and area of squares and rectangles, they may not be as successful with triangles or other parallelograms. Understanding which measurements to use as well as understanding how to use the correct formula can be challenging.

- The *Middle School Mathematics Formula Sheet* should be used in tandem with this standard to help students use the appropriate formula when solving stand-alone problems or problems in context.

- Students may have to infer from the situation whether they need to find area or perimeter. Students need plenty of opportunities to solve problems involving contextual situations that require the application of perimeter or area. Students should be encouraged to use a consistent problem-solving strategy to ensure their answer is reasonable and that they are answering the question being asked. Several examples and misconceptions are included below:

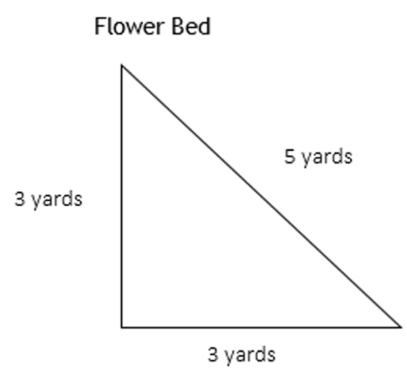
    - Matt has a triangular shaped piece of cloth to use for small flags. He needs 80 square inches of cloth for each small flag. How many flags can he make with his triangular shaped piece of cloth?



Students may have difficulty understanding why they should multiply the product of the base and height by one-half. Use grid paper to model that multiplying by one-half is the same as dividing by 2. Students might also need to review multiplying fractions using a model.

When determining the number of small flags that can be made with the given material, some students may think the answer is 2.875 or $2\frac{7}{8}$. Review similar division situations that require interpreting the remainder with students to help them realize in the situation they will need to focus on the whole number in the quotient to determine the number of small flags that can be made.

- Laura is building a flower bed in the shape of a triangle. She drew the plan below.



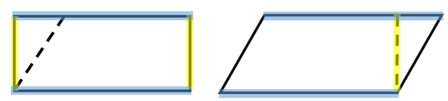
  - She needs to build a border around the flower bed. How long is the border around the flower bed?
  - She needs to buy enough mulch to cover the area of the flower bed. How many square feet will need to be covered by mulch?

Students should have plenty of experiences working with a variety of triangles. To help solidify the understanding that the area of a triangle is half of the area of a rectangle or square, have students draw rectangles and squares on grid paper and physically cut or fold them in half. This provides a concrete way to model the formula for the area of a triangle.

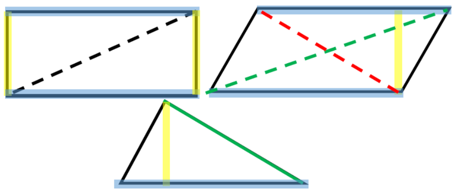
- Questions to further elicit students' understanding of this standard are –
    - How do you determine the perimeter and area of a triangle?
    - How do you determine the perimeter and area of a parallelogram?
    - Is there a difference between *A* = *lw* or *A* = *bh*?

Mathematical Connections: Students must develop the formula for determining the area of parallelograms and triangles using pictorial representations and concrete manipulatives. For example, start with a rectangle to derive the formulas for parallelograms and triangles –

- Give students three 3 x 5 index cards, scissors, tape, and two markers of different colors. Students will highlight the lengths and widths of each index card.
    - Rectangle (Index Card 1): Students will determine that the area of any rectangle is *A* = *lw* or *A* = *bh*, where *l* is the length, *w* is the width; or *b* is the base and *h* is the height.
    - Parallelogram (Index Card 2): Students will determine that the area of any parallelogram is *A* = *bh*, where *b* is the base and *h* is the height by manipulating the rectangle into a parallelogram. To do this, draw a straight line from any point on the top edge, to a bottom vertex. Cut along this line (students should cut off a triangle). Have the students slide the triangle to the opposite side and then tape it in place.



    - Triangle (Index Card 3): Students will determine that the area of any triangle is determined by computing one-half of the product of the base and the height using $A = \frac{1}{2}bh$, where *b* is the base and *h* is the height. Have students take out the third index card. Ask them what the area is of it and then have them use a marker or highlighter to mark the lengths (both) and the widths (both). Have them color along the edges of the card. Using their rulers, have the students draw in a diagonal of the figure (see pictures below). If students use the rectangle, then they will create two right triangles. If they use the parallelogram, then they will create either acute or obtuse triangles depending on whether they cut on the green diagonal, or the red one. *They should not cut along both diagonal*s.



- Questions to further elicit students' understanding and developing connections are –
  - How are the areas of a triangle and rectangle related? Explain and provide your reasoning.
  - How are the areas of rectangles and parallelograms related?
  - What is the relationship between the area of a triangle and the area of a rectangle?
  - What is the relationship between the area of a rectangle and a parallelogram?

Concepts and Connections

Concepts

Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.

Connections

- *Within the grade level/course*:
  - 6.MG.1 – The student will identify the characteristics of circles and solve problems, including those in context, involving circumference and area.
- *Vertical Progression*:
  - 5.MG.2 – The student will use multiple representations to solve problems, including those in context, involving perimeter, area, and volume.
  - 7.MG.1 – The student will investigate and determine the volume formula for right cylinders and the surface area formulas for rectangular prisms and right cylinders and apply the formulas in context.
  - 7.MG.3 – The student will compare and contrast quadrilaterals based on their properties and determine unknown side lengths and angle measures of quadrilaterals.
- *Digital Learning Integration*
  - 6-8 CT.A. Students create, identify, explore, and solve problems using technology-assisted methods such as data analysis, modeling, or algorithmic thinking.
  - 6-8 CT.B. Students find or organize data and use appropriate technologies to interpret, analyze, and represent data to construct models, predict outcomes, solve problems, and make evidence-based decisions.

Across Content Areas

Reference 6.MG.1

Resources to Support Local Curriculum

- A list of approved textbooks and instructional materials is posted on the VDOE website.
- Middle School Mathematics Formula Sheet (PDF)
- Contextual Problems Involving Area and Perimeter Exemplar Mathematics Instructional Plan (Word | PDF)

6.MG.3

The student will describe the characteristics of the coordinate plane and graph ordered pairs.

Students will demonstrate the following Knowledge and Skills:

a) Identify and label the axes, origin, and quadrants of a coordinate plane.

b) Identify and describe the location (quadrant or the axis) of a point given as an ordered pair. Ordered pairs will be limited to coordinates expressed as integers.

c) Graph ordered pairs in the four quadrants and on the axes of a coordinate plane. Ordered pairs will be limited to coordinates expressed as integers.

d) Identify ordered pairs represented by points in the four quadrants and on the axes of the coordinate plane. Ordered pairs will be limited to coordinates expressed as integers.

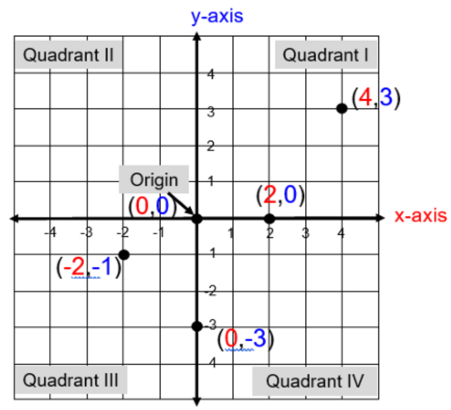
e) Relate the coordinates of a point to the distance from each axis and relate the coordinates of a single point to another point on the same horizontal or vertical line. Ordered pairs will be limited to coordinates expressed as integers.

f) Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to determine the length of a side joining points with the same first coordinate or the same second coordinate. Ordered pairs will be limited to coordinates expressed as integers. Apply these techniques in the context of solving contextual and mathematical problems.

Understanding the Standard

- In a coordinate plane, the coordinates of a point are typically represented by the ordered pair (x, y), where x is the first coordinate and y is the second coordinate.
- Any given point is defined by only one ordered pair in the coordinate plane.
- The grid lines on a coordinate plane are perpendicular.
- The axes of the coordinate plane are the two intersecting perpendicular lines that divide the coordinate plane into four quadrants. The x-axis is the horizontal axis, and the y-axis is the vertical axis.
- The quadrants of a coordinate plane are the four regions created by the two intersecting perpendicular lines (x- and y-axes). Quadrants are named in counterclockwise order. The signs on the ordered pairs for quadrant I are (+,+); for quadrant II (–,+); for quadrant III (–, –); and for quadrant IV (+,–).
- In a coordinate plane, the origin is the point at the intersection of the x-axis and y-axis; the coordinates of this point are (0, 0).
- For all points on the x-axis, the y-coordinate is 0. For all points on the y-axis, the x-coordinate is 0.

Coordinate Plane



ordered pair (x,y)

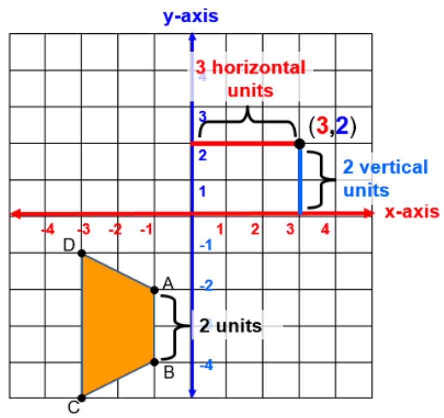
- The coordinates may be used to name a point (e.g., the point (2, 7)). It is not necessary to say, "The point whose coordinates are (2, 7)." The first coordinate tells the location or distance of the point to the left or right of the *y*-axis and the second coordinate tells the location or distance of the point above or below the *x*-axis. For example, (2, 7) is two units to the right of the *y*-axis and seven units above the *x*-axis.
- Coordinates of points having the same *x*-coordinate are located on the same vertical line. For example, (2, 4) and (2, -3) are both two units to the right of the *y*-axis and are vertically seven units from each other.
- Coordinates of points having the same *y*-coordinate are located on the same horizontal line. For example, (-4, -2) and (2, -2) are both two units below the *x*-axis and are horizontally six units from each other.

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

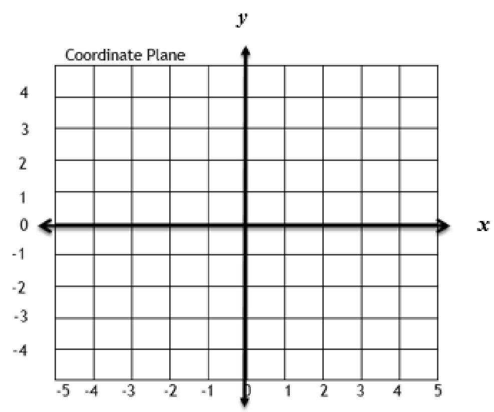
Mathematical Reasoning: As students draw polygons in the coordinate plane given coordinates for the vertices and use coordinates to determine the length of a side joining points with the same first coordinate or the same second coordinate, it is important to consider misconceptions students may have in determining the distance between coordinates. When looking at the length of a line segment between given points, students may add the *x*- or *y*-coordinates to determine the distance rather than considering the absolute value of the distance of a point from the *x*- or *y*-axis and each other. For example –

- What is the length of side AB in the figure ABCD?



  - A(-1,-2) and B(-1,-4)
  - The length of AB is $|-2-(-4)|$ or $|-4-(-2)|$
  - The length of side AB is 2 units

- Parker needs to draw a model of his backyard on the coordinate plane. Use the ordered pairs to draw the model of his backyard: {A(-2, 3), B(4, 0), C(4, -3), and D(-2, 0)}. What are the lengths of $\overline{AD}$ and $\overline{DB}$?



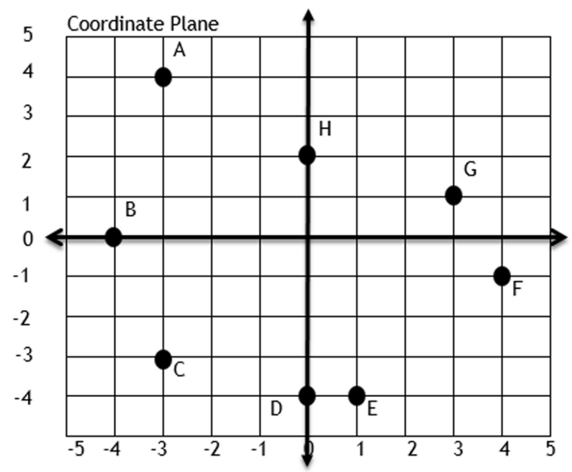
Teachers may wish to encourage students to trace the length from point A to point D with a colored pencil and count the units as they are tracing. Students should associate this length with how many units each point is away from the axis.

Mathematical Representations: Students must practice identifying and labeling the axes, origin, and quadrants of a coordinate plane. One common error in labeling the quadrants is that students may go clockwise around the coordinate plane to label the quadrants as I, II, III, and IV rather than counterclockwise. To help students, have them create various models of the coordinate plane with labels to make the vocabulary more meaningful. For example –

- Label the classroom as a giant coordinate plane by using tape to mark the axes and vocabulary cards to note the key terms. Label the four corners of your room as the four quadrants and have students play four corners to review which quadrant is which and later which quadrant a given ordered pair belongs to.

- Students often confuse the *x*- and *y*-axes and the terms horizontal and vertical. Making a large model of the coordinate plane in the classroom and labeling the axes as both *x*- and *y*-, as well as horizontal and vertical, can provide a powerful visual to help students distinguish between and identify the axes correctly. After working with a larger model, having students create their own models with geoboards and/or grid paper with labels will provide a concrete example that students may refer to as needed. Additionally, referring to objects in the class and asking students to describe features of the objects as horizontal or vertical will provide real world practice (e.g., Is the bottom edge of a bulletin board running horizontally or vertically?).

- Ask students to name the quadrant or axis on which each point is positioned:



Some students may incorrectly label points on the x or y axis in one of the four quadrants. It may be beneficial to have a class discussion to discuss why these points would not be located in a quadrant. Relating to the idea that quadrant coordinates are always positive and/or negative, while points on the axis include a coordinate that is zero (which is neither positive nor negative) would be beneficial.

- As students work through this standard, the following questions may be helpful in addressing misconceptions –
  - How can you determine the quadrant in which an ordered pair should be placed without plotting the point?
  - What is the same about the four quadrants? How are the four quadrants different from each other?
  - How do you graph a particular point in a coordinate plane?
  - How do you identify the ordered pair of a particular point in a coordinate plane? How do you know you are correct?
  - Where is the origin located on a coordinate plane?
  - How can you find the distance between points on the same horizontal or vertical line?
  - Can any given point be represented by more than one ordered pair?
  - In naming a point in the coordinate plane, does the order of the two coordinates matter?
  - How can you determine whether a polygon on a coordinate plane is a regular polygon using the ordered pairs?

Concepts and Connections

Concepts

Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.

Connections

- *Within the grade level/course*:
  - 6.CE.2b – Add, subtract, multiply, and divide two integers.
- *Vertical Progression*:
  - There are no formal standards that address describing the characteristics of the coordinate plane and graphing ordered pairs in previous grade levels.
  - 7.MG.4 – The student will apply dilations of polygons in the coordinate plane.

Across Content Areas

Reference 6.MG.1

Resources to Support Local Curriculum

- A list of approved textbooks and instructional materials is posted on the VDOE website.
- Middle School Mathematics Formula Sheet (PDF)

6.MG.4

The student will determine congruence of segments, angles, and polygons.

Students will demonstrate the following Knowledge and Skills:

a) Identify regular polygons.

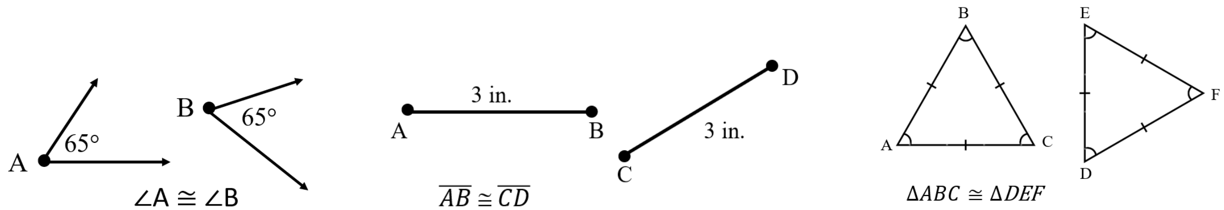
b) Draw lines of symmetry to divide regular polygons into two congruent parts.

c) Determine the congruence of segments, angles, and polygons given their properties.

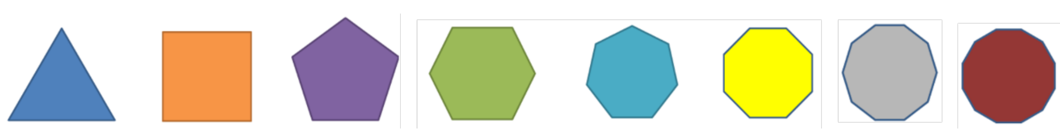
d) Determine whether polygons are congruent or noncongruent according to the measures of their sides and angles.

Understanding the Standard

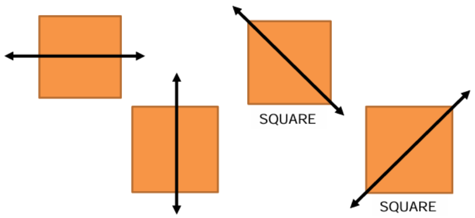
- The symbol for congruency is ≅.
- Congruent figures are the same size and the same shape. Angles are congruent if they have the same measure. Line segments are congruent if they are the same length. Polygons are congruent if they have an equal number of sides, and all the corresponding sides and angles are congruent.



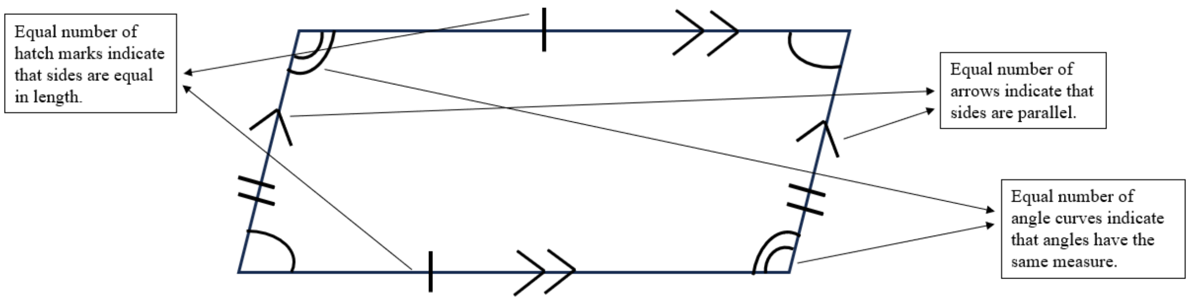
- A polygon is a closed plane figure composed of at least three line segments that do not cross.
- A regular polygon has congruent sides and congruent interior angles. The number of lines of symmetry of a regular polygon is equal to the number of sides of the polygon.



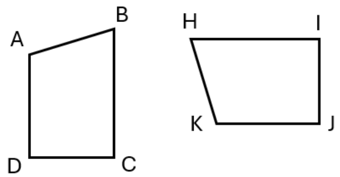
- A line of symmetry divides a figure into two congruent parts, each of which is the mirror image of the other. Lines of symmetry are not limited to horizontal and vertical lines.



- Noncongruent figures may have the same shape but not the same size.
- Students should be familiar with geometric markings in figures to indicate congruence of sides and angles and to indicate parallel sides. An equal number of hatch (hash) marks indicate that those sides are equal in length. An equal number of arrows indicate that those sides are parallel. An equal number of angle curves indicates that those angles have the same measure. See the diagram below.



- The determination of the congruence or noncongruence of two figures can be accomplished by placing one figure on top of the other or by comparing the measurements of all corresponding sides and angles.
- The order of the letters matter when naming congruent polygons. While quadrilateral *ABCD* is congruent to quadrilateral *KHIJ*, it is not congruent to quadrilateral *HIJK*.



$ABCD \cong KHIJ$

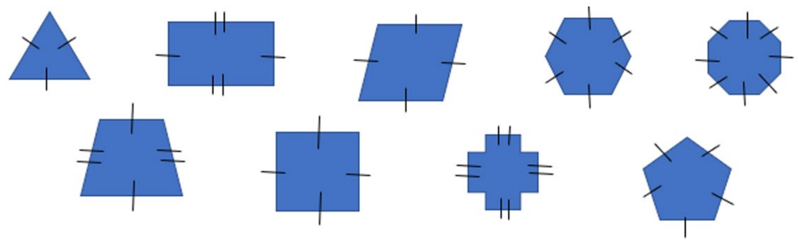
- Construction of congruent angles, line segments, and polygons helps students understand congruency.

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

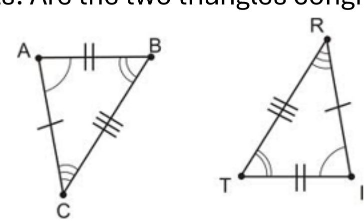
Mathematical Reasoning:

- Students may think polygons can be classified as regular based only on the congruency of their sides. They may overlook that interior angles must also be congruent in regular polygons. Completing a sorting activity in which students are asked to compare the characteristics of polygons, while also identifying which characteristics are common to all regular polygons, can be beneficial for students who struggle with this concept. For example, students can name polygons from a given set of shapes and use geometric markings to determine congruency. They can then be asked to identify which shapes are regular polygons and explain the reasoning behind their choices –



- When determining congruence or noncongruence of two figures, students need practice comparing figures or components of figures and place them on top of the other or compare the measurements of all corresponding sides and angles. An example has been provided with common misconceptions –
    - Look at the triangles below and complete the statements. Are the two triangles congruent?

$\angle C \cong \angle$ ______
$\overline{AB} \cong$ ______
$\angle T \cong \angle$ ______
$\overline{RT} \cong$ ______



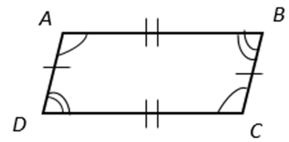
Some students may have difficulty visualizing the corresponding congruent sides and angles when shapes are rotated or reflected. This may indicate that a student struggles with spatial relationships and potentially with geometric markings that indicate congruency. Teachers may wish to utilize the word wall cards and/or co-create anchor charts with the students to help solidify this information. Exploring congruent polygons with manipulatives and being able to place them on top of each other may assist students in understanding the characteristics that make them congruent. Students could also be encouraged to color code corresponding sides and angles to assist with determining congruency.

Mathematical Representations:

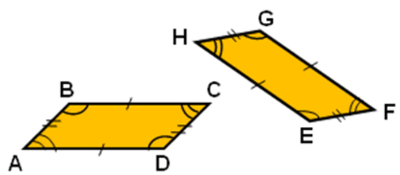
- When determining congruence of segments, angles, and polygons, students may have a difficult time understanding that the shape and the angles must be the same to be congruent. Further, students may confuse congruent with similar. It is important for students to fully understand that line segments and other shapes may be congruent even if they look different because they are oriented differently.

    - Compare the pair of polygons by tracing them, cutting them out, and placing one polygon on top of the other. If they are an exact match in size and shape, then the two polygons are congruent; if the two polygons differ in size and/or shape, then the two polygons are not congruent.

    - Discuss the geometric markings on figures that indicate congruence of length (hash marks), angle measure (arcs) and parallel sides (arrows).

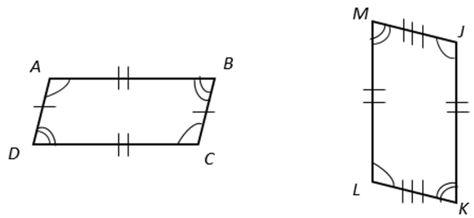


    - Discuss the geometric markings for congruence (≅). For example, figure ABCD is congruent to figure HGFE (ABCD ≅ HGFE).



▱ABCD ≅ ▱HGFE

    - Discuss whether the following two parallelograms are congruent to each other:



Some students may think parallelograms ABCD and JKLM are congruent. This may indicate that a student is basing their decision on the fact that each figure has two sets of congruent angles without regard to the geometric markings indicating that the shorter side lengths of the parallelograms are not denoted with the same congruent markings. Exploring congruent polygons with manipulatives and being able to place them on top of each other may assist students in understanding the characteristics that make them congruent. Students should be encouraged to color code corresponding sides and angles to assist with determining congruency.

- **Drawing lines of symmetry to divide regular polygons into two congruent parts:** Students must understand that lines of symmetry are not limited to horizontal and vertical lines. For example, if given the following polygons, ask students to draw all lines of symmetry.

Students struggling with drawing the lines of symmetry may need hands-on experiences with paper folding to help them conceptualize the two congruent parts created by the line of symmetry. Many times, students see the polygon in only one way, giving them a limited number of lines of symmetry that are visible. For example, they may draw a line from the top of the triangle to the bottom, not realizing that there are two additional lines if they turn the triangle. Hands on exploration helps students visualize the multiple lines of symmetry. Once students complete the exploration with a few regular polygons, they usually discover that the number of lines of symmetry for regular polygons is the same as the number of sides and angles.

Concepts and Connections

Concepts

Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.

Connections

- *Within the grade level/course*:
    - There are no horizontal connections.

- *Vertical Progression*:
  - 2.MG.3 – The student will identify, describe, and create plane figures (including circles, triangles, squares, and rectangles) that have at least one line of symmetry and explain its relationship with congruency.
  - 7.MG.2 – The student will solve problems and justify relationships of similarity using proportional reasoning.
- *Digital Learning Integration*
  - 6-8 CT.A. Students create, identify, explore, and solve problems using technology-assisted methods such as data analysis, modeling, or algorithmic thinking.

ACROSS CONTENT AREAS

Reference 6.MG.1

Resources to Support Local Curriculum

- A list of approved textbooks and instructional materials is posted on the VDOE website.
- Middle School Mathematics Formula Sheet (PDF)

Probability and Statistics

In K-12 mathematics, probability and statistics introduce students to the concepts of uncertainty and data analysis. Probability involves understanding the likelihood of events occurring, often using concepts such as experiments, outcomes, and the use of fractions and percentages. The formal study or probability begins in Grade 4. Statistics focuses on collecting, organizing, and interpreting data and includes a basic understanding of graphs and charts. Probability and statistics help students analyze and make sense of real-world data and are fundamental for developing critical thinking skills and making informed decisions using data.

In Grade 6, students will understand that the world can be investigated through posing questions and collecting, representing, analyzing, and interpreting data to describe and predict events and real-world phenomena. At this grade level, students use statistical investigation to determine experimental and theoretical probability and apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graphs. Further, students represent the mean as a balance point and describe how statistical measures are affected when a data value is added, removed, or changed.

6.PS.1

The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graphs.

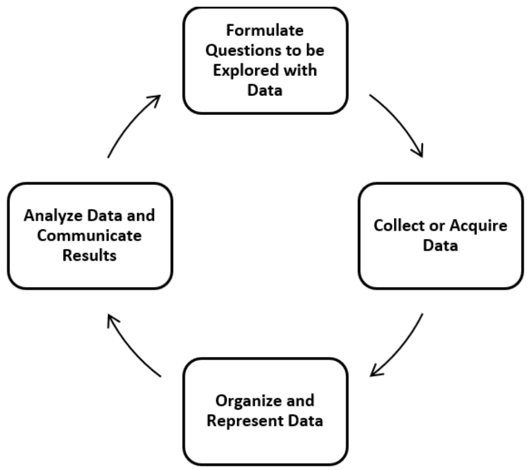
Students will demonstrate the following Knowledge and Skills:

a) Formulate questions that require the collection or acquisition of data with a focus on circle graphs.
b) Determine the data needed to answer a formulated question and collect the data (or acquire existing data) using various methods (e.g., observations, measurement, surveys, experiments).
c) Determine the factors that will ensure that the data collected is a sample that is representative of a larger population.
d) Organize and represent data using circle graphs, with and without the use of technology tools. The number of data values should be limited to allow for comparisons that have denominators of 12 or less or those that are factors of 100 (e.g., in a class of 20 students, 7 choose apples as a favorite fruit, so the comparison is 7 out of 20, $\frac{7}{20}$, or 35%).
e) Analyze data represented in a circle graph by making observations and drawing conclusions.
f) Compare data represented in a circle graph with the same data represented in other graphs, including but not limited to bar graphs, pictographs, and line plots (dot plots), and justify which graphical representation best represents the data.

Understanding the Standard

- Students should explore the entire data cycle with a question and set of data that has been collected or acquired. Student reflection should occur throughout the data cycle. The data cycle includes the following steps: formulating questions to be explored with data, collecting or acquiring data, organizing and representing data, and analyzing and communicating results.

Data Cycle
Grade K-Algebra 2

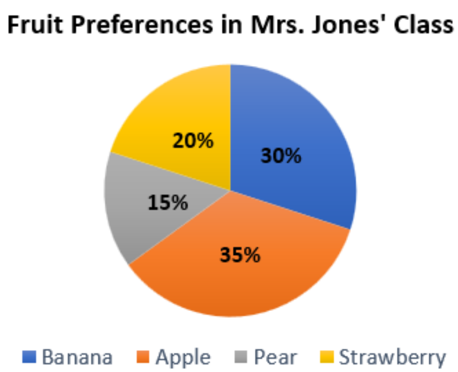


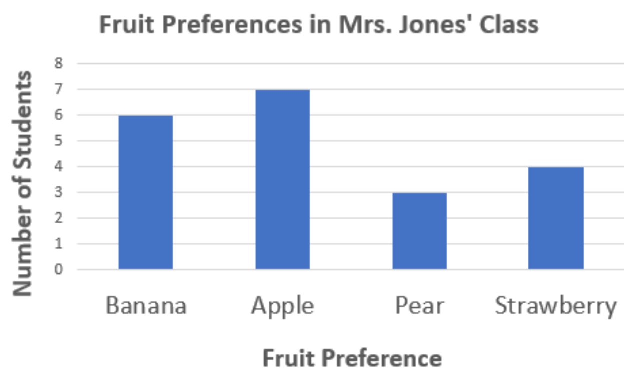
- There are many methods to collect data for any problem situation. These may include experiments, surveys, observations, or other data-gathering strategies. The data can be organized, displayed, analyzed, and interpreted to solve the problem.
- The teacher can provide data sets to students in addition to students engaging in their own data collection or acquisition.
- A population is the entire set of individuals or items from which data is drawn for a statistical study.
- A sample is a data set obtained from a subset that represents an entire population. This data set can be used to make reasonable assumptions about the whole population.
- Sampling is the process of selecting a suitable sample, or representative part of a population, for the purpose of determining characteristics of the whole population. A cursory overview of sampling is intended for Grade 6.
- An example of a population would be the entire student body at a school, whereas a sample might be selecting a subset of students from each grade level. A sample may or may not be representative of the population as a whole. Questions to consider when determining whether an identified sample is representative of the population include:
  - What is the target population of the formulated question?
  - Who or what is the subject or context of the formulated question?
- Examples of questions to consider in building good samples:

- What is the context of the data to be collected?
- Who is the audience?
- What amount of data should be collected?

- A circle graph is used for categorical and discrete numerical data. Circle graphs are used for data showing a relationship of the parts to the whole.
  - Example: The favorite fruit of 20 students in Mrs. Jones' class was recorded in a table. Compare the same data displayed in both a circle graph and a bar graph.

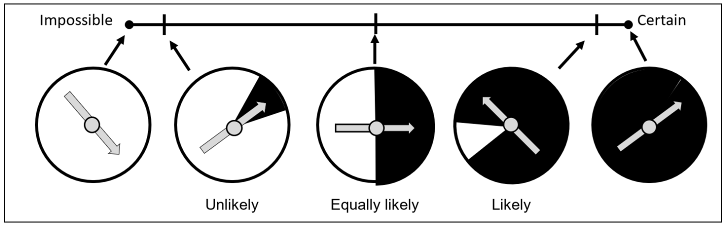
Fruit Preference	# of students
banana	6
apple	7
pear	3
strawberry	4





- Circle graphs can represent percent or frequency.
- Circle graphs are not effective for representing data with large numbers of categories.
- Teachers should be reasonable about the selection of data values. The number of data values can affect how a circle graph is constructed (e.g., 10 out of 25 would be 40%, but 7 out of 9 would be $77.\overline{7}\%$, making the construction of a circle graph more complex). Students should have experience constructing circle graphs, but a focus should be placed on the analysis of circle graphs.

- Students are not expected to construct circle graphs by multiplying the percentage of data in a category by 360° in order to determine the central angle measure. Limiting comparisons to fraction parameters noted in the standard will assist students in constructing circle graphs.
- Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data.
- Circle graphs must include a title, percent or number labels for data categories, and a key. A key is essential to explain how to read the graph. A title is essential to explain what the graph represents.
- Circle graphs can be created in programs such as Excel or Google spreadsheets. Some programs refer to circle graphs as pie charts.
- In previous grades, students had experience with pictographs, bar graphs, line graphs, line plots and stem-and-leaf plots. In Grade 6, students are not expected to construct these graphs.
  - A pictograph is used to show categorical data. Pictographs are used to show frequency and compare categories.
  - A bar graph is used for categorical data and is used to show comparisons between categories.
  - A line graph is used to show how numerical data changes over time.
  - A line plot provides an ordered display of all values in a data set and shows the frequency of data on a number line. Line plots are used to show the spread of the data, to include clusters (groups of data points) and gaps (large spaces between data points), and quickly identify the range, mode, and any extreme data values.
  - A stem-and-leaf plot uses columns to display a summary of discrete numerical data while maintaining the individual data points. A stem-and-leaf plot displays data to show its shape and distribution.
- Different situations call for different types of graphs (e.g., visual representations). The way data are displayed is often dependent upon what question is being investigated and what someone is trying to communicate.
- Comparing different types of representations (e.g., charts, graphs, line plots) provides students with opportunities to learn how different graphs can show different aspects of the same data. Following the construction of representations, discussions around what information each representation provides or does not provide should occur.
- The information displayed in different graphs may be examined to determine how data are or are not related, differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or questions such as "What could happen if…" (inferences).
- Connections can be made with probability and drawing conclusions from a circle graph.
- In general, if all outcomes of an event are equally likely, the probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes in the sample space.
- The probability of an event occurring can be represented as a ratio or equivalent fraction, decimal, or percent.
- Based on the data in the circle graph, the likelihood of an event can be determined as impossible, unlikely, equally likely, likely, and certain.



Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Communication: There are many good questions students can ask about circle graphs. Not all questions can be answered with the types of data displays that are the focus at this grade.

- As students are formulating questions, they should consider –
  - What type of data can be collected for this question?
  - Would the data collected make sense to display in a circle graph?
  - Is the data easily categorized – *What is your favorite flavor of ice cream? Choose from the given list. Which video game is your favorite?*
  - OR is the data discrete numerical data – *How many siblings do you have? How many hours per day do you spend on a screen?*

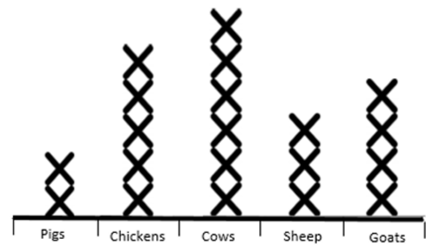
  - Circle graphs require a clear "whole."
    - Do I know what the "whole" is for this situation?
    - Can my question be answered with percentages?

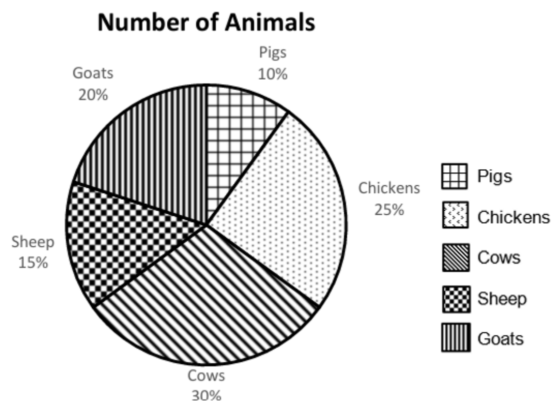
  - Data displays are intended to provide a simplified display of the data. (It is challenging to digest the raw data.) However, most data displays will lose details.
    - Does a circle graph address your question? The labels are key to communication.
    - What title and labels are necessary to clearly communicate?
    - Do I need to include totals, or just percentages?

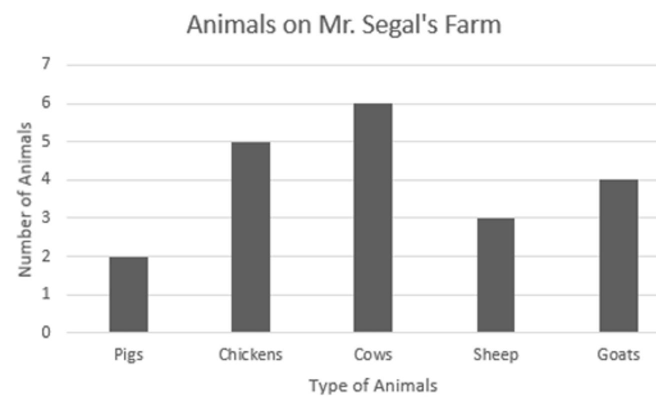
Mathematical Reasoning: Students can find real-life examples of circle graphs in newspapers, magazines, or online sources. They should analyze the data represented in the circle graphs and write a short paragraph explaining the insights they gained from the graphs. At this grade, it would be appropriate to formulate a hypothesis about the relationship between two variables based on the information presented in a circle graph and design an experiment to test it. This will require students to draw conclusions about a given circle graph and explain how the data supports their conclusions. Further, students should assess the effectiveness of using a circle graph to represent a specific set of data and justify their evaluation.

Mathematical Representations: Students should be given opportunities to look at the same data presented in different graphs and determine which is the most effective representation. Students could also consider which question types would be best represented by different types of graphs. Students should return to their question to see if their data answers the question and if not, consider at which point in the cycle they would want to modify their process to gather better data or represent the data in a more meaningful way. For example –

Types of Animals on Mr. Segal's Farm







Consider the following questions while comparing the data represented in each graph:
- Which graph(s) shows the type of animal that is most common on Mr. Segal's farm?
- Which graph best shows how many pigs are on Mr. Segal's farm?
- Which graph best helps determine the total number of animals on Mr. Segal's farm?
- Mr. Segal has at least 4 of which types of animals? In which graph(s) is this most clearly seen?
- Which graph helps you determine the percentage of Mr. Segal's animals that have four legs?

Concepts and Connections

Concepts

The world can be investigated through posing questions and collecting, representing, analyzing, and interpreting data to describe and predict events and real-world phenomena.

Connections

- *Within the grade level/course*:
  - 6.PS.2 – The student will represent the mean as a balance point and determine the effect on statistical measures when a data point is added, removed, or changed.
- *Vertical Progression*:
  - 5.PS.1 – The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on line plots (dot plots) and stem-and-leaf plots.
  - 7.PS.1 – The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on histograms.
- *Digital Learning Integration:*
  - 6-8 CT.B. Students find or organize data and use appropriate technologies to interpret, analyze, and represent data to construct models, predict outcomes, solve problems, and make evidence-based decisions.

Across Content Areas [Theme – Using Data]

- *Computer Science*:
  - Data and Statistics concepts include storing and process data; analyzing large datasets; and designing graphs and charts.
  - Statistics and Probability highlights include artificial intelligence and machine learning – probability is used in decision making algorithms; randomized algorithms – randomized sorting algorithms; and cybersecurity and encryption – cryptographic key generation.
- *Science:*

- 6.3 The student will investigate and understand that there is a relationship between the sun, Earth, and the moon. To meet this standard, students are expected to analyze tide tables.
- 6.4 The student will investigate and understand that there are basic sources of energy, and that energy can be transformed. To meet this standard, students expected to plan and conduct an investigation on the transfer of radiant energy.
- 6.6 The student will investigate and understand that water has unique physical properties and has a role in the natural and human-made environment. To meet this standard, students expected to plan and conduct investigations about water properties.
- 6.7 The student will investigate and understand that air has properties and that Earth's atmosphere has structure and is dynamic. To meet this standard. students are expected to analyze data about atmosphere characteristics and altitude.

Resources to Support Local Curriculum

- A list of approved textbooks and instructional materials is posted on the VDOE website.
- Middle School Mathematics Formula Sheet (PDF)
- Data Cycle and Circle Graphs Exemplar Mathematics Instructional Plan (Word | PDF)
- Data Cycle Teacher Resource (PPT | PDF)

6.PS.2

The student will represent the mean as a balance point and determine the effect on statistical measures when a data point is added, removed, or changed.

Students will demonstrate the following Knowledge and Skills:

a) Represent the mean of a set of data graphically as the balance point represented in a line plot (dot plot).

b) Determine the effect on measures of center when a single value of a data set is added, removed, or changed.

c) Observe patterns in data to identify outliers and determine their effect on mean, median, mode, or range.

Understanding the Standard

- Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data.
- Measures of center are types of averages for a data set. They represent numbers that describe a data set. Mean, median, and mode are measures of center that are useful for describing different situations.
- Mean may be appropriate for sets of data where there are no values much higher or lower than those in the rest of the data set.
- Median may be appropriate when data sets have some values that are much higher or lower than most of the other values in the data set. The median is the middle value of a data set in ranked order. If there are an odd number of pieces of data, the median is the middle value. If there are an even number of pieces of data, the median is the numerical average of the two middle values.

6, 7, 8, 9, 9 — 8 = median

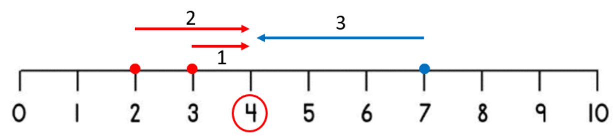
5, 6, 8, 9, 11, 12 — 8.5 = median

- Mode may be appropriate when the set of data has some identical values, when data is categorical, or when the data reflect the most popular option. The mode is the piece of data that occurs most frequently. If no value occurs more often than any other, there is no mode. If there are multiple values that occur most often, each of these values is a mode. When there are exactly two modes, the data set is bimodal.

Data Sets	Mode
2, 3, 3, 3, 5, 5, 9, 10	3
5.2, 5.4, 5.5, 5.6, 5.8, 5.9, 6.0	none
1, 1, 2, 5, 6, 7, 7, 9, 11, 12	1, 7

bimodal

- Mean can be defined as the point on a number line where the data distribution is balanced. This requires that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances from the mean of all the data points below the mean. This is the concept of mean as the balance point.
  - Example: Given the data set: 2, 3, 7, the mean value of 4 can be represented on a number line as the balance point:



$$\frac{2+3+4+7}{4} = \frac{16}{4} = 4$$

- The mean can also be found by calculating the numerical average of the data set.
- In Grade 5 mathematics, students had experiences defining the mean as fair share.
- Defining mean as the balance point is a prerequisite for understanding standard deviation, which is addressed in high school level mathematics.
- The range is the spread of a set of data. The range of a set of data is the difference between the greatest and least values in the data set. It is determined by subtracting the least number in the data set from the greatest number in the data set. An example is ordering test scores from least to greatest: 73, 77, 84, 87, 89, 91, 94. The greatest score in the data set is 94 and the least score is 73, so the least score is subtracted from the greatest score or 94 - 73 = 21. The range of these test scores is 21.
- An outlier can be identified by sorting the data in ascending order. A data value that is an abnormal distance relative to the other values in the data set is an outlier. It represents a value that "lies outside" (is much smaller or larger than) most of the other values in a set of data. Outliers have a greater effect on the mean and range of a data set but have less of an effect on the median or mode.
- In Grade 6, students are not expected to mathematically determine outliers. Instead, at this level, they are expected to visually determine outliers when provided a representation of a data set.

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Reasoning: Although students at this grade level are not expected to mathematically determine outliers, they are expected to visually determine outliers when given a data set. Students must understand that an outlier lies outside (much smaller or larger than) most of the values in the data set. Outliers impact the mean and range of the data set but have less of an effect on the median or mode. Students should engage in problems that have outliers and determine the effect on measures of center when a single value of a data set is added, removed, or changed. For example –

Amber took six tests in science this semester. Her scores on these tests are 93, 99, 83, 93, 100, and 65.

- What is the mean, median, and mode of Amber's science test scores?

  Some students may obtain incorrect measures of center. Some students may not realize that it is possible to have no mode for a data set or will represent "no mode" by stating that the mode is 0. Students may not put the numbers in ascending order before determining the median. These students may need additional review on how to calculate measures of center and what they mean in terms of representing a data set.

- The teacher removes the outlier from the data and recalculates the measures of center and spread. When the outlier is removed from the data, what happens to the mean, median, mode, and range?

  Engage students in a discussion about which data point is an outlier and why. Representing Amber's scores on a line plot can help students visualize the outlier in the data set. Students should determine that removing the outlier of 65 would result in an increase to both the range and the mean. However, the median and the mode would stay the same.

- Amber retook the test on which she received an 83 and earned a 95. How did changing this one test score affect the mean, median, mode, and range of the original data set?

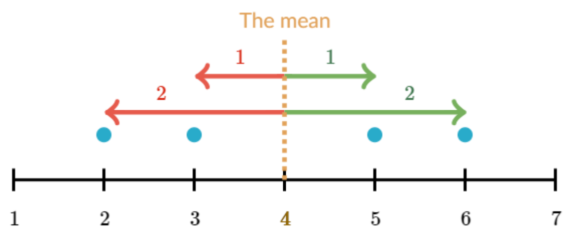
  In changing the data point from 83 to 95 students may accidentally add the 95 rather than replacing the 83. The use of manipulatives to model a situation like this can aid students as they will physically remove one data point and add another. The students should indicate that the mean and median both increased, while the mode and range stayed the same. The teacher may wish to engage students in discussions about why this happened. *Does this always happen? What would happen if a different data point was changed?*

Mathematical Communication: As students engage with this standard, use the following questions to elicit student discourse as students examine data sets, interpret the data rendered, and share out their results –

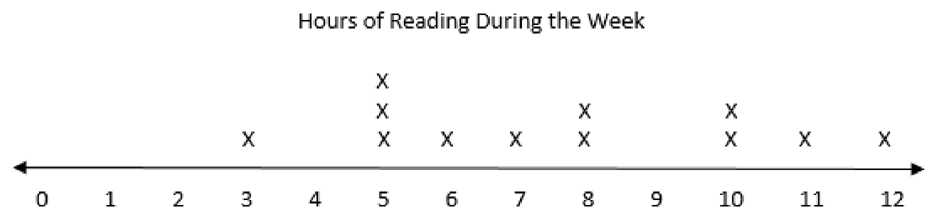
- How does the mean balance the distribution of a data set?
- How does the mean summarize the center of a distribution?
- How are the mean, median, mode, and range effective in describing a data set?
- Are there limitations to the measures of center (mean, median, and mode)? If so, what?
- Is there a limitation to the measure of variability (or spread) (range)? If so, what?
- How can a line plot (dot plot) help us make sense of this interpretation of the mean as a balance point?
- Could another value, besides the mean, balance a data distribution? How can we tell?

Mathematical Representations:

- It is important for students to think of the mean as the balance point. The total distance from the mean to the data points below the mean is equal to the total distance from the mean to the data points above the mean. For example, when determining the mean of {2, 3, 5, 6} (which is 4), it can be observed that the total distance from the mean to the data points below the mean is equal to the total distance from the mean to the data points above the mean because **1** + **2** = **1** + **2**. It is always true that the total distance below the mean is equal to the total distance above the mean.



- When students develop line plots, it is important for students to understand that the mean is not always a whole number (a common misconception). For example –
    - Ms. Rogers made a line plot of how many hours each of her students read during the week. She organized the data into the line plot below. What is the balance point for the data? Each X represents one student.



Some students may identify the balance point as 7 or 8, not realizing that it could be a decimal. In this example, the balance point lies between two of the points on the line plot (7 and 8), and the balance point is 7.5. Provide students with experience locating a balance point in which the result is not always a whole number. Students also may find the mean using computation (algorithm) or a calculator and note the balance point using that quotient. The intent of this standard is for students to define mean as the balance point as it is a prerequisite for understanding standard deviation, which is addressed in high school level mathematics.

Concepts and Connections

Concepts

The world can be investigated through posing questions and collecting, representing, analyzing, and interpreting data to describe and predict events and real-world phenomena.

Connections

- *Within the grade level/course*:
  - 6.PS.1 – The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graph.
- *Vertical Progression*:
  - 5.PS.2 – The student will solve contextual problems using measures of center and the range.
  - A2.ST.1h – Determine the solution to problems involving the relationship of the mean, standard deviation, and *z*-score of a data set represented by a smooth or normal curve.
- *Digital Learning Integration*
  - 6-8 CT.B. Students find or organize data and use appropriate technologies to interpret, analyze, and represent data to construct models, predict outcomes, solve problems, and make evidence-based decisions.

Across Content Areas

Reference 6.PS.1

Resources to Support Local Curriculum

- A list of approved textbooks and instructional materials is posted on the VDOE website.
- Middle School Mathematics Formula Sheet (PDF)

Patterns, Functions, and Algebra

In K-12 mathematics, the patterns, functions, and algebra strand focuses on the recognition, description, and analysis of patterns, functions, and algebraic concepts. Students develop an understanding of mathematical relationships and represent these using symbols, tables, graphs, and rules. In later grades, students use models to solve equations and inequalities and develop an understanding of functions. This strand is designed to develop students' algebraic thinking and problem-solving skills, laying the foundation for more advanced mathematical concepts.

In Grade 6, students learn that proportional relationships can be described, and generalizations can be made using patterns, relations, and functions. Algebraic equations and inequalities can be used to represent and solve real world problems. At this grade level, students will use ratios to represent relationships between quantities; identify and represent proportional relationships between two quantities; create and solve one-step linear equations in one-variable; and represent a contextual situation using a linear inequality in one variable with symbols and graphs on a number line.

6.PFA.1

The student will use ratios to represent relationships between quantities, including those in context.

Students will demonstrate the following Knowledge and Skills:

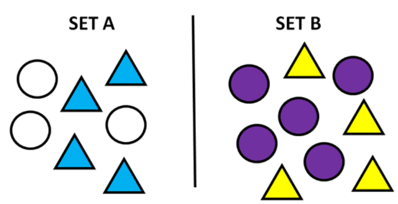
a) Represent a relationship between two quantities using ratios.
b) Represent a relationship in context that makes a comparison by using the notations $\frac{a}{b}$, *a*:*b*, and *a* to *b*.
c) Represent different comparisons within the same quantity or between different quantities (e.g., part to part, part to whole, whole to whole).
d) Create a relationship in words for a given ratio expressed symbolically.
e) Create a table of equivalent ratios to represent a proportional relationship between two quantities, when given a ratio.
f) Create a table of equivalent ratios to represent a proportional relationship between two quantities, when given a contextual situation.

Understanding the Standard

- A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between quantities. Ratios are used in contextual situations when there is a need to compare quantities.
- In the elementary grades, students are taught that fractions represent a part-to-whole relationship. However, fractions may also express a measurement, an operator (multiplication), a quotient, or a ratio. Examples of fraction interpretations include:

- fractions as parts of wholes: $\frac{3}{4}$ represents three parts of a whole, where the whole is separated into four equal parts;
- fractions as measurement: the notation $\frac{3}{4}$ can be interpreted as three one-fourths of a unit;
- fractions as an operator: $\frac{3}{4}$ represents a multiplier of three-fourths of the original magnitude;
- fractions as a quotient: $\frac{3}{4}$ represents the result obtained when three is divided by four; and
- fractions as a ratio: $\frac{3}{4}$ is a comparison of 3 of a quantity to the whole quantity of 4.

- A ratio may be written using a colon (*a:b*), the word "to" (*a* to *b*), or fraction notation $\frac{a}{b}$.

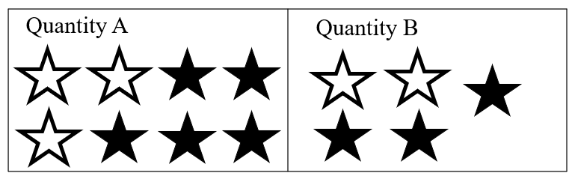


▲ to ○	**4 to 3 or 4:3**
▲ to all of set A	**4 to 7 or 4:7 or $\frac{4}{7}$**
○ (set A) to ● (set B)	**3 to 5 or 3:5**
set B to set A	**9 to 7 or 9:7**

- The order of the values in a ratio is directly related to the order in which the quantities are compared. For example, in a certain class, there is a ratio of 3 girls to 4 boys (3:4).
- Another comparison that could represent the relationship between these quantities is the ratio of 4 boys to 3 girls (4:3). Both ratios give the same information about the number of girls and boys in the class, but they are distinct ratios. When you switch the order of comparison (girls to boys vs. boys to girls), there are two different ratios expressed.
- Fractions may be used when determining equivalent ratios.
  - Example: The ratio of girls to boys in a class is 3:4, this can be interpreted as:
    - number of girls = $\frac{3}{4} \cdot$ number of boys;
    - in a class with 16 boys, number of girls = $\frac{3}{4} \cdot (16) = 12$ girls.
  - Example: A similar comparison could compare the ratio of boys to girls in the class as being 4:3, which can be interpreted as:
    - number of boys = $\frac{4}{3} \cdot$ number of girls;
    - in a class with 12 girls, number of boys = $\frac{4}{3} \cdot (12) = 16$ boys.
- A ratio can compare two real-world quantities (e.g., miles per gallon, unit rate, and circumference to diameter of a circle).
- Ratios may or may not be written in simplest form.
- A ratio can represent different comparisons within the same quantity or between different quantities.

Ratio	Comparison
part-to-whole (within the same quantity)	compare part of a whole to the entire whole
part-to-part (within the same quantity)	compare part of a whole to another part of the same whole
whole-to-whole (different quantities)	compare all of one whole to all another whole
part-to-part (different quantities)	compare part of one whole to part of another whole

- For example:



Ratio	Example	Ratio Notation(s)
part-to-whole (within the same quantity)	compare the number of unfilled stars to the total number of stars in Quantity A	3:8; 3 to 8; or $\frac{3}{8}$
part-to-part (within the same quantity)	compare the number of unfilled stars to the number of filled stars in Quantity A	3:5 or 3 to 5

Ratio	Example	Ratio Notation(s)
whole-to-whole (different quantities)	compare the number of stars in Quantity A to the number of stars in Quantity B	8:5 or 8 to 5
part-to-part (different quantities)	compare the number of unfilled stars in Quantity A to the number of unfilled stars in Quantity B	3:2 or 3 to 2

- Part-to-part comparisons and whole-to-whole comparisons are ratios that are not typically represented in fraction notation except in certain contexts, such as determining whether two different ratios are equivalent.
- Equivalent ratios are created by multiplying each value in a ratio by the same constant value. For example, the ratio of 4:2 would be equivalent to the ratio 8:4, since each value in the first ratio could be multiplied by 2 to obtain the second ratio.
- Students will begin to make the connection between equivalent ratios and proportionality. A proportional relationship consists of two quantities where there exists a constant number such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.
- Proportional thinking requires students to think multiplicatively, rather than additively. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e., one quantity is the result of multiplying the other quantity by a value). See the examples below.

Additive relationship:

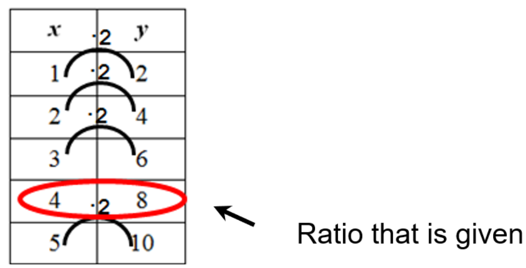
x		y
2	+8 →	10
3	+8 →	11
4	+8 →	12
5	+8 →	13

Multiplicative relationship:

x		y
2	·5 →	10
3	·5 →	15
4	·5 →	20
5	·5 →	25

- In the additive relationship, *y* is the result of adding 8 to *x*.
- In the multiplicative relationship, *y* is the result of multiplying *x* times 5.
- The ordered pair (2, 10) is a quantity in both relationships; however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative.

- It is important to use contextual situations to model proportional relationships. Context can help students to see the relationship between two quantities.
- In the elementary grades, students had experiences with tables of values (input/output tables that are additive and multiplicative). The concept of a ratio table should be connected to students' prior knowledge of representing number patterns in tables.
- A ratio table is a table of values representing a proportional relationship that includes pairs of values that represent equivalent rates or ratios. A constant exists that can be multiplied by the measure of one quantity to get the measure of the other quantity for every ratio pair. The same proportional relationship exists between each pair of quantities in a ratio table.
- Example: Given that the ratio of *y* to *x* in a proportional relationship is 8:4, create a ratio table that includes three additional equivalent ratios.



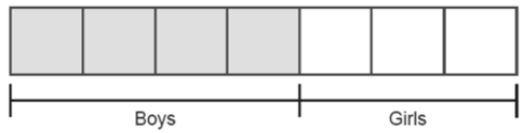
Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Problem Solving: Expand students' ability to identify relevant information in word problems by presenting problem information differently. It is essential to include problems that vary the unknown quantity to help students understand the mathematical structure in each problem type. Other problems that look different may require additional steps to solve or include irrelevant numerical information or information on a chart, graph, or diagram. Problem types related to ratios are provided below –

- **Ratio problem with ratio given**: Rena bought some food at the farmer's market. For every 1 cucumber she bought, she bought 3 tomatoes. If she bought 12 tomatoes, how many cucumbers did she buy?

- **Ratio problem with a diagram (chart or graph):** In Mr. Lardin's class, there are more boys than girls. Below is a diagram representing the number of boys to girls. If there are 12 boys in the class, how many girls are there?



- **Ratio problem with irrelevant information:** Roe loves to garden. She keeps 5 flower gardens and 1 vegetable garden. In the flower garden, for every 5 daisies she plants, she also plants 1 rose. If she planted 3 roses, how many daisies did she plant?

- **Ratio problem with multiple steps:** Carolina loves to plant flowers in her garden. For every 5 daisies she plants, she also plants 1 rose. If she planted 3 roses, how many flowers did she plant altogether?

Mathematical Connections: When creating a table of equivalent ratios to represent a proportional relationship between two quantities either when given a ratio or contextually, students must make the connection that proportional thinking requires students to thinking multiplicatively, rather than additively.

- A step-by-step process to help students is to (a) identify the ratio provided in the problem; (b) create equivalent ratios by either multiplying or dividing both sides of the ratio by the same number; and (c) repeating (b) until a table is filled out with equivalent ratios by putting all the numbers on the left or top of the ratio in the left column and all numbers on the right or bottom of the ratio in the other column. For example –

  The ratio of apple trees to peach trees at Field Family Farm is 5 to 2. Create a table of equivalent ratios and determine the type of proportional relationship given.

  - Identify the ratio provided in the problem: In this problem, the ratio provided is $\frac{5 \text{ apple trees}}{2 \text{ peach trees}}$

  - To create equivalent ratios, multiply or divide both sides of the ratio by the same number: In this problem, students will multiply. Multiply both sides of the ratio by 2. The first equivalent ratio is 10 apple trees to 4 peach trees.

$$\frac{5 \cdot 2}{2 \cdot 2} = \frac{10 \text{ apple trees}}{4 \text{ peach trees}}$$

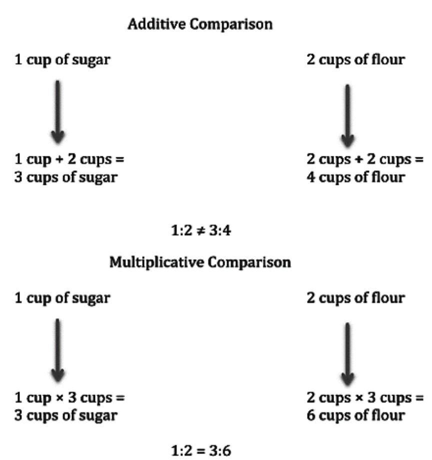
  - Repeat the process to fill out the table with equivalent ratios:

Apple Trees	Peach Trees
5	2
10	4
15	6
20	8
25	10

- The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative. Students may not understand that 4 more than (additive comparison) has a different meaning than 4 times (multiplicative comparison) –

A recipe calls for 2 cups of flour for every 1 cup of sugar. How many cups of flour are needed if a recipe is increased to 3 cups of sugar?

Students can think 1 cup of sugar for 2 cups of flour, 2 cups of sugar for 4 cups of flour, and 3 cups of sugar for 6 cups of flour. Therefore, the amount of flour now needed is 6 cups, which maintains the same relationship as the original one given (1:2). Students who think ratios are additive would have mistakenly thought that 3 cups of sugar is an increase of 2 cups from the original 1 cup. They would then add to the original 2 cups of flour 2 more cups to get 4 cups needed flour.



- As students explore ratios and make connections between and among them, engaging in the following will help to elicit their understanding of this concept –
  - Create a real-world situation in which ratios have been used. Swap with a partner and have your partner solve.
  - Explain how fractions and ratios are similar and how understanding one concept can assist you in understanding the other.
  - Explain how to translate a ratio that is written symbolically into words and vice versa.

Mathematical Representations: A ratio can represent different comparisons within the same quantity or between different quantities. A common misconception that students may exhibit is not understanding the type of relationship that exists between different quantities (whether it be part-to-part, part-to-whole, whole-to-whole, or whole-to-part). Make sure students understand that a ratio is a comparison of any two quantities and that it is used to represent a relationship within or between sets. Emphasize that the two quantities in a ratio *must be ordered in the same order as the quantities in the relationship*. A ratio may be written using a colon (*a*:*b*), the word *to* (*a* to *b*), or fraction notation $\frac{a}{b}$. Part-to-part comparisons and whole-to-whole comparisons are ratios that are not typically represented in fraction notation, except in certain contexts, such as determining whether two different ratios are equivalent. The following contextual examples are provided as well as the common misconceptions students may experience –

- **Distinguishing between a part-to-part and a part-to-whole relationship:**

  The Linwood High School Basketball Team wins 28 of the games they play. They play a total of 36 games. What is the ratio of the games they win to the games they lose? How do you know?

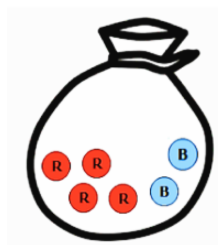
  A common error for some students is to describe a part-to-whole relationship rather than a part-to-part relationship. These students may have the misconception that all the numbers used in the ratio will be included in the problem. They may not realize that they need to use the details in the problem to determine how many games were lost. Provide students with opportunities to explore ratio relationships with counters and part-part-whole mats to model scenarios concretely and then write statements to represent them. As students model different part-part-whole combinations, ask them to use ratios to describe part-to-part relationships and part-to-whole relationships. This will help students develop the understanding that ratios can reflect different types of relationships.

- **Creating proportional relationships from context:**

  - **Without a model:** Create a set of items with squares and stars. The set of items should represent a 2:3 ratio for the number of squares to the number of stars. The set created should contain more than 5 items.

A common error for some students is to switch the order of the ratio when representing the relationship presented. These students may have the misconception that the order of the numbers or items does not matter. Additionally, some students may be able to create a 2:3 ratio with 5 counters but may struggle to extend it to a larger set of items. It may be helpful for these students to use concrete objects to represent relationships, focusing on matching the objects named to a ratio relationship. Then ask students how they could extend this ratio relationship to a larger set. As students become more confident, they may build several different sets of items showing the same ratio relationship.

- **With a model:** There are 4 red marbles and 2 blue marbles shown in the bag. What is the least number of red and blue marbles that can be added to the bag to create a ratio of 3 red marbles to 1 blue marble?



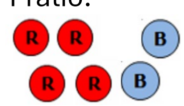
Key: 

= red 

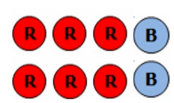
= blue

In this scenario, the least number of marbles that needs to be added to the bag to create a 3 to 1, red to blue ratio, would be two red marbles and zero blue marbles. Some students may reverse the order of the ratio to blue to red and then add items to the set to support the reversal. In this case students might add ten blue marbles and zero red marbles. Some students may also add more marbles than needed and disregard the "least" criteria to create a 3 to 1 (or incorrectly create a 1 to 3 ratio). It may be helpful for students to arrange concrete counters linearly to see the proportional relationship they are creating. For example –

Given set before creating a red to blue 3 to 1 ratio:



Linear concrete arrangement to see the least amount needed (2 more red marbles) to create a red to blue, 3 to 1, ratio:



Concepts and Connections

Concepts

Proportional relationships can be described, and generalizations can be made using patterns, relations, and functions. Algebraic equations and inequalities can be used to represent and solve real world problems.

Connections

- *Within the grade level/course*:
  - 6.PS.1 – The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graphs.
- *Vertical Progression*:
  - 7.PFA.1 – The student will investigate and analyze proportional relationships between two quantities using verbal descriptions, tables, equations in $y = mx$ form, and graphs, including problems in context.
- *Digital Learning Integration:*
  - 6-8 CT.B. Students find or organize data and use appropriate technologies to interpret, analyze, and represent data to construct models, predict outcomes, solve problems, and make evidence-based decisions.

Across Content Areas [Theme – Modeling]

- *Computer Science*:
  - Logic and Conditional Statements concepts include conditional statements and Boolean logic.
  - Geometry and Transformations concepts include manipulating objects in computer graphics; applying geometric calculations in game development and simulations; and developing user interfaces with symmetrical and proportional layouts.
  - Ratios, Proportions, and Scaling concepts include image and game object scaling and normalizing data for machine learning models.
  - Ratios and Proportional Reasoning highlights include scaling in graphics and simulations – scale images in graphic design, adjusting resolution in digital media, and resizing elements for UX design; algorithm efficiency and performance analysis – utilized in application to define of runtime or memory usage changes as input size grows; data encoding and compression – proportional reasoning helps in lossy compression techniques.
  - Geometry highlights include game development and graphics programming – 2D and 3D coordinate systems used in positioning objects, animations, and rendering; computer vision and image processing – shape detection, edge recognition, and transformations (rotation, scaling, and translation); and geographic information systems (GIS) – coordinate systems are used to map and analyze real-world data.

- *Science:*
    - 6.2 The student will investigate and understand that the solar system is organized and the various bodies in the solar system interact. To meet this standard, students are expected to make a model of solar system through Jupiter.
    - 6.3 The student will investigate and understand that there is a relationship between the sun, Earth, and the moon. To meet this standard, students are expected to develop and use a model to explain the moon phases and eclipses.
    - 6.4 The student will investigate and understand that there are basic sources of energy, and that energy can be transformed. To meet this standard, students are expected to use a model to explain the source of energy sources.
    - 6.7 The student will investigate and understand that air has properties and that Earth's atmosphere has structure and is dynamic. To meet this standard, students are expected to use models to explain air movement and weather conditions.
    - 6.8 The student will investigate and understand that land and water have roles in watershed systems. To meet this standard, students are expected to use models to identify components of watershed systems.

Resources to Support Local Curriculum

- A list of approved textbooks and instructional materials is posted on the VDOE website.
- Middle School Mathematics Formula Sheet (PDF)

6.PFA.2

The student will identify and represent proportional relationships between two quantities, including those in context (unit rates are limited to positive values).

Students will demonstrate the following Knowledge and Skills:

a) Identify the unit rate of a proportional relationship represented by a table of values, a contextual situation, or a graph.

b) Determine a missing value in a ratio table that represents a proportional relationship between two quantities using a unit rate.

c) Determine whether a proportional relationship exists between two quantities, when given a table of values, context, or graph.

d) When given a contextual situation representing a proportional relationship, find the unit rate and create a table of values or a graph.

e) Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs.

Understanding the Standard

- A rate is a ratio that involves two different units and how they relate to each other. Relationships between two units of measure are also rates (e.g., inches per foot).
- A unit rate describes how many units of the first quantity of a ratio correspond to one unit of the second quantity.
  - Example: If it costs $10 for 5 items at a store (a ratio of 10:5 comparing cost to the number of items), then the unit rate would be $2.00/per item (a ratio of 2:1 comparing cost to number of items).

Unit Rate — Given ratio

# of items (x)	1	2	5	10
Cost in $ (y)	$2.00	$4.00	$10.00	$20.00

- Any ratio can be converted into a unit rate by writing the ratio as a fraction and then dividing the numerator and denominator each by the value of the denominator.
  - Example: It costs $8 for 16 gourmet cookies at a bake sale. What is the price per cookie (unit rate) represented by this situation?
  - $\frac{8}{16} = \frac{8 \div 16}{16 \div 16} = \frac{0.5}{1}$
  - It would cost $0.50 per cookie, which would be the unit rate.

- Examples such as $\frac{8}{16}$ and 40 to 10 are ratios but are not unit rates. However, $\frac{0.5}{1}$ and 4 to 1 are examples of unit rates.
- Example of a proportional relationship:
  - Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges $8 for each medium pizza. This ratio table represents the cost (*y*) per number of pizzas ordered (*x*).

x number of pizzas	1	2	3	4
y total cost	8	16	24	32

x8 x8

  - In this relationship, the ratio of *y* (cost in $) to *x* (number of pizzas) in each ordered pair is the same:

$$\frac{8}{1} = \frac{16}{2} = \frac{24}{3} = \frac{32}{4}$$

- Example of a non-proportional relationship:
  - Uptown Pizza sells medium pizzas for $7 each but charges a $3 delivery fee per order. This table represents the cost per number of pizzas ordered.

x number of pizzas	1	2	3	4
y total cost	10	17	24	31

  - The ratios represented in the table above are not equivalent.
  - In this relationship, the ratio of *y* to *x* in each ordered pair is not the same:

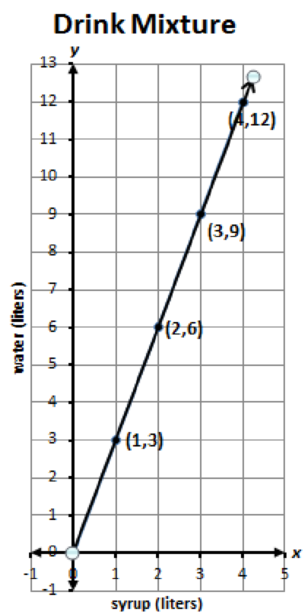
$$\frac{10}{1} \neq \frac{17}{2} \neq \frac{24}{3} \neq \frac{31}{4}$$

- Other non-proportional relationships will be studied in later mathematics courses.
- Proportional relationships involve collections of pairs of equivalent ratios that may be graphed in the coordinate plane. The graph of a proportional relationship includes ordered pairs (*x, y*) that represent pairs of values that may be represented in a ratio table.

- Proportional relationships can be expressed using verbal descriptions, tables, and graphs. When describing proportional relationships verbally, the phrases “for each,” “for every,” and “per” are used.
- Example: (verbal description) To make a drink, mix 1 liter of syrup with 3 liters of water. If *x* represents how many liters of syrup are in the mixture and *y* represents how many liters of water are in the mixture, this proportional relationship can be represented using a ratio table:

Syrup (liters) *x*	1	2	3	4
Water (liters) *y*	3	6	9	12

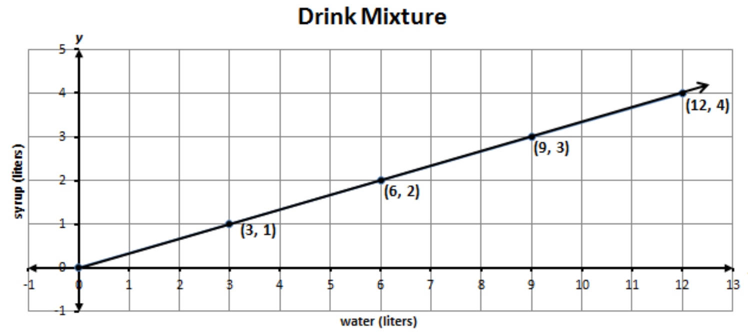
  - The ratio of the amount of water (*y*) to the amount of syrup (*x*) is 3:1. Additionally, the proportional relationship may be graphed using the ordered pairs in the table.



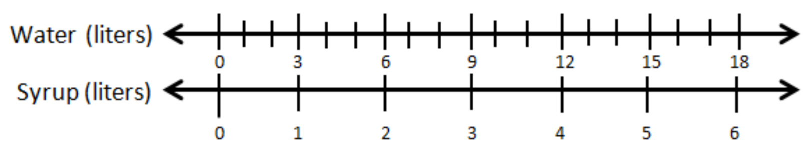
- The representation of the ratio between two quantities may depend upon the order in which the quantities are being compared. For example: In the mixture example above, we could also compare the ratio of the liters of syrup per liters of water, as shown:

Water (liters) x	3	6	9	12
Syrup (liters) y	1	2	3	4

- In this comparison, the ratio of the amount of syrup (y) to the amount of water (x) would be 1:3.
- The following graph could represent this relationship:

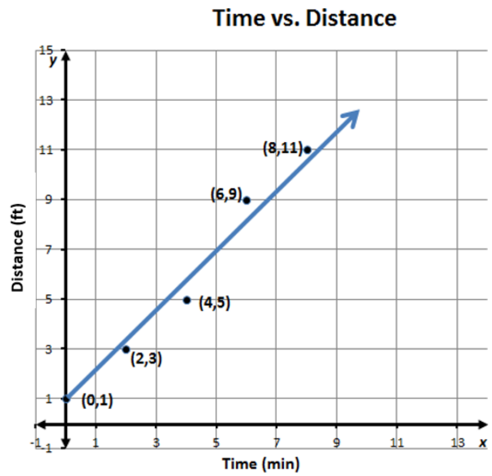


- Students should be aware of how the order in which quantities are compared affects the way in which the relationship is represented as a table of equivalent ratios or as a graph.
- Double number line diagrams can also be used to represent proportional relationships and determine pairs of equivalent ratios. See the example below.



  - In this proportional relationship, there are three liters of water for each liter of syrup represented on the number lines.
- A graph representing a proportional relationship includes ordered pairs that lie in a straight line that, if extended, would pass through (0, 0). The context of the problem and the type of data represented by the graph must be considered when determining whether the points are to be connected by a straight line on the graph.

- Example of the graph of a non-proportional relationship:



  - The relationship of distance (*y*) to time (*x*) is non-proportional. The ratio of *y* to *x* for each ordered pair is not equivalent. That is,

$$\frac{11}{8} \neq \frac{9}{6} \neq \frac{5}{4} \neq \frac{3}{2} \neq \frac{1}{0}$$

  - The points of the graph do not lie in a straight line. Additionally, the line does not pass through the point (0, 0), thus the relationship of *y* to *x* cannot be considered proportional.
- Contextual situations that model proportional relationships can typically be represented by graphs in the first quadrant, since in most contextual situations, the values for *x* and *y* are positive. Additionally, unit rates are typically positive in contextual situations involving proportional relationships.
- A unit rate could be used to find missing values in a ratio table.
  - Example: A store advertises a price of $25 for 5 DVDs. What would be the cost to purchase 2 DVDs? 3 DVDs? 4 DVDs?

# DVDs	1	2	3	4	5
Cost	$5	?	?	?	$25

The ratio of $25 per 5 DVDs is also equivalent to a ratio of $5 per 1 DVD, which would be the unit rate for this relationship. This unit rate could be used to find the missing value of the ratio table above. If we multiply the number of DVDs by a constant multiplier of 5 (the unit rate) we obtain the total cost. Thus, 2 DVDs would cost $10, 3 DVDs would cost $15, and 4 DVDs would cost $20.

- At this level, students should build a conceptual understanding of proportional relationships and unit rates before moving to more abstract representations and complex computations in higher grade levels. Students are not expected to use formal calculations for slope and unit rates (e.g., slope formula) in Grade 6.

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Reasoning:

- Use ratio and rate reasoning to solve real-world and mathematical problems. To do this, make tables of equivalent ratios relating quantities with whole number measurements; find missing values in the tables; and plot the pairs of values on the coordinate plane.
- When working to find a pattern in a table, students mistakenly look at only the pattern from row to row rather than thinking about how two quantities vary together. The table below represents a proportional relationship:

x	*y*
1	$\frac{7}{8}$
3	$2\frac{5}{8}$
5	$4\frac{3}{8}$
10	?

A common misconception for students is to use the increase in the values in the y-column from the completed rows in the table to determine the missing value for 10. This may indicate that a student is seeing the progression as an additive relationship. Help students move toward multiplicative reasoning by allowing them to explore both types of relationships. Use context in conjunction with manipulatives to model the proportional relationship. Double number line diagrams can also be used to represent proportional relationships and create collections of pairs of equivalent ratios.

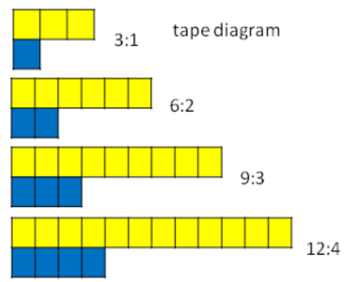
- To help students understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio a:b, use rate language in the context of a ratio relationship. For instance, *"This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is* $\frac{3}{4}$ *cup of flour for each cup of sugar."* or *"We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*

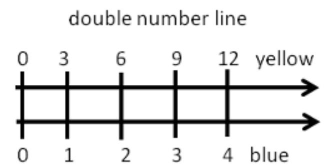
Mathematical Connections:

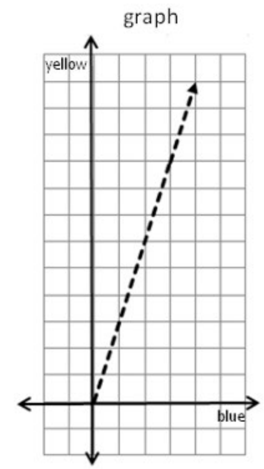
- By presenting representations simultaneously, students can make connections about how symbols represent actions or relationships embodied in physical models or diagrams. For example, the ratio of gallons of yellow paint to gallons of blue paint is 3:1 and could be represented in the following ways:

table

yellow	**3**	**6**	**9**	**12**
blue	**1**	**2**	**3**	**4**



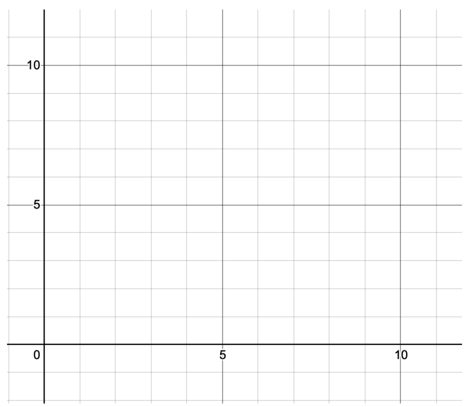




- In a related way, generalization questions ask students to identify patterns and use those patterns to make conjectures or generalizations. From the introduction of a topic to the final lesson in the instructional sequence, these guiding statements provide opportunities for students to think deeply about significant ideas. To use generalization questions, apply the following guidelines:
    - Have students identify patterns that they notice.
    - Ask students to name representations they are familiar with that work for the new problem.
    - Have students identify a strategy that they have learned that can be used to solve the problem.

Mathematical Representations:

- Use activities that promote the use of multiple representations. Have students publicly share their processes or solution approaches. Encourage students to show their thinking rather than only sharing an algorithm or step-by-step process. Consider options for students to critique each other's presentations.
  - **Example 1:** Complete the missing parts of the given scenario using the information provided about the proportional relationship.

<table>
<tr><th>Verbal Description</th><th>Table</th></tr>
<tr><td>I use $\frac{2}{5}$ cup of sugar per 2 cups of flour.</td><td><table><tr><th>Sugar (cups)</th><th>Flour (cups)</th></tr><tr><td>$\frac{2}{5}$</td><td>2</td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td></tr>
<tr><th>Graph</th><th>Connection</th></tr>
<tr><td>
</td><td>For this problem, the verbal description connects to the graph because:</td></tr>
</table>

Several misconceptions can be present as students work to connect a verbal description/ratio table to a graph. Some students may omit the ordered pair (0,0) when describing or identifying a proportional relationship from a table or a graph. Additionally, students may struggle with choosing which variable is on the *x*-axis and which variable goes on the *y*-axis when graphing ratios. Making the connection between the ordered pairs in the table and plotting them on the graph using their labeled axes proves to be challenging when connecting the two different representations.

- **Example 2:** Have students to work with a partner to complete a table like the following –

Number of hours worked	1	2	3	4	5	6			*x*
Amount of money earned	$7.50	$15.00	$22.50	$30.00			$75.00	$150.00	

- Analyzing authentic issues can help engage students in meaningful and exciting ways, which will help them to see how these representations connect. Allow for experiences where students are provided with data and decide how to represent that data in a graph. Ask students, "*How did you decide on the title of the x-axis? The y-axis? What would happen if you switched them?"*

- As students compare graphs that show proportional relationships, ask follow-up questions, such as, *"What do you notice about each graph? What do they have in common? What is different?"* Students also may benefit from concrete experiences with manipulatives, such as a geoboard, prior to drawing in the coordinate points.

Concepts and Connections

Concepts

Proportional relationships can be described, and generalizations can be made using patterns, relations, and functions. Algebraic equations and inequalities can be used to represent and solve real world problems.

Connections

- *Within the grade level/course*:
  - 6.PS.1 – The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graph.
  - 6.PFA.1 –The student will use ratios to represent relationships between quantities, including those in context.
- *Vertical Progression*:
  - 7.PFA.1 – The student will investigate and analyze proportional relationships between two quantities using verbal descriptions, tables, equations in $y = mx$ form, and graphs, including problems in context.

- *Digital Learning Integration:*
    - 6-8 CT.B. Students find or organize data and use appropriate technologies to interpret, analyze, and represent data to construct models, predict outcomes, solve problems, and make evidence-based decisions.

Across Content Areas

Reference 6.PFA.1

Resources to Support Local Curriculum

- A list of approved textbooks and instructional materials is posted on the VDOE website.
- Middle School Mathematics Formula Sheet (PDF)

6.PFA.3

The student will write and solve one-step linear equations in one variable, including contextual problems that require the solution of a one-step linear equation in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Identify and develop examples of the following algebraic vocabulary: equation, variable, expression, term, and coefficient.

b) Represent and solve one-step linear equations in one variable, using a variety of concrete manipulatives and pictorial representations (e.g., colored chips, algebra tiles, weights on a balance scale).

c) Apply properties of real numbers and properties of equality to solve a one-step equation in one variable. Coefficients are limited to integers and unit fractions. Numeric terms are limited to integers.

d) Confirm solutions to one-step linear equations in one variable using a variety of concrete manipulatives and pictorial representations (e.g., colored chips, algebra tiles, weights on a balance scale).

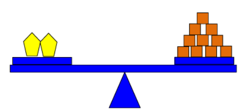
e) Write a one-step linear equation in one variable to represent a verbal situation, including those in context.

f) Create a verbal situation in context given a one-step linear equation in one variable.

Understanding the Standard

- An algebraic equation is a mathematical statement that says two expressions are equal (e.g., $2x + 7 = 15$).

$$2x = 10$$



- An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an "equal sign (=)" (e.g., 7, $\frac{3}{4}$, $5x$, $140 - 38.2$, $18 \cdot 21$, $5 + x$). An expression cannot be solved.

- A variable is a symbol used to represent an unknown quantity.

$$3 + x = 2.08 \qquad A = \pi r^2$$

- A term is a number, variable, product, or quotient in an expression of sums and/or differences. In the expression $7x^2 + 5x - 3$, there are three terms, $7x^2$, $5x$, and 3.

$$\underbrace{\tfrac{2}{3}a}_{\text{1 term}} \qquad \underbrace{-5x + (-2)}_{\text{2 terms}} \qquad \underbrace{3y^2 + 2y - 8}_{\text{3 terms}}$$

- A coefficient is the numerical factor in a term. In the term $3xy^2$, 3 is the coefficient; in the term z, 1 is the coefficient.

$$(-4) = 2x \qquad -7y \qquad \frac{1}{3}a = -5$$

- A one-step linear equation may include, but not be limited to, equations such as the following:
  - $2x = 5$
  - $y - 3 = -6$
  - $\frac{1}{5}x = {}^{-}3$
  - $a - (-4) = 11$
- A variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable.
- An algebraic expression is an expression that contains at least one variable (e.g., $x - 3$).
- A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used when the numbers are unknown. Example, the verbal expression "a number multiplied by 5" could be represented by the variable expression "$n \cdot 5$" or "$5n$."
- A verbal sentence is a complete word statement (e.g., "The sum of a number and two is five" could be represented by "$n + 2 = 5$").

Verbal	Algebraic
A number multiplied by 5	$5n$
The sum of negative two and a number	$-2 + n$
The sum of a number and two is five	$y + 2 = 5$
Negative three is one-fifth of a number	$-3 = \frac{1}{5}x$

- The solution to an equation is a value that makes it a true statement. Many equations have one solution and are represented as a point on a number line. Solving an equation or inequality involves a process of determining which value(s) from a specified set, if any, make the equation or inequality a true statement. Substitution can be used to determine whether a given value(s) makes an equation or inequality true.

- Word choice and language are very important when representing verbal situations in context using mathematical operations, equality, and variables. When presented with an equation or context, student choice of language should reflect the situation being modeled. Identifying the mathematical operation(s) to represent the action(s) and the variable to represent the unknown quantity will help students to write equations that represent the contextual situation.
- Properties of real numbers and properties of equality can be used to solve equations, justify equation solutions, and express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard).
  - Commutative property of addition: $a + b = b + a$
  - Commutative property of multiplication: $a \cdot b = b \cdot a$
  - Subtraction and division are neither commutative nor associative.
  - Identity property of addition (additive identity property): $a + 0 = a$ and $0 + a = a$
  - Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a$ and $1 \cdot a = a$
  - The additive identity is zero (0) because any number added to zero is equal to the number. The multiplicative identity is one (1) because any number multiplied by one is equal to the number. There are no identity elements for subtraction and division.
  - Inverses are numbers that combine with other numbers and result in identity elements.
  - Inverse property of addition (additive inverse property): $a + (-a) = 0 \; and \; (-a) + a = 0$ (e.g., 5 + (-5) = 0)
  - Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$ (e.g., $5 \cdot \frac{1}{5} = 1$)
  - Zero has no multiplicative inverse.
  - Multiplicative property of zero: $a \cdot 0 = 0$ and $0 \cdot a = 0$
  - Division by zero is not a possible mathematical operation. It is undefined.
  - Addition property of equality: If $a = b$, then $a + c = b + c$
  - Subtraction property of equality: If $a = b$, then $a - c = b - c$
  - Multiplication property of equality: If $a = b$, then $a \cdot c = b \cdot c$
  - Division property of equality: If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$
  - Substitution property: If $a = b$, then *b* can be substituted for *a* in any expression, equation, or inequality

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Problem Solving:

- General algorithms exist for solving many kinds of equations. These algorithms are broadly applicable for solving a wide range of similar equations. Students should be familiar with and apply the properties of real numbers and properties of equality to solve problems.
- Linear equations can be solved by symbolic, graphical, or numerical methods. Students may have trouble moving from solving using the concrete materials to solving numerically. It is important to link the two methods together before releasing students to solve solely using algebraic methods.
- Facilitate a discussion with students to help them translate or make sense of equations. Consider the following example –
  - Ashton determines that 32 is the solution to the equation $\frac{1}{4}x = 8$. How can Ashton confirm his solution is correct?

  A common misconception some students may have is thinking that 32 must equal the product of $\frac{1}{4}x$. This may indicate that a student interpreted eight as the value of x and incorrectly multiplied eight times four to obtain a value of thirty-two in the denominator, ($\frac{x}{4} \cdot 8$) = 32. It may be beneficial to have students write the given equation as $\frac{x}{4}$ = 8. Have students think about a context that might fit the equation. For example, how many objects would be needed to make four equal groups of eight objects? Each of eight students received the same number of crayons, how many total crayons would be needed for each student to receive four crayons?

Mathematical Reasoning: Equations are solved as a process of reasoning using properties of equality, which can justify each step of the process. Provide students with example like the following and when doing so, ask students to justify each step in the process and their solutions. Common misconceptions are provided below –

- Solve each equation and justify your solution: -10 = *h* + 14 2*b*= 48 $\frac{n}{5}$ = -10
  - A common misconception some students may have when solving -10 = *h* + 14 is to add fourteen to both sides of the equation.
  - A common misconception some student may have when solving $\frac{n}{5} = -10$ is to divide both sides of the equation by 5.

  Each of these misconceptions may indicate that the student sees an addition and division type equation, *h* + 14 and $\frac{n}{5}$, and uses the same operation to solve the equation instead of using an inverse operation.

  - A common misconception that some students may have when solving 2*b* = 48 is to subtract two from both sides of the equation. This may indicate that a student interprets the coefficient as a +2 but thinks subtracting two is the appropriate inverse operation to solve the equation.

It may be beneficial to have students use models to represent the equations when applicable. It would also be helpful to have students work with balance scales in conjunction with open sentences to develop the connection between the equal sign and the expression on each side of the equal sign. In addition, as students are solving equations, include verbal descriptions that explain the meaning of the equation. Encourage students to explain their thinking and even try to determine more than one way to solve each equation.

- Hillary determines that 32 is the solution to the equation $\frac{1}{4}x$ = 8. How can Hillary confirm her solution is correct?

A common misconception some students may make is to interpret that 32 must equal the product of $\frac{1}{4}x$. This may indicate that a student interprets eight as the value of *x* and incorrectly multiplies eight times four to obtain a value of 32 in the denominator, $\left(\frac{1}{4 \cdot 8}\right)$ = 32. It may be beneficial to have students write the given equation as $\frac{x}{4} = 8$. The teacher can facilitate a discussion with students to help them translate or make sense of this equation. Have students think about a context that might fit the equation. Ask students, "*How many objects would be needed to make four equal groups of eight objects? If each of eight students received the same number of crayons, how many total crayons would be needed for each student to receive four crayons?"*

Mathematical Connections: Writing words to represent equations and expressions can be especially challenging for students. To help students make the connection between verbal expressions and sentences, use concrete materials, such as algebra tiles, balance scales, or unifix cubes. Starting with contextual problems, students can use sticky notes or chart paper to add labels to the models they are creating. For example, consider creating the following chart and having students fill in the missing information:

Verbal Expression/Sentence	Algebraic Expression/Equation
Marco is twice as old as Ella.	
x is 5 fewer than y	
	$\frac{r}{3}$
	$z = 5 + p$

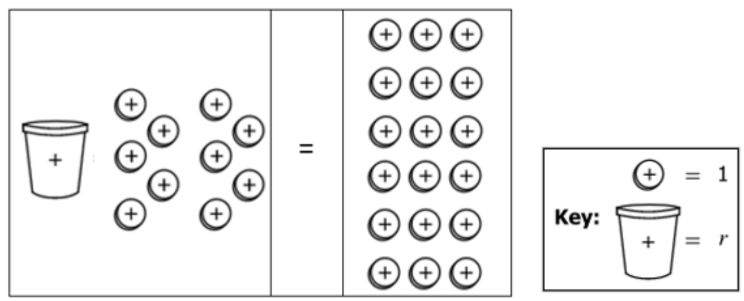
Students might write the equation 2M = E in the first row, or x – 5 = y in the second row. Ask students to create models and discuss "*What exactly is showing us Marco's age?" "Ella's age?" "How does your representation show the relationship between the two?"*

Additionally, it is helpful to have the students say the meaning of the equation or expression in their own words. What is the relationship between Marco and Ella's ages? What is the relationship between x and y? How do you know? Which is fewer/greater?

Lastly, students can work to correct this misconception by comparing examples and non-examples. Have students engage in discussions about why they decided certain examples were correct or incorrect. For example, for x = 5 + y, students could compare two different written descriptions: Paula is 5 years older than Zoe and Zoe is 5 years older than Paula. Which one makes sense and why? Again, to support students further, manipulatives can be used to model each scenario with labels to strengthen students' discussion.

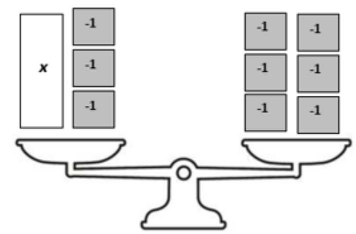
Mathematical Representations: Students must have opportunities to use concrete and pictorial representations of solving equations before proceeding to the algorithm. Remember, concrete manipulatives like algebra tiles make mathematics visual and provide the conceptual understanding that students need to successfully transition to the algorithm. As students solve equations, have them write out what is happening as they are using manipulatives to solve an equation (e.g., making sure to say [and make students say], "2 times *x* equals 6" as opposed to "2*x* = 6"). Examples with common misconceptions follow –

- **Equation Mat:** Write an equation to represent the model shown.



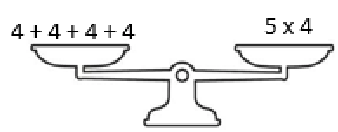
Students may incorrectly interpret the left side of the equation mat as 10*r*. This may indicate a student believes the number of unit counters on the left side of the mat represents the coefficient of *r*. It may be beneficial to have students think about the shapes that are used in the model and replace each shape in the mat with the value as designated in the available key. Making a connection between like shapes in the equation mat and like terms of an equation would be helpful as well.

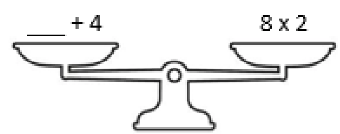
- **Multiple representations (contextual situation, model, equation, and justifying a solution):** Complete the missing parts of the grid using the information provided about the contextual problem.

<table>
<tr><th>Contextual Problem</th><th>Model</th></tr>
<tr><td>The temperature dropped three degrees. Now the thermometer says -6 degrees. What was the temperature before it dropped?</td><td>
</td></tr>
<tr><th>Equation</th><th>Solution</th></tr>
<tr><td></td><td>$x =$ ___________</td></tr>
<tr><th colspan="2">Explain and justify your answer:</th></tr>
<tr><td colspan="2"></td></tr>
</table>

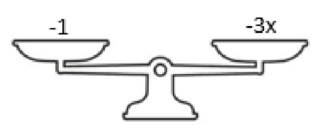
In justifying a solution for this problem, students must be able to make sense of the model and context to explain why their solution makes sense. Confirming solutions can prove to be difficult for students if they are only thinking procedurally.

If students need more support in connecting models and equations, consider using true/false sentences. Use a balance scale with a modeled equation or concrete objects to represent an equation (such as algebra tiles) and ask, "*Is this true or false? How do you know*? Students should support their reasoning by explaining their thinking. Here are some examples: *What would make this tilt? Which way would it tilt? What would make it balance?"*





Students could benefit from using balance scales when solving equations. Having a concrete tool to model if something is only performed to one side of the balance scale; it changes the balance (relationship). To maintain balance, you would also have to make a change to the other side. Consider giving scales with variables to students and asking students, *"How could we change these to keep the balance?"* For example, here we would divide both sides by -3:



Lastly, plotting the value of *x* (or any variable) on a number line might help students to see the value of the variable, keeping in mind that sometimes variables can be multiple values and sometimes only one value makes the equation true.

Concepts and Connections

Concepts

Proportional relationships can be described, and generalizations can be made using patterns, relations, and functions. Algebraic equations and inequalities can be used to represent and solve real world problems.

Connections

- *Within the grade level/course*:
  - 6.PFA.4 – The student will represent a contextual situation using a linear inequality in one variable with symbols and graphs on a number line.
- *Vertical Progression*:
  - 5.PFA.2 – The student will investigate and use variables in contextual problems.
  - 7.PFA.2 – The student will simplify numerical expressions, simplify and generate equivalent algebraic expressions in one variable, and evaluate algebraic expressions for given replacement values of the variables.

Across Content Areas

Reference 6.PFA.1

Resources to Support Local Curriculum

- A list of approved textbooks and instructional materials is posted on the VDOE website.
- Middle School Mathematics Formula Sheet (PDF)

6.PFA.4

The student will represent a contextual situation using a linear inequality in one variable with symbols and graphs on a number line.

Students will demonstrate the following Knowledge and Skills:

a) Given the graph of a linear inequality in one variable on a number line, represent the inequality in two equivalent ways (e.g., $x < -5$ or $-5 > x$) using symbols. Symbols include <, >, ≤, ≥.

b) Write a linear inequality in one variable to represent a given constraint or condition in context or given a graph on a number line.

c) Given a linear inequality in one variable, create a corresponding contextual situation or create a number line graph.

d) Use substitution or a number line graph to justify whether a given number in a specified set makes a linear inequality in one variable true.

e) Identify a numerical value(s) that is part of the solution set of a given inequality in one variable.

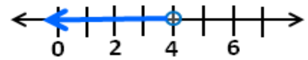
Understanding the Standard

- The solution set to an inequality is the set of all numbers that make the inequality true.
- Inequalities can represent contextual situations.
  - Example: Jaxon works at least 4 hours per week mowing lawns. Write an inequality representing this situation and graph the solution.

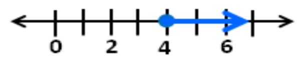
$$x \geq 4 \text{ or } 4 \leq x$$

  - Students might then be asked: "Would Jaxon ever work 3 hours in a week? 6 hours?
- The variable in an inequality may represent values that are limited by the context of the problem or situation. For example, if the variable represents all children in a classroom who are taller than 36 inches, the variable will be limited to have a minimum and maximum value based on the heights of the children. Students are not expected to represent these situations with a compound inequality (e.g., $36 < x < 70$) but only recognize that the values satisfying the single inequality, $x > 36$, will be limited by the context of the situation.
- Inequalities using the < or > symbols are represented on a number line with an open circle on the number and a shaded ray in the direction of the solution set.

- Example: When graphing $x < 4$, use an open circle on the 4 to indicate that the 4 is not included in the solution set.

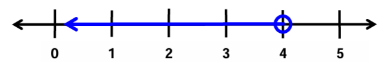


- Inequalities using the ≤ or ≥ symbols are represented on a number line with a closed circle on the number and a shaded ray in the direction of the solution set.
  - Example: When graphing $x \geq 4$, fill in the circle on the 4 to indicate that the 4 is included in the solution set.



- It is important for students to see inequalities written with the variable before the inequality symbol and after. Example: $y < 4$ is not the same relationship as $4 < y$. However, $y < 4$ is the same relationship as $4 > y$.

$y < 4$ or $4 > y$



Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Reasoning: There are several misconceptions that students may experience when writing a linear inequality in one variable to represent a given constraint or condition in context or given a graph on a number line. Examples with common misconceptions follow –

- Joe sent at least 100 texts yesterday. Write an inequality to represent the number of texts Joe might have sent and explain your reasoning. Then, give three possible numbers that could represent a solution to the inequality and explain why these solutions would work.

A common error made by some students is to use the less than or equal to sign, rather than the greater than or equal to sign because they associate "at least" with less than. To assist students in understanding what "at least" means conceptually, ask them questions like: *If Joe sent at least 100 texts yesterday, could he have sent 101 texts? Could he have sent 105 texts? Could he have sent 90 texts?* Substituting possible values for the solutions will help students conceptualize "at least." Additionally, having students examine "at most" in the same fashion will also help them to understand the difference between these two phrases and the associated inequalities.

- Callie sold 22 tickets to the school talent show. She sold more than twice the number of tickets that Ava sold. Write an inequality to show how many tickets Ava might have sold. Explain your reasoning.

Writing inequalities that match the words in story problems is a common challenge related to algebraic thinking. Here, a student might write an inequality that has “22*c*” or might not understand that Ava sold more. To help students make the connection between verbal descriptions and inequalities, use concrete materials, such as algebra tiles, balance scales, or unifix cubes. Ask questions like, “*What is the relationship between tickets sold by Callie and tickets sold by Ava? Who sold more? How do you know?”*

Starting with contextual problems, students can use post-its or larger paper to add labels to the models they are creating: “*What exactly is showing us Callie’s tickets? Ava’s tickets? How does your representations show the relationship between Ava and Callie’s tickets?”* They can also do this by substituting in numbers for the inequalities that would make sense: “*What are the possibilities that would make it true?”*

Mathematical Representations: Students should have intentional exposure to creating an inequality from a verbal description, graphing the inequality using the appropriate symbols, and explaining their answers. A four-part graphic organizer like the one below, when given the verbal description, and having students to complete the remaining areas is an example of capturing the intention of this standard –

Verbal description: Luca has at most 10 brownies to share with his friends.	**Inequality (using symbols):**
Explain:	**Graph of the Inequality:**

- This problem is designed to uncover student understanding about comparison vocabulary. Students are most likely familiar with “less” and “more” but may have misunderstandings about how those vocabulary terms connect to the symbols. If student explanations demonstrate a misunderstanding of the comparison vocabulary, have class discussions about what it means to say, “less than” or “less than or equal to,” having students debate which signs make the most sense given specific situations. Here, they can also use a pictorial representation such as a number line to prove their solutions. Ask questions like: “*If this is the inequality, would Luca be able to have 11 brownies? How do you know? Does this inequality make sense with the context of the story? How do you know? How would the context change with each that does not match?”*

- As students continue their work in writing inequalities to match verbal descriptions, have them write an example and non-example for each. Conversely, give similar inequalities and have students write the verbal expression or story context that would make sense for each. Ask, "*How are these alike? How are they different?*"

- Consider the given graphic organizer and then replacing another section such as the graph of the inequality, and having students complete the remaining sections to draw not only representations, but also mathematical connections.

Concepts and Connections

Concepts

Proportional relationships can be described, and generalizations can be made using patterns, relations, and functions. Algebraic equations and inequalities can be used to represent and solve real world problems.

Connections

- *Within the grade level/course*:
  - 6.PFA.3 – The student will write and solve one-step linear equations in one variable, including contextual problems that require the solution of a one-step linear equation in one variable.
- *Vertical Progression*:
  - 5.PFA.2 – The student will investigate and use variables in contextual problems.
  - 7.PFA.2 – The student will simplify numerical expressions, simplify and generate equivalent algebraic expressions in one variable, and evaluate algebraic expressions for given replacement values of the variables.

Across Content Areas

Reference 6.PFA.1

Resources to Support Local Curriculum

- A list of approved textbooks and instructional materials is posted on the VDOE website.
- Middle School Mathematics Formula Sheet (PDF)