

# Mathematics Playbook: Instructional Structures and Supports



Implementation Guidance Document



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# Part 1: Supporting Divisions in Implementing the Mathematics *Standards of Learning*

## Chapter 1: Overview

Every student in a Virginia classroom deserves access to exceptional mathematics instruction, which includes developing and utilizing the mathematics process goals as they build and demonstrate mastery of the mathematical concepts encompassed in the 2023 Mathematics *Standards of Learning*. The Virginia 2023 Mathematics *Standards of Learning* are “best in class” and are benchmarked against NAEP, ACT, and SAT. These clear, rigorous learning standards set expectations for mathematics instruction needed to prepare graduates with the mathematical skills and concepts for post-graduate success. Standards outline the expectations for students; however, teachers are critical in the effective implementation of the standards. Students need access to a curriculum that builds on a systematic progression in foundational skills and builds coherently grade-to-grade and content-to-content on mathematical concepts. Instruction that includes evidence-based strategies, differentiation to meet the needs of each student, scaffolding of mathematics concepts, and the application of the mathematical processes requires endorsed teachers, instructional resources, and professional development.

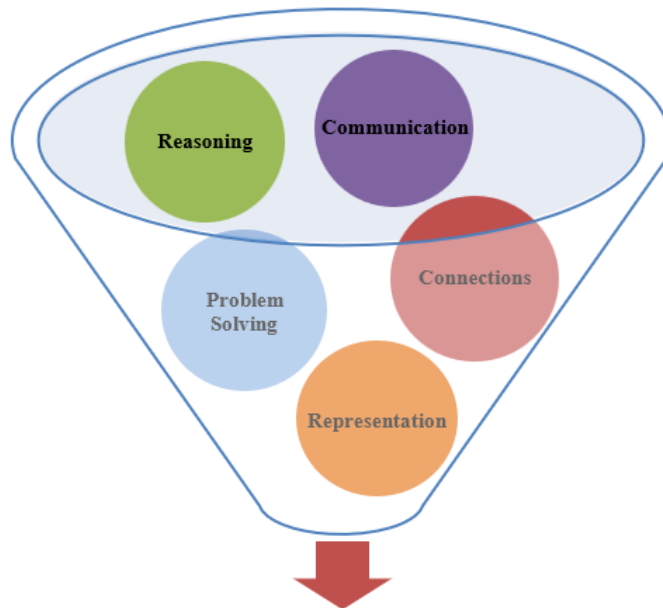
### *The 2023 Mathematics Standards of Learning*

The *Standards of Learning* provide a framework for instructional content to raise the academic achievement of all students in Virginia and prepare them for college and careers.

The 2023 *Mathematics Standards* were revised with input from parents, teachers, the business community, school administrators, representatives from higher education, and state mathematics education organizations. The *Standards* set clear and rigorous academic expectations for students. Parents and families are encouraged to work with their children, their children’s teachers, and their children’s schools to help them achieve these academic standards.

### *The Mathematical Process Goals*

Students learn and apply the five mathematical process goals as they work to achieve the content of the Mathematics *Standards*. These processes support students in becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret contextual situations. Contextual situations include real-world problems and problems that model real-world situations.



## **Mathematical Understanding**

### *Mathematical Problem Solving*

Students apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students should recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

### *Mathematical Communication*

Students communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics helps students clarify their thinking and deepen their understanding of the mathematical concepts being studied. Mathematical communication becomes visible when learning involves participation in mathematical discussions.

### *Mathematical Reasoning*

Students recognize reasoning and proof as fundamental aspects of mathematics. Students should learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

### *Mathematical Connections*

Students build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the application of content and process skills, students make connections among different areas of mathematics, between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

### *Mathematical Representations*

Students represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students should understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

## Chapter 2: Establishing a Vision for Mathematics Instruction

### *Mathematics Identity*

An initial phase in formulating a mathematics vision involves assessing the current state of mathematics within the division. This begins with an examination of stakeholders' perspectives and beliefs about mathematics. For instance, posing the question, "Who considers themselves a 'math person'?" often elicits a varied response, highlighting differing levels of confidence and identification with mathematics.

Students' dispositions and deeply ingrained beliefs regarding their aptitude for engaging with and succeeding in mathematics have a substantial impact on their application of mathematical concepts across diverse situations throughout their lives. Teachers, peers, and parents significantly influence the formation of students' mathematical identity. Educational resources, instructional materials, and pedagogical strategies structure learning experiences that enable students to cultivate the five mathematical process goals: becoming proficient problem solvers, communicating mathematically effectively, reasoning logically, establishing meaningful mathematical connections, and employing mathematical representations to model and interpret practical scenarios (National Research Council, 2001).

Educators' mathematics identities may impact student outcomes in divisions, schools, and classrooms. It is important for leaders to reflect on their own experiences and identities as mathematicians before starting to build a vision for strong instruction in mathematics and the new standards. Reflection questions to engage and to frame discussions with teachers and others that support mathematics instruction can be –

- What did you experience as a student of mathematics?
- What personal successes and/or struggles did you have in mathematics classes?
- What do you value in mathematics and in your experiences?
- Who was the first educator or mathematics teacher that made an impact on you?
- What were pivotal experiences and/or decisions that led to where you are today in your mathematics educator journey?

*What comes naturally to children in mathematics?* The statements articulated by cognitive scientist, Daniel Willingham (2010) may challenge how stakeholders view the teaching and learning of mathematics –

“They have a natural sense that allows them to understand and manipulate very small quantities with precision, and much larger quantities in approximation.”

“Those abilities are a far cry from the abilities teachers hope to develop in their students, but they are the base upon which teachers can build.”

“For educators, that means we should not expect students will learn mathematics with ease. Rather, we should expect that mathematical proficiency will require careful cultivation and will develop slowly. At the same time, we should keep in mind that students are born with the ability to learn math, and we should not let students give up by concluding that they’re just no good at math.”

No one is born knowing mathematics, and no one is born lacking the ability to learn mathematics. Research indicates that all humans are born with some innate sense of quantity and a sense that numbers and space are related. These studies conclude that everyone can learn mathematics with the right teaching and messaging.

Furthermore, Willingham (2010) clarifies that the notion of a distinct "math person" is inaccurate. While variations in aptitude exist, most individuals possess the capacity to learn mathematics at advanced levels.

The dispositions and deeply held beliefs that students develop about their ability to participate in and perform well in mathematics impact and influence their use of mathematics in a variety of contexts throughout their lifetime. Teachers, peers, and parents influence how the mathematics identity of students develops. Resources, materials, and lessons frame the learning experiences to support students in using the five process goals of becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations (National Research Council, 2001). Conceptual understanding, procedural fluency, and procedural flexibility represent distinct yet interconnected mathematical competencies; mathematical proficiency emerges when students develop these and form connections between them. In short, everyone is a "math person", and educators must lean into this identity when supporting students in the classroom. The capacity for mathematical thinking is universal, and educators should actively foster this identity in all students within the classroom.

### *Developing a Vision for Mathematics Instruction*

Following the educators' and leaders' reflection on their personal mathematics identities, the subsequent step involves collaboratively developing and implementing a division-wide vision for mathematics education. This process should include a comprehensive analysis

of current mathematics instructional practices across the division. Key questions to consider during this analysis include:

- Do our students and educators perceive themselves as capable in mathematics?
- What prevailing pedagogical approaches are evident in our mathematics classrooms? Are there discernible patterns in mathematics instruction across different schools or throughout the division?
- How do the existing mindsets of teachers, mathematics specialists, and school and division leaders influence instructional practices?

The responses to these inquiries can provide valuable insights into the prevailing school culture, specifically concerning mathematics instruction. School culture, encompassing the beliefs and attitudes that shape all facets of a school's operation, can be leveraged by instructional leaders as a tool to foster collaboration among staff and exert a positive influence on student achievement.

Research suggests that a common culture prevalent in mathematics classrooms is centered on "answer-getting." Conversely, effective mathematics instruction necessitates a deliberate emphasis on cultivating a culture of "Mathematizing" rather than solely focusing on obtaining correct answers. Harris (2020) characterizes a mathematizing classroom culture as one where educators encourage students to engage in discussion, collaborate, connect new concepts to prior knowledge, and develop models and justifications as integral components of the learning process. In contrast, an "answer-getting" approach utilizes instructional tasks primarily to teach students the procedural steps required to arrive at a correct solution. A classroom culture that prioritizes answer-getting often results in students who can follow specific procedures to solve familiar problems but struggle to apply those procedures effectively in unfamiliar or novel contexts.

A mathematizing culture fosters greater student agency as they investigate mathematical concepts, apply the mathematics process goals to problem-solving, and cultivate a deeper conceptual understanding.

In classrooms that prioritize mathematizing over answer-getting, students actively engage with both the mathematical content and the underlying processes, cultivate a collaborative learning environment through peer interaction, and apply their comprehension to challenging, grade-level appropriate tasks. Teachers sustain the inherent mathematical complexity through their skillful facilitation of learning. Furthermore, within a mathematizing classroom, students enhance their mathematical learning by reflecting on newly acquired knowledge and establishing connections to previous learning and experiences.

### *The Importance of a Division Mathematics Vision*



To transform mathematics instruction, division and school leaders should articulate a clear vision to teachers, building administrators, and staff. This transparent communication fosters buy-in and ensures a unified pursuit of common objectives. It also aids in reducing the natural resistance to change often associated with the introduction of new standards and instructional resources.

As divisions formulate their mathematics vision, it is essential to consider the current and desired mathematics culture, as well as evidence-based best practices in mathematics instruction. A comprehensive division mathematics vision integrates educator mindsets, the prevailing school culture, student needs, aspirational student outcomes, and community priorities. The following reflection questions can facilitate the development of this vision and guide discussions with teachers and other stakeholders who support mathematics instruction:

- What attributes does the division value in mathematically literate graduates?
- How do we define mathematical success for our students?
- What do our data reveal regarding gaps in the current mathematics program?
- How does our professional development plan support teachers in implementing new standards and utilizing instructional materials effectively?
- What does effective instruction look, sound, and feel like for both teachers and students?
- What critical elements must be included in the vision for mathematics within our school division?

#### Resources:

The following are key resources on the importance of mathematics identity –

- Aguirre, J. et al (2013). The impact of identity in K-8 mathematics.
- National Research Council. (2001). Adding it up: Helping children learn mathematics. J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.
- Willingham, D. T. (2010). [Ask the cognitive scientist: It is true that some people just can't do math?](#)

The following are key resources on the importance of a vision for mathematics –

- Education Elements (2024). [From Vision to Practice: A Comprehensive Guide to High Quality Instructional Materials.](#)
- Harris (2020). [Answer-Getting Culture.](#)
- TNTP (2021). [Vision of Strong Instruction in Mathematics.](#)

## Chapter 3: Critical Supports for Mathematics Instruction

The 2023 Mathematics *Standards of Learning* reflect best-in-class standards for students in Virginia. Mastery of these standards is dependent on effective, evidence-based instruction that is differentiated to the needs of the individual learners in the classroom. Multiple factors come into play when developing learning environments that support students, including

- Endorsed teachers
- High-quality instructional materials (HQIM)
- Professional development
- Consistent principal and division support and feedback

The absence of any of these factors will impact student outcomes.

### *Endorsed Teachers*

Endorsed teachers are critical in creating learning environments that meet the needs of students. Teachers who hold high expectations and believe students can meet grade-level standards create classroom environments where students are engaged in doing the math. Teachers create learning environments that allow students to justify how they reach a solution to a mathematical scenario, communicate with their peers and the teacher on the steps they utilize during the problem-solving process and when making connections between and among concepts, utilize representations to problem solve and model their understanding, when using the data cycle to collect, analyze, and interpret data, and when they apply their reasoning skills when solving contextual problems.

### *High-Quality Instructional Materials*

High-quality instructional materials (HQIM) play a crucial role in supporting and enhancing instruction by integrating grade-level tasks, promoting effective teaching through evidence-based strategies, fostering opportunities for deep student engagement in mathematics, and aligning with the rigorous expectations of the 2023 Mathematics *Standards of Learning*. These materials serve as a primary resource for teachers in implementing effective instructional practices.

HQIM is anchored in the content of the standards and incorporates evidence-based strategies that teachers can readily employ within the learning environment. By providing well-designed lesson plans and resources, HQIM allows educators to redirect time typically spent on lesson development or resource curation towards focused lesson implementation and the planning of differentiated instruction to address the diverse learning needs within their classrooms.

### *Professional Development*

Sustained professional development is critical in developing teacher pedagogy and in implementing any instructional tool, particularly HQIM. Ongoing professional development on the use of HQIM allows teachers to implement HQIM and utilize the evidence-based practices needed for students to demonstrate content mastery. This professional

development is critical for school and division leadership as they support teachers. Divisions should launch their mathematics vision with a clear plan and all necessary supports in place. This will ensure that teachers and leaders are prepared to tackle the challenging task of standard implementation.

An effective plan of support will include professional learning to familiarize teachers and leaders with new materials and evidence-based mathematics instructional practices, and regular support and development structures such as weekly lesson planning and preparation cycles and frequent HQIM-based classroom walk-throughs, coaching, and feedback.

Professional development to support teachers with the implementation of standards and HQIM should focus on deep dives into the specific standards, hands-on practice with the HQIM materials, collaborative HQIM unit reviews, and ongoing feedback to ensure teachers are effectively aligning instruction with the learning objectives outlined in the standards, all while utilizing the strengths of the chosen HQIM. Key components of the sustained professional development should include:

- Standards-based curriculum analysis:
  - Thorough review of the mathematics standards to understand the depth and complexity of each standard
  - Identifying key concepts and the mathematical processes within the standards
  - Discussions on how the standards may be interconnected across different subject areas, particularly in Grades K-5
- HQIM deep dive:
  - Exploring the structures of the HQIM materials, including lesson plans, assessments, and supporting resources
  - Examining how the HQIM aligns with specific standards and determining any gap in content that may exist between the HQIM and the Virginia standard expectations
  - Identify key instructional strategies embedded in the materials
- Collaborative lesson planning
  - Working in teams to develop standards-aligned lessons using the HQIM materials to address gaps in standard expectations
  - Discussing how to differentiate instruction to meet the needs of all learners within the lesson plan
  - Share best practices and strategies to support the effective implementation process

If teachers are provided instructional materials, professional learning communities can shift from individual and collaborative lesson planning to lesson implementation. The lesson implementation cycle starts with collaboration, often conducted within professional learning communities. Teachers engage with the materials and plan for implementation, deliver lessons, and use feedback (data from student performance, principal walkthrough discussions, etc.) to inform future instruction.

### *Consistent principal and division support and feedback*

Principals, mathematics coaches, and division mathematics specialists are critical for teacher and student success. These personnel provide expertise in mathematics instruction, and their feedback should consistently support and reinforce the mathematics vision. The VDOE Mathematics Walkthrough Tool was created to support division leaders, principals, mathematics coaches, and other instructional leaders with classroom observations and inform teacher feedback.

The lesson implementation cycle includes lesson feedback to promote a mathematics vision. Classroom observations are to be used to construct feedback cycles to ensure instruction and pedagogical methodologies are aligned with the guiding principles of the 2023 Mathematics *Standards of Learning*, and to inform the teachers on how to identify areas of focus for planning, acting, and continuously improving.

#### **Lesson Preparation Meeting**

Teachers and others prepare for daily lesson delivery of approved high-quality instructional materials (HQIM).

#### **Lesson Feedback**

School leaders attend lesson feedback sessions to support teachers, discussing what worked well and what is needed for better implementation of HQIM in future lessons.



#### **Lesson Delivery**

Teachers deliver lessons grounded in HQIM; School leaders walk through lesson delivery regularly.

#### **Lesson Walk Throughs**

School leaders conduct non-evaluative lesson walk throughs to support teachers on lesson delivery of HQIM grounded lessons followed by lesson feedback sessions.

## Chapter 4: Shifting Mindsets and Defining Clear Roles and Responsibilities

### *Shifting Mindsets*

Transforming mathematics teaching and learning necessitates a corresponding shift in the mindsets of educators, students, and other stakeholders. Modifying instructional practices requires changing the prevailing attitudes of teachers, leaders, families, and students. The existing mindset must evolve towards a growth-oriented perspective that aligns with the division's vision for mathematics education.

For a mathematics vision to be successfully achieved, it is imperative that divisions explicitly define the roles and responsibilities of all stakeholders involved. Deficiencies in communication or unclear role assignments can significantly hinder the implementation process. To illustrate, if division leadership has well-articulated responsibilities concerning the implementation of High-Quality Instructional Materials (HQIM), yet school leaders lack clearly communicated roles, the effectiveness of the implementation is likely to be compromised.

A vision for effective mathematics instruction is best realized through endorsed teachers, high-quality instructional materials (HQIM), targeted professional development, and consistent support and feedback from principals and division leaders. A critical initial step in implementing a mathematics program is establishing a clear vision that articulates expectations for exemplary teaching and learning. While materials are essential for enacting the division's vision, the capacity of teachers to internalize and implement effective instruction is equally central to achieving that vision and improving student outcomes.

The adoption of instructional materials transcends mere budgetary allocation and procurement. It necessitates a complex system of interconnected changes to ensure coherence among all stakeholders, including students, educators, families, and the community, as they work collaboratively towards a shared vision. Consequently, the adoption of new materials will necessitate adjustments across various elements of the educational system.

One main shift that HQIM allows for is a move away from lesson planning towards lesson preparation. When teachers participate in lesson planning, they spend significant time determining what will be completed in each class period. Using HQIM allows teachers to focus on lesson preparation where they develop deeper content knowledge and spend more time focusing on learning outcomes and differentiation. Lesson preparation provides an opportunity for teachers to study and grow content knowledge, practice instructional moves and routines, receive feedback from peers and instructional leaders, and reflect on learning and practices.



### Study & Grow Content Knowledge

- Use curricular materials as primary source of content
- Facilitator leads participants in collaborative IP for upcoming lessons
- Facilitator creates opportunities to build either teacher content knowledge or pedagogical knowledge (or both!)

### Practice

- Facilitator creates opportunities for rehearsal of planning routines, instructional moves/routines, facilitating classroom discourse, use of precise language, or key representations that are aligned with relevant upcoming lessons

### Feedback

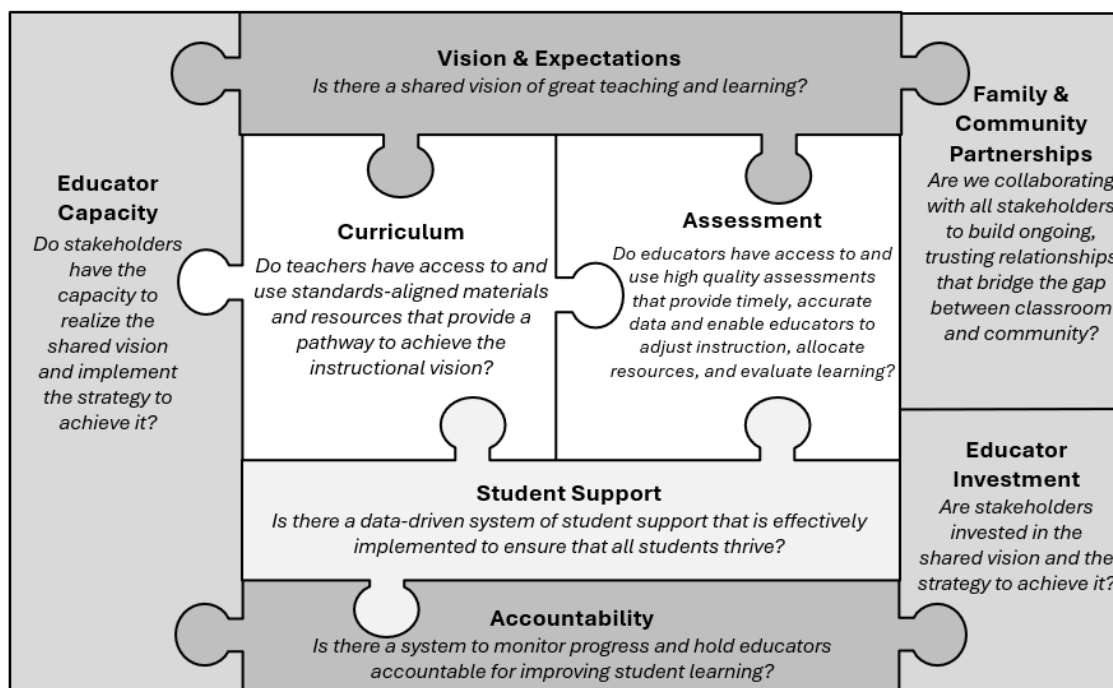
- Facilitator provides individual or batch feedback to participants in relation to practice or recent classroom observations
- Facilitator provides opportunities for peer-to-peer feedback
- Participants have the opportunity to make adjustments in response to feedback

### Reflection

- Participants have the opportunity to reflect on their learning or instructional practices

### Recommended Roles and Responsibilities in HQIM Implementation

In preparation for the initial implementation of High-Quality Instructional Materials (HQIM), divisions should prioritize actively engaging and investing stakeholders in the HQIM transition. This includes carefully planning the processes and procedures necessary to establish the conditions conducive to successful HQIM implementation.



Below is a list of the key responsibilities for all division stakeholders in implementing HQIM and developing effective mathematics programs. Additional support for implementing HQIM can be found in the <Virginia Mathematics Instructional Implementation Tool>.

### **Superintendent**

Role: Vision setter, Investment Builder, Expectation Setter, Accountability

Responsibilities:

- Provide feedback on academic experiences and goals.
- Understand the HQIM research; Build investment and understanding in the need for HQIM adoption.
- Establish a division-level HQIM lead (e.g. CAO, Mathematics/Curriculum Director, etc.) to spearhead adoption and implementation efforts.
- Work with HQIM lead to:
  - Assess current state of instruction
  - Establish a division-wide vision for strong instruction in mathematics
  - Establish mathematics/HQIM priorities and goals
  - Develop an HQIM-based strategy
- Approve and purchase selected HQIM.
- Communicate adoption decision. Begin to build investment in the adoption.
- Set and communicate clear expectations pertaining to HQIM use as a key driver toward the division’s vision for strong instruction in mathematics.
- Ensure time, structures, and supports are put in place to allow teachers and leaders sufficient opportunity to understand and build investment in new HQIM.

### **Division Mathematics/ HQIM Lead (Key Instructional Architect)**

Role: Same as Division Leader plus – Strategy developer, Lead HQIM trainer, HQIM

Adoption Committee Leader

Responsibilities:

- Understand the HQIM research; build investment and understanding in the need for HQIM adoption.
- Assess current state of instruction and develop an HQIM--based strategy.
- Communicate the division’s vision for strong mathematics instruction, priorities, goals, and strategy.
- Establish and train curriculum adoption committee/s consisting of key stakeholders. Solicit input from a wide array of stakeholders.
- With adoption committee, review curricula and make adoption recommendation.
- Design structures and supports for unit and lesson preparation, collaboration, learning, and feedback for teachers and leaders.
- Create HQIM-based development plans for teachers, school leaders and other support staff
- Set clear expectations around HQIM use. Communicate expectations, roles, and responsibilities to all stakeholders.

### **School Leadership (Principal and Mathematics Support)**

Role: School-level Investment Builder, Expectation Setter, Implementation Lead

Responsibilities:

- Understand the HQIM research; build investment and understanding across teachers in the need for HQIM adoption.
- Communicate the division’s vision for strong mathematics instruction, priorities, goals, and strategy.
- Understand and develop expertise in the design and research of adopted HQIM.
- Put in place structures, supports, and roles to enable successful HQIM implementation.
- Develop expertise (your own or others) in supporting lesson preparation and implementation of adopted HQIM.
- Lead school-level development to promote understanding of adopted HQIM
- Develop progress monitoring plan.
- Ensure understanding of any new progress monitoring tools, etc.

### **Teachers**

Role: Expertise Builder

Responsibilities:

- Understand the research behind HQIM and why quality materials matter.
- Provide input and feedback on HQIM selection.
- Through training, understand the design of adopted HQIM and how the materials align to the division’s vision for strong instruction in mathematics.
- Commit to using adopted HQIM with consistency, intentionality, and integrity.
- Through training, understand any new planning procedures, tools, and expectations such as unit- and lesson-level HQIM-based lesson preparation and implementation.
- Carefully review and organize new materials, ensuring understanding prior to implementation.
- Ask questions and collaborate to build understanding of new materials.

### **Students**

Role: Input Provider, Owner of Learning

Responsibilities:

- Provide input and feedback on HQIM selection.
- With guided support, understand learning expectations for the grade and how adopted HQIM will prepare students for these expectations.
- With support, understand how academic expectations may shift with new, more rigorous HQIM.

Resources:

The following resources informed the content of this chapter –

- Virginia Department of Education (2023). [Virginia Literacy Act Implementation Playbook](#).

- TNTP (2022). [The Key to High-Quality Learning Acceleration for Students.](#)

## Part 2: VDOE Instructional Resources

### *Mathematics Instructional Guides*

The [Mathematics Instructional Guides](#), companion documents to the 2023 Mathematics *Standards of Learning*, amplify the standards by defining the core knowledge and skills in practice, supporting teachers and their instruction, and serving to transition classroom instruction from the 2016 Mathematics *Standards of Learning* to the newly adopted 2023 Mathematics *Standards of Learning*. Grades K through Algebra 2 each have a VDOE Mathematics Instructional Guide. *Only Understanding the Standards documents for upper-level mathematics courses (e.g., Discrete Mathematics) are provided.*

The Mathematics Instructional Guides are for mathematics educators and for those who support them (e.g., principals, central office personnel, directors, Superintendents). The Mathematics Instructional Guides were developed by the Virginia Department of Education Mathematics Team in collaboration with mathematics educators and parents from across the Commonwealth. Focus groups, comprised of divisions from each Superintendent’s Region, were also conducted to obtain feedback and inform revisions.

The Mathematics Instructional Guides reflect an increase in rigor and depth – a major key in the implementation of the four key resources and vision for strong instruction in mathematics as shared in previous chapters:

- Global revisions to increase rigor and depth include improving coherence, horizontal and vertical articulation, and pedagogical support for educators. While the five process goals (reasoning, communication, problem solving, connections, and representations) are expected to be embedded in each standard, the most prevalent process goals in relation to the content are presented in the Skills in Practice component.
- The contents of the Mathematics Instructional Guides were informed by mathematics national and international research organizations and the U.S. Department of Education's Institute of Education Sciences (IES), What Works Clearinghouse, as a central, trusted source of scientific evidence for what works in education. Sample questions reflect applicable and aligned content from the Virginia Department of Education's published assessment items, Mathematics Item Maps, and National Association of Educational Progress (NAEP) assessment questions.

The Mathematics Instructional Guides are divided into three sections: Understanding the Standard, Skills in Practice, and Concepts and Connections aligned to the Standard. The purpose of each is explained below –



**Understanding the Standard:** This section includes mathematics understandings and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, or examples regarding information sources that support the content. They describe what students should know (core knowledge) as a result of the instruction specific to the course/grade level and include evidence-based practices to approaching the Standard.

There are also possible misconceptions and common student errors for each standard to help teachers plan their instruction.



**Skills in Practice:** This section outlines supporting questions and skills that are specifically linked to the standard. They frame student inquiry, promote critical thinking, and assist in learning transfer through the lens of the five process goals. Curriculum writers and teachers should use them to plan instruction to deepen understanding of the broader unit and course objectives.



**Concepts and Connections:** This section outlines concepts that transcend grade levels and thread through the mathematics program as appropriate. Concept connections reflect connections to prior grade-level concepts as content and practices build within the discipline as well as potential connections across disciplines.

### *Mathematics Concepts and Connections Articulation Guide*

As a supplement to the Mathematics Instructional Guides, the [Mathematics Concepts and Connections Articulation Guide](#) provides support in identifying concepts aligned to the 2023 Mathematics *Standards of Learning* that articulate across mathematics grade levels or courses. This guide connects prerequisite learning within the context of new learning in such a way that allows teachers to build and support connections between the relevant prerequisite (subsumed) skills and grade level/content area work in support of students' access to content. The Concepts are interrelated and support the Connections to illustrate commonalities and connections.

The Mathematics Concepts and Connections Articulation Guide is divided into five strands: Number and Number Sense (NS); Computation and Estimation (CE); Measurement and Geometry (MG); Probability and Statistics (PS); and, Patterns, Functions, and Algebra (PFA). A description of the overarching strand summary is provided for each strand. Where appropriate and applicable, content from EOC courses – Algebra 1, Geometry, and Algebra 2 – are connected within the strands. Intentional and purposeful connections are made related to End Of Course mathematics content.

### Number and Number Sense (NS) Strand Summary

There are multiple representations of numbers and relationships among numbers that provide meaning and structure and allow for sense-making.

Strand Concept 1: Exponents, Squares, and Square Roots		
Grade Level/ Content Area	Standard	Description
<a href="#">Grade 6</a>	6.NS.3	The student will recognize and represent patterns with whole number exponents and perfect squares.
<a href="#">Grade 7</a>	7.NS.3	The student will recognize and describe the relationship between square roots and perfect squares.
<a href="#">Grade 8</a>	8.NS.1a	Estimate and identify the two consecutive natural numbers between which the positive square root of a given number lies and justify which natural number is the better approximation. Numbers are limited to natural numbers from 1 to 400.
	8.NS.1b	Use rational approximations (to the nearest hundredth) of irrational numbers to compare, order, and locate values on a number line. Radicals may include both positive and negative square roots of values from 0 to 400 yielding an irrational number.
<a href="#">Algebra 1</a>	A.EO.3	The student will derive and apply the laws of exponents.
	A.EO.4	The student will simplify and determine equivalent radical expressions involving square roots of whole numbers and cube roots of integers.

At the end of each Strand section of the guide, a Comprehensive Strand Summary is presented to help teachers and those that support mathematics instruction to see the vertical and horizontal coherence articulation across and among grade-levels and content areas.

Number and Number Sense (NS) Comprehensive Strand Summary								
Grade Level/ Content Area	Strand Concept 1: Exponents, Squares, and Square Roots	Strand Concept 2: Flexible Counting Strategies	Strand Concept 3: Money <i>Money is explored contextually through numeration and number</i>	Strand Concept 4: Number Sets and Characteristics of Numbers	Strand Concept 5: Place Value	Strand Concept 6: Rational Numbers <i>(Identify, represent, model, order, and compare quantities)</i>	Strand Concept 7: Rational Numbers <i>Equivalencies</i>	Strand Concept 8: Whole Numbers <i>Identify, represent, order, and compare quantities</i>
<a href="#">Kindergarten</a>		K.NS.1	K.NS.1j		K.NS.1c, j			K.NS.2
<a href="#">Grade 1</a>		1.NS.1	1.NS.1f-g		1.NS.1e 1.NS.2c	1.NS.3		1.NS.2
<a href="#">Grade 2</a>		2.NS.1	2.NS.4	2.NS.1h-j	2.NS.2	2.NS.3		2.NS.2

### Exemplar Mathematics Instructional Plans

These [exemplar documents](#) provide educators with sample instructional plans aligned with the 2023 Mathematics *Standards of Learning*. These examples of how the knowledge, skills, and processes found in the standards can be presented to students in the classroom can guide the development and review of high-quality instructional materials

Number and Number Sense (NS) Comprehensive Strand Summary								
Grade Level/ Content Area	Strand Concept 1: Exponents, Squares, and Square Roots	Strand Concept 2: Flexible Counting Strategies	Strand Concept 3: Money <i>Money is explored contextually through numeration and number</i>	Strand Concept 4: Number Sets and Characteristics of Numbers	Strand Concept 5: Place Value	Strand Concept 6: Rational Numbers <i>(identify, represent, model, order, and compare quantities)</i>	Strand Concept 7: Rational Numbers <i>Equivalencies</i>	Strand Concept 8: Whole Numbers <i>Identify, represent, order, and compare quantities</i>
<a href="#">Kindergarten</a>		K.NS.1	K.NS.1j		K.NS.1c, j			K.NS.2
<a href="#">Grade 1</a>		1.NS.1	1.NS.1f-g		1.NS.1e 1.NS.2c	1.NS.3		1.NS.2
<a href="#">Grade 2</a>		2.NS.1	2.NS.4	2.NS.1h-j	2.NS.2	2.NS.3		2.NS.2

(e.g., student/teacher actions, extension and connections, etc.).

Mathematics Instructional Plan – Grade 2

### Race to a Dollar or Two!

**Strand:** Number and Number Sense  
**Topic:** Counting, comparing, and representing coin values  
**Primary SOL:** **2.NS.4** The student will solve problems that involve counting and representing money amounts up to \$2.00.

b) Count by ones, fives, tens, and twenty-fives to determine the value of a collection of mixed coins and one-dollar bills whose total value is \$2.00 or less.  
c) Construct a set of coins and/or bills to total a given amount of money whose value is \$2.00 or less.  
d) Represent the value of a collection of coins and one-dollar bills (limited to \$2.00 or less) using the cent (¢) and dollar (\$) symbols and decimal point (.)

**Materials**

- Race to a Dollar or Two recording sheet, one per pair (attached)
- Race to a Dollar or Two Game Board, one per student (attached; copy on cardstock or place in a protective sleeve)
- Pennies, nickels, dimes, quarters, dollar bills whose value is 100–200 cents (one set per pair of students)
- Number cubes (one per pair)

**Vocabulary**  
bills, change, cent, dime, dollar, equal to, equivalent, fives, greater than, less than, nickel, ones, penny, quarter, tens, twenty-fives, value

**Student/Teacher Actions: What should students be doing? What should teachers be doing?**

1. Have students review coin values and make statements about how they compare to one another. For instance, hold a dime in one hand and a quarter in the other; ask students the value of each coin, and ask them to make a statement comparing the value (e.g., “the dime has a value less than the quarter” or “the quarter has a value that is greater than the dime”).
2. Introduce the game to the students by playing as a whole class or as teacher vs. class. Use one game board for the class and one game board for the teacher. (See directions in 4-6 below.)
3. Students should work with a partner. Explain that students will play a coin-value game called “Race to a Dollar or Two.” Give each pair two game boards (one for each player), one recording sheet (if desired), one number cube, two \$1 bills, and a coin bag containing a collection of pennies, nickels, dimes, and quarters whose value is 100–200 cents (depending on whether they will race to \$1 or \$2). It may be appropriate to have students play the game a few times before having them record the values on the recording sheet.

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### Mathematics Walkthrough Tool

The Virginia Grade K-12 Mathematics Walkthrough Protocol (Appendix A) reflects a vision for mathematics teaching and learning, grounded in the guiding principles of the 2023 Mathematics *Standards of Learning*: ensuring every student builds strong foundational skills by building automaticity and procedural fluency; mastering critical content through the application of skills and conceptual understanding; fostering the application of the five mathematics process goals (reasoning, communication, problem solving, connections, and representation) to set students up to recognize and see mathematics in real-world situations; and, coherently integrating mathematics within and across content areas. The Mathematics Walkthrough Tool will be piloted in Fall 2025 and with anticipated release to all divisions in Spring 2026. Professional development will be provided to support division and school leaders on the use of the Mathematics Walkthrough Tool.

## Mathematics and NAEP Framework Analysis

### Summary of Analysis between Virginia Standards of Learning and 2026 NAEP Framework



National Assessment of Educational Progress (NAEP) is a national assessment that sets the benchmark for what students should know and provides information about the

progress of the nation's students. An external party reviewed the 2023 Mathematics *Standards of Learning* in comparison to the [2026 NAEP Mathematics Framework](#). Crosswalk and analysis found overall strong alignment between Virginia's 2023 Mathematics *Standards of Learning* and the 2026 NAEP Mathematics Framework. The [Summary of Analysis between Virginia Standards of Learning and 2026 NAEP Framework](#) provides areas where alignment may not be as strong so that stakeholders can consider how to approach these areas. The areas are divided into "significant areas to consider" as well as "minor areas to consider" for each NAEP testing grade.

### Mathematics Item Maps

Virginia assessment [item maps](#) provide descriptions and examples of test items students were likely to answer correctly based on the vertical scaled score they achieved on their state assessment. A vertical scaled score is a score that allows comparisons between Virginia Growth Assessments and *Standards of Learning* tests. Each item map provides examples of test question descriptions of test questions at difference score points within the vertical scaled score range for a specific grade and subject.

The image shows a sample item from the Grade 6 Mathematics Item Map. It includes the Virginia Department of Education logo and the title "Grade 6 Mathematics Item Map: A Resource to Understanding Student Scores". The item number is 1705, and the description is "Determine solutions of a linear inequality in one variable, given the graph of the inequality on a number line. (Probability, Statistics, Patterns, Functions, and Algebra)". The graph shows a number line from -9 to -4 with tick marks every 1 unit. There is an open circle at -7 and a closed circle at -4. The region between -7 and -4 is shaded. Below the graph, the question asks "Which values are solutions of the inequality shown on the graph?" and provides four multiple-choice options: A. -1 and 10, B. -7 and -4, C. -7 and 7, and D. -9 and -8.

### Mathematics Practice Items and Tools

Exemplar [Mathematics Practice Items](#) and the Mathematics Tools Practice, along with the Guided Practice Suggestions, will be updated annually to include new item types aligned to the 2023 Mathematics *Standards of Learning* (field tested in Spring 2024). Test Blueprints and Formula Sheets aligned to the 2023 Mathematics *Standards of Learning* can be found on the VDOE website.

### Virginia Mathematics Instructional Materials Implementation Tool

The Virginia Mathematics Textbooks and Instructional Materials Implementation Tool is intended to support division leaders with the selection and implementation of HQIM. Within each phase of the implementation process, essential questions are provided to allow division leaders to frame the work and determine the steps needed to proceed

within the respective phase. Key actions are provided to support division leaders, school leaders, and teachers within each phase of implementation.

#### Resources:

The following are key resources that will help us to implement the 2023 Mathematics *Standards of Learning* –

- National Assessment Governing Board (2021). [2026 NAEP Mathematics Framework](#).
- Virginia Department of Education (2023). [Mathematics Standards of Learning for Virginia Public Schools](#).
- Virginia Department of Education (2023). [2023 Virginia Mathematics Standards of Learning Overview of Revisions](#).
- Virginia Department of Education (2023). [Exemplar Mathematics Instructional Plans](#).
- Virginia Department of Education (2024). [Mathematics Concepts and Connections Articulation Guide](#).
- Virginia Department of Education (2024). [Mathematics Instructional Guides](#).
- Virginia Department of Education (2024). [Mathematics Instructional Guides – FAQ Document](#).
- Virginia Department of Education (2024). [Mathematics Item Maps](#).
- Virginia Department of Education (2024). [Mathematics Practice Items and Tools](#).
- Virginia Department of Education (2024). [Summary of Analysis between Virginia Standards of Learning and the 2026 Mathematics Framework](#).

#### Closing

The VDOE Mathematics Instruction Team looks forward to continuing to support and partner with school divisions as they continue to implement the 2023 Mathematics *Standards of Learning* and embark on a long-term HQIM-based strategy in mathematics. Implementation of new standards is not easy, but it is the work that must be done to ensure that all students across Virginia graduate high school with the mathematical knowledge and skills needed to be ready for college, career, and life.



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