Just in Time Quick Check

Standard of Learning 3.NS.3

Strand: Number and Number Sense

Standard of Learning 3.NS.3

The student will use mathematical reasoning and justification to represent and compare fractions (proper and improper) and mixed numbers with denominators of 2, 3, 4, 5, 6, 8, and 10), including those in context.

Students will demonstrate the following Knowledge and Skills:

a) Represent, name, and write a given fraction (proper or improper) or mixed number with denominators of 2, 3, 4, 5, 6, 8, and 10 using:
   i) region/area models (e.g., pie pieces, pattern blocks, geoboards);
   ii) length models (e.g., paper fraction strips, fraction bars, rods, number lines); and
   iii) set models (e.g., chips, counters, cubes).

b) Identify a fraction represented by a model as the sum of unit fractions.

c) Use a model of a fraction greater than one to count the fractional parts to name and write it as an improper fraction and as a mixed number (e.g., $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4} = 1\frac{1}{4}$).

d) Compose and decompose fractions (proper and improper) with denominators of 2, 3, 4, 5, 6, 8, and 10 in multiple ways (e.g., $\frac{7}{4} = \frac{4}{4} + \frac{3}{4}$ or $\frac{4}{6} = \frac{3}{6} + \frac{1}{6} = \frac{2}{6} + \frac{2}{6}$) with models.

e) Compare a fraction, less than or equal to one, to the benchmarks of 0, $\frac{1}{2}$, and 1 using area/region models, length models, and without models.

f) Compare two fractions (proper or improper) and/or mixed numbers with like numerators of 2, 3, 4, 5, 6, 8, and 10 (e.g., $\frac{2}{3} > \frac{2}{8}$) using words (*greater than, less than*, *equal to*) and/or symbols (>, <, =), using area/region models, length models, and without models.

g) Compare two fractions (proper or improper) and/or mixed numbers with like denominators of 2, 3, 4, 5, 6, 8, and 10 (e.g., $\frac{3}{6} < \frac{4}{6}$) using words (*greater than, less than*, *equal to*) and/or symbols (>, <, =), using area/region models, length models, and without models.

h) Represent equivalent fractions with denominators of 2, 3, 4, 5, 6, 8, or 10, using region/area models and length models.

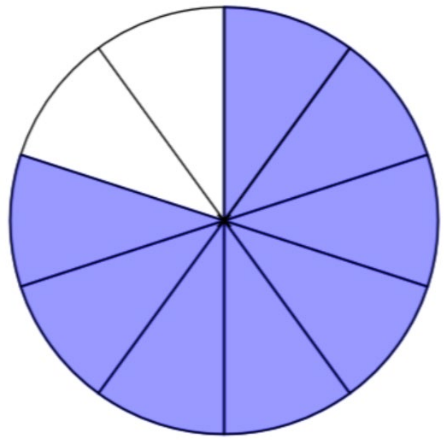
[Just in Time Quick Check]

[Just in Time Quick Check Teacher Notes]

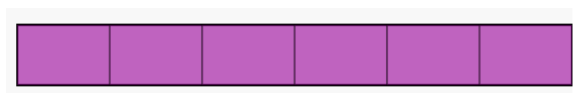
Supporting and Prerequisite SOL: 2.NS.3

Just in Time Quick Check 3.NS.3

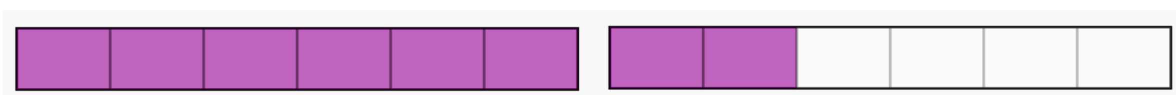
1. What fraction of the circle is shaded?



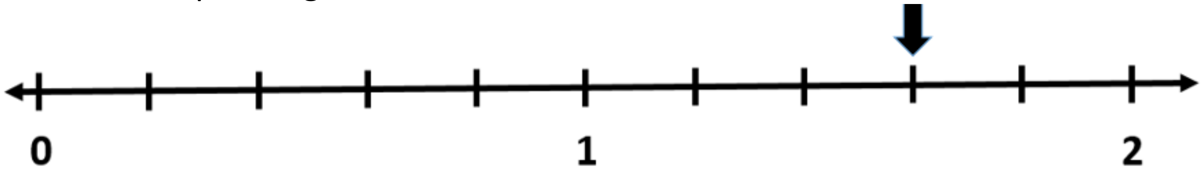
2. This picture represents one whole.



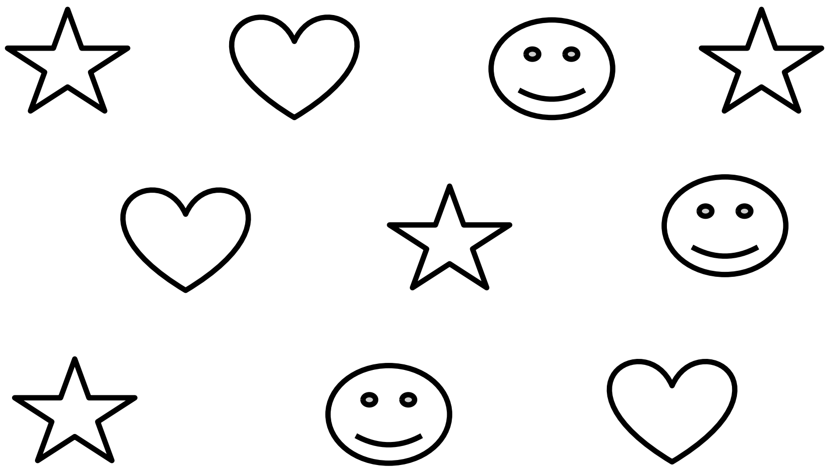
Write the fraction that is shaded in the picture below.



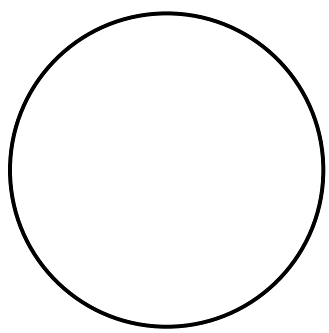
3. The arrow is pointing to a fraction on the number line. Name this fraction.



4. What fraction of this set is stars?

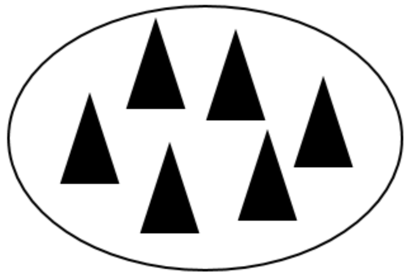


5. This circle represents one whole.



Draw a model to show $\frac{7}{4}$.

6. This set of triangles is one whole.

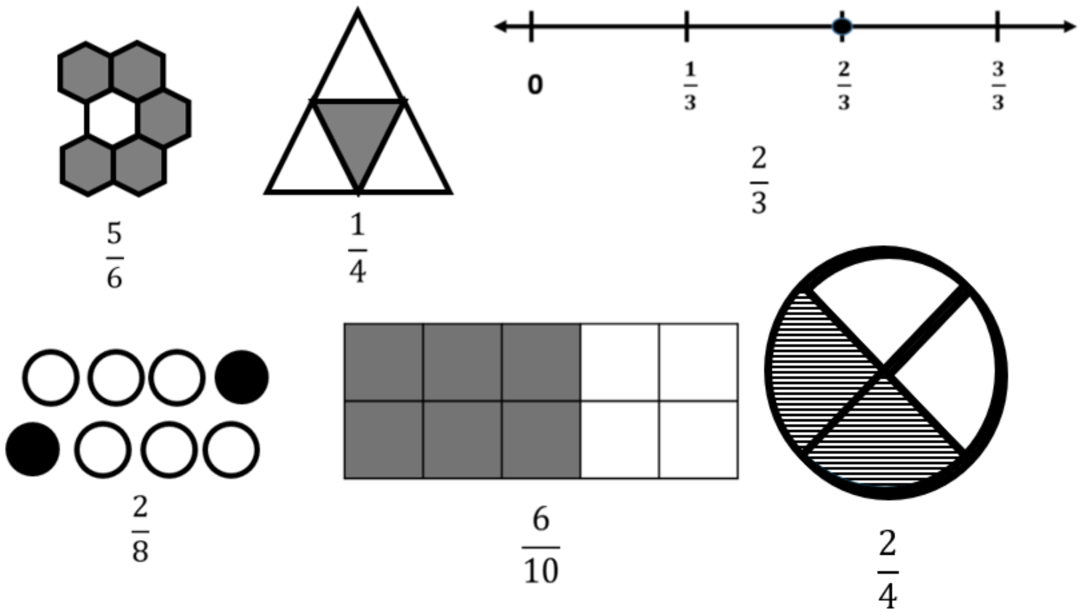


Draw a model of $2\frac{2}{6}$.

7. Liam has a chocolate bar that is divided into 8 equal pieces. Create two different ways that Liam can split his chocolate bar into two or more pieces.

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

8. Look at these fraction models.

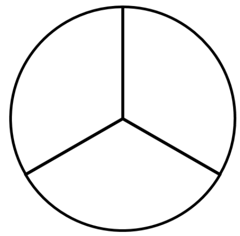


a) Use the fraction models to identify the fractions that will make each number sentence true in the table below. Write each of these fractions in the table under the number sentence it will make true.

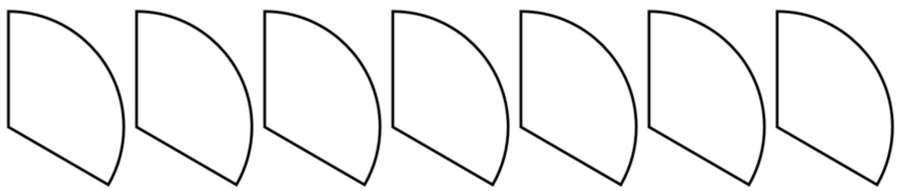
b) Circle the fraction in the table that is closest to 1.

______ $< \frac{1}{2}$	________ $= \frac{1}{2}$	________ $> \frac{1}{2}$

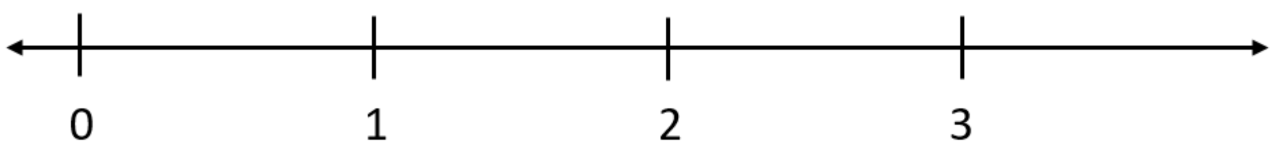
9. This circle is one whole. Each part of the circle is exactly the same size.



Each of these pieces is a fractional part of the circle. Write an addition sentence to show the sum of these fractional parts.



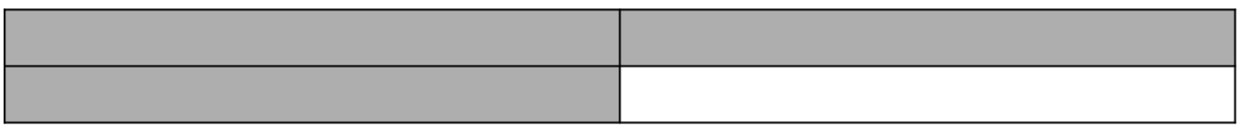
10. The locations of 0, 1, 2, and 3 are labeled on this number line. On the same number line, label the location of $\frac{1}{3}$ and the location of $\frac{5}{3}$.



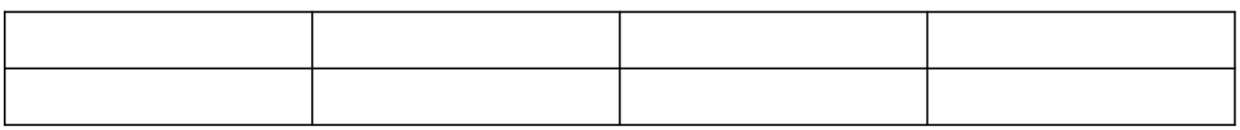
11. Use the symbols <, >, or = to compare the fractions.

$\frac{2}{6}$ ______ $\frac{4}{6}$	$\frac{8}{6}$ ______ $\frac{3}{6}$	$1\frac{1}{5}$ ______ $1\frac{3}{5}$
$\frac{10}{4}$ ______ $\frac{10}{10}$	$\frac{5}{8}$ ______ $\frac{5}{6}$	$\frac{3}{2}$ ______ $\frac{3}{8}$

12. The picture below shows $\frac{3}{4}$ of the rectangle shaded.



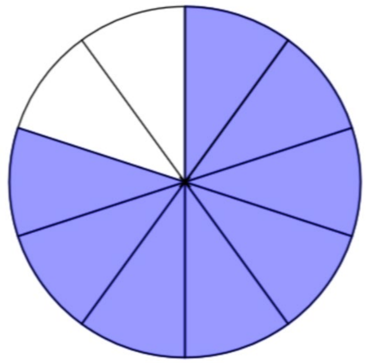
Shade in the rectangle to show a fraction that is equivalent to $\frac{3}{4}$.



3.NS.3 Just in Time Quick Check Teacher Notes

Common Errors/Misconceptions and their Possible Indications

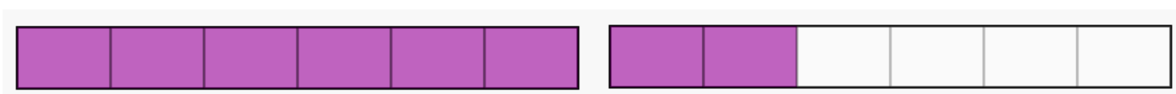
1. What fraction of the circle is shaded?



Students may write $\frac{2}{8}$ or $\frac{8}{2}$, where the numbers used as the numerator and the denominator each represent a part of the circle (shaded part or unshaded part). This may indicate a lack of understanding that the denominator represents the total number of equal parts in the whole. To help students develop conceptual understanding of the numerator and denominator in a fraction, ask students to name the number of pieces described in the model (in this example, the number of pieces shaded) and record that number as the numerator. Then ask students to record the total number of equal pieces in the whole as the denominator. Practice naming examples and non-examples (the fraction shaded in this circle is $\frac{8}{10}$ and not $\frac{8}{2}$) may also help students discriminate between correct and incorrect names for fractions.

2. This picture represents one whole.

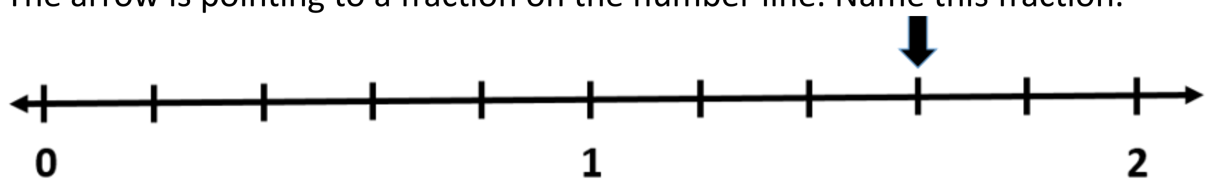
Write the fraction that is shaded in the picture below.



Students who do not recognize the whole, even though it is described at the beginning, may name the picture as $\frac{8}{12}$, counting the twelve equal parts that make up the two wholes as the number of parts in one whole and then counting the 8 shaded parts to get $\frac{8}{12}$. These students may not recognize that there are two wholes, each having 6 equal parts, with only one whole completely shaded and the other only having two parts shaded, thus creating the fractions $\frac{8}{6}$ and $1\frac{2}{6}$.

Students may say the fraction is $\frac{2}{6}$ because they are excluding the whole on the left and only counting the $\frac{2}{6}$ of the second whole on the right. They may only think of proper fractions and will need to develop an understanding that there can be more counting pieces in the numerator than the denominator, thus creating an improper fraction. Students who have difficulty naming improper fractions like $\frac{8}{6}$ may benefit from counting by unit fractions past the whole (e.g., $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}, \frac{7}{6}, \frac{8}{6}$). The use of a variety of manipulatives like pie circles, bars, and pattern blocks in addition to counting by fractions on a number line may also help students to understand the relationship between improper fractions and mixed numbers. Students may find it easier to move from improper to mixed numbers than to move from mixed numbers to improper fractions.

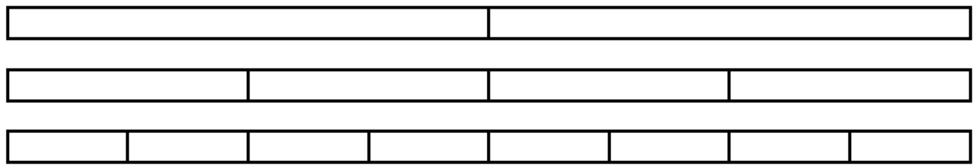
3. The arrow is pointing to a fraction on the number line. Name this fraction.



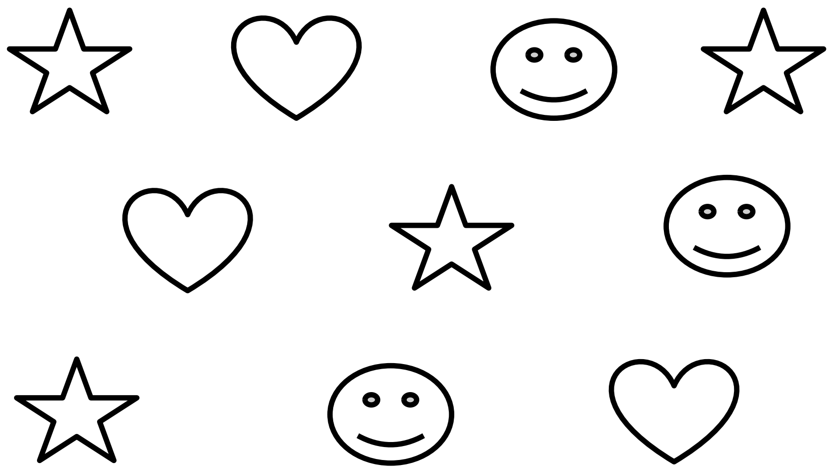
When working with number lines, students may count all the tick marks instead of the spaces between them, over counting the number of parts making the whole. In this example, students may name the fraction as $\frac{9}{11}$ (starting with zero, the arrow is on the ninth tick mark and there are eleven tick marks from zero to two, inclusive) or as $1\frac{4}{6}$ (starting with the tick mark at 1, the arrow is on the fourth tick mark and there are six tick marks from one to two, inclusive).

Some students may think the arrow is at one and a half because there are two tick marks on either side of the tick mark labeled with the arrow. Other students may think that it is representing $1\frac{3}{4}$, because the arrow is at the third tick mark out of 4 tick marks before the 2 on the number line. These students are counting the marks between 1 and 2, instead of counting the equal spaces between the tick marks on the number line.

Students may need experience with folding strips of paper that represent a whole and then marking each fold as a fraction. For example, the first fold would represent half, and the next fold now makes four equal parts or represents fourths ($\frac{1}{4}$ and $\frac{3}{4}$, while $\frac{1}{2}$ is equal to $\frac{2}{4}$). Then one last fold creates eighths. Once students see the equal parts are counted and not the lines for fractions between zero and one, they can explore the same activity between zero and two. In this activity, the middle line in the first fold becomes one and the lines in the next strip become $\frac{1}{2}$ and $1\frac{1}{2}$. The last strip would represent fourths.



4. What fraction of this set is stars?

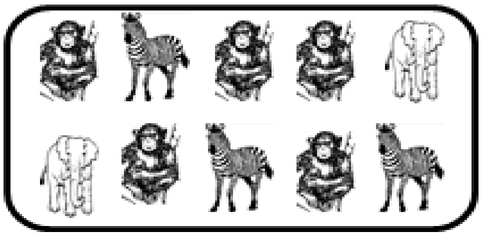


Students may have difficulty identifying the whole when a set has different types of items. In this example, students may think the fraction is $\frac{4}{6}$ which indicates that they see the four stars and the other six items as separate groups. Students may be unable to recognize that the entire set of ten items is one whole, and this whole includes both the four stars and the other six items. These students would benefit from experiences creating a set given the description of that set that uses fractions. For example, students could be asked to draw a set of circles where $\frac{3}{8}$ of the set of circles is shaded and $\frac{5}{8}$ of the set is unshaded.

Other students may name the fraction as $\frac{4}{3}$, which may indicate that they see the four stars, and they recognize that there are three hearts and three smiley faces. Since both types of items left are the same amount, students may decide this must be the denominator. Again, they are not viewing the ten items as a whole set. Experiences with sets having only two different types of items should be provided before experiences with sets having more than two different types of items.

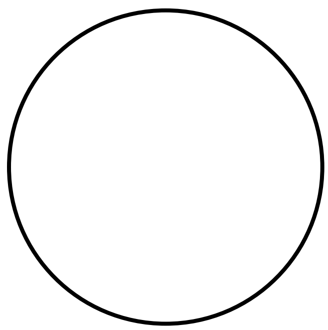
Another misconception is that the students may see the answer as four. These students are using whole number knowledge and are counting the stars but not considering them as part of the set. They are seeing each item as an individual item, rather than as a fractional part of a set. It is important for students to understand the vocabulary of fractions, so they know if the problem is asking for a whole number answer, a fraction of a set, or the fractional part that is shaded or being used.

In a set model, each member of the set is an equivalent part of the set. In set models, the whole needs to be defined, but members of the set may have different sizes and shapes. For instance, if a whole is defined as a set of ten animals, the animals within the set may be different. For example, students should be able to identify apes as representing $\frac{5}{10}$ of the animals in the set shown:



Students may benefit from experience with sets that are comprised of congruent parts (e.g., twelve eggs in a carton) before working with sets that have noncongruent parts.

5. This circle represents one whole.



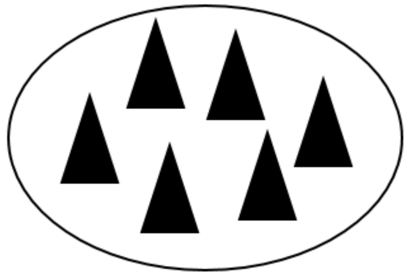
Draw a model to show $\frac{7}{4}$.

Students may think that you cannot have $\frac{7}{4}$ because the numerator is larger than the denominator. They may draw the fourths in one circle but only shade in the four-fourths of the first circle to represent a whole. They may not know how to represent the other 3 parts that are needed to make $\frac{7}{4}$. Students may benefit from experience with manipulatives that include counting fractions past one whole to a given improper fraction (e.g., $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}$).

Some students may draw one circle with 7 parts and shade 4 of the parts. They may do this because they feel more familiar with $\frac{4}{7}$, using their knowledge of proper fractions. These students may benefit from opportunities to name paired models of proper and improper fractions in which the numerator and denominator are reversed (i.e., reciprocals, such as $\frac{7}{4}$ and $\frac{4}{7}$).

Some students may draw 11 whole circles and shade 7, while others may draw 7 circles and shade 4. This may indicate that students are considering the numerator and denominator as whole numbers. Additional opportunities to name given models and to create models of fractions greater than one may be helpful.

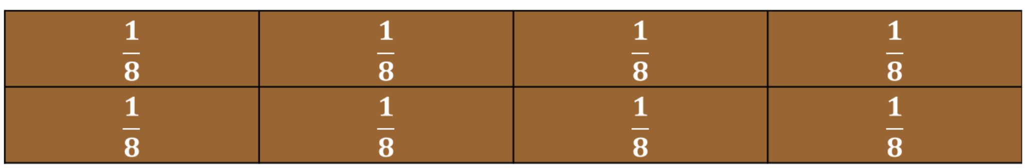
6. This set of triangles is one whole.



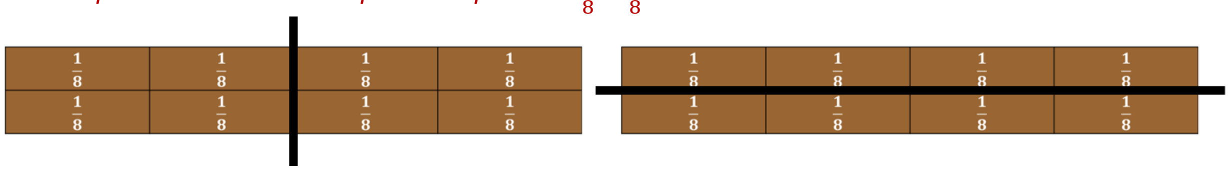
Draw a model of $2\frac{2}{6}$.

Students may draw a model of $\frac{2}{6}$ because they are unsure how to represent the whole number with a set model. Other students may draw the 2 whole sets but do not know how to show the $\frac{2}{6}$. Students may benefit from experience with set models, first identifying the number of pieces in the whole and then counting like-size pieces past the whole to represent and name an improper fraction and its equivalent mixed number.

7. Liam has a chocolate bar that is divided into 8 equal pieces. Create two different ways that Liam can split his chocolate bar into two or more pieces.

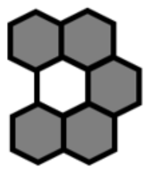


A common error is for students to respond with the number of pieces instead of using fractions. For example, students might say Liam can split his chocolate bar into 2 pieces + 2 pieces + 4 pieces. Students should be encouraged to then translate that statement into one that uses eighths (i.e., $\frac{2}{8}+\frac{2}{8}+\frac{4}{8}$). Another common error is for students to create an expression that does not represent the entire chocolate bar. For example, students may write $\frac{1}{8}+\frac{1}{8}$, and neglect to account for the other 6 pieces of the chocolate bar. Students who draw a picture may create the same expression, represented differently in a picture, as shown in the example below where each picture represents $\frac{4}{8}+\frac{4}{8}$.

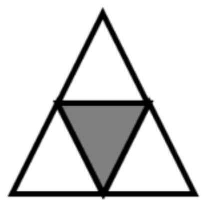


Problems like this allow students to demonstrate their understanding of decomposing fractions. Teachers can support student learning by having students showcase various ways of splitting the chocolate bar into two, three, or more pieces. For example, it may be helpful to show $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$, making connections to the sum of unit fractions, and *to show $\frac{4}{8}+\frac{4}{8}$, making connections to the benchmark fraction of $\frac{1}{2}$. Providing students with opportunities to see other students' work and to hear about their explanation of their strategies to decompose the chocolate bar will be beneficial for all students.*

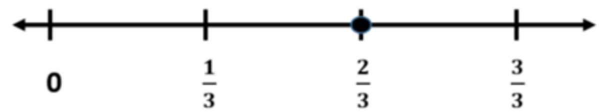
8. Look at these fraction models.



$\frac{5}{6}$

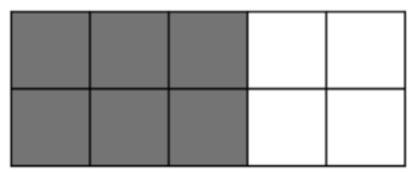


$\frac{1}{4}$



$\frac{2}{3}$

$\frac{2}{8}$



$\frac{6}{10}$

$\frac{2}{4}$

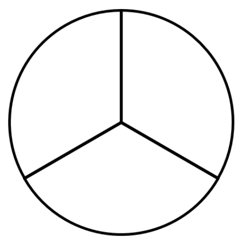
a) Use the fraction models to identify the fractions that will make each number sentence true in the table below. Write each of these fractions in the table under the number sentence it will make true.
b) Circle the fraction in the table that is closest to 1.

_____ $< \frac{1}{2}$	_____ $= \frac{1}{2}$	_____ $> \frac{1}{2}$

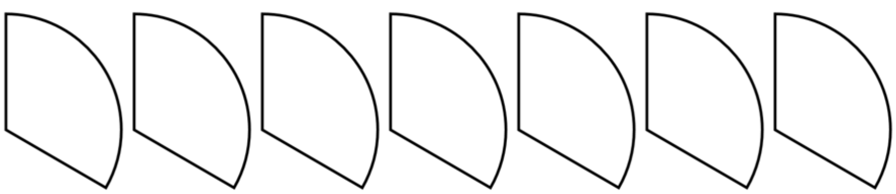
Students may think that the fractions with the greater numbers in the denominators are greater than one-half and those with lesser numbers in the denominators are less than one-half. Students might also think that because 3 and 4 are closer to 1 than the other denominators, $\frac{2}{3}$ and $\frac{1}{4}$ are closer to 1 and therefore both must be greater than $\frac{1}{2}$, which is true for $\frac{2}{3}$ but not for $\frac{1}{4}$. Students may be using whole number understanding rather than fraction thinking and would benefit from opportunities to use manipulatives to directly compare fractions with different denominators to $\frac{1}{2}$.

Throughout fraction instruction, teachers are encouraged to have students talk about and visualize comparisons to one-half (e.g., "How many eighths would it take to make one-half?" or "Is this fraction more than one-half or less than one-half? Can you draw a picture to show that?") to build conceptual understanding for using $\frac{1}{2}$ as a benchmark comparison. Paper folding activities (as described in problem #3) combined with placing the fractions being compared to $\frac{1}{2}$ on the same number line as $\frac{1}{2}$ can help students develop this understanding.

9. This circle is one whole. Each part of the circle is exactly the same size.



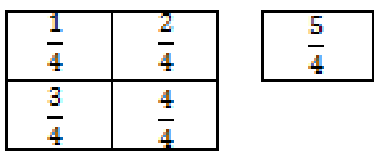
Each of these pieces is a fractional part of the circle. Write an addition sentence to show the sum of these fractional parts.



Students who write the addition sentence $1 + 1 + 1 + 1 + 1 + 1 + 1 = 7$ are counting each fractional piece as one whole. These students would benefit from opportunities to count fractions on a number line, counting past one, using fraction vocabulary when counting (e.g., one-third, two-thirds, three-thirds, four-thirds, five-thirds, six-thirds, seven-thirds). It may be helpful for students to label each piece as $\frac{1}{3}$, which will help them see the sum of the unit fractions ($\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$). Anchor charts that display models of pictorial representations of fractions and a corresponding addition equation using unit fractions may provide support as students develop this understanding.

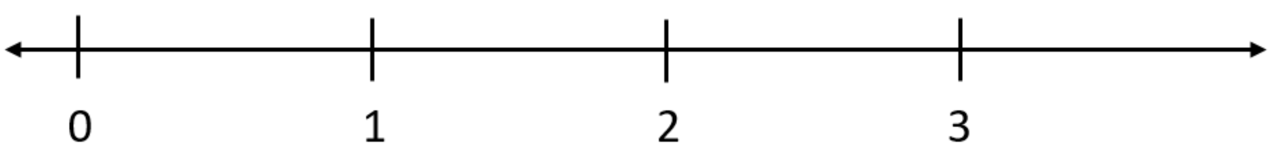
Students may name the sum as 2 with 1 left over, seeing that the first two sets of 3 pieces can each be combined to make 2 whole circles, but they may be unable to name the one piece remaining as one-third of the circle. Students may need more experience with counting various fractional pieces having like sizes that can be combined, such as pattern blocks or linking cubes, to create a mixed number and its equivalent improper fraction (e.g., $\frac{1}{3}, \frac{2}{3}$, 1 whole, $1\frac{1}{3}, 1\frac{2}{3}$, 2 wholes, $2\frac{1}{3}$).

In all the above tasks, students need opportunities to use models to count fractional parts that go beyond a whole. For instance, if students are counting five slices of cake and building the cake as they count, where each slice is equivalent to one-fourth, they might say "one-fourth, two-fourths, three-fourths, four-fourths, five-fourths." As a result of building the whole while they are counting, students begin to realize that four-fourths make one whole and the fifth-fourth starts another whole, and they begin to develop flexibility in naming this amount in different ways (e.g., five-fourths or one and one-fourth). Students will begin to generalize that when the numerator and the denominator are the same, there is one whole and when the numerator is larger than the denominator, there is more than one whole. They also will begin to see a fraction as the sum of unit fractions (e.g., three-fourths contains three one-fourths or four-fourths contains four one-fourths which is equal to one whole). This provides students with a visual, as in the example below, for when one whole is reached, and develops a greater understanding of numerator and denominator.



$\frac{5}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ and $\frac{5}{4} = 1\frac{1}{4}$

10. The locations of 0, 1, 2, and 3 are labeled on this number line. On the same number line, label the location of $\frac{1}{3}$ and the location of $\frac{5}{3}$.



Students may add a label of $\frac{1}{3}$ underneath the existing label for 1, which indicates students are using each interval that represents one whole as one-third. These students may benefit from experience adding tick marks between whole numbers on a given number line to create the equal fractional pieces needed. It is important that experiences with number lines extend beyond the numbers needed. This number line extends from zero to three since thirds are given as the denominator of the fractions that need to be labeled.

Students may struggle to place $\frac{5}{3}$ on the number line. They may extend the number line to include 4 and 5, and label $\frac{5}{3}$ on the number 5. This may indicate that students are using whole number understanding and may have only focused on the numerator when determining where to place $\frac{5}{3}$. For some students, it may be helpful to convert $\frac{5}{3}$ from an improper fraction to a mixed number. Seeing $\frac{5}{3}$ as $1\frac{2}{3}$ may help students better understand where this point will be on the number line.

11. Use the symbols <, >, or = to compare the fractions.

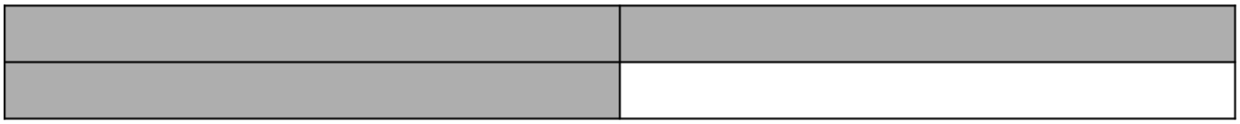
$\frac{2}{6}$ ____ $\frac{4}{6}$	$\frac{8}{6}$ ____ $\frac{3}{6}$	$1\frac{1}{5}$ ____ $1\frac{3}{5}$
$\frac{10}{4}$ ____ $\frac{10}{10}$	$\frac{5}{8}$ ____ $\frac{5}{6}$	$\frac{3}{2}$ ____ $\frac{3}{8}$

Students may struggle to compare fractions if they do not have strong conceptual understanding of fractions and instead try to apply memorized rules. Comparing fractions with the same denominator and different numerators is often easier for students than comparing fractions with the same numerator and different denominators because the sizes of the pieces are the same. It may also be helpful to encourage students to think about comparing fractions with the same denominator to benchmark fractions such as $\frac{1}{2}$ or 1 whole. For example, when comparing $\frac{8}{6}$ and $\frac{3}{6}$, students should recognize that $\frac{8}{6}$ is greater than 1 whole and $\frac{3}{6}$ is less than 1 whole. Similarly, students should recognize that $\frac{2}{6}$ is less than $\frac{1}{2}$, and $\frac{4}{6}$ is greater than $\frac{1}{2}$.

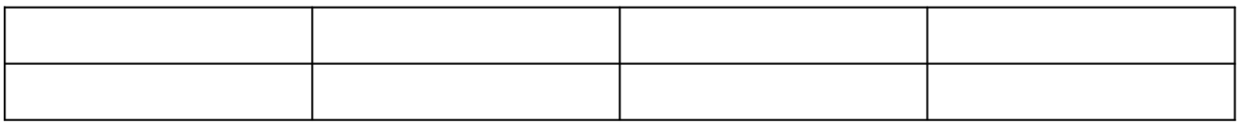
When comparing fractions with the same numerator and different denominators, students may think that the larger denominator means a larger fraction. This may lead students to incorrectly state that $\frac{5}{8} > \frac{5}{6}$, because they see that 8 is greater than 6 and assume that eighths are greater than sixths. This may indicate that students are applying whole number thinking to fractions and are only focusing on the size of the denominator. It may be helpful to use visual models to demonstrate that the more pieces a whole is cut into, the smaller each piece will be. Additionally, asking questions such as, "Which pieces are bigger? How do you know?" will help elicit student thinking.

For each of these comparisons, it may be beneficial for students to create their own representations using region models (e.g., rectangles or circles) or number lines to help them compare the fractions. Students should also be encouraged to compare fractions by reasoning about the size of pieces and the number of pieces, rather than comparing fractions by just looking at the numbers. Having students explain and justify their reasoning will help deepen their conceptual understanding of fractions.

12.The picture below shows $\frac{3}{4}$ of the rectangle shaded.



Shade in the rectangle to show a fraction that is equivalent to $\frac{3}{4}$.



A common error is for students to shade in 3 of the boxes, creating a fraction that represents $\frac{3}{8}$. This may indicate that students focused on the numerator of 3 and believed that shading in 3 boxes would produce a fraction equivalent to $\frac{3}{4}$. Students may not understand that equivalent fractions represent the same amount of the whole, even though the number of parts changes. Another common error is for students to shade too much or too little of the rectangle, resulting in a shaded amount that is not equivalent to $\frac{3}{4}$. It may be helpful to elicit student thinking by asking questions such as, “Does this look like the same amount shaded?” “If we laid the two rectangles on top of each other, would they match?”