Just in Time Quick Check

Standard of Learning G.RLT.1

Strand: Reasoning, Lines, and Transformations

Standard of Learning G.RLT.1

The student will translate logic statements, identify conditional statements, and use and interpret Venn diagrams.

Students will demonstrate the following Knowledge and Skills:

a) Translate propositional statements and compound statements into symbolic form, including negations ($\sim p$, read “not p”), conjunctions ($p \wedge q$, read “p and q”), disjunctions ($p \vee q$, read “p or q”), conditionals ($p \rightarrow q$, read “if p then q”), and biconditionals ($p \leftrightarrow q$, read “p if and only if q”), including statements representing geometric relationships.
b) Identify and determine the validity of the converse, inverse, and contrapositive of a conditional statement, and recognize the connection between a biconditional statement and a true conditional statement with a true converse, including statements representing geometric relationships.
c) Use Venn diagrams to represent set relationships, including union, intersection, subset, and negation.
d) Interpret Venn diagrams, including those representing contextual situations.

[Just in Time Quick Check]

[Just in Time Quick Check Teacher Notes]

Supporting and Prerequisite SOL: A.ST.1

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1. Read the statement below. Then write the converse of the statement.

   If a figure is a rectangle, then it has four right angles.

2. Write the inverse of the following statements.

   a) If it is not raining, then we will have our baseball game.

   b) If $x = 2$, then $x + 4 = 6$.

3. Statement: *A six-sided polygon is a hexagon.*

   a) Write the statement in if-then form.

   b) Write the contrapositive of the given statement.

4. Given: $p \rightarrow \sim q$.

   a) What is the converse of this statement?

   b) What is the inverse of this statement?

   c) What is the contrapositive of this statement?

5. Let *a* represent: *Line segments JK and XY are congruent.* Let *b* represent: *Line segments JK and XY are the same length.*

Translate the following argument into symbolic form.

Line segments *JK* and *XY* are congruent if and only if they are the same length.	
Line segments *JK* and *XY* are different lengths.	
Therefore, line segments *JK* and *XY* are not congruent.	

6. Let *p* represent: $\angle A$ *and* $\angle B$ *are supplementary*. Let *q* represent: $m\angle A + m\angle B = 180°$.

Translate the following statement into symbolic form:

$m\angle A + m\angle B = 180°$ *and are supplementary*.

7. Let *p* represent: $a^2 + b^2 = c^2$. Let *q* represent: $a + b > c$.

Create a verbal argument by translating the symbolic representations.

$p \rightarrow q$	
$\sim q$	
$\therefore \sim p$	

8. Provide a counterexample for each of the following.

   a) If you are eating a red fruit, then you are eating an apple.

   b) If $x + 4 > 10$, then x is greater than 8.

9. Determine if the following arguments are valid. Explain your reasoning.

Argument 1	Argument 2
All squares are rectangles. ABCD is a square. Therefore, ABCD is a rectangle.	All squares are rectangles. ABCD is a rectangle. Therefore, ABCD is a square.

10. Determine if this argument is valid. If so, indicate the law(s) of logic used.

    If a figure has three sides, then it is a triangle.

    If a figure is a triangle, then the sum of the interior angles is 180°.

    Therefore, if the sum of the interior angles is not 180 degrees, then the figure does not have three sides.

11. Given the following statement, apply the law of contrapositive to provide a valid conclusion.

If Sarah gets an A in the class, then she will not have to take the final exam.

12. Suppose the symbolic statements $a \rightarrow b$ and $b \rightarrow c$ are true. Determine two other statements that must also be true.

G.RLT.1 Just in Time Quick Check Teacher Notes

Common Errors/Misconceptions and their Possible Indications

1. Read the statement below. Then write the converse of the statement.

   If a figure is a rectangle, then it has four right angles.

   A common misconception some students may have is confusing the converse and the inverse. Another common misconception is that some students may confuse the converse and the contrapositive. This might indicate that students do not properly understand the vocabulary terms. Teachers may consider phrases or other connections to help students remember the difference between converse and inverse and between converse and contrapositive. To help students understand the contrapositive, have them think about it as the "inverse of the converse." They are changing the order of the hypothesis and the conclusion of the inverse statement. Students may benefit from additional practice in matching sentence strips to create the converse, inverse, and contrapositive to a given conditional statement. For students having difficulty discerning the hypothesis and conclusion, one strategy that may be helpful for students is to underline the hypothesis and circle the conclusion.

2. Write the inverse of the following statements.

   a) If it is not raining, then we will have our baseball game.

   b) If $x = 2$, then $x + 4 = 6$.

   A common misconception students might have for part a) would be to think the word "not" must be included in both the hypothesis and conclusion. This may indicate that students are not aware that they must adjust the hypothesis and conclusion to represent the opposite of each component. In part b), some students may have difficulty expressing the inverse when mathematical symbols are involved. This may indicate that students do not understand writing the inverse of a conditional statement in symbolic form. Students would benefit from examples that include the word "not" as well as examples that include mathematical equations. A strategy that could be used with students is to have them create statements and then partner with another student to negate the statement and then progress to writing the inverse of the statement. Teachers are also encouraged to provide real world examples which may resonate more with students.

3. Statement: *A six-sided polygon is a hexagon.*

   a) Write the statement in if-then form.

   b) Write the contrapositive of the given statement.

A common misconception some students may have is incorrectly identifying the hypothesis and conclusion, in a statement that is not in if-then form. This may indicate that students do not understand how to transfer a statement into a conditional statement written in if-then form. A strategy that might benefit some students is to start with an if-then statement and transition backwards into a non-conditional statement. An additional strategy is to have students create their own conditional statement and rewrite it as a non-conditional statement and then identify the hypothesis and conclusion in both statements.

Some students may confuse the contrapositive with the converse. Students may benefit from additional practice in matching sentence strips to create the converse, inverse, and contrapositive to a given conditional statement. For students having difficulty discerning the hypothesis and conclusion, one strategy that may be helpful for students is to underline the hypothesis and circle the conclusion.

4. Given: $p \rightarrow \sim q$.
   a) What is the converse of this statement?
   b) What is the inverse of this statement?
   c) What is the contrapositive of this statement?

   Some students may understand how to write the converse, inverse, and contrapositive, but struggle when they must write the statement in symbolic form. In addition, the concept of negating the symbolic form is difficult for some students. A strategy that may be used to help students is to use index cards and have a card with each of the following: p, q, →, ~. Teachers may write a conditional and then use the notecards to place the symbols above the words. Once students are familiar with transferring the written to the symbolic, they can progress to all symbolic notations. Using the notecards may also help some students when they must negate a 'not'. They can transfer the knowledge of two negatives (minus a negative) means a positive from algebra.

5. Let *a* represent: *Line segments JK and XY are congruent.* Let *b* represent: *Line segments JK and XY have the same length.*

   Translate the following argument into symbolic form.

Line segments *JK* and *XY* are congruent if and only if they have the same length.	
Line segments *JK* and *XY* have different lengths.	
Therefore, line segments *JK* and *XY* are not congruent.	

A common error students may make would be to improperly identify the second statement because it is negating using the opposite form but does not use the word "not". This may indicate students only *recognize negation when the word "not" is being used. Teachers are encouraged to use examples of negation where opposite statements are represented without the use of the word "not."*

6. Let *p* represent: $\angle A$ *and* $\angle B$ *are supplementary*. Let *q* represent: $m\angle A + m\angle B = 180°$.

   Translate the following statement into symbolic form.

   $m\angle A + m\angle B = 180°$ *and are supplementary*.

   A common error students might make is to use the symbol for "or," ∨, rather than "and," ∧, when representing the symbolic form. This may indicate that some students have improperly applied their symbols.

7. Let *p* represent: $a^2 + b^2 = c^2$. Let *q* represent: $a + b > c$.

   Create a verbal argument by translating the symbolic representations.

$p \rightarrow q$	
$\sim q$	
$\therefore \sim p$	

A common error students may make is not correctly negating symbols such as $=$, $\neq$, $<$, and $>$. This may indicate that students do not understand how to negate statements involving equalities and inequalities. Teachers are encouraged to use examples including these types of symbols to allow students to practice negation in other forms. It is important that teachers explain the negation of symbols, for example the negation of $>$ is actually $\leq$. Another error students might make would be writing the verbal arguments when given symbolic forms. For example, students may forget if or then in their first statement. This can become a more common error when lengthy statements are involved. Teachers could have the related argument pieces on index cards and have students place them in the correct order to create the entire statement to represent each symbolic form given.

8. Provide a counterexample for each of the following.

   a) If you are eating a red fruit, then you are eating an apple.

   b) If $x + 4 > 10$, then x is greater than 8.

   A common error students might make would be providing an example to make the hypothesis of the statement false. This would indicate that students may be confused about which part of the statement they are providing an example for to prove the conjecture false. Teachers should encourage students to label the hypothesis and conclusion of each statement and/or highlight the *part that needs the false example. A strategy teachers could use might be to put students in small groups where some of the students in the group create one or two statements that are not always true and the other students in the group find counterexamples.*

9. Determine if the following arguments are valid. Explain your reasoning.

Argument 1	Argument 2
All squares are rectangles. ABCD is a square. Therefore, ABCD is a rectangle	All squares are rectangles. ABCD is a rectangle. Therefore, ABCD is a square.

A common error students might make would be to choose both statements that are valid. This may indicate students do not recognize how to apply deductive reasoning when statements are not in "if-then" form. Some students may be able to guess the right conclusion but may not be able to explain their reasoning. This may also indicate students cannot properly apply law of detachment. Some students may think it is valid when the conclusion follows the conditional. Teachers should encourage students to label their statements.

10. Determine if this argument is valid. If so, indicate the law(s) of logic used.

If a figure has three sides, then it is a triangle.

If a figure is a triangle, then the sum of the interior angles is 180°.

Therefore, if the sum of the interior angles is not 180 degrees, then the figure does not have three sides.

A common error students might make would be to conclude that this argument is invalid. Students may think that the only valid conclusion may be "If the figure has three sides, then the sum of interior angles is 180 degrees." This may indicate that students do not recognize that multiple laws can be used to make a valid argument. Teachers should provide examples of how multiple laws can be used to draw valid conclusions.

11. Given the following statement, apply the law of contrapositive to provide a valid conclusion.

If Sarah gets an A in the class, then she will not have to take the final exam.

A common error students might make would be to say that "Sarah did not take the final exam. Therefore, Sarah got an A in the class." This would indicate that students may not be able to properly negate when "not" is already in the statement. Another common error students might make would be to say "Sarah did not get an A in the class. Therefore, Sarah will have to take the final exam." This may indicate that students are improperly identifying the law of contrapositive as $\sim p, \therefore \sim q$*, instead* *of $\sim q, \therefore \sim p$. Teachers should remind students that the law of contrapositive should follow the same order as when writing the contrapositive of a statement.*

12. Suppose the symbolic statements $a \rightarrow b$ and $b \rightarrow c$ are true. Determine two other statements that must also be true.

    A common error students might make would be to only recognize one other statement to be true such as $a \rightarrow c$, using law of syllogism. This may indicate students are not comfortable identifying conclusions that do not follow the traditional alignment. Students may also have difficulty recognizing laws when using symbolic form or when using symbols other that p and q. This may indicate students need more practice with symbolic form.