Just in Time Quick Check

Standard of Learning G.TR.1

Strand: Triangles

Standard of Learning G.TR.1

The student will determine the relationships between the measures of angles and lengths of sides in triangles, including problems in context.

Students will demonstrate the following Knowledge and Skills:

a) Given the lengths of three segments, determine whether a triangle could be formed.
b) Given the lengths of two sides of a triangle, determine the range in which the length of the third side must lie.
c) Order the sides of a triangle by their lengths when given information about the measures of the angles.
d) Order the angles of a triangle by their measures when given information about the lengths of the sides.
e) Solve for interior and exterior angles of a triangle, when given two angles.

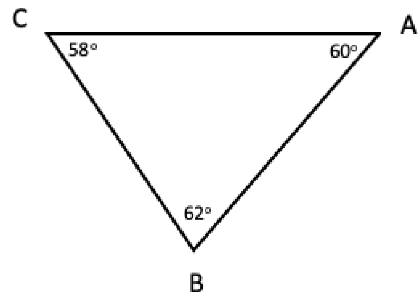
Just in Time Quick Check

Just in Time Quick Check Teacher Notes

Supporting and Prerequisite SOL: 8.MG.1

Just in Time Quick Check G.TR.1

1. Given the angle measures of ΔABC, order the sides from greatest to least. Place your response in the blanks provided below. The figure is not drawn to scale.



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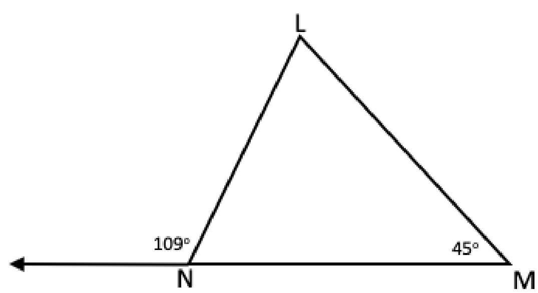
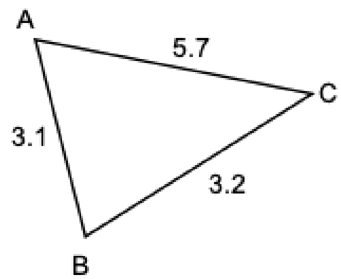
2. Determine the longest side of the ΔLMN. The figure is not drawn to scale. Justify your reasoning.



3. In any triangle with an angle measure of 110°, is the longest side always opposite this angle? Justify your reasoning.

4. Three sprinkler heads are placed on a lawn at each vertex of a triangular pattern shown. Order the angles that are formed by the triangular pattern from smallest to largest angle using the blanks provided. The figure is not drawn to scale.



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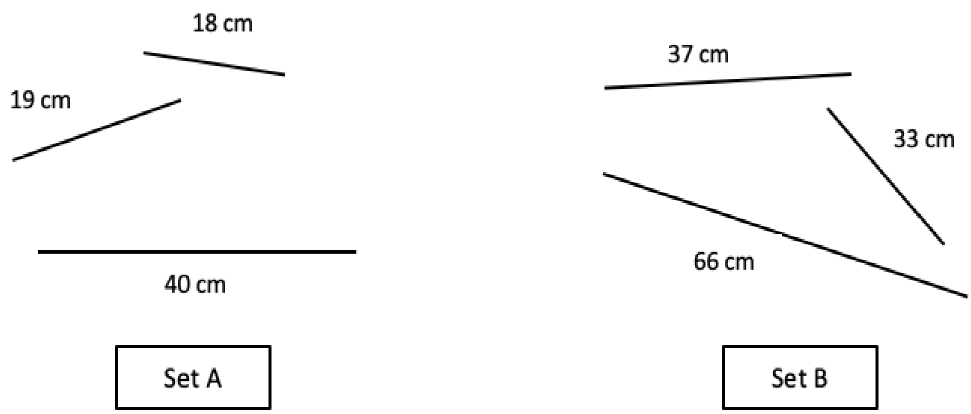
5. Three stages at a music festival are arranged in a triangle. The distances between the centers of each stage are given:

- Stage A and Stage B: 100 yards
- Stage B and Stage C: 135 yards
- Stage C and Stage A: 210 yards

Using the information above, list the angles created at Stage A, Stage B, and Stage C in descending order.

6. ΔPQR has a perimeter of 48 cm. $\overline{PQ} = x$, $\overline{QR} = \frac{4}{3}\overline{PR}$, and $\overline{PR} = \frac{3}{5}\overline{PQ}$. List the interior angle measures of ΔPQR, ordered from greatest to least.

7. Ms. Jones asked her students to determine which of the following sets of segments could form the three sides of a triangle.



Jessica states the sum of the lengths of the two shorter sides of Set A is less than the length of the longest side. Therefore, the line segments in Set A would form the sides of a triangle.

Ashley states the sum of the lengths of the two shorter sides of Set B is greater than the length of the longest side. Therefore, the line segments in Set B would form the sides of a triangle.

Who is correct, Jessica or Ashley? Justify your reasoning.

8. Could a triangle have side lengths of 16 ft, 44 ft, and 28 ft? Justify your reasoning.

9. A triangle has side lengths of 2 units and 8 units. Identify all possible integral lengths of the third side. Justify your reasoning.

10. Anthony stated that a triangle with side lengths of 3 cm and 4 cm could have a third side with a side length of 7 cm. Is Anthony correct? Explain. Create a statement about the length of the third side.

11. The lengths of two sides of a triangle are 30 inches and 50 inches. What is the range of possible lengths for the third side of this triangle? Justify your reasoning.

12. Michele is creating a triangle by connecting three wooden rods. If the lengths of two of the wooden rods are 15 centimeters and 22 centimeters, what is the range of possible lengths of a third wooden rod that will create a triangle? Justify your reasoning.

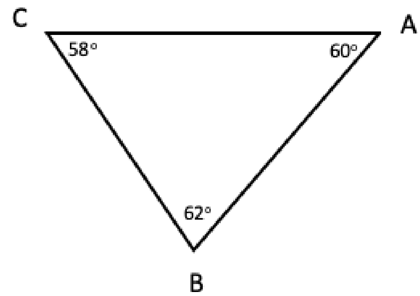
13. The measures of two sides of a triangle are 8 feet and 12 feet. Use values to fill in the blanks to create a true statement about the length of the third side of the triangle.

    The third side of the triangle must be a value larger than _______ but smaller than _______.

G.TR.1 Just in Time Quick Check Teacher Notes

Common Errors/Misconceptions and their Possible Indications

1. Given the angle measures of ΔABC, order the sides from greatest to least. Place your response in the blanks provided below. The figure is not drawn to scale.



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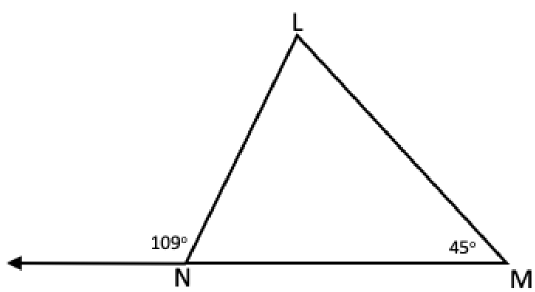
A common misconception that students may have is not understanding the relationship between the lengths of the sides and measures of the angles of a triangle. This may indicate that students do not understand that the longest side of a triangle is opposite the largest angle of the triangle and the shortest side is opposite the smallest angle. Further, this may indicate that students may not be able to name the line segment opposite an angle in a triangle (e.g., the segment opposite ∠C is $\overline{AB}$). Some students may find it easier to identify the longest side in a triangle, by first ordering the angles from smallest to largest and then using that list to order the sides. Teachers are encouraged to help students use the ordered list of angles to order the sides. Teachers should ask scaffolding questions such as, "Which side is opposite $\angle C$? Opposite $\angle A$? Opposite $\angle B$?" to help students order the sides.

A common error that students may make is not using the correct symbolic notation to denote a line segment (e.g., $\overline{AB}$ versus $\overleftrightarrow{AB}$ versus AB).

2. Determine the longest side of the ΔLMN. The figure is not drawn to scale. Justify your reasoning.

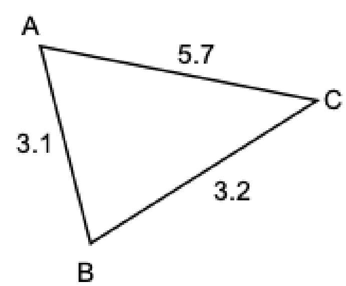


A common error students make is failing to apply their understanding of supplementary angles and the Triangle Sum Theorem when determining the order of the sides in a triangle. Students may misidentify angles by confusing exterior angles with interior angles or by selecting the adjacent interior angle instead of the two remote interior angles. They may also misunderstand the angle–side *relationship—forgetting that the longest side lies opposite the largest angle—and incorrectly assume that the side next to the largest angle is the longest. Some students mistakenly apply the Triangle Sum Theorem when they should use the Exterior Angle Theorem, rely on misleading diagrams rather than calculating the correct relationships, or solving the wrong angle because they forget the goal of the problem. Teachers can support students by using clear visual models, such as color-coding exterior angles and remote interior angles, and by reinforcing key vocabulary and angle relationships, including linear pairs and the fact that an exterior angle equals the sum of the two remote interior angles. Because an exterior angle forms a linear pair with the unknown interior angle of ΔLMN, teachers should demonstrate that subtracting the exterior angle from 180° yields the missing interior angle. Once all angle measures are known, teachers can model how to order the angles from least to greatest and show that the side opposite the greatest angle is the longest. Additional color-coding of missing angle measures and their opposite sides can further support student understanding.*

3. In any triangle with an angle measure of 110°, is the longest side always opposite this angle? Justify your reasoning.

   A common misconception that students may have is to reason that this is a false statement. This may indicate that students fail to recognize that if one angle in a triangle measures 110°, then the sum of the measures of the other two angles is 70°. Therefore, the side opposite the 110° angle must be the longest side as this angle serves as the greatest angle measure. Teachers may want to use concrete manipulatives or dynamic Geometry software to create triangles with various angle measures and then determine the order of the side lengths in both ascending and descending order.

4. Three sprinkler heads are placed on a lawn at each vertex of a triangular pattern shown. Order the angles that are formed by the triangular pattern from smallest to largest angle using the blanks provided. The figure is not drawn to scale.



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The common error students may make is to list the sides in the order from smallest to largest $\overline{AB}, \overline{BC}, \overline{AC}$. This may indicate that students do not recognize the angle that is opposite each side of the triangle. Teachers are encouraged to demonstrate this concept using color-coded diagrams while *going over examples with students, such as using the same color to outline the smallest side and angle, etc. Students may benefit from teachers drawing an arrow from one side to the opposite angle so that they may visualize any angle and its opposite side. Teachers may wish to reinforce the use of appropriate geometric notation (e.g., writing $\angle A, \angle B, and\ \angle C$ instead of A, B, C to represent angles).*

5. Three stages at a music festival are arranged in a triangle. The distances between the centers of each stage are given:
   - Stage A and Stage B: 100 yards
   - Stage B and Stage C: 135 yards
   - Stage C and Stage A: 210 yards

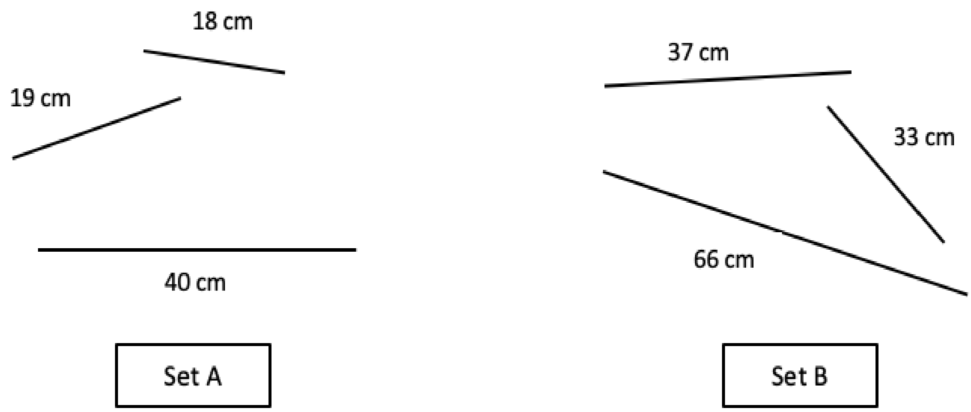
   Using the information above, list the angles created at Stage A, Stage B, and Stage C in descending order.

   A common error students may make is to list the angles from smallest to largest. This may indicate that students do not understand the meaning of descending vs. ascending. Teachers may consider using visual aids (such as arranging straws with various lengths) or kinesthetic activities (such as asking students to stand up or sit down from their seats) to illustrate the meaning of ascending (to climb up) and descending (to climb down).

6. ΔPQR has a perimeter of 48 cm. $\overline{PQ} = x$, $\overline{QR} = \frac{4}{3}\overline{PR}$, and $\overline{PR} = \frac{3}{5}\overline{PQ}$. List the interior angle measures of ΔPQR, ordered from greatest to least.

   A common error some students may make is to base their list of the angles in descending order according to the coefficients of the given side length measures, $\overline{QR}$ being the longest side and $\overline{PR}$ being the shortest side. This may indicate that some students are not considering that $\overline{QR} = \frac{4}{3}\overline{PR}$ is equivalent to $\frac{4}{3} \cdot \frac{3}{5}x$, which represents the side between the longest and shortest side of the triangle. Teachers should encourage students to draw out the triangle and label each side based on the given information before ordering the angles. Then, engage students in color-coding as described in previous teacher notes to help them to differentiate between the longest side lengths and the relationship to the angle measures.

7. Ms. Jones asked her students to determine which of the following sets of segments could form a triangle.



Jessica states the sum of the lengths of the two shorter sides of Set A is less than the length of the longest side. Therefore, the line segments in Set A would form the sides of a triangle.

Ashley states the sum of the lengths of the two shorter sides of Set B is greater than the length of the longest side. Therefore, the line segments in Set B would form the sides of a triangle.

Who is correct, Jessica or Ashley? Justify your reasoning.

A common error students may make is agreeing with Jessica's reasoning. This may indicate that students have not applied the Triangle Inequality Theorem appropriately where they are to determine that the sum of the two shorter side lengths must be greater than the longest side length to form a triangle. This may also indicate that students have ignored criteria such as the "longest side length." Students with this misconception would benefit from first verifying whether the sum of any two sides of a triangle is greater than the measure of the third side. As soon as the students discover that the sum of two sides is less than (or equal to) the measure of a third side, then the sides cannot form a triangle. Teachers should demonstrate that the sum of the lengths of the two shorter sides of a triangle must always be greater than the length of the third and longest side. Teachers are encouraged to provide examples of side lengths using visual representations or concrete manipulatives to determine whether a triangle can be formed. Teachers may adjust the units for the use of manipulatives.

8. Could a triangle have side lengths of 16 ft, 44 ft, and 28 ft? Justify your reasoning.

   A common error that students may make is stating that the triangle could be formed as the sum of the two shorter sides is greater than or equal to the longest side. This may indicate that students did not verify each aspect of the Triangle Inequality Theorem (AB + BC > AC; BC + AC > AB; and AC + AB > BC). Students may benefit from assigning each side length measure to a variable (i.e., AB, BC, or AC). Then, using the substitution property, evaluate each inequality statement. It would be helpful if teachers encouraged students to examine the pair of side lengths that would determine whether a triangle is formed – the sum of the two shorter side lengths (16 ft and 28 ft) must be greater than the longest side length (44 ft). Teachers may give students multiple side lengths to examine using visual representations, concrete manipulatives, or dynamic software to determine whether a triangle can be formed (units may be adjusted as appropriate).

9. A triangle has side lengths of 2 units and 8 units. Identify all possible integral lengths of the third side. Justify your reasoning.

   A common error students may make is selecting 3, 4, 5, or 6; however, these values do not satisfy the Triangle Inequality Theorem. This may indicate that students have not applied the Triangle Inequality Theorem appropriately. Reference question 1 for strategies to support students who demonstrate this misconception.

10. Anthony stated that a triangle with side lengths of 3 cm and 4 cm could have a third side with a side length of 7 cm. Is Anthony correct? Explain. Create a statement about the length of the third side.

    Some students will indicate that Anthony is correct. This may indicate a common misconception that $AB + BC \geq AC$. Further, a common error that students may make is not recognizing that a triangle cannot be formed if the length of the third side is equal to the sum of the lengths of the other two sides. The third side length must be greater than the sum of the other two sides for a triangle to exist. Therefore, in this example, the third side length must be greater than 1 and less than 7 as no other length would form a triangle. Teachers are encouraged to emphasize the Triangle Inequality Theorem and how to verify whether a triangle exists by demonstrating that the sum of the shorter side lengths is equal to the longest side length per this example; hence, the triangle cannot exist based on the given side lengths. Reference question 1 for additional strategies to support students who demonstrate this misconception.

11. The lengths of two sides of a triangle are 30 inches and 50 inches. What is the range of possible lengths for the third side of this triangle? Explain your thinking.

    A common misconception students may have is to determine that the possible lengths ***are*** *20 inches or 80 inches instead of a range of values* ***between*** *these two parameters. This may indicate that students correctly found the values of the upper and lower bounds of the range by finding the sum and difference of 50 inches and 30 inches but do not understand that there are an infinite number of values that can represent the length of the third side. Teachers are encouraged to emphasize with students that the length of each side must be* ***within the range*** *that is determined by the lengths of the other two sides. Teachers are encouraged to use concrete manipulatives such as pieces of string, straws, or other manipulatives to model or to illustrate the range of values that can be used to form triangles when two side lengths are given.*

12. Michele is creating a triangle by connecting three wooden rods. If the lengths of two of the wooden rods are 15 centimeters and 22 centimeters, what is the range of possible lengths of a third wooden rod that will create a triangle? Justify your reasoning.

    A common error students may make is writing the compound inequality incorrectly to express the range of values. Some students may be able to explain this concept in words but be unable to write the symbolic form for the compound inequality that represents the range of possible values. Teachers may want to ask students about lengths that would not fit the parameters of this example such as 0 centimeters or 99 centimeters, so that students may discover that another constraint is needed. Teachers are encouraged to show students how to write their solution as a compound inequality;

and, then using the substitution property, substituting multiple values that would satisfy the compound inequality.

13. The measures of two sides of a triangle are 8 feet and 12 feet. Use values to fill in the blanks to create a true statement about the length of the third side of the triangle.

    The third side of the triangle must be a value larger than _______ but smaller than _______.

    A common error students may make is identifying the given side lengths as the constraints of the compound inequality ($8 < x < 12$). This may indicate that students do not understand that to determine the range of the measure of the third side, students must find the sum and difference of the given side lengths. Teachers are encouraged to use concrete manipulatives such as pieces of string, straws, or other manipulatives to model or to illustrate the range of values that can be used to form triangles when two side lengths are given. Teachers may also use dynamic software to model the range in which the length of the third side must lie.