Just in Time Quick Check

Standard of Learning A2.EI.2

Strand: Equations and Inequalities

Standard of Learning A2.EI.2

The student will represent, solve, and interpret the solution to quadratic equations in one variable over the set of complex numbers and solve quadratic inequalities in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Create a quadratic equation or inequality in one variable to model a contextual situation.

b) Solve a quadratic equation in one variable over the set of complex numbers algebraically.

c) Determine the solution to a quadratic inequality in one variable over the set of real numbers algebraically.

d) Verify possible solution(s) to quadratic equations or inequalities in one variable algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.

Just in Time Quick Check

Just in Time Quick Check Teacher Notes

Supporting and Prerequisite SOL: A.EI.3

Just in Time Quick Check A2.EI.2

1. Find the solution(s) of the equation $x^2 + 36 = 0$.

2. What is the solution set for $(x - 16)^2 + 13 = 0$?

3. What are the roots of $2 - 4x = -3x^2$?

4. The value of the discriminant of a quadratic equation is -7. Describe the nature of the roots of this quadratic equation.

5. Donelle and Susan are trying to determine the number and type of solutions for the equation $2x^2 - 3x - 5 = 0$ by using the discriminant. Their work is shown below.

Donelle's Work	Susan's Work
$D = b^2 - 4a$	$D = b^2 - 4ac$
$D = (-3)^2 - 4(2)(-5)$	$D = (-3)^2 - 4(2)(-5)$
$D = 9 - 40$	$D = 9 + 40$
$D = -31$	$D = 49$
Because the discriminant is less than zero, there are two imaginary solutions.	Because the discriminant is greater than zero, there are two real solutions.

Describe and correct the errors made.

6. A quadratic equation with real coefficients has $x = -6 + 5i\sqrt{2}$ as one solution. What other value of *x* must also be a solution to this quadratic equation?

7. A baseball is thrown in the air. Its height, *h,* in feet after *t* seconds is given by the function:

$$h(t) = -16t^2 + 24t + 3$$

a) When does the baseball reach its maximum height?

b) What is the baseball's maximum height?

8. Solve the inequality over the set of real numbers and justify your solution.

$$x^2 - 5x - 6 \leq 0$$

A2.EI.2 Just in Time Quick Check Teacher Notes

Common Errors/Misconceptions and their Possible Indications

1. Find the solution(s) of the equation $x^2 + 36 = 0$.

   A common error some students may make is to find only one imaginary solution when solving $x^2 = \sqrt{-36}$. This may indicate that students do not understand that a quadratic equation has two solutions. It may be helpful to emphasize that the $\pm$ is essential and represents two distinct solutions. Teachers may wish to model solving this equation using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$. In addition, it may be beneficial for teachers to review simplifying square roots involving imaginary numbers, particularly the role of i, to help students correctly find the two solutions. Teachers could also have students compare the solutions of $x^2 = 36$ and $x^2 = -36$ by solving algebraically or graphically.

2. What is the solution set for $(x - 16)^2$ + 13 = 0?

   A common error some students may make is to state the solution set as $\{4 - i\sqrt{13},\ 4 + i\sqrt{13}\}$. This error may indicate that some students incorrectly simplified $\sqrt{(x-16)^2}$, believing that it results in x – 4. It may be helpful to have students rewrite $\sqrt{(x-16)^2}$ as $\sqrt{(x-16)(x-16)}$. This may elicit prior knowledge from Algebra 1, where students simplified square roots by identifying factor pairs. This strategy may reinforce the idea that the quantity being squared is the entire binomial, not just the variable.

3. What are the roots of $2 - 4x = -3x^2$?

   A common student error is incorrectly identifying the coefficients as $a = -3$, $b = -4$, and $c = 2$ when using the quadratic formula. This suggests that some students may not recognize the need to first rewrite the equation in standard form, $Ax^2 + Bx + C = 0$, by moving all terms to one side. Emphasizing this step supports students in more accurately factoring and determining the roots of the equation. Students may also benefit from using a graphic organizer to reinforce this consistent first step when solving quadratic equations.

4. The value of the discriminant of a quadratic equation is -7. Describe the nature of the roots of this quadratic equation.

   A common error some students may make is to state that this quadratic equation has two irrational roots, rather than two complex roots. This may indicate that students do not understand that the discriminant of -7 represents the radicand of the quadratic formula and yields two nonreal solutions. Teachers may find it beneficial to have students create a graphic organizer demonstrating the value of the discriminant, the graphical representation that is associated with the discriminant value, and a description of the nature of the roots. Visualizing how the discriminant relates to both the graph and the quadratic formula may help students correctly interpret nonreal solutions and distinguish them from irrational real solutions.

5. Donelle and Susan are trying to determine the number and type of solutions for the equation $2x^2 - 3x - 5 = 0$ by using the discriminant. Their work is shown below.

Donelle's Work	Susan's Work
$D = b^2 - 4a$	$D = b^2 - 4ac$
$D = (-3)^2 - 4(2)(-5)$	$D = (-3)^2 - 4(2)(-5)$
$D = 9 - 40$	$D = 9 + 40$
$D = -31$	$D = 49$
Because the discriminant is less than zero, there are two imaginary solutions.	Because the discriminant is greater than zero, there are two real solutions.

Describe and correct the errors made.

A common error some students may make is not recognizing that Donelle's work is not simplified correctly. Some students may incorrectly simplify the discriminant as $(-3)^2 - 40$ instead of $(-3)^2 + 40$. This may indicate that some students do not recognize that the product of –(4)(2)(-5) yields +40. Teachers may encourage students to graph the equation to verify the number of solutions, which would support their algebraic thinking.

6. A quadratic equation with real coefficients has $x = -6 + 5i\sqrt{2}$ as one solution. What other value of *x* must also be a solution to this quadratic equation?

A common error some students may make is stating that the other solution is $x = 6 - 5i\sqrt{2}$. This error may indicate that students believe the conjugate is found by taking the opposite of the real part and the imaginary part of the given solution. Teachers may wish to remind students when solving quadratic equations, each complex solution will always have a conjugate pair such as a + bi and a – bi, where a and b are real numbers. Teachers may find it helpful to have students first write their solutions in complex form and then circle the operations to show they are opposite.

7. A baseball is thrown in the air. Its height, *h*, in feet after *t* seconds is given by the function:

$$h(t) = -16t^2 + 24t + 3$$

a) When does the baseball reach its maximum height?

b) What is the baseball's maximum height?

A common error students may make when working with a quadratic function is misreading the function as linear and expressing it as: h(t) = -16t + 24t + 3. The presence of t^2 in the first term is what makes the function quadratic and produces a parabolic shape, necessary for determining the maximum height.

Some students may struggle with the concept of negative time and negative height, incorrectly believing negative time or height values are valid solutions. Teachers should discuss domain restrictions as they relate to time and height where $t \geq 0$ and $h \geq 0$ and use real-world examples to discuss negative values to help students understand projectile motion. Teachers should also *encourage students to check the reasonableness of their solutions and discuss why negative times and heights are not appropriate solutions for the given scenario.*

8. Solve the inequality over the set of real numbers and justify your solution.

$$x^2 - 5x - 6 \leq 0$$

A common error students may make when solving this inequality is incorrectly factoring $x^2 - 5x - 6$*, such as writing* $(x-6)(x-1)$ *instead of the correct factors* $(x-6)(x+1)$*. Another common error students may make is treating the inequality like an equation by identifying the zeros* $x = -1$ *and* $x = 6$ *but listing only these values as the solution. Students may also misinterpret the inequality symbol by writing the solution as* $x \leq -1$ *or* $x \geq 6$*, rather than writing the interval between the roots. Teachers can address these errors by reinforcing systematic factoring strategies and having students check their factors through multiplication, explicitly distinguishing between solving equations and solving inequalities, and emphasizing the need to test intervals to determine where the expression is positive or negative. Additionally, graphing the inequality can further support understanding by providing a visual that the parabola in this example opens upward and lies below or on the x-axis between* $x = 1$ *and* $x = 6$*, helping students justify why the solution includes all real numbers within that interval.*