Just in Time Quick Check

Standard of Learning A2.EI.3

Strand: Equations and Inequalities

Standard of Learning A2.EI.3

The student will solve a system of equations in two variables containing a quadratic expression.

Students will demonstrate the following Knowledge and Skills:

a) Create a linear-quadratic or quadratic-quadratic system of equations to model a contextual situation.

b) Determine the number of solutions to a linear-quadratic and quadratic-quadratic system of equations in two variables.

c) Solve a linear-quadratic and quadratic-quadratic system of equations algebraically and graphically, including situations in context.

d) Verify possible solution(s) to linear-quadratic or quadratic-quadratic system of equations algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.

Just in Time Quick Check

Just in Time Quick Check Teacher Notes

Supporting and Prerequisite SOL: A.EI.2, A2.EI.2

Just in Time Quick Check A2.EI.3

1. What are the *y*-coordinates for the solutions to the system of equations shown?

$$\begin{cases} 2x = -y + 4 \\ y + 8 = x^2 + 2x \end{cases}$$

2. A ball is tossed upward from a balcony. The height, *h,* in feet, of the ball after *t* seconds is shown by the equation $h = -16t^2 + 20t + 40$. At the same time, a drone is flying at a constant height of 35 feet. Create a system of equations to determine when the ball reaches the drone's height.

3. Determine the number of solutions for the following system of equations. Use the discriminant to verify the number of solutions.

$$\begin{cases} y = x^2 + 4x + 3 \\ y = 2x - 1 \end{cases}$$

4. Solve the following system of equations algebraically, then verify your solutions graphically.

$$\begin{cases} y = -x^2 + 6x \\ y = x^2 - 2x + 4 \end{cases}$$

5. Solve the following system of equations:

$$\begin{cases} y = x^2 - 2x \\ y = 3x - 4 \end{cases}$$

A2.EI.3 Just in Time Quick Check Teacher Notes

Common Errors/Misconceptions and their Possible Indications

1. What are the *y*-coordinates for the solutions to the system of equations shown?

$$\begin{cases} 2x = -y + 4 \\ y + 8 = x^2 + 2x \end{cases}$$

A common error some students may make is stating the x-coordinates instead of the y-coordinates when asked for the y-values of the points of intersection. For example, students may say the y-coordinates are -6 and 2, instead of 16 and 0. This may indicate students can solve the system algebraically but may not accurately interpret which coordinate is needed. It may be beneficial to have students write the ordered pair solutions first as (x, y) and then circle or highlight the y-coordinate. Students could also check their solutions graphically or by substituting back into both equations to verify the y-values.

2. A ball is tossed upward from a balcony. The height, *h,* in feet, of the ball after *t* seconds is shown by the equation $h = -16t^2 + 20t + 40$. At the same time, a drone is flying at a constant height of 35 feet. Create a system of equations to determine when the ball reaches the drone's height.

A common error students may make is to represent the height of the ball as an expression, $-16t^2 + 20t + 40$, *rather than as an equation,* $h = -16t^2 + 20t + 40$. *Another common error is to only write one equation,* $-16t^2 + 20t + 40 = 35$, *without representing the drone's height as a separate equation,* $h = 35$. *This can lead to students misunderstanding the structure of a system of equations. Students should be encouraged to identify each situation separately, translating verbal descriptions into the appropriate mathematical models (e.g., linear for constant rate, quadratic for acceleration), and explicitly writing the system of equations. Using graphic organizers to compare models and providing examples and non-examples of mathematical models can help students accurately represent and solve these types of problems.*

3. Determine the number of solutions for the following system of equations.
Use the discriminant to verify the number of solutions.

$$\begin{cases} y = x^2 + 4x + 3 \\ y = 2x - 1 \end{cases}$$

A common error students may make when determining the number of solutions for this system of equations is to fail to set the equation equal before analyzing the system. For example, students may attempt to work with $y = x^2 + 4x + 3$ *without first writing* $x^2 + 4x + 3 = 2x - 1$. *Students may also make sign errors or algebraic errors when combining like terms, such as simplifying* $x^2 + 4x + 3 - 2x - 1$ *as* $x^2 + 6x + 2 = 0$. *Students may also misidentify coefficients a, b, and c when computing the discriminant, such as using* $a = 1$, $b = 6$, $c = 2$, *which leads to an incorrect discriminant and may result in an inaccurate conclusion about the number of solutions.*

It may be helpful for teachers to explicitly model each step: how to rewrite the equation in standard form, how to correctly label a, b, and c, and how to substitute these values in the discriminant formula. Finally, connecting the negative discriminant ($D = 2^2 - 4(1)(4) = -12$) to a graph of the parabola and line reinforces why the system has no real solutions and helps students see the connections between the algebraic and graphical representations of the system of equations.

4. Solve the following system of equations algebraically, then verify your solutions graphically.

$$\begin{cases} y = -x^2 + 6x \\ y = x^2 - 2x + 4 \end{cases}$$

Students may neglect to set the equations equal and instead try to solve them separately, rather than writing $-x^2 + 6x = x^2 - 2x + 4$. They may make sign errors or combine like terms incorrectly when rewriting in standard form, such as simplifying $-x^2 + 6x - x^2 + 2x - 4 = 0$ as $-2x^2 + 4x - 4 = 0$ instead of the correct simplified equation of $-2x^2 + 8x - 4 = 0$. Students may also solve for x correctly but forget to substitute back to find corresponding y-values.

Teachers can support students by modeling the correct setup step by step, demonstrating that setting the equations to be equal leads to $-2x^2 + 8x - 4 = 0$, then dividing by -2 results in $x^2 - 4x + 2 = 0$. Students should also be encouraged to verify solutions graphically by plotting both equations and confirming that the intersection points match the algebraic solutions, providing a visual connection between the algebraic and graphical representations.

5. Solve the following system of equations:

$$\begin{cases} y = x^2 - 2x \\ y = 3x - 4 \end{cases}$$

A common error students may make when solving this system is forgetting to check their solutions in both original equations. For example, after solving $x^2 - 2x = 3x - 4$ and finding $x = 1$ and $x = 4$, students may compute the y-values from only one equation or they may substitute incorrectly, which can lead to reporting incorrect points. Students may also make sign errors or mistakes when combining like terms, such as simplifying $x^x - 2x - 3x + 4$ incorrectly, resulting in an incorrect quadratic. It is also common for students to report only the x-values or only the y-values instead of full ordered pairs (x, y) as their solutions. This may indicate that students do not recognize that the solutions to the system of equations represent points of intersection of the given parabola and line.

Teachers can support students by modeling each step: setting the equations to be equal, simplifying correctly, solving for x, substituting back to find y, and writing the solutions as ordered pairs. It may also be helpful to emphasize the connection between the algebraic solutions and the graphical representation of the parabola and line to help students understand the meaning of solutions. Providing students with a verification checklist can help students ensure they have completed all the steps and may help reduce the number of errors made.