Just in Time Quick Check

Standard of Learning A2.EI.4

Strand: Equations and Inequalities

Standard of Learning A2.EI.4

The student will represent, solve, and interpret the solution to an equation containing rational algebraic expressions.

Students will demonstrate the following Knowledge and Skills:

a) Create an equation containing a rational expression to model a contextual situation.

b) Solve rational equations with real solutions containing factorable algebraic expressions algebraically and graphically. Algebraic expressions should be limited to linear and quadratic expressions.

c) Verify possible solution(s) to rational equations algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.

d) Justify why a possible solution to an equation containing a rational expression might be extraneous.

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Just in Time Quick Check Teacher Notes

Supporting and Prerequisite SOL: A.EI.3, A2.EO.1, A2.EO.3, A2.EI.2d

Just in Time Quick Check A2.EI.4

1. Find the solution for the following equation: $\frac{x-3}{x^2-5x+6}+\frac{x-3}{2x-6}=1$

2. Find the solution for the equation: $\frac{6}{x-1}=\frac{4}{x-2}+\frac{2}{x+1}$

3. A cyclist travels a 30-mile route. Her speed in the first half is *x* miles per hour (mph). In the second half, her speed increases by 4 mph. Write an equation that represents a total ride time of 2.5 hours.

4. Solve and verify the solution to the following equation algebraically.

$$\frac{4x}{x^2-1}=1$$

A2.EI.4 Just in Time Quick Check Teacher Notes

Common Errors/Misconceptions and their Possible Indications

1. Find the solution for the following equation: $\frac{x-3}{x^2-5x+6} + \frac{x-3}{2x-6} = 1$

   A common error some students may make is to simplify the rational expression and think the least common denominator (LCD) is x-2. Other students may simplify correctly and identify the LCD but neglect to multiply both sides of the equation by the LCD, therefore setting the equation equal to 1. This may indicate that students do not know how to correctly identify and/or multiply by the LCD. Teachers may wish to use algebra tiles to model how 2 and x – 2 are different values, and both must be included as part of the LCD. Some students may also benefit from the visual of using an equation mat to indicate how the equation is unbalanced if they do not multiply both sides by the LCD. Teachers may wish to have students create a graphic organizer including questions like: What factors do we need in every denominator? What do we need to multiply by the first denominator to get the common factors? What do we need to multiply by the second denominator to get the common factors? Teachers could also encourage students to use a graphing utility such as Desmos to graph their simplified expressions and the given expression to determine if they are equivalent.

2. Find the solution for the equation: $\frac{6}{x-1} = \frac{4}{x-2} + \frac{2}{x+1}$

   A common error students may make is failing to check extraneous solutions. This type of error may indicate that some students assume that all answers found are actual solutions to a problem. Teachers may want to use Desmos to show students where asymptotes exist and how to graphically identify extraneous solutions. Students may benefit from exploring how the restricted domain values might impact extraneous solutions, $x \neq -1, x \neq 1,\ x \neq 2$. It may be beneficial for students to create a graphic organizer that includes checking solutions as a final step.

3. A cyclist travels a 30-mile route. Her speed in the first half is *x* miles per hour (mph). In the second half, her speed increases by 4 mph. Write an equation that represents a total ride time of 2.5 hours.

   When creating a rational equation to model this contextual situation, students may misunderstand how distance, speed, and time are related and neglect to use the formula, time = distance ÷ speed. A common error students may make is to use the total distance of 30 miles for each part of the trip, leading to an equation such as $\frac{30}{x} + \frac{30}{x+4} = 2.5$, rather than recognizing that each half of the route is 15 miles. Students may also incorrectly combine the numerators and denominators, resulting in the equation $\frac{30}{x} + \frac{30}{x+4} = \frac{60}{2x+4}$. It may be helpful to encourage students to annotate units when translating verbal expressions to algebraic equations (miles ÷ miles per hour = hours), use diagrams or tables to represent each half of the trip, and explicitly connect the equation setup to the reasoning that total time equals the sum of the times for each section. It may also be beneficial for teachers to model how to break the situation into parts before writing the equation.

4. Solve and verify the solution (s) to the following equation algebraically.

$$\frac{4x}{x^2 - 1} = 1$$

A common error students may make when solving this rational equation is forgetting to consider domain restrictions and treating x = ±1 as possible solutions, even though these values make the denominator equal to zero and must be excluded. Another common error may occur when students try to clear the denominator by multiplying each side by $x^2 - 1$. Students may neglect to use parentheses around $x^2 - 1$, leading to incorrect use of the distributive property (e.g., $4x = x^2 - 1 + 1$).

After obtaining a quadratic equation, students may make sign errors when rearranging terms or they may misapply the quadratic formula by substituting incorrect values for a, b, and c. For example, students may think that a = 1, b = 4, and c = 0, which would lead to an incorrect discriminant or lost solutions.

Teachers can address these errors by encouraging students to state domain restrictions prior to beginning algebraic manipulation of the equation. It may also be helpful to model the correct use of parentheses when clearing denominators and to review the correct use of the distributive property. Students should also be encouraged to verify their solutions by substituting their solutions back into the original equation, which may reinforce procedural accuracy and help develop conceptual understanding of rational equations.