Just in Time Quick Check

Standard of Learning A2.EI.6

Strand: Equations and Inequalities

Standard of Learning A2.EI.6

The student will represent, solve, and interpret the solution to a polynomial equation.

Students will demonstrate the following Knowledge and Skills:

a) Determine a factored form of a polynomial equation, of degree three or higher, given its zeros or the *x*-intercepts of the graph of its related function.

b) Determine the number and type of solutions (real or imaginary) of a polynomial equation of degree three or higher.

c) Solve a polynomial equation over the set of complex numbers.

d) Verify possible solution(s) to polynomial equations of degree three or higher algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions in context.

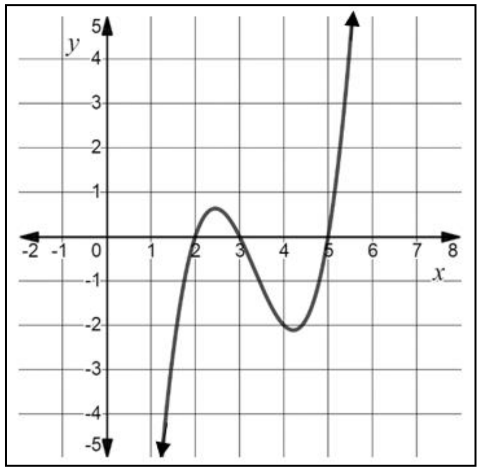
Just in Time Quick Check

Just in Time Quick Check Teacher Notes

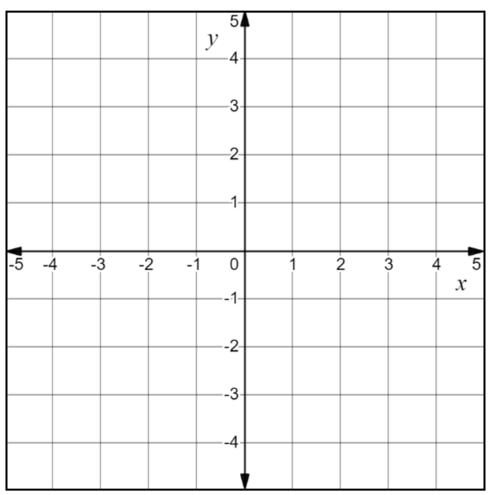
Supporting and Prerequisite SOL: A2.EI.2, A2.EI.5

Just in Time Quick Check A2.EI.6

1. The graph of a function is shown. Write the equation that best represents this function.



2. Create a polynomial function, $f(x)$, that has roots of -2, -1, and 1 with multiplicity 2.



3. Write the factored form of a polynomial equation given the zeros are $x = 2,\ x = -1$, and $x = 3$.

4. Given the following polynomial equation, determine the number of solutions and types of solutions (real or imaginary).

$$f(x) = x^3 - 2x^2 + x - 5$$

5. Solve the following polynomial equation.

$$x^3 - x^2 - 4x + 4 = 0$$

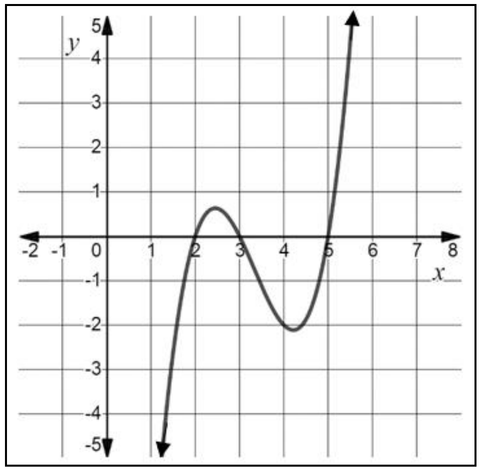
6. Verify the solutions to the following equation algebraically and graphically.

$$(x - 2)(x + 1)(x - 3) = 0$$

A2.EI.6 Just in Time Quick Check Teacher Notes

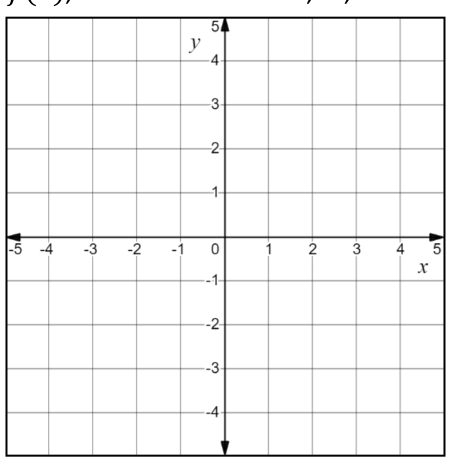
Common Errors/Misconceptions and their Possible Indications

1. The graph of a function is shown. Write the equation that best represents this function.



A common error some students may make is to correctly identify the zeros as 2, 3, and 5 but then write the factors as (x + 2)(x + 3)(x + 5). This error reflects a misunderstanding of the relationship between zeros and factors. Specifically, if $x = k$ is a zero of a polynomial, then $(x - k)$, not $(x + k)$, is a corresponding factor, as stated by the Factor Theorem. Students may benefit from a review of the Zero-Product Property and Factor Theorem, emphasizing how $(x - k) = 0$ leads directly to the zero $x = k$. Teachers may wish to have students practice constructing polynomial functions from given zeros and then graphing those functions to verify their work. Throughout this process, students should explicitly connect each x-intercept on the graph to its corresponding linear factor. Additionally, the use of graphing technology can further support students in visualizing how factors, zeros, and graphs are related.

2. Create a polynomial function, $f(x)$, that has roots of -2, -1, and 1 with multiplicity 2.



A common error is for students to draw a graph that passes through the x-axis at $x = -2, x = -1$, and $x = 1$, resulting in a cubic polynomial. This error may indicate a lack of understanding of *multiplicity, specifically that a root with even multiplicity does not cross the x-axis. Instead, the graph touches x-axis at that x-value and turns around, rather than passing through it. In this problem, the root at x=1 has a multiplicity 2, so the graph should be tangent to the x-axis at $x = 1$. It may be helpful to use a graphing utility to compare polynomial functions that differ only in the exponent of the repeated factor. For example, graphing $f(x) = (x - 1)$ and $g(x) = (x - 1)^2$ allows students to notice that $x - 1)$ crosses the x-axis at $x = 1$, while $(x - 2)^2$ touches the x-axis at $x = 1$ without crossing. This visual comparison may help students make the connection between even multiplicities and tangential behavior at the x-axis.*

3. Write the factored form of a polynomial equation given the zeros are $x = 2$, $x = -1$, and $x = 3$.

   A common error students may make is to correctly identify the zeros but incorrectly write the corresponding factors by using the same sign instead of the opposite sign. For example, students may write $(x + 2)(x - 1)(x + 3) = 0$ rather than the correct form $(x - 2)(x + 1)(x - 3) = 0$. This error may indicate confusion about how zeros relate to factors. Another common error occurs when students list the numerical zeros 2, -1, and 3 as the factors themselves, rather than expressing them in terms of x. Teachers can support student understanding by modeling the relationship between zeros and factors using the Factor Theorem. For example, demonstrating that $x = 2$ corresponds to the factor $(x - 2), x = -1$ corresponds to $(x + 1)$, and $x = 3$ corresponds to $(x - 3)$ helps students clearly see why the sign changes occur. Repeated practice with this reasoning can strengthen students' conceptual understanding and may help to reduce sign errors.

4. Given the following polynomial equation, determine the number of solutions and types of solutions (real or imaginary).

$$f(x) = x^3 - 2x^2 + x - 5$$

A common error occurs when students attempt to factor the polynomial even though it is not factorable over the set of integers. For example, students may try to factor this as $x^3 - 2x^2 + x - 5 = (x - 5)(x^2 + 1)$ without verifying the result by expanding the factors.

A common misconception arises when students misapply the Fundamental Theorem of Algebra by assuming that a degree-three polynomial must have three real solutions, even though the graph only shows one x-intercept. This error indicates confusion between the total number of solutions (real and imaginary) and the number of real solutions. While the Fundamental Theorem of Algebra states that a polynomial of degree three has exactly three solutions in the complex number system, it does not guarantee that all solutions are real. In this case, because there is only one real root and thus one real solution, the remaining two solutions must be complex (imaginary) solutions that occur as a conjugate pair. Teachers can reinforce this concept by using graphing technology to compare the number of real x-intercepts with the total number of solutions predicted by the theorem.

5. Solve the following polynomial equation.

$$x^3 - x^2 - 4x + 4 = 0$$

Common errors students when solving $x^3 - x^2 - 4x + 4 = 0$ often occur during the factoring process. One common mistake is grouping incorrectly. For example, students may rewrite the expression as $(x^3 - 4x) - (x^2 - 4)$ and then stop, not recognizing that this grouping does not produce a common binomial factor, which prevents further factoring. Another common error is failing to factor completely. For example, some students may stop at $(x - 1)(x^2 - 4) = 0$, not recognizing that $(x^2 - 4)$ is a difference of squares and can be factored further into $(x - 2)(x + 2)$. Students may also incorrectly list the solutions $x = 1$ and $x = 2$, failing to recognize that the factor $(x + 2)$ produces the solution $x = -2$. Teachers can help prevent these errors by emphasizing the importance of factoring completely and applying the Zero-Product Property carefully to every factor.

6. Verify the solutions to the following equation algebraically and graphically.

$$(x - 2)(x + 1)(x - 3) = 0$$

Common errors when verifying solutions often stem from misunderstandings of solving linear factors and applying the Zero-Product Property. For example, students may incorrectly conclude that $x = 1$ is a solution from the factor $x + 1 = 0$, rather than recognizing the correct solution of $x = -1$. Another common misconception is the belief that all factors must equal zero at the same time. This may indicate confusion about the Zero-Product Property, which states that if a product equals zero, then at least one factor must be zero – not necessarily all of them simultaneously. Additionally, errors can occur during algebraic verification when students substitute values inconsistently, such as checking $x = 3$ but failing to replace x in every factor, resulting in expressions like $(3 - 2)(3 + 1)(x - 3)$ instead of $(3 - 2)(3 + 1)(3 - 3)$. Teachers can address these issues by emphasizing careful substitution and encouraging students to verify each solution by fully evaluating the original expression.