Just in Time Quick Check

Standard of Learning A2.EO.3

Strand: Expressions and Operations

Standard of Learning A2.EO.3

The student will perform operations on polynomial expressions and factor polynomial expressions in one and two variables.

Students will demonstrate the following Knowledge and Skills:

a) Determine sums, differences, and products of polynomials in one and two variables.
b) Factor polynomials completely in one and two variables with no more than four terms over the set of integers.
c) Determine the quotient of polynomials in one and two variables, using monomial, binomial, and factorable trinomial divisors.
d) Represent and demonstrate equality of polynomial expressions written in different forms and verify polynomial identities including the difference of squares, sum and difference of cubes, and perfect square trinomials.

Just in Time Quick Check

Just in Time Quick Check Teacher Notes

Supporting and Prerequisite SOL: A.EO.2

Just in Time Quick Check A2.EO.3

1. Factor the expression completely.

$$12x^2 + 7xy - 10y^2$$

2. Factor the expression completely.

$$27a^9 - 64b^6$$

3. What is the complete factorization of $x^4 - 5x - 36$?

4. Factor the expression completely.

$$-11x^2 + 24xy - 4y^2$$

5. Student A factored the expression $8x^2 - 30x + 7$. Their work is shown below. Is their answer correct? If yes, state it is correct. If not, find the error and factor correctly.

Student A

$$8x^2 - 30x + 7$$

$$(8x^2 - 2x) + (-28x + 7)$$

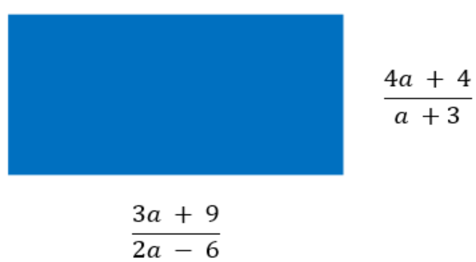
$$2x(4x - 1) + 7(-4x + 1)$$

$$(4x - 1)(2x + 7)$$

6. Divide the following. Assume the denominator does not equal zero.

$$(x^2y + 5xy + 6y) \div (x + 2)$$

7. Write an expression for the area of the rectangle given below. The expression $\frac{4a + 4}{a + 3}$ represents the length of the rectangle, and the expression $\frac{3a + 9}{2a - 6}$ represents the width of the rectangle. The denominators of the length and width do not equal zero.



A2.EO.3 Just in Time Quick Check Teacher Notes

Common Errors/Misconceptions and their Possible Indications

1. Factor the expression completely.

$$12x^2 + 7xy - 10y^2$$

A common error students may make when factoring trinomials is choosing an incorrect factor pair for $a \cdot c = -120y^2$. For example, students may use the factor pair $-15y$ and $8y$, which multiply to $-120y^2$ but add to $-7y$, rather than the required middle coefficient of $+7y$. This error indicates that the chosen factors do not correctly decompose the middle term. This can lead to another common error of splitting the middle term incorrectly, such as rewriting the expression as $12x^2 - 15xy + 8xy - 10y^2$, which does not simplify back into the original polynomial. Even when students correctly rewrite the trinomial as $12x^2 + 15xy - 8xy - 10y^2$, they may neglect to factor out the greatest common factor for each pair leading to an expression. This can result in expressions such as $2x(6x - 4y) + 5y(3x - 2y)$, which contain mismatched binomials and prevent students from completely factoring the polynomial expression. Teachers can support students by explicitly teaching a structured factoring process. This process includes identifying the values of a, b, and c, calculating $a \cdot c \ = 12(-10y^2) = -120y^2$, and verifying that the selected factor pair both multiplies correctly and adds to the middle coefficient. Modeling error analysis, such as demonstrating why the factor pair -15 and $8y$ fails, can help students deepen their understanding of the factoring process.

2. Factor the expression completely.

$$27a^9 - 64b^6$$

A common error students may make when completely factoring this expression is they misidentify the expression as a difference of squares instead of a difference of cubes. This may result in students attempting to factor using square roots, such as by writing $27a^9 - 64b^6 = (\sqrt{27a^9} - \sqrt{64b^6}\,)(\sqrt{27a^9} + \sqrt{64b^6})$. Another common error occurs when students rewrite the powers incorrectly, such as expressing $27a^9$ as $(3a^2)^3$ insted of the correct form $(3a^3)^3$. Some students incorrectly use the difference of cubes formula by applying incorrect exponents or variables in the second factor, resulting in expressions that do not multiply back to the original polynomial. For example, after incorrectly applying the formula, students may get a result of $(3a^3 - 4b^2)(3a^6 + 4a^3b^2 + 16b^4)$. Teachers can support students by explicitly modeling how to rewrite each term as a cube, $27a^9 = (3a^3)^3$ and $64b^4 = (4b^2)^3$ prior to applying the difference of cubes formula. It may also be beneficial to have students verify their results by multiplying the factors back to the original expression. This will allow students to identify and correct any errors made when factoring.

3. What is the complete factorization of $x^4 - 5x^2 - 36$?

A common error students make is they may only factor this expression into the product of two binomials, $(x^2 - 9)(x^2 + 4)$, without checking whether either factor can be factored further. This may indicate that students do not recognize that $x^2 - 9$ is a difference of squares and can be factored further. It may be helpful to emphasize that "factoring completely" means continuing the factoring process until no factor can be factored further. Students may benefit from creating a graphic organizer that prompts students to reexamine each factor for special products, such as differences of squares. Teachers may want to provide students with questions where they must select *all factors of an expression to help students understand that an expression can have more than two factors in its complete factorization.*

4. Factor the expression completely.

$$-11x^2 + 24xy - 4y^2$$

When factoring this expression, students may have difficulty with the negative leading coefficient and should be encouraged to first factor out -1, rather than trying to use -11 and -4. This will likely help to reduce sign errors and will make the trinomial easier to factor.

A common error occurs when students do not factor out -1 and attempt to choose factor pairs directly. They may select numbers that multiply to $a \cdot c = 44$ but add incorrectly to match the middle term. For example, using -22 and -2 without accounting for the negative may produce the split $11x^2 - 22xy - 2xy + 4y^2$, which leads to middle terms that sum to $-24xy$, which does not match the original polynomial.

Another common error is omitting the y variable when factoring. This may indicate students do not understand how to factor expressions with two variables. Highlighting the variables may help students keep track of the variables throughout their work. Additionally, teachers can support students by modeling a structured factoring process to include the following steps:

- *a) Factor out a negative leading coefficient if present.*
- *b) Identify a, b, and c and compute $a \cdot c$.*
- *c) Select factor pairs that multiply to a ·c and sum to the middle coefficient.*
- *d) Split the middle term and factor by grouping.*
- *e) Verify that the factored form multiplies back to the original polynomial.*

5. Student A factored the expression $8x^2 - 30x + 7$. Their work is shown below. Is their answer correct? If yes, state it is correct. If not, find the error and factor correctly.

Student A

$$8x^2 - 30x + 7$$

$$(8x^2 - 2x) + (-28x + 7)$$

$$2x(4x - 1) + 7(-4x + 1)$$

$$(4x - 1)(2x + 7)$$

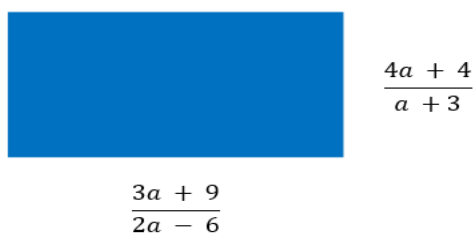
A common error students may make is to state that Student A's work is correct. This may indicate that students do not recognize that the greatest common factor (GCF) of the second binomial, $-28x + 7$, should be negative so that a matching binomial will be produced. When the negative is not factored out, students may incorrectly treat the binomials $4x - 1$ and $-4x + 1$ as equivalent, even though they differ by a factor of -1. Circling the negatives with any leading coefficient may help students identify the GCF should be negative. Teachers should emphasize that binomials must match exactly, not just appear to be similar. Strategies such as highlighting negative signs when factoring and encouraging students to multiply their factors to verify equivalence to the original expressions can reinforce accuracy and support deeper understanding.

6. Divide the following. Assume the denominator does not equal zero.

$$(x^2y + 5xy + 6y) \div (x + 2)$$

A common error students may make is to divide only the first term of the numerator by the divisor and to ignore the remaining terms, which results in an incomplete and incorrect answer. Another common error is to attempt to factor the numerator but misapply the greatest common factor, such as writing $x^2y + 5xy + 6y = y(x^2 + 5x + 6y)$, which changes the original expression. Students may also attempt to cancel individual terms instead of common factors. Teachers can address these errors by modeling how to factor the numerator correctly, obtaining $\frac{(x^2y+5xy+6y)}{(x+2)} = \frac{y(x^2+5x+6)}{(x+2)}$, and then modeling how to factor the quadratic completely, obtaining $x^2 + 5x + 6 = (x + 2)(x + 3)$. Emphasizing that only common factors, not individual terms, can be canceled may help students understand why $\frac{y(x+2)(x+3)}{(x+2)}$ simplifies to $y(x + 3)$. Additionally, encouraging students to verify their work by rewriting each step algebraically can further reinforce accurate division of polynomial expressions.

7. Write an expression for the area of the rectangle given below. The expression $\frac{4a + 4}{a + 3}$ represents the length of the rectangle, and the expression $\frac{3a + 9}{2a - 6}$ represents the width of the rectangle. The denominators of the length and width do not equal zero.



A common error students may make when multiplying rational polynomial expressions is to multiply the numerators and denominators incorrectly. For example, students may incorrectly write $\frac{4a+4}{a+3} \cdot \frac{3a+9}{2a-6} = \frac{12a+36}{2a-6}$, which indicates that they multiplied parts of the expressions, rather than the entire factored forms. Students may also engage in factoring errors, such as canceling terms instead of factors, such as crossing out the $+4$ in $4a + 4$, or leaving the product as $\frac{(4a+4)(3a+9)}{(a+3)(2a-6)}$, without factoring and simplifying. Teachers can support students by emphasizing the importance of factoring first: $4a + 4 = 4(a + 1), 3a + 9 = 3(a + 3)$, and $2a - 6 = 2(a - 3)$. Then the area can be rewritten as $\frac{4(a+1)\cdot 3(a+3)}{(a+3)\cdot 2(a-3)} = \frac{12(a+1)}{2(a-3)} = \frac{6(a+1)}{a-3}$. Teachers should also remind students to state the restrictions on the denominator, $a \neq -3, 3$.