Just in Time Quick Check

Standard of Learning A2.F.1

Strand: Functions

Standard of Learning A2.F.1

The student will investigate, analyze, and compare square root, cube root, rational, exponential, and logarithmic function families, algebraically and graphically, using transformations.

Students will demonstrate the following Knowledge and Skills:

a) Distinguish between the graphs of parent functions for square root, cube root, rational, exponential, and logarithmic function families.

b) Write the equation of a square root, cube root, rational, exponential, and logarithmic function, given a graph, using transformations of the parent function, including $f(x) + k$; $f(kx)$; $f(x + k)$; and $kf(x)$, where k is limited to rational values. Transformations of exponential and logarithmic functions, given a graph, should be limited to a single transformation.

c) Graph a square root, cube root, rational, exponential, and logarithmic function, given the equation, using transformations of the parent function including $f(x) + k$; $f(kx)$; $f(x + k)$; and $kf(x)$, where k is limited to rational values. Use technology to verify transformations of the functions.

d) Determine when two variables are directly proportional, inversely proportional, or neither, given a table of values. Write an equation and create a graph to represent a direct or inverse variation, including situations in context.

e) Compare and contrast the graphs, tables, and equations of square root, cube root, rational, exponential, and logarithmic functions.

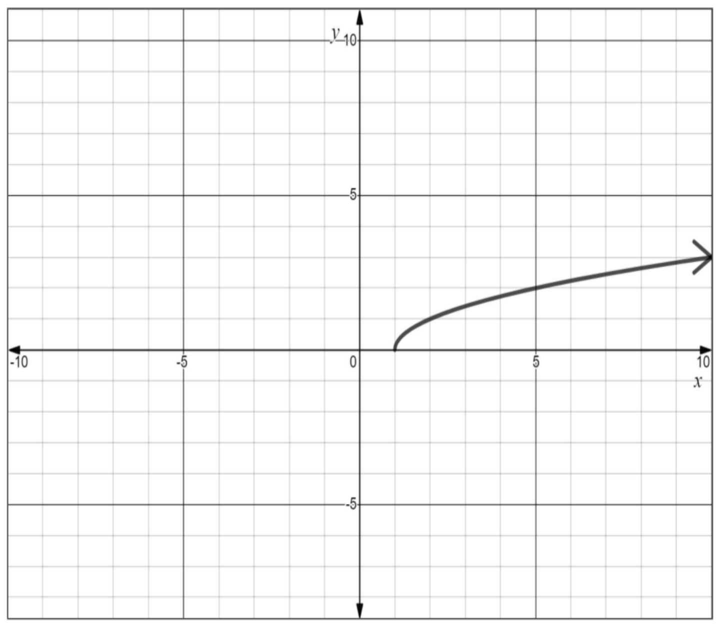
Just in Time Quick Check

Just in Time Quick Check Teacher Notes

Supporting and Prerequisite SOL: A.F.1, A2.F.2

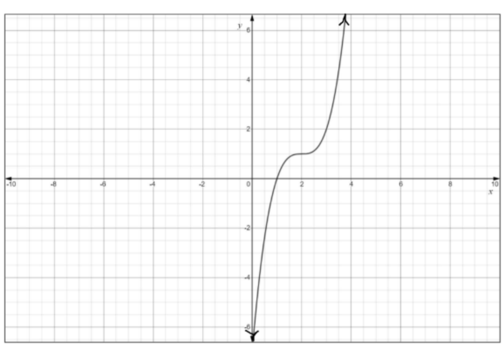
Just in Time Quick Check A2.F.1

1. The graph of *f(x)* is shown. Determine whether *f(x)* appears to be a square root function or a logarithmic function. Justify your reasoning.

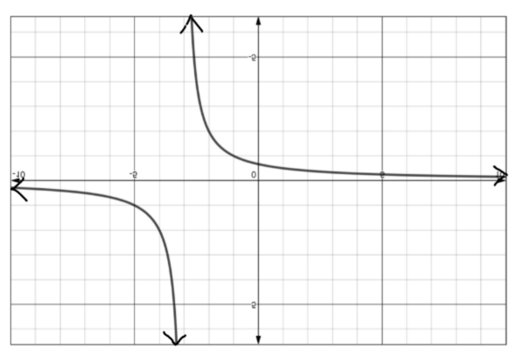


2. Identify the function family to which each graph appears to belong. Justify your answers.

a)



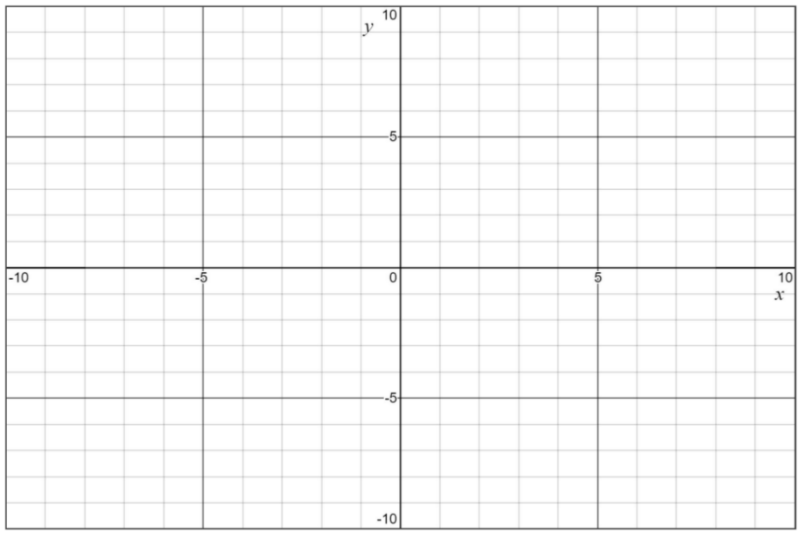
b)



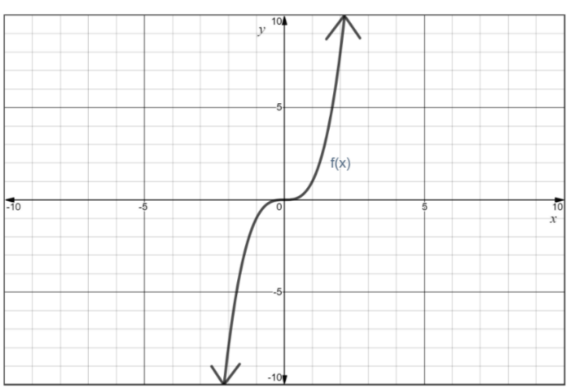
3. Create and graph an exponential function *g(x)* that represents a transformation of its parent exponential function.

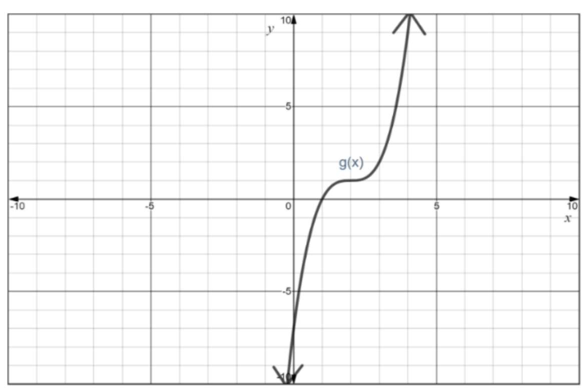
   a) Graph both the parent function and the transformed function on the same coordinate plane, label at least three points on the graph.

   b) Write the equation for *g(x)* and describe how it is transformed from the parent function.



4. A graph of the function $f(x) = x^3$, and the function after being transformed $g(x)$, are shown.





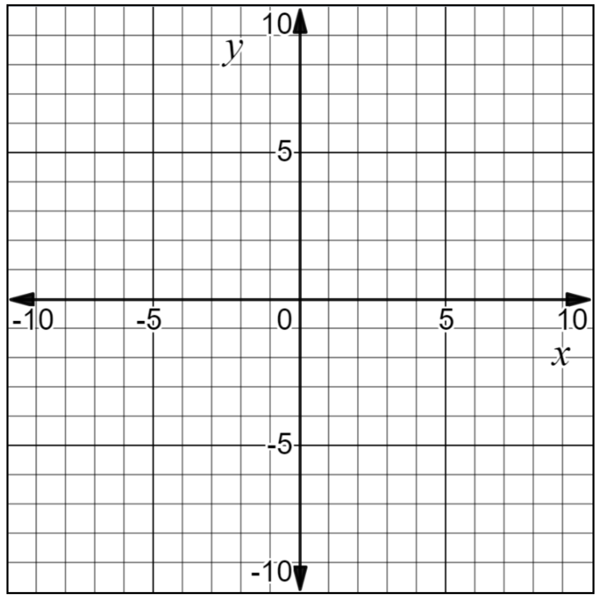
   a) Describe the transformation of $f(x)$ to produce $g(x)$.

   b) Write the equation to represent the graph of $g(x)$.

5. Graph the equation $g(x) = \sqrt{4 - x}$.

   a) Label at least three points with integral coordinates.

   b) Explain how the graph of function $g(x) = \sqrt{4 - x}$ differs from the parent function $f(x) = \sqrt{x}$.



6. A local furniture company has scheduled drivers to deliver furniture to customers on a fixed route. The table below shows the relationship between the number of drivers, *d,* working on the route and the time*, t,* in hours it takes to complete the delivery route.

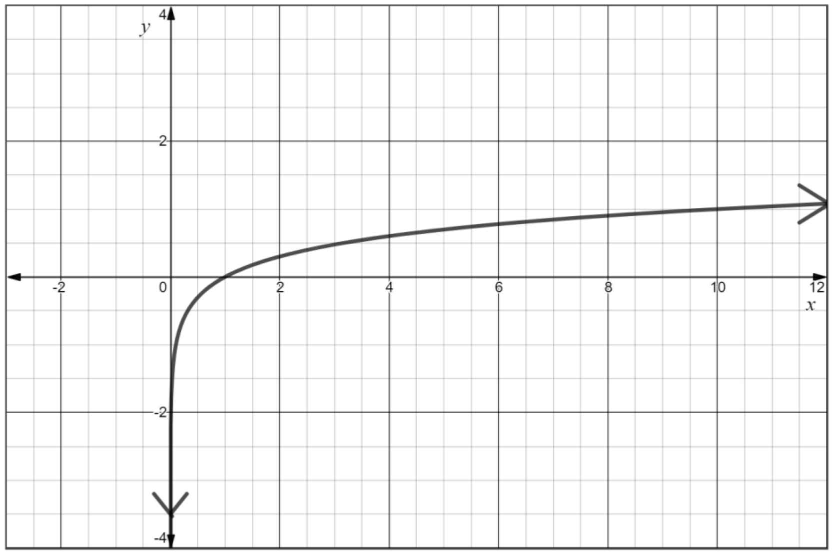
   a) Determine whether the relationship represents a directly proportional relationship, an inversely proportional relationship, or neither.

   b) Write an equation that represents the relationship.

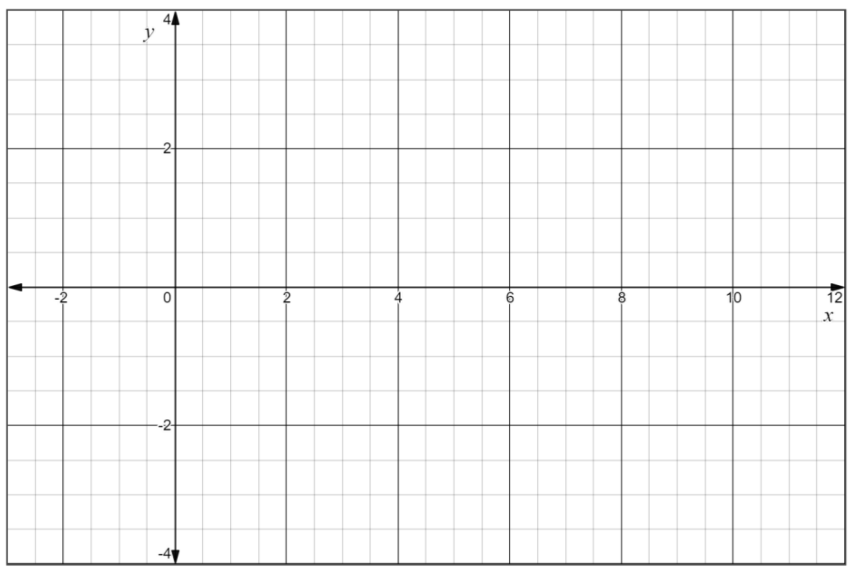
   c) Explain what the relationship means in the context of the problem.

Number of Drivers *d*	**Time in hours *t***
2	24
3	16
4	12
6	8
8	6
12	4

7. The graph of the logarithmic function $f(x) = \log x$ is shown.



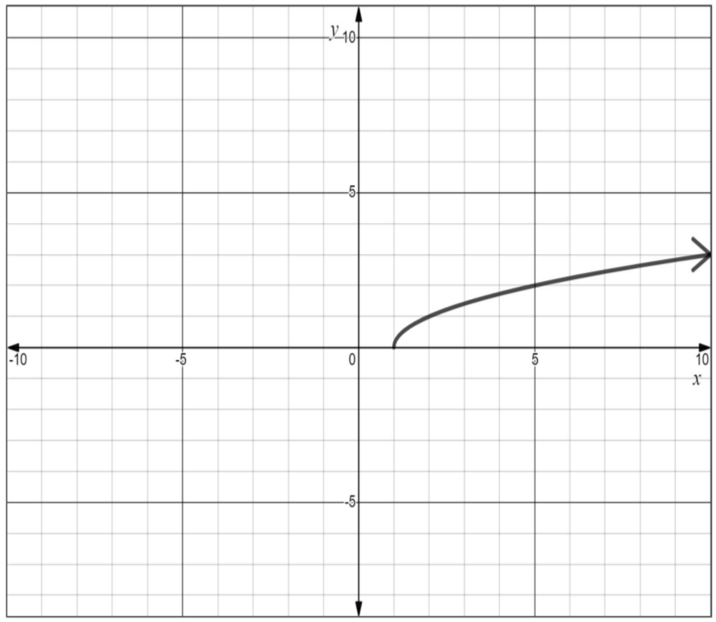
Use the grid below to graph the function $g(x) = \log(x - 2)$.



A2.F.1 Just in Time Quick Check Teacher Notes

Common Errors/Misconceptions and their Possible Indications

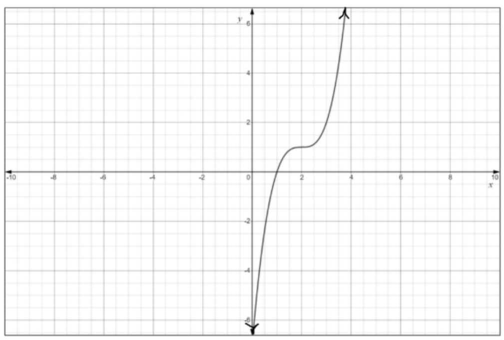
1. The graph of *f(x)* is shown. Determine whether *f(x)* appears to be a square root function or a logarithmic function. Justify your reasoning.



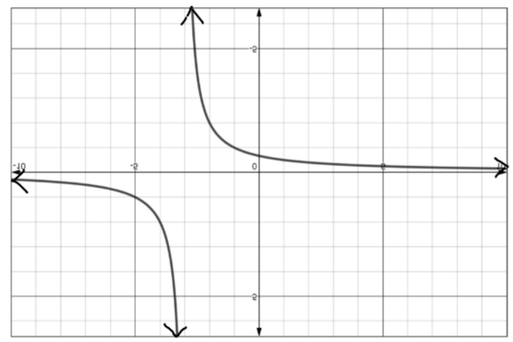
A common misconception is that some students may believe that logarithmic functions have a restricted range. However, logarithmic functions have a restricted domain and an unrestricted range. This misunderstanding may stem from not recognizing the inverse relationship between logarithmic and exponential functions. Another common error is confusing an endpoint with a vertical asymptote. Square root functions begin at an endpoint and are undefined to the left of that point, while logarithmic functions do not start a point; instead, they approach a vertical asymptote. It may be helpful for teachers to encourage students to identify whether the graph has an endpoint or a vertical asymptote and to notice how the graph behaves near its starting location. Focusing on these graphical features may help students distinguish between square root and logarithmic function families.

2. Identify the function family to which each graph appears to belong. Justify your answers.

a)



b)

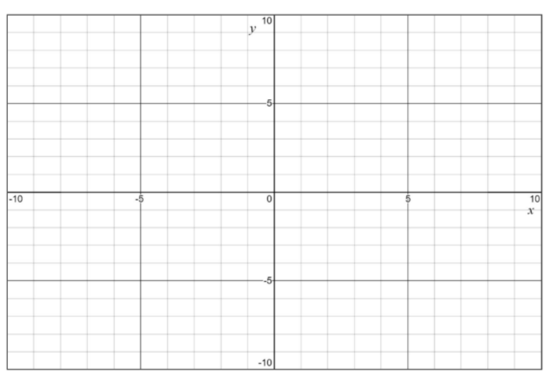


Some students may have difficulty recognizing the similarities and differences among function families. While students may attempt to guess possible equations for a given graph, they may overlook important structural features that define each function family. For example, students may not recognize that cube root functions are continuous and have no asymptotes, while rational functions often contain vertical and/or horizontal asymptotes and may be split into multiple branches. Students should be encouraged to look for family-defining characteristics (e.g., continuity, asymptotes, end behavior, symmetry) when determining to which function family a graph belongs. It may also be helpful for teachers to use anchor charts or interactive notes that display the parent graphs of each function family so students can visually compare them.

3. Create and graph an exponential function *g(x)* that represents a transformation of its parent exponential function, *f(x)*.

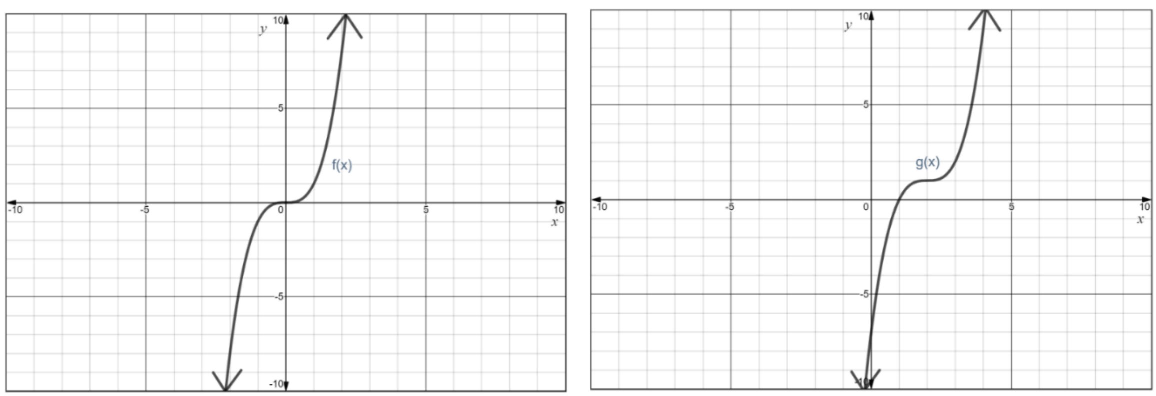
   a) Graph both the parent function and the transformed function on the same coordinate plane, label at least three points on the graph.

   b) Write the equation for *g(x)* and describe how it is transformed from the parent function.



A common error is that students confuse horizontal and vertical transformations of exponential functions. For example, students may incorrectly write an expression such as $2^x - 2$ *when representing a horizontal shift, instead of* 2^{x-2}*. This error may indicate confusion about changes made inside the function (affecting the input and causing horizontal shifts) and changes made outside the function (affecting the output and causing vertical shifts). Students may also incorrectly* *graph horizontal transformations by adjusting y-values rather than x-values, or by neglecting to modify key features such as the horizontal asymptote after a vertical shift. Teachers can support students by modeling how replacing the input value within the function changes the graph differently than adding or subtracting values outside of the function. It may also be helpful for students to generate a table of values and map points from the parent function to the transformed function. These strategies strengthen student understanding of the connection between function notation, transformations, and graphical representations of exponential functions.*

4. A graph of the function $f(x) = x^3$, and the function after being transformed, $g(x)$, are shown.



a) Describe the transformation of $f(x)$ to produce $g(x)$.

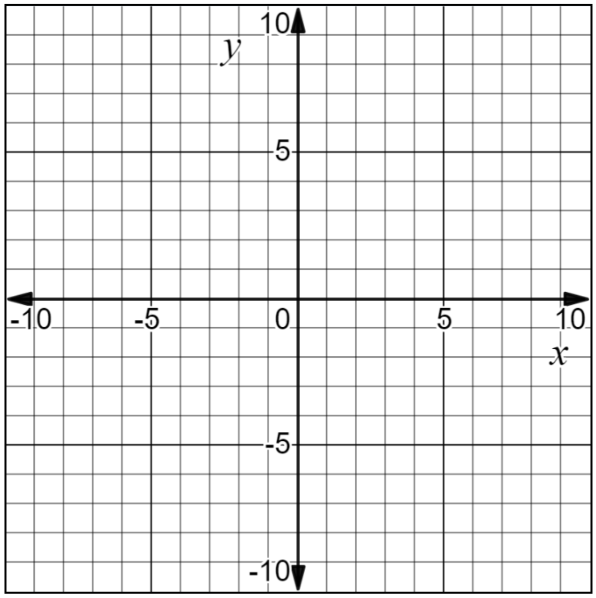
b) Write the equation to represent the graph of $g(x)$.

Some students may misinterpret how horizontal translations are represented symbolically. For example, students may assume that adding a positive value inside the function shifts the graph to the right, when in fact, this would shift the graph to the left (e.g., writing $g(x) = (x + 2)^3 + 1$ would shift the graph to the left 2 units). This error may indicate that students do not fully understand the inverse relationship between the sign inside the function and the direction of a horizontal translation. Teachers can support student learning by having students explore functions of the form $f(x) = (x + a)^3 + b$ using graphing technology. Students should be encouraged to examine the effect of changing a while keeping $b = 0$ to isolate horizontal shifts, and the effect of changing b while keeping $a = 0$ to isolate vertical shifts.

5. Graph the equation $g(x) = \sqrt{4 - x}$.

   a) Label at least three points with integral coordinates.

   b) Explain how the graph of function $g(x) = \sqrt{4 - x}$ differs from the parent function $f(x) = \sqrt{x}$.



A common misconception that some students may have is to misinterpret how changes to the x-value inside a square root function affect reflections. For example, students may not recognize that replacing x with -x results in a reflection across the y-axis, not the x-axis. This may lead students to incorrectly associate negative signs with vertical reflections.

Students may also have difficulty performing the horizontal translation correctly. Some students may apply the reflection and translation in the wrong order or incorrectly assume that $4 - x$ produces the same result as $x - 4$. It may be helpful to have students explore graphs of the equations $y = \sqrt{x}$, $y = \sqrt{-x}$, and $y = -\sqrt{x}$ using a graphing utility. Creating a table of values for each graph and explicitly discussing the domain of each function may help students distinguish between reflections over the y-axis versus the x-axis and horizontal versus vertical translations.

6. A local furniture company has scheduled drivers to deliver furniture to customers on a fixed route. The table below shows the relationship between the number of drivers, *d,* working on the route and the time, *t,* in hours it takes to complete the delivery route.
   a) Determine whether the relationship represents a directly proportional relationship, an inversely proportional relationship, or neither.
   b) Write an equation that represents the relationship.
   c) Explain what the relationship means in the context of the problem.

Number of Drivers *d*	**Time in hours *t***
2	24
3	16
4	12
6	8
8	6
12	4

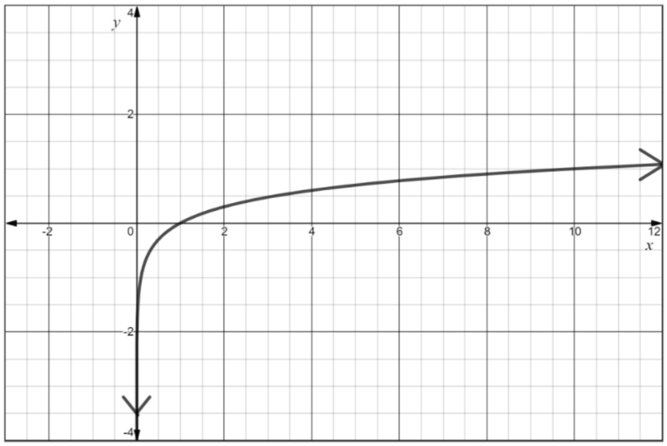
A common error students may make is to assume the values are directly proportional. They may calculate a ratio such as $\frac{t}{d} = \frac{24}{2} = 12$ and then incorrectly conclude the equation is $t = 12d$. This may indicate that students did not check whether the ratio is consistent for all data points.

Another error occurs when students correctly recognize the relationship as inversely proportional but write the equation incorrectly, such as $t = \frac{d}{48}$ instead of the correct equation $t = \frac{k}{d} = \frac{48}{d}$. This error may indicate confusion about the standard equation for an inversely proportional relationship, $y = \frac{k}{x}$.

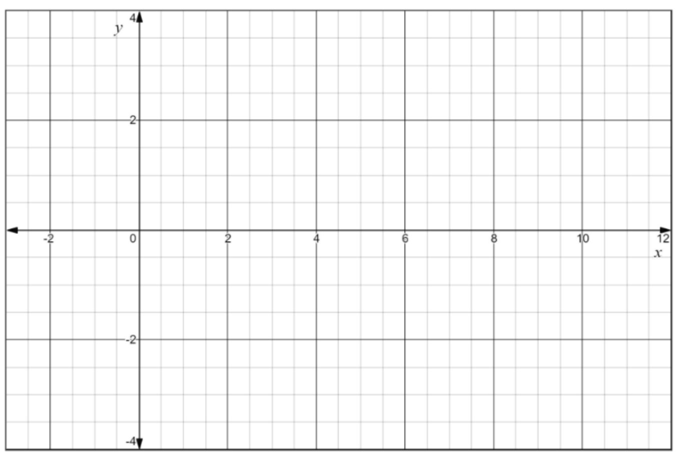
Students may also incorrectly model the data with a linear equation. For example, the equation $t = -2d + 28$ fits the ordered pair (2, 24). If students do not verify all the given ordered pairs, they may think this is the correct equation. However, this equation implies the graph is a straight line, rather than the curved graph associated two inversely proportional variables.
Teachers can support student learning by encouraging students to compare the ratio $\frac{t}{d}$ and the product $d \cdot t$, emphasizing that a constant product indicates an inverse variation. Teachers can also encourage students to verify their equations by substituting all ordered pairs from the table. Additionally, students can compare equations such as $t = \frac{48}{d}$ and $t = -2d + 28$, focusing on whether the variable appears in the denominator and how that placement affects the shape of the graph. These strategies may help students see connections between tables, equations, and graphs,

and may help strengthen their understanding of directly proportional and inversely proportional variables.

7. The graph of the logarithmic function $f(x) = \log x$ is shown.



Use the grid below to graph the function $g(x) = \log(x-2)$.



A common misconception for some students is to misunderstand the direction of translations. Students who sketch g(x) with a left translation or a downward shift may not understand the role of constants grouped with x and how they affect the graph. For example, students may incorrectly assume that "-2" means "move two units to the left," when in fact, $\log(x-2)$ *moves the graph two units to the right.*

Another common error is for students to neglect to move the vertical asymptote, leaving it at $x = 0$ *instead of shifting it to* $x = 2$*. This may indicate that students do not understand the connections between algebraic expressions and key graphical features of logarithmic functions.*

It may be helpful for students to use graphing technology to explore functions of the form $y = \log(x-a)$*. Students should track how the asymptote, domain, and a known reference point (such as (1,0)) change together. This will help reinforce the correct interpretation of horizontal translations and strengthen conceptual understanding of logarithmic transformations.*