

Just in Time Quick Check
Standard of Learning A2.ST.3
Strand: Statistics

Standard of Learning A2.ST.3

The student will compute and distinguish between permutations and combinations.

Students will demonstrate the following Knowledge and Skills:

- a) Compare and contrast permutations and combinations to count the number of ways that events can occur.
- b) Calculate the number of permutations of n objects taken r at a time.
- c) Calculate the number of combinations of n objects taken r at a time.
- d) Use permutations and combinations as counting techniques to solve contextual problems.
- e) Calculate and verify permutations and combinations using technology.

Just in Time Quick Check

Just in Time Quick Check Teacher Notes

Supporting and Prerequisite SOL: 8.PS.1

Just in Time Quick Check A2.ST.3

1. Mr. Martin needs to select 4 students from his class to serve on a planning committee. All members of the committee have equal responsibilities. There are 24 students in Mr. Martin's class.
 - a) Does this represent a permutation or a combination? Explain your reasoning.

 - b) How many different committees can be formed?

2. Nina is creating a password using four different letters from the alphabet, with no repeats.
 - a) Does this represent a permutation or a combination? Explain your reasoning.

 - b) How many ways can the same four letters be arranged to create unique passwords?

3. There are 14 members on the academic team. The academic team has 8 boys and 6 girls. The coach needs 3 boys and 1 girl for the academic team competition. How many different teams of 3 boys and 1 girl can be formed?

4. Consider the two scenarios below.
- A coach selects five players from a team of 10 to be on the field. How many different groups of players are possible?
 - A coach assigns five specific positions to players from the same team of ten players. How many different assignments are possible?
 - a) What role do positions play in making the problems different?
 - b) Which problem results in more possible outcomes? Why?
5. Create a problem that represents a permutation.
6. Create a problem that represents a combination.

A2.ST.2 Just in Time Quick Check Teacher Notes

Common Errors/Misconceptions and their Possible Indications

1. Mr. Martin needs to select 4 students from his class to serve on a planning committee. All members of the committee have equal responsibilities. There are 24 students in Mr. Martin's class.

- a) Does this represent a permutation or a combination? Explain your reasoning.

- b) How many different committees can be formed?

A common misconception some students may have is difficulty distinguishing between permutations and combinations. In this problem, some students may incorrectly assume the scenario represents a permutation simply because the question asks for "different" committees. When distinguishing between permutations and combinations, the important idea is whether order matters, not whether the outcome is different. Since all committee members have equal roles and responsibilities, the order in which the students are selected does not matter, making this a combination.

It may be beneficial to use concrete or visual examples. For example, have four students stand in a row in front of the class and ask whether changing their order creates a new group or the same group. Visual models, anchor charts, or word wall cards that explicitly compare permutations and combinations can further support student understanding.

2. Nina is creating a password using four different letters from the alphabet, with no repeats.

- a) Does this represent a permutation or a combination? Explain your reasoning.

- b) How many ways can the same four letters be arranged to create unique passwords?

A common error students may have is to state that this scenario represents a combination. This indicates that students may not recognize that the order of the letters in a password matters; even when the same four letters are used, changing their order creates a completely different password (e.g., abcd is not the same password as dcba). To address this error, instruction should emphasize that permutations are used when both the selection of items and their arrangement are important. It may help to clarify this idea by discussing real-world situations in which order matters such as passwords, PIN numbers, rankings, or seating arrangements. It may also be beneficial to use visual strategies, for example, having students stand in a line at the front of the classroom and asking whether rearranging their order creates a new outcome. Have students discuss a scenario where students are simply forming a group with no assigned roles compared to a scenario when students are assigned specific roles (e.g., president, vice president, secretary) to help reinforce the concept that when positions or order matter, the scenario represents a permutation.

3. There are 14 members on the academic team. The academic team has 8 boys and 6 girls. The coach needs 3 boys and 1 girl for the academic team competition. How many different teams of 3 boys and 1 girl can be formed?

Some students may incorrectly state there are 62 different teams, which indicates that students recognize the scenario as a combination problem, but have incorrectly applied the Fundamental Counting Principle by adding the number of ways instead of multiplying. These students may treat the selection of boys and girls as separate, unrelated events rather than understanding that both selections must occur together to form a valid team.

Another common error is to state that there could be 1001 different teams. These students may have correctly computed $\binom{14}{4}$, demonstrating an understanding of selecting 4 students from 14, but they failed to account for the constraints of the problem (i.e., that the team must include exactly 3 boys and 1 girl). This error indicates that students may be confused about choosing correct values for “n” and “r.” It may be helpful to use counterexamples, such as teams formed from 4 students that could include all boys or could include all girls, can help students see why choosing 4 from 14 is not appropriate or accurate in this scenario. Additional experience with structured, multistep counting problems and the use of anchor charts or vocabulary cards demonstrating combinations, permutations, and the Fundamental Counting Principle can further enhance student understanding.

4. Consider the two scenarios below.
- A coach selects five players from a team of 10 to be on the field. How many different groups of players are possible?
 - A coach assigns five specific positions to players from the same team of ten players. How many different assignments are possible?
 - a) What role do positions play in making the problems different?
 - b) Which problem results in more possible outcomes? Why?

Some students may incorrectly use the same formula for both scenarios, indicating difficulty distinguishing between combinations and permutations. These students may benefit from explicit discussion about what changes when positions are introduced and how positions create an order among the selected players. Modeling this example in the classroom could assist students with seeing the difference between the two situations.

Other students may believe that positions don't count as order or that order does not matter because all team members are equally important. This misconception may stem from a narrow understanding of order as the physical location rather than the role or assignment. It may be helpful to emphasize that order can be created by positions, titles, or responsibilities, even when all individuals have equal value. It may also be beneficial to model both scenarios side by side to help students see the difference. For example, listing a small number of players and assigning or

reassigning positions demonstrates how the number of outcomes increases when order matters. Reinforcing these ideas with anchor charts or vocabulary cards can support student understanding.

5. Create a problem that represents a permutation.

Some students may create a scenario that represents a combination, rather than a permutation. This error occurs when students fail to include specific roles, positions, or ordered outcomes, resulting in a scenario where the order does not matter. This may indicate that students have difficulty distinguishing between permutations and combinations at a conceptual level.

It may be helpful to emphasize that a valid permutation scenario must clearly show that order or assignment matters, such as ranked outcomes, specific jobs, or labeled positions. Students may benefit from creating a list of real-world situations where order matters (e.g., passwords, officer positions, race results) and contrasting these with situations where order does not matter (e.g., forming a team or committee). Encouraging students to check their answers by asking, "Would switching two people or items change the outcome?" can help reinforce the correct identification of permutations versus combinations.

6. Create a problem that represents a combination.

Some students may create a scenario that represents a permutation, rather than a combination. This error may occur when students include specific roles, titles, or ordered outcomes (such as assigning positions or ranks) which causes the order to matter. These students may understand the context of selecting items but struggle to distinguish when order is important. Other students may use vague language that does not clearly indicate whether order matters. For example, neglecting to specify that all selected individuals have the same role may introduce ambiguity into the scenario.

It may be helpful to emphasize that a valid combination problem must clearly show that the order does not matter and that each selected individual or item has equal importance. Encouraging students to check their scenarios by asking, "If I rearrange the selected people or objects, do I still have the same group?" can help reinforce the correct identification of combinations versus permutations. Additionally, providing side by side examples of combinations (committees, teams with no positions, selecting objects) and permutations (officers, rankings, passwords) can help solidify student understanding.