Just in Time Quick Check

Standard of Learning A2.F.2

Strand: Functions

Standard of Learning A2.F.2

The student will investigate and analyze characteristics of square root, cube root, rational, polynomial, exponential, logarithmic, and piecewise-defined functions algebraically and graphically.

Students will demonstrate the following Knowledge and Skills:

a) Determine and identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically, including graphs with discontinuities.

b) Compare and contrast the characteristics of square root, cube root, rational, polynomial, exponential, logarithmic, and piecewise-defined functions.

c) Determine the intervals on which the graph of a function is increasing, decreasing, or constant.

d) Determine the location and value of absolute (global) maxima and absolute (global) minima of a function.

e) Determine the location and value of relative (local) maxima or relative (local) minima of a function.

f) For any value, x, in the domain of f, determine $f(x)$ using a graph or equation. Explain the meaning of x and $f(x)$ in context, where applicable.

g) Describe the end behavior of a function.

h) Determine the equations of any vertical and horizontal asymptotes of a function using a graph or equation (rational, exponential, and logarithmic).

i) Determine the inverse of a function algebraically and graphically, given the equation of a linear or quadratic function (linear, quadratic, and square root). Justify and explain why two functions are inverses of each other.

j) Graph the inverse of a function as a reflection over the line $y = x$.

k) Determine the composition of two functions algebraically and graphically.

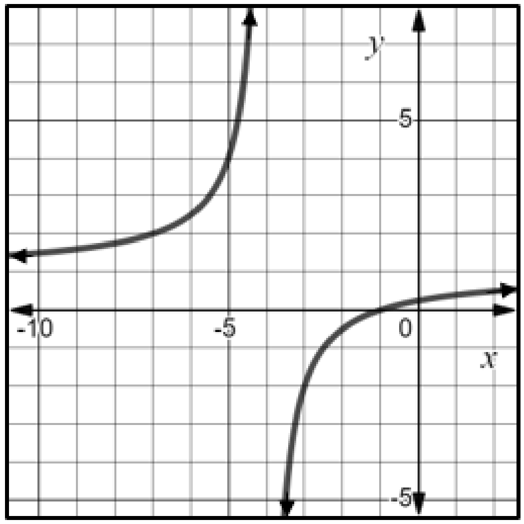
Just in Time Quick Check

Just in Time Quick Check Teacher Notes

Supporting and Prerequisite SOL: A2.F.1, A.F.2

Just in Time Quick Check A2.F.2

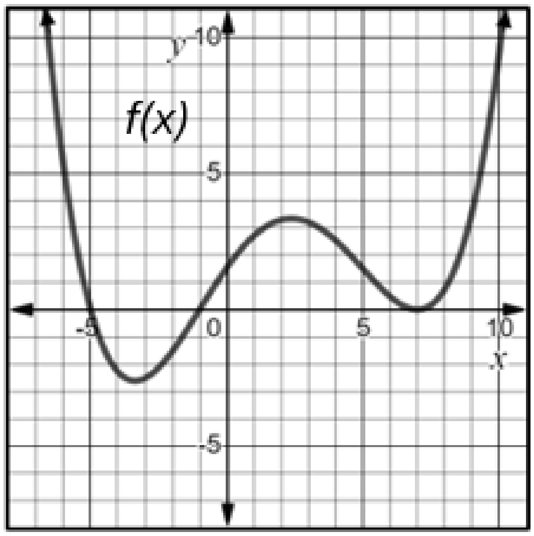
1. What is the domain of the relation shown? Use set or interval notation.



2. Identify the range of the function $f(x) = \sqrt{x-2}$. Use interval notation.

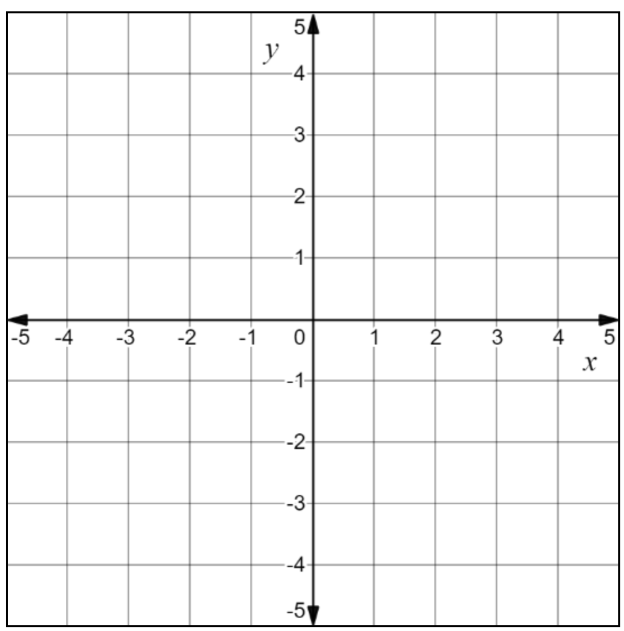
3. Analyze the polynomial $f(x) = x^4 + 5x^3 - x^2 - 5x + 2$. Identify any global maximum or global minimum values or explain why they do not exist.

4. The graph of function *f(x)* is shown. What appears to be the value of the relative minimum of *f(x)* on the interval (4, 8)?



5. Sketch the graph of a function $g(x)$ that is:

- decreasing throughout the interval $(-\infty, 2)$
- increasing throughout the interval $(2, 3)$
- decreasing throughout the interval $(3, \infty)$



6. Identify the intervals on which the function $f(x) = (x-1)(x+2)^2$ appears to be always increasing. Select all correct intervals.

$4 < x < 8$	$-2 < x < 0$	$-\infty < x < -2$	$-\infty < x < \infty$

7. What appears to be the zero of the function $f(x) = \log(2x-5)$?

8. The function $f(x)$ is given. Circle all the intercepts of the function.

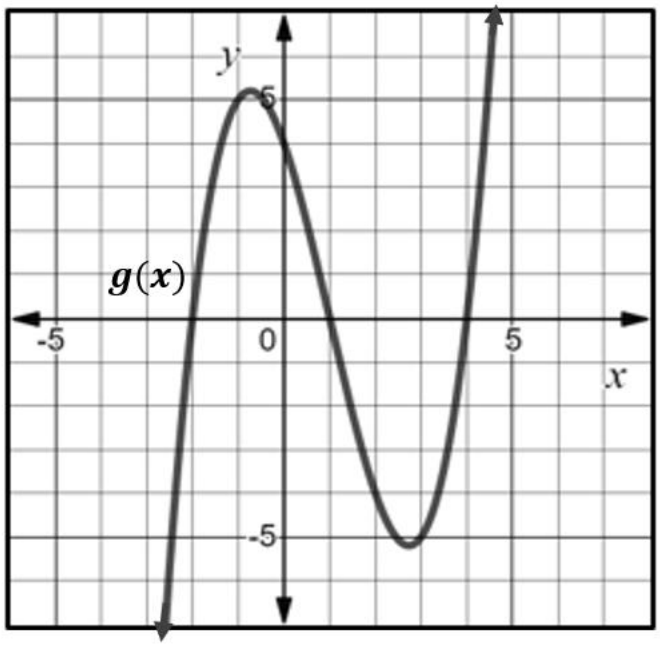
$$f(x) = x^3 - 2x^2 - 5x + 6$$

$(0, 3)$	$(1, 0)$	$(3, 0)$	$(6, 0)$
$(-2, 0)$	$(0, -2)$	$(0, 6)$	$(0, 1)$

9. Find the values of the range of $f(x) = x^3 - 3x^2 + x + 1$ when the domain is $\{-1, 2, 4\}$.

10. If $h(x) = \frac{2}{x-3} + 4$, what is the value of $h(6)$?

11. The graph of the function $g(x)$ is shown. What is the approximate value of $g(0)$?

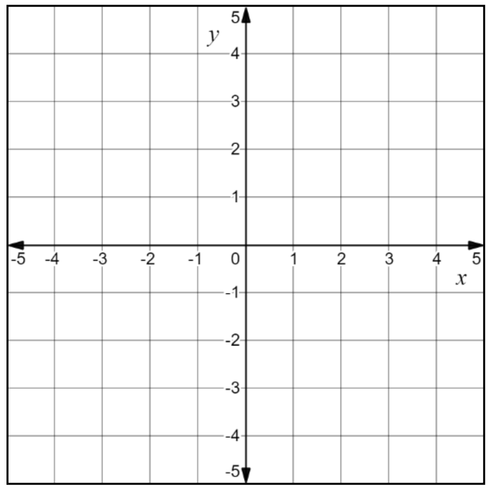


12. Given the following polynomial and rational functions $f(x) = x^3 - 4x$ and $g(x) = \frac{1}{x+2} + 7$:
    a) Determine the behavior of $f(x)$ as *x* approaches negative infinity and positive infinity.
    b) Determine the behavior of $g(x)$ as *x* approaches negative infinity and positive infinity.
    c) Compare the two functions.
    d) Write the equation of the vertical and horizontal asymptotes.

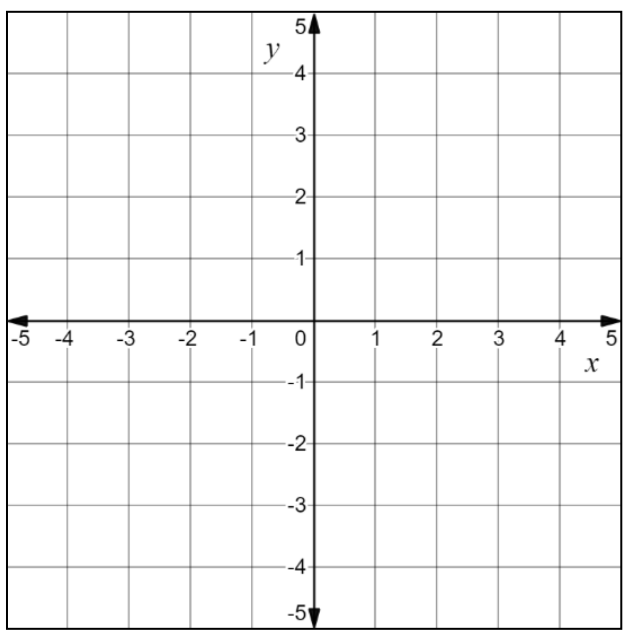
13. Select all the true statements about the end behavior of the function $f(x) = -x^4 - 5x^3 + 5x + 3$.

- As x approaches positive infinity, *f(x)* approaches positive infinity.
- As x approaches negative infinity, *f(x)* approaches negative infinity.
- As x approaches positive infinity, *f(x)* approaches negative infinity.
- As x approaches negative infinity, *f(x)* approaches positive infinity.

14. Given $f(x) = 4x - 7$, determine the inverse algebraically. Graph $f(x)$ and $f^{-1}(x)$ on the coordinate plane below to verify that $f(f^{-1}(x)) = x$.



15. Graph the inverse of the function $f(x) = x^2$. Identify the domain and range for $f(x)$ and $f^{-1}(x)$.



16. Given $f(x) = 3x^2 - 12$ and $g(x) = 2x + 1$. Find $g(f(x))$.

17. Given $f(x) = \frac{1}{2}x^2 + 5$ and $g(x) = \sqrt{2x - 6}$. Find $(f \circ g)(x)$.

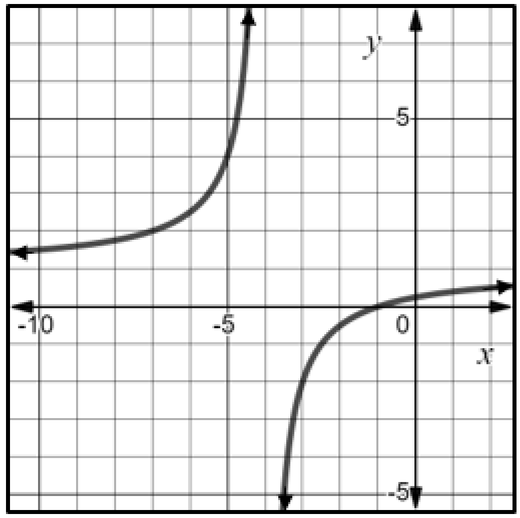
18. Given $f(x) = 216x^3 - 5$ and $g(x) = \frac{\sqrt[3]{x+5}}{216}$. Determine whether $f(x)$ and $g(x)$ are inverses of each other. Justify your thinking.

19. What is the equation(s) of the vertical asymptote(s) of the graph of the function $y = \frac{x+4}{x^2-16}$?

A2.F.2 Just in Time Quick Check Teacher Notes

Common Errors/Misconceptions and their Possible Indications

1. What is the domain of the relation shown? Use set or interval notation.



A common error that some students may make is to list that the domain is all real numbers, even though the graph contains a vertical asymptote at x = -4. This may indicate that a student believes that the domain only pertains to the beginning and ending values of the function's domain. To address this misconception, teachers should emphasize that the domain includes only the x-values for which the graph exists. A vertical asymptote indicates an x-value where the function is undefined and therefore must be excluded from the domain.

Teachers can support student learning by having students identify and draw the vertical line the graph approaches but never crosses. Additionally, it may be helpful to ask guiding questions such as, "Is there a vertical line where the graph breaks apart?" "Does the graph ever touch that line?" "Which x-value must be excluded from the domain?"

2. Identify the range of the function $f(x) = \sqrt{x-2}$. Use interval notation.

A common error that some students may make is to list the range as $(0, \infty)$, excluding 0. This may indicate that students are unsure about when to use parentheses versus brackets when writing interval notation. To address this, teachers should emphasize that the range includes all output values the function attains. Since $f(2) = \sqrt{0} = 0$, the value 0 is produced by the function and should be included in the range. It may also be helpful to review the meaning of parentheses (value not included) versus brackets (value included) in interval notation and connecting interval notation to number line graphs and graphing inequalities (e.g., using open circles for excluded values and closed circles for included values). These strategies may help strength the connection between algebraic reasoning, graphical representations of square root functions, and the use of interval notation.

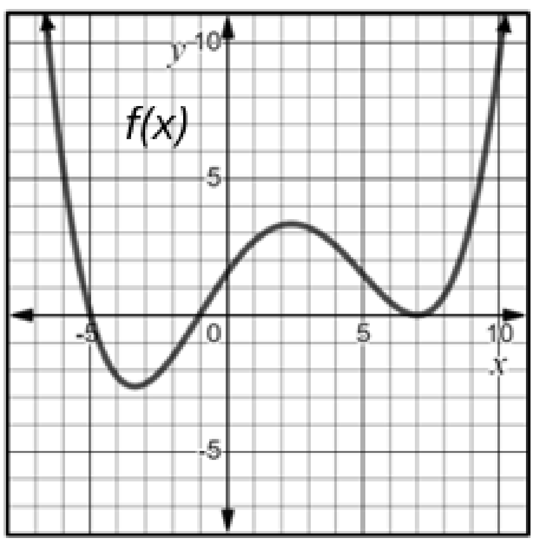
3. Analyze the polynomial $f(x) = x^4 + 5x^3 - x^2 - 5x + 2$. Identify any global maximum or global minimum values or explain why they do not exist.

A common error students make is to assume all polynomial functions behave like quadratics and therefore have exactly one highest point or lowest point. Students may also incorrectly believe that identifying end behavior alone is enough to determine global extrema without analyzing the entire graph. For the given function, some students may state that there is no global minimum because the graph has multiple turning points. Another common error is for students confuse local extrema with global extrema and state that the local maximum is the global maximum.

To analyze the given function, students should first consider end behavior. Because the polynomial has even degree and a positive leading coefficient, as $x \to \pm\infty, f(x) \to \infty$. This means the function cannot have a global maximum, but it must have a global minimum.

Teachers can support student learning by emphasizing the distinction between local and global extrema and reinforcing that end behavior tells whether global extrema are possible and critical points or graphing are necessary to determine where global extrema occur. Students can use graphing technology to see that while the function has more than one turning point, only one minimum is the lowest point on the graph. It may also be beneficial to have students compare this quartic function to quadratic functions (always have one global maximum or minimum point) and cubic functions (opposite end behavior) to further strengthen student understanding of how the degree of a polynomial affects global behavior.

4. The graph of function *f(x)* is shown. What appears to be the value of the relative minimum of *f(x)* on the interval (4, 8)?

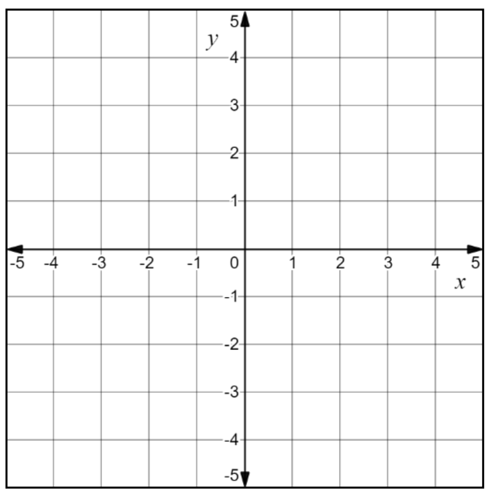


A common error some students may make is to state there is a relative minimum at 7, which is the x-value where the relative minimum occurs. Instead, students should give the value of the relative minimum, which refers to the y-value of the function at that point. This error may indicate confusion between the location of a relative minimum and the value of the function at that location. Finding the value of a function means substituting an x-value into the function to obtain a y-value. Thus, it may be helpful for students to connect this idea to input-output tables *or to a graphical representation where students identify the x-coordinate of interest and then determine the corresponding y-coordinate from the graph.*

Teachers can support student learning by encouraging students to label relative extrema on graphs with ordered pairs and encouraging students to restate answers using complete sentences (e.g., "The relative minimum occurs at x = 7 and its value is 0."). Additionally, students may benefit from exposure to the different ways in which questions could be asked, and to which values those questions are referring. Examples would include:

- *What is the value of the extrema? – This question refers to the y-values.*
- *What is the interval where the extreme occurs? – This question refers to x-values.*

5. Sketch the graph of a function $g(x)$ that is:
    - decreasing throughout the interval $(-\infty, 2)$
    - increasing throughout the interval $(2, 3)$
    - decreasing throughout the interval $(3, \infty)$



While there are infinitely many graphs that could be correct, a common error some students may make is to place turning points (x, 2) and (x, 3), which may indicate that students are incorrectly using y-values rather than x-values to describe intervals of increase and decrease. To address this error, teachers should emphasize that intervals describing increasing or decreasing behavior are written in terms of x-values and that turning points occur at specific x-values where the function changes direction. Additionally, it may be helpful for students to draw vertical lines at x = 2 and x = 3. This visually separates the domain into three intervals and provides a visual to show where the function's behavior must change. It may also be beneficial for students to verbally describe the function's behavior as they move from left to right along the x-axis or to use graphing technology to explore functions and observe how changing the behavior of the function at specific x-values affects the overall shape. These strategies may help students connect domain values to the graph's increasing and decreasing behavior.

6. Identify the intervals on which the function $f(x) = (x-1)(x+2)^2$ appears to be always increasing. Select all correct intervals.

$4 < x < 8$	$-2 < x < 0$	$-\infty < x < -2$	$-\infty < x < \infty$

A common error some students might make is not selecting $4 < x < 8$ as an increasing interval. This may occur because students incorrectly believe increasing intervals must be bounded by turning points or special values such as zeros. Teachers can address this error by emphasizing that an interval is increasing if the y-values consistently increase as x increases, regardless of whether a turning point occurs within that interval. It may be helpful for students to use an input-output table for values between 4 and 8, showing that each successive x-value produces a larger y-value.

Another common error some students may make is to assume that a function with a repeated factor (such as $(x+2)^2$) must always decrease before that x-value. In this case, students may incorrectly determine that the function decreases on (−∞, −2). Students may also believe that increasing intervals must lie between turning points. However, a function can be increasing over an interval that extends beyond critical points, if the y-values consistently increase as x increases. The use of a sign chart or input-output table for values less than -2 may be helpful to show that y-values increase as x increases.

7. What appears to be the zero of the function $f(x) = \log(2x-5)$?

A common error some students may make is to state that the zero of the function is located at $\left(\frac{5}{2}, 0\right)$. This may indicate that students incorrectly set the expression inside the logarithm, 2x – 5, equal to zero, rather than recognizing that a logarithmic function equals zero when its argument equals 1. This misconception may reflect an overgeneralization from solving polynomial equations, where zeros are often found by setting factors equal to zero. Teachers should emphasize that different types of functions require different strategies for determining zeros. For example, it may be helpful to have students using a graphing utility to compare logarithmic functions and exponential functions to reinforce inverse relationships. It may also be helpful to review the key property that $\log(1) = 0$, and to encourage students to verify their work graphically (e.g., visually confirming that the zero occurs at $x = 3$, not $x = \frac{5}{2}$) or using substitution (e.g., $f(3) = log(2(3) - 5) = log(1) = 0$.

8. The function $f(x)$ is given. Circle all the intercepts of the function.

$$f(x) = x^3 - 2x^2 - 5x + 6$$

$(0, 3)$	$(1, 0)$	$(3, 0)$	$(6, 0)$
$(-2, 0)$	$(0, -2)$	$(0, 6)$	$(0, 1)$

A common error students may make is to circle: $(-2, 0)$, $(1, 0)$, and $(3, 0)$ but omit $(0, 6)$. This may indicate a misunderstanding that intercepts only refer to the x-intercepts of a function, rather than recognizing that intercepts include both x- and y-intercepts. Another common error is for students to circle any point that includes a coordinate of 0, believing it must be an intercept without first determining whether it satisfies the function. It may be helpful for teachers to review the definitions of x-intercepts, which occur where f(x) = 0, and y-intercepts, which occur where x = 0. Students should be encouraged to graph the function and visually identify (by circling, highlighting, or labeling) where the function crosses the x-axis and the y-axis. Additionally, students can also verify selected points by substituting the coordinates into the equation.

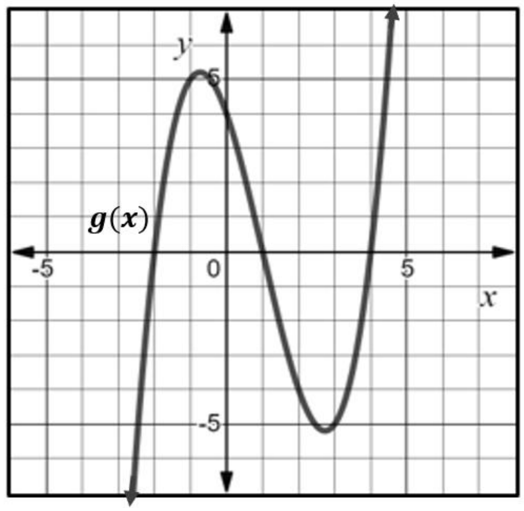
9. Find the values of the range of $f(x) = x^3 - 3x^2 + x + 1$ when the domain is $\{-1, 2, 4\}$.

A common error that some students may make is to substitute multiple domain values at one time. For example, a student may write and evaluate $(-1)^3 - 3(2)^2 + (4) + 1$, which combines inputs incorrectly. This may indicate a student does not understand that each input value will have exactly one corresponding output value and that the function evaluation must be completed with one input at a time. It may be helpful for students to rewrite the question as $f(-1), f(2), f(4)$ to highlight that three separate domain elements are given and that three corresponding range values need to be calculated. The use of a table with domain values in one column and the corresponding range values in another column may help students to organize their work. Additionally, students can verify their range values by graphing the function and checking the y-values for the given domain values.

10. If $h(x) = \frac{2}{x-3} + 4$, what is the value of $h(6)$?

A common error some students may make is to replace h(x) with the value of 6 resulting in the equation, $6 = \frac{2}{x-3} + 4$, instead of correctly evaluating h(6). This may indicate that students think 6 is the output of the function, h(x), rather than the input value being substituted for x. Teachers should emphasize that h(6) means "the value of the function when x = 6." It may be helpful for students to highlight or circle the variable, x, in the function and the number 6 in h(6), then explicitly replace each x with 6, leading to students rewriting the function as $h(6) = \frac{2}{6-3} + 4$ before simplifying. These strategies may support accurate substitution and help students distinguish between inputs and outputs when evaluating functions.

11. The graph of the function $g(x)$ is shown. What is the approximate value of $g(0)$?



Common errors students make is they think $g(0) = -2, g(0) = 1$ and $(0) = 4$ are all solutions. This may indicate that a student interprets finding the value of a function as finding the zero of the function when given a graph. Since $g(0)$ means to find the value of the function when $x = 0$, a strategy that might be helpful for students is to draw a vertical line representing $x = 0$ and determine the y-coordinate of the point where the vertical line intersects the graph of the function provided. Teachers may help by providing additional practice allowing students to

12. Given the following polynomial and rational functions $f(x) = x^3 - 4x$ and $g(x) = \frac{1}{x+2} + 7$:
   a) Determine the behavior of $f(x)$ as *x* approaches negative infinity and positive infinity.
   b) Determine the behavior of $g(x)$ as *x* approaches negative infinity and positive infinity.
   c) Compare the two functions.
   d) Write the equation of the vertical and horizontal asymptotes.

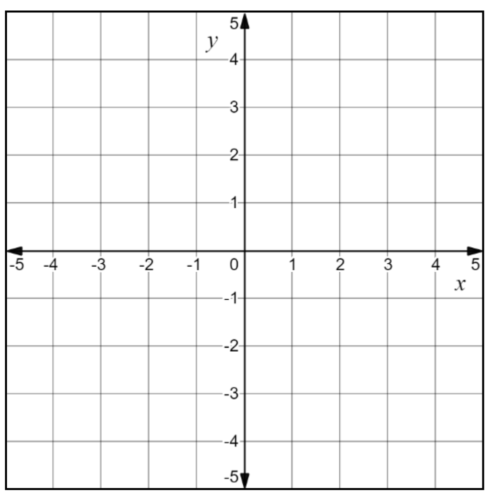
Common errors occur when students confuse the end behavior of polynomial and rational functions, incorrectly stating that both functions increase or decrease without bound as x approaches positive or negative infinity. This may indicate a lack of attention to the structure of the function, particularly the role of leading terms. Students may also make algebraic mistakes when identifying asymptotes, such as solving the entire rational equation for zero instead of setting the denominator equal to zero to find the vertical asymptote or incorrectly stating that the horizontal asymptote is $y = 0$, rather than recognizing that the function approaches $y = 7$ due to the vertical shift. Teachers can support student learning by emphasizing end behavior rules. For example, for polynomials, focus on the leading term to determine end behavior. For rational functions, teachers should explain that as $x \to \pm\infty$, the ratio approaches zero. It may also be helpful to require students to identify vertical and horizontal asymptotes separately and to clearly state what each one represents. Class discussions that focus on comparison questions that highlight the differences between function families (e.g., "Why does one function grow without bound while the other approaches a constant?") may help students deepen their understanding of function families.

13. Select all the true statements about the end behavior of the function $f(x) = -x^4 - 5x^3 + 5x + 3$.

- As x approaches positive infinity, *f(x)* approaches positive infinity.
- As x approaches negative infinity, *f(x)* approaches negative infinity.
- As x approaches positive infinity, *f(x)* approaches negative infinity.
- As x approaches negative infinity, *f(x)* approaches positive infinity.

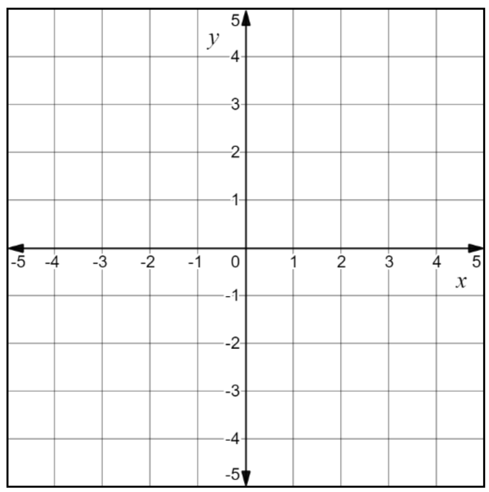
A common error some students may make is failing to select, "As x approaches positive infinity, f(x) approaches negative infinity." This may indicate a misconception that as x increases, a function must always increase. Students may overgeneralize from functions with positive leading coefficients and not consider the sign of the leading term. Another common error is for students to assume that opposite ends of the graph must go in opposite directions. This may indicate that students do not understand that for even-degree polynomials, both ends of the function behave the same way and the direction depends upon whether the leading coefficient is positive or negative. Teachers should encourage students to analyze the leading term prior to any graphing or computation. It may also be helpful to encourage students to create a table of values and evaluate the function at increasingly large positive and negative values such as $x = \pm 5, x = \pm 50, x = \pm 100$ *and* $x = \pm 1000$*. It would also be beneficial for students to graph the function using a graphing utility so they can make the connection both algebraically and graphically to determine that as x gets larger,* $f(x)$ *will continue to decrease. Similarly, as x gets smaller,* $f(x)$ *also decreases.*

14. Given $f(x) = 4x - 7$, determine the inverse algebraically. Graph $f(x)$ and $f^{-1}(x)$ on the coordinate plane below to verify that $f(f^{-1}(x)) = x$.



A common error students make when finding the inverse of $f(x) = 4x - 7$ *is forgetting to interchange the roles of x and y before solving, leading to an expression such as* $x = \frac{y+7}{4}$*, rather than correctly rewriting the inverse as a function of x, where* $y = \frac{x+7}{4}$*. Students may also make algebraic errors when isolating y, such as adding 7 correctly but forgetting to divide both terms* *by 4, resulting in the incorrect solution of $y = x + \frac{7}{4}$. Students may also not understand how to verify inverse functions by making the arithmetic error $f\left(f^{-1}(x)\right) = 4\left(\frac{x+7}{4}\right) - 7 = x + 7 - 7 = 0$ suggesting that students may not understand that the goal of the verification process is to simplify the expression to x, not to zero. Teachers can support student learning by modeling the correct steps to find the inverse, such as demonstrating how to interchange the x and y, solving $y = 4x - 7 \Rightarrow x = 4y - 7 \Rightarrow y = \frac{x+7}{4}$. Students can also verify the inverse using compositions to show their understanding that $f(f^{-1}(x)) = 4\left(\frac{x+7}{4}\right) - 7 = x$. Additionally, students can also use graphing technology to illustrate that $f(x)$ and $f^{-1}(x)$ are reflections across $y = x$, reinforcing the concept visually and algebraically.*

15. Graph the inverse of the function $f(x) = x^2$. Identify the domain and range for $f(x)$ and $f^{-1}(x)$.



Common errors students make when finding the invers of $f(x) = x^2$ is they forget to interchange the roles of x and y before solving. This leads to an expression such as $x = \sqrt{y}$ rather than correctly rewriting the inverse as a function of x, where $y = \sqrt{x}$. Another common error is for students to ignore the domain restriction required for the inverse of the quadratic function, writing $f^{-1}(x) = \sqrt{x}$ without restricting the domain. Without this restriction, the inverse would not be a function. Students may also incorrectly state that the domain and range for both $f(x)$ and $f^{-1}(x)$ are (-∞, ∞). Teachers can support student learning by modeling the correct algebraic process for interchanging x and y. For example, demonstrating $y = x^2 \Rightarrow x = y^2 \Rightarrow y = \sqrt{x}$. It may also be helpful to clearly explain domain restrictions by showing that for the inverse to be a function, the original domain must be limited to $x \geq 0$, so the inverse has a domain $x \geq 0$ and a range $y \geq 0$. Students can also use compositions to verify that $f(f^{-1}(x)) = (\sqrt{x})^2 = x,$ for $x \geq 0$ and use a graphing utility to show the reflection across $y = x$.

16. Given $f(x) = 3x^2 - 12$ and $g(x) = 2x + 1$. Find $g(f(x))$.

A common error that some students may make is to calculate $f(g(x)) = 3(2x+1)^2 - 12$ instead of calculating $g(f(x)) = 2(3x^2 - 12) + 1$. This may indicate that students do not understand that function composition is not commutative, when in fact the order of composition matters. The teacher may wish to have students read the composition aloud (e.g., "g of f of x") and to create a mapping of the process x → f → g, where students must work from the inside out by first substituting the given value into the inner function f in place of x and calculate its output. Next, students must take the resulting value and substitute it into the outer function g in place of x. The final step requires students to calculate the output g, which is the final value of the composition function. This may help the students visualize the proper order of evaluating. It may also be helpful for students to practice composition with numerical inputs before using algebraic expressions. An example may include finding $f(g(5))$ and $g(f(5))$ given $f(x) = 2x + 5, g(x) = x^2 - 3$. This results in $f(g(5)) = 49$ and $g(f(5)) = 222$. Teachers should also have students compare other examples of f(g(x)) and g(f(x)) to show that they generally produce different results.

17. Given $f(x) = \frac{1}{2}x^2 + 5$ and $g(x) = \sqrt{2x - 6}$. Find $(f \circ g)(x)$.

A common error that some students may make is to fail to substitute the entire expression for $g(x)$ into the function $f(x)$. For example, students may square $\sqrt{2x - 6}$ correctly but then apply the coefficient $\frac{1}{2}$ to only part of the resulting expression, leading to an incorrect simplification such as $x - 1$ instead of the correct result $x + 2$. This error may indicate that students do not recognize that the coefficient $\frac{1}{2}$ multiplies the entire quantity $(2x - 6)$ after squaring. It may be helpful to encourage students to use parentheses placeholders before substitution, such as $f(g(x)) = \frac{1}{2}(\qquad)^2 + 5$. Using a highlighter to note $g(x)$ is the replacement expression that may help students recognize the full expression will be substituted into the function.

18. Given $f(x) = 216x^3 - 5$ and $g(x) = \frac{\sqrt[3]{x+5}}{216}$. Determine whether $f(x)$ and $g(x)$ are inverses of each other. Justify your thinking.

A common error some students may make is to claim that the functions are inverses because cubic and cube root functions are inverse families. This may indicate a misunderstanding that the numbers within the function have no bearing on the function nor its inverse. Students could graph each of the functions using Desmos to determine if the functions are mirror images of each other. If they are not, then $f(x)$ and $g(x)$ are not inverse functions. An algebraic approach would be to determine if $f(g(x)) = x$ and $g(f(x)) = x$. If either is not true, then they are not inverse functions of each other.

19. What is the equation(s) of the vertical asymptote(s) of the graph of the function $y = \frac{x+4}{x^2-16}$?

A common error that some students may make is to state that the equations of the vertical asymptotes are $x = 4$ and $x = -4$. This may indicate that students have a misconception about the removable discontinuities (holes) in a rational function that occurs when the numerator and denominator share a common factor. When a common factor cancels, the function is still undefined *at that value of x, but the graph does not approach* $\pm\infty$*. Instead, a hole appears in the graph. Teachers should encourage students to factor the numerator and denominator completely and identify which values cause the denominator to be zero before and after simplification. Students can then distinguish between values that produce vertical asymptotes and values that produce holes. Additionally, graphing both the original and simplified equations can help students see that the graphs appear the same except for the missing point at the hole where* $x = -4$.