Ranking Volume and Surface Area of Cylinders and Rectangular Prism

STRAND: Measurement and Geometry

STRAND CONCEPT: Volume and Surface Area

SOL 7.4a, 7.4b

Remediation Plan Summary

Students devise a method for measuring the volume and surface area of a gelatin box and a toilet paper roll.

Common Errors and Misconceptions

Students have difficulty with volume measurement. They treat three-dimensional figures as two-dimensional ones. Students have difficulty with creating the arrays that fill the prism. Students often don't understand the relationship of the formula and the counting of the cubes.

Materials

- Empty gelatin boxes
- Toilet paper rolls
- Grid paper
- Construction paper
- Scissors
- Glue
- Centimeter cubes
- Centimeter rulers
- Copies of the attached worksheets

Introductory Activity

- Display a gelatin box and toilet paper roll, and ask students what types of solids they represent (rectangular prism and cylinder). Sketch each solid on the board, and write their dimensions in centimeters (make sure you are getting length, height, and width of the rectangular prism and height diameter and circumference of the cylinders). Ask which solid has the greater volume, that is, if both were filled with centimeter cubes, which one would hold more cubes?

Plan for Instruction

Measuring Volume

1. Give each pair of students a gelatin box and a toilet paper roll. Ask the class how they could measure the exact volume of each object without filling it? Discuss methods of figuring out how many cubic centimeters would fill each container. Students might

suggest stacking the bases to fill the containers. Record all student-developed formulas, and circle those that would work.

2. Demonstrate how to find the volume of the each object by measuring the area of its base and multiplying this area by its height.
3. Have students compare each exact volume measurement found using a formula to the estimate obtained by filling each container with rice during the warm-up. Ask whether the class estimates were reasonable.
4. Ask how measuring the volume of a rectangular prism and measuring the volume of a cylinder are similar and different.

Measuring Surface Area

5. Ask the students to look at the *surface* of each object. Ask which object would take more paper to wrap it with no overlaps. Have students write a hypothesis and a rationale and then share their hypotheses and reasoning.
6. Ask the class how the amount of paper needed to cover each figure could be measured. Have students share their ideas.
7. Suggest that one method to determine the surface area of each solid is by constructing a net for the solid and counting how many square units make up the net.
8. Have students trace the faces of a gelatin box on centimeter grid paper, cut out the six tracings, arrange them on construction paper into a net that will fold up into a box, and glue them down on the construction paper.
9. Have students devise a method for calculating the number of square units that make up the net, and have them write down their calculations and methods in words in a box next to the net.
10. Have students trace the two circular bases of the toilet paper roll on grid paper. Then, have them cut open the lateral curved face of the roll, flatten it out, and trace it on the grid paper. Have them cut out these three tracings, arrange them on construction paper into a net, and glue them down on the construction paper.
11. Have students devise a method for calculating the number of square units that make up the net, and have them write down their calculations and methods in words in a box next to the net. If students have difficulty with this, prompt by asking how they might use their knowledge of calculating the area of a circle.
12. Ask students to explain how they calculated the surface area of each object, and write on the board the different methods used. Discuss the methods, asking whether each one would always work. Circle the ones that will always work, and show how to translate the methods expressed in words to formulas:

    $SA_{\text{surface area prism}} = 2lw + 2lh + 2wh$

    $SA_{\text{surface area cylinder}} = 2\pi rh + 2\pi r^2$

13. Show how to organize the formula for the surface area of a rectangular prism into a graphic organizer. Have students use the worksheet or their math logs to practice calculating the surface areas of several different rectangular prisms.

14. Use the attached worksheets for additional practice in follow-up lessons, as needed by your students.

Pulling It All Together (Reflection)

- Have students answer the “Reflection” worksheet. After they are finished, have students pair up to do a Think-Pair-Share. Then, ask for volunteers to share their solutions with calculations. If time permits, have each pair write another problem by changing the dimensions of the box, and have pairs exchange problems to solve.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Name: ____

Ranking Prisms and Cylinders by Volume

1. Given four solids (prisms and cylinders) lettered A through D, estimate the volume of each, and rank them from least to greatest by volume. Record your ranking:

Least → ***Greatest***				
Solid:	_______	_______	_______	_______

2. Sketch each of the four solids in the appropriate boxes in the chart below.
3. Measure the dimensions of the solids, and record the measurements in the chart.
4. Label the units you used to measure.
5. Find the area of the base of each solid, and record.
6. Multiply the area of the base by the height to find the volume (use $\pi \approx 3.14$), and record.
7. Rank the four solids from least to greatest by volume.

Sketch	Dimensions	Area of base (square units)	Height	Volume (cubic units)	Rank L to G by volume
Solid A					
Solid B					
Solid C					
Solid D					

8. Did your ranking change after you found the volume of each solid? ______ Did any of the measurements surprise you? ______. Explain:

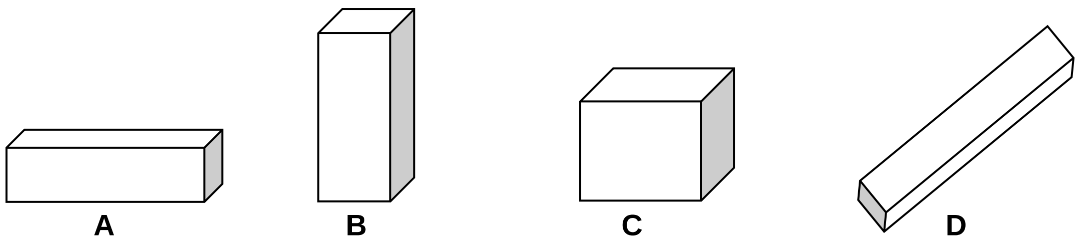
9. Explain how measuring the volume of a prism and measuring the volume of a cylinder are similar and different.

AR Remediation Plan –Volume and Surface Area

Name: ____

Which Prism Holds the Most?

1. The boxes shown below are examples of rectangular prisms. Note that they are *not* drawn to scale, so you must look at their dimensions in the chart. Circle the box that you think would hold the *most*. Explain your choice.



Rectangular Prism	Length (cm)	Width (cm)	Height (cm)
A	9	2	3
B	4	3	8
C	5	5	4
D	12	2	2

2. Rank of the prisms from *greatest to least* by estimated volume.

Greatest → ***Least***

Solid: _______ _______ _______ _______

3. How can you calculate the exact volume of each rectangular prism?

____

4. Test your estimates by calculating the volume of each prism. Record your calculations in the chart, including the correct unit of measurement for volume, and rank the prisms from greatest to least by actual volume.

Rectangular Prism	Actual volume	Rank G to L by actual volume
A		
B		
C		
D		

5. Which of the actual volumes surprised you? Explain.

____

____

6. Which two prisms were closest in volume? ______________

Name: _____

Finding the Volume of a Rectangular Prism

Find how many cubic units fill each of the following prisms. Label the units in each situation.

1. A tissue box that measures 9 inches by 5 inches by 3½ inches:

   ____

2. A CD case that measures 14 cm by 12 cm by 1 cm:

   ____

3. A box of cereal that measures 8 inches by 4 inches by 13.5 inches:

   ____

4. A shoe box that measures 12 inches by 6 inches by 5 inches:

   ____

5. A package of computer paper that measures 8½ inches x 11 inches x 4 inches:

   ____

6. A ring box that measures 6 cm by 5 cm by 3 cm:

   ____

7. Explain why the formula for volume of a rectangular prism ($l \times w \times h$) makes sense to you.

   ____

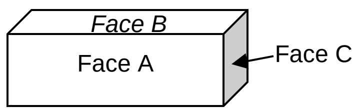
   ____

Name: ____

Finding the Surface Area of a Rectangular Prism I

Deconstruct each figure below to find the total surface area in square units.

1. The dimensions of the rectangular prism below are: l = 7 cm; w = 2 cm; h = 4 cm



Area of Face A = ____________________

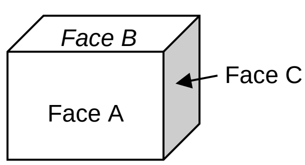
Area of Face B = ____________________

Area of Face C = ____________________

Because the opposite faces are congruent, the total surface area of all 6 faces is

____

2. The dimensions of the rectangular prism below are: l = 6 in.; w = 3 in.; h = 4 in.



Area of Face A = ____________________

Area of Face B = ____________________

Area of Face C = ____________________

Because the opposite faces are congruent, the total surface area of all 6 faces is

____

Name: _____

Finding the Surface Area of a Rectangular Prism II

In each situation below, find the total surface area of the figure. Explain your strategy or formula in words, and show your calculations. You may include sketches as well as numbers in your work. Include appropriate units with your numbers!

1. What is the minimum amount of cardboard needed to make a cereal box with dimensions 8 inches by 4 inches by 13 inches?

   Strategy: ____

   Calculations: ____

2. How much wrapping paper is needed to cover a dress box that measures 20 cm x 15 cm x 5 cm?

   Strategy: ____

   Calculations: ____

3. A board game and a drawing kit need to be gift wrapped. The board game measures 12 inches by 10 inches by 4 inches, while the drawing kit measures 18 inches by 10 inches by 2 inches. Which box will take the most wrapping paper to cover?

   Strategy: ____

   Calculations: ____

AR Remediation Plan –Volume and Surface Area

4. A chocolate sheet cake measures 15 inches x 11 inches x 3 inches high, and a cherry sheet cake measures 12 inches x 12 inches x 4 inches high. Which cake will need the most icing to cover its top and sides but not its base?

   Strategy: ______________________________

   Calculations: ______________________________

   ____

   ____

   ____

   ____

   ____

5. Extra for Experts: A round cake has a radius of 4½ inches and a height of 8 inches, while a rectangular cake measures 13 inches by 9 inches by 4 inches high. Which cake will take the most icing to cover its top and sides but not its base? Use $\pi \approx 3.14$.

   Strategy: ______________________________

   Calculations: ______________________________

   ____

   ____

   ____

   ____

   ____

Covering a Surface

1. How much metal makes up a soup can with a height of 10 cm and a radius of 3 cm?

   Draw the surfaces that make up the can, and label each one. On each piece, write a formula or method of finding its surface area (*SA*).

   Find the area of each piece, using $\pi \approx 3.14$, and add the totals together.

2. On the Grade 7 Mathematics Formula Sheet, find the formula for the surface area (*SA*) of a cylinder, and write it below.

   Explain how this formula makes sense to you. Which parts of the soup can match which parts of the formula?

3. Write the number that you would use for each variable in the formula.

   Using the formula for the surface area of a cylinder from step 2, write the formula for the surface area of the soup can in step 1.

   Enter this formula into your calculator to find the answer.

   Answer: ________________

   Does the measurement match what you got by adding the surfaces together in step 2?

   ____

4. Now try a different cylinder—a golf-ball can with a radius of 2 cm and a height of 19 cm. Do you think the surface area of this can is greater than or less than the surface area of the soup can? ______________________

   Sketch the golf-ball can in the space at right.

   Find the surface area in square centimeters.
   Show your work, and explain your formula in words.

   ____

   Using the formula for the surface area of a cylinder from step 2, write the formula for the surface area of the golf-ball can.

   ____

   Enter this formula into your calculator to find the answer.

   Answer: __________________

   Is the surface area of the golf-ball can greater than or less than the surface area of the soup can? ___________________________

5. Explain how you use the formula for surface area if the radius changes or the height changes.

   ____

6. List key words that let you know that you are measuring surface area.

   ____

   ____

   ____

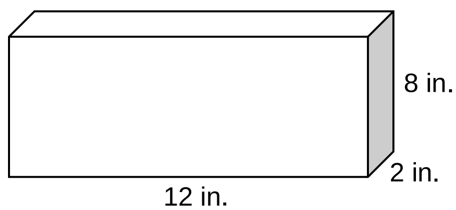
Name: ____

Reflection

Answer the following released SOL test questions about volume and surface area:

1. A cylinder-shaped barrel has a diameter of 3 feet and a height of 4.5 feet. How much water will be needed to fill up the cylinder?

2. Carl wants to cover this rectangular-prism-shaped box with wrapping paper. It has the dimensions shown:



What is the minimum amount of wrapping paper Carl needs to cover the entire box?