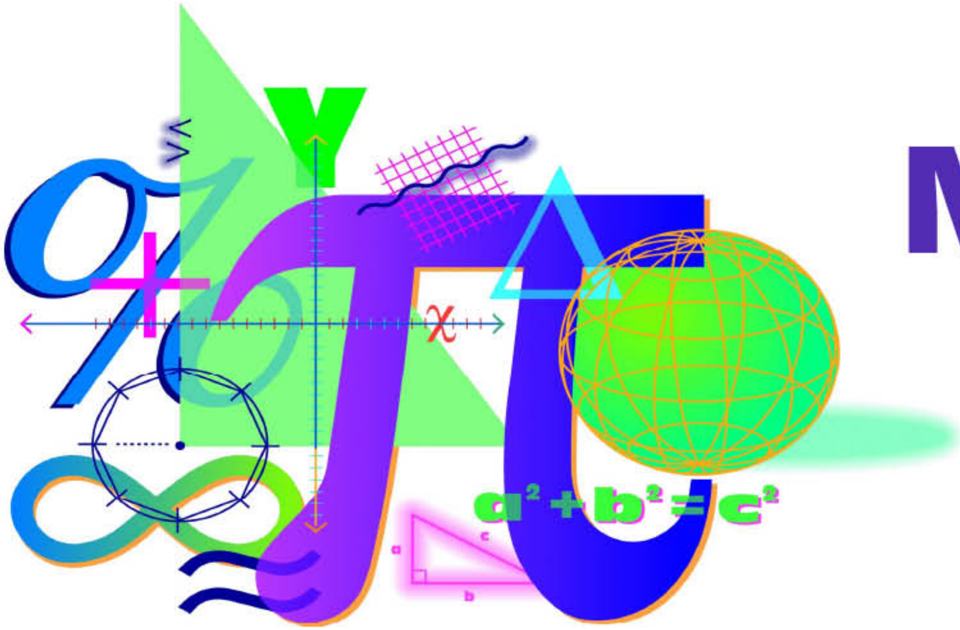

Mathematics

Standards of Learning

Curriculum Framework 2009

Grade 8

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The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia *Mathematics Standards of Learning* Curriculum Framework 2009
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students in the middle grades focus on mastering rational numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to most middle school mathematics topics.
- Students develop an understanding of integers and rational numbers by using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational-number representations is encouraged when students solve problems.
- Students develop an understanding of the properties of operations on real numbers through experiences with rational numbers and by applying the order of operations.
- Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

8.1 The student will

a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and

b) compare and order decimals, fractions, percents, and numbers written in scientific notation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• *Expression* is a word used to designate any symbolic mathematical phrase that may contain numbers and/or variables. Expressions do not contain equal or inequality signs. • The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* does not equal zero (e.g., $\sqrt{25}$, $\frac{1}{4}$ -2.3, 75%, $4.\overline{59}$). • A rational number is any number that can be written in fraction form. • A numerical expression contains only numbers and the operations on those numbers. • Expressions are simplified using the order of operations and the properties for operations with real numbers, i.e., associative, commutative, and distributive and inverse properties. • The order of operations, a mathematical convention, is as follows: Complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols (embedded), do the innermost operation first. Evaluate all exponential expressions. Multiply and/or divide in order from left to right. Add and/or subtract in order from left to right. *Parentheses ( ), brackets [ ], braces { }, the absolute value $\vert\ \vert$, division/fraction bar −, and the square root symbol $\sqrt{\ }$ should be treated as grouping symbols.	• What is the role of the order of operations when simplifying numerical expressions? The order of operations prescribes the order to use to simplify a numerical expression. • How does the different ways rational numbers can be represented help us compare and order rational numbers? Numbers can be represented as decimals, fractions, percents, and in scientific notation. It is often useful to convert numbers to be compared and/or ordered to one representation (e.g., fractions, decimals or percents). • What is a rational number? A rational number is any number that can be written in fraction form. • When are numbers written in scientific notation? Scientific notation is used to represent very large and very small numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Simplify numerical expressions containing: 1) exponents (where the base is a rational number and the exponent is a positive whole number); 2) fractions, decimals, integers and square roots of perfect squares; and 3) grouping symbols (no more than 2 embedded grouping symbols). Order of operations and properties of operations with real numbers should be used. • Compare and order no more than five fractions, decimals, percents, and numbers written in scientific notation using positive and negative exponents. Ordering may be in ascending or descending order.

8.1 The student will

a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and

b) compare and order decimals, fractions, percents, and numbers written in scientific notation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A power of a number represents repeated multiplication of the number. For example, $(-5)^4$ means $(-5) \cdot (-5) \cdot (-5) \cdot (-5)$. The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In this example, (–5) is the base, and 4 is the exponent. The product is 625. Notice that the base appears inside the grouping symbols. The meaning changes with the removal of the grouping symbols. For example, -5^4 means $5\cdot5\cdot5\cdot5$ negated which results in a product of -625. The expression $-(5)^4$ means to take the opposite of $5\cdot5\cdot5\cdot5$ which is -625. Students should be exposed to all three representations. • Scientific notation is used to represent very large or very small numbers. • A number written in scientific notation is the product of two factors: a decimal greater than or equal to one but less than 10 multiplied by a power of 10 (e.g., $3.1 \times 10^5 = 310{,}000$ and $3.1 \times 10^{-5} = 0.000031$). • Any real number raised to the zero power is 1. The only exception to this rule is zero itself. Zero raised to the zero power is undefined. • All state approved scientific calculators use algebraic logic (follow the order of operations).		

8.2 The student will describe orally and in writing the relationships between the subsets of the real number system.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The set of real numbers includes natural numbers, counting numbers, whole numbers, integers, rational and irrational numbers. • The set of natural numbers is the set of counting numbers {1, 2, 3, 4, ...}. • The set of whole numbers includes the set of all the natural numbers or counting numbers and zero {0, 1, 2, 3…}. • The set of integers includes the set of whole numbers and their opposites {…-2, -1, 0, 1, 2…}. • The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where a and b are integers and b does not equal zero (e.g., $\sqrt{25}$, $\frac{1}{4}$, -2.3, 75%, $4.\overline{59}$) . • The set of irrational numbers is the set of all nonrepeating, nonterminating decimals. An irrational number cannot be written in fraction form {e.g., π , $\sqrt{2}$, 1.232332333…}.	• How are the real numbers related? Some numbers can appear in more than one subset, e.g., 4 is an integer, a whole number, a counting or natural number and a rational number. The attributes of one subset can be contained in whole or in part in another subset.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe orally and in writing the relationships among the sets of natural or counting numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers. • Illustrate the relationships among the subsets of the real number system by using graphic organizers such as Venn diagrams. Subsets include rational numbers, irrational numbers, integers, whole numbers, and natural or counting numbers. • Identify the subsets of the real number system to which a given number belongs. • Determine whether a given number is a member of a particular subset of the real number system, and explain why. • Describe each subset of the set of real numbers and include examples and nonexamples. • Recognize that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.[†] [†]Revised March 2011

In the middle grades, the focus of mathematics learning is to
- build on students' concrete reasoning experiences developed in the elementary grades;
- construct through active learning experiences a more advanced understanding of mathematics;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense.
- Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate.
- Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonableness of answers.
- Students reinforce skills with operations with whole numbers, fractions, and decimals through problem solving and application activities.

8.3 The student will
a) solve practical problems involving rational numbers, percents, ratios, and proportions; and
b) determine the percent increase or decrease for a given situation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Practical problems may include, but not be limited to, those related to economics, sports, science, social sciences, transportation, and health. Some examples include problems involving the amount of a pay check per month, the discount price on a product, temperature, simple interest, sales tax and installment buying. • A percent is a special ratio with a denominator of 100. • A discount is a percent of the original price. The discount price is the original price minus the discount. • Simple interest for a number of years is determined by multiplying the principle by the rate of interest by the number of years of the loan or investment $\left(I = prt\right)$. • The total value of an investment is equal to the sum of the original investment and the interest earned. • The total cost of a loan is equal to the sum of the original cost and the interest paid. • Percent increase and percent decrease are both percents of change. • Percent of change is the percent that a quantity increases or decreases. • Percent increase determines the rate of growth and may be calculated using a ratio. $\frac{\text{Change (new – original)}}{\text{original}}$	• What is the difference between percent increase and percent decrease? Percent increase and percent decrease are both percents of change measuring the percent a quantity increases or decreases. Percent increase shows a growing change in the quantity while percent decrease shows a lessening change. • What is a percent? A percent is a special ratio with a denominator of 100.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write a proportion given the relationship of equality between two ratios. • Solve practical problems by using computation procedures for whole numbers, integers, fractions, percents, ratios, and proportions. Some problems may require the application of a formula. • Maintain a checkbook and check registry for five or fewer transactions. • Compute a discount or markup and the resulting sale price for one discount or markup. • Compute the percent increase or decrease for a one-step equation found in a real life situation. • Compute the sales tax or tip and resulting total. • Substitute values for variables in given formulas. For example, use the simple interest formula $I = prt$ to determine the value of any missing variable when given specific information. • Compute the simple interest and new balance earned in an investment or on a loan for a given number of years.

8.3 The student will
a) solve practical problems involving rational numbers, percents, ratios, and proportions; and
b) determine the percent increase or decrease for a given situation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• For percent increase, the change will result in a positive number. • Percent decrease determines the rate of decline and may be calculated using the same ratio as percent increase. However, the change will result in a negative number.		

8.4 The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Algebraic expressions use operations with algebraic symbols (variables) and numbers. • Algebraic expressions are evaluated by substituting numbers for variables and applying the order of operations to simplify the resulting expression. • The order of operations is as follows: Complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols (embedded), do the innermost operation first. Evaluate all exponential expressions. Multiply and/or divide in order from left to right. Add and/or subtract in order from left to right. * Parentheses ( ), brackets [ ], braces { }, the absolute value $\mid\ \mid$, division/fraction bar −, and the square root symbol $\sqrt{\ }$ should be treated as grouping symbols.	• What is the role of the order of operations when evaluating expressions? Using the order of operations assures only one correct answer for an expression.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Substitute numbers for variables in algebraic expressions and simplify the expressions by using the order of operations. Exponents are positive and limited to whole numbers less than 4. Square roots are limited to perfect squares. • Apply the order of operations to evaluate formulas. Problems will be limited to positive exponents. Square roots may be included in the expressions but limited to perfect squares.

8.5 The student will
a) determine whether a given number is a perfect square; and
b) find the two consecutive whole numbers between which a square root lies.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A perfect square is a whole number whose square root is an integer (e.g., The square root of 25 is 5 and -5; thus, 25 is a perfect square). • The square root of a number is any number which when multiplied by itself equals the number. • Whole numbers have both positive and negative roots. • Any whole number other than a perfect square has a square root that lies between two consecutive whole numbers. • The square root of a whole number that is not a perfect square is an irrational number (e.g., $\sqrt{2}$ is an irrational number). An irrational number cannot be expressed exactly as a ratio. • Students can use grid paper and estimation to determine what is needed to build a perfect square.	• How does the area of a square relate to the square of a number? The area determines the perfect square number. If it is not a perfect square, the area provides a means for estimation. • Why do numbers have both positive and negative roots? The square root of a number is any number which when multiplied by itself equals the number. A product, when multiplying two positive factors, is always the same as the product when multiplying their opposites (e.g., $7 \cdot 7 = 49$ and $-7 \cdot -7 = 49$).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the perfect squares from 0 to 400. • Identify the two consecutive whole numbers between which the square root of a given whole number from 0 to 400 lies (e.g., $\sqrt{57}$ lies between 7 and 8 since $7^2 = 49$ and $8^2 = 64$). • Define a perfect square. • Find the positive or positive and negative square roots of a given whole number from 0 to 400. (Use the symbol $\sqrt{\ }$ to ask for the positive root and $-\sqrt{\ }$ when asking for the negative root.)

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- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop the measurement skills that provide a natural context and connection among many mathematics concepts. Estimation skills are developed in determining length, weight/mass, liquid volume/capacity, and angle measure. Measurement is an essential part of mathematical explorations throughout the school year.
- Students continue to focus on experiences in which they measure objects physically and develop a deep understanding of the concepts and processes of measurement. Physical experiences in measuring various objects and quantities promote the long-term retention and understanding of measurement. Actual measurement activities are used to determine length, weight/mass, and liquid volume/capacity.
- Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, pyramids, and cones.

8.6 The student will
a) verify by measuring and describe the relationships among vertical angles, adjacent angles, supplementary angles, and complementary angles; and
b) measure angles of less than 360°.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Vertical angles are (all nonadjacent angles) formed by two intersecting lines. Vertical angles are congruent and share a common vertex. • Complementary angles are any two angles such that the sum of their measures is 90°. • Supplementary angles are any two angles such that the sum of their measures is 180°. • Reflex angles measure more than 180°. • Adjacent angles are any two non-overlapping angles that share a common side and a common vertex.	• How are vertical, adjacent, complementary and supplementary angles related? Adjacent angles are any two non-overlapping angles that share a common side and a common vertex. Vertical angles will always be nonadjacent angles. Supplementary and complementary angles may or may not be adjacent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Measure angles of less than 360° to the nearest degree, using appropriate tools. • Identify and describe the relationships between angles formed by two intersecting lines. • Identify and describe the relationship between pairs of angles that are vertical. • Identify and describe the relationship between pairs of angles that are supplementary. • Identify and describe the relationship between pairs of angles that are complementary. • Identify and describe the relationship between pairs of angles that are adjacent. • Use the relationships among supplementary, complementary, vertical, and adjacent angles to solve practical problems.[†] [†]Revised March 2011

8.7 The student will
a) investigate and solve practical problems involving volume and surface area of prisms, cylinders, cones, and pyramids; and
b) describe how changing one measured attribute of the figure affects the volume and surface area.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A polyhedron is a solid figure whose faces are all polygons. • A pyramid is a polyhedron with a base that is a polygon and other faces that are triangles with a common vertex. • The area of the base of a pyramid is the area of the polygon which is the base. • The total surface area of a pyramid is the sum of the areas of the triangular faces and the area of the base. • The volume of a pyramid is $\frac{1}{3}Bh$, where B is the area of the base and h is the height. • The area of the base of a circular cone is πr^2. • The surface area of a right circular cone is $\pi r^2 + \pi rl$, where l represents the slant height of the cone. • The volume of a cone is $\frac{1}{3}\pi r^2 h$, where h is the height and πr^2 is the area of the base. • The surface area of a right circular cylinder is $2\pi r^2 + 2\pi rh$. • The volume of a cylinder is the area of the base of the cylinder multiplied by the height. • The surface area of a rectangular prism is the sum of the areas of the six faces. • The volume of a rectangular prism is calculated by multiplying the length, width and height of the prism.	• How does the volume of a three-dimensional figure differ from its surface area? Volume is the amount a container holds. Surface area of a figure is the sum of the area on surfaces of the figure. • How are the formulas for the volume of prisms and cylinders similar? For both formulas you are finding the area of the base and multiplying that by the height. • How are the formulas for the volume of cones and pyramids similar? For cones you are finding $\frac{1}{3}$ of the volume of the cylinder with the same size base and height. For pyramids you are finding $\frac{1}{3}$ of the volume of the prism with the same size base and height. • In general what effect does changing one attribute of a prism by a scale factor have on the volume of the prism? When you increase or decrease the length, width or height of a prism by a factor greater than 1, the volume of the prism is also increased by that factor.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Distinguish between situations that are applications of surface area and those that are applications of volume. • Investigate and compute the surface area of a square or triangular pyramid by finding the sum of the areas of the triangular faces and the base using concrete objects, nets, diagrams and formulas. • Investigate and compute the surface area of a cone by calculating the sum of the areas of the side and the base, using concrete objects, nets, diagrams and formulas. • Investigate and compute the surface area of a right cylinder using concrete objects, nets, diagrams and formulas. • Investigate and compute the surface area of a rectangular prism using concrete objects, nets, diagrams and formulas. • Investigate and compute the volume of prisms, cylinders, cones, and pyramids, using concrete objects, nets, diagrams, and formulas. • Solve practical problems involving volume and surface area of prisms, cylinders, cones, and pyramids.

8.7 The student will

a) investigate and solve practical problems involving volume and surface area of prisms, cylinders, cones, and pyramids; and

b) describe how changing one measured attribute of the figure affects the volume and surface area.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A prism is a solid figure that has a congruent pair of parallel bases and faces that are parallelograms. The surface area of a prism is the sum of the areas of the faces and bases. • When one attribute of a prism is changed through multiplication or division the volume increases by the same factor that the attribute increased by. For example, if a prism has a volume of 2 x 3 x 4, the volume is 24. However, if one of the attributes are doubled, the volume doubles. • The volume of a prism is *Bh*, where *B* is the area of the base and h is the height of the prism. • Nets are two-dimensional representations that can be folded into three-dimensional figures.		• Compare and contrast the volume and surface area of a prism with a given set of attributes with the volume of a prism where one of the attributes has been increased by a factor of 2, 3, 5 or 10. • Describe the two-dimensional figures that result from slicing three-dimensional figures parallel to the base (e.g., as in plane sections of right rectangular prisms and right rectangular pyramids).[†] [†]Revised March 2011

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- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students expand the informal experiences they have had with geometry in the elementary grades and develop a solid foundation for the exploration of geometry in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.
- Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provides experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.
- Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.
- Students explore and develop an understanding of the Pythagorean Theorem. Mastery of the use of the Pythagorean Theorem has far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)
- **Level 4: Deduction.** Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)

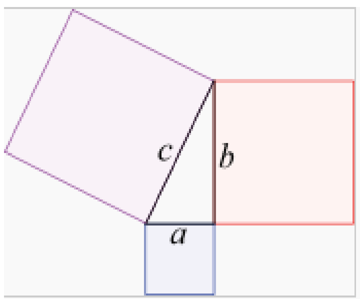
8.8 The student will
a) apply transformations to plane figures; and
b) identify applications of transformations.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A rotation of a geometric figure is a clockwise or counterclockwise turn of the figure around a fixed point. The point may or may not be on the figure. The fixed point is called the *center of rotation.* • A reflection of a geometric figure moves all of the points of the figure across an axis. Each point on the reflected figure is the same distance from the axis as the corresponding point in the original figure. • A translation of a geometric figure moves all the points on the figure the same distance in the same direction. • A dilation of a geometric figure is a transformation that changes the size of a figure by a scale factor to create a similar figure. • Practical applications may include, but are not limited to, the following: – A rotation of the hour hand of a clock from 2:00 to 3:00 shows a turn of 30° clockwise; – A reflection of a boat in water shows an image of the boat flipped upside down with the water line being the line of reflection; – A translation of a figure on a wallpaper pattern shows the same figure slid the same distance in the same direction; and – A dilation of a model airplane is the production model of the airplane. • The image of a polygon is the resulting polygon after a transformation. The preimage is the original polygon before the transformation. • A transformation of preimage point A can be denoted as the image A' (read as "A prime").	• How does the transformation of a figure on the coordinate grid affect the congruency, orientation, location and symmetry of an image? Translations, rotations and reflections maintain congruence between the preimage and image but change location. Dilations by a scale factor other than 1 produce an image that is not congruent to the pre-image but is similar. Rotations and reflections change the orientation of the image.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Demonstrate the reflection of a polygon over the vertical or horizontal axis on a coordinate grid. • Demonstrate 90°, 180°, 270°, and 360°clockwise and counterclockwise rotations of a figure on a coordinate grid. The center of rotation will be limited to the origin. • Demonstrate the translation of a polygon on a coordinate grid. • Demonstrate the dilation of a polygon from a fixed point on a coordinate grid. • Identify practical applications of transformations including, but not limited to, tiling, fabric, and wallpaper designs, art and scale drawings. • Identify the type of transformation in a given example.

8.9 The student will construct a three-dimensional model, given the top or bottom, side, and front views.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Three-dimensional models of geometric solids can be used to understand perspective and provide tactile experiences in determining two-dimensional perspectives. • Three-dimensional models of geometric solids can be represented on isometric paper. • The top view is a mirror image of the bottom view.	• How does knowledge of two-dimensional figures inform work with three-dimensional objects? It is important to know that a three-dimensional object can be represented as a two-dimensional model with views of the object from different perspectives.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct three-dimensional models, given the top or bottom, side, and front views. • Identify three-dimensional models given a two-dimensional perspective.

8.10 The student will
a) verify the Pythagorean Theorem; and
b) apply the Pythagorean Theorem.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the legs (altitude and base). This relationship is known as the Pythagorean Theorem: $a^2 + b^2 = c^2$.  • The Pythagorean Theorem is used to find the measure of any one of the three sides of a right triangle if the measures of the other two sides are known. • Whole number triples that are the measures of the sides of right triangles, such as (3,4,5), (6,8,10), (9,12,15), and (5,12,13), are commonly known as Pythagorean triples. • The hypotenuse of a right triangle is the side opposite the right angle. • The hypotenuse of a right triangle is always the longest side of the right triangle. • The legs of a right triangle form the right angle.	• How can the area of squares generated by the legs and the hypotenuse of a right triangle be used to verify the Pythagorean Theorem? For a right triangle, the area of a square with one side equal to the measure of the hypotenuse equals the sum of the areas of the squares with one side each equal to the measures of the legs of the triangle.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the parts of a right triangle (the hypotenuse and the legs). • Verify a triangle is a right triangle given the measures of its three sides. • Verify the Pythagorean Theorem, using diagrams, concrete materials, and measurement. • Find the measure of a side of a right triangle, given the measures of the other two sides. • Solve practical problems involving right triangles by using the Pythagorean Theorem.

8.11 The student will solve practical area and perimeter problems involving composite plane figures.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A polygon is a simple, closed plane figure with sides that are line segments. • The perimeter of a polygon is the distance around the figure. • The area of any composite figure is based upon knowing how to find the area of the composite parts such as triangles and rectangles. • The area of a rectangle is computed by multiplying the lengths of two adjacent sides ($A = lw$). • The area of a triangle is computed by multiplying the measure of its base by the measure of its height and dividing the product by 2 ($A = \frac{1}{2}bh$). • The area of a parallelogram is computed by multiplying the measure of its base by the measure of its height ($A = bh$). • The area of a trapezoid is computed by taking the average of the measures of the two bases and multiplying this average by the height [$A = \frac{1}{2}h(b_1 + b_2)$]. • The area of a circle is computed by multiplying Pi times the radius squared ($A = \pi r^2$). • The circumference of a circle is found by multiplying Pi by the diameter or multiplying Pi by 2 times the radius ($C = \pi d$ or $C = 2\pi r$). • An estimate of the area of a composite figure can be made by subdividing the polygon into triangles, rectangles, squares, trapezoids and semicircles, estimating their areas, and adding the areas together.	• How does knowing the areas of polygons assist in calculating the areas of composite figures? The area of a composite figure can be found by subdividing the figure into triangles, rectangles, squares, trapezoids and semi-circles, calculating their areas, and adding the areas together.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Subdivide a figure into triangles, rectangles, squares, trapezoids and semicircles. Estimate the area of subdivisions and combine to determine the area of the composite figure. • Use the attributes of the subdivisions to determine the perimeter and circumference of a figure. • Apply perimeter, circumference and area formulas to solve practical problems.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop an awareness of the power of data analysis and probability by building on their natural curiosity about data and making predictions.
- Students explore methods of data collection and use technology to represent data with various types of graphs. They learn that different types of graphs represent different types of data effectively. They use measures of center and dispersion to analyze and interpret data.
- Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability.
- Students explore experimental and theoretical probability through experiments and simulations by using concrete, active learning activities.

8.12 The student will determine the probability of independent and dependent events with and without replacement.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Two events are either dependent or independent. • If the outcome of one event does not influence the occurrence of the other event, they are called independent. If events are independent, then the second event occurs regardless of whether or not the first occurs. For example, the first roll of a number cube does not influence the second roll of the number cube. Other examples of independent events are, but not limited to: flipping two coins; spinning a spinner and rolling a number cube; flipping a coin and selecting a card; and choosing a card from a deck, replacing the card and selecting again. • The probability of three independent events is found by using the following formula: $P(A \, and \, B \, and \, C) = P(A) \cdot P(B) \cdot P(C)$ Ex: When rolling three number cubes simultaneously, what is the probability of rolling a 3 on one cube, a 4 on one cube, and a 5 on the third? $P(3 \, and \, 4 \, and \, 5) = P(3) \cdot P(4) \cdot P(5) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$	• How are the probabilities of dependent and independent events similar? Different? If events are dependent then the second event is considered only if the first event has already occurred. If events are independent, then the second event occurs regardless of whether or not the first occurs.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the probability of no more than three independent events. • Determine the probability of no more than two dependent events without replacement. • Compare the outcomes of events with and without replacement.

8.12 The student will determine the probability of independent and dependent events with and without replacement.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• If the outcome of one event has an impact on the outcome of the other event, the events are called dependent. If events are dependent then the second event is considered only if the first event has already occurred. For example, if you are dealt a King from a deck of cards and you do not place the King back into the deck before selecting a second card, the chance of selecting a King the second time is diminished because there are now only three Kings remaining in the deck. Other examples of dependent events are, but not limited to: choosing two marbles from a bag but not replacing the first after selecting it; and picking a sock out of a drawer and then picking a second sock without replacing the first. • The probability of two dependent events is found by using the following formula: $P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$ Ex: You have a bag holding a blue ball, a red ball, and a yellow ball. What is the probability of picking a blue ball out of the bag on the first pick then *without* replacing the blue ball in the bag, picking a red ball on the second pick? $P(\text{blue } \textit{and} \text{ red}) = P(\text{blue}) \cdot P(\text{red } \textit{after} \text{ blue}) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$		

8.13 The student will
a) make comparisons, predictions, and inferences, using information displayed in graphs; and
b) construct and analyze scatterplots.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Comparisons, predictions, and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. • The information displayed in different graphs may be examined to determine how data are or are not related, ascertaining differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inferences). • A scatterplot illustrates the relationship between two sets of data. A scatterplot consists of points. The coordinates of the point represent the measures of the two attributes of the point. • Scatterplots can be used to predict trends and estimate a line of best fit. • In a scatterplot, each point is represented by an independent and dependent variable. The independent variable is graphed on the horizontal axis and the dependent is graphed on the vertical axis.	• Why do we estimate a line of best fit for a scatterplot? A line of best fit helps in making interpretations and predictions about the situation modeled in the data set. • What are the inferences that can be drawn from sets of data points having a positive relationship, a negative relationship, and no relationship? Sets of data points with positive relationships demonstrate that the values of the two variables are increasing. A negative relationship indicates that as the value of the independent variable increases, the value of the dependent variable decreases.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect, organize, and interpret a data set of no more than 20 items using scatterplots. Predict from the trend an estimate of the line of best fit with a drawing. • Interpret a set of data points in a scatterplot as having a positive relationship, a negative relationship, or no relationship.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students extend their knowledge of patterns developed in the elementary grades and through life experiences by investigating and describing functional relationships.
- Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable, term, coefficient, exponent, expression, equation, inequality*, *domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.
- Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.
- Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.

8.14 The student will make connections between any two representations (tables, graphs, words, and rules) of a given relationship.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A relation is any set of ordered pairs. For each first member, there may be many second members. • A function is a relation in which there is one and only one second member for each first member. • As a table of values, a function has a unique value assigned to the second variable for each value of the first variable. • As a graph, a function is any curve (including straight lines) such that any vertical line would pass through the curve only once. • Some relations are functions; all functions are relations. • Graphs of functions can be discrete or continuous. • In a discrete function graph there are separate, distinct points. You would not use a line to connect these points on a graph. The points between the plotted points have no meaning and cannot be interpreted. • In a graph of continuous function every point in the domain can be interpreted therefore it is possible to connect the points on the graph with a continuous line as every point on the line answers the original question being asked. • Functions can be represented as tables, graphs, equations, physical models, or in words.	• What is the relationship among tables, graphs, words, and rules in modeling a given situation? Any given relationship can be represented by all four.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Graph in a coordinate plane ordered pairs that represent a relation. • Describe and represent relations and functions, using tables, graphs, words, and rules. Given one representation, students will be able to represent the relation in another form. • Relate and compare different representations for the same relation.

8.15 The student will
a) solve multistep linear equations in one variable on one and two sides of the equation;
b) solve two-step linear inequalities and graph the results on a number line; and
c) identify properties of operations used to solve an equation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A multistep equation is an equation that requires more than one different mathematical operation to solve. • A two-step inequality is defined as an inequality that requires the use of two different operations to solve (e.g., $3x - 4 > 9$). • In an equation, the equal sign indicates that the value on the left is the same as the value on the right. • To maintain equality, an operation that is performed on one side of an equation must be performed on the other side. • When both expressions of an inequality are multiplied or divided by a negative number, the inequality sign reverses. • The commutative property for addition states that changing the order of the addends does not change the sum (e.g., $5 + 4 = 4 + 5$). • The commutative property for multiplication states that changing the order of the factors does not change the product (e.g., $5 \cdot 4 = 4 \cdot 5$). • The associative property of addition states that regrouping the addends does not change the sum [e.g., $5 + (4 + 3) = (5 + 4) + 3$]. • The associative property of multiplication states that regrouping the factors does not change the product [e.g., $5 \cdot (4 \cdot 3) = (5 \cdot 4) \cdot 3$].	• How does the solution to an equation differ from the solution to an inequality? While a linear equation has only one replacement value for the variable that makes the equation true, an inequality can have more than one.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve two- to four-step linear equations in one variable using concrete materials, pictorial representations, and paper and pencil illustrating the steps performed. • Solve two-step inequalities in one variable by showing the steps and using algebraic sentences. • Graph solutions to two-step linear inequalities on a number line. • Identify properties of operations used to solve an equation from among: - the commutative properties of addition and multiplication; - the associative properties of addition and multiplication; - the distributive property; - the identity properties of addition and multiplication; - the zero property of multiplication; - the additive inverse property; and - the multiplicative inverse property.

8.15 The student will

a) solve multistep linear equations in one variable on one and two sides of the equation;

b) solve two-step linear inequalities and graph the results on a number line; and

c) identify properties of operations used to solve an equation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Subtraction and division are neither commutative nor associative. • The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number [e.g., $5 \cdot (3 + 7) = (5 \cdot 3) + (5 \cdot 7)$, or $5 \cdot (3 – 7) = (5 \cdot 3) – (5 \cdot 7)$]. • Identity elements are numbers that combine with other numbers without changing the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). There are no identity elements for subtraction and division. • The additive identity property states that the sum of any real number and zero is equal to the given real number (e.g., $5 + 0 = 5$). • The multiplicative identity property states that the product of any real number and one is equal to the given real number (e.g., $8 \cdot 1 = 8$). Inverses are numbers that combine with other numbers and result in identity elements [e.g., $5 + (–5) = 0$; $\frac{1}{5} \cdot 5 = 1$]. • The additive inverse property states that the sum of a number and its additive inverse always equals zero [e.g., $5 + (–5) = 0$]. • The multiplicative inverse property states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., $4 \cdot \frac{1}{4} = 1$).		

8.15 The student will
a) solve multistep linear equations in one variable on one and two sides of the equation;
b) solve two-step linear inequalities and graph the results on a number line; and
c) identify properties of operations used to solve an equation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Zero has no multiplicative inverse. • The multiplicative property of zero states that the product of any real number and zero is zero. • Division by zero is not a possible arithmetic operation. • Combining like terms means to combine terms that have the same variable and the same exponent (e.g., $8x + 11 – 3x$ can be $5x +11$ by combining the like terms of $8x$ and $-3x$).		

8.16 The student will graph a linear equation in two variables.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A linear equation is an equation in two variables whose graph is a straight line, a type of continuous function (see SOL 8.14). • A linear equation represents a situation with a constant rate. For example, when driving at a rate of 35 mph, the distance increases as the time increases, but the rate of speed remains the same. • Graphing a linear equation requires determining a table of ordered pairs by substituting into the equation values for one variable and solving for the other variable, plotting the ordered pairs in the coordinate plane, and connecting the points to form a straight line. • The axes of a coordinate plane are generally labeled x and y; however, any letters may be used that are appropriate for the function.	• What types of real life situations can be represented with linear equations? Any situation with a constant rate can be represented by a linear equation.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct a table of ordered pairs by substituting values for x in a linear equation to find values for y. • Plot in the coordinate plane ordered pairs (x, y) from a table. • Connect the ordered pairs to form a straight line (a continuous function). • Interpret the unit rate of the proportional relationship graphed as the slope of the graph, and compare two different proportional relationships represented in different ways.† †Revised March 2011

8.17 The student will identify the domain, range, independent variable or dependent variable in a given situation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)

- The domain is the set of all the input values for the independent variable in a given situation.
- The range is the set of all the output values for the dependent variable in a given situation.
- The independent variable is the input value.
- The dependent variable depends on the independent variable and is the output value.
- Below is a table of values for finding the circumference of circles, $C = \pi d$, where the value of π is approximated as 3.14.

Diameter	Circumference
1 in.	3.14 in.
2 in.	6.28 in.
3 in.	9.42 in.
4 in.	12.56 in.

- The independent variable, or input, is the diameter of the circle. The values for the diameter make up the domain.
- The dependent variable, or output, is the circumference of the circle. The set of values for the circumference makes up the range.

ESSENTIAL UNDERSTANDINGS

- What are the similarities and differences among the terms domain, range, independent variable and dependent variable?
  The value of the dependent variable changes as the independent variable changes. The domain is the set of all input values for the independent variable. The range is the set of all possible values for the dependent variable.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Apply the following algebraic terms appropriately: *domain, range, independent variable,* and *dependent variable.*
- Identify examples of domain, range, independent variable, and dependent variable.
- Determine the domain of a function.
- Determine the range of a function.
- Determine the independent variable of a relationship.
- Determine the dependent variable of a relationship.