Just In Time Quick Check

Standard of Learning (SOL) AII.5

Strand: Functions

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The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including writing the first n terms, determining the n^{th} term, and evaluating summation formulas. Notation will include $\sum$ and a_n.

Grade Level Skills:

- Distinguish between a sequence and a series.
- Generalize patterns in a sequence using explicit and recursive formulas.
- Use and interpret the notations $\sum$, n, n^{th} term, and a_n.
- Given the formula, determine a_n (the n^{th} term) for an arithmetic or a geometric sequence.
- Given formulas, write the first n terms and determine the sum, S_n, of the first n terms of an arithmetic or geometric series.
- Given the formula, determine the sum of a convergent infinite series.
- Model practical situations using sequences and series.

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Just in Time Quick Check Teacher Notes

Supporting Resources:

- VDOE Mathematics Instructional Plans (MIPS)
  - AII.5 – Arithmetic and Geometric Sequence and Series (Word) / PDF Version
- VDOE Word Wall Cards: Algebra II (Word) | PDF
  - Arithmetic Sequence
  - Geometric Sequence

Supporting and Prerequisite SOL: A.1b, 8.14a

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1.

a) Write an equation for the *n*th term of the arithmetic sequence whose seventh term is 21 and has a common difference of 5.

b) Use the equation from part a) to find the 123rd term of the sequence.

2. Write an equation for the *n*th term of the geometric sequence: 18, -12, 8, $-\frac{16}{3}, \frac{32}{9}$

3. Evaluate:

$$\sum_{n=1}^{6}(-3)^{n-1}$$

4. Which of the following represent an infinite geometric series whose sum does not approach a finite value? Select all that apply and justify your choices.

a) $150 + 30 + 6 + \cdots$
b) $-10 - 20 - 40 - \cdots$
c) $2.2 + 2.42 + 2.662 + \cdots$
d) $\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \cdots$

5. Write the first four terms of an infinite geometric series where $a_1 = 60$ and $r = -\frac{2}{3}$. If the series converges, find the sum.

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Common Errors/Misconceptions and their Possible Indications

1.

a) Write an equation for the *n*th term of the arithmetic sequence whose seventh term is 21 and has a common difference of 5.

b) Use the equation from part a) to find the 123rd term of the sequence.

A common error that some students may make is to inadvertently switch a_n and a_1 when using the formula $a_n = a_1 + (n-1)d$ and write $a_7 = 21 + (7-1)5$. A student may then divide by 7 to solve for the value of a_1. This may indicate that the student does not have a conceptual understanding of the concept of the formula. One teaching strategy might be to have students list out the terms of the arithmetic sequence without using the formula. For example, start with ___,_____,_____,_____,_____,______, 21 and work backwards using the common difference to find the first term.

2. Write an equation for the *n*th term of the geometric sequence: 18, -12, 8, $-\frac{16}{3}, \frac{32}{9}$

A common error that some students may make is to obtain the equation $a_n = -12^{n-1}$ by incorrectly multiplying the first term 18, and the common ratio $-\frac{2}{3}$. This may indicate that students do not have a conceptual understanding of the derivation of the equation representing the nth term of a geometric sequence $a_n = a_1 r^{n-1}$. Consider having students start by identifying the first term, the number of terms, and the common ratio. Once they have determined an equation, have them substitute n values into their rule to see if the original sequence can be generated.

3. Evaluate:

$$\sum_{n=1}^{6} (-3)^{n-1}$$

A common error that some students may make is to fail to start with 1 as the first term and obtain a sum of 546. This may indicate that the students misunderstood the summation notation and evaluated the sum $(-3)^n$ as the first term. A beneficial teaching strategy may be to have the students list out the six terms using a table of values.

4. Which of the following represent an infinite geometric series whose sum does not approach a finite value? Select all that apply and justify your choices.

a) $150 + 30 + 6 + \cdots$

b) $-10 - 20 - 40 - \cdots$

c) $2.2 + 2.42 + 2.662 + \cdots$

d) $\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots$

A common misconception that some students may have is to assume that all infinite geometry series have a finite sum. This may indicate that students do not recognize that $|r| \geq 1$, the series is divergent and will not approach a finite value. A teaching strategy may be to use a table of values to show that as n gets larger, the sum of the series approach infinity and has no limit.

5. Write the first four terms of an infinite geometric series where $a_1 = 60$ and $r = -\frac{2}{3}$. If the series converges, find the sum.

A common error some students may make is to find the sum of only the first four terms. This may indicate that the students do not recognize that to find the sum of a convergent series, they must divide the first term by the difference between 1 and the common ratio. One teaching strategy might be to have students discern between a geometry sequence, which has a finite number of terms, and a geometric series. The VDOE Vocabulary Word Wall cards for Algebra II may provide additional instructional support.