Probability – A Co-Teaching Lesson Plan

Co-Teaching Approaches

A "(Y)" in front of the following list items indicates the approach is outlined in the lesson. An "(N)" in front of the following list items indicates the approach is not outlined in the lesson.

- (N) Parallel Teaching
- (Y) Team Teaching
- (Y) Station Teaching
- (N) One Teach/One Observe
- (N) Alternative Teaching
- (Y) One Teach/One Assist

Subject

Algebra, Functions, and Data Analysis (AFDA)

Strand

AFDA.6 Probability

Topic

Probability

SOL

AFDA.6 The student will calculate probabilities. Key concepts include
a) Conditional probability;
b) Dependent and independent events;
c) Mutually exclusive events;
d) Counting techniques (permutations and combinations); and
e) Law of Large Numbers.

Outcomes

Students will identify examples of complementary, dependent, independent, and mutually exclusive events. Students will use the addition rule for calculating the probabilities of mutually exclusive events. Students will determine probabilities from Venn diagrams. Students will calculate the number of possible events using the concept of permutations. Students recognize that as an experiment is repeated over and over, the relative frequency probability tends to approach the actual probability.

Materials

- Applications: NCTM Illuminations for spinner simulator
- Name the Event worksheet (attached)
- Activity II: Teacher Notes – Students with an Earring; Band and Choir (Venn Diagram Analysis) (attached)
- Students with an Earring worksheet (attached)
- Students in Band, Students in Choir worksheet (attached)
- Activity III: Teacher's Notes - Permutations (attached)
- Permutations (attached)
- Activity VI: Teacher's Notes – Law of Large Numbers (attached)
- Law of Large Numbers (attached)
- Sample Resource (attached)

Vocabulary

complementary events, complements, conditional probability, dependent events, experimental probability, factorial notation, factorials, independent events, Law of Large Numbers, mutually exclusive events, permutation, probability notation, relative frequency, sample space, Venn diagrams

Co-Teacher Actions

Lesson Component	Co-Teaching Approach(es)	General Educator (GE)	Special Educator (SE)
Anticipatory Set	Team Teaching	Teachers facilitate a discussion with students about what they know/remember about probability. Use props to help aid the discussion (dice, cards, coins, spinners, etc.) Explain that students need to understand the difference between *independent, dependent, mutually exclusive,* and *complementary events.* Explain that two events, *A* and *B*, are *independent* if the occurrence of one does not affect the probability of the occurrence of the other. If events *A* and *B* are **not** *independent*,	Teachers facilitate discussion with students about what they know/remember about probability. Use props to help aid discussion (dice, cards, coins, spinners, etc.). Explain that students need to understand the difference between *independent, dependent, mutually exclusive,* and *complementary events.* Explain that two events, *A* and *B*, are *independent* if the occurrence of one does not affect the probability of the occurrence of the other. If events *A* and *B* are **not**

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		then they are said to be *dependent.* Therefore, two events are *dependent* if the outcome of the first affects the outcome of the second. Explain that *mutually exclusive* events cannot occur simultaneously. The *complement* of Event *A* consists of **all** outcomes in which event *A* does not occur. *Complementary events* are always *mutually exclusive*, but *mutually exclusive events* are not necessarily complementary. Give an experiment rolling two dice, the event of the dice dots having a sum of six and the event of the dice dots having a sum of eight are *mutually exclusive*. In that same experiment, the event of the dice dots having an even sum is the *complement* of the event of the dice dots having an odd sum. Give another experiment involving two dice where the probability of the first die showing six dots does not affect the probability of the second die showing six dots. Therefore, the rolls of each die are independent; one roll does not affect the other. Give a third experiment involving two dice where the probability of rolling a sum greater than ten greatly improves when the first roll produces a six. Therefore, the	*independent*, then they are said to be *dependent.* Therefore, two events are *dependent* if the outcome of the first affects the outcome of the second. Explain that *mutually exclusive* events cannot occur simultaneously. The *complement* of Event *A* consists of **all** outcomes in which event *A* does not occur. *Complementary events* are always *mutually exclusive*, but *mutually exclusive events* are not necessarily complementary. Give an experiment rolling two dice, the event of the dice dots having a sum of six and the event of the dice dots having a sum of eight are *mutually exclusive*. In that same experiment, the event of the dice dots having an even sum is the *complement* of the event of the dice dots having an odd sum. Give another experiment involving two dice where the probability of the first die showing six dots does not affect the probability of the second die showing six dots. Therefore, the rolls of each die are independent; one roll does not affect the other. Give a third experiment involving two dice where the probability of rolling a sum greater than ten greatly improves

Lesson Component	**Co-Teaching Approach(es)**	**General Educator (GE)**	**Special Educator (SE)**
		probability of rolling a sum greater than ten depends on the number from the first roll.	when the first roll produces a six. Therefore, the probability of rolling a sum greater than ten depends on the number from the first roll.
Lesson Activities/ Procedures	Team Teach	Students now complete 1-10 of Name the Event worksheet on their own. GE reassembles the class to discuss the answers. Once all students have the correct answers, they complete Name the Event worksheet with a partner. Teachers walk around and assist as needed. GE reassembles class to share each partner's choice for 15 on the Name the Event worksheet.	Students now complete 1-10 of Name the Event worksheet on their own. GE reassembles the class to discuss the answers. Once all students have the correct answers, they complete Name the Event worksheet with a partner. Teachers walk around and assist as needed.
Guided/ Independent Practice	Station Teaching	GE and SE set up stations – approx. 20 minutes at each station. Station 1: Since students have an idea of probability and Venn diagrams from previous math classes, Venn diagrams are an independent station. If you feel your class needs a review, go over a few examples first. Station 3: Led by GE. Lead students through the Law of Large Numbers activity.	GE and SE set up stations – approx. 20 minutes at each station. Station 1: Since students have an idea of probability and Venn diagrams from previous math classes, Venn diagrams are an independent station. If you feel your class needs a review, go over a few examples first. Station 2: Led by SE. Lead students through the Permutations activity.
Closure	One teach/One assist	GE explains the exit ticket instructions: • On a blank sheet of paper, write down a new situation that requires	SE walks around and assist.

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		a permutation. Switch with a partner and complete. GE walks around and assist.	
Formative Assessment Strategies	Team Teaching	GE checks for understanding throughout the lesson. GE collects and grades station work. GE assesses exit ticket.	SE checks for understanding throughout the lesson. GE collects and grades station work. GE assesses exit ticket.
Homework	Team Teaching	No homework is assigned.	No homework is assigned.

Specially Designed Instruction

- Use multisensory strategies: visual and tactile (dice, cards, etc.) and auditory (discussion)
- Station teaching reduces the teacher-to-student ratio and assures understanding through observation and discussion
- Break the concepts into smaller chunks. Introduce new concepts as smaller chunks are mastered.

Accommodations

- Provide oral and written instructions, per students' IEP or 504 accommodations.
- Allow extra time for written work.
- Allow discussion response for students with written expression deficits.
- Reduce the number of problems on the activity sheets

Modifications

- For those students who require modifications, the depth of the content can be reduced to theoretical probability with simple events.
- "Special educator" as noted in this lesson plan might be an EL teacher, speech pathologist, or other specialist co-teaching with a general educator.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Activity I: Teacher's Notes--Name the Event

Students need to understand the difference between *independent, dependent, mutually exclusive,* and *complementary events.* Two events, *A* and *B*, are *independent* if the occurrence of one does not affect the probability of the occurrence of the other. If events *A* and *B* are **not** *independent*, then they are said to be *dependent.* Therefore, two events are *dependent* if the outcome of the first affects the outcome of the second.

- *Mutually exclusive* events cannot occur simultaneously. The *complement* of event *A* consists of **all** outcomes in which event *A* does not occur.
- *Complementary events* are always *mutually exclusive,* but *mutually exclusive events* are not necessarily complementary. Given an experiment involving rolling two dice, the event of the dice dots having a sum of six and the event of the dice dots having a sum of eight are *mutually exclusive*. In that same experiment, the event of the dice dots having an even sum is the *complement* of the event of the dice dots having an odd sum.
- Given another experiment involving two dice, the probability of the first die showing six dots does not affect the probability of the second die showing six dots. Therefore, the rolls of each die are independent; one roll does not affect the other.
- Given a third experiment involving two dice, the probability of rolling a sum greater than ten greatly improves when the first roll produces a six. Therefore, the probability of rolling a sum greater than ten depends on the number from the first roll.

Students may need additional examples to understand the difference between these four types of events. Once students grasp the differences, they should be able to complete the first page of **Name the Event**. Review these answers before students begin creating their own event pairings. Sports, cars, course schedules, and characteristics of students are topics in which students could find event pairings. Finally, students need partners to share their event pairings. They should avoid reading them in order to make the challenge of identifying the relationship more realistic for their partners. Each partnership then selects one of each kind of relationship to add to a class list. Discuss the nuances of each event pairing that places it in a distinct category.

Name the Event

Given the following events, identify each as *dependent, independent, or mutually exclusive*. If they are *mutually exclusive*, say whether or not they are complementary.

1. A National Football Conference team wins the Super Bowl and an American Football Conference team wins the Super Bowl.

2. The Washington Redskins win the Super Bowl, and the Washington Wizards win the National Basketball Association championship.

3. The Washington Redskins win the Super Bowl, and Washington, DC has a parade.

4. The first two numbers of your Pick 3 lottery ticket match the winning numbers and the third number does not match.

5. Gus takes the bus to school, and he receives a speeding ticket on his way to school.

6. Caitlin sings in the school choir, and she is on her school's soccer team.

7. Hope sings in the school choir, and she sings in a concert.

8. Alex is a waiter at Olive Garden, and he is a chef at Olive Garden.

9. Karissa drives at night, and she hits a deer.

10. Lance's girlfriend has a cold sore, and Lance has a cold sore.

11. Create two different situations that illustrate independent events.
    a.

    b.

12. Create two different situations that illustrate dependent events.
    a.

    b.

13. Create two different situations that illustrate mutually exclusive events.
    a.

    b.

14. With a partner, take turns reading your examples while the other person determines whether each example illustrates **independent** events, **dependent** events, or **mutually exclusive** events.

    Record your guesses:
    a.

    b.

    c.

    d.

    e.

    f.

15. With your partner, pick one of each type to share with the whole class.
    a. Independent:

    b. Dependent:

    c. Mutually Exclusive:

Activity II: Teacher Notes--Students with an Earring; Band and Choir (Venn diagram Analysis)

Students should already have a basic understanding of probability from earlier math courses. This activity will guide students into calculating the probabilities of conditional events. Introduce the probability notation ***P* (*E*)**, which refers to the probability that the event E occurs.

Students should already know that $P(\text{event}) = \dfrac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

Set theory is used to represent relationships among events. In general, if ***A*** and ***B*** are two events in the sample space, ***S***, then

$A \cup B$ is said "*A* union *B*" which means that either *A* or *B* occurs or both occur

$A \cap B$ is said "*A* intersection *B*" which means that both *A* and *B* occur

A' is said "the complement of *A*" which means that *A* does not occur

If *A* and *B* are mutually exclusive then $P(A \cup B) = P(A) + P(B)$.

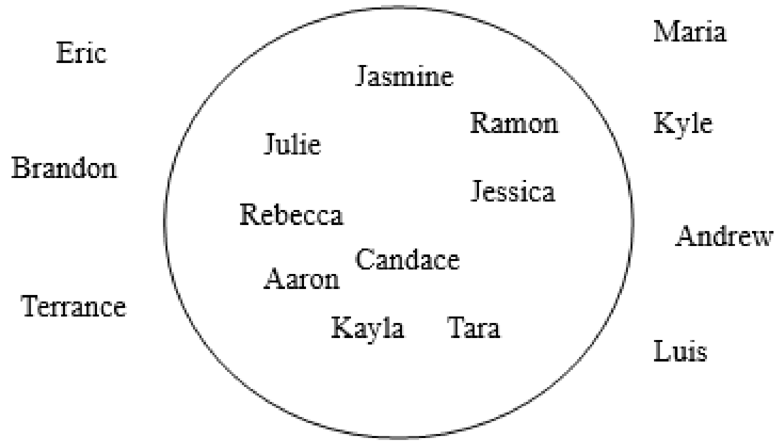
If *B* is A', then $P(A \cup A') = P(A) + P(A') = 1$.

If *A* and *B* are independent events, then $P(A \cap B) = P(A)P(B)$.

If *B* is dependent on *A*, then $P(B \mid A)$ is the probability that *B* will occur given that *A* has already occurred. $P(B \mid A)$ is called the conditional probability of *B* given *A*.

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \quad \Rightarrow \quad P(A \cap B) = P(A)P(B \mid A)$$

Students with an Earring



Answer the following questions using the Venn diagram.

1. How many students are in the class?

2. How many students have earrings?

3. How many students do not have earrings?

If one student is picked at random, find the following probabilities.

4. P(girl)

5. P(not a girl)

6. P(boy)

7. P(student with earring)

8. P(student without earring)

9. P(boy with earring)

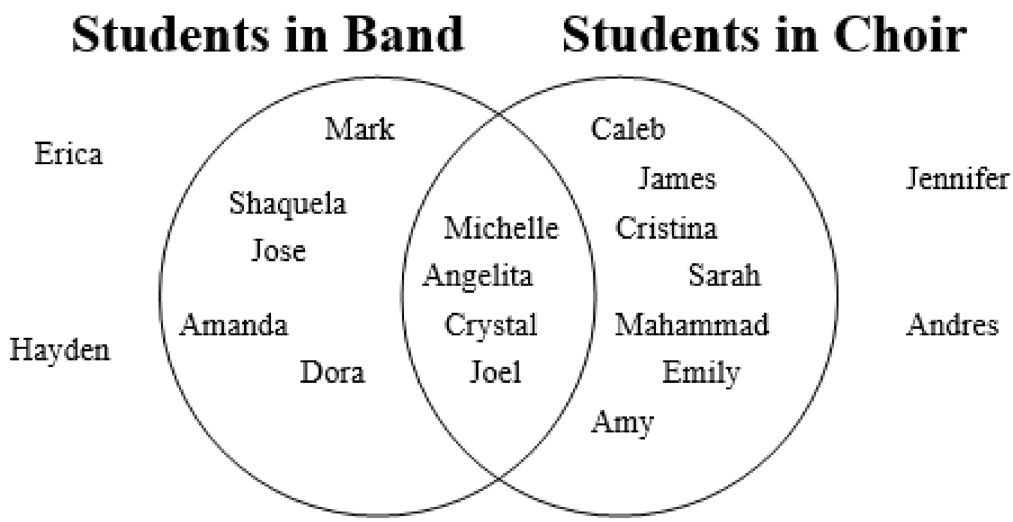
10. P(girl without earring)

11. P(girl with earring)

12. P(boy with earring $\cup$ girl with name starting with J)

13. P(name starting with A $\cup$ name starting with T)

14. Write a probability question which has an answer of $\frac{5}{8}$.



15. How many students are in the sample?

16. How many students are only in band?

17. How many students are in band and in choir?

If one student is picked at random, find the following probabilities.

18. P(student in choir)

19. P(student in neither band nor choir)

20. P(boy in band)

21. P(girl in band and choir)

22. P(boy in band or choir)

23. P(name starts with C and in choir)

24. P(student in band | boy)

25. P(girl | student in band)

26. P(boy | student in both band and choir)

27. Write a probability question which has an answer of $\frac{1}{2}$.

28. Write a probability question which has an answer of $\frac{4}{7}$.

Activity III: Teacher's Notes--Permutations

Students may need a review of the Fundamental Counting Principle. The handout will lead students to a discussion of factorials. Following the introduction to factorials, students will explore permutations, including the formula ${}_nP_r = \frac{n!}{(n-r)!}$. Emphasize the need for the objects to be arranged, as opposed to grouped.

Permutations

Mr. Vandergoobergooten has decided to put Adam, Brianna, Chase, Destiny, and Eduardo in the front row of his classroom. He has five desks in the front row but is undecided about which student should sit at which desk.

1. How many different students could he place in the rightmost desk?

2. After selecting a student for the rightmost desk, how many different students could he place in the next desk?

3. After selecting a student for the first two desks, how many different students could he place in the next desk?

4. After selecting a student for the first three desks, how many different students could he place in the next desk?

5. After selecting a student for the right four desks, how many different students could he place in the leftmost desk?

6. Using the fundamental counting principle, write an expression that would find the number of different ways Mr. Vandergoobergooten could choose to arrange these five students.

7. How many different ways can he arrange the students?

8. If Mr. Vandergoobergooten had six desks in the front row and wanted to put Felicia in the front row too, write an expression that would find the number of different arrangements that are now possible.

9. How many different arrangements are now possible?

The process of multiplying every counting number from 1 to *n* can be abbreviated as *n*! which is said "*n* factorial." Therefore $3 \cdot 2 \cdot 1 = 3! = 6$, $4 \cdot 3 \cdot 2 \cdot 1 = 4! = 24$, and $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 8! = 40{,}320$.

10. Write 2! as a multiplication problem and then find the value of 2!.

11. Write 7! as a multiplication problem and then find the value of 7!.

Since $2! = 2$ and $3! = 6$, then $3! = 3 \cdot 2! = 3 \cdot 2$
Since $3! = 6$ and $4! = 24$, then $4! = 4 \cdot 3! = 4 \cdot 6$
Since $4! = 24$ and $5! = 120$, then $5! = 5 \cdot 4! = 5 \cdot 24$

12. Since $6! =$_________ and $7! =$_________, then $7! =$____________ = __________

13. Since $10! = 3,628,800$ and $11! = 39,916,800$, then $11! =$____________$\cdot 10!$

14. Mr. Vandergoobergooten has 25 students and 25 desks in his class. Using factorials, how many different ways could he create a seating chart?

15. Mr. Vandergoobergooten has 12 poster-size pictures of his cat that he wants to line up on the wall above his chalkboard. Using factorials, how many different ways could he hang them on the wall?

16. Mr. Vandergoobergooten discovered that his cat posters are too big to fit all 12 on the wall. Instead, he will only be able to hang 4 of his posters.
   a) How many different posters can he select for the first position on the wall?

   b) After selecting the first poster, how many different posters can he select for the second position?

   c) After selecting the first two posters, how many different posters can he select for the third position?

   d) After selecting the first three posters, how many different posters can he select for the fourth and final position?

   e) Using the fundamental counting principle, write an expression for the number of ways Mr. Vandergoobergooten can hang four cat posters.

   f) How many different ways can Mr. Vandergoobergooten hang four of these posters?

17. For the back wall, Mr. Vandergoobergooten has room for three of his seven poster-size pictures of his goldfish. How many different ways can Mr. Vandergoobergooten hang three of these posters?

18. How many goldfish posters did Mr. Vandergoobergooten not use?

19. How many ways can seven posters be arranged?

20. How many ways can the posters that were not used be arranged?

21. What do you find if you divide your answer from 19 by your answer from 20?

22. Using this idea, what would be another way to calculate 16f (Mr. Vandergoobergooten's cat posters)?

Permutations are used to calculate the number of possible arrangements of *n* objects taken *r* at a time. For example, 12 posters are taken 4 at a time or ${}_{12}P_4$.

$${}_nP_r = \frac{n!}{(n-r)!}$$

For example, ${}_{12}P_4 = \frac{12!}{(12-4)!} = \frac{12!}{8!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8!} = 12 \cdot 11 \cdot 10 \cdot 9 = 11,880$

23. Mr. Vandergoobergooten has five desks in the front row of this classroom and 25 students in the class. How many different ways can he seat students in that first row so that every seat is filled?

24. Ms. Pi has eight seats in the first row of her classroom and she has 14 students in class. How many different ways can she seat students in that first row so that every seat is filled?

25. Joseph has six classes he still has to take to graduate. He will take four of them in the fall semester and then the other two, plus two electives in the spring. How many different schedules could Joseph have in the fall semester?

Activity VI: Teacher's Notes--Law of Large Numbers

Students need a spinner simulator to complete this activity. A simulation tool is available at http://illuminations.nctm.org/ActivityDetail.aspx?ID=79.

At the end of the investigation, discuss students' conclusions from the last step. Be sure that everyone understands that as a procedure is repeated again and again, the relative frequency probability of an event **tends** to **approach** the actual probability.

Law of Large Numbers

Directions: Use a spinner simulator on a calculator or computer. A simulation tool is available at http://illuminations.nctm.org/ActivityDetail.aspx?ID=79.
Set the number of sections on the spinner to 5. Change one of the actual (theoretical) values to 50%.

1. What has to be true of the other four actual (theoretical) values?

2. What happened to the appearance of the spinner?

Select any five probability values that you would like. Record these actual (theoretical) probability values as percentages.

Section:	1	2	3	4	5
3. Theoretical Values	_____	_____	_____	_____	_____

Using the calculator, graph the experimental probability values.

4. Experimental Probability (Relative Frequency) as a percentage:

after 1 spin	_____	_____	_____	_____	_____
after 2 spins	_____	_____	_____	_____	_____
after 5 spins	_____	_____	_____	_____	_____
after 10 spins	_____	_____	_____	_____	_____

Change the Trial Set or Number of Spins to 10.

after 20 spins	_____	_____	_____	_____	_____

5. Compare the relative frequency (experimental) probabilities in #4 with the actual (theoretical) probabilities from #3. Are you surprised by your results? Why or why not?

6. How many spins do you think it will take for the two types of probabilities to be equal? Explain.

7. Probability from Relative Frequency

after 30 spins	_____	_____	_____	_____	_____
after 40 spins	_____	_____	_____	_____	_____
after 50 spins	_____	_____	_____	_____	_____
after 100 spins	_____	_____	_____	_____	_____

8. Change the Trial Set or Number of Spins to 100.

after 200 spins	_____	_____	_____	_____	_____
after 300 spins	_____	_____	_____	_____	_____
after 400 spins	_____	_____	_____	_____	_____
after 500 spins	_____	_____	_____	_____	_____

9. Change the Trial Set or Number of Spins to 500.

after 1000 spins	_____	_____	_____	_____	_____
after 2000 spins	_____	_____	_____	_____	_____
after 3000 spins	_____	_____	_____	_____	_____

10. Round each of your relative frequency (experimental) probabilities from 3000 spins to the nearest whole percentage.

_____ _____ _____ _____ _____

11. Copy your actual (theoretical) probabilities from step 3.

_____ _____ _____ _____ _____

12. Compare the relative frequency (experimental) probabilities in step 23 with the actual (theoretical) probabilities from step 24.

13. Look at the results from three other people. What conclusion can you make about the relationship between relative frequency probability and actual probability as the number of experiments increases?

Sample Resource

The Virginia Lottery
http://www.valottery.com

Illuminations
http://illuminations.nctm.org/

- Sinner Simulations http://illuminations.nctm.org/ActivityDetail.aspx?ID=79
- Random Drawing Tool http://illuminations.nctm.org/ActivityDetail.aspx?id=67
- Exploration with Chance http://illuminations.nctm.org/LessonDetail.aspx?ID=L290