Simplifying Cube Roots of Integers

Strand: Expressions and Operations

Topic: Simplifying cube roots of integers

Primary SOL: A.3 The student will simplify
b) cube roots of integers;

Related SOL: A.2, A.4b, A.3c

Materials

- Graphing calculator
- Self-stick notes
- I Have … Who Has? activity sheet

Vocabulary

cube root, integer, *n*th *root, perfect cube, radical, radicand, simplest radical form*

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Begin by writing the three vocabulary terms, *integer, cube root,* and *perfect cube* on a board or chart paper around the room and ask students to recall or create a definition/meaning for each term. Have students write their definitions on sticky notes and place them under the appropriate term. Students can work with a partner to create a definition. If there are students who are struggling, ask them probing questions, such as "What do you think the difference between a whole number and an integer is?" or "What is a square root, and how do you think a square root can be compared to a cube root?" and "Recall a perfect square: What do you think the difference between a perfect square and a perfect cube is?" Encourage students to create a definition even if they don't know the correct one. Once students have finished, share some of the ideas written on sticky notes and come up with a consensus as a class of the proper meanings for each term.
2. Ask students to simplify $\sqrt[3]{8}$ using any method they choose. Have students share their methods for simplifying the expression. If prime factorization does not emerge as a strategy, the teacher should be prepared to model this as a possible method, where students are looking for groups of three common prime factors. Give students $\sqrt[3]{27}$, $\sqrt[3]{64}$, $\sqrt[3]{125}$ to simplify. Allow students to work together and discuss their ideas. Remind students of their methods used to simplify square root expressions and how similar methods can be used to simplify cube root expressions. Ask students to think of the perfect cubes, such as $2 \cdot 2 \cdot 2 = 2^3 = 8$.
3. Have students explore equivalency in cube roots of integers. In partners, or by separating the class into two groups (A and B), have the groups evaluate the following on their calculators one at a time, and then share their decimal equivalents at the same time.

   Students may also use a graphic organizer to organize their thinking.

A	B
$\sqrt[3]{3} \cdot \sqrt[3]{2}$	$\sqrt[3]{6}$
$\sqrt[3]{9} \cdot \sqrt[3]{5}$	$\sqrt[3]{45}$
$\sqrt[3]{-375}$	$-5\sqrt[3]{3}$

As with square root expressions, students should see the commonalities that will occur with simplifying cube roots and that there are different ways to represent a cube root without changing the value of it.

4. Have students work with a partner to represent $\sqrt[3]{-16}$, $\sqrt[3]{24}$, and $\sqrt[3]{-8}$ in as many ways as they can. If any of the students can express $\sqrt[3]{24}$ using a whole number, ask those students to share their method of simplifying. Ask students what they noticed about simplifying $\sqrt[3]{-16}$ and $\sqrt[3]{-8}$. This will allow the conversation to occur that the cube root of a nonperfect cube lies between two consecutive integers and that the cube root of a perfect cube is an integer.
5. To summarize the main ideas of simplifying cube roots, write or display to the students "A cube root of a number b is a number y such that $y^3 = b$" and "a cube root in simplest form is one in which the radicand has no perfect cube factors other than one." Use $\sqrt[3]{24}$ and $\sqrt[3]{-8}$ completely simplified as examples.
6. Separate the class into two to three groups to complete I Have … Who Has? activity. In each group, students work with a partner and receive two or more "I have … who has?" cards. The pair in each group who has "–1" will begin. The students will work to represent cube root expressions in their simplest form and answer the "who has?" questions. *(Note: There may be several different strategies for simplifying these cube roots. Some students may use prime factorization to get the cubic factors, and others may use their knowledge of perfect cubes. It is important that you accept these strategies and any other mathematically sound strategies that students offer.)*
7. End the lesson by asking each student to explain, in complete sentences, and show how to simplify $\sqrt[3]{-40}$.

Assessment

- **Questions**
  - What is the inverse of cubing a number?
  - What is a perfect cube?
  - What is the cube root of a perfect cube?
  - How could you find the two consecutive integers between which a cube root lies?
  - How do you know when a cube root is in its simplest form?
- **Journal/writing prompts**
  - Explain how simplifying a cube root expression is like simplifying a square root expression.

  - Explain how simplifying a cube root expression is different than simplifying a square root expression.
  - Why are square root expressions limited to whole numbers and cube root expressions can be integers?

- **Other Assessments**
  - Give students the cube root of an integer simplified incorrectly and have students explain why the work shown is incorrect and how to correct the incorrect expression.
  - Give students five partially simplified cube roots of integers and ask them to place them in order from least to greatest. Examples: $2\sqrt[3]{-16}$, $3\sqrt[3]{96}$, $4\sqrt[3]{76}$, $5\sqrt[3]{-8}$, $6\sqrt[3]{24}$.

Extensions and Connections (for all students)

- Have students simplify cube roots of integers and square roots of monomial algebraic expressions with a partner, alternating between cube roots and square roots.
- Represent cube root expressions in exponential form and demonstrate the expansion of the radicand using exponents.
- Ask students what they know about volume and why they think volume is expressed in cubic units. Allow students time discuss what they know about volume and how would they calculate the volume of a cube. How would they find the length, width, or height of a cube if they only knew its volume?

Strategies for Differentiation

- Have students create an organized table or linear expansion of radicands to show the prime factorization of the radicand.
- For visual learners, have students circle groups of three when simplifying cube roots.
- Allow students the use of dry-erase surfaces with multiple colored markers to demonstrate each step of their work.
- Ask students to verbally explain how they simplify a cube root and what it means to take the cube root of an integer.
- Ask students to create a Venn diagram demonstrating the similarities and differences between simplifying a square root monomial and the cube root of an integer.
- Modify the radicand values in the I Have … Who Has? activity to display integer values that are closer to zero.
- Create multiple versions of the I Have … Who Has? activity to accommodate for students of different ability levels.
- For Student/Teacher Action 7, have students explain orally to a partner how to simplify the expression.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

I Have … Who Has?

I have –1. Who has $\sqrt[3]{8}$?	I have 2. Who has $\sqrt[3]{16}$?	I have $2\sqrt[3]{2}$. Who has $\sqrt[3]{-343}$?
I have –7. Who has $\sqrt[3]{108}$?	I have $3\sqrt[3]{4}$. Who has $\sqrt[3]{192}$?	I have $4\sqrt[3]{3}$. Who has $\sqrt[3]{343}$?
I have 7. Who has $\sqrt[3]{500}$?	I have $5\sqrt[3]{4}$. Who has $\sqrt[3]{-320}$?	I have $-4\sqrt[3]{5}$. Who has $\sqrt[3]{432}$?
I have $6\sqrt[3]{2}$. Who has $\sqrt[3]{-48}$?	I have $-2\sqrt[3]{6}$. Who has $\sqrt[3]{3584}$?	I have $8\sqrt[3]{7}$. Who has $\sqrt[3]{2744}$?

I HAVE 14

WHO HAS $\sqrt[3]{162}$?

I HAVE $3\sqrt[3]{6}$?

WHO HAS $\sqrt[3]{-648}$?

I HAVE $-6\sqrt[3]{3}$

WHO HAS $\sqrt[3]{2662}$?

I HAVE $11\sqrt[3]{2}$?

WHO HAS $\sqrt[3]{-5184}$?

I HAVE $-12\sqrt[3]{3}$

WHO HAS $\sqrt[3]{4394}$?

I HAVE $13\sqrt[3]{2}$

WHO HAS $\sqrt[3]{-864}$?

I HAVE $-6\sqrt[3]{4}$

WHO HAS $\sqrt[3]{-1000}$?

I HAVE -10

WHO HAS $\sqrt[3]{5488}$?

I HAVE $14\sqrt[3]{2}$

WHO HAS $\sqrt[3]{128}$?

I HAVE $4\sqrt[3]{2}$

WHO HAS $\sqrt[3]{-192}$?

I HAVE $-4\sqrt[3]{3}$

WHO HAS $\sqrt[3]{3645}$?

I HAVE $9\sqrt[3]{5}$

WHO HAS $\sqrt[3]{-1}$?