Permutations and Combinations

Strand: Statistics

Topic: Counting using permutation and combinations

Primary SOL: AII.12 The student will compute and distinguish between permutations and combinations.

Materials
- Counting Exploration activity sheet (attached)
- Permutations activity sheet (attached)
- Combinations activity sheet (attached)
- Counting Sort activity sheet (attached)
- Graphing utility

Vocabulary

combination, counting, factorial, Fundamental Counting Principle, *permutation*

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Time: 90 minutes

1. Distribute the Counting Exploration activity sheet. Have students work in groups to complete it, then discuss responses with the class. (*Note: This activity reviews critical concepts necessary for understanding and interpreting permutations and combinations; therefore, a thorough discussion of the results is important. This may also include an introduction to factorials and reinforcing some of the simplification methods by expanding factorials.*)
2. Then, describe a *permutation* as an arrangement of objects in which order is important. Discuss which examples on the Counting Exploration activity sheet were permutations.

   Now present the problem: "There are seven marching bands in a parade, and all of them want to be at or near front of the parade?" Put seven blank spaces, left to right, on the board to represent the seven band positions in the parade. Ask: *"How many bands do I have to choose from for the first position in the parade?" "Once I choose that one, how many are left to choose from for the second position?" "For the third position? Fourth? Fifth? Sixth? Seventh?"* As each question is answered, write the number in the appropriate space. Ask students to calculate the total number of possible arrangements using the Fundamental Counting Principle. Connect the counting of these permutations to *factorials*.
3. Next, present this problem; "Twenty bands have applied to march in the parade, but only seven spots are available. How many permutations of seven bands are possible for their order in the parade?" Again, put seven blanks on the board, and ask: *"How many bands can I choose from for the first spot in the parade?" "Once I choose that one, how many are left to choose from for the second spot? Third? Fourth? Fifth? Sixth? Seventh?"* The blank spaces will be filled as follows: 20, 19, 18, 17, 16, 15, 14. Have students state that the total number of permutations is the product of these seven

numbers. Connect this value to the permutation formula:

$$\frac{20!}{(20-7)!}=\frac{20!}{13!}=20\bullet19\bullet18\bullet17\bullet16\bullet15\bullet14$$

4. Distribute the Permutations activity sheet. Have students work in pairs to complete the activity. When they are finished, have them share their work with the class.
5. Next, describe a *combination* as a collection of objects in which the order is not important. Present the problem: "A student gets to select two of the six gifts in a gift box. How many different pairs of gifts can she select?" Have students list all possible combinations of two out of the six gifts. Students can use alphabet letters to represent the different prizes.
6. Once students have listed the different combinations, discuss how these are similar or different from a permutation. Show students that the number of combinations in this case are half as many as the number of permutations. Emphasize to students that now we are not counting the ones that have the same letters but just in a different arrangement. Arrangement doesn't matter now, so we are going to divide by the repeats. In this case, since we're only picking two items, we divide our number of permutations by the number of ways we can arrange two items, which is 2!.
7. Present the problem, "A high school class has 18 members and must select three members to serve on a student council committee. How many different committees could they choose from the class?" Ask, *"How many different arrangements there could be of three students from the class of 18?"* Remind them that this is the permutation. Then ask, *"How many of these repeat?"* Have students look at just three things and how many there are in that set to show that all of those count as just one option, so we can divide by that. The goal is that students will understand that the combination of *r* objects selected from a group of *n* is the same as $\frac{{}_nP_r}{r!}$ because the order of the *r* objects is not important. This expression also describes the combination formula.
8. Distribute the Combinations activity sheet. Have students work in pairs to complete it. When they are finished, have them share their work with the class.
9. Distribute the Counting Sort activity sheet. Have students sort the problems as permutations or combinations and solve each.

Assessment

- **Questions**
  - What are some examples of counting problems for which you would use a permutation? What are some examples of which you would use a combination?
  - What are some words you associate with permutation? What are some words you associate with combination?
- **Journal/writing prompts**
  - Explain why a locker combination is called a combination when it is actually a permutation.
  - Explain why 0! is equal to 1.

- Explain the difference between permutations and combinations and how we calculate the possibilities.
- Create a combination problem and write a letter to a friend explaining how to find the number of possibilities for the problem.

- **Other Assessments**
  - Have students create their own permutation or combination problem and trade the problem with a classmate to solve.
  - Have students sort and solve a group of permutation and combination problems.

Extensions and Connections

- Several lottery games have odds calculated by using permutations and combinations. Have students investigate one such game and determine whether a permutation or a combination is used to calculate the odds.
- Show students how Pascal’s triangle can be used as a combination table.
- Have students explore permutations of letters for a word. Discuss with students what would happen if you had a word where letters repeated.

Strategies for Differentiation

- Have students physically arrange themselves in order to complete questions 1 and 2 on the Counting Exploration activity sheet.
- Provide opportunities for students to model permutations and combinations physically.
- Provide struggling students with problem situations that require a smaller number of arrangements or outcomes.
- Use vocabulary cards for related vocabulary listed above.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Counting Exploration

1. Three students—Andy, Beth, and Cara—must arrange their desks in a row in their classroom. Your job is to figure out how many different arrangements they can make. Make a list of all possible arrangements. How many arrangements are possible?

2. Dana is added to the group of students in problem 1, so you now have to arrange four students in a row. How many arrangements are possible? Justify your answer.

3. Mr. Smith's class wants to elect a president, a vice president, and a treasurer. The names of students in his class are: Arlene, Bud, Cathy, David, Eli, Fred, George, Harry, Isaiah, Jack, Kim, Larry, Marlin, Ned, Oprah, Perry, Quentin, Ralph, and Sara. How many different slates of officers are possible?

4. Mr. Smith's class wants to create a dress code committee composed of three students. Using student names in problem 3, how many different committees are possible?

5. How is the answer to problem 3 related to the answer to problem 4? Explain.

6. Bill has three hats, five shirts, 10 pairs of pants, and six pairs of shoes. If an outfit consists of a hat, a shirt, a pair of pants, and a pair of shoes, how many different outfits does Bill have?

7. Joe's Pizza has the following optional toppings: pepperoni, sausage, peppers, and onions. All pizzas come with cheese. How many combinations of toppings are possible? Show all of them or describe them in such a way that you show you have accounted for all possibilities.

8. n factorial, written as $n!$, is equal to $n \cdot (n - 1) \cdot (n - 2) \cdot \ldots \cdot 2 \cdot 1$. Complete the factorial table at the right and simplify the factorial expressions.

a. $\frac{5!}{3!}$

b. $\frac{7!}{4!}$

c. $\frac{8!}{5!3!}$

d. $\frac{10!}{5!5!}$

$n!$	
1!	
2!	
3!	
4!	
5!	
6!	
7!	
8!	
9!	
10!	

Permutations

Complete the following table by providing the missing information.

Problem	**Filling in the blanks: How many ways can you fill each space?**	**Using the permutation formula** $_nP_r = \frac{n!}{(n-r)!}$	**Answer**
The manager of a baseball team has named the nine starters for a game. He needs to determine the batting order. How many batting orders are possible?	____ ____ ____ ____ ____ ____ ____ ____ ____	$_9P_9 = \frac{9!}{(9-9)!} = \frac{9!}{0!} = 9!$	362,880
There are 33 cars in a car race. The first-, second-, and third-place finishers win a prize. How many different arrangements of the first three positions are possible?	____ ____ ____		
	<u>10</u> <u>9</u> <u>8</u> <u>7</u>		5,040
The license plates in a certain country contain a permutation of exactly five letters. If no letter can be repeated, how many letter permutations are possible?			
		$_7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!}$	

Combinations

Complete the following table by providing missing information.

Problem	**Example of one possible combination**	**Using the combination formula** $_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$	**Answer**
The manager of a baseball team has to select the nine starters for a game. She has 11 players to choose from. How many combinations of nine players can she choose?		$_{11}C_9 = \frac{11!}{9!(11-9)!} = \frac{11!}{9!2!}$	55
A committee of five students is to be selected from a class of 20 students. How many different committees are possible?	Students 1, 5, 7, 19, 20		15,504
		$_9C_3 = \frac{9!}{3!(9-3)!} = \frac{9!}{3!6!}$	
The special combo pizza at Ciro's Pizza includes any five toppings from the list of 16 toppings. How many different combo pizzas are possible?			
		$_7C_4 = \frac{7!}{4!(7-4)!} = \frac{7!}{4!3!}$	

Counting Sort

Identify each of the following as a permutation or combination and solve.

A person can select three presents from a pile of 10. How many ways can this be done?	A museum has 10 paintings. The curator wants to hang five on a wall. How many arrangements are possible?	Ten runners compete in a race. How many ways can first, second, and third place occur?
How many ways can the letters in algebra be arranged?	How many five-digit ZIP codes can be made using the digits 0–9?	A club has 22 members. How many ways can a president, vice president, and treasurer be selected from the members?
The general manager of Walmart must select five stores from 12 for a promotional program. How many possible ways can this selection be done?	At a restaurant there are five appetizers, seven entrees, and three desserts. How many different meals are possible?	A test has seven true-false questions. How many different ways can the test be answered?
How many ways can a coach make a batting order of nine players?	There are 20 people in a class. How many different groups of four can be selected?	A computer password is made of four letters. How many possible passwords are there?
A dice is rolled twice, and a coin is flipped three times. How many outcomes are possible?	A child is allowed to pick two puppies from a litter of nine. How many ways can this be done?	A telephone number consists of seven digits from 0–9 but cannot start with zero. How many possible phone numbers are there?
Twenty-five girls try out for the soccer team. How many different ways can a team of 18 be formed?	Seven books are arranged on a shelf. How many arrangements are possible?	A smoothie shop has 10 different fruits for smoothies. How many different three-fruit smoothies can be made, assuming no fruits are repeated?