Absolute Value Equations and Inequalities

Strand: Equations and Inequalities

Topic: Solving absolute value equations and inequalities

Primary SOL: AII.3 The student will solve
a) The student will solve absolute value equations and inequalities.

Related SOL: AII.6

Materials:

- Absolute Value Matching Cards activity sheet (attached)
- Absolute Value Equations/Inequalities activity sheet (attached)
- Absolute Value Stations Review activity sheet (attached)
- Graphing utility
- Colored pencils or highlighters.

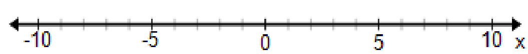
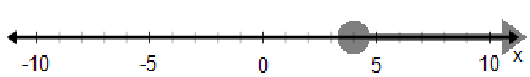
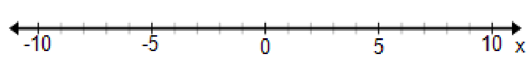
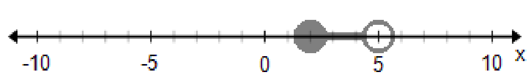
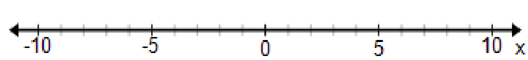
Vocabulary

absolute value inequality, compound inequality, compound statement, intersection, linear equation, linear inequality, interval notation, union, set-builder notation, solution set

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Time: 90 minutes

1. Review graphing and writing inequalities with students. Have students work to fill in the table following in cooperative groups.

In words	Number Line	Set Notation	Interval Notation
All real numbers less than 2			(-∞, 2)
			
		$\{x \mid x \le 3\}$	
			[2, 5)
All real numbers greater than 1 or less than – 5.			

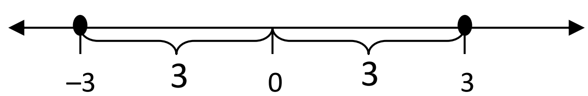
2. Invoke students' prior learning by asking questions about absolute value. Use questions and equations such as the following:

- *"What is absolute value?"*
- *"How can the absolute value of a number be modeled using a number line or another representation?"* Demonstrate representation(s) of absolute value.
- *"What are the solutions to* $|x|=5$ *?"*
- *"What are some solutions to* $|x|<5$ *?"*
- *"What are some solutions to* $|x-2|\geq 4$ *?"*

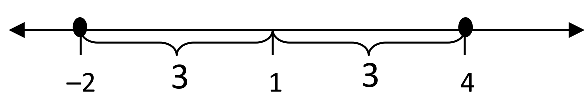
Explain to students that the objective of the lesson is to learn two methods for solving absolute value equations and inequalities, which will give them greater insight into the process of finding the solution set.

<u>Method 1: Absolute Value as Distance</u>

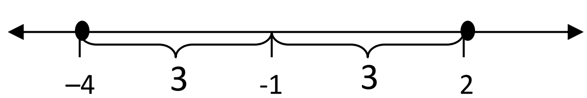
3. Discuss the statement, "Absolute value represents distance." Ask students whether this is true, and if so, what it means. Have the class discuss $|-5|=5$ in terms of distance.
4. Ask what values make $|x|=3$ true, and discuss this in terms of distance. Summarize responses by saying, "We can describe the solution set as the set of points whose distance from the origin is equal to 3." Show the solution set on a number line.



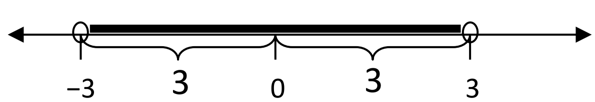
5. Ask what changes in the solution set when we want to solve $|x-1|=3$. Summarize responses by saying, "We can describe the solution set as the set of points whose distance from 1, rather than 0, is equal to 3." Discuss that when we have two points on a number line, we subtract them to find the distance. So, the general form for an absolute value uses that subtraction. It is important to show the solution set on a number line.



6. Discuss how the process changes if we want to solve $|x+1|=3$. Summarize responses by saying, "This equations is saying, 'The distance between x and -1 is 3,' so now we're starting at -1 and going out 3 from there."



7. Ask, *"If the solution to* $|x|=3$ *is the set of points whose distance from the origin is 'equal' to 3, then what is the solution set to* $|x|<3$ *?"* Discuss, and ask students to identify values of *x* that make $|x|<3$ true. They should come to see that the solution set is the set of points whose distance from the origin is *less than* 3. Show the solution set on a number line.



Ask students how we might represent these points as a solution set. Discuss that we want the points between 3 and -3. So, we'd want the points where $x > -3$ *and* $x < 3$. We can write this as the compound inequality $\{x \mid -3 < x < 3\}$, or we can use interval notation to write the solution set as (-3, 3).

8. Ask how the graph of this solution set would be different if you were solving $|x| \le 3$? If you were solving $|x| > 3$? If you were solving $|x| \ge 3$?
9. Instruct students to use the distance method to solve the following examples:

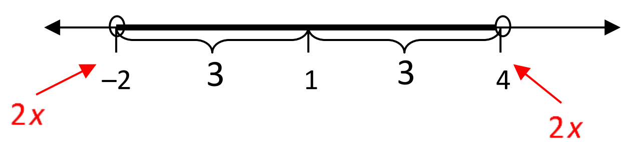
$$|x-2| \le 3 \qquad |x+5| > 7 \qquad |x-4| \ge \frac{3}{4} \qquad |x-a| < b$$

Then, have students state the solution set of each example, using the sentence frame in the box below. Finally, have students graph each solution set on a number line.

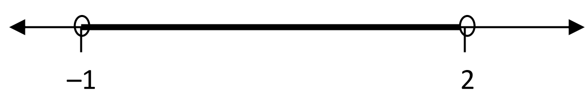
Sentence Frame for Solution Set

" The solution set to a *given equation or inequality* is the set of points whose distance from ______________ is (*equal to, less than, etc.).*"

10. Ask how the solution set might change if we were trying to solve $|2x-1| < 3$. Summarize the responses by saying, "Now the equation says that the distance between 2x and 1 is 3." So, we can start at 1, go out three in each direction and these solutions would represent 2x.



Divide each of the terms by two to find the solution.

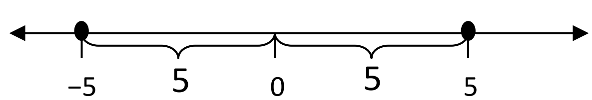


Therefore, we can write the solution set in set-builder notation as $\{x \mid -1 < x < 2\}$ or in interval notation as $(-1, 2)$.

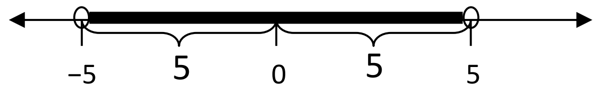
Method 2: Absolute Value as Compound Inequality

11. Explain the method of writing absolute value equations and inequalities as compound statements. Students should make the connection quickly if this is the second method discussed.

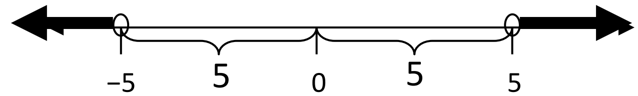
$|x| = 5 \Rightarrow x = 5 \text{ OR } x = -5$



$|x|<5 \Rightarrow x<5 \text{ and } x>-5$



$|x|>5 \Rightarrow x>5 \text{ or } x<-5$

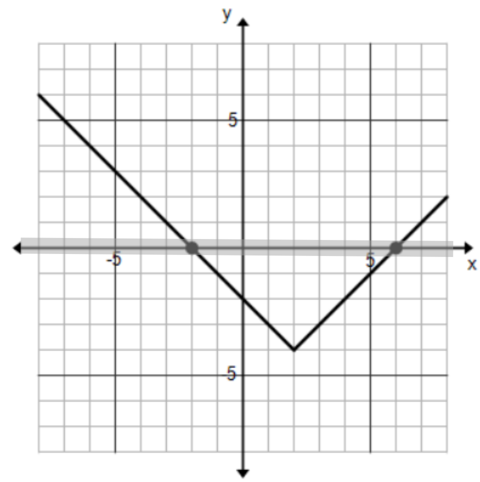


12. Have students write the following as compound statements, solve each branch, and then graph the solution set to each:

$|x+2|=6$ $\qquad$ $|2x-3|\le 7$ $\qquad$ $|3x-5|\ge 4$

Method 3: Using Graphs of Absolute Value Equations

13. Give students the equation $|x-2|-3=0$. Have the students graph the absolute value and discuss what the solutions to the equation would be using the graph. Review the idea of the zero or root of a function using the equation and the graph.



14. Give the students the equation $|x-2|-3=1$. Discuss how the equation might be transformed to apply the idea of the roots to solve the equation.

15. Give the students the inequality $|x-2|-3<1$. Transform the equation so that it would be less than zero. Have students graph $y=|x-2|-4$ and $y=0$. Ask, *"Where is the graph of the absolute value less than 0?"* Have students use colored pencils or highlighters to shade the area below the axis. Ask, *"What are the values of x that make that true?"* Students should respond with the x-values between –2 and 6. So, $|x-2|-4<0$ or $|x-2|-3<1$ when $-2<x<6$.

16. Give pairs of students a set of Absolute Value Matching Cards. Explain that student pairs will play a game to match an inequality with the graph of its solution set, the statement describing its solution set, and the corresponding compound statement.

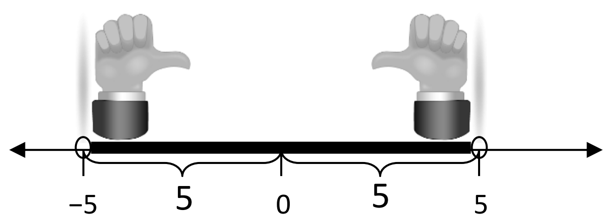
    Have pairs shuffle their cards and deal eight cards each. In turn, each player places one card on the table. If there is a corresponding card already on the table, the player places his/her card on top of that card to create a stack; if not, the player starts a new stack. At the end of the game, six stacks should have been created. When a student places the fourth card on any stack, that student collects that stack. The player with the most stacks wins.

    Once the game becomes too repetitive with these cards, have students create their own game cards in the same manner by writing inequalities, accompanying statements describing the solution sets, accompanying graphs, and accompanying compound inequalities.

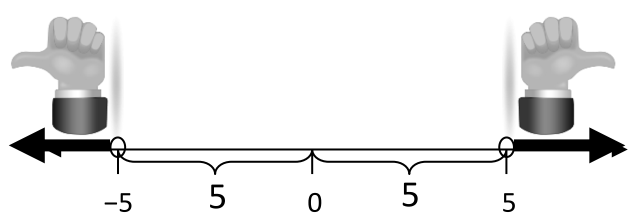
17. Distribute copies of the Absolute Value Equations/Inequalities activity sheet. Have the students complete the problems using whichever method they choose. Questions 16–18 provide an opportunity for students to apply their understanding of absolute value equations and inequalities by writing equations and inequalities for a given solution set. Additional discussion within the class may be necessary with these questions. Or, the questions can be assigned to small groups. Have groups share and explain their problems to other groups.
18. The Absolute Value Stations Review is designed as a review and practice session on these topics, with students assisting each other through the review process. Set up six workstations, with each station containing multiple sheets showing one set of problems. Be sure that each station has enough sheets for every student to have one. Place a poster at each station with the answers and full solution sets to the problems. Put students into groups at the six stations. Give each group seven to 10 minutes per station to complete the problems found there, working together and checking answers. Set a timer so that the groups know when to stop working and rotate to the next station.

Assessment

- **Questions**
  - You have noticed when you graph certain absolute value inequalities that the solution set is everything within an interval between two points, as demonstrated below:



    In other absolute value inequalities, the solution set is everything on the number line outside of an interval, as demonstrated below:



    What types of absolute value inequalities lead to each of these kinds of solution sets? Why?
  - Given any interval between two given numbers, how can you write an absolute value inequality to produce that interval as its solution set?
- **Journal/writing prompts**
  - Discuss the two ways of solving absolute value equations and inequalities, including which one you think makes the most sense and why.

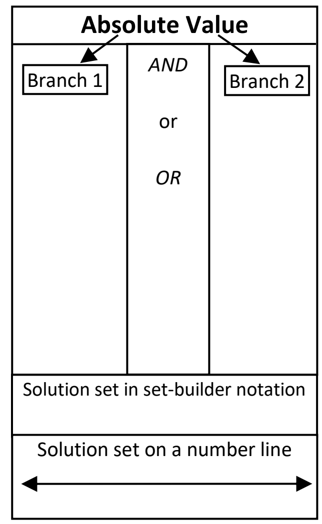
- Explain the steps you would use to solve an absolute value inequality, using your method of choice.
- Write a fictitious story about the history of absolute value.

Extensions and Connections

- Have students solve problems where the argument of the absolute value is quadratic.
- Have students work in groups to reteach one of the methods, and record a video for student review.
- Have students develop their own mnemonic to remember the connecting compound word, *and* or *or*, for each absolute value equation or inequality type.
- Have students solve $2 \le |x| \le 5$, using a distance argument.

Strategies for Differentiation

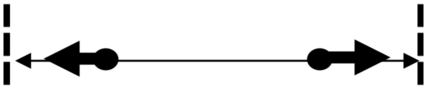
- When teaching the compound statement method, consider using a template like the one shown at right.
- Provide additional sentence frames to help students articulate solution sets to absolute value inequalities.
- Restrict the number of game cards to two representations for students who struggle with multiple representations.
- Construct a number line on the floor, and have students physically represent distances from zero and solution sets to absolute value equations and inequalities.
- Allow students to use talking graphing utilities as they interpret graphs and points of intersection.

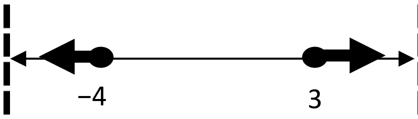


Note: The following pages are intended for classroom use for students as a visual aid to learning.

Absolute Value Matching Cards

Copy cards on cardstock, and cut apart on the dotted lines.

$\lvert x + 5\rvert \leq 4$	The set of points whose distance from −5 is less than or equal to 4	
$x + 5 \geq -4$ *and* $x + 5 \leq 4$	The set of points whose distance from 5 is less than 4	
$\lvert x - 5\rvert < 4$	$x - 5 > -4$ *and* $x - 5 < 4$	
$\lvert x + 4\rvert \geq 5$	The set of points whose distance from −4 is greater than or equal to 5	$x + 4 \leq -5$ *or* $x + 4 \geq 5$

$\lvert 2x - 1 \rvert \le 7$	The set of points whose distance from $\frac{1}{2}$ is less than or equal to $\frac{7}{2}$	
$2x - 1 \ge -7$ *and* $2x - 1 \le 7$	$\lvert 2x + 1 \rvert \ge 7$	The set of points whose distance from $-\frac{1}{2}$ is greater than or equal to $\frac{7}{2}$
	$2x + 1 \le -7$ *or* $2x + 1 \ge 7$	$\lvert x - 2 \rvert \le 7$
The set of points whose distance from 2 is less than or equal to 7		$x - 2 \ge -7$ *and* $x - 2 \le 7$

Absolute Value Equations/Inequalities

Graph each of the following on a number line.

1. $3 < x \leq 5$
2. $(-\infty, -5) \cup (1, \infty)$
3. $[0, 4)$
4. $x \geq 4$ and $x < 7$
5. $x > -1$ or $x < 3$

Solve each of the following. Graph the solution on a number line and state the solution set in set-builder notation and interval notation.

6. $|x| = 5$
7. $|x| = 0$
8. $|x| \leq 8$
9. $|x| \geq 6$
10. $|x| \geq -1$
11. $|x| \leq 0$
12. $|x - 3| \leq 5$
13. $|x + 3| \leq 6$
14. $|x - 4| \geq 3$
15. $|2x - 5| \leq 16$

Write an absolute value inequality with the given solution set.

16. $x < -5$ or $x > 1$
17. $(-\infty, 3) \cap (-2, \infty)$
18. $x \leq 5$ *or* $x > -1$

Absolute Value Stations Review

Station 1 Problems

Solve and graph the following:

1. $3(x - 2) \geq 5$
2. $6 - 4x \leq 10$
3. $\frac{x}{3} + 5 \leq 7$
4. $-5 \leq 2x + 1 \leq 3$
5. $7x - (3 - 2x) \geq x - 3$

Station 2 Problems

Solve and graph the following:

1. $3x + 6 \geq 4$ or $6x - 2 \geq 8$
2. $4x + 2 \geq 10$ or $-4x - 2 \geq 10$
3. $\frac{x}{-4} + 6 > 10$ and $\frac{2x - 1}{3} > 2$
4. $-5x < 10$ or $4x < 16$
5. Graph: $x > 5$ and $x > 9$

Station 3 Problems

Solve and graph the following:

1. $|2x + 5| = 3$
2. $|5 - x| + 4 = 10$
3. $-2|x + 6| = -10$
4. $|2x - 3| \leq 5$
5. $|11 - 3x| + 6 > 10$

Station 4 Problems

Solve and graph the following:

1. $3|2x - 2| + 8 = 23$
2. $|3x - 5| \geq 4$
3. $4|-x + 4| < 12$
4. $|5 - (2x + 3)| \geq 4$
5. $|2 - x| < 9$

Station 5 Problems

Write an inequality for each of the following:

1. You have $20,000 available to invest in stocks A and B. Write an inequality stating the restriction on A if at least $3,000 must be invested in each stock.

2. The largest egg of any bird is that of the ostrich. An ostrich egg can reach 8 inches in length. The smallest egg is that of the hummingbird. Its eggs are approximately 0.4 inches in length. Write an inequality that represents the range of lengths of bird eggs.

3. On Pennsylvania's interstate highway, the speed limit is 55 mph. The minimum speed limit is 45 mph. Write a compound inequality that represents the allowable speeds.

4. You have $60 to spend and a coupon that allows you to take $10 off any purchase of $50 or more at the department store. Write an inequality that describes the possible retail value of the items you can buy if you use this coupon. Write an inequality that describes the different amounts of money you can spend if you use the coupon.

5. Develop a context for the inequality $0 < 2x - 5 < 30$.

6. Develop a context for the inequality $|x - 7| > 12$.

Station 6 Problems

Set up and solve each of the following inequalities:

1. Maria has grades of 79, 67, 83, and 90 on four tests. What is the lowest grade she can make on the fifth test in order to have an average of at least 82?

2. Most fish can adjust to a change in water temperature of up to 15°F if the change is not too sudden. Suppose lake trout live comfortably in water that is 58°F. Write an inequality that represents the range of temperatures at which the lake trout can survive. Write an absolute value inequality that represents the same information.